Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.4-f-x-^m-d+e-x^2-^q-a+b-x^2+c-x^4-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [413]. This is test number [41].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (413)	% 0.00 (0)
Mathematica	% 96.85 (400)	% 3.15 (13)
Maple	% 91.04 (376)	% 8.96 (37)
Maxima	% 41.89 (173)	% 58.11 (240)
Fricas	% 62.47 (258)	% 37.53 (155)
Sympy	% 30.51 (126)	% 69.49 (287)
Giac	% 63.92 (264)	% 36.08 (149)
Mupad	% 52.78 (218)	% 47.22 (195)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

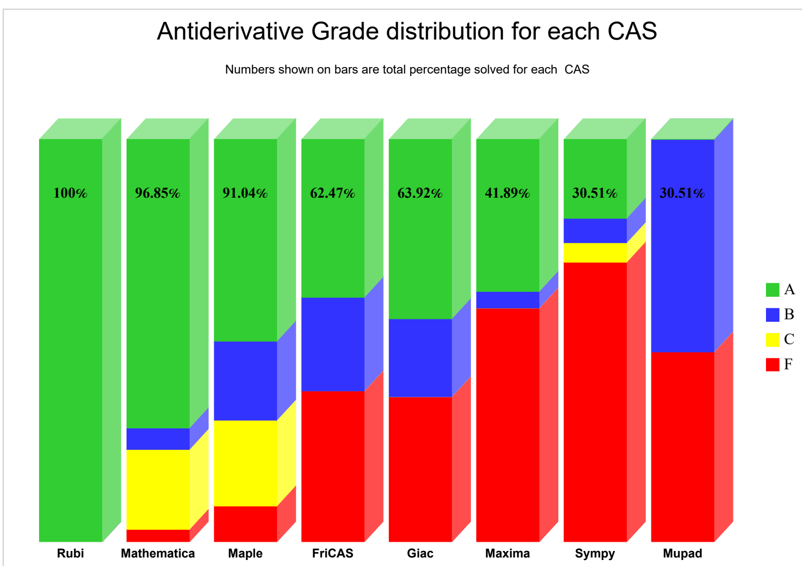
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

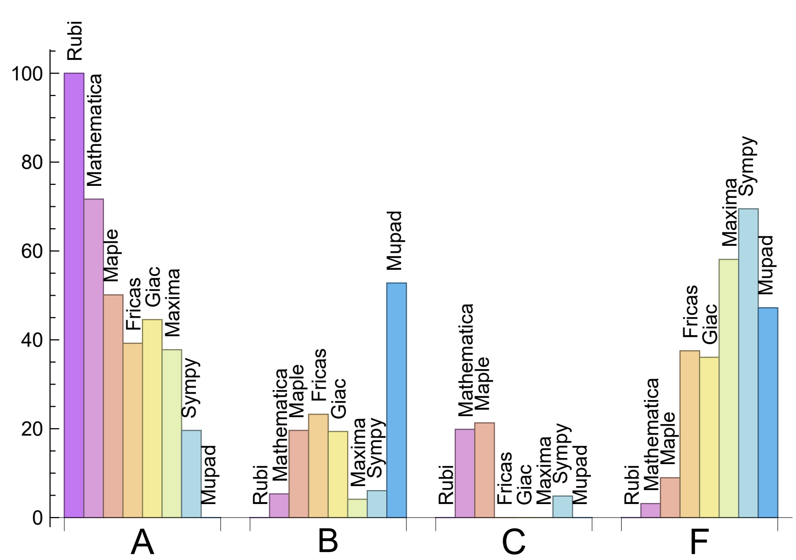
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	71.67	5.33	19.85	3.15
Maple	50.12	19.61	21.31	8.96
Maxima	37.77	4.12	0.00	58.11
Fricas	39.23	23.24	0.00	37.53
Sympy	19.61	6.05	4.84	69.49
Giac	44.55	19.37	0.00	36.08
Mupad	0.00	52.78	0.00	47.22

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	13	53.85 %	46.15 %	0.00 %
Maple	37	100.00 %	0.00 %	0.00 %
Maxima	240	83.33 %	0.00 %	16.67 %
Fricas	155	72.90 %	27.10 %	0.00 %
Sympy	287	65.16 %	34.84 %	0.00 %
Giac	149	78.52 %	7.38 %	14.09 %
Mupad	195	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

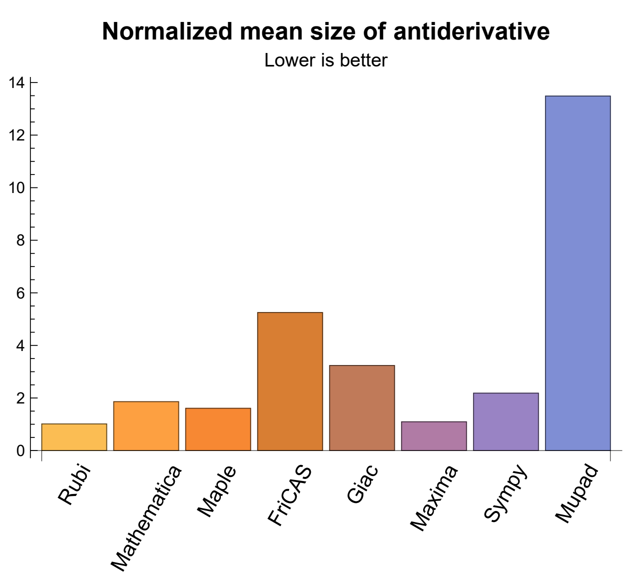
1.3 Performance

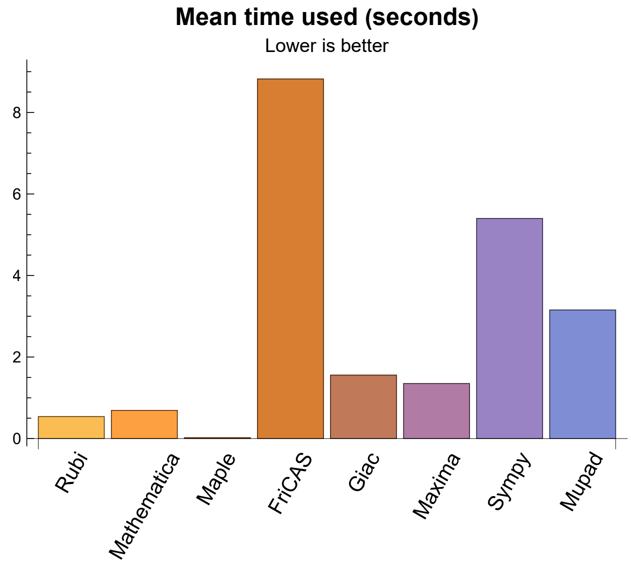
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.54	229.96	1.01	189.00	1.00
Mathematica	0.69	547.14	1.86	156.00	0.96
Maple	0.02	386.39	1.61	203.00	1.05
Maxima	1.35	134.41	1.09	107.00	0.98
Fricas	8.82	1512.98	5.25	226.00	2.08
Sympy	5.40	281.32	2.18	119.00	1.19
Giac	1.56	834.20	3.24	144.00	1.08
Mupad	3.15	4535.03	13.49	169.00	1.95

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {151, 152, 153, 154, 155, 163, 164, 166, 167, 168, 189, 190, 191, 192, 193, 199, 200, 202, 203, 204, 205, 206, 207, 209, 211, 212, 213, 214, 215, 217, 219, 224, 225, 226, 227, 228, 354, 361, 362, 364, 365, 366, 371, 372, 373, 374, 375, 381, 383, 384, 385, 394, 395, 396, 397, 398, 399}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

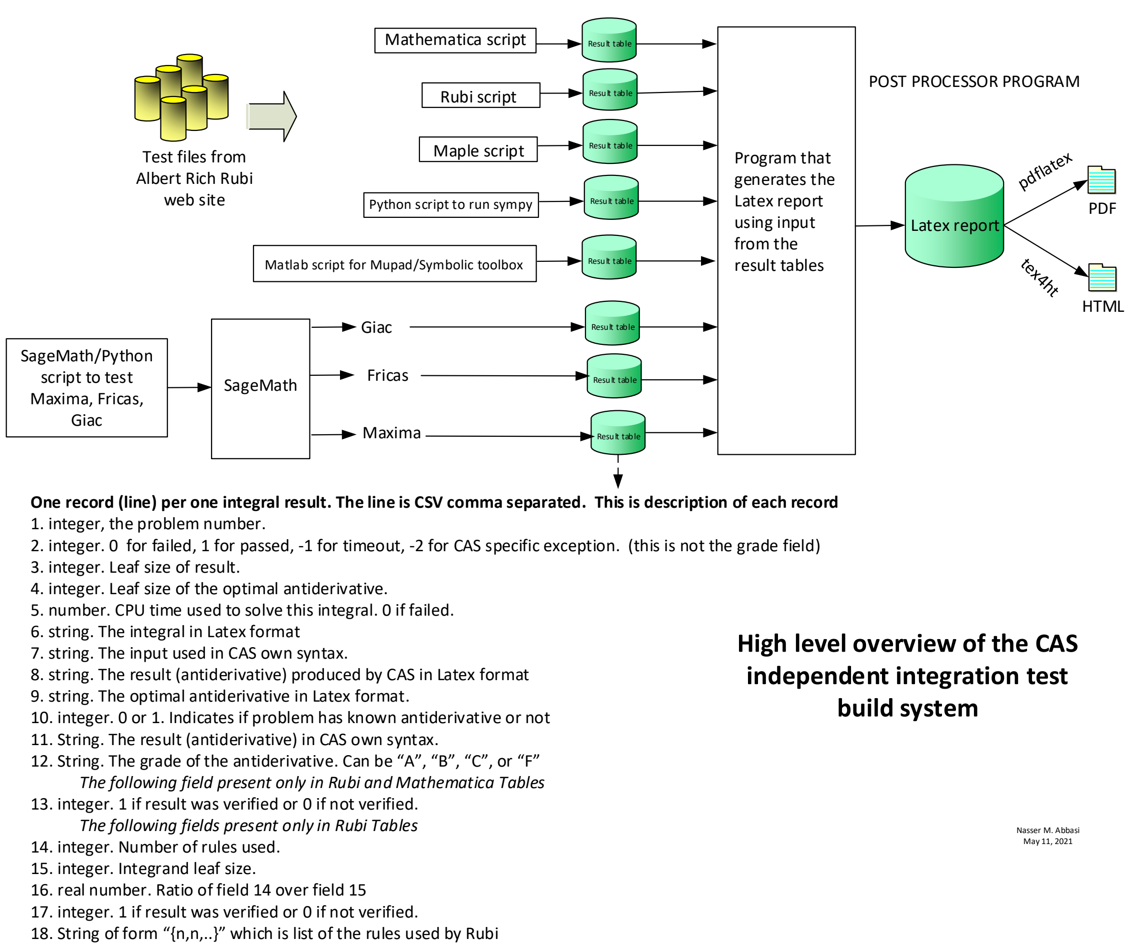
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 55, 57, 59, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 175, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 196, 197, 198, 204, 205, 206, 207, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 262, 263, 264, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 322, 323, 324, 325, 326, 332, 333, 334, 335, 336, 342, 343, 344, 345, 346, 347, 355, 356, 357, 358, 359, 360, 367, 368, 369, 370, 376, 377, 378, 379, 380, 386, 387, 388, 389, 390, 391, 392, 393, 401, 402, 403, 404, 405, 406, 413 }

B grade: { 56, 58, 60, 66, 68, 354, 361, 362, 363, 364, 365, 366, 371, 372, 373, 374, 375, 381, 382, 383, 384, 394 }

C grade: { 15, 16, 17, 18, 19, 24, 25, 26, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 49, 50, 51, 52, 53, 54, 151, 152, 153, 154, 155, 163, 164, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 202, 203, 224, 259, 260, 265, 266, 310, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 385, 395, 396, 397, 398, 399 }

F grade: { 165, 201, 208, 210, 216, 218, 400, 407, 408, 409, 410, 411, 412 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 114, 115, 129, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 263, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 297, 298, 299, 300, 332, 333, 335, 336 }

B grade: { 55, 56, 58, 60, 65, 66, 68, 70, 87, 88, 102, 107, 108, 109, 110, 111, 112, 113, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 169, 170, 176, 195, 220, 221, 222, 262, 264, 295, 296, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 322, 323, 324, 325, 326, 334, 342, 343, 344, 345, 346, 347, 376, 377, 378, 379, 380, 385 }

C grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 259, 260, 265, 266, 310, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 413 }

F grade: { 90, 91, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 12, 13, 14, 20, 24, 25, 26, 32, 33, 37, 38, 44, 45, 46, 47, 48, 49, 55, 57, 59, 61, 62, 63, 64, 65, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 100, 101, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 196, 197, 198, 220, 221, 222, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

B grade: { 8, 9, 10, 11, 21, 22, 23, 34, 35, 36, 56, 58, 60, 66, 68, 70, 92 }

C grade: { }

F grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 90, 91, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 175, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 198, 229, 230, 231, 232, 233, 234, 235, 244, 245, 246, 247, 248, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 297, 298, 299, 312, 313, 314, 315, 336 }

B grade: { 55, 56, 58, 60, 65, 66, 68, 70, 87, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 196, 197, 220, 221, 222, 237, 238, 239, 240, 241, 242, 243, 252, 253, 254, 255, 256, 257, 305, 306, 332, 333, 334, 335, 342, 343, 344, 345, 346, 347, 356, 357, 358, 361, 362, 363, 364, 365, 366, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 388, 389, 390, 391, 413 }

C grade: { }

F grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 90, 91, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 236, 249, 250, 251, 258, 259, 260, 261, 262, 263, 264, 265, 266, 295, 296, 300, 301, 302, 303, 304, 307, 308, 309, 310, 311, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 359, 360, 367, 368, 369, 370, 371, 372, 386, 387, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 92, 95, 96, 97, 98, 99, 100, 101, 109, 137, 138, 139, 140, 141, 220, 221, 222, 273, 274, 275, 276, 277, 278, 279, 281, 282, 284, 285, 288, 289, 290, 291, 292, 293 }

B grade: { 21, 22, 47, 48, 49, 56, 58, 60, 66, 68, 70, 102, 103, 104, 114, 115, 126, 127, 128, 129, 280, 283, 286, 287, 294 }

C grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54 }

F grade: { 55, 65, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 105, 106, 107, 108, 110, 111, 112, 113, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 20, 21, 22, 23, 24, 32, 33, 34, 35, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 84, 86, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 113, 114, 115, 116, 117, 124, 125, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 156, 158, 159, 160, 161, 169, 170, 171, 173, 181, 182, 183, 184, 185, 194, 195, 196, 197, 198, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 315, 334, 336, 357, 361, 362, 370, 371, 372, 373, 386, 387, 388, 394, 413 }

B grade: { 13, 14, 25, 26, 36, 37, 38, 55, 56, 58, 60, 65, 66, 68, 70, 87, 88, 89, 93, 94, 107, 108, 109, 110, 111, 118, 119, 120, 121, 122, 123, 126, 132, 133, 134, 135, 136, 147, 148, 149, 150, 157, 162, 174, 175, 186, 187, 188, 220, 221, 222, 303, 304, 305, 306, 307, 308, 309, 343, 344, 345, 347, 354, 355, 356, 358, 360, 367, 368, 369, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

C grade: { }

F grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 81, 83, 85, 90, 91, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 172, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 337, 338, 339, 340, 341, 342, 346, 348, 349, 350, 351, 352, 353, 359, 363, 364, 365, 366, 374, 375, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 20, 21, 22, 23, 24, 25, 26, 30, 32, 33, 34, 35, 36, 37, 38, 42, 44, 45, 46, 47, 48, 49, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 78, 80, 82, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 158, 171, 172, 173, 184, 185, 186, 195, 196, 220, 221, 222, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 354, 355, 356, 357, 358, 359, 360, 367, 368, 369, 370, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

C grade: { }

F grade: { 15, 16, 17, 19, 27, 28, 29, 31, 39, 40, 41, 43, 50, 51, 52, 54, 75, 77, 79, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 187, 188, 189, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 361, 362, 363, 364, 365, 366, 371, 372, 373, 374, 375, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	149	126	125	125	151	131	125
normalized size	1	1.00	1.00	0.85	0.84	0.84	1.01	0.88	0.84
time (sec)	N/A	0.220	0.016	0.010	0.431	0.547	0.145	0.261	0.306

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	149	126	125	125	155	131	125
normalized size	1	1.00	1.00	0.85	0.84	0.84	1.04	0.88	0.84
time (sec)	N/A	0.098	0.004	0.000	0.463	0.496	0.088	0.193	0.069

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	146	125	124	124	150	130	124
normalized size	1	1.00	1.64	1.40	1.39	1.39	1.69	1.46	1.39
time (sec)	N/A	0.077	0.003	0.003	0.441	0.432	0.089	0.180	0.071

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	141	122	121	121	148	127	121
normalized size	1	1.00	1.00	0.87	0.86	0.86	1.05	0.90	0.86
time (sec)	N/A	0.082	0.003	0.001	0.465	0.623	0.089	0.221	0.070

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	142	123	125	122	150	131	122
normalized size	1	1.00	1.00	0.87	0.88	0.86	1.06	0.92	0.86
time (sec)	N/A	0.111	0.012	0.014	0.518	0.577	0.250	0.197	0.109

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	139	122	121	127	143	127	121
normalized size	1	1.00	1.00	0.88	0.87	0.91	1.03	0.91	0.87
time (sec)	N/A	0.082	0.007	0.017	0.522	0.673	0.244	0.195	0.094

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	142	123	125	129	150	142	122
normalized size	1	1.00	1.00	0.87	0.88	0.91	1.06	1.00	0.86
time (sec)	N/A	0.121	0.008	0.009	0.504	0.686	0.279	0.208	0.072

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	50	53	102	48	97	54	42
normalized size	1	1.00	0.75	0.79	1.52	0.72	1.45	0.81	0.63
time (sec)	N/A	0.050	0.053	0.046	1.012	0.494	6.015	0.233	0.370

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	46	93	43	70	45	37
normalized size	1	1.00	0.86	0.90	1.82	0.84	1.37	0.88	0.73
time (sec)	N/A	0.031	0.026	0.006	1.016	0.571	4.307	0.233	0.392

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	34	67	34	53	38	32
normalized size	1	1.00	0.82	0.77	1.52	0.77	1.20	0.86	0.73
time (sec)	N/A	0.021	0.030	0.008	1.325	0.557	3.083	0.197	0.137

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	49	99	56	83	76	45
normalized size	1	1.00	0.98	0.84	1.71	0.97	1.43	1.31	0.78
time (sec)	N/A	0.055	0.059	0.016	1.561	0.731	15.588	0.213	0.146

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	61	88	72	83	91	51
normalized size	1	1.00	1.00	1.03	1.49	1.22	1.41	1.54	0.86
time (sec)	N/A	0.056	0.044	0.013	1.408	0.848	7.460	0.245	0.787

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	75	91	72	76	129	56
normalized size	1	1.00	0.94	1.19	1.44	1.14	1.21	2.05	0.89
time (sec)	N/A	0.055	0.067	0.016	1.237	0.646	6.042	0.244	0.423

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	72	52	59	59	63	116	43
normalized size	1	1.00	1.24	0.90	1.02	1.02	1.09	2.00	0.74
time (sec)	N/A	0.047	0.034	0.013	1.612	0.638	5.956	0.232	0.681

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	82	192	0	0	78	0	-1
normalized size	1	1.00	0.39	0.92	0.00	0.00	0.38	0.00	-0.00
time (sec)	N/A	0.124	0.034	0.090	0.000	0.696	2.333	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	68	180	0	0	78	0	-1
normalized size	1	1.00	0.35	0.94	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.097	0.024	0.020	0.000	0.663	2.141	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	48	168	0	0	76	0	-1
normalized size	1	1.00	0.27	0.95	0.00	0.00	0.43	0.00	-0.01
time (sec)	N/A	0.064	0.010	0.012	0.000	0.728	2.016	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	53	167	0	0	78	0	61
normalized size	1	1.00	0.31	0.98	0.00	0.00	0.46	0.00	0.36
time (sec)	N/A	0.067	0.019	0.018	0.000	0.683	2.304	0.000	0.410

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	54	170	0	0	83	0	-1
normalized size	1	1.00	0.28	0.89	0.00	0.00	0.43	0.00	-0.01
time (sec)	N/A	0.089	0.020	0.017	0.000	0.684	2.500	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	73	127	58	131	80	52
normalized size	1	1.00	0.87	0.88	1.53	0.70	1.58	0.96	0.63
time (sec)	N/A	0.059	0.049	0.025	1.349	0.710	14.314	0.247	0.319

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	58	118	53	124	71	47
normalized size	1	1.00	0.81	0.87	1.76	0.79	1.85	1.06	0.70
time (sec)	N/A	0.040	0.036	0.009	1.133	0.627	11.572	0.205	0.430

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	46	95	48	109	57	42
normalized size	1	1.00	0.93	0.77	1.58	0.80	1.82	0.95	0.70
time (sec)	N/A	0.030	0.027	0.014	1.597	0.749	8.193	0.242	0.177

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	67	75	138	67	114	90	55
normalized size	1	1.00	0.86	0.96	1.77	0.86	1.46	1.15	0.71
time (sec)	N/A	0.075	0.034	0.020	1.187	0.762	33.672	0.261	0.180

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	71	75	122	78	114	102	64
normalized size	1	1.00	0.88	0.93	1.51	0.96	1.41	1.26	0.79
time (sec)	N/A	0.075	0.053	0.016	1.330	0.680	11.391	0.258	0.765

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	60	73	123	82	133	146	71
normalized size	1	1.00	0.70	0.85	1.43	0.95	1.55	1.70	0.83
time (sec)	N/A	0.077	0.027	0.020	1.372	0.748	12.755	0.255	0.552

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	60	73	112	82	148	158	82
normalized size	1	1.00	0.73	0.89	1.37	1.00	1.80	1.93	1.00
time (sec)	N/A	0.075	0.025	0.023	1.346	0.699	12.558	0.270	0.946

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	74	216	0	0	160	0	-1
normalized size	1	1.00	0.31	0.92	0.00	0.00	0.68	0.00	-0.00
time (sec)	N/A	0.134	0.042	0.020	0.000	0.581	3.969	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	68	204	0	0	160	0	-1
normalized size	1	1.00	0.31	0.93	0.00	0.00	0.73	0.00	-0.00
time (sec)	N/A	0.120	0.027	0.013	0.000	0.666	3.654	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	49	192	0	0	158	0	-1
normalized size	1	1.00	0.25	0.97	0.00	0.00	0.80	0.00	-0.01
time (sec)	N/A	0.082	0.010	0.011	0.000	0.639	3.629	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	53	192	0	0	160	0	48
normalized size	1	1.00	0.27	0.96	0.00	0.00	0.80	0.00	0.24
time (sec)	N/A	0.088	0.021	0.018	0.000	0.727	4.257	0.000	0.533

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	54	192	0	0	163	0	-1
normalized size	1	1.00	0.27	0.96	0.00	0.00	0.81	0.00	-0.00
time (sec)	N/A	0.086	0.021	0.017	0.000	0.711	4.121	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	44	51	104	43	85	46	38
normalized size	1	1.00	0.66	0.76	1.55	0.64	1.27	0.69	0.57
time (sec)	N/A	0.058	0.028	0.018	1.314	0.796	7.072	0.189	0.590

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	39	76	34	66	37	32
normalized size	1	1.00	0.69	0.76	1.49	0.67	1.29	0.73	0.63
time (sec)	N/A	0.042	0.022	0.012	1.218	0.699	5.519	0.216	0.309

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	32	65	33	53	33	27
normalized size	1	1.00	0.97	0.91	1.86	0.94	1.51	0.94	0.77
time (sec)	N/A	0.026	0.017	0.007	1.158	0.512	4.041	0.216	0.491

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	20	42	26	22	26	19
normalized size	1	1.00	1.00	0.83	1.75	1.08	0.92	1.08	0.79
time (sec)	N/A	0.017	0.019	0.010	1.118	0.663	2.131	0.196	0.294

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	30	67	41	31	61	30
normalized size	1	1.00	1.00	0.79	1.76	1.08	0.82	1.61	0.79
time (sec)	N/A	0.038	0.016	0.013	1.173	0.701	6.008	0.205	0.615

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	47	47	31	66	31
normalized size	1	1.00	1.00	0.74	1.12	1.12	0.74	1.57	0.74
time (sec)	N/A	0.037	0.021	0.014	1.257	0.768	3.597	0.216	0.329

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	43	59	50	88	114	43
normalized size	1	1.00	0.84	0.74	1.02	0.86	1.52	1.97	0.74
time (sec)	N/A	0.051	0.023	0.019	1.118	0.575	14.329	0.217	0.694

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	74	168	0	0	75	0	-1
normalized size	1	1.00	0.40	0.91	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.085	0.029	0.023	0.000	0.683	2.551	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	66	155	0	0	75	0	-1
normalized size	1	1.00	0.40	0.93	0.00	0.00	0.45	0.00	-0.01
time (sec)	N/A	0.066	0.021	0.013	0.000	0.691	2.360	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	48	146	0	0	73	0	-1
normalized size	1	1.00	0.31	0.94	0.00	0.00	0.47	0.00	-0.01
time (sec)	N/A	0.046	0.011	0.010	0.000	0.559	1.705	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	53	158	0	0	75	0	48
normalized size	1	1.00	0.31	0.91	0.00	0.00	0.43	0.00	0.28
time (sec)	N/A	0.062	0.023	0.019	0.000	0.715	1.819	0.000	0.504

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	54	170	0	0	80	0	-1
normalized size	1	1.00	0.29	0.90	0.00	0.00	0.42	0.00	-0.01
time (sec)	N/A	0.086	0.021	0.024	0.000	0.834	2.082	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	50	89	62	66	45	97
normalized size	1	1.00	0.88	0.86	1.53	1.07	1.14	0.78	1.67
time (sec)	N/A	0.046	0.024	0.021	1.211	0.549	14.277	0.207	1.108

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	37	63	58	48	39	89
normalized size	1	1.00	1.02	0.82	1.40	1.29	1.07	0.87	1.98
time (sec)	N/A	0.039	0.021	0.014	1.414	0.673	12.336	0.232	0.894

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	41	34	54	52	39	33	82
normalized size	1	1.00	1.17	0.97	1.54	1.49	1.11	0.94	2.34
time (sec)	N/A	0.027	0.017	0.007	1.227	0.770	10.675	0.254	0.841

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	31	31	16	16
normalized size	1	1.00	1.00	0.85	1.10	1.55	1.55	0.80	0.80
time (sec)	N/A	0.016	0.009	0.003	1.486	0.716	7.856	0.245	0.164

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	40	56	61	212	61	40
normalized size	1	1.00	1.00	0.87	1.22	1.33	4.61	1.33	0.87
time (sec)	N/A	0.043	0.032	0.020	1.405	0.539	19.618	0.253	0.475

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	47	68	77	228	82	47
normalized size	1	1.00	0.69	0.72	1.05	1.18	3.51	1.26	0.72
time (sec)	N/A	0.055	0.025	0.014	1.098	0.587	12.982	0.243	0.537

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	70	168	0	0	75	0	-1
normalized size	1	1.00	0.36	0.86	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.085	0.039	0.025	0.000	0.925	5.590	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	68	168	0	0	75	0	-1
normalized size	1	1.00	0.38	0.95	0.00	0.00	0.42	0.00	-0.01
time (sec)	N/A	0.069	0.029	0.017	0.000	0.663	5.095	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	66	168	0	0	73	0	-1
normalized size	1	1.00	0.37	0.93	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.061	0.022	0.014	0.000	0.694	5.075	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	71	180	0	0	75	0	48
normalized size	1	1.00	0.36	0.92	0.00	0.00	0.38	0.00	0.24
time (sec)	N/A	0.082	0.051	0.021	0.000	0.587	7.203	0.000	0.456

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	54	192	0	0	80	0	-1
normalized size	1	1.00	0.25	0.90	0.00	0.00	0.37	0.00	-0.00
time (sec)	N/A	0.108	0.024	0.020	0.000	0.747	8.146	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	189	2295	372	1571	0	3752	1539
normalized size	1	1.00	0.70	8.53	1.38	5.84	0.00	13.95	5.72
time (sec)	N/A	0.161	0.557	0.027	0.885	0.731	0.000	0.459	1.777

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	153	130	129	132	134	143	121
normalized size	1	1.00	2.43	2.06	2.05	2.10	2.13	2.27	1.92
time (sec)	N/A	0.197	0.022	0.002	0.591	0.396	0.096	0.278	0.094

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	153	130	129	133	141	144	123
normalized size	1	1.00	1.00	0.85	0.84	0.87	0.92	0.94	0.80
time (sec)	N/A	0.117	0.023	0.000	0.667	0.446	0.095	0.296	0.121

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	151	130	129	133	136	144	123
normalized size	1	1.00	3.36	2.89	2.87	2.96	3.02	3.20	2.73
time (sec)	N/A	0.123	0.017	0.000	0.654	0.645	0.098	0.400	0.078

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	153	130	129	133	139	144	123
normalized size	1	1.00	1.00	0.85	0.84	0.87	0.91	0.94	0.80
time (sec)	N/A	0.085	0.017	0.002	0.499	0.536	0.097	0.288	0.081

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	149	130	129	133	133	144	123
normalized size	1	1.00	5.14	4.48	4.45	4.59	4.59	4.97	4.24
time (sec)	N/A	0.049	0.012	0.002	0.645	0.635	0.096	0.314	0.077

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	143	127	125	130	134	141	120
normalized size	1	1.00	1.00	0.89	0.87	0.91	0.94	0.99	0.84
time (sec)	N/A	0.074	0.016	0.001	0.576	0.542	0.098	0.362	0.079

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	149	132	130	127	131	145	121
normalized size	1	1.00	1.60	1.42	1.40	1.37	1.41	1.56	1.30
time (sec)	N/A	0.055	0.025	0.003	0.478	0.640	0.328	0.267	0.129

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	141	129	125	131	124	139	119
normalized size	1	1.00	1.00	0.91	0.89	0.93	0.88	0.99	0.84
time (sec)	N/A	0.082	0.026	0.005	0.504	0.562	0.321	0.361	0.081

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	147	131	130	133	131	156	120
normalized size	1	1.00	1.00	0.89	0.88	0.90	0.89	1.06	0.82
time (sec)	N/A	0.136	0.034	0.007	0.698	0.756	0.386	0.263	0.084

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	122	1121	192	759	0	1848	1483
normalized size	1	1.00	0.60	5.52	0.95	3.74	0.00	9.10	7.31
time (sec)	N/A	0.073	0.044	0.013	0.972	0.834	0.000	0.619	1.251

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	85	62	61	61	76	61	61
normalized size	1	1.00	2.50	1.82	1.79	1.79	2.24	1.79	1.79
time (sec)	N/A	0.047	0.002	0.001	0.637	0.494	0.072	0.339	0.062

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	62	61	61	75	61	61
normalized size	1	1.00	1.00	0.75	0.73	0.73	0.90	0.73	0.73
time (sec)	N/A	0.031	0.002	0.003	0.762	0.596	0.071	0.414	0.059

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	83	62	61	61	75	61	61
normalized size	1	1.00	3.61	2.70	2.65	2.65	3.26	2.65	2.65
time (sec)	N/A	0.022	0.002	0.001	0.563	0.485	0.073	0.362	0.060

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	62	61	61	75	61	61
normalized size	1	1.00	1.00	0.75	0.73	0.73	0.90	0.73	0.73
time (sec)	N/A	0.027	0.001	0.000	0.880	0.547	0.071	0.304	0.059

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	62	61	61	71	76	61
normalized size	1	1.00	1.00	5.64	5.55	5.55	6.45	6.91	5.55
time (sec)	N/A	0.002	0.002	0.001	0.782	0.577	0.070	0.314	0.058

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	57	57	68	57	57
normalized size	1	1.00	1.00	0.79	0.78	0.78	0.93	0.78	0.78
time (sec)	N/A	0.022	0.001	0.001	0.977	0.558	0.074	0.274	0.058

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	59	62	58	75	62	58
normalized size	1	1.00	1.00	0.74	0.78	0.72	0.94	0.78	0.72
time (sec)	N/A	0.033	0.003	0.003	0.821	0.491	0.107	0.230	0.061

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	60	59	62	66	59	59
normalized size	1	1.00	1.00	0.82	0.81	0.85	0.90	0.81	0.81
time (sec)	N/A	0.025	0.003	0.004	0.826	0.638	0.101	0.297	0.060

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	61	62	64	75	69	60
normalized size	1	1.00	1.00	0.76	0.78	0.80	0.94	0.86	0.75
time (sec)	N/A	0.040	0.003	0.005	0.736	0.674	0.112	0.357	0.061

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	80	90	54	129	90	101	-1
normalized size	1	1.00	0.55	0.62	0.37	0.89	0.62	0.70	-0.01
time (sec)	N/A	0.090	0.083	0.039	1.519	0.766	0.365	0.430	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	51	55	31	29	27	42	103
normalized size	1	1.00	0.61	0.66	0.37	0.35	0.33	0.51	1.24
time (sec)	N/A	0.072	0.021	0.010	0.779	0.795	0.277	0.383	0.876

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	69	62	33	98	82	59	-1
normalized size	1	1.00	0.71	0.64	0.34	1.01	0.85	0.61	-0.01
time (sec)	N/A	0.047	0.027	0.007	1.507	0.828	0.317	0.265	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	54	57	35	33	26	61	83
normalized size	1	1.00	0.59	0.62	0.38	0.36	0.28	0.66	0.90
time (sec)	N/A	0.072	0.021	0.010	0.734	0.716	0.709	0.381	0.766

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	72	67	37	105	82	62	-1
normalized size	1	1.00	0.71	0.66	0.37	1.04	0.81	0.61	-0.01
time (sec)	N/A	0.063	0.030	0.010	1.216	0.690	0.372	0.343	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	70	79	48	48	41	131	125
normalized size	1	1.00	0.51	0.58	0.35	0.35	0.30	0.96	0.91
time (sec)	N/A	0.098	0.033	0.013	0.784	0.903	0.726	0.386	0.810

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	108	188	125	300	0	0	-1
normalized size	1	1.00	0.71	1.23	0.82	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.060	0.019	1.489	0.752	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	45	38	65	42	0	40	48
normalized size	1	1.00	0.58	0.49	0.84	0.55	0.00	0.52	0.62
time (sec)	N/A	0.066	0.019	0.007	0.636	0.689	0.000	0.507	0.179

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	108	186	124	301	0	0	-1
normalized size	1	1.00	0.69	1.19	0.79	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.051	0.014	1.670	0.754	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	92	133	88	119	0	96	-1
normalized size	1	1.00	0.57	0.83	0.55	0.74	0.00	0.60	-0.01
time (sec)	N/A	0.118	0.041	0.021	0.888	0.664	0.000	0.528	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	124	206	134	334	0	0	-1
normalized size	1	1.00	0.65	1.08	0.71	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.069	0.023	1.403	0.675	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	130	249	138	205	0	144	-1
normalized size	1	1.00	0.58	1.12	0.62	0.92	0.00	0.65	-0.00
time (sec)	N/A	0.184	0.070	0.025	0.841	0.652	0.000	0.403	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	160	1099	491	853	0	2213	-1
normalized size	1	1.00	0.40	2.75	1.23	2.13	0.00	5.53	-0.00
time (sec)	N/A	0.243	0.220	0.010	0.815	0.747	0.000	0.687	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	112	495	243	381	0	1013	-1
normalized size	1	1.00	0.41	1.79	0.88	1.38	0.00	3.67	-0.00
time (sec)	N/A	0.153	0.110	0.009	0.964	1.030	0.000	0.524	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	86	131	75	94	0	269	-1
normalized size	1	1.00	0.56	0.86	0.49	0.61	0.00	1.76	-0.01
time (sec)	N/A	0.076	0.055	0.006	0.811	0.860	0.000	0.322	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	78	0	0	0	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.067	0.094	0.000	0.629	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	101	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.072	0.029	0.000	0.674	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	25	40	86	47	165	32	59
normalized size	1	1.00	0.74	1.18	2.53	1.38	4.85	0.94	1.74
time (sec)	N/A	0.029	0.008	0.002	0.722	0.634	9.597	0.289	0.142

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	45	62	135	92	0	196	108
normalized size	1	1.00	0.52	0.72	1.57	1.07	0.00	2.28	1.26
time (sec)	N/A	0.089	0.024	0.007	0.737	0.519	0.000	0.427	0.171

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	68	99	196	140	0	331	169
normalized size	1	1.00	0.53	0.77	1.53	1.09	0.00	2.59	1.32
time (sec)	N/A	0.129	0.035	0.006	0.698	0.685	0.000	0.385	0.202

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	166	226	166	193	202	193	169
normalized size	1	1.00	1.00	1.36	1.00	1.16	1.22	1.16	1.02
time (sec)	N/A	0.393	0.050	0.000	0.604	0.633	0.102	0.338	0.076

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	166	226	166	193	204	193	169
normalized size	1	1.00	1.00	1.36	1.00	1.16	1.23	1.16	1.02
time (sec)	N/A	0.154	0.054	0.000	0.726	0.582	0.101	0.270	0.096

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	154	226	166	193	199	193	169
normalized size	1	1.00	0.93	1.36	1.00	1.16	1.20	1.16	1.02
time (sec)	N/A	0.287	0.056	0.000	0.768	0.543	0.102	0.287	0.048

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	161	223	163	189	199	189	165
normalized size	1	1.00	1.00	1.39	1.01	1.17	1.24	1.17	1.02
time (sec)	N/A	0.118	0.046	0.000	0.730	0.599	0.102	0.264	0.049

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	162	191	167	164	199	193	166
normalized size	1	1.00	1.00	1.18	1.03	1.01	1.23	1.19	1.02
time (sec)	N/A	0.227	0.056	0.005	0.787	0.618	0.309	0.335	0.103

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	156	186	162	168	185	185	163
normalized size	1	1.00	1.00	1.19	1.04	1.08	1.19	1.19	1.04
time (sec)	N/A	0.108	0.082	0.004	0.707	0.573	0.307	0.306	0.051

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	162	190	167	170	197	212	166
normalized size	1	1.00	1.00	1.17	1.03	1.05	1.22	1.31	1.02
time (sec)	N/A	0.226	0.071	0.007	0.785	0.746	0.401	0.404	0.056

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	126	261	0	421	620	126	1343
normalized size	1	1.00	0.95	1.96	0.00	3.17	4.66	0.95	10.10
time (sec)	N/A	0.207	0.064	0.007	0.000	0.810	43.892	1.873	0.458

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	93	175	0	312	434	91	979
normalized size	1	1.00	0.96	1.80	0.00	3.22	4.47	0.94	10.09
time (sec)	N/A	0.116	0.068	0.004	0.000	0.838	10.545	1.812	0.646

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	98	0	219	287	67	606
normalized size	1	1.00	1.00	1.38	0.00	3.08	4.04	0.94	8.54
time (sec)	N/A	0.070	0.046	0.003	0.000	0.575	3.589	1.715	0.498

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	128	105	0	249	0	78	2424
normalized size	1	1.00	1.64	1.35	0.00	3.19	0.00	1.00	31.08
time (sec)	N/A	0.139	0.103	0.007	0.000	0.798	0.000	1.610	4.476

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	186	191	0	385	0	124	3729
normalized size	1	1.00	1.66	1.71	0.00	3.44	0.00	1.11	33.29
time (sec)	N/A	0.245	0.218	0.010	0.000	1.007	0.000	1.874	4.855

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	327	825	0	5140	0	4391	10177
normalized size	1	1.00	1.25	3.16	0.00	19.69	0.00	16.82	38.99
time (sec)	N/A	1.489	0.405	0.053	0.000	2.886	0.000	3.789	1.594

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	251	560	0	2632	0	3179	6366
normalized size	1	1.00	1.21	2.69	0.00	12.65	0.00	15.28	30.61
time (sec)	N/A	0.528	0.163	0.026	0.000	0.879	0.000	3.498	1.256

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	173	328	0	1569	314	1400	4109
normalized size	1	1.00	1.01	1.91	0.00	9.12	1.83	8.14	23.89
time (sec)	N/A	0.201	0.094	0.020	0.000	0.874	16.959	2.416	0.997

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	206	353	0	2914	0	2805	6335
normalized size	1	1.00	1.09	1.87	0.00	15.42	0.00	14.84	33.52
time (sec)	N/A	0.400	0.291	0.030	0.000	0.993	0.000	3.568	1.354

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	267	611	0	5442	0	2870	10101
normalized size	1	1.00	0.99	2.25	0.00	20.08	0.00	10.59	37.27
time (sec)	N/A	0.653	0.316	0.033	0.000	2.318	0.000	3.346	2.187

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	208	689	0	1323	0	239	2282
normalized size	1	1.00	0.98	3.25	0.00	6.24	0.00	1.13	10.76
time (sec)	N/A	0.381	0.282	0.020	0.000	1.039	0.000	1.649	0.834

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	160	286	0	849	0	194	1527
normalized size	1	1.00	1.09	1.95	0.00	5.78	0.00	1.32	10.39
time (sec)	N/A	0.175	0.186	0.014	0.000	0.859	0.000	1.617	1.217

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	111	158	0	538	394	120	283
normalized size	1	1.00	1.04	1.48	0.00	5.03	3.68	1.12	2.64
time (sec)	N/A	0.113	0.080	0.012	0.000	0.681	5.380	1.723	0.323

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	101	127	0	474	374	102	264
normalized size	1	1.00	1.07	1.35	0.00	5.04	3.98	1.09	2.81
time (sec)	N/A	0.088	0.074	0.007	0.000	0.689	3.363	1.367	0.301

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	243	361	0	1014	0	201	7119
normalized size	1	1.00	1.62	2.41	0.00	6.76	0.00	1.34	47.46
time (sec)	N/A	0.331	0.330	0.018	0.000	1.432	0.000	1.710	7.884

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	379	622	0	1635	0	250	10034
normalized size	1	1.00	1.70	2.79	0.00	7.33	0.00	1.12	45.00
time (sec)	N/A	0.419	0.564	0.023	0.000	3.384	0.000	1.647	9.088

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	455	1507	0	7252	0	5681	16604
normalized size	1	1.00	1.07	3.55	0.00	17.06	0.00	13.37	39.07
time (sec)	N/A	3.675	1.202	0.043	0.000	5.997	0.000	6.490	4.358

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	362	1030	0	4658	0	4538	12396
normalized size	1	1.00	1.08	3.07	0.00	13.86	0.00	13.51	36.89
time (sec)	N/A	1.717	0.853	0.036	0.000	2.368	0.000	6.288	5.184

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	298	733	0	3467	0	3776	9444
normalized size	1	1.00	1.08	2.66	0.00	12.56	0.00	13.68	34.22
time (sec)	N/A	0.553	0.664	0.029	0.000	1.258	0.000	4.777	4.411

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	304	1761	0	4885	0	4426	12349
normalized size	1	1.00	1.04	6.01	0.00	16.67	0.00	15.11	42.15
time (sec)	N/A	0.846	0.786	0.110	0.000	3.329	0.000	5.965	4.839

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	382	1252	0	7583	0	5408	17591
normalized size	1	1.00	0.98	3.22	0.00	19.49	0.00	13.90	45.22
time (sec)	N/A	1.220	1.039	0.040	0.000	7.298	0.000	6.160	5.380

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	487	1653	0	10190	0	6327	21554
normalized size	1	1.00	0.93	3.17	0.00	19.52	0.00	12.12	41.29
time (sec)	N/A	1.365	1.199	0.047	0.000	18.852	0.000	8.150	5.698

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	435	2054	0	3196	0	598	4501
normalized size	1	1.00	1.19	5.63	0.00	8.76	0.00	1.64	12.33
time (sec)	N/A	1.455	0.663	0.031	0.000	1.270	0.000	5.883	4.660

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	354	723	0	2167	0	466	3062
normalized size	1	1.00	1.39	2.85	0.00	8.53	0.00	1.83	12.06
time (sec)	N/A	0.404	0.479	0.026	0.000	0.869	0.000	6.311	5.151

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	261	398	0	1378	775	318	593
normalized size	1	1.00	1.79	2.73	0.00	9.44	5.31	2.18	4.06
time (sec)	N/A	0.139	0.268	0.017	0.000	0.562	102.043	6.463	0.694

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	233	411	0	1369	833	268	625
normalized size	1	1.00	1.26	2.22	0.00	7.40	4.50	1.45	3.38
time (sec)	N/A	0.262	0.236	0.020	0.000	0.619	44.844	6.587	0.680

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	172	379	0	1226	789	228	587
normalized size	1	1.00	1.01	2.23	0.00	7.21	4.64	1.34	3.45
time (sec)	N/A	0.163	0.204	0.017	0.000	0.859	21.367	6.057	0.660

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	142	262	0	1109	661	208	517
normalized size	1	1.00	1.02	1.88	0.00	7.98	4.76	1.50	3.72
time (sec)	N/A	0.124	0.135	0.010	0.000	0.592	12.404	5.570	0.587

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	396	1161	0	2494	0	421	11674
normalized size	1	1.00	1.57	4.61	0.00	9.90	0.00	1.67	46.33
time (sec)	N/A	0.543	0.691	0.029	0.000	4.988	0.000	6.445	11.572

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	642	1862	0	3956	0	648	16265
normalized size	1	1.00	1.77	5.13	0.00	10.90	0.00	1.79	44.81
time (sec)	N/A	0.771	1.500	0.039	0.000	10.165	0.000	6.371	15.906

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	644	2015	0	9636	0	3987	22911
normalized size	1	1.00	1.16	3.64	0.00	17.39	0.00	7.20	41.36
time (sec)	N/A	11.195	2.398	0.069	0.000	14.453	0.000	8.280	5.047

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	543	1631	0	7060	0	7578	19041
normalized size	1	1.00	1.18	3.54	0.00	15.31	0.00	16.44	41.30
time (sec)	N/A	4.622	2.021	0.048	0.000	5.167	0.000	11.662	3.951

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	447	1283	0	5650	0	3162	16688
normalized size	1	1.00	1.18	3.38	0.00	14.87	0.00	8.32	43.92
time (sec)	N/A	1.415	1.696	0.049	0.000	4.426	0.000	8.261	3.487

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	436	1335	0	7270	0	7267	18992
normalized size	1	1.00	1.00	3.05	0.00	16.60	0.00	16.59	43.36
time (sec)	N/A	1.090	1.646	0.046	0.000	6.678	0.000	11.773	3.916

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	516	11936	0	9909	0	4609	22914
normalized size	1	1.00	1.12	25.95	0.00	21.54	0.00	10.02	49.81
time (sec)	N/A	1.353	2.189	0.281	0.000	16.221	0.000	8.537	4.614

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	17	19	17
normalized size	1	1.00	1.00	0.72	0.68	0.68	0.68	0.76	0.68
time (sec)	N/A	0.018	0.009	0.006	0.718	0.667	0.121	0.305	0.057

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	25	17	17	19	17
normalized size	1	1.00	1.00	0.72	1.00	0.68	0.68	0.76	0.68
time (sec)	N/A	0.029	0.006	0.004	0.739	0.656	0.115	0.283	0.029

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	37	30	32
normalized size	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86
time (sec)	N/A	0.035	0.013	0.003	1.401	0.546	0.120	0.282	0.212

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	53	30	37	30	32
normalized size	1	1.00	1.00	0.84	1.43	0.81	1.00	0.81	0.86
time (sec)	N/A	0.042	0.006	0.003	1.597	0.655	0.121	0.372	0.034

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	41	0	47	44	38	41
normalized size	1	1.00	1.00	0.91	0.00	1.04	0.98	0.84	0.91
time (sec)	N/A	0.049	0.026	0.006	0.000	0.618	0.152	0.944	0.052

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	71	91	104	61	0	102	102
normalized size	1	1.00	0.70	0.89	1.02	0.60	0.00	1.00	1.00
time (sec)	N/A	0.079	0.029	0.029	0.672	0.843	0.000	0.465	0.490

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	66	74	87	56	0	88	85
normalized size	1	1.00	0.81	0.91	1.07	0.69	0.00	1.09	1.05
time (sec)	N/A	0.056	0.021	0.015	0.593	0.705	0.000	0.363	0.433

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	61	57	70	51	0	74	67
normalized size	1	1.00	0.82	0.77	0.95	0.69	0.00	1.00	0.91
time (sec)	N/A	0.044	0.017	0.011	0.653	0.655	0.000	0.342	0.287

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	92	85	89	95	0	98	86
normalized size	1	1.00	0.98	0.90	0.95	1.01	0.00	1.04	0.91
time (sec)	N/A	0.083	0.038	0.015	1.496	0.676	0.000	0.478	0.426

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	104	89	112	0	138	84
normalized size	1	1.00	1.00	1.07	0.92	1.15	0.00	1.42	0.87
time (sec)	N/A	0.083	0.041	0.016	1.350	0.681	0.000	0.515	0.878

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	97	121	106	112	0	169	-1
normalized size	1	1.00	0.98	1.22	1.07	1.13	0.00	1.71	-0.01
time (sec)	N/A	0.083	0.039	0.016	1.426	0.652	0.000	0.572	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	74	118	99	90	0	189	-1
normalized size	1	1.00	0.82	1.31	1.10	1.00	0.00	2.10	-0.01
time (sec)	N/A	0.068	0.026	0.015	1.453	0.735	0.000	0.532	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	82	135	116	95	0	233	-1
normalized size	1	1.00	0.74	1.22	1.05	0.86	0.00	2.10	-0.01
time (sec)	N/A	0.086	0.027	0.017	1.736	0.631	0.000	0.457	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	84	152	133	100	0	255	-1
normalized size	1	1.00	0.64	1.15	1.01	0.76	0.00	1.93	-0.01
time (sec)	N/A	0.109	0.037	0.019	1.730	0.722	0.000	0.563	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	237	260	0	0	0	0	-1
normalized size	1	1.00	0.74	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.274	0.378	0.119	0.000	0.814	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	234	243	0	0	0	0	-1
normalized size	1	1.00	0.77	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.256	0.014	0.000	0.682	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	229	226	0	0	0	0	-1
normalized size	1	1.00	0.82	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.122	0.243	0.012	0.000	0.749	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	231	225	0	0	0	0	-1
normalized size	1	1.00	0.81	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.128	0.262	0.020	0.000	0.691	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	237	228	0	0	0	0	-1
normalized size	1	1.00	0.78	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.158	0.260	0.020	0.000	0.775	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	81	138	135	71	0	207	-1
normalized size	1	1.00	0.64	1.09	1.06	0.56	0.00	1.63	-0.01
time (sec)	N/A	0.096	0.035	0.033	0.585	0.841	0.000	0.541	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	76	121	118	66	0	179	-1
normalized size	1	1.00	0.72	1.14	1.11	0.62	0.00	1.69	-0.01
time (sec)	N/A	0.072	0.031	0.016	0.878	0.573	0.000	0.587	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	71	104	101	61	0	151	127
normalized size	1	1.00	0.72	1.05	1.02	0.62	0.00	1.53	1.28
time (sec)	N/A	0.058	0.025	0.016	0.666	0.642	0.000	0.452	0.530

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	104	117	120	106	0	113	-1
normalized size	1	1.00	0.87	0.98	1.01	0.89	0.00	0.95	-0.01
time (sec)	N/A	0.106	0.058	0.015	1.396	0.685	0.000	0.498	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	107	117	120	122	0	153	-1
normalized size	1	1.00	0.88	0.96	0.98	1.00	0.00	1.25	-0.01
time (sec)	N/A	0.107	0.054	0.021	1.746	0.594	0.000	0.559	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	107	117	137	122	0	190	-1
normalized size	1	1.00	0.84	0.92	1.08	0.96	0.00	1.50	-0.01
time (sec)	N/A	0.109	0.062	0.020	1.461	0.796	0.000	0.654	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	107	117	154	122	0	227	-1
normalized size	1	1.00	0.84	0.92	1.21	0.96	0.00	1.79	-0.01
time (sec)	N/A	0.107	0.051	0.022	1.585	0.624	0.000	0.681	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	249	294	0	0	0	0	-1
normalized size	1	1.00	0.70	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	0.270	0.021	0.000	0.732	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	244	277	0	0	0	0	-1
normalized size	1	1.00	0.74	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.214	0.254	0.017	0.000	0.729	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	0	260	0	0	0	0	-1
normalized size	1	1.00	0.00	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.000	0.014	0.000	0.659	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	235	260	0	0	0	0	-1
normalized size	1	1.00	0.75	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.151	0.261	0.019	0.000	0.575	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	247	260	0	0	0	0	-1
normalized size	1	1.00	0.79	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.164	0.294	0.020	0.000	0.636	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	244	259	0	0	0	0	-1
normalized size	1	1.00	0.74	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.281	0.021	0.000	0.529	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	139	286	0	315	0	138	-1
normalized size	1	1.00	0.91	1.87	0.00	2.06	0.00	0.90	-0.01
time (sec)	N/A	0.203	0.111	0.030	0.000	0.836	0.000	0.544	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	101	176	0	233	0	98	-1
normalized size	1	1.00	1.01	1.76	0.00	2.33	0.00	0.98	-0.01
time (sec)	N/A	0.093	0.047	0.015	0.000	0.835	0.000	0.449	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	78	93	0	178	0	69	92
normalized size	1	1.00	1.03	1.22	0.00	2.34	0.00	0.91	1.21
time (sec)	N/A	0.064	0.025	0.012	0.000	1.084	0.000	0.461	1.046

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	89	76	0	517	0	0	81
normalized size	1	1.00	0.99	0.84	0.00	5.74	0.00	0.00	0.90
time (sec)	N/A	0.094	0.028	0.013	0.000	1.068	0.000	0.000	0.760

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	104	0	197	0	124	103
normalized size	1	1.00	1.02	1.30	0.00	2.46	0.00	1.55	1.29
time (sec)	N/A	0.083	0.035	0.018	0.000	1.152	0.000	0.538	0.776

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	107	194	0	255	0	339	-1
normalized size	1	1.00	0.86	1.56	0.00	2.06	0.00	2.73	-0.01
time (sec)	N/A	0.145	0.075	0.019	0.000	0.868	0.000	0.516	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	148	311	0	339	0	571	-1
normalized size	1	1.00	0.84	1.76	0.00	1.92	0.00	3.23	-0.01
time (sec)	N/A	0.239	0.108	0.020	0.000	0.964	0.000	0.639	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	532	815	0	0	0	0	-1
normalized size	1	1.00	1.32	2.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	2.211	0.023	0.000	0.488	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	479	607	0	0	0	0	-1
normalized size	1	1.00	1.43	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	1.373	0.011	0.000	0.731	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	302	362	0	0	0	0	-1
normalized size	1	1.00	1.07	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.083	0.254	0.010	0.000	0.764	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	448	386	0	0	0	0	-1
normalized size	1	1.00	1.44	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.131	1.070	0.020	0.000	0.686	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	373	656	0	0	0	0	-1
normalized size	1	1.00	0.99	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.227	0.695	0.023	0.000	0.817	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	66	87	90	56	0	60	-1
normalized size	1	1.00	0.67	0.89	0.92	0.57	0.00	0.61	-0.01
time (sec)	N/A	0.087	0.028	0.019	0.926	0.550	0.000	0.370	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	70	73	51	0	53	-1
normalized size	1	1.00	0.79	0.91	0.95	0.66	0.00	0.69	-0.01
time (sec)	N/A	0.066	0.020	0.014	0.988	0.592	0.000	0.344	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	56	46	0	46	-1
normalized size	1	1.00	1.00	0.95	1.00	0.82	0.00	0.82	-0.02
time (sec)	N/A	0.045	0.016	0.013	1.040	0.536	0.000	0.348	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	36	39	39	0	39	35
normalized size	1	1.00	1.00	0.73	0.80	0.80	0.00	0.80	0.71
time (sec)	N/A	0.032	0.009	0.012	0.923	0.898	0.000	0.468	0.523

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	52	58	75	0	78	56
normalized size	1	1.00	1.00	0.75	0.84	1.09	0.00	1.13	0.81
time (sec)	N/A	0.062	0.013	0.012	1.929	0.828	0.000	0.435	1.009

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	49	51	78	0	101	83
normalized size	1	1.00	1.00	0.79	0.82	1.26	0.00	1.63	1.34
time (sec)	N/A	0.050	0.015	0.015	2.021	0.519	0.000	0.386	0.663

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	66	68	83	0	145	-1
normalized size	1	1.00	0.81	0.80	0.82	1.00	0.00	1.75	-0.01
time (sec)	N/A	0.070	0.022	0.013	2.000	0.701	0.000	0.521	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	77	83	85	90	0	167	-1
normalized size	1	1.00	0.74	0.80	0.82	0.87	0.00	1.61	-0.01
time (sec)	N/A	0.088	0.024	0.017	1.981	0.540	0.000	0.507	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	229	226	0	0	0	0	-1
normalized size	1	1.00	0.77	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.175	0.271	0.018	0.000	0.722	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	222	208	0	0	0	0	-1
normalized size	1	1.00	0.82	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.120	0.244	0.013	0.000	0.646	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	159	194	0	0	0	0	-1
normalized size	1	1.00	0.62	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.077	0.119	0.012	0.000	1.235	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	224	211	0	0	0	0	-1
normalized size	1	1.00	0.81	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.123	0.247	0.023	0.000	0.684	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	237	228	0	0	0	0	-1
normalized size	1	1.00	0.78	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.263	0.018	0.000	0.729	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	72	91	73	86	0	52	-1
normalized size	1	1.00	0.94	1.18	0.95	1.12	0.00	0.68	-0.01
time (sec)	N/A	0.058	0.018	0.022	0.923	0.557	0.000	0.383	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	95	56	81	0	46	52
normalized size	1	1.00	0.96	1.70	1.00	1.45	0.00	0.82	0.93
time (sec)	N/A	0.043	0.121	0.013	0.933	0.613	0.000	0.512	0.312

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	32	46	0	21	21
normalized size	1	1.00	1.00	0.88	1.28	1.84	0.00	0.84	0.84
time (sec)	N/A	0.019	0.092	0.006	0.938	0.736	0.000	0.363	0.238

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	67	65	107	0	78	-1
normalized size	1	1.00	1.00	1.02	0.98	1.62	0.00	1.18	-0.02
time (sec)	N/A	0.057	0.024	0.019	2.006	0.724	0.000	0.556	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	88	84	82	124	0	122	-1
normalized size	1	1.00	0.98	0.93	0.91	1.38	0.00	1.36	-0.01
time (sec)	N/A	0.071	0.018	0.016	2.088	0.873	0.000	0.492	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	219	240	0	0	0	0	-1
normalized size	1	1.00	0.71	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.258	0.024	0.000	0.753	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	219	240	0	0	0	0	-1
normalized size	1	1.00	0.77	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.123	0.252	0.016	0.000	0.807	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	0	240	0	0	0	0	-1
normalized size	1	1.00	0.00	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.109	0.000	0.017	0.000	0.595	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	228	257	0	0	0	0	-1
normalized size	1	1.00	0.74	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.164	0.270	0.026	0.000	0.778	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	234	274	0	0	0	0	-1
normalized size	1	1.00	0.72	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.275	0.023	0.000	0.781	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	430	0	0	0	0	0	-1
normalized size	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.385	0.983	0.128	0.000	0.591	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	386	0	0	0	0	0	-1
normalized size	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.323	5.714	0.067	0.000	0.982	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	386	0	0	0	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.321	0.709	0.059	0.000	0.753	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	370	0	0	0	0	0	-1
normalized size	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	0.859	0.062	0.000	0.564	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.347	0.000	0.062	0.000	0.677	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	490	0	0	0	0	0	-1
normalized size	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.354	6.144	0.069	0.000	0.642	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.351	0.000	0.058	0.000	0.808	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	447	0	0	0	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.348	0.993	0.065	0.000	0.586	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	354	0	0	0	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.328	0.589	0.051	0.000	0.559	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	242	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.322	5.141	0.027	0.000	0.666	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	241	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.325	0.195	0.025	0.000	0.506	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	356	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.325	0.618	0.058	0.000	0.587	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.344	0.000	0.027	0.000	0.693	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	397	0	0	0	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	5.700	0.026	0.000	0.676	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.350	0.000	0.039	0.000	0.951	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	460	0	0	0	0	0	-1
normalized size	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.351	0.973	0.134	0.000	0.569	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	191	1935	408	1357	11538	2816	769
normalized size	1	1.00	0.79	7.96	1.68	5.58	47.48	11.59	3.16
time (sec)	N/A	0.176	0.298	0.009	1.388	0.742	12.378	0.750	1.058

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	117	783	230	573	4190	1178	429
normalized size	1	1.00	0.75	5.05	1.48	3.70	27.03	7.60	2.77
time (sec)	N/A	0.099	0.123	0.008	1.221	0.643	5.444	0.398	0.598

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	59	221	104	171	1056	350	171
normalized size	1	1.00	0.71	2.66	1.25	2.06	12.72	4.22	2.06
time (sec)	N/A	0.047	0.049	0.004	1.058	0.909	1.862	0.419	0.339

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	156	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.299	0.244	0.037	0.000	0.519	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	392	358	160	0	0	0	0	0	-1
normalized size	1	0.91	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.646	0.226	0.028	0.000	0.852	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	466	0	0	0	0	0	-1
normalized size	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	0.531	0.012	0.000	0.695	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	267	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.361	0.198	0.011	0.000	0.816	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	267	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.264	0.014	0.000	0.864	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	307	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.388	0.405	0.012	0.000	0.682	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	134	122	120	277	0	121	181
normalized size	1	1.00	1.00	0.91	0.90	2.07	0.00	0.90	1.35
time (sec)	N/A	0.179	0.063	0.014	1.998	8.361	0.000	0.309	0.873

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	99	108	107	212	0	105	166
normalized size	1	1.00	0.84	0.92	0.91	1.80	0.00	0.89	1.41
time (sec)	N/A	0.152	0.094	0.010	2.018	3.473	0.000	0.307	0.722

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	77	92	89	170	0	90	138
normalized size	1	1.00	0.73	0.88	0.85	1.62	0.00	0.86	1.31
time (sec)	N/A	0.137	0.034	0.009	1.998	1.963	0.000	0.356	0.989

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	66	83	82	145	0	86	944
normalized size	1	1.00	0.69	0.86	0.85	1.51	0.00	0.90	9.83
time (sec)	N/A	0.094	0.040	0.007	1.985	0.835	0.000	0.393	1.936

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	67	83	82	146	0	85	328
normalized size	1	1.00	0.70	0.86	0.85	1.52	0.00	0.89	3.42
time (sec)	N/A	0.064	0.037	0.007	1.985	0.947	0.000	0.354	1.020

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	134	101	101	201	0	102	527
normalized size	1	1.00	1.18	0.89	0.89	1.76	0.00	0.89	4.62
time (sec)	N/A	0.125	0.071	0.010	1.966	11.290	0.000	0.289	0.961

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	169	119	120	265	0	132	820
normalized size	1	1.00	1.31	0.92	0.93	2.05	0.00	1.02	6.36
time (sec)	N/A	0.150	0.101	0.013	1.997	68.844	0.000	0.302	1.381

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	209	145	145	0	0	168	1017
normalized size	1	1.00	1.34	0.93	0.93	0.00	0.00	1.08	6.52
time (sec)	N/A	0.183	0.093	0.016	2.050	0.000	0.000	0.342	1.868

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	344	405	294	4414	0	363	6097
normalized size	1	1.00	0.96	1.13	0.82	12.30	0.00	1.01	16.98
time (sec)	N/A	0.346	0.360	0.017	2.048	21.106	0.000	0.541	2.056

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	373	387	289	4354	0	333	5908
normalized size	1	1.00	1.08	1.12	0.84	12.62	0.00	0.97	17.12
time (sec)	N/A	0.302	0.238	0.008	2.089	3.582	0.000	0.444	1.827

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	233	363	268	4040	0	327	5111
normalized size	1	1.00	0.69	1.08	0.80	12.02	0.00	0.97	15.21
time (sec)	N/A	0.274	0.151	0.009	2.627	1.678	0.000	0.434	2.199

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	232	351	275	3892	0	336	4720
normalized size	1	1.00	0.69	1.04	0.82	11.55	0.00	1.00	14.01
time (sec)	N/A	0.267	0.126	0.009	1.427	1.200	0.000	0.377	1.589

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	234	363	268	4084	0	339	4802
normalized size	1	1.00	0.70	1.08	0.80	12.15	0.00	1.01	14.29
time (sec)	N/A	0.273	0.145	0.008	1.090	2.282	0.000	0.416	1.672

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	389	390	292	4362	0	348	5761
normalized size	1	1.00	1.12	1.12	0.84	12.53	0.00	1.00	16.55
time (sec)	N/A	0.300	0.253	0.012	1.274	6.206	0.000	0.398	1.999

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	367	406	297	4442	0	364	5972
normalized size	1	1.00	1.02	1.13	0.82	12.34	0.00	1.01	16.59
time (sec)	N/A	0.301	0.401	0.012	2.123	19.675	0.000	0.395	2.257

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	135	305	220	555	0	251	305
normalized size	1	1.00	0.80	1.80	1.30	3.28	0.00	1.49	1.80
time (sec)	N/A	0.367	0.210	0.020	2.075	31.512	0.000	0.364	1.302

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	142	260	197	457	0	223	647
normalized size	1	1.00	0.95	1.73	1.31	3.05	0.00	1.49	4.31
time (sec)	N/A	0.247	0.112	0.019	2.047	15.332	0.000	0.351	1.487

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	153	120	252	192	487	0	220	528
normalized size	1	0.99	0.77	1.63	1.24	3.14	0.00	1.42	3.41
time (sec)	N/A	0.248	0.153	0.016	2.077	6.531	0.000	0.331	1.520

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	148	114	247	186	492	0	188	527
normalized size	1	0.99	0.77	1.66	1.25	3.30	0.00	1.26	3.54
time (sec)	N/A	0.187	0.141	0.015	2.027	6.692	0.000	0.281	1.411

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	117	255	196	458	0	199	649
normalized size	1	1.00	0.77	1.69	1.30	3.03	0.00	1.32	4.30
time (sec)	N/A	0.181	0.133	0.017	2.030	15.717	0.000	0.383	1.493

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	241	309	228	0	0	279	1082
normalized size	1	1.00	1.15	1.48	1.09	0.00	0.00	1.33	5.18
time (sec)	N/A	0.239	0.183	0.023	2.054	0.000	0.000	0.368	2.577

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	248	332	278	0	0	344	1337
normalized size	1	1.00	1.05	1.41	1.18	0.00	0.00	1.46	5.67
time (sec)	N/A	0.261	0.427	0.025	2.007	0.000	0.000	0.353	2.942

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	278	363	332	0	0	350	1545
normalized size	1	1.00	1.05	1.37	1.25	0.00	0.00	1.32	5.83
time (sec)	N/A	0.327	0.391	0.025	2.082	0.000	0.000	0.367	3.477

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	431	873	504	9856	0	581	18343
normalized size	1	1.00	0.61	1.23	0.71	13.84	0.00	0.82	25.76
time (sec)	N/A	0.671	0.304	0.017	2.064	37.529	0.000	0.573	2.864

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	428	852	476	9822	0	595	17909
normalized size	1	1.00	0.62	1.24	0.69	14.30	0.00	0.87	26.07
time (sec)	N/A	0.602	0.369	0.019	2.079	22.081	0.000	0.587	2.825

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	685	423	848	490	9678	0	586	17180
normalized size	1	1.00	0.62	1.24	0.72	14.13	0.00	0.86	25.08
time (sec)	N/A	0.609	0.272	0.016	2.095	18.399	0.000	0.466	4.873

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	685	428	852	472	9774	0	603	17812
normalized size	1	1.00	0.62	1.24	0.69	14.27	0.00	0.88	26.00
time (sec)	N/A	0.560	0.283	0.017	2.148	20.924	0.000	0.502	2.867

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	429	873	506	9892	0	603	17945
normalized size	1	1.00	0.62	1.27	0.73	14.36	0.00	0.88	26.04
time (sec)	N/A	0.601	0.291	0.018	2.060	40.137	0.000	0.466	2.732

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	745	745	499	911	521	10188	0	639	24015
normalized size	1	1.00	0.67	1.22	0.70	13.68	0.00	0.86	32.23
time (sec)	N/A	0.772	0.370	0.020	2.111	98.080	0.000	0.452	5.161

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	513	932	543	0	0	628	20828
normalized size	1	1.00	0.68	1.24	0.72	0.00	0.00	0.84	27.73
time (sec)	N/A	0.686	0.386	0.021	2.110	0.000	0.000	0.497	5.219

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	40	110	0	0	0	0	-1
normalized size	1	1.00	0.57	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.096	0.074	0.000	1.140	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	36	112	0	0	0	0	-1
normalized size	1	1.00	0.51	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.092	0.023	0.000	1.194	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	46	96	0	0	0	0	-1
normalized size	1	1.00	0.46	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.106	0.019	0.000	1.189	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	37	143	0	0	0	0	-1
normalized size	1	1.00	0.61	2.34	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.079	0.033	0.000	0.796	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	54	99	0	0	0	0	-1
normalized size	1	1.00	0.48	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.075	0.017	0.000	0.822	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	134	0	0	0	0	-1
normalized size	1	1.00	0.61	2.35	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.072	0.020	0.000	0.662	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	60	168	0	0	0	0	-1
normalized size	1	1.00	0.81	2.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.091	0.039	0.000	1.277	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	56	115	0	0	0	0	-1
normalized size	1	1.00	0.76	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.088	0.020	0.000	0.900	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	142	159	124	206	0	156	-1
normalized size	1	1.00	0.58	0.65	0.51	0.85	0.00	0.64	-0.00
time (sec)	N/A	0.130	0.184	0.011	0.918	0.635	0.000	0.376	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	56	51	50	50	0	68	-1
normalized size	1	1.00	0.52	0.47	0.46	0.46	0.00	0.63	-0.01
time (sec)	N/A	0.101	0.030	0.004	0.898	0.927	0.000	0.336	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	121	119	81	155	0	109	-1
normalized size	1	1.00	0.68	0.67	0.46	0.87	0.00	0.61	-0.01
time (sec)	N/A	0.076	0.113	0.007	1.080	0.975	0.000	0.461	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	83	80	45	123	0	84	-1
normalized size	1	1.00	0.55	0.53	0.30	0.81	0.00	0.55	-0.01
time (sec)	N/A	0.095	0.046	0.007	1.453	0.874	0.000	0.445	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	122	128	59	134	0	116	-1
normalized size	1	1.00	0.69	0.72	0.33	0.76	0.00	0.66	-0.01
time (sec)	N/A	0.091	0.109	0.013	1.080	0.804	0.000	0.471	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	90	133	83	141	0	100	-1
normalized size	1	1.00	0.51	0.75	0.47	0.80	0.00	0.56	-0.01
time (sec)	N/A	0.119	0.045	0.012	1.225	0.685	0.000	0.443	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	73	72	79	76	79	73
normalized size	1	1.00	0.92	0.94	0.92	1.01	0.97	1.01	0.94
time (sec)	N/A	0.135	0.026	0.001	1.106	0.542	0.079	0.267	0.038

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	73	72	79	82	79	73
normalized size	1	1.00	1.00	0.94	0.92	1.01	1.05	1.01	0.94
time (sec)	N/A	0.064	0.016	0.001	1.189	0.880	0.079	0.388	0.028

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	72	73	72	79	76	79	73
normalized size	1	1.00	0.96	0.97	0.96	1.05	1.01	1.05	0.97
time (sec)	N/A	0.131	0.022	0.000	1.211	0.834	0.077	0.269	0.029

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	76	78	76	70
normalized size	1	1.00	1.00	0.96	0.95	1.04	1.07	1.04	0.96
time (sec)	N/A	0.044	0.015	0.002	1.173	0.547	0.077	0.354	0.029

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	77	73	70	73	79	70
normalized size	1	1.00	1.00	1.04	0.99	0.95	0.99	1.07	0.95
time (sec)	N/A	0.090	0.020	0.003	1.120	0.802	0.171	0.263	0.034

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	75	69	74	73	74	70
normalized size	1	1.00	1.00	1.06	0.97	1.04	1.03	1.04	0.99
time (sec)	N/A	0.048	0.032	0.003	1.070	0.553	0.164	0.361	0.033

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	71	76	73	76	71	97	70
normalized size	1	1.00	0.96	1.03	0.99	1.03	0.96	1.31	0.95
time (sec)	N/A	0.096	0.042	0.007	1.094	0.896	0.263	0.379	0.038

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	165	214	165	426	320	160	251
normalized size	1	1.00	0.98	1.27	0.98	2.54	1.90	0.95	1.49
time (sec)	N/A	0.234	0.138	0.013	2.487	0.937	1.177	0.321	0.325

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	133	176	130	350	189	125	179
normalized size	1	1.00	0.99	1.30	0.96	2.59	1.40	0.93	1.33
time (sec)	N/A	0.159	0.078	0.012	2.451	0.679	1.084	0.277	0.322

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	102	141	95	302	162	91	95
normalized size	1	1.00	0.96	1.33	0.90	2.85	1.53	0.86	0.90
time (sec)	N/A	0.106	0.065	0.010	2.517	0.844	0.966	0.335	0.342

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	118	84	268	153	75	77
normalized size	1	1.00	1.06	1.42	1.01	3.23	1.84	0.90	0.93
time (sec)	N/A	0.093	0.054	0.010	2.335	0.814	0.770	0.406	0.357

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	86	89	121	87	267	155	83	81
normalized size	1	0.97	1.00	1.36	0.98	3.00	1.74	0.93	0.91
time (sec)	N/A	0.118	0.061	0.013	2.462	0.978	1.118	0.293	0.367

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	105	146	103	316	167	94	98
normalized size	1	1.00	0.99	1.38	0.97	2.98	1.58	0.89	0.92
time (sec)	N/A	0.137	0.063	0.013	2.385	0.935	1.527	0.261	0.357

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	135	183	139	360	284	131	128
normalized size	1	1.00	0.99	1.35	1.02	2.65	2.09	0.96	0.94
time (sec)	N/A	0.252	0.088	0.015	2.463	0.894	2.133	0.327	0.383

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	166	221	174	436	328	164	156
normalized size	1	1.00	0.99	1.32	1.04	2.61	1.96	0.98	0.93
time (sec)	N/A	0.330	0.095	0.016	2.570	0.896	2.679	0.417	0.403

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	170	239	175	504	235	160	223
normalized size	1	1.00	0.98	1.38	1.01	2.91	1.36	0.92	1.29
time (sec)	N/A	0.321	0.111	0.015	2.466	0.968	3.583	0.359	0.354

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	141	202	139	462	212	125	137
normalized size	1	1.00	0.99	1.41	0.97	3.23	1.48	0.87	0.96
time (sec)	N/A	0.210	0.087	0.013	2.509	0.675	3.370	0.471	0.344

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	122	179	126	421	201	107	118
normalized size	1	1.00	0.98	1.44	1.02	3.40	1.62	0.86	0.95
time (sec)	N/A	0.138	0.104	0.011	2.461	0.815	2.621	0.263	0.386

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	110	131	121	391	196	101	112
normalized size	1	1.00	0.96	1.14	1.05	3.40	1.70	0.88	0.97
time (sec)	N/A	0.116	0.096	0.010	2.462	0.767	1.501	0.312	0.379

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	124	124	182	129	421	202	110	118
normalized size	1	0.98	0.98	1.43	1.02	3.31	1.59	0.87	0.93
time (sec)	N/A	0.204	0.135	0.013	2.650	0.697	2.142	0.316	0.390

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	141	207	147	476	214	128	138
normalized size	1	1.00	0.99	1.46	1.04	3.35	1.51	0.90	0.97
time (sec)	N/A	0.217	0.085	0.016	2.594	0.881	2.924	0.340	0.398

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	173	245	183	514	330	164	168
normalized size	1	1.00	1.01	1.43	1.07	3.01	1.93	0.96	0.98
time (sec)	N/A	0.373	0.114	0.016	2.453	0.946	3.757	0.347	0.412

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	228	538	0	0	0	236	7024
normalized size	1	1.00	0.99	2.34	0.00	0.00	0.00	1.03	30.54
time (sec)	N/A	0.487	0.243	0.016	0.000	0.000	0.000	1.728	69.941

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	186	408	0	0	0	194	2304
normalized size	1	1.00	0.98	2.16	0.00	0.00	0.00	1.03	12.19
time (sec)	N/A	0.329	0.187	0.010	0.000	0.000	0.000	2.185	15.207

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	139	289	0	421	0	157	1853
normalized size	1	1.00	0.88	1.83	0.00	2.66	0.00	0.99	11.73
time (sec)	N/A	0.261	0.106	0.010	0.000	144.356	0.000	1.843	11.051

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	114	176	0	321	0	133	3704
normalized size	1	1.00	0.86	1.33	0.00	2.43	0.00	1.01	28.06
time (sec)	N/A	0.157	0.070	0.008	0.000	27.802	0.000	1.720	9.751

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	112	176	0	321	0	134	2434
normalized size	1	1.00	0.84	1.32	0.00	2.41	0.00	1.01	18.30
time (sec)	N/A	0.123	0.068	0.010	0.000	19.134	0.000	1.908	8.707

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	242	298	0	0	0	172	6285
normalized size	1	1.00	1.45	1.78	0.00	0.00	0.00	1.03	37.63
time (sec)	N/A	0.306	0.321	0.011	0.000	0.000	0.000	1.917	17.199

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	331	430	0	0	0	237	5368
normalized size	1	1.00	1.61	2.10	0.00	0.00	0.00	1.16	26.19
time (sec)	N/A	0.470	0.343	0.019	0.000	0.000	0.000	1.962	62.948

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	426	584	0	0	0	332	10300
normalized size	1	1.00	1.59	2.18	0.00	0.00	0.00	1.24	38.43
time (sec)	N/A	0.597	0.434	0.020	0.000	0.000	0.000	1.462	144.755

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	463	1449	0	0	0	12506	41755
normalized size	1	1.00	1.20	3.74	0.00	0.00	0.00	32.32	107.89
time (sec)	N/A	4.032	0.614	0.042	0.000	0.000	0.000	12.649	7.134

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	385	1098	0	0	0	11030	33892
normalized size	1	1.00	1.19	3.40	0.00	0.00	0.00	34.15	104.93
time (sec)	N/A	1.366	0.520	0.036	0.000	0.000	0.000	13.868	6.446

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	323	764	0	15553	0	8658	25202
normalized size	1	1.00	1.15	2.73	0.00	55.55	0.00	30.92	90.01
time (sec)	N/A	0.894	0.334	0.026	0.000	9.379	0.000	11.236	5.800

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	277	478	0	12269	0	6921	19401
normalized size	1	1.00	1.10	1.90	0.00	48.88	0.00	27.57	77.29
time (sec)	N/A	0.451	0.498	0.024	0.000	3.757	0.000	8.453	4.959

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	274	480	0	0	0	7650	23640
normalized size	1	1.00	1.08	1.89	0.00	0.00	0.00	30.12	93.07
time (sec)	N/A	0.520	0.248	0.027	0.000	0.000	0.000	9.134	5.614

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	340	817	0	0	0	10058	33644
normalized size	1	1.00	1.14	2.74	0.00	0.00	0.00	33.75	112.90
time (sec)	N/A	0.960	0.399	0.030	0.000	0.000	0.000	12.818	5.890

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	410	1160	0	0	0	12268	42882
normalized size	1	1.00	1.18	3.33	0.00	0.00	0.00	35.25	123.22
time (sec)	N/A	1.549	0.525	0.040	0.000	0.000	0.000	12.000	6.726

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	866	866	267	336	0	0	0	0	43112
normalized size	1	1.00	0.31	0.39	0.00	0.00	0.00	0.00	49.78
time (sec)	N/A	2.509	0.377	0.096	0.000	0.000	0.000	0.000	6.836

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	267	1049	0	0	0	0	-1
normalized size	1	1.00	0.98	3.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.573	0.435	0.043	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	205	887	0	1231	0	0	-1
normalized size	1	1.00	0.99	4.26	0.00	5.92	0.00	0.00	-0.00
time (sec)	N/A	0.315	0.262	0.010	0.000	46.630	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	167	757	0	1050	0	0	-1
normalized size	1	1.00	0.99	4.51	0.00	6.25	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.128	0.005	0.000	3.691	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	179	851	0	2367	0	0	-1
normalized size	1	1.00	0.96	4.58	0.00	12.73	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.149	0.017	0.000	162.637	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	165	1009	0	1094	0	216	-1
normalized size	1	1.00	0.46	2.80	0.00	3.03	0.00	0.60	-0.00
time (sec)	N/A	0.509	0.315	0.020	0.000	1.490	0.000	0.523	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	619	209	528	0	0	0	0	-1
normalized size	1	1.46	0.49	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.561	0.259	0.101	0.000	1.047	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	591	204	509	0	0	0	0	-1
normalized size	1	1.42	0.49	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	0.195	0.009	0.000	0.986	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	470	127	341	0	0	0	0	-1
normalized size	1	1.23	0.33	0.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	0.109	0.006	0.000	1.083	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	208	511	0	0	0	0	-1
normalized size	1	1.00	0.52	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.190	0.013	0.000	1.027	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	154	448	0	0	0	0	-1
normalized size	1	1.00	0.43	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.194	0.173	0.021	0.000	0.982	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	224	549	0	0	0	0	-1
normalized size	1	1.00	0.41	1.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	0.238	0.020	0.000	1.154	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	545	2068	0	0	0	0	-1
normalized size	1	1.00	1.13	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.103	0.976	0.062	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	344	1696	0	0	0	0	-1
normalized size	1	1.00	0.96	4.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.697	0.600	0.014	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	255	1411	0	0	0	0	-1
normalized size	1	1.00	0.95	5.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.456	0.351	0.007	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	251	1270	0	0	0	0	-1
normalized size	1	1.00	0.72	3.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.573	0.534	0.036	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	240	1207	0	0	0	0	-1
normalized size	1	1.00	0.43	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.924	0.503	0.033	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	875	214	547	0	0	0	0	-1
normalized size	1	1.89	0.46	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.672	0.272	0.036	0.000	1.323	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	602	209	377	0	0	0	0	-1
normalized size	1	1.41	0.49	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.149	0.007	0.000	1.281	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	722	722	213	528	0	0	0	0	-1
normalized size	1	1.00	0.30	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	0.206	0.016	0.000	1.155	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	625	625	219	530	0	0	0	0	-1
normalized size	1	1.00	0.35	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	0.218	0.017	0.000	1.261	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	224	549	0	0	0	0	-1
normalized size	1	1.00	0.41	0.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	0.245	0.018	0.000	1.083	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	171	267	0	1364	0	0	-1
normalized size	1	1.00	0.99	1.54	0.00	7.88	0.00	0.00	-0.01
time (sec)	N/A	0.314	0.367	0.020	0.000	54.253	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	133	204	0	1084	0	0	-1
normalized size	1	1.00	0.97	1.49	0.00	7.91	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.118	0.010	0.000	3.574	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	87	165	0	357	0	75	-1
normalized size	1	1.00	1.01	1.92	0.00	4.15	0.00	0.87	-0.01
time (sec)	N/A	0.084	0.018	0.007	0.000	1.318	0.000	0.486	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	134	207	0	1097	0	0	-1
normalized size	1	1.00	0.97	1.50	0.00	7.95	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.140	0.010	0.000	1.385	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	175	276	0	1414	0	208	-1
normalized size	1	1.00	0.80	1.27	0.00	6.49	0.00	0.95	-0.00
time (sec)	N/A	0.266	0.367	0.011	0.000	2.933	0.000	0.488	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	127	222	0	0	0	0	-1
normalized size	1	1.00	0.30	0.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.207	0.022	0.000	1.216	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	99	134	0	0	0	0	-1
normalized size	1	1.00	0.40	0.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.126	0.124	0.009	0.000	1.376	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	80	70	0	0	0	0	-1
normalized size	1	1.00	0.33	0.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.111	0.057	0.006	0.000	1.249	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	147	178	0	0	0	0	-1
normalized size	1	1.00	0.37	0.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.347	0.218	0.016	0.000	1.313	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	219	260	0	0	0	0	-1
normalized size	1	1.00	0.52	0.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.517	0.184	0.014	0.000	1.363	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	271	720	0	4901	0	0	-1
normalized size	1	1.00	1.15	3.05	0.00	20.77	0.00	0.00	-0.00
time (sec)	N/A	0.474	0.821	0.063	0.000	151.802	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	204	613	0	1381	0	458	-1
normalized size	1	1.00	1.22	3.67	0.00	8.27	0.00	2.74	-0.01
time (sec)	N/A	0.294	0.616	0.015	0.000	1.971	0.000	0.720	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	162	506	0	1349	0	441	-1
normalized size	1	1.00	1.02	3.18	0.00	8.48	0.00	2.77	-0.01
time (sec)	N/A	0.191	0.172	0.011	0.000	2.139	0.000	0.641	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	167	454	0	1379	0	454	-1
normalized size	1	1.00	1.01	2.73	0.00	8.31	0.00	2.73	-0.01
time (sec)	N/A	0.171	0.149	0.008	0.000	2.158	0.000	0.618	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	236	612	0	4909	0	0	-1
normalized size	1	1.00	0.89	2.30	0.00	18.45	0.00	0.00	-0.00
time (sec)	N/A	0.388	0.731	0.024	0.000	9.527	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	350	863	0	6486	0	762	-1
normalized size	1	1.00	0.84	2.06	0.00	15.48	0.00	1.82	-0.00
time (sec)	N/A	0.564	1.477	0.028	0.000	22.070	0.000	2.736	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	566	199	603	0	0	0	0	-1
normalized size	1	1.26	0.44	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.347	0.277	0.046	0.000	1.152	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	503	199	586	0	0	0	0	-1
normalized size	1	1.19	0.47	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	0.205	0.010	0.000	1.492	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	501	199	561	0	0	0	0	-1
normalized size	1	1.19	0.47	1.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.206	0.010	0.000	1.375	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	503	199	536	0	0	0	0	-1
normalized size	1	1.19	0.47	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.180	0.010	0.000	1.105	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	501	199	366	0	0	0	0	-1
normalized size	1	1.19	0.47	0.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.137	0.007	0.000	1.209	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	644	211	553	0	0	0	0	-1
normalized size	1	1.38	0.45	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.445	0.225	0.018	0.000	1.306	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	943	496	0	0	0	928	11195
normalized size	1	1.00	2.32	1.22	0.00	0.00	0.00	2.29	27.57
time (sec)	N/A	8.593	10.842	0.059	0.000	0.000	0.000	0.650	2.476

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	591	332	0	0	0	745	8222
normalized size	1	1.00	1.82	1.02	0.00	0.00	0.00	2.30	25.38
time (sec)	N/A	3.529	7.251	0.033	0.000	0.000	0.000	0.819	1.994

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	308	275	0	2435	0	619	5705
normalized size	1	1.00	1.05	0.94	0.00	8.34	0.00	2.12	19.54
time (sec)	N/A	3.600	0.547	0.026	0.000	134.689	0.000	0.782	2.342

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	179	177	0	1085	0	228	717
normalized size	1	1.00	0.89	0.88	0.00	5.37	0.00	1.13	3.55
time (sec)	N/A	0.363	0.328	0.016	0.000	19.819	0.000	0.555	1.723

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	241	294	0	3126	0	717	10964
normalized size	1	1.00	0.86	1.05	0.00	11.12	0.00	2.55	39.02
time (sec)	N/A	1.350	0.814	0.030	0.000	147.577	0.000	0.675	6.875

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	370	349	401	0	0	0	0	19959
normalized size	1	0.97	0.91	1.05	0.00	0.00	0.00	0.00	52.25
time (sec)	N/A	4.134	1.389	0.030	0.000	0.000	0.000	0.000	5.460

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	466	655	0	0	0	1055	33925
normalized size	1	1.00	0.84	1.19	0.00	0.00	0.00	1.91	61.46
time (sec)	N/A	4.244	1.967	0.035	0.000	0.000	0.000	0.905	7.300

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	10915	290	0	6534	0	53	-1
normalized size	1	1.00	27.99	0.74	0.00	16.75	0.00	0.14	-0.00
time (sec)	N/A	2.919	6.397	0.040	0.000	62.645	0.000	2.038	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	7768	224	0	3260	0	27	-1
normalized size	1	1.00	23.98	0.69	0.00	10.06	0.00	0.08	-0.00
time (sec)	N/A	1.517	6.171	0.027	0.000	10.794	0.000	1.832	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	2585	161	0	985	0	0	-1
normalized size	1	1.00	10.77	0.67	0.00	4.10	0.00	0.00	-0.00
time (sec)	N/A	0.318	5.125	0.018	0.000	2.980	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	4644	272	0	2402	0	0	-1
normalized size	1	1.00	15.96	0.93	0.00	8.25	0.00	0.00	-0.00
time (sec)	N/A	0.671	6.322	0.029	0.000	5.057	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	7777	322	0	4095	0	0	-1
normalized size	1	1.00	20.85	0.86	0.00	10.98	0.00	0.00	-0.00
time (sec)	N/A	2.527	6.391	0.038	0.000	23.172	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	10933	503	0	5773	0	0	-1
normalized size	1	1.00	21.35	0.98	0.00	11.28	0.00	0.00	-0.00
time (sec)	N/A	4.943	6.586	0.038	0.000	53.957	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	457	490	0	0	0	857	16951
normalized size	1	1.00	0.99	1.07	0.00	0.00	0.00	1.86	36.85
time (sec)	N/A	5.084	0.966	0.033	0.000	0.000	0.000	1.246	3.413

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	324	279	0	0	0	649	12392
normalized size	1	1.00	0.99	0.85	0.00	0.00	0.00	1.98	37.90
time (sec)	N/A	1.456	0.568	0.021	0.000	0.000	0.000	1.089	4.129

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	333	388	0	0	0	827	28434
normalized size	1	1.00	0.96	1.12	0.00	0.00	0.00	2.39	82.18
time (sec)	N/A	1.744	1.379	0.029	0.000	0.000	0.000	1.024	7.673

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	416	380	555	0	0	0	433	35855
normalized size	1	1.00	0.91	1.33	0.00	0.00	0.00	1.04	85.98
time (sec)	N/A	3.243	1.600	0.043	0.000	0.000	0.000	0.725	6.097

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	595	595	18689	516	0	0	0	104	-1
normalized size	1	1.00	31.41	0.87	0.00	0.00	0.00	0.17	-0.00
time (sec)	N/A	3.285	6.489	0.039	0.000	0.000	0.000	1.869	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	14032	382	0	0	0	58	-1
normalized size	1	1.00	28.58	0.78	0.00	0.00	0.00	0.12	-0.00
time (sec)	N/A	1.802	6.261	0.033	0.000	0.000	0.000	1.975	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	9290	217	0	7721	0	27	-1
normalized size	1	1.00	19.08	0.45	0.00	15.85	0.00	0.06	-0.00
time (sec)	N/A	1.572	6.160	0.027	0.000	57.446	0.000	1.968	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	432	7789	360	0	4059	0	0	-1
normalized size	1	1.66	29.96	1.38	0.00	15.61	0.00	0.00	-0.00
time (sec)	N/A	0.855	6.302	0.036	0.000	29.108	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	9321	511	0	7830	0	0	-1
normalized size	1	1.00	17.82	0.98	0.00	14.97	0.00	0.00	-0.00
time (sec)	N/A	2.617	6.408	0.036	0.000	144.243	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	354	2134	0	3615	0	4637	917
normalized size	1	1.00	1.26	7.59	0.00	12.86	0.00	16.50	3.26
time (sec)	N/A	7.336	0.542	0.099	0.000	16.517	0.000	4.484	1.450

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	276	1223	0	2053	0	4060	776
normalized size	1	1.00	1.21	5.34	0.00	8.97	0.00	17.73	3.39
time (sec)	N/A	1.751	0.387	0.063	0.000	6.333	0.000	4.104	1.311

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	169	1167	0	871	0	591	649
normalized size	1	1.00	0.93	6.41	0.00	4.79	0.00	3.25	3.57
time (sec)	N/A	0.268	0.243	0.051	0.000	2.843	0.000	4.242	1.288

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	212	2099	0	1232	0	3639	669
normalized size	1	1.00	0.88	8.71	0.00	5.11	0.00	15.10	2.78
time (sec)	N/A	1.645	0.432	0.064	0.000	13.247	0.000	3.674	1.298

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	292	2770	0	2799	0	1675	825
normalized size	1	1.00	1.01	9.55	0.00	9.65	0.00	5.78	2.84
time (sec)	N/A	2.362	0.762	0.085	0.000	38.359	0.000	7.120	1.410

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	10606	222	0	2860	0	1710	1024
normalized size	1	1.00	32.63	0.68	0.00	8.80	0.00	5.26	3.15
time (sec)	N/A	5.391	6.307	0.042	0.000	5.754	0.000	6.686	1.300

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	7543	175	0	1430	0	3580	870
normalized size	1	1.00	28.68	0.67	0.00	5.44	0.00	13.61	3.31
time (sec)	N/A	2.135	6.140	0.023	0.000	2.718	0.000	4.734	1.272

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	2266	130	0	759	0	641	989
normalized size	1	1.00	10.30	0.59	0.00	3.45	0.00	2.91	4.50
time (sec)	N/A	0.294	5.418	0.013	0.000	1.255	0.000	5.122	1.268

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	2661	217	0	1998	0	3965	1234
normalized size	1	1.00	10.04	0.82	0.00	7.54	0.00	14.96	4.66
time (sec)	N/A	0.780	4.946	0.027	0.000	1.861	0.000	5.037	1.205

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	743	160	0	290	0	209	383
normalized size	1	1.00	7.74	1.67	0.00	3.02	0.00	2.18	3.99
time (sec)	N/A	0.200	0.496	0.088	0.000	1.087	0.000	0.718	1.500

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	461	377	0	0	0	105	-1
normalized size	1	1.00	0.96	0.79	0.00	0.00	0.00	0.22	-0.00
time (sec)	N/A	1.858	1.868	0.034	0.000	0.000	0.000	2.027	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	355	269	0	0	0	55	-1
normalized size	1	1.00	0.97	0.73	0.00	0.00	0.00	0.15	-0.00
time (sec)	N/A	1.173	1.030	0.026	0.000	0.000	0.000	2.140	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	292	200	0	11094	0	27	-1
normalized size	1	1.00	0.98	0.67	0.00	37.23	0.00	0.09	-0.00
time (sec)	N/A	0.724	0.588	0.022	0.000	44.771	0.000	2.049	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	227	161	0	3395	0	0	-1
normalized size	1	1.00	0.95	0.67	0.00	14.15	0.00	0.00	-0.00
time (sec)	N/A	0.304	0.478	0.017	0.000	10.218	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	229	151	0	4557	0	0	-1
normalized size	1	1.00	0.94	0.62	0.00	18.75	0.00	0.00	-0.00
time (sec)	N/A	0.178	0.406	0.016	0.000	23.398	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	271	197	0	6431	0	0	-1
normalized size	1	1.00	0.97	0.70	0.00	22.97	0.00	0.00	-0.00
time (sec)	N/A	0.602	1.013	0.025	0.000	46.072	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	320	248	0	0	0	0	-1
normalized size	1	1.00	0.94	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.741	0.686	0.029	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	383	350	0	0	0	0	-1
normalized size	1	1.00	0.86	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.432	1.636	0.036	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	350	507	10968	480	0	0	0	75	-1
normalized size	1	1.45	31.34	1.37	0.00	0.00	0.00	0.21	-0.00
time (sec)	N/A	4.328	11.262	0.040	0.000	0.000	0.000	2.254	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	2162	338	0	0	0	0	-1
normalized size	1	1.00	6.01	0.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.267	7.897	0.034	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	2119	252	0	0	0	0	-1
normalized size	1	1.00	6.36	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.658	6.719	0.029	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	2112	246	0	0	0	0	-1
normalized size	1	1.00	6.19	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.774	7.152	0.024	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	462	2158	387	0	0	0	0	-1
normalized size	1	1.36	6.37	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.837	6.774	0.037	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	419	647	2218	541	0	0	0	0	-1
normalized size	1	1.54	5.29	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.566	6.796	0.042	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.650	0.206	0.086	0.000	1.408	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	272	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.939	0.791	0.096	0.000	1.593	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	211	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	0.343	0.075	0.000	1.222	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	183	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.204	0.092	0.000	0.845	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	168	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.357	0.264	0.097	0.000	0.867	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	218	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.502	0.351	0.070	0.000	1.333	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	259	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.663	0.348	0.076	0.000	0.826	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.628	0.494	0.073	0.000	0.834	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.531	0.235	0.099	0.000	0.964	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.314	0.099	0.088	0.000	1.206	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.294	0.035	0.073	0.000	1.049	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	0.205	0.095	0.000	0.769	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.620	0.500	0.096	0.000	0.531	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	44	101	0	120	0	42	-1
normalized size	1	1.10	1.10	2.52	0.00	3.00	0.00	1.05	-0.02
time (sec)	N/A	0.072	0.052	0.059	0.000	1.408	0.000	0.222	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [252] had the largest ratio of [.5000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	20	0.100
2	A	2	1	1.00	20	0.050
3	A	4	3	1.00	18	0.167
4	A	2	1	1.00	17	0.059
5	A	3	2	1.00	20	0.100
6	A	2	1	1.00	20	0.050
7	A	3	2	1.00	20	0.100
8	A	5	5	1.00	20	0.250
9	A	4	4	1.00	20	0.200
10	A	4	4	1.00	18	0.222
11	A	7	7	1.00	20	0.350
12	A	7	7	1.00	20	0.350
13	A	7	7	1.00	20	0.350
14	A	6	6	1.00	20	0.300
15	A	6	5	1.00	20	0.250
16	A	5	5	1.00	20	0.250
17	A	4	4	1.00	17	0.235
18	A	4	4	1.00	20	0.200
19	A	5	5	1.00	20	0.250
20	A	6	5	1.00	20	0.250
21	A	5	4	1.00	20	0.200
22	A	5	4	1.00	18	0.222
23	A	8	7	1.00	20	0.350
24	A	8	8	1.00	20	0.400
25	A	8	7	1.00	20	0.350
26	A	8	8	1.00	20	0.400
27	A	7	5	1.00	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	6	5	1.00	20	0.250
29	A	5	4	1.00	17	0.235
30	A	5	5	1.00	20	0.250
31	A	5	4	1.00	20	0.200
32	A	5	4	1.00	20	0.200
33	A	4	4	1.00	20	0.200
34	A	3	3	1.00	20	0.150
35	A	3	3	1.00	18	0.167
36	A	6	6	1.00	20	0.300
37	A	5	5	1.00	20	0.250
38	A	6	6	1.00	20	0.300
39	A	5	4	1.00	20	0.200
40	A	4	4	1.00	20	0.200
41	A	3	3	1.00	17	0.176
42	A	4	4	1.00	20	0.200
43	A	5	4	1.00	20	0.200
44	A	4	4	1.00	20	0.200
45	A	4	4	1.00	20	0.200
46	A	3	3	1.00	20	0.150
47	A	2	2	1.00	18	0.111
48	A	6	6	1.00	20	0.300
49	A	6	6	1.00	20	0.300
50	A	5	5	1.00	20	0.250
51	A	4	4	1.00	20	0.200
52	A	4	4	1.00	17	0.235
53	A	5	5	1.00	20	0.250
54	A	6	5	1.00	20	0.250
55	A	3	2	1.00	25	0.080
56	A	4	3	1.00	23	0.130
57	A	3	2	1.00	23	0.087
58	A	4	3	1.00	23	0.130
59	A	3	2	1.00	23	0.087
60	A	4	3	1.00	21	0.143
61	A	3	2	1.00	20	0.100
62	A	5	4	1.00	23	0.174
63	A	3	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	4	3	1.00	23	0.130
65	A	3	2	1.00	23	0.087
66	A	4	3	1.00	21	0.143
67	A	3	2	1.00	21	0.095
68	A	4	3	1.00	21	0.143
69	A	3	2	1.00	21	0.095
70	A	2	2	1.00	19	0.105
71	A	3	2	1.00	18	0.111
72	A	4	3	1.00	21	0.143
73	A	3	2	1.00	21	0.095
74	A	4	3	1.00	21	0.143
75	A	4	4	1.00	33	0.121
76	A	4	4	1.00	31	0.129
77	A	3	3	1.00	30	0.100
78	A	4	3	1.00	33	0.091
79	A	3	3	1.00	33	0.091
80	A	4	3	1.00	33	0.091
81	A	4	4	1.00	33	0.121
82	A	3	3	1.00	31	0.097
83	A	4	4	1.00	30	0.133
84	A	4	3	1.00	33	0.091
85	A	5	4	1.00	33	0.121
86	A	4	3	1.00	33	0.091
87	A	3	2	1.00	35	0.057
88	A	3	2	1.00	35	0.057
89	A	3	2	1.00	35	0.057
90	A	3	3	1.00	35	0.086
91	A	3	3	1.00	35	0.086
92	A	2	2	1.00	29	0.069
93	A	5	4	1.00	31	0.129
94	A	5	4	1.00	31	0.129
95	A	3	2	1.00	25	0.080
96	A	2	1	1.00	25	0.040
97	A	3	2	1.00	23	0.087
98	A	2	1	1.00	22	0.045
99	A	3	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
100	A	2	1	1.00	25	0.040
101	A	3	2	1.00	25	0.080
102	A	7	6	1.00	25	0.240
103	A	6	6	1.00	25	0.240
104	A	5	5	1.00	23	0.217
105	A	7	6	1.00	25	0.240
106	A	7	6	1.00	25	0.240
107	A	5	3	1.00	25	0.120
108	A	4	3	1.00	25	0.120
109	A	3	2	1.00	22	0.091
110	A	4	3	1.00	25	0.120
111	A	5	3	1.00	25	0.120
112	A	7	7	1.00	25	0.280
113	A	6	6	1.00	25	0.240
114	A	4	4	1.00	25	0.160
115	A	4	4	1.00	23	0.174
116	A	8	7	1.00	25	0.280
117	A	8	7	1.00	25	0.280
118	A	6	4	1.00	25	0.160
119	A	5	4	1.00	25	0.160
120	A	4	3	1.00	25	0.120
121	A	4	3	1.00	22	0.136
122	A	5	4	1.00	25	0.160
123	A	6	4	1.00	25	0.160
124	A	8	7	1.00	25	0.280
125	A	7	6	1.00	25	0.240
126	A	5	5	1.00	25	0.200
127	A	5	5	1.00	25	0.200
128	A	5	5	1.00	25	0.200
129	A	5	5	1.00	23	0.217
130	A	9	7	1.00	25	0.280
131	A	9	7	1.00	25	0.280
132	A	7	4	1.00	25	0.160
133	A	6	4	1.00	25	0.160
134	A	5	3	1.00	25	0.120
135	A	5	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	5	3	1.00	22	0.136
137	A	4	3	1.00	21	0.143
138	A	5	4	1.00	22	0.182
139	A	5	5	1.00	17	0.294
140	A	6	6	1.00	18	0.333
141	A	5	5	1.00	22	0.227
142	A	6	6	1.00	25	0.240
143	A	5	5	1.00	25	0.200
144	A	5	5	1.00	23	0.217
145	A	7	6	1.00	25	0.240
146	A	7	6	1.00	25	0.240
147	A	7	6	1.00	25	0.240
148	A	5	5	1.00	25	0.200
149	A	6	6	1.00	25	0.240
150	A	7	6	1.00	25	0.240
151	A	6	5	1.00	25	0.200
152	A	5	5	1.00	25	0.200
153	A	4	4	1.00	22	0.182
154	A	4	4	1.00	25	0.160
155	A	5	5	1.00	25	0.200
156	A	7	6	1.00	25	0.240
157	A	6	5	1.00	25	0.200
158	A	6	5	1.00	23	0.217
159	A	8	6	1.00	25	0.240
160	A	8	7	1.00	25	0.280
161	A	8	6	1.00	25	0.240
162	A	8	7	1.00	25	0.280
163	A	7	5	1.00	25	0.200
164	A	6	5	1.00	25	0.200
165	A	5	4	1.00	22	0.182
166	A	5	5	1.00	25	0.200
167	A	5	4	1.00	25	0.160
168	A	6	5	1.00	25	0.200
169	A	5	5	1.00	27	0.185
170	A	4	4	1.00	27	0.148
171	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
172	A	6	5	1.00	27	0.185
173	A	4	4	1.00	27	0.148
174	A	5	5	1.00	27	0.185
175	A	6	5	1.00	27	0.185
176	A	5	4	1.00	27	0.148
177	A	4	4	1.00	27	0.148
178	A	3	3	1.00	24	0.125
179	A	4	4	1.00	27	0.148
180	A	5	4	1.00	27	0.148
181	A	6	5	1.00	25	0.200
182	A	5	5	1.00	25	0.200
183	A	4	4	1.00	25	0.160
184	A	4	4	1.00	23	0.174
185	A	6	5	1.00	25	0.200
186	A	4	4	1.00	25	0.160
187	A	5	5	1.00	25	0.200
188	A	6	5	1.00	25	0.200
189	A	5	4	1.00	25	0.160
190	A	4	4	1.00	25	0.160
191	A	3	3	1.00	22	0.136
192	A	4	4	1.00	25	0.160
193	A	5	4	1.00	25	0.160
194	A	5	5	1.00	25	0.200
195	A	4	4	1.00	25	0.160
196	A	2	2	1.00	23	0.087
197	A	5	5	1.00	25	0.200
198	A	5	5	1.00	25	0.200
199	A	5	5	1.00	25	0.200
200	A	4	4	1.00	25	0.160
201	A	4	4	1.00	22	0.182
202	A	5	5	1.00	25	0.200
203	A	6	5	1.00	25	0.200
204	A	6	3	1.00	31	0.097
205	A	6	3	1.00	31	0.097
206	A	6	3	1.00	31	0.097
207	A	6	3	1.00	31	0.097

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
208	A	6	3	1.00	31	0.097
209	A	6	3	1.00	31	0.097
210	A	6	3	1.00	31	0.097
211	A	6	3	1.00	31	0.097
212	A	6	3	1.00	31	0.097
213	A	6	3	1.00	31	0.097
214	A	6	3	1.00	31	0.097
215	A	6	3	1.00	31	0.097
216	A	6	3	1.00	31	0.097
217	A	6	3	1.00	31	0.097
218	A	6	3	1.00	31	0.097
219	A	6	3	1.00	31	0.097
220	A	2	1	1.00	27	0.037
221	A	2	1	1.00	27	0.037
222	A	2	1	1.00	25	0.040
223	A	3	2	1.00	27	0.074
224	A	4	3	0.91	27	0.111
225	A	6	3	1.00	29	0.103
226	A	6	3	1.00	29	0.103
227	A	6	3	1.00	29	0.103
228	A	6	3	1.00	29	0.103
229	A	6	5	1.00	22	0.227
230	A	6	5	1.00	22	0.227
231	A	6	5	1.00	22	0.227
232	A	6	5	1.00	22	0.227
233	A	6	6	1.00	20	0.300
234	A	6	5	1.00	22	0.227
235	A	6	5	1.00	22	0.227
236	A	6	5	1.00	22	0.227
237	A	12	8	1.00	22	0.364
238	A	12	8	1.00	22	0.364
239	A	12	8	1.00	22	0.364
240	A	12	8	1.00	22	0.364
241	A	12	8	1.00	19	0.421
242	A	12	8	1.00	22	0.364
243	A	12	8	1.00	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
244	A	7	6	1.00	22	0.273
245	A	7	6	1.00	22	0.273
246	A	7	6	0.99	22	0.273
247	A	7	6	0.99	22	0.273
248	A	7	6	1.00	20	0.300
249	A	8	6	1.00	22	0.273
250	A	8	6	1.00	22	0.273
251	A	8	6	1.00	22	0.273
252	A	24	11	1.00	22	0.500
253	A	23	10	1.00	22	0.454
254	A	23	10	1.00	22	0.454
255	A	23	10	1.00	22	0.454
256	A	22	9	1.00	19	0.474
257	A	22	9	1.00	22	0.409
258	A	22	9	1.00	22	0.409
259	A	4	4	1.00	20	0.200
260	A	4	4	1.00	22	0.182
261	A	6	6	1.00	22	0.273
262	A	3	3	1.00	24	0.125
263	A	7	7	1.00	20	0.350
264	A	4	4	1.00	22	0.182
265	A	4	4	1.00	22	0.182
266	A	4	4	1.00	24	0.167
267	A	6	6	1.00	37	0.162
268	A	4	3	1.00	35	0.086
269	A	5	5	1.00	34	0.147
270	A	6	6	1.00	37	0.162
271	A	5	5	1.00	37	0.135
272	A	6	6	1.00	37	0.162
273	A	3	2	1.00	25	0.080
274	A	2	1	1.00	25	0.040
275	A	3	2	1.00	23	0.087
276	A	2	1	1.00	22	0.045
277	A	3	2	1.00	25	0.080
278	A	2	1	1.00	25	0.040
279	A	3	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	4	3	1.00	25	0.120
281	A	4	3	1.00	25	0.120
282	A	4	3	1.00	25	0.120
283	A	3	3	1.00	22	0.136
284	A	3	3	0.97	25	0.120
285	A	4	3	1.00	25	0.120
286	A	4	3	1.00	25	0.120
287	A	4	3	1.00	25	0.120
288	A	5	4	1.00	25	0.160
289	A	5	4	1.00	25	0.160
290	A	4	4	1.00	25	0.160
291	A	3	3	1.00	22	0.136
292	A	4	4	0.98	25	0.160
293	A	5	3	1.00	25	0.120
294	A	5	4	1.00	25	0.160
295	A	7	6	1.00	27	0.222
296	A	7	6	1.00	27	0.222
297	A	7	6	1.00	27	0.222
298	A	7	6	1.00	27	0.222
299	A	7	7	1.00	25	0.280
300	A	7	6	1.00	27	0.222
301	A	7	6	1.00	27	0.222
302	A	7	6	1.00	27	0.222
303	A	6	3	1.00	27	0.111
304	A	6	3	1.00	27	0.111
305	A	6	3	1.00	27	0.111
306	A	6	3	1.00	27	0.111
307	A	6	3	1.00	24	0.125
308	A	6	3	1.00	27	0.111
309	A	6	3	1.00	27	0.111
310	A	19	12	1.00	31	0.387
311	A	8	7	1.00	29	0.241
312	A	7	6	1.00	29	0.207
313	A	7	6	1.00	27	0.222
314	A	9	6	1.00	29	0.207
315	A	21	8	1.00	29	0.276

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
316	A	17	9	1.46	29	0.310
317	A	13	8	1.42	29	0.276
318	A	7	6	1.23	26	0.231
319	A	8	7	1.00	29	0.241
320	A	7	6	1.00	29	0.207
321	A	13	9	1.00	29	0.310
322	A	9	7	1.00	29	0.241
323	A	8	6	1.00	29	0.207
324	A	8	7	1.00	27	0.259
325	A	14	8	1.00	29	0.276
326	A	24	9	1.00	29	0.310
327	A	19	9	1.89	29	0.310
328	A	12	7	1.41	26	0.269
329	A	13	10	1.00	29	0.345
330	A	13	9	1.00	29	0.310
331	A	15	9	1.00	29	0.310
332	A	7	6	1.00	29	0.207
333	A	6	5	1.00	29	0.172
334	A	3	3	1.00	27	0.111
335	A	7	4	1.00	29	0.138
336	A	10	5	1.00	29	0.172
337	A	4	4	1.00	29	0.138
338	A	3	3	1.00	29	0.103
339	A	3	3	1.00	26	0.115
340	A	6	6	1.00	29	0.207
341	A	7	7	1.00	29	0.241
342	A	7	6	1.00	29	0.207
343	A	5	5	1.00	29	0.172
344	A	5	5	1.00	29	0.172
345	A	5	5	1.00	27	0.185
346	A	11	6	1.00	29	0.207
347	A	15	7	1.00	29	0.241
348	A	10	8	1.26	29	0.276
349	A	8	7	1.19	29	0.241
350	A	8	7	1.19	29	0.241
351	A	8	7	1.19	29	0.241

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	8	7	1.19	26	0.269
353	A	15	10	1.38	29	0.345
354	A	7	5	1.00	29	0.172
355	A	7	5	1.00	29	0.172
356	A	6	5	1.00	29	0.172
357	A	5	4	1.00	27	0.148
358	A	8	6	1.00	29	0.207
359	A	10	7	0.97	29	0.241
360	A	13	7	1.00	29	0.241
361	A	10	7	1.00	29	0.241
362	A	9	6	1.00	29	0.207
363	A	11	6	1.00	26	0.231
364	A	8	5	1.00	29	0.172
365	A	12	7	1.00	29	0.241
366	A	15	7	1.00	29	0.241
367	A	7	5	1.00	29	0.172
368	A	6	5	1.00	27	0.185
369	A	8	6	1.00	29	0.207
370	A	10	7	1.00	29	0.241
371	A	17	9	1.00	29	0.310
372	A	16	8	1.00	29	0.276
373	A	13	7	1.00	26	0.269
374	A	16	8	1.66	29	0.276
375	A	19	10	1.00	29	0.345
376	A	7	5	1.00	29	0.172
377	A	6	5	1.00	29	0.172
378	A	5	4	1.00	27	0.148
379	A	8	6	1.00	29	0.207
380	A	8	6	1.00	29	0.207
381	A	9	6	1.00	29	0.207
382	A	8	5	1.00	29	0.172
383	A	9	5	1.00	26	0.192
384	A	8	5	1.00	29	0.172
385	A	8	6	1.00	25	0.240
386	A	17	7	1.00	29	0.241
387	A	13	7	1.00	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
388	A	10	6	1.00	29	0.207
389	A	6	3	1.00	29	0.103
390	A	5	3	1.00	26	0.115
391	A	9	5	1.00	29	0.172
392	A	11	6	1.00	29	0.207
393	A	14	6	1.00	29	0.207
394	A	14	7	1.45	29	0.241
395	A	8	5	1.00	29	0.172
396	A	8	5	1.00	29	0.172
397	A	8	5	1.00	26	0.192
398	A	12	8	1.36	29	0.276
399	A	15	8	1.54	29	0.276
400	A	6	3	1.00	29	0.103
401	A	5	3	1.00	27	0.111
402	A	5	3	1.00	27	0.111
403	A	5	3	1.00	27	0.111
404	A	5	3	1.00	25	0.120
405	A	8	5	1.00	27	0.185
406	A	9	5	1.00	27	0.185
407	A	12	8	1.00	27	0.296
408	A	10	6	1.00	27	0.222
409	A	6	3	1.00	27	0.111
410	A	5	3	1.00	24	0.125
411	A	10	6	1.00	27	0.222
412	A	12	6	1.00	27	0.222
413	A	5	5	1.10	28	0.179

Chapter 3

Listing of integrals

3.1 $\int x^3 \left(d+ex^2\right) \left(a+cx^4\right)^5 \, dx$

Optimal. Leaf size=149

$$\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{9}a^2c^3ex^{18}+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+$$

```
[Out] 1/4*a^5*d*x^4+1/6*a^5*e*x^6+5/8*a^4*c*d*x^8+1/2*a^4*c*e*x^10+5/6*a^3*c^2*d*
x^12+5/7*a^3*c^2*e*x^14+5/8*a^2*c^3*d*x^16+5/9*a^2*c^3*e*x^18+1/4*a*c^4*d*x
^20+5/22*a*c^4*e*x^22+1/24*c^5*d*x^24+1/26*c^5*e*x^26
```

Rubi [A] time = 0.22, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1252, 766}

$$\frac{5}{8}a^2c^3dx^{16}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{9}a^2c^3ex^{18}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a
^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3
*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^
5*e*x^26)/26
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int x^3\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx = \frac{1}{2}\,\text{Subst}\left(\int x(d+ex)\left(a+cx^2\right)^5\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(a^5dx + a^5ex^2 + 5a^4cdx^3 + 5a^4cex^4 + 10a^3c^2dx^5 + 10a^3c^2ex^6 + 10a^2c\right.\right.$$
$$= \frac{1}{4}a^5dx^4 + \frac{1}{6}a^5ex^6 + \frac{5}{8}a^4cdx^8 + \frac{1}{2}a^4cex^{10} + \frac{5}{6}a^3c^2dx^{12} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{8}a^2c^3dx^{16}$$

Mathematica [A] time = 0.02, size = 149, normalized size = 1.00

$$\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{9}a^2c^3ex^{18}+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+\frac{1}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a
^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3
*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^
5*e*x^26)/26
```

fricas [A] time = 0.55, size = 125, normalized size = 0.84

$$\frac{1}{26}x^{26}ec^5+\frac{1}{24}x^{24}dc^5+\frac{5}{22}x^{22}ec^4a+\frac{1}{4}x^{20}dc^4a+\frac{5}{9}x^{18}ec^3a^2+\frac{5}{8}x^{16}dc^3a^2+\frac{5}{7}x^{14}ec^2a^3+\frac{5}{6}x^{12}dc^2a^3+\frac{1}{2}x^{10}eca^4+\frac{5}{8}x^8dca^4+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/26*x^26*e*c^5 + 1/24*x^24*d*c^5 + 5/22*x^22*e*c^4*a + 1/4*x^20*d*c^4*a +
5/9*x^18*e*c^3*a^2 + 5/8*x^16*d*c^3*a^2 + 5/7*x^14*e*c^2*a^3 + 5/6*x^12*d*c
^2*a^3 + 1/2*x^10*e*c*a^4 + 5/8*x^8*d*c*a^4 + 1/6*x^6*e*a^5 + 1/4*x^4*d*a^5
```

giac [A] time = 0.26, size = 131, normalized size = 0.88

$$\frac{1}{26}c^5x^{26}e+\frac{1}{24}c^5dx^{24}+\frac{5}{22}ac^4x^{22}e+\frac{1}{4}ac^4dx^{20}+\frac{5}{9}a^2c^3x^{18}e+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{7}a^3c^2x^{14}e+\frac{5}{6}a^3c^2dx^{12}+\frac{1}{2}a^4cx^{10}e+\frac{5}{8}a^4c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/26*c^5*x^26*e + 1/24*c^5*d*x^24 + 5/22*a*c^4*x^22*e + 1/4*a*c^4*d*x^20 +
5/9*a^2*c^3*x^18*e + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*x^14*e + 5/6*a^3*c^2*
d*x^12 + 1/2*a^4*c*x^10*e + 5/8*a^4*c*d*x^8 + 1/6*a^5*x^6*e + 1/4*a^5*d*x^4
```

maple [A] time = 0.01, size = 126, normalized size = 0.85

$$\frac{1}{26}c^5e\,x^{26}+\frac{1}{24}c^5d\,x^{24}+\frac{5}{22}a\,c^4e\,x^{22}+\frac{1}{4}a\,c^4d\,x^{20}+\frac{5}{9}a^2c^3e\,x^{18}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{8}a^4c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] 1/4*a^5*d*x^4+1/6*a^5*e*x^6+5/8*a^4*c*d*x^8+1/2*a^4*c*e*x^10+5/6*a^3*c^2*d*
x^12+5/7*a^3*c^2*e*x^14+5/8*a^2*c^3*d*x^16+5/9*a^2*c^3*e*x^18+1/4*a*c^4*d*x
^20+5/22*a*c^4*e*x^22+1/24*c^5*d*x^24+1/26*c^5*e*x^26
```

maxima [A] time = 0.43, size = 125, normalized size = 0.84

$$\frac{1}{26}c^5ex^{26}+\frac{1}{24}c^5dx^{24}+\frac{5}{22}ac^4ex^{22}+\frac{1}{4}ac^4dx^{20}+\frac{5}{9}a^2c^3ex^{18}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{6}a^3c^2dx^{12}+\frac{1}{2}a^4cex^{10}+\frac{5}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/26*c^5*e*x^26 + 1/24*c^5*d*x^24 + 5/22*a*c^4*e*x^22 + 1/4*a*c^4*d*x^20 +
5/9*a^2*c^3*e*x^18 + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*e*x^14 + 5/6*a^3*c^2*
d*x^12 + 1/2*a^4*c*e*x^10 + 5/8*a^4*c*d*x^8 + 1/6*a^5*e*x^6 + 1/4*a^5*d*x^4
```

mupad [B] time = 0.31, size = 125, normalized size = 0.84

$$\frac{e\,a^5\,x^6}{6}+\frac{d\,a^5\,x^4}{4}+\frac{e\,a^4\,c\,x^{10}}{2}+\frac{5\,d\,a^4\,c\,x^8}{8}+\frac{5\,e\,a^3\,c^2\,x^{14}}{7}+\frac{5\,d\,a^3\,c^2\,x^{12}}{6}+\frac{5\,e\,a^2\,c^3\,x^{18}}{9}+\frac{5\,d\,a^2\,c^3\,x^{16}}{8}+\frac{5\,e\,a\,c^4\,x^{22}}{22}+\frac{d\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (c^5*d*x^24)/24 + (c^5*e*x^26)/26 + (5*a^3*
c^2*d*x^12)/6 + (5*a^2*c^3*d*x^16)/8 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*e*
x^18)/9 + (5*a^4*c*d*x^8)/8 + (a*c^4*d*x^20)/4 + (a^4*c*e*x^10)/2 + (5*a*c^
4*e*x^22)/22
```

sympy [A] time = 0.15, size = 151, normalized size = 1.01

$$\frac{a^5dx^4}{4}+\frac{a^5ex^6}{6}+\frac{5a^4cdx^8}{8}+\frac{a^4cex^{10}}{2}+\frac{5a^3c^2dx^{12}}{6}+\frac{5a^3c^2ex^{14}}{7}+\frac{5a^2c^3dx^{16}}{8}+\frac{5a^2c^3ex^{18}}{9}+\frac{ac^4dx^{20}}{4}+\frac{5ac^4ex^{22}}{22}+\frac{c^5dx^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**4/4 + a**5*e*x**6/6 + 5*a**4*c*d*x**8/8 + a**4*c*e*x**10/2 + 5*a*
*3*c**2*d*x**12/6 + 5*a**3*c**2*e*x**14/7 + 5*a**2*c**3*d*x**16/8 + 5*a**2*
c**3*e*x**18/9 + a*c**4*d*x**20/4 + 5*a*c**4*e*x**22/22 + c**5*d*x**24/24 +
 c**5*e*x**26/26
```

3.2 $\int x^2 \left(d+ex^2\right) \left(a+cx^4\right)^5 \, dx$

Optimal. Leaf size=149

$$\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+$$

```
[Out] 1/3*a^5*d*x^3+1/5*a^5*e*x^5+5/7*a^4*c*d*x^7+5/9*a^4*c*e*x^9+10/11*a^3*c^2*d
*x^11+10/13*a^3*c^2*e*x^13+2/3*a^2*c^3*d*x^15+10/17*a^2*c^3*e*x^17+5/19*a*c
^4*d*x^19+5/21*a*c^4*e*x^21+1/23*c^5*d*x^23+1/25*c^5*e*x^25
```

Rubi [A] time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1262}

$$\frac{2}{3}a^2c^3dx^{15}+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{17}a^2c^3ex^{17}+\frac{10}{13}a^3c^2ex^{13}+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (10
*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10*a
^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x^23
)/23 + (c^5*e*x^25)/25
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(d+ex^2\right) \left(a+cx^4\right)^5 \, dx &= \int \left(a^5dx^2+a^5ex^4+5a^4cdx^6+5a^4cex^8+10a^3c^2dx^{10}+10a^3c^2ex^{12}+10a^2c^3dx^{14}\right. \\ &= \frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15} \end{aligned}$$

Mathematica [A] time = 0.00, size = 149, normalized size = 1.00

$$\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (10
*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10*a
^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x^23
)/23 + (c^5*e*x^25)/25
```

fricas [A] time = 0.50, size = 125, normalized size = 0.84

$$\frac{1}{25}x^{25}ec^5+\frac{1}{23}x^{23}dc^5+\frac{5}{21}x^{21}ec^4a+\frac{5}{19}x^{19}dc^4a+\frac{10}{17}x^{17}ec^3a^2+\frac{2}{3}x^{15}dc^3a^2+\frac{10}{13}x^{13}ec^2a^3+\frac{10}{11}x^{11}dc^2a^3+\frac{5}{9}x^9eca^4+\frac{5}{7}x^7dc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*e*c^5 + 1/23*x^23*d*c^5 + 5/21*x^21*e*c^4*a + 5/19*x^19*d*c^4*a +
 10/17*x^17*e*c^3*a^2 + 2/3*x^15*d*c^3*a^2 + 10/13*x^13*e*c^2*a^3 + 10/11*x
^11*d*c^2*a^3 + 5/9*x^9*e*c*a^4 + 5/7*x^7*d*c*a^4 + 1/5*x^5*e*a^5 + 1/3*x^3
*d*a^5
```

giac [A] time = 0.19, size = 131, normalized size = 0.88

$$\frac{1}{25}c^5x^{25}e+\frac{1}{23}c^5dx^{23}+\frac{5}{21}ac^4x^{21}e+\frac{5}{19}ac^4dx^{19}+\frac{10}{17}a^2c^3x^{17}e+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{13}a^3c^2x^{13}e+\frac{10}{11}a^3c^2dx^{11}+\frac{5}{9}a^4cx^9e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/25*c^5*x^25*e + 1/23*c^5*d*x^23 + 5/21*a*c^4*x^21*e + 5/19*a*c^4*d*x^19 +
 10/17*a^2*c^3*x^17*e + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*x^13*e + 10/11*a
^3*c^2*d*x^11 + 5/9*a^4*c*x^9*e + 5/7*a^4*c*d*x^7 + 1/5*a^5*x^5*e + 1/3*a^5
*d*x^3
```

maple [A] time = 0.00, size = 126, normalized size = 0.85

$$\frac{1}{25}c^5e\,x^{25}+\frac{1}{23}c^5d\,x^{23}+\frac{5}{21}a\,c^4e\,x^{21}+\frac{5}{19}a\,c^4d\,x^{19}+\frac{10}{17}a^2c^3e\,x^{17}+\frac{2}{3}a^2c^3d\,x^{15}+\frac{10}{13}a^3c^2e\,x^{13}+\frac{10}{11}a^3c^2d\,x^{11}+\frac{5}{9}a^4ce\,x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] 1/3*a^5*d*x^3+1/5*a^5*e*x^5+5/7*a^4*c*d*x^7+5/9*a^4*c*e*x^9+10/11*a^3*c^2*d
*x^11+10/13*a^3*c^2*e*x^13+2/3*a^2*c^3*d*x^15+10/17*a^2*c^3*e*x^17+5/19*a*c
^4*d*x^19+5/21*a*c^4*e*x^21+1/23*c^5*d*x^23+1/25*c^5*e*x^25
```

maxima [A] time = 0.46, size = 125, normalized size = 0.84

$$\frac{1}{25}c^5ex^{25}+\frac{1}{23}c^5dx^{23}+\frac{5}{21}ac^4ex^{21}+\frac{5}{19}ac^4dx^{19}+\frac{10}{17}a^2c^3ex^{17}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{13}a^3c^2ex^{13}+\frac{10}{11}a^3c^2dx^{11}+\frac{5}{9}a^4cex^9+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/25*c^5*e*x^25 + 1/23*c^5*d*x^23 + 5/21*a*c^4*e*x^21 + 5/19*a*c^4*d*x^19 +
 10/17*a^2*c^3*e*x^17 + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*e*x^13 + 10/11*a
^3*c^2*d*x^11 + 5/9*a^4*c*e*x^9 + 5/7*a^4*c*d*x^7 + 1/5*a^5*e*x^5 + 1/3*a^5
*d*x^3
```

mupad [B] time = 0.07, size = 125, normalized size = 0.84

$$\frac{e\,a^5\,x^5}{5}+\frac{d\,a^5\,x^3}{3}+\frac{5\,e\,a^4\,c\,x^9}{9}+\frac{5\,d\,a^4\,c\,x^7}{7}+\frac{10\,e\,a^3\,c^2\,x^{13}}{13}+\frac{10\,d\,a^3\,c^2\,x^{11}}{11}+\frac{10\,e\,a^2\,c^3\,x^{17}}{17}+\frac{2\,d\,a^2\,c^3\,x^{15}}{3}+\frac{5\,e\,a\,c^4\,x^{21}}{21}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (c^5*d*x^23)/23 + (c^5*e*x^25)/25 + (10*a^3
*c^2*d*x^11)/11 + (2*a^2*c^3*d*x^15)/3 + (10*a^3*c^2*e*x^13)/13 + (10*a^2*c
^3*e*x^17)/17 + (5*a^4*c*d*x^7)/7 + (5*a*c^4*d*x^19)/19 + (5*a^4*c*e*x^9)/9
 + (5*a*c^4*e*x^21)/21
```

sympy [A] time = 0.09, size = 155, normalized size = 1.04

$$\frac{a^5dx^3}{3}+\frac{a^5ex^5}{5}+\frac{5a^4cdx^7}{7}+\frac{5a^4cex^9}{9}+\frac{10a^3c^2dx^{11}}{11}+\frac{10a^3c^2ex^{13}}{13}+\frac{2a^2c^3dx^{15}}{3}+\frac{10a^2c^3ex^{17}}{17}+\frac{5ac^4dx^{19}}{19}+\frac{5ac^4ex^{21}}{21}+\frac{c^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**3/3 + a**5*e*x**5/5 + 5*a**4*c*d*x**7/7 + 5*a**4*c*e*x**9/9 + 10*
a**3*c**2*d*x**11/11 + 10*a**3*c**2*e*x**13/13 + 2*a**2*c**3*d*x**15/3 + 10
*a**2*c**3*e*x**17/17 + 5*a*c**4*d*x**19/19 + 5*a*c**4*e*x**21/21 + c**5*d*
x**23/23 + c**5*e*x**25/25
```

3.3 $\int x \left(d+ex^2\right) \left(a+cx^4\right)^5 \, dx$

Optimal. Leaf size=89

$$\frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{22}c^5dx^{22}$$

```
[Out] 1/2*a^5*d*x^2+5/6*a^4*c*d*x^6+a^3*c^2*d*x^10+5/7*a^2*c^3*d*x^14+5/18*a*c^4*
d*x^18+1/22*c^5*d*x^22+1/24*e*(c*x^4+a)^6/c
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1248, 641, 194}

$$\frac{5}{7}a^2c^3dx^{14}+a^3c^2dx^{10}+\frac{5}{6}a^4cdx^6+\frac{1}{2}a^5dx^2+\frac{5}{18}ac^4dx^{18}+\frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{22}c^5dx^{22}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^2)/2 + (5*a^4*c*d*x^6)/6 + a^3*c^2*d*x^10 + (5*a^2*c^3*d*x^14)/7 +
 (5*a*c^4*d*x^18)/18 + (c^5*d*x^22)/22 + (e*(a + c*x^4)^6)/(24*c)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x \left(d+ex^2\right) \left(a+cx^4\right)^5 \, dx &= \frac{1}{2}\text{Subst}\left(\int (d+ex) \left(a+cx^2\right)^5 \, dx,x,x^2\right)\\ &= \frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{2}d\,\text{Subst}\left(\int \left(a+cx^2\right)^5 \, dx,x,x^2\right)\\ &= \frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{2}d\,\text{Subst}\left(\int \left(a^5+5a^4cx^2+10a^3c^2x^4+10a^2c^3x^6+5ac^4x^8+c^5\right.\right.\\ &= \frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{1}{22}c^5dx^{22}+\frac{e\left(a+c\right.}{24c}\end{aligned}$$

Mathematica [A] time = 0.00, size = 146, normalized size = 1.64

$$\frac{1}{2}a^5dx^2+\frac{1}{4}a^5ex^4+\frac{5}{6}a^4cdx^6+\frac{5}{8}a^4cex^8+a^3c^2dx^{10}+\frac{5}{6}a^3c^2ex^{12}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{8}a^2c^3ex^{16}+\frac{5}{18}ac^4dx^{18}+\frac{1}{4}ac^4ex^{20}+\frac{1}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (5*a^4*c*d*x^6)/6 + (5*a^4*c*e*x^8)/8 + a^3
*c^2*d*x^10 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*d*x^14)/7 + (5*a^2*c^3*e*x^
16)/8 + (5*a*c^4*d*x^18)/18 + (a*c^4*e*x^20)/4 + (c^5*d*x^22)/22 + (c^5*e*x
^24)/24
```

fricas [A] time = 0.43, size = 124, normalized size = 1.39

$$\frac{1}{24}x^{24}ec^5+\frac{1}{22}x^{22}dc^5+\frac{1}{4}x^{20}ec^4a+\frac{5}{18}x^{18}dc^4a+\frac{5}{8}x^{16}ec^3a^2+\frac{5}{7}x^{14}dc^3a^2+\frac{5}{6}x^{12}ec^2a^3+x^{10}dc^2a^3+\frac{5}{8}x^8eca^4+\frac{5}{6}x^6dca^4+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*e*c^5 + 1/22*x^22*d*c^5 + 1/4*x^20*e*c^4*a + 5/18*x^18*d*c^4*a +
5/8*x^16*e*c^3*a^2 + 5/7*x^14*d*c^3*a^2 + 5/6*x^12*e*c^2*a^3 + x^10*d*c^2*a
^3 + 5/8*x^8*e*c*a^4 + 5/6*x^6*d*c*a^4 + 1/4*x^4*e*a^5 + 1/2*x^2*d*a^5
```

giac [A] time = 0.18, size = 130, normalized size = 1.46

$$\frac{1}{24}c^5x^{24}e+\frac{1}{22}c^5dx^{22}+\frac{1}{4}ac^4x^{20}e+\frac{5}{18}ac^4dx^{18}+\frac{5}{8}a^2c^3x^{16}e+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{6}a^3c^2x^{12}e+a^3c^2dx^{10}+\frac{5}{8}a^4cx^8e+\frac{5}{6}a^4cdx^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*c^5*x^24*e + 1/22*c^5*d*x^22 + 1/4*a*c^4*x^20*e + 5/18*a*c^4*d*x^18 +
5/8*a^2*c^3*x^16*e + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*x^12*e + a^3*c^2*d*x^
10 + 5/8*a^4*c*x^8*e + 5/6*a^4*c*d*x^6 + 1/4*a^5*x^4*e + 1/2*a^5*d*x^2
```

maple [A] time = 0.00, size = 125, normalized size = 1.40

$$\frac{1}{24}c^5e\,x^{24}+\frac{1}{22}c^5d\,x^{22}+\frac{1}{4}a\,c^4e\,x^{20}+\frac{5}{18}a\,c^4d\,x^{18}+\frac{5}{8}a^2c^3e\,x^{16}+\frac{5}{7}a^2c^3d\,x^{14}+\frac{5}{6}a^3c^2e\,x^{12}+a^3c^2d\,x^{10}+\frac{5}{8}a^4ce\,x^8+\frac{5}{6}a^4cd\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] 1/24*c^5*e*x^24+1/22*c^5*d*x^22+1/4*a*c^4*e*x^20+5/18*a*c^4*d*x^18+5/8*a^2*
c^3*e*x^16+5/7*a^2*c^3*d*x^14+5/6*a^3*c^2*e*x^12+a^3*c^2*d*x^10+5/8*a^4*c*e
*x^8+5/6*a^4*c*d*x^6+1/4*a^5*e*x^4+1/2*a^5*d*x^2
```

maxima [A] time = 0.44, size = 124, normalized size = 1.39

$$\frac{1}{24}c^5ex^{24}+\frac{1}{22}c^5dx^{22}+\frac{1}{4}ac^4ex^{20}+\frac{5}{18}ac^4dx^{18}+\frac{5}{8}a^2c^3ex^{16}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{6}a^3c^2ex^{12}+a^3c^2dx^{10}+\frac{5}{8}a^4cex^8+\frac{5}{6}a^4cdx^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*c^5*e*x^24 + 1/22*c^5*d*x^22 + 1/4*a*c^4*e*x^20 + 5/18*a*c^4*d*x^18 +
5/8*a^2*c^3*e*x^16 + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*e*x^12 + a^3*c^2*d*x^
10 + 5/8*a^4*c*e*x^8 + 5/6*a^4*c*d*x^6 + 1/4*a^5*e*x^4 + 1/2*a^5*d*x^2
```

mupad [B] time = 0.07, size = 124, normalized size = 1.39

$$\frac{e\,a^5\,x^4}{4}+\frac{d\,a^5\,x^2}{2}+\frac{5\,e\,a^4\,c\,x^8}{8}+\frac{5\,d\,a^4\,c\,x^6}{6}+\frac{5\,e\,a^3\,c^2\,x^{12}}{6}+d\,a^3\,c^2\,x^{10}+\frac{5\,e\,a^2\,c^3\,x^{16}}{8}+\frac{5\,d\,a^2\,c^3\,x^{14}}{7}+\frac{e\,a\,c^4\,x^{20}}{4}+\frac{5\,d\,a\,c^4\,x^1}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (c^5*d*x^22)/22 + (c^5*e*x^24)/24 + a^3*c^2
*d*x^10 + (5*a^2*c^3*d*x^14)/7 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*e*x^16)/
8 + (5*a^4*c*d*x^6)/6 + (5*a*c^4*d*x^18)/18 + (5*a^4*c*e*x^8)/8 + (a*c^4*e*
x^20)/4
```

sympy [A] time = 0.09, size = 150, normalized size = 1.69

$$\frac{a^5dx^2}{2}+\frac{a^5ex^4}{4}+\frac{5a^4cdx^6}{6}+\frac{5a^4cex^8}{8}+a^3c^2dx^{10}+\frac{5a^3c^2ex^{12}}{6}+\frac{5a^2c^3dx^{14}}{7}+\frac{5a^2c^3ex^{16}}{8}+\frac{5ac^4dx^{18}}{18}+\frac{ac^4ex^{20}}{4}+\frac{c^5dx^{22}}{22}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**2/2 + a**5*e*x**4/4 + 5*a**4*c*d*x**6/6 + 5*a**4*c*e*x**8/8 + a**
3*c**2*d*x**10 + 5*a**3*c**2*e*x**12/6 + 5*a**2*c**3*d*x**14/7 + 5*a**2*c**
3*e*x**16/8 + 5*a*c**4*d*x**18/18 + a*c**4*e*x**20/4 + c**5*d*x**22/22 + c*
*5*e*x**24/24
```

3.4 $\int \left(d+ex^2\right)\left(a+cx^4\right)^5 \, dx$

Optimal. Leaf size=141

$$a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5$$

```
[Out] a^5*d*x+1/3*a^5*e*x^3+a^4*c*d*x^5+5/7*a^4*c*e*x^7+10/9*a^3*c^2*d*x^9+10/11*
a^3*c^2*e*x^11+10/13*a^2*c^3*d*x^13+2/3*a^2*c^3*e*x^15+5/17*a*c^4*d*x^17+5/
19*a*c^4*e*x^19+1/21*c^5*d*x^21+1/23*c^5*e*x^23
```

Rubi [A] time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1154}

$$\frac{10}{13}a^2c^3dx^{13}+\frac{10}{9}a^3c^2dx^9+\frac{2}{3}a^2c^3ex^{15}+\frac{10}{11}a^3c^2ex^{11}+a^4cdx^5+\frac{5}{7}a^4cex^7+a^5dx+\frac{1}{3}a^5ex^3+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*x
^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^15
)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5*e*
x^23)/23
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)\left(a+cx^4\right)^5 \, dx &= \int \left(a^5d+a^5ex^2+5a^4cdx^4+5a^4cex^6+10a^3c^2dx^8+10a^3c^2ex^{10}+10a^2c^3dx^{12}+10a\right. \\ &= a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2\end{aligned}$$

Mathematica [A] time = 0.00, size = 141, normalized size = 1.00

$$a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*x
^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^15
)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5*e*
x^23)/23
```

fricas [A] time = 0.62, size = 121, normalized size = 0.86

$$\frac{1}{23}x^{23}ec^5+\frac{1}{21}x^{21}dc^5+\frac{5}{19}x^{19}ec^4a+\frac{5}{17}x^{17}dc^4a+\frac{2}{3}x^{15}ec^3a^2+\frac{10}{13}x^{13}dc^3a^2+\frac{10}{11}x^{11}ec^2a^3+\frac{10}{9}x^9dc^2a^3+\frac{5}{7}x^7eca^4+x^5dca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*e*c^5 + 1/21*x^21*d*c^5 + 5/19*x^19*e*c^4*a + 5/17*x^17*d*c^4*a +
 2/3*x^15*e*c^3*a^2 + 10/13*x^13*d*c^3*a^2 + 10/11*x^11*e*c^2*a^3 + 10/9*x^
9*d*c^2*a^3 + 5/7*x^7*e*c*a^4 + x^5*d*c*a^4 + 1/3*x^3*e*a^5 + x*d*a^5
```

giac [A] time = 0.22, size = 127, normalized size = 0.90

$$\frac{1}{23}c^5x^{23}e+\frac{1}{21}c^5dx^{21}+\frac{5}{19}ac^4x^{19}e+\frac{5}{17}ac^4dx^{17}+\frac{2}{3}a^2c^3x^{15}e+\frac{10}{13}a^2c^3dx^{13}+\frac{10}{11}a^3c^2x^{11}e+\frac{10}{9}a^3c^2dx^9+\frac{5}{7}a^4cx^7e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/23*c^5*x^23*e + 1/21*c^5*d*x^21 + 5/19*a*c^4*x^19*e + 5/17*a*c^4*d*x^17 +
 2/3*a^2*c^3*x^15*e + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*x^11*e + 10/9*a^
3*c^2*d*x^9 + 5/7*a^4*c*x^7*e + a^4*c*d*x^5 + 1/3*a^5*x^3*e + a^5*d*x
```

maple [A] time = 0.00, size = 122, normalized size = 0.87

$$\frac{1}{23}c^5e\,x^{23}+\frac{1}{21}c^5d\,x^{21}+\frac{5}{19}a\,c^4e\,x^{19}+\frac{5}{17}a\,c^4d\,x^{17}+\frac{2}{3}a^2c^3e\,x^{15}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{9}a^3c^2d\,x^9+\frac{5}{7}a^4ce\,x^7+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] a^5*d*x+1/3*a^5*e*x^3+a^4*c*d*x^5+5/7*a^4*c*e*x^7+10/9*a^3*c^2*d*x^9+10/11*
a^3*c^2*e*x^11+10/13*a^2*c^3*d*x^13+2/3*a^2*c^3*e*x^15+5/17*a*c^4*d*x^17+5/
19*a*c^4*e*x^19+1/21*c^5*d*x^21+1/23*c^5*e*x^23
```

maxima [A] time = 0.46, size = 121, normalized size = 0.86

$$\frac{1}{23}c^5ex^{23}+\frac{1}{21}c^5dx^{21}+\frac{5}{19}ac^4ex^{19}+\frac{5}{17}ac^4dx^{17}+\frac{2}{3}a^2c^3ex^{15}+\frac{10}{13}a^2c^3dx^{13}+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{9}a^3c^2dx^9+\frac{5}{7}a^4cex^7+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/23*c^5*e*x^23 + 1/21*c^5*d*x^21 + 5/19*a*c^4*e*x^19 + 5/17*a*c^4*d*x^17 +
 2/3*a^2*c^3*e*x^15 + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*e*x^11 + 10/9*a^
3*c^2*d*x^9 + 5/7*a^4*c*e*x^7 + a^4*c*d*x^5 + 1/3*a^5*e*x^3 + a^5*d*x
```

mupad [B] time = 0.07, size = 121, normalized size = 0.86

$$\frac{e\,a^5\,x^3}{3}+d\,a^5\,x+\frac{5\,e\,a^4\,c\,x^7}{7}+d\,a^4\,c\,x^5+\frac{10\,e\,a^3\,c^2\,x^{11}}{11}+\frac{10\,d\,a^3\,c^2\,x^9}{9}+\frac{2\,e\,a^2\,c^3\,x^{15}}{3}+\frac{10\,d\,a^2\,c^3\,x^{13}}{13}+\frac{5\,e\,a\,c^4\,x^{19}}{19}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*e*x^3)/3 + (c^5*d*x^21)/21 + (c^5*e*x^23)/23 + a^5*d*x + (10*a^3*c^2*d
*x^9)/9 + (10*a^2*c^3*d*x^13)/13 + (10*a^3*c^2*e*x^11)/11 + (2*a^2*c^3*e*x^
15)/3 + a^4*c*d*x^5 + (5*a*c^4*d*x^17)/17 + (5*a^4*c*e*x^7)/7 + (5*a*c^4*e*
x^19)/19
```

sympy [A] time = 0.09, size = 148, normalized size = 1.05

$$a^5dx+\frac{a^5ex^3}{3}+a^4cdx^5+\frac{5a^4cex^7}{7}+\frac{10a^3c^2dx^9}{9}+\frac{10a^3c^2ex^{11}}{11}+\frac{10a^2c^3dx^{13}}{13}+\frac{2a^2c^3ex^{15}}{3}+\frac{5ac^4dx^{17}}{17}+\frac{5ac^4ex^{19}}{19}+\frac{c^5dx^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x + a**5*e*x**3/3 + a**4*c*d*x**5 + 5*a**4*c*e*x**7/7 + 10*a**3*c**2
*d*x**9/9 + 10*a**3*c**2*e*x**11/11 + 10*a**2*c**3*d*x**13/13 + 2*a**2*c**3
*e*x**15/3 + 5*a*c**4*d*x**17/17 + 5*a*c**4*e*x**19/19 + c**5*d*x**21/21 +
c**5*e*x**23/23
```

3.5 $\int \frac{(d+ex^2)(a+cx^4)^5}{x} dx$

Optimal. Leaf size=142

$$a^5d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+$$

```
[Out] 1/2*a^5*e*x^2+5/4*a^4*c*d*x^4+5/6*a^4*c*e*x^6+5/4*a^3*c^2*d*x^8+a^3*c^2*e*x
^10+5/6*a^2*c^3*d*x^12+5/7*a^2*c^3*e*x^14+5/16*a*c^4*d*x^16+5/18*a*c^4*e*x^
18+1/20*c^5*d*x^20+1/22*c^5*e*x^22+a^5*d*ln(x)
```

Rubi [A] time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1252, 766}

$$\frac{5}{6}a^2c^3dx^{12}+\frac{5}{4}a^3c^2dx^8+\frac{5}{7}a^2c^3ex^{14}+a^3c^2ex^{10}+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+a^5d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4
 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*
d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*
d*Log[x]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)(a+cx^4)^5}{x} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)(a+cx^2)^5}{x} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^5e+\frac{a^5d}{x}+5a^4cdx+5a^4cex^2+10a^3c^2dx^3+10a^3c^2ex^4+10a^2c^3dx\right.\right.\\ &= \frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\end{aligned}$$

Mathematica [A] time = 0.01, size = 142, normalized size = 1.00

$$a^5d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4
 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*
d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*
d*Log[x]
```

fricas [A] time = 0.58, size = 122, normalized size = 0.86

$$\frac{1}{22}\,c^5ex^{22}+\frac{1}{20}\,c^5dx^{20}+\frac{5}{18}\,ac^4ex^{18}+\frac{5}{16}\,ac^4dx^{16}+\frac{5}{7}\,a^2c^3ex^{14}+\frac{5}{6}\,a^2c^3dx^{12}+a^3c^2ex^{10}+\frac{5}{4}\,a^3c^2dx^8+\frac{5}{6}\,a^4cex^6+\frac{5}{4}\,a^4cdx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + a^5*d*log(x)
```

giac [A] time = 0.20, size = 131, normalized size = 0.92

$$\frac{1}{22}\,c^5x^{22}e+\frac{1}{20}\,c^5dx^{20}+\frac{5}{18}\,ac^4x^{18}e+\frac{5}{16}\,ac^4dx^{16}+\frac{5}{7}\,a^2c^3x^{14}e+\frac{5}{6}\,a^2c^3dx^{12}+a^3c^2x^{10}e+\frac{5}{4}\,a^3c^2dx^8+\frac{5}{6}\,a^4cx^6e+\frac{5}{4}\,a^4cdx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*c^5*x^22*e + 1/20*c^5*d*x^20 + 5/18*a*c^4*x^18*e + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*x^14*e + 5/6*a^2*c^3*d*x^12 + a^3*c^2*x^10*e + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*x^6*e + 5/4*a^4*c*d*x^4 + 1/2*a^5*x^2*e + 1/2*a^5*d*log(x^2)
```

maple [A] time = 0.01, size = 123, normalized size = 0.87

$$\frac{c^5e\,x^{22}}{22}+\frac{c^5d\,x^{20}}{20}+\frac{5a\,c^4e\,x^{18}}{18}+\frac{5a\,c^4d\,x^{16}}{16}+\frac{5a^2c^3e\,x^{14}}{7}+\frac{5a^2c^3d\,x^{12}}{6}+a^3c^2e\,x^{10}+\frac{5a^3c^2d\,x^8}{4}+\frac{5a^4ce\,x^6}{6}+\frac{5a^4cd\,x^4}{4}+\frac{a^5e}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x,x)
```

```
[Out] 1/2*a^5*e*x^2+5/4*a^4*c*d*x^4+5/6*a^4*c*e*x^6+5/4*a^3*c^2*d*x^8+a^3*c^2*e*x
^10+5/6*a^2*c^3*d*x^12+5/7*a^2*c^3*e*x^14+5/16*a*c^4*d*x^16+5/18*a*c^4*e*x^
18+1/20*c^5*d*x^20+1/22*c^5*e*x^22+a^5*d*ln(x)
```

maxima [A] time = 0.52, size = 125, normalized size = 0.88

$$\frac{1}{22}\,c^5ex^{22}+\frac{1}{20}\,c^5dx^{20}+\frac{5}{18}\,ac^4ex^{18}+\frac{5}{16}\,ac^4dx^{16}+\frac{5}{7}\,a^2c^3ex^{14}+\frac{5}{6}\,a^2c^3dx^{12}+a^3c^2ex^{10}+\frac{5}{4}\,a^3c^2dx^8+\frac{5}{6}\,a^4cex^6+\frac{5}{4}\,a^4cdx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + 1/2*a^5*d*log(x^2)
```

mupad [B] time = 0.11, size = 122, normalized size = 0.86

$$\frac{a^5\,e\,x^2}{2}+\frac{c^5\,d\,x^{20}}{20}+\frac{c^5\,e\,x^{22}}{22}+a^5\,d\,\ln(x)+\frac{5\,a^3\,c^2\,d\,x^8}{4}+\frac{5\,a^2\,c^3\,d\,x^{12}}{6}+a^3\,c^2\,e\,x^{10}+\frac{5\,a^2\,c^3\,e\,x^{14}}{7}+\frac{5\,a^4\,c\,d\,x^4}{4}+\frac{5\,a\,c^4\,d\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^4)^5*(d + e*x^2))/x,x)
```

```
[Out] (a^5*e*x^2)/2 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*d*log(x) + (5*a^3*c
^2*d*x^8)/4 + (5*a^2*c^3*d*x^12)/6 + a^3*c^2*e*x^10 + (5*a^2*c^3*e*x^14)/7
+ (5*a^4*c*d*x^4)/4 + (5*a*c^4*d*x^16)/16 + (5*a^4*c*e*x^6)/6 + (5*a*c^4*e*
x^18)/18
```

sympy [A] time = 0.25, size = 150, normalized size = 1.06

$$a^5 d\log(x) + \frac{a^5 e x^2}{2} + \frac{5a^4 c d x^4}{4} + \frac{5a^4 c e x^6}{6} + \frac{5a^3 c^2 d x^8}{4} + a^3 c^2 e x^{10} + \frac{5a^2 c^3 d x^{12}}{6} + \frac{5a^2 c^3 e x^{14}}{7} + \frac{5a c^4 d x^{16}}{16} + \frac{5a c^4 e x^{18}}{18} + \frac{c^5 d}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x,x)
```

```
[Out] a**5*d*log(x) + a**5*e*x**2/2 + 5*a**4*c*d*x**4/4 + 5*a**4*c*e*x**6/6 + 5*a
**3*c**2*d*x**8/4 + a**3*c**2*e*x**10 + 5*a**2*c**3*d*x**12/6 + 5*a**2*c**3
*e*x**14/7 + 5*a*c**4*d*x**16/16 + 5*a*c**4*e*x**18/18 + c**5*d*x**20/20 +
c**5*e*x**22/22
```

3.6 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx$

Optimal. Leaf size=139

$$-\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3ex^{13}+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5dx^1$$

```
[Out] -a^5*d/x+a^5*e*x+5/3*a^4*c*d*x^3+a^4*c*e*x^5+10/7*a^3*c^2*d*x^7+10/9*a^3*c^
2*e*x^9+10/11*a^2*c^3*d*x^11+10/13*a^2*c^3*e*x^13+1/3*a*c^4*d*x^15+5/17*a*c
^4*e*x^17+1/19*c^5*d*x^19+1/21*c^5*e*x^21
```

Rubi [A] time = 0.08, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1262}

$$\frac{10}{11}a^2c^3dx^{11}+\frac{10}{7}a^3c^2dx^7+\frac{10}{13}a^2c^3ex^{13}+\frac{10}{9}a^3c^2ex^9+\frac{5}{3}a^4cdx^3+a^4cex^5-\frac{a^5d}{x}+a^5ex+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5dx^{19}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x^
7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13)/
13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x^21
)/21
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx &= \int \left(a^5e+\frac{a^5d}{x^2}+5a^4cdx^2+5a^4cex^4+10a^3c^2dx^6+10a^3c^2ex^8+10a^2c^3dx^{10}+10a^2c^3\right.\\ &= -\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3\end{aligned}$$

Mathematica [A] time = 0.01, size = 139, normalized size = 1.00

$$-\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3ex^{13}+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5dx^1$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x^
7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13)/
13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x^21
)/21
```

fricas [A] time = 0.67, size = 127, normalized size = 0.91

$$\frac{138567\,c^5ex^{22}+153153\,c^5dx^{20}+855855\,ac^4ex^{18}+969969\,ac^4dx^{16}+2238390\,a^2c^3ex^{14}+2645370\,a^2c^3dx^{12}+3}{2909907}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/2909907*(138567*c^5*e*x^22 + 153153*c^5*d*x^20 + 855855*a*c^4*e*x^18 + 96
9969*a*c^4*d*x^16 + 2238390*a^2*c^3*e*x^14 + 2645370*a^2*c^3*d*x^12 + 32332
30*a^3*c^2*e*x^10 + 4157010*a^3*c^2*d*x^8 + 2909907*a^4*c*e*x^6 + 4849845*a
^4*c*d*x^4 + 2909907*a^5*e*x^2 - 2909907*a^5*d)/x
```

giac [A] time = 0.20, size = 127, normalized size = 0.91

$$\frac{1}{21}c^5x^{21}e+\frac{1}{19}c^5dx^{19}+\frac{5}{17}ac^4x^{17}e+\frac{1}{3}ac^4dx^{15}+\frac{10}{13}a^2c^3x^{13}e+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{9}a^3c^2x^9e+\frac{10}{7}a^3c^2dx^7+a^4cx^5e+\frac{5}{3}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*c^5*x^21*e + 1/19*c^5*d*x^19 + 5/17*a*c^4*x^17*e + 1/3*a*c^4*d*x^15 +
10/13*a^2*c^3*x^13*e + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*x^9*e + 10/7*a^3
*c^2*d*x^7 + a^4*c*x^5*e + 5/3*a^4*c*d*x^3 + a^5*x*e - a^5*d/x
```

maple [A] time = 0.02, size = 122, normalized size = 0.88

$$\frac{c^5e\,x^{21}}{21}+\frac{c^5d\,x^{19}}{19}+\frac{5a\,c^4e\,x^{17}}{17}+\frac{a\,c^4d\,x^{15}}{3}+\frac{10a^2c^3e\,x^{13}}{13}+\frac{10a^2c^3d\,x^{11}}{11}+\frac{10a^3c^2e\,x^9}{9}+\frac{10a^3c^2d\,x^7}{7}+a^4ce\,x^5+\frac{5a^4cd\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x^2,x)
```

```
[Out] -a^5*d/x+a^5*e*x+5/3*a^4*c*d*x^3+a^4*c*e*x^5+10/7*a^3*c^2*d*x^7+10/9*a^3*c^
2*e*x^9+10/11*a^2*c^3*d*x^11+10/13*a^2*c^3*e*x^13+1/3*a*c^4*d*x^15+5/17*a*c
^4*e*x^17+1/19*c^5*d*x^19+1/21*c^5*e*x^21
```

maxima [A] time = 0.52, size = 121, normalized size = 0.87

$$\frac{1}{21}c^5ex^{21}+\frac{1}{19}c^5dx^{19}+\frac{5}{17}ac^4ex^{17}+\frac{1}{3}ac^4dx^{15}+\frac{10}{13}a^2c^3ex^{13}+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{9}a^3c^2ex^9+\frac{10}{7}a^3c^2dx^7+a^4cex^5+\frac{5}{3}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*c^5*e*x^21 + 1/19*c^5*d*x^19 + 5/17*a*c^4*e*x^17 + 1/3*a*c^4*d*x^15 +
10/13*a^2*c^3*e*x^13 + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*e*x^9 + 10/7*a^3
*c^2*d*x^7 + a^4*c*e*x^5 + 5/3*a^4*c*d*x^3 + a^5*e*x - a^5*d/x
```

mupad [B] time = 0.09, size = 121, normalized size = 0.87

$$\frac{c^5\,d\,x^{19}}{19}-\frac{a^5\,d}{x}+\frac{c^5\,e\,x^{21}}{21}+a^5\,e\,x+\frac{10\,a^3\,c^2\,d\,x^7}{7}+\frac{10\,a^2\,c^3\,d\,x^{11}}{11}+\frac{10\,a^3\,c^2\,e\,x^9}{9}+\frac{10\,a^2\,c^3\,e\,x^{13}}{13}+\frac{5\,a^4\,c\,d\,x^3}{3}+\frac{a\,c^4\,d\,x^{15}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^4)^5*(d + e*x^2))/x^2,x)
```

```
[Out] (c^5*d*x^19)/19 - (a^5*d)/x + (c^5*e*x^21)/21 + a^5*e*x + (10*a^3*c^2*d*x^7
)/7 + (10*a^2*c^3*d*x^11)/11 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*e*x^13)/1
3 + (5*a^4*c*d*x^3)/3 + (a*c^4*d*x^15)/3 + a^4*c*e*x^5 + (5*a*c^4*e*x^17)/1
7
```

sympy [A] time = 0.24, size = 143, normalized size = 1.03

$$-\frac{a^5d}{x}+a^5ex+\frac{5a^4cdx^3}{3}+a^4cex^5+\frac{10a^3c^2dx^7}{7}+\frac{10a^3c^2ex^9}{9}+\frac{10a^2c^3dx^{11}}{11}+\frac{10a^2c^3ex^{13}}{13}+\frac{ac^4dx^{15}}{3}+\frac{5ac^4ex^{17}}{17}+\frac{c^5dx^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x**2,x)
```

```
[Out] -a**5*d/x + a**5*e*x + 5*a**4*c*d*x**3/3 + a**4*c*e*x**5 + 10*a**3*c**2*d*x
**7/7 + 10*a**3*c**2*e*x**9/9 + 10*a**2*c**3*d*x**11/11 + 10*a**2*c**3*e*x*
*13/13 + a*c**4*d*x**15/3 + 5*a*c**4*e*x**17/17 + c**5*d*x**19/19 + c**5*e*
x**21/21
```

3.7 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx$

Optimal. Leaf size=142

$$-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}$$

```
[Out] -1/2*a^5*d/x^2+5/2*a^4*c*d*x^2+5/4*a^4*c*e*x^4+5/3*a^3*c^2*d*x^6+5/4*a^3*c^
2*e*x^8+a^2*c^3*d*x^10+5/6*a^2*c^3*e*x^12+5/14*a*c^4*d*x^14+5/16*a*c^4*e*x^
16+1/18*c^5*d*x^18+1/20*c^5*e*x^20+a^5*e*ln(x)
```

Rubi [A] time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1252, 766}

$$a^2c^3dx^{10}+\frac{5}{3}a^3c^2dx^6+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{4}a^3c^2ex^8+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}c^5$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] -(a^5*d)/(2*x^2) + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^6
)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/6 + (5*a*c^
4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x^18)/18 + (c^5*e*x^20)/20 + a^
5*e*Log[x]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)(a+cx^2)^5}{x^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(5a^4cd+\frac{a^5d}{x^2}+\frac{a^5e}{x}+5a^4cex+10a^3c^2dx^2+10a^3c^2ex^3+10a^2c^3dx^4\right.\right.\\ &= -\frac{a^5d}{2x^2}+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\end{aligned}$$

Mathematica [A] time = 0.01, size = 142, normalized size = 1.00

$$-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] -1/2*(a^5*d)/x^2 + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^6
)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/6 + (5*a*c^
4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x^18)/18 + (c^5*e*x^20)/20 + a^
5*e*Log[x]
```

fricas [A] time = 0.69, size = 129, normalized size = 0.91

$$\frac{252\, c^5 e x^{22} + 280\, c^5 d x^{20} + 1575\, a c^4 e x^{18} + 1800\, a c^4 d x^{16} + 4200\, a^2 c^3 e x^{14} + 5040\, a^2 c^3 d x^{12} + 6300\, a^3 c^2 e x^{10} + 8400}{5040\, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*c^5*e*x^22 + 280*c^5*d*x^20 + 1575*a*c^4*e*x^18 + 1800*a*c^4*d*
x^16 + 4200*a^2*c^3*e*x^14 + 5040*a^2*c^3*d*x^12 + 6300*a^3*c^2*e*x^10 + 84
00*a^3*c^2*d*x^8 + 6300*a^4*c*e*x^6 + 12600*a^4*c*d*x^4 + 5040*a^5*e*x^2*lo
g(x) - 2520*a^5*d)/x^2
```

giac [A] time = 0.21, size = 142, normalized size = 1.00

$$\frac{1}{20}\, c^5 x^{20} e + \frac{1}{18}\, c^5 d x^{18} + \frac{5}{16}\, a c^4 x^{16} e + \frac{5}{14}\, a c^4 d x^{14} + \frac{5}{6}\, a^2 c^3 x^{12} e + a^2 c^3 d x^{10} + \frac{5}{4}\, a^3 c^2 x^8 e + \frac{5}{3}\, a^3 c^2 d x^6 + \frac{5}{4}\, a^4 c x^4 e + \frac{5}{2}\, a^4 c d x^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*c^5*x^20*e + 1/18*c^5*d*x^18 + 5/16*a*c^4*x^16*e + 5/14*a*c^4*d*x^14 +
 5/6*a^2*c^3*x^12*e + a^2*c^3*d*x^10 + 5/4*a^3*c^2*x^8*e + 5/3*a^3*c^2*d*x^
6 + 5/4*a^4*c*x^4*e + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*(a^5*x^2*e
 + a^5*d)/x^2
```

maple [A] time = 0.01, size = 123, normalized size = 0.87

$$\frac{c^5 e\, x^{20}}{20}+\frac{c^5 d\, x^{18}}{18}+\frac{5 a\, c^4 e\, x^{16}}{16}+\frac{5 a\, c^4 d\, x^{14}}{14}+\frac{5 a^2 c^3 e\, x^{12}}{6}+a^2 c^3 d\, x^{10}+\frac{5 a^3 c^2 e\, x^8}{4}+\frac{5 a^3 c^2 d\, x^6}{3}+\frac{5 a^4 c e\, x^4}{4}+\frac{5 a^4 c d\, x^2}{2}+a^5 e \ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x^3,x)
```

```
[Out] -1/2*a^5*d/x^2+5/2*a^4*c*d*x^2+5/4*a^4*c*e*x^4+5/3*a^3*c^2*d*x^6+5/4*a^3*c^
2*e*x^8+a^2*c^3*d*x^10+5/6*a^2*c^3*e*x^12+5/14*a*c^4*d*x^14+5/16*a*c^4*e*x^
16+1/18*c^5*d*x^18+1/20*c^5*e*x^20+a^5*e*ln(x)
```

maxima [A] time = 0.50, size = 125, normalized size = 0.88

$$\frac{1}{20}\, c^5 e x^{20} + \frac{1}{18}\, c^5 d x^{18} + \frac{5}{16}\, a c^4 e x^{16} + \frac{5}{14}\, a c^4 d x^{14} + \frac{5}{6}\, a^2 c^3 e x^{12} + a^2 c^3 d x^{10} + \frac{5}{4}\, a^3 c^2 e x^8 + \frac{5}{3}\, a^3 c^2 d x^6 + \frac{5}{4}\, a^4 c e x^4 + \frac{5}{2}\, a^4 c d x^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*c^5*e*x^20 + 1/18*c^5*d*x^18 + 5/16*a*c^4*e*x^16 + 5/14*a*c^4*d*x^14 +
 5/6*a^2*c^3*e*x^12 + a^2*c^3*d*x^10 + 5/4*a^3*c^2*e*x^8 + 5/3*a^3*c^2*d*x^
6 + 5/4*a^4*c*e*x^4 + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*a^5*d/x^2
```

mupad [B] time = 0.07, size = 122, normalized size = 0.86

$$\frac{c^5\, d\, x^{18}}{18}-\frac{a^5\, d}{2\, x^2}+\frac{c^5\, e\, x^{20}}{20}+a^5\, e\, \ln(x)+\frac{5\, a^3\, c^2\, d\, x^6}{3}+a^2\, c^3\, d\, x^{10}+\frac{5\, a^3\, c^2\, e\, x^8}{4}+\frac{5\, a^2\, c^3\, e\, x^{12}}{6}+\frac{5\, a^4\, c\, d\, x^2}{2}+\frac{5\, a\, c^4\, d\, x^{14}}{14}+5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^4)^5*(d + e*x^2))/x^3,x)
```

```
[Out] (c^5*d*x^18)/18 - (a^5*d)/(2*x^2) + (c^5*e*x^20)/20 + a^5*e*log(x) + (5*a^3
*c^2*d*x^6)/3 + a^2*c^3*d*x^10 + (5*a^3*c^2*e*x^8)/4 + (5*a^2*c^3*e*x^12)/6
 + (5*a^4*c*d*x^2)/2 + (5*a*c^4*d*x^14)/14 + (5*a^4*c*e*x^4)/4 + (5*a*c^4*e
*x^16)/16
```

sympy [A] time = 0.28, size = 150, normalized size = 1.06

$$-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5a^4cdx^2}{2}+\frac{5a^4cex^4}{4}+\frac{5a^3c^2dx^6}{3}+\frac{5a^3c^2ex^8}{4}+a^2c^3dx^{10}+\frac{5a^2c^3ex^{12}}{6}+\frac{5ac^4dx^{14}}{14}+\frac{5ac^4ex^{16}}{16}+\frac{c^5dx}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x**3,x)
```

```
[Out] -a**5*d/(2*x**2) + a**5*e*log(x) + 5*a**4*c*d*x**2/2 + 5*a**4*c*e*x**4/4 +
5*a**3*c**2*d*x**6/3 + 5*a**3*c**2*e*x**8/4 + a**2*c**3*d*x**10 + 5*a**2*c*
*3*e*x**12/6 + 5*a*c**4*d*x**14/14 + 5*a*c**4*e*x**16/16 + c**5*d*x**18/18
+ c**5*e*x**20/20
```

3.8 $\int x^5 \left(2+3x^2\right) \sqrt{5+x^4}\, dx$

Optimal. Leaf size=67

$$\frac{3}{10}\left(x^4+5\right)^{3/2}x^4-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5}{8}\sqrt{x^4+5}\,x^2-\frac{1}{4}\left(4-x^2\right)\left(x^4+5\right)^{3/2}$$

```
[Out] 3/10*x^4*(x^4+5)^(3/2)-1/4*(-x^2+4)*(x^4+5)^(3/2)-25/8*arcsinh(1/5*x^2*5^(1
/2))-5/8*x^2*(x^4+5)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1252, 833, 780, 195, 215}

$$\frac{3}{10}\left(x^4+5\right)^{3/2}x^4-\frac{5}{8}\sqrt{x^4+5}\,x^2-\frac{1}{4}\left(4-x^2\right)\left(x^4+5\right)^{3/2}-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (-5*x^2*Sqrt[5 + x^4])/8 + (3*x^4*(5 + x^4)^(3/2))/10 - ((4 - x^2)*(5 + x^4
)^(3/2))/4 - (25*ArcSinh[x^2/Sqrt[5]])/8
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^5\left(2+3x^2\right)\sqrt{5+x^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(2+3x)\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= \frac{3}{10}x^4\left(5+x^4\right)^{3/2}+\frac{1}{10}\operatorname{Subst}\left(\int x(-30+10x)\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= \frac{3}{10}x^4\left(5+x^4\right)^{3/2}-\frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2}-\frac{5}{4}\operatorname{Subst}\left(\int\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= -\frac{5}{8}x^2\sqrt{5+x^4}+\frac{3}{10}x^4\left(5+x^4\right)^{3/2}-\frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2}-\frac{25}{8}\operatorname{Subst}\left(\int\frac{1}{\sqrt{5+\right.\\ &= -\frac{5}{8}x^2\sqrt{5+x^4}+\frac{3}{10}x^4\left(5+x^4\right)^{3/2}-\frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2}-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.75

$$\frac{1}{40}\sqrt{x^4+5}\left(12x^8+10x^6+20x^4+25x^2-200\right)-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(-200 + 25*x^2 + 20*x^4 + 10*x^6 + 12*x^8))/40 - (25*ArcSinh
[x^2/Sqrt[5]])/8
```

fricas [A] time = 0.49, size = 48, normalized size = 0.72

$$\frac{1}{40}\left(12x^8+10x^6+20x^4+25x^2-200\right)\sqrt{x^4+5}+\frac{25}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/40*(12*x^8 + 10*x^6 + 20*x^4 + 25*x^2 - 200)*sqrt(x^4 + 5) + 25/8*log(-x^
2 + sqrt(x^4 + 5))
```

giac [A] time = 0.23, size = 54, normalized size = 0.81

$$\frac{1}{8}\left(2x^4+5\right)\sqrt{x^4+5}\,x^2+\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}-\frac{5}{2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{25}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(3
/2) + 25/8*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.05, size = 53, normalized size = 0.79

$$\frac{\left(x^4+5\right)^{\frac{3}{2}}x^2}{4}-\frac{5\sqrt{x^4+5}\,x^2}{8}-\frac{25\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}+\frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3x^4-10\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 1/10*(x^4+5)^(3/2)*(3*x^4-10)+1/4*x^2*(x^4+5)^(3/2)-5/8*x^2*(x^4+5)^(1/2)-2
5/8*arcsinh(1/5*x^2*5^(1/2))
```

maxima [B] time = 1.01, size = 102, normalized size = 1.52

$$\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}-\frac{5}{2}\left(x^4+5\right)^{\frac{3}{2}}-\frac{25\left(\frac{\sqrt{x^4+5}}{x^2}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}-\frac{25}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{25}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(3/2) - 25/8*(sqrt(x^4 + 5)/x^2 + (x^4
 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 25/16*log(sqrt(x
^4 + 5)/x^2 + 1) + 25/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.37, size = 42, normalized size = 0.63

$$\sqrt{x^4+5}\,\left(\frac{3\,x^8}{10}+\frac{x^6}{4}+\frac{x^4}{2}+\frac{5\,x^2}{8}-5\right)-\frac{25\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((5*x^2)/8 + x^4/2 + x^6/4 + (3*x^8)/10 - 5) - (25*asinh((5
^(1/2)*x^2)/5))/8
```

sympy [A] time = 6.01, size = 97, normalized size = 1.45

$$\frac{x^{10}}{4\sqrt{x^4+5}}+\frac{3x^8\sqrt{x^4+5}}{10}+\frac{15x^6}{8\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{25x^2}{8\sqrt{x^4+5}}-5\sqrt{x^4+5}-\frac{25\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**10/(4*sqrt(x**4 + 5)) + 3*x**8*sqrt(x**4 + 5)/10 + 15*x**6/(8*sqrt(x**4
+ 5)) + x**4*sqrt(x**4 + 5)/2 + 25*x**2/(8*sqrt(x**4 + 5)) - 5*sqrt(x**4 +
5) - 25*asinh(sqrt(5)*x**2/5)/8
```

3.9 $\int x^3 \left(2+3x^2\right) \sqrt{5+x^4}\, dx$

Optimal. Leaf size=51

$$-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{15}{16}\sqrt{x^4+5}\,x^2+\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}$$

```
[Out] 1/24*(9*x^2+8)*(x^4+5)^(3/2)-75/16*arcsinh(1/5*x^2*5^(1/2))-15/16*x^2*(x^4+
5)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1252, 780, 195, 215}

$$-\frac{15}{16}\sqrt{x^4+5}\,x^2+\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (-15*x^2*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 - (75*ArcSinh
[x^2/Sqrt[5]])/16
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int x^3 \left(2+3x^2\right) \sqrt{5+x^4}\, dx = \frac{1}{2} \text{Subst}\left(\int x(2+3x)\sqrt{5+x^2}\, dx, x, x^2\right)$$
$$= \frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2} - \frac{15}{8} \text{Subst}\left(\int \sqrt{5+x^2}\, dx, x, x^2\right)$$
$$= -\frac{15}{16} x^2 \sqrt{5+x^4} + \frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2} - \frac{75}{16} \text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\, dx, x, x^2\right)$$
$$= -\frac{15}{16} x^2 \sqrt{5+x^4} + \frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2} - \frac{75}{16} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.86

$$\frac{1}{48}\left(\sqrt{x^4+5}\left(18x^6+16x^4+45x^2+80\right) - 225 \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(80 + 45*x^2 + 16*x^4 + 18*x^6) - 225*ArcSinh[x^2/Sqrt[5]])/
48
```

fricas [A] time = 0.57, size = 43, normalized size = 0.84

$$\frac{1}{48}\left(18\,x^6+16\,x^4+45\,x^2+80\right)\sqrt{x^4+5} + \frac{75}{16} \log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 + 45*x^2 + 80)*sqrt(x^4 + 5) + 75/16*log(-x^2 + sqrt(
x^4 + 5))
```

giac [A] time = 0.23, size = 45, normalized size = 0.88

$$\frac{3}{16}\left(2\,x^4+5\right)\sqrt{x^4+5}\,x^2 + \frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}} + \frac{75}{16} \log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 3/16*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 1/3*(x^4 + 5)^(3/2) + 75/16*log(-x^2 +
 sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 46, normalized size = 0.90

$$\frac{3\left(x^4+5\right)^{\frac{3}{2}} x^2}{8} - \frac{15\sqrt{x^4+5}\; x^2}{16} - \frac{75 \operatorname{arcsinh}\left(\frac{\sqrt{5}\, x^2}{5}\right)}{16} + \frac{\left(x^4+5\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 3/8*(x^4+5)^(3/2)*x^2-15/16*(x^4+5)^(1/2)*x^2-75/16*arcsinh(1/5*5^(1/2)*x^2
)+1/3*(x^4+5)^(3/2)
```

maxima [B] time = 1.02, size = 93, normalized size = 1.82

$$\frac{1}{3}\left(x^{4}+5\right)^{\frac{3}{2}}-\frac{75\left(\frac{\sqrt{x^{4}+5}}{x^{2}}+\frac{\left(x^{4}+5\right)^{\frac{3}{2}}}{x^{6}}\right)}{16\left(\frac{2\left(x^{4}+5\right)}{x^{4}}-\frac{\left(x^{4}+5\right)^{2}}{x^{8}}-1\right)}-\frac{75}{32}\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}+1\right)+\frac{75}{32}\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) - 75/16*(sqrt(x^4 + 5)/x^2 + (x^4 + 5)^(3/2)/x^6)/(2*(x
^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 75/32*log(sqrt(x^4 + 5)/x^2 + 1) + 75/
32*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.39, size = 37, normalized size = 0.73

$$\sqrt{x^{4}+5}\,\left(\frac{3\,x^{6}}{8}+\frac{x^{4}}{3}+\frac{15\,x^{2}}{16}+\frac{5}{3}\right)-\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^{2}}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((15*x^2)/16 + x^4/3 + (3*x^6)/8 + 5/3) - (75*asinh((5^(1/2
)*x^2)/5))/16
```

sympy [A] time = 4.31, size = 70, normalized size = 1.37

$$\frac{3x^{10}}{8\sqrt{x^{4}+5}}+\frac{45x^{6}}{16\sqrt{x^{4}+5}}+\frac{75x^{2}}{16\sqrt{x^{4}+5}}+\frac{\left(x^{4}+5\right)^{\frac{3}{2}}}{3}-\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^{2}}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) + 45*x**6/(16*sqrt(x**4 + 5)) + 75*x**2/(16*sqrt
(x**4 + 5)) + (x**4 + 5)**(3/2)/3 - 75*asinh(sqrt(5)*x**2/5)/16
```

3.10 $\int x\left(2+3x^2\right)\sqrt{5+x^4}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}\left(x^4+5\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}\,x^2$$

```
[Out] 1/2*(x^4+5)^(3/2)+5/2*arcsinh(1/5*x^2*5^(1/2))+1/2*x^2*(x^4+5)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1248, 641, 195, 215}

$$\frac{1}{2}\sqrt{x^4+5}\,x^2+\frac{1}{2}\left(x^4+5\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x^2*Sqrt[5 + x^4])/2 + (5 + x^4)^(3/2)/2 + (5*ArcSinh[x^2/Sqrt[5]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int x\left(2+3x^2\right)\sqrt{5+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int (2+3x)\sqrt{5+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\left(5+x^4\right)^{3/2}+\text{Subst}\left(\int \sqrt{5+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}x^2\sqrt{5+x^4}+\frac{1}{2}\left(5+x^4\right)^{3/2}+\frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}x^2\sqrt{5+x^4}+\frac{1}{2}\left(5+x^4\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.82

$$\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}\,\left(x^4+x^2+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(5 + x^2 + x^4))/2 + (5*ArcSinh[x^2/Sqrt[5]])/2
```

fricas [A] time = 0.56, size = 34, normalized size = 0.77

$$\frac{1}{2}\left(x^4+x^2+5\right)\sqrt{x^4+5}-\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/2*(x^4 + x^2 + 5)*sqrt(x^4 + 5) - 5/2*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.20, size = 38, normalized size = 0.86

$$\frac{1}{2}\sqrt{x^4+5}\,x^2+\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}-\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^4 + 5)*x^2 + 1/2*(x^4 + 5)^(3/2) - 5/2*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 34, normalized size = 0.77

$$\frac{\sqrt{x^4+5}\,x^2}{2}+\frac{5\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5)^(1/2),x)
[Out] 1/2*(x^4+5)^(3/2)+5/2*arcsinh(1/5*5^(1/2)*x^2)+1/2*(x^4+5)^(1/2)*x^2
```

maxima [B] time = 1.32, size = 67, normalized size = 1.52

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{5\sqrt{x^4+5}}{2\,x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
[Out] 1/2*(x^4 + 5)^(3/2) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 5/4*log
(sqrt(x^4 + 5)/x^2 + 1) - 5/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.14, size = 32, normalized size = 0.73

$$\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}+\sqrt{x^4+5}\,\left(\frac{x^4}{2}+\frac{x^2}{2}+\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] (5*asinh((5^(1/2)*x^2)/5))/2 + (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 + 5/2)
```

sympy [**A**] time = 3.08, size = 53, normalized size = 1.20

$$\frac{x^6}{2\sqrt{x^4+5}}+\frac{5x^2}{2\sqrt{x^4+5}}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{2}+\frac{5\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) + 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/2 +
 5*asinh(sqrt(5)*x**2/5)/2
```

3.11 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx$

Optimal. Leaf size=58

$$-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{4}\sqrt{x^4+5}\,\left(3x^2+4\right)$$

```
[Out] 15/4*arcsinh(1/5*x^2*5^(1/2))-arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+1/
4*(3*x^2+4)*(x^4+5)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1252, 815, 844, 215, 266, 63, 207}

$$\frac{1}{4}\sqrt{x^4+5}\,\left(3x^2+4\right)+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*ArcSinh[x^2/Sqrt[5]])/4 - Sqrt[5]*ArcTa
nh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(2+3x^2\right)\sqrt{5+x^4}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{20+15x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}+\frac{15}{4}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)+5\,\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,\right.\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{5}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+5\,\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.98

$$\frac{1}{4}\left(-4\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+15\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\sqrt{x^4+5}\left(3x^2+4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4] + 15*ArcSinh[x^2/Sqrt[5]] - 4*Sqrt[5]*ArcTanh[Sq
rt[5 + x^4]/Sqrt[5]])/4
```

fricas [A] time = 0.73, size = 56, normalized size = 0.97

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2)
 - 15/4*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.21, size = 76, normalized size = 1.31

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)-\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5)
)/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 15/4*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.02, size = 49, normalized size = 0.84

$$\frac{3\sqrt{x^4+5}\,x^2}{4}+\frac{15\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}-\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)+\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x,x)
```

```
[Out] 3/4*(x^4+5)^(1/2)*x^2+15/4*arcsinh(1/5*5^(1/2)*x^2)+(x^4+5)^(1/2)-5^(1/2)*a
rctanh(5^(1/2)/(x^4+5)^(1/2))
```

maxima [B] time = 1.56, size = 99, normalized size = 1.71

$$\frac{1}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\sqrt{x^4+5}+\frac{15\sqrt{x^4+5}}{4x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sqr
t(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/8*log(sqrt(x
^4 + 5)/x^2 + 1) - 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.15, size = 45, normalized size = 0.78

$$\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}-\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)+\sqrt{x^4+5}\,\left(\frac{3x^2}{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x,x)
```

```
[Out] (15*asinh((5^(1/2)*x^2)/5))/4 - 5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5)
+ (x^4 + 5)^(1/2)*((3*x^2)/4 + 1)
```

sympy [A] time = 15.59, size = 83, normalized size = 1.43

$$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{15x^2}{4\sqrt{x^4+5}}+\sqrt{x^4+5}+\frac{\sqrt{5}\log\left(x^4\right)}{2}-\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)+\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x,x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) + s
qrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1) + 15*asinh(sqrt(5)*x
**2/5)/4
```

3.12 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx$

Optimal. Leaf size=59

$$-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}\,(2-3x^2)}{2x^2}$$

```
[Out] arcsinh(1/5*x^2*5^(1/2))-3/2*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/2
*(-3*x^2+2)*(x^4+5)^(1/2)/x^2
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1252, 813, 844, 215, 266, 63, 207}

$$-\frac{\sqrt{x^4+5}\,(2-3x^2)}{2x^2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] -((2 - 3*x^2)*Sqrt[5 + x^4])/(2*x^2) + ArcSinh[x^2/Sqrt[5]] - (3*Sqrt[5]*Ar
cTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{-30-4x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \frac{15}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right) + \operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx\right.\\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{2}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 1.00

$$\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{2}\left(\frac{(3x^2-2)\sqrt{x^4+5}}{x^2} - 3\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] ArcSinh[x^2/Sqrt[5]] + (((-2 + 3*x^2)*Sqrt[5 + x^4])/x^2 - 3*Sqrt[5]*ArcTan
h[Sqrt[5 + x^4]/Sqrt[5]])/2
```

fricas [A] time = 0.85, size = 72, normalized size = 1.22

$$\frac{3\sqrt{5}\,x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 2\,x^2\log\left(-x^2+\sqrt{x^4+5}\right) - 2\,x^2 + \sqrt{x^4+5}\,(3\,x^2-2)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2*log(-x^2 + s
qrt(x^4 + 5)) - 2*x^2 + sqrt(x^4 + 5)*(3*x^2 - 2))/x^2
```

giac [A] time = 0.25, size = 91, normalized size = 1.54

$$\frac{3}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}+\frac{10}{\left(x^2-\sqrt{x^4+5}\right)^2-5}-\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 3/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
+ 5))) + 3/2*sqrt(x^4 + 5) + 10/((x^2 - sqrt(x^4 + 5))^2 - 5) - log(-x^2 +
sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 61, normalized size = 1.03

$$\frac{\sqrt{x^4+5}\,x^2}{5}+\operatorname{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{5x^2}+\frac{3\sqrt{x^4+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^3,x)
```

```
[Out] 3/2*(x^4+5)^(1/2)-3/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/5/x^2*(x^4+5
)^(3/2)+1/5*(x^4+5)^(1/2)*x^2+arcsinh(1/5*5^(1/2)*x^2)
```

maxima [A] time = 1.41, size = 88, normalized size = 1.49

$$\frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}-\frac{\sqrt{x^4+5}}{x^2}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/2
*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*l
og(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.79, size = 51, normalized size = 0.86

$$\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{3\sqrt{x^4+5}}{2}-\frac{\sqrt{x^4+5}}{x^2}+\frac{\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)3\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^3,x)
```

```
[Out] asinh((5^(1/2)*x^2)/5) + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5)*3i)/
2 + (3*(x^4 + 5)^(1/2))/2 - (x^4 + 5)^(1/2)/x^2
```

sympy [A] time = 7.46, size = 83, normalized size = 1.41

$$-\frac{x^2}{\sqrt{x^4+5}}+\frac{3\sqrt{x^4+5}}{2}+\frac{3\sqrt{5}\log\left(x^4\right)}{4}-\frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}+\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)-\frac{5}{x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**3,x)
```

```
[Out] -x**2/sqrt(x**4 + 5) + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*log(x**4)/4 - 3*sqrt(
5)*log(sqrt(x**4/5 + 1) + 1)/2 + asinh(sqrt(5)*x**2/5) - 5/(x**2*sqrt(x**4
+ 5))
```

3.13 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx$

Optimal. Leaf size=63

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}\,(3x^2+1)}{2x^4}$$

```
[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))-1/10*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2
)-1/2*(3*x^2+1)*(x^4+5)^(1/2)/x^4
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1252, 811, 844, 215, 266, 63, 207}

$$-\frac{\sqrt{x^4+5}\,(3x^2+1)}{2x^4}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

```
[Out] -((1 + 3*x^2)*Sqrt[5 + x^4])/(2*x^4) + (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh
[Sqrt[5 + x^4]/Sqrt[5]]/(2*Sqrt[5])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
```

```
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x^2\right)\sqrt{5+x^4}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\left(1+3x^2\right)\sqrt{5+x^4}}{2x^4} - \frac{1}{40}\operatorname{Subst}\left(\int \frac{-20-60x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\left(1+3x^2\right)\sqrt{5+x^4}}{2x^4} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right) + \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,d\right.\\
&= -\frac{\left(1+3x^2\right)\sqrt{5+x^4}}{2x^4} + \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{\left(1+3x^2\right)\sqrt{5+x^4}}{2x^4} + \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{\left(1+3x^2\right)\sqrt{5+x^4}}{2x^4} + \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.94

$$\frac{1}{10}\left(-\sqrt{5}\tanh^{-1}\left(\sqrt{\frac{x^4}{5}+1}\right)+15\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5\sqrt{x^4+5}\left(3x^2+1\right)}{x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

```
[Out] ((-5*(1 + 3*x^2)*Sqrt[5 + x^4])/x^4 + 15*ArcSinh[x^2/Sqrt[5]] - Sqrt[5]*Arc
Tanh[Sqrt[1 + x^4/5]])/10
```

fricas [A] time = 0.65, size = 72, normalized size = 1.14

$$\frac{\sqrt{5}\,x^4\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-15\,x^4\log\left(-x^2+\sqrt{x^4+5}\right)-15\,x^4-5\,\sqrt{x^4+5}\left(3\,x^2+1\right)}{10\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^4*log(-x^2 + s
qrt(x^4 + 5)) - 15*x^4 - 5*sqrt(x^4 + 5)*(3*x^2 + 1))/x^4
```

giac [B] time = 0.24, size = 129, normalized size = 2.05

$$\frac{1}{10}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5x^2-5\sqrt{x^4+5}-75}{\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2}-\frac{3}{2}\log\left(-x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + ((x^2 - sqrt(x^4 + 5))^3 + 15*(x^2 - sqrt(x^4 + 5))^2 + 5*x^2 - 5
*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5))^2 - 5)^2 - 3/2*log(-x^2 + sqrt(
x^4 + 5))
```

maple [A] time = 0.02, size = 75, normalized size = 1.19

$$\frac{3\sqrt{x^4+5}\,x^2}{10}+\frac{3\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}-\frac{3\left(x^4+5\right)^{\frac{3}{2}}}{10x^2}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{10x^4}+\frac{\sqrt{x^4+5}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^5,x)
```

```
[Out] -1/10/x^4*(x^4+5)^(3/2)+1/10*(x^4+5)^(1/2)-1/10*5^(1/2)*arctanh(5^(1/2)/(x^
4+5)^(1/2))-3/10*(x^4+5)^(3/2)/x^2+3/10*(x^4+5)^(1/2)*x^2+3/2*arcsinh(1/5*5
^(1/2)*x^2)
```

maxima [A] time = 1.24, size = 91, normalized size = 1.44

$$\frac{1}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{2\,x^2}-\frac{\sqrt{x^4+5}}{2\,x^4}+\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3/
2*sqrt(x^4 + 5)/x^2 - 1/2*sqrt(x^4 + 5)/x^4 + 3/4*log(sqrt(x^4 + 5)/x^2 + 1
) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.42, size = 56, normalized size = 0.89

$$\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{3\,\sqrt{x^4+5}}{2\,x^2}-\frac{\sqrt{x^4+5}}{2\,x^4}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,1\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^5,x)
```

```
[Out] (3*asinh((5^(1/2)*x^2)/5))/2 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5
)*1i)/10 - (3*(x^4 + 5)^(1/2))/(2*x^2) - (x^4 + 5)^(1/2)/(2*x^4)
```

sympy [A] time = 6.04, size = 76, normalized size = 1.21

$$-\frac{3x^2}{2\sqrt{x^4+5}}-\frac{\sqrt{5}\,\mathrm{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{\sqrt{1+\frac{5}{x^4}}}{2x^2}-\frac{15}{2x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**5,x)
```

```
[Out] -3*x**2/(2*sqrt(x**4 + 5)) - sqrt(5)*asinh(sqrt(5)/x**2)/10 + 3*asinh(sqrt(
5)*x**2/5)/2 - sqrt(1 + 5/x**4)/(2*x**2) - 15/(2*x**2*sqrt(x**4 + 5))
```

3.14 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7} dx$

Optimal. Leaf size=58

$$-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{4\sqrt{5}}-\frac{\left(x^4+5\right)^{3/2}}{15x^6}$$

```
[Out] -1/15*(x^4+5)^(3/2)/x^6-3/20*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-3/4
*(x^4+5)^(1/2)/x^4
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1252, 807, 266, 47, 63, 207}

$$-\frac{\left(x^4+5\right)^{3/2}}{15x^6}-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

```
[Out] (-3*Sqrt[5 + x^4])/(4*x^4) - (5 + x^4)^(3/2)/(15*x^6) - (3*ArcTanh[Sqrt[5 +
 x^4]/Sqrt[5]])/(4*Sqrt[5])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{2}\operatorname{Subst}\left(\int \frac{\sqrt{5+x^2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{4}\operatorname{Subst}\left(\int \frac{\sqrt{5+x}}{x^2}\,dx, x, x^4\right)\\
&= -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} - \frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{4\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.24

$$-\frac{3\left(5x^4+\sqrt{5}\sqrt{x^4+5}\,x^4\tanh^{-1}\left(\sqrt{\frac{x^4}{5}+1}\right)+25\right)}{20x^4\sqrt{x^4+5}}-\frac{(x^4+5)^{3/2}}{15x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

```
[Out] -1/15*(5 + x^4)^(3/2)/x^6 - (3*(25 + 5*x^4 + Sqrt[5]*x^4*Sqrt[5 + x^4]*ArcT
anh[Sqrt[1 + x^4/5]]))/(20*x^4*Sqrt[5 + x^4])
```

fricas [A] time = 0.64, size = 59, normalized size = 1.02

$$\frac{9\sqrt{5}\,x^6\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-4\,x^6-\left(4\,x^4+45\,x^2+20\right)\sqrt{x^4+5}}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(9*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 4*x^6 - (4*x^4 +
45*x^2 + 20)*sqrt(x^4 + 5))/x^6
```

giac [B] time = 0.23, size = 116, normalized size = 2.00

$$\frac{3}{20}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{9\left(x^2-\sqrt{x^4+5}\right)^5+12\left(x^2-\sqrt{x^4+5}\right)^4-225\,x^2+225\sqrt{x^4+5}+100}{6\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 3/20*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + 1/6*(9*(x^2 - sqrt(x^4 + 5))^5 + 12*(x^2 - sqrt(x^4 + 5))^4 - 225
*x^2 + 225*sqrt(x^4 + 5) + 100)/((x^2 - sqrt(x^4 + 5))^2 - 5)^3
```

maple [A] time = 0.01, size = 52, normalized size = 0.90

$$-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{20}-\frac{3\left(x^4+5\right)^{\frac{3}{2}}}{20x^4}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{15x^6}+\frac{3\sqrt{x^4+5}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^7,x)
```

```
[Out] -1/15*(x^4+5)^(3/2)/x^6-3/20*(x^4+5)^(3/2)/x^4+3/20*(x^4+5)^(1/2)-3/20*5^(1
/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

maxima [A] time = 1.61, size = 59, normalized size = 1.02

$$\frac{3}{40}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{4\,x^4}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{15\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3/
4*sqrt(x^4 + 5)/x^4 - 1/15*(x^4 + 5)^(3/2)/x^6
```

mupad [B] time = 0.68, size = 43, normalized size = 0.74

$$-\frac{3\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{20}-\frac{3\sqrt{x^4+5}}{4\,x^4}-\frac{\left(x^4+5\right)^{3/2}}{15\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^7,x)
```

```
[Out] - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/20 - (3*(x^4 + 5)^(1/2))/(
4*x^4) - (x^4 + 5)^(3/2)/(15*x^6)
```

sympy [A] time = 5.96, size = 63, normalized size = 1.09

$$-\frac{\sqrt{1+\frac{5}{x^4}}}{15}-\frac{3\sqrt{5}\,\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{20}-\frac{3\sqrt{1+\frac{5}{x^4}}}{4x^2}-\frac{\sqrt{1+\frac{5}{x^4}}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**7,x)
```

```
[Out] -sqrt(1 + 5/x**4)/15 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/20 - 3*sqrt(1 + 5/x**4
)/(4*x**2) - sqrt(1 + 5/x**4)/(3*x**4)
```

3.15 $\int x^4 \left(2+3x^2\right) \sqrt{5+x^4} \, dx$

Optimal. Leaf size=208

$$\frac{20}{21}\sqrt{x^4+5}\,x+\frac{2}{3}\sqrt{x^4+5}\,x^3-\frac{10\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}-\frac{5\sqrt[4]{5}\left(21+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{21\sqrt{x^4+5}}+\frac{10\sqrt[4]{5}}{}$$

```
[Out] 20/21*x*(x^4+5)^(1/2)+2/3*x^3*(x^4+5)^(1/2)+1/21*x^5*(7*x^2+6)*(x^4+5)^(1/2
)-10*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+10*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))
^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4)
)),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
-5/21*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(
3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2
1+2*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1274, 1280, 1198, 220, 1196}

$$\frac{1}{21}\left(7x^2+6\right)\sqrt{x^4+5}\,x^5+\frac{2}{3}\sqrt{x^4+5}\,x^3-\frac{10\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{20}{21}\sqrt{x^4+5}\,x-\frac{5\sqrt[4]{5}\left(21+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F}{21\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (20*x*Sqrt[5 + x^4])/21 + (2*x^3*Sqrt[5 + x^4])/3 - (10*x*Sqrt[5 + x^4])/(S
qrt[5] + x^2) + (x^5*(6 + 7*x^2)*Sqrt[5 + x^4])/21 + (10*5^(1/4)*(Sqrt[5] +
 x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2]
)/Sqrt[5 + x^4] - (5*5^(1/4)*(21 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4
)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(21*Sqrt[5 + x^4]
)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\begin{aligned}
\int x^4\left(2+3x^2\right)\sqrt{5+x^4}\,dx &= \frac{1}{21}x^5\left(6+7x^2\right)\sqrt{5+x^4}+\frac{10}{63}\int\frac{x^4\left(18+21x^2\right)}{\sqrt{5+x^4}}\,dx\\
&= \frac{2}{3}x^3\sqrt{5+x^4}+\frac{1}{21}x^5\left(6+7x^2\right)\sqrt{5+x^4}-\frac{2}{63}\int\frac{x^2\left(315-90x^2\right)}{\sqrt{5+x^4}}\,dx\\
&= \frac{20}{21}x\sqrt{5+x^4}+\frac{2}{3}x^3\sqrt{5+x^4}+\frac{1}{21}x^5\left(6+7x^2\right)\sqrt{5+x^4}+\frac{2}{189}\int\frac{-450-945x^2}{\sqrt{5+x^4}}\,dx\\
&= \frac{20}{21}x\sqrt{5+x^4}+\frac{2}{3}x^3\sqrt{5+x^4}+\frac{1}{21}x^5\left(6+7x^2\right)\sqrt{5+x^4}+\left(10\sqrt{5}\right)\int\frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\\
&= \frac{20}{21}x\sqrt{5+x^4}+\frac{2}{3}x^3\sqrt{5+x^4}-\frac{10x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{21}x^5\left(6+7x^2\right)\sqrt{5+x^4}+\frac{10\sqrt[4]{5}\left(\sqrt{\right.}}{}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 82, normalized size = 0.39

$$\frac{1}{21}x\left(-30\sqrt{5}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)-35\sqrt{5}\,x^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)+6\left(x^4+5\right)^{3/2}+7\left(x^4+5\right)^{3/2}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x*(6*(5 + x^4)^(3/2) + 7*x^2*(5 + x^4)^(3/2) - 30*Sqrt[5]*Hypergeometric2F
1[-1/2, 1/4, 5/4, -1/5*x^4] - 35*Sqrt[5]*x^2*Hypergeometric2F1[-1/2, 3/4, 7
/4, -1/5*x^4]))/21
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(3\,x^6+2\,x^4\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)*sqrt(x^4 + 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+5}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4, x)
```

maple [C] time = 0.09, size = 192, normalized size = 0.92

$$\frac{\sqrt{x^4+5}\,x^7}{3}+\frac{2\sqrt{x^4+5}\,x^5}{7}+\frac{2\sqrt{x^4+5}\,x^3}{3}+\frac{20\sqrt{x^4+5}\,x}{21}-\frac{4\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\mathrm{EllipticF}\left(\right.}{21\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 1/3*x^7*(x^4+5)^(1/2)+2/3*x^3*(x^4+5)^(1/2)-2*I/(I*5^(1/2))^(1/2)*(25-5*I*5
^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x
*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
+2/7*x^5*(x^4+5)^(1/2)+20/21*x*(x^4+5)^(1/2)-4/21*5^(1/2)/(I*5^(1/2))^(1/2)
*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*Ellipt
icF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+5}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\sqrt{x^4+5}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] int(x^4*(x^4 + 5)^(1/2)*(3*x^2 + 2), x)
```

sympy [C] time = 2.33, size = 78, normalized size = 0.38

$$\frac{3\sqrt{5}\,x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**
4*exp_polar(I*pi)/5)/(2*gamma(9/4))
```

3.16 $\int x^2 (2+3x^2) \sqrt{5+x^4}\, dx$

Optimal. Leaf size=192

$$\frac{10}{7}\sqrt{x^4+5}\,x+\frac{4\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{\sqrt[4]{5}\,(14-5\sqrt{5})\,(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{x^4+5}}-\frac{4\sqrt[4]{5}\,(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5}}}}{\sqrt{x^4+}}$$

```
[Out] 10/7*x*(x^4+5)^(1/2)+1/35*x^3*(15*x^2+14)*(x^4+5)^(1/2)+4*x*(x^4+5)^(1/2)/(
x^2+5^(1/2))-4*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(
1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^
(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/7*5^(1/4)*(cos(2*arc
tan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*a
rctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(14-5*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x
^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1274, 1280, 1198, 220, 1196}

$$\frac{1}{35}\left(15x^2+14\right)\sqrt{x^4+5}\,x^3+\frac{4\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{10}{7}\sqrt{x^4+5}\,x+\frac{\sqrt[4]{5}\,(14-5\sqrt{5})\,(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right.}{7\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (10*x*Sqrt[5 + x^4])/7 + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x^3*(14 + 1
5*x^2)*Sqrt[5 + x^4])/35 - (4*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[
5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*
(14 - 5*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
F[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
```

```
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\int x^2\left(2+3x^2\right)\sqrt{5+x^4}\,dx = \frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4}+\frac{2}{7}\int\frac{x^2\left(14+15x^2\right)}{\sqrt{5+x^4}}\,dx$$

$$= \frac{10}{7}x\sqrt{5+x^4}+\frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4}-\frac{2}{21}\int\frac{75-42x^2}{\sqrt{5+x^4}}\,dx$$

$$= \frac{10}{7}x\sqrt{5+x^4}+\frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4}-\left(4\sqrt{5}\right)\int\frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{7}\left(2\left(25-\right.\right.$$

$$= \frac{10}{7}x\sqrt{5+x^4}+\frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4}-\frac{4\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{}{(}}}{}$$

Mathematica [C] time = 0.02, size = 68, normalized size = 0.35

$$\frac{1}{21}x\left(-45\sqrt{5}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+14\sqrt{5}x^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)+9\left(x^4+5\right)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(2 + 3*x^2)*Sqrt[5 + x^4],x]
[Out] (x*(9*(5 + x^4)^(3/2) - 45*Sqrt[5]*Hypergeometric2F1[-1/2, 1/4, 5/4, -1/5*x
^4] + 14*Sqrt[5]*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -1/5*x^4]))/21
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(3\,x^4+2\,x^2\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
[Out] integral((3*x^4 + 2*x^2)*sqrt(x^4 + 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^4+5}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2, x)
```

maple [C] time = 0.02, size = 180, normalized size = 0.94

$$\frac{3\sqrt{x^4+5}\,x^5}{7}+\frac{2\sqrt{x^4+5}\,x^3}{5}+\frac{10\sqrt{x^4+5}\,x}{7}-\frac{2\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\mathrm{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{4i\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 3/7*(x^4+5)^(1/2)*x^5+10/7*(x^4+5)^(1/2)*x-2/7*5^(1/2)/(I*5^(1/2))^(1/2)*(-
5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*Elliptic
F(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)+2/5*(x^4+5)^(1/2)*x^3+4/5*I/(I*5^(1/2)
)^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2
)*(EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^
(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+5}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{x^4+5}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] int(x^2*(x^4 + 5)^(1/2)*(3*x^2 + 2), x)
```

sympy [C] time = 2.14, size = 78, normalized size = 0.41

$$\frac{3\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*
exp_polar(I*pi)/5)/(2*gamma(7/4))
```

3.17 $\int \left(2+3x^2\right)\sqrt{5+x^4}\,dx$

Optimal. Leaf size=176

$$\frac{1}{15}\left(9x^2+10\right)\sqrt{x^4+5}\,x+\frac{6\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{\sqrt[4]{5}\left(9+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}-\frac{6\sqrt[4]{5}\left(x^2+\right.}{}$$

```
[Out] 1/15*x*(9*x^2+10)*(x^4+5)^(1/2)+6*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-6*5^(1/4)*(
cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Elliptic
E(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(
1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/3*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(
1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/
2*2^(1/2))*(x^2+5^(1/2))*(9+2*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4
+5)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1177, 1198, 220, 1196}

$$\frac{1}{15}\left(9x^2+10\right)\sqrt{x^4+5}\,x+\frac{6\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{\sqrt[4]{5}\left(9+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}-\frac{6\sqrt[4]{5}\left(x^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (6*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x*(10 + 9*x^2)*Sqrt[5 + x^4])/15 - (
6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*Arc
Tan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(9 + 2*Sqrt[5])*(Sqrt[5] + x
^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/
(3*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \left(2+3x^2\right)\sqrt{5+x^4}\,dx &= \frac{1}{15}x\left(10+9x^2\right)\sqrt{5+x^4}+\frac{1}{15}\int \frac{100+90x^2}{\sqrt{5+x^4}}\,dx\\ &= \frac{1}{15}x\left(10+9x^2\right)\sqrt{5+x^4}-\left(6\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx+\frac{1}{3}\left(2\left(10+9\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\ &= \frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{15}x\left(10+9x^2\right)\sqrt{5+x^4}-\frac{6\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)\right.}{\sqrt{5+x^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.27

$$\sqrt{5}\,x\left(2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+x^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)*Sqrt[5 + x^4],x]
[Out] Sqrt[5]*x*(2*Hypergeometric2F1[-1/2, 1/4, 5/4, -1/5*x^4] + x^2*Hypergeometr
ic2F1[-1/2, 3/4, 7/4, -1/5*x^4])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{x^4+5}\left(3\,x^2+2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+5}\left(3\,x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

maple [C] time = 0.01, size = 168, normalized size = 0.95

$$\frac{3\sqrt{x^4+5}\;x^3}{5}+\frac{2\sqrt{x^4+5}\;x}{3}+\frac{4\sqrt{5}\;\sqrt{-5i\sqrt{5}\;x^2+25}\;\sqrt{5i\sqrt{5}\;x^2+25}\;\mathrm{EllipticF}\left(\frac{\sqrt{5}\;\sqrt{i\sqrt{5}}\;x}{5},i\right)}{15\sqrt{i\sqrt{5}}\;\sqrt{x^4+5}}+\frac{6i\sqrt{-5i\sqrt{5}\;x^2+25}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 3/5*(x^4+5)^(1/2)*x^3+6/5*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(
5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/2)*(I*5^(1/2))^
(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))+2/3*(x^4+5)^(1/2)*
x+4/15*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x
^2+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+5}\left(3\,x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^4+5}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] int((x^4 + 5)^(1/2)*(3*x^2 + 2), x)
```

sympy [C] time = 2.02, size = 76, normalized size = 0.43

$$\frac{3\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp
_polar(I*pi)/5)/(2*gamma(5/4))
```

3.18 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx$

Optimal. Leaf size=171

$$\frac{4\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}-\frac{(2-x^2)\sqrt{x^4+5}}{x}+\frac{\sqrt[4]{5}\,(2+\sqrt{5})\,(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{4\sqrt[4]{5}\,(x^2+\sqrt{5})\sqrt{\frac{x^4}{(x^2+}}}{\sqrt{x}}$$

```
[Out] -(-x^2+2)*(x^4+5)^(1/2)/x+4*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-4*5^(1/4)*(cos(2*
arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(
2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^
2)^(1/2)/(x^4+5)^(1/2)+5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2
*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))
*(2+5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1272, 1198, 220, 1196}

$$\frac{4\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}-\frac{(2-x^2)\sqrt{x^4+5}}{x}+\frac{\sqrt[4]{5}\,(2+\sqrt{5})\,(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{4\sqrt[4]{5}\,(x^2+\sqrt{5})\sqrt{\frac{x^4}{(x^2+}}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^2,x]
```

```
[Out] -(((2 - x^2)*Sqrt[5 + x^4])/x) + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (4*5
^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan
[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(2 + Sqrt[5])*(Sqrt[5] + x^2)*S
qrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[
5 + x^4]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
```

```
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = -\frac{(2-x^2)\sqrt{5+x^4}}{x} - \frac{2}{3}\int \frac{-15-6x^2}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{(2-x^2)\sqrt{5+x^4}}{x} - \left(4\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \left(2\left(5+2\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{(2-x^2)\sqrt{5+x^4}}{x} + \frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{4\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right.}{\sqrt{5+x^4}}$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.31

$$3\sqrt{5}\,x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right) - \frac{2\sqrt{5}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^2,x]
[Out] (-2*Sqrt[5]*Hypergeometric2F1[-1/2, -1/4, 3/4, -1/5*x^4])/x + 3*Sqrt[5]*x*H
ypergeometric2F1[-1/2, 1/4, 5/4, -1/5*x^4]
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5}\,(3\,x^2+2)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+5}\,(3\,x^2+2)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

maple [C] time = 0.02, size = 167, normalized size = 0.98

$$\sqrt{x^4+5}\,x + \frac{2\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\mathrm{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}} - \frac{2\sqrt{x^4+5}}{x} + \frac{4i\sqrt{-5i\sqrt{5}\,x^2+25}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^2,x)
```

```
[Out] (x^4+5)^(1/2)*x+2/5*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(
5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(
1/2)*x,I)-2*(x^4+5)^(1/2)/x+4/5*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(
1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/2)*(I*5^(
1/2))^(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

mupad [B] time = 0.41, size = 61, normalized size = 0.36

$$\frac{3\,x\,\sqrt{x^4+5}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{x^4}{5}\right)}{\sqrt{\frac{x^4}{5}+1}}+\frac{2\,\sqrt{x^4+5}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{4};\ \frac{3}{4};\ -\frac{5}{x^4}\right)}{x\,\sqrt{\frac{5}{x^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^2,x)
```

```
[Out] (3*x*(x^4 + 5)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -x^4/5))/(x^4/5 + 1)^(1/2)
 + (2*(x^4 + 5)^(1/2)*hypergeom([-1/2, -1/4], 3/4, -5/x^4))/(x*(5/x^4 + 1)^
(1/2))
```

sympy [C] time = 2.30, size = 78, normalized size = 0.46

$$\frac{3\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{2x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**2,x)
```

```
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi)/5)/(
4*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_po
lar(I*pi)/5)/(2*x*gamma(3/4))
```

3.19 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx$

Optimal. Leaf size=192

$$-\frac{6\sqrt{x^4+5}}{x}+\frac{6\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{5}\sqrt{x^4+5}}-\frac{6\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}}{\sqrt{x^4+5}}$$

```
[Out] -6*(x^4+5)^(1/2)/x-1/3*(-9*x^2+2)*(x^4+5)^(1/2)/x^3+6*x*(x^4+5)^(1/2)/(x^2+
5^(1/2))-6*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*
x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2
))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/15*(cos(2*arctan(1/5*x*5
^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x
*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2+9*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^
2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1272, 1282, 1198, 220, 1196}

$$\frac{6\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}-\frac{6\sqrt{x^4+5}}{x}-\frac{\left(2-9x^2\right)\sqrt{x^4+5}}{3x^3}+\frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{5}\sqrt{x^4+5}}-\frac{6\sqrt[4]{5}\left(x^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^4,x]
```

```
[Out] (-6*Sqrt[5 + x^4])/x - ((2 - 9*x^2)*Sqrt[5 + x^4])/(3*x^3) + (6*x*Sqrt[5 +
x^4])/(Sqrt[5] + x^2) - (6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5]
+ x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + ((2 + 9*Sqrt
[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x
/5^(1/4)], 1/2])/(3*5^(1/4)*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x^2\right)\sqrt{5+x^4}}{x^4}\,dx &= -\frac{\left(2-9x^2\right)\sqrt{5+x^4}}{3x^3} - \frac{2}{3}\int \frac{-45-2x^2}{x^2\sqrt{5+x^4}}\,dx \\
&= -\frac{6\sqrt{5+x^4}}{x} - \frac{\left(2-9x^2\right)\sqrt{5+x^4}}{3x^3} + \frac{2}{15}\int \frac{10+45x^2}{\sqrt{5+x^4}}\,dx \\
&= -\frac{6\sqrt{5+x^4}}{x} - \frac{\left(2-9x^2\right)\sqrt{5+x^4}}{3x^3} - \left(6\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \frac{1}{3}\left(2\left(2+9\sqrt{5}\right)\right)\int \frac{}{\sqrt{5}} \\
&= -\frac{6\sqrt{5+x^4}}{x} - \frac{\left(2-9x^2\right)\sqrt{5+x^4}}{3x^3} + \frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{6\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\,\mathrm{ta}\right.}{\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.28

$$-\frac{\sqrt{5}\left(2\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{x^4}{5}\right)+9x^2\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^4,x]
```

```
[Out] -1/3*(Sqrt[5]*(2*Hypergeometric2F1[-3/4, -1/2, 1/4, -1/5*x^4] + 9*x^2*Hyper
geometric2F1[-1/2, -1/4, 3/4, -1/5*x^4]))/x^3
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

maple [C] time = 0.02, size = 170, normalized size = 0.89

$$\frac{4\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\;\mathrm{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5}, i\right)}{75\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{3\sqrt{x^4+5}}{x}-\frac{2\sqrt{x^4+5}}{3x^3}+\frac{6i\sqrt{-5i\sqrt{5}\,x^2+25}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^4,x)
```

```
[Out] -2/3*(x^4+5)^(1/2)/x^3+4/75*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)
^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/2)*(I*5^
(1/2))^(1/2)*x,I)-3*(x^4+5)^(1/2)/x+6/5*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x
^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/2
)*(I*5^(1/2))^(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4+5}\,\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^4,x)
```

```
[Out] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^4, x)
```

sympy [C] time = 2.50, size = 83, normalized size = 0.43

$$\frac{3\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, -\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4x\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4}, -\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**4,x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_polar(I*pi)/5)/(
4*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), x**4*exp_
polar(I*pi)/5)/(2*x**3*gamma(1/4))
```

3.20 $\int x^5 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=83

$$\frac{3}{14}\left(x^4+5\right)^{5/2}x^4-\frac{125}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5}{24}\left(x^4+5\right)^{3/2}x^2-\frac{25}{16}\sqrt{x^4+5}\,x^2-\frac{1}{42}\left(18-7x^2\right)\left(x^4+5\right)^{5/2}$$

```
[Out] -5/24*x^2*(x^4+5)^(3/2)+3/14*x^4*(x^4+5)^(5/2)-1/42*(-7*x^2+18)*(x^4+5)^(5/
2)-125/16*arcsinh(1/5*x^2*5^(1/2))-25/16*x^2*(x^4+5)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1252, 833, 780, 195, 215}

$$\frac{3}{14}\left(x^4+5\right)^{5/2}x^4-\frac{5}{24}\left(x^4+5\right)^{3/2}x^2-\frac{25}{16}\sqrt{x^4+5}\,x^2-\frac{1}{42}\left(18-7x^2\right)\left(x^4+5\right)^{5/2}-\frac{125}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (-25*x^2*Sqrt[5 + x^4])/16 - (5*x^2*(5 + x^4)^(3/2))/24 + (3*x^4*(5 + x^4)^
(5/2))/14 - ((18 - 7*x^2)*(5 + x^4)^(5/2))/42 - (125*ArcSinh[x^2/Sqrt[5]])/
16
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int x^5 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} \, dx = \frac{1}{2} \text{Subst}\left(\int x^2 (2+3x) \left(5+x^2\right)^{3/2} \, dx, x, x^2\right)$$
$$= \frac{3}{14} x^4 \left(5+x^4\right)^{5/2} + \frac{1}{14} \text{Subst}\left(\int x(-30+14x) \left(5+x^2\right)^{3/2} \, dx, x, x^2\right)$$
$$= \frac{3}{14} x^4 \left(5+x^4\right)^{5/2} - \frac{1}{42} \left(18-7x^2\right) \left(5+x^4\right)^{5/2} - \frac{5}{6} \text{Subst}\left(\int \left(5+x^2\right)^{3/2} \, dx, x, x$$
$$= -\frac{5}{24} x^2 \left(5+x^4\right)^{3/2} + \frac{3}{14} x^4 \left(5+x^4\right)^{5/2} - \frac{1}{42} \left(18-7x^2\right) \left(5+x^4\right)^{5/2} - \frac{25}{8} \text{Subst}$$
$$= -\frac{25}{16} x^2 \sqrt{5+x^4} - \frac{5}{24} x^2 \left(5+x^4\right)^{3/2} + \frac{3}{14} x^4 \left(5+x^4\right)^{5/2} - \frac{1}{42} \left(18-7x^2\right) \left(5+x^4$$
$$= -\frac{25}{16} x^2 \sqrt{5+x^4} - \frac{5}{24} x^2 \left(5+x^4\right)^{3/2} + \frac{3}{14} x^4 \left(5+x^4\right)^{5/2} - \frac{1}{42} \left(18-7x^2\right) \left(5+x^4$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.87

$$\frac{3}{14} \left(x^4-2\right) \left(x^4+5\right)^{5/2} + \frac{1}{6} x^2 \left(x^4+5\right)^{5/2} - \frac{5}{48} \left(75 \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \sqrt{x^4+5} \left(2x^4+25\right) x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (x^2*(5 + x^4)^(5/2))/6 + (3*(-2 + x^4)*(5 + x^4)^(5/2))/14 - (5*(x^2*Sqrt[
5 + x^4]*(25 + 2*x^4) + 75*ArcSinh[x^2/Sqrt[5]]))/48
```

fricas [A] time = 0.71, size = 58, normalized size = 0.70

$$\frac{1}{336} \left(72 x^{12} + 56 x^{10} + 576 x^8 + 490 x^6 + 360 x^4 + 525 x^2 - 3600\right) \sqrt{x^4+5} + \frac{125}{16} \log\left(-x^2 + \sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/336*(72*x^12 + 56*x^10 + 576*x^8 + 490*x^6 + 360*x^4 + 525*x^2 - 3600)*sq
rt(x^4 + 5) + 125/16*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.25, size = 80, normalized size = 0.96

$$\frac{3}{14} \left(x^4+5\right)^{\frac{7}{2}} + \frac{1}{48} \left(2 \left(4 x^4+5\right) x^4 - 75\right) \sqrt{x^4+5}\, x^2 + \frac{5}{8} \left(2 x^4+5\right) \sqrt{x^4+5}\, x^2 - \frac{3}{2} \left(x^4+5\right)^{\frac{5}{2}} + \frac{125}{16} \log\left(-x^2 + \sqrt{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 3/14*(x^4 + 5)^(7/2) + 1/48*(2*(4*x^4 + 5)*x^4 - 75)*sqrt(x^4 + 5)*x^2 + 5/
8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 - 3/2*(x^4 + 5)^(5/2) + 125/16*log(-x^2 + s
qrt(x^4 + 5))
```

maple [A] time = 0.02, size = 73, normalized size = 0.88

$$\frac{\sqrt{x^4+5}\, x^{10}}{6} + \frac{35 \sqrt{x^4+5}\, x^6}{24} + \frac{25 \sqrt{x^4+5}\, x^2}{16} - \frac{125 \,\mathrm{arcsinh}\left(\frac{\sqrt{5}\, x^2}{5}\right)}{16} + \frac{3 \sqrt{x^4+5} \left(x^4-2\right) \left(x^8+10x^4+25\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 3/14*(x^4+5)^(1/2)*(x^4-2)*(x^8+10*x^4+25)+1/6*x^10*(x^4+5)^(1/2)+35/24*x^6
*(x^4+5)^(1/2)+25/16*(x^4+5)^(1/2)*x^2-125/16*arcsinh(1/5*5^(1/2)*x^2)
```

maxima [A] time = 1.35, size = 127, normalized size = 1.53

$$\frac{3}{14}\left(x^4+5\right)^{\frac{7}{2}}-\frac{3}{2}\left(x^4+5\right)^{\frac{5}{2}}-\frac{125\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6}-\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{48\left(\frac{3\left(x^4+5\right)}{x^4}-\frac{3\left(x^4+5\right)^2}{x^8}+\frac{\left(x^4+5\right)^3}{x^{12}}-1\right)}-\frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/14*(x^4 + 5)^(7/2) - 3/2*(x^4 + 5)^(5/2) - 125/48*(3*sqrt(x^4 + 5)/x^2 -
8*(x^4 + 5)^(3/2)/x^6 - 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 +
 5)^2/x^8 + (x^4 + 5)^3/x^12 - 1) - 125/32*log(sqrt(x^4 + 5)/x^2 + 1) + 125
/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.32, size = 52, normalized size = 0.63

$$\sqrt{x^4+5}\left(\frac{3\,x^{12}}{14}+\frac{x^{10}}{6}+\frac{12\,x^8}{7}+\frac{35\,x^6}{24}+\frac{15\,x^4}{14}+\frac{25\,x^2}{16}-\frac{75}{7}\right)-\frac{125\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((25*x^2)/16 + (15*x^4)/14 + (35*x^6)/24 + (12*x^8)/7 + x^1
0/6 + (3*x^12)/14 - 75/7) - (125*asinh((5^(1/2)*x^2)/5))/16
```

sympy [A] time = 14.31, size = 131, normalized size = 1.58

$$\frac{x^{14}}{6\sqrt{x^4+5}}+\frac{3x^{12}\sqrt{x^4+5}}{14}+\frac{55x^{10}}{24\sqrt{x^4+5}}+\frac{12x^8\sqrt{x^4+5}}{7}+\frac{425x^6}{48\sqrt{x^4+5}}+\frac{15x^4\sqrt{x^4+5}}{14}+\frac{125x^2}{16\sqrt{x^4+5}}-\frac{75\sqrt{x^4+5}}{7}-\frac{12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**14/(6*sqrt(x**4 + 5)) + 3*x**12*sqrt(x**4 + 5)/14 + 55*x**10/(24*sqrt(x*
*4 + 5)) + 12*x**8*sqrt(x**4 + 5)/7 + 425*x**6/(48*sqrt(x**4 + 5)) + 15*x**
4*sqrt(x**4 + 5)/14 + 125*x**2/(16*sqrt(x**4 + 5)) - 75*sqrt(x**4 + 5)/7 -
125*asinh(sqrt(5)*x**2/5)/16
```

3.21 $\int x^3 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=67

$$-\frac{375}{32}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{20}\left(5x^2+4\right)\left(x^4+5\right)^{5/2}-\frac{5}{16}x^2\left(x^4+5\right)^{3/2}-\frac{75}{32}x^2\sqrt{x^4+5}$$

```
[Out] -5/16*x^2*(x^4+5)^(3/2)+1/20*(5*x^2+4)*(x^4+5)^(5/2)-375/32*arcsinh(1/5*x^2
*5^(1/2))-75/32*x^2*(x^4+5)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1252, 780, 195, 215}

$$\frac{1}{20}\left(5x^2+4\right)\left(x^4+5\right)^{5/2}-\frac{5}{16}x^2\left(x^4+5\right)^{3/2}-\frac{75}{32}x^2\sqrt{x^4+5}-\frac{375}{32}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (-75*x^2*Sqrt[5 + x^4])/32 - (5*x^2*(5 + x^4)^(3/2))/16 + ((4 + 5*x^2)*(5 +
 x^4)^(5/2))/20 - (375*ArcSinh[x^2/Sqrt[5]])/32
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int x^3\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int x(2+3x)\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)$$
$$= \frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2}-\frac{5}{4}\,\text{Subst}\left(\int\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)$$
$$= -\frac{5}{16}x^2\left(5+x^4\right)^{3/2}+\frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2}-\frac{75}{16}\,\text{Subst}\left(\int\sqrt{5+x^2}\,dx,x,x^2\right)$$
$$= -\frac{75}{32}x^2\sqrt{5+x^4}-\frac{5}{16}x^2\left(5+x^4\right)^{3/2}+\frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2}-\frac{375}{32}\,\text{Subst}\left(\int\right.$$
$$= -\frac{75}{32}x^2\sqrt{5+x^4}-\frac{5}{16}x^2\left(5+x^4\right)^{3/2}+\frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2}-\frac{375}{32}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right.$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.81

$$\frac{1}{160}\left(\sqrt{x^4+5}\left(40x^{10}+32x^8+350x^6+320x^4+375x^2+800\right)-1875\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(800 + 375*x^2 + 320*x^4 + 350*x^6 + 32*x^8 + 40*x^10) - 187
5*ArcSinh[x^2/Sqrt[5]])/160
```

fricas [A] time = 0.63, size = 53, normalized size = 0.79

$$\frac{1}{160}\left(40\,x^{10}+32\,x^8+350\,x^6+320\,x^4+375\,x^2+800\right)\sqrt{x^4+5}+\frac{375}{32}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/160*(40*x^10 + 32*x^8 + 350*x^6 + 320*x^4 + 375*x^2 + 800)*sqrt(x^4 + 5)
+ 375/32*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.20, size = 71, normalized size = 1.06

$$\frac{1}{32}\left(2\left(4\,x^4+5\right)x^4-75\right)\sqrt{x^4+5}\,x^2+\frac{15}{16}\left(2\,x^4+5\right)\sqrt{x^4+5}\,x^2+\frac{1}{5}\left(x^4+5\right)^{\frac{5}{2}}+\frac{375}{32}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(2*(4*x^4 + 5)*x^4 - 75)*sqrt(x^4 + 5)*x^2 + 15/16*(2*x^4 + 5)*sqrt(x^
4 + 5)*x^2 + 1/5*(x^4 + 5)^(5/2) + 375/32*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 58, normalized size = 0.87

$$\frac{\sqrt{x^4+5}\,x^{10}}{4}+\frac{35\sqrt{x^4+5}\,x^6}{16}+\frac{75\sqrt{x^4+5}\,x^2}{32}-\frac{375\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{32}+\frac{\left(x^4+5\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 1/4*(x^4+5)^(1/2)*x^10+35/16*(x^4+5)^(1/2)*x^6+75/32*(x^4+5)^(1/2)*x^2-375/
32*arcsinh(1/5*5^(1/2)*x^2)+1/5*(x^4+5)^(5/2)
```

maxima [B] time = 1.13, size = 118, normalized size = 1.76

$$\frac{1}{5}\left(x^4+5\right)^{\frac{5}{2}}-\frac{125\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6}-\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{32\left(\frac{3\left(x^4+5\right)}{x^4}-\frac{3\left(x^4+5\right)^2}{x^8}+\frac{\left(x^4+5\right)^3}{x^{12}}-1\right)}-\frac{375}{64}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{375}{64}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(x^4 + 5)^(5/2) - 125/32*(3*sqrt(x^4 + 5)/x^2 - 8*(x^4 + 5)^(3/2)/x^6 -
 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 + 5)^2/x^8 + (x^4 + 5)^3
/x^12 - 1) - 375/64*log(sqrt(x^4 + 5)/x^2 + 1) + 375/64*log(sqrt(x^4 + 5)/x
^2 - 1)
```

mupad [B] time = 0.43, size = 47, normalized size = 0.70

$$\sqrt{x^4+5}\left(\frac{x^{10}}{4}+\frac{x^8}{5}+\frac{35\,x^6}{16}+2\,x^4+\frac{75\,x^2}{32}+5\right)-\frac{375\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((75*x^2)/32 + 2*x^4 + (35*x^6)/16 + x^8/5 + x^10/4 + 5) -
(375*asinh((5^(1/2)*x^2)/5))/32
```

sympy [B] time = 11.57, size = 124, normalized size = 1.85

$$\frac{x^{14}}{4\sqrt{x^4+5}}+\frac{55x^{10}}{16\sqrt{x^4+5}}+\frac{x^8\sqrt{x^4+5}}{5}+\frac{425x^6}{32\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{3}+\frac{375x^2}{32\sqrt{x^4+5}}+\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{3}-\frac{10\sqrt{x^4+5}}{3}-\frac{375\,\mathrm{a}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**14/(4*sqrt(x**4 + 5)) + 55*x**10/(16*sqrt(x**4 + 5)) + x**8*sqrt(x**4 +
5)/5 + 425*x**6/(32*sqrt(x**4 + 5)) + x**4*sqrt(x**4 + 5)/3 + 375*x**2/(32*
sqrt(x**4 + 5)) + 5*(x**4 + 5)**(3/2)/3 - 10*sqrt(x**4 + 5)/3 - 375*asinh(s
qrt(5)*x**2/5)/32
```

3.22 $\int x\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=60

$$\frac{3}{10}\left(x^4+5\right)^{5/2}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{4}x^2\left(x^4+5\right)^{3/2}+\frac{15}{8}x^2\sqrt{x^4+5}$$

```
[Out] 1/4*x^2*(x^4+5)^(3/2)+3/10*(x^4+5)^(5/2)+75/8*arcsinh(1/5*x^2*5^(1/2))+15/8
*x^2*(x^4+5)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1248, 641, 195, 215}

$$\frac{3}{10}\left(x^4+5\right)^{5/2}+\frac{1}{4}x^2\left(x^4+5\right)^{3/2}+\frac{15}{8}x^2\sqrt{x^4+5}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (15*x^2*Sqrt[5 + x^4])/8 + (x^2*(5 + x^4)^(3/2))/4 + (3*(5 + x^4)^(5/2))/10
 + (75*ArcSinh[x^2/Sqrt[5]])/8
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\int x\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int (2+3x)\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{3}{10}\left(5+x^4\right)^{5/2}+\text{Subst}\left(\int \left(5+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{15}{4}\text{Subst}\left(\int \sqrt{5+x^2}\,dx,x,x^2\right)\\
&= \frac{15}{8}x^2\sqrt{5+x^4}+\frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{75}{8}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,\right.\\
&= \frac{15}{8}x^2\sqrt{5+x^4}+\frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.93

$$\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}\left(\frac{3x^8}{5}+\frac{x^6}{2}+6x^4+\frac{25x^2}{4}+15\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(15 + (25*x^2)/4 + 6*x^4 + x^6/2 + (3*x^8)/5))/2 + (75*ArcSi
nh[x^2/Sqrt[5]])/8
```

fricas [A] time = 0.75, size = 48, normalized size = 0.80

$$\frac{1}{40}\left(12\,x^8+10\,x^6+120\,x^4+125\,x^2+300\right)\sqrt{x^4+5}-\frac{75}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(12*x^8 + 10*x^6 + 120*x^4 + 125*x^2 + 300)*sqrt(x^4 + 5) - 75/8*log(-
x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.24, size = 57, normalized size = 0.95

$$\frac{1}{8}\left(2\,x^4+5\right)\sqrt{x^4+5}\,x^2+\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}+\frac{5}{2}\sqrt{x^4+5}\,x^2-\frac{75}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 3/10*(x^4 + 5)^(5/2) + 5/2*sqrt(x^4 + 5
)*x^2 - 75/8*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 46, normalized size = 0.77

$$\frac{\sqrt{x^4+5}\,x^6}{4}+\frac{25\sqrt{x^4+5}\,x^2}{8}+\frac{75\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}+\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 3/10*(x^4+5)^(5/2)+1/4*(x^4+5)^(1/2)*x^6+25/8*(x^4+5)^(1/2)*x^2+75/8*arcsin
h(1/5*5^(1/2)*x^2)
```

maxima [B] time = 1.60, size = 95, normalized size = 1.58

$$\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}+\frac{25\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5)^(5/2) + 25/8*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 + 5)^(3/2)/x^6)/(
2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 75/16*log(sqrt(x^4 + 5)/x^2 + 1) -
 75/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.18, size = 42, normalized size = 0.70

$$\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}+\sqrt{x^4+5}\,\left(\frac{3\,x^8}{10}+\frac{x^6}{4}+3\,x^4+\frac{25\,x^2}{8}+\frac{15}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] (75*asinh((5^(1/2)*x^2)/5))/8 + (x^4 + 5)^(1/2)*((25*x^2)/8 + 3*x^4 + x^6/4
 + (3*x^8)/10 + 15/2)
```

sympy [B] time = 8.19, size = 109, normalized size = 1.82

$$\frac{x^{10}}{4\sqrt{x^4+5}}+\frac{3x^8\sqrt{x^4+5}}{10}+\frac{35x^6}{8\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{125x^2}{8\sqrt{x^4+5}}+\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{2}-5\sqrt{x^4+5}+\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**10/(4*sqrt(x**4 + 5)) + 3*x**8*sqrt(x**4 + 5)/10 + 35*x**6/(8*sqrt(x**4
+ 5)) + x**4*sqrt(x**4 + 5)/2 + 125*x**2/(8*sqrt(x**4 + 5)) + 5*(x**4 + 5)*
*(3/2)/2 - 5*sqrt(x**4 + 5) + 75*asinh(sqrt(5)*x**2/5)/8
```

3.23 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x} \, dx$

Optimal. Leaf size=78

$$-5\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}+\frac{5}{16}\left(9x^2+16\right)\sqrt{x^4+5}$$

```
[Out] 1/24*(9*x^2+8)*(x^4+5)^(3/2)+225/16*arcsinh(1/5*x^2*5^(1/2))-5*arctanh(1/5*
(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+5/16*(9*x^2+16)*(x^4+5)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1252, 815, 844, 215, 266, 63, 207}

$$\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}+\frac{5}{16}\left(9x^2+16\right)\sqrt{x^4+5}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-5\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
```

```
[Out] (5*(16 + 9*x^2)*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 + (225
*ArcSinh[x^2/Sqrt[5]])/16 - 5*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(2+3x^2\right)\left(5+x^4\right)^{3/2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\left(5+x^2\right)^{3/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}+\frac{1}{8}\operatorname{Subst}\left(\int \frac{(40+45x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)\\
&= \frac{5}{16}\left(16+9x^2\right)\sqrt{5+x^4}+\frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}+\frac{1}{16}\operatorname{Subst}\left(\int \frac{400+225x}{x\sqrt{5+x^2}}\,dx, x\right.\\
&= \frac{5}{16}\left(16+9x^2\right)\sqrt{5+x^4}+\frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}+\frac{225}{16}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x,\right.\\
&= \frac{5}{16}\left(16+9x^2\right)\sqrt{5+x^4}+\frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{25}{2}\operatorname{Subst}\\
&= \frac{5}{16}\left(16+9x^2\right)\sqrt{5+x^4}+\frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+25\operatorname{Subst}\\
&= \frac{5}{16}\left(16+9x^2\right)\sqrt{5+x^4}+\frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-5\sqrt{5}\tanh
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.86

$$
\frac{1}{48}\left(-240\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+675\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\sqrt{x^4+5}\left(18x^6+16x^4+225x^2+320\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[5 + x^4]*(320 + 225*x^2 + 16*x^4 + 18*x^6) + 675*ArcSinh[x^2/Sqrt[5]]
 - 240*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/48
```

fricas [A] time = 0.76, size = 67, normalized size = 0.86

$$
\frac{1}{48}\left(18\,x^6+16\,x^4+225\,x^2+320\right)\sqrt{x^4+5}+5\,\sqrt{5}\,\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-\frac{225}{16}\,\log\left(-x^2+\sqrt{x^4+5}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 + 225*x^2 + 320)*sqrt(x^4 + 5) + 5*sqrt(5)*log(-(sqrt
(5) - sqrt(x^4 + 5))/x^2) - 225/16*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.26, size = 90, normalized size = 1.15

$$\frac{1}{48}\sqrt{x^4+5}\left(\left(2\left(9x^2+8\right)x^2+225\right)x^2+320\right)+5\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)-\frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 + 225)*x^2 + 320) + 5*sqrt(5)*log(-(
x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 225/16*lo
g(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.02, size = 75, normalized size = 0.96

$$\frac{3\sqrt{x^4+5}\,x^6}{8}+\frac{\sqrt{x^4+5}\,x^4}{3}+\frac{75\sqrt{x^4+5}\,x^2}{16}+\frac{225\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}-5\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)+\frac{20\sqrt{x^4+5}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x,x)
```

```
[Out] 3/8*(x^4+5)^(1/2)*x^6+75/16*(x^4+5)^(1/2)*x^2+225/16*arcsinh(1/5*5^(1/2)*x^
2)+1/3*x^4*(x^4+5)^(1/2)+20/3*(x^4+5)^(1/2)-5*5^(1/2)*arctanh(5^(1/2)/(x^4+
5)^(1/2))
```

maxima [B] time = 1.19, size = 138, normalized size = 1.77

$$\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}+\frac{5}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+5\sqrt{x^4+5}+\frac{75\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) + 5/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) +
 sqrt(x^4 + 5))) + 5*sqrt(x^4 + 5) + 75/16*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 +
5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*log(sqrt(x^4
 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.18, size = 55, normalized size = 0.71

$$\frac{225\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}-5\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)+\sqrt{x^4+5}\,\left(\frac{3x^6}{8}+\frac{x^4}{3}+\frac{75x^2}{16}+\frac{20}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x,x)
```

```
[Out] (225*asinh((5^(1/2)*x^2)/5))/16 - 5*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))
/5) + (x^4 + 5)^(1/2)*((75*x^2)/16 + x^4/3 + (3*x^6)/8 + 20/3)
```

sympy [A] time = 33.67, size = 114, normalized size = 1.46

$$\frac{3x^{10}}{8\sqrt{x^4+5}}+\frac{105x^6}{16\sqrt{x^4+5}}+\frac{375x^2}{16\sqrt{x^4+5}}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{3}+5\sqrt{x^4+5}+\frac{5\sqrt{5}\log\left(x^4\right)}{2}-5\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)+\frac{225\,\mathrm{as}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x,x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) + 105*x**6/(16*sqrt(x**4 + 5)) + 375*x**2/(16*sq
rt(x**4 + 5)) + (x**4 + 5)**(3/2)/3 + 5*sqrt(x**4 + 5) + 5*sqrt(5)*log(x**4
)/2 - 5*sqrt(5)*log(sqrt(x**4/5 + 1) + 1) + 225*asinh(sqrt(5)*x**2/5)/16
```

3.24 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=81

$$-\frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(2-x^2)(x^4+5)^{3/2}}{2x^2}+\frac{3}{2}(x^2+5)\sqrt{x^4+5}$$

```
[Out] -1/2*(-x^2+2)*(x^4+5)^(3/2)/x^2+15/2*arcsinh(1/5*x^2*5^(1/2))-15/2*arctanh(
1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+3/2*(x^2+5)*(x^4+5)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1252, 813, 815, 844, 215, 266, 63, 207}

$$-\frac{(2-x^2)(x^4+5)^{3/2}}{2x^2}+\frac{3}{2}(x^2+5)\sqrt{x^4+5}+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

```
[Out] (3*(5 + x^2)*Sqrt[5 + x^4])/2 - ((2 - x^2)*(5 + x^4)^(3/2))/(2*x^2) + (15*A
rcSinh[x^2/Sqrt[5]])/2 - (15*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
```

```
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{(-30-12x)\sqrt{5+x^2}}{x}\,dx,x,x^2\right)\\
&= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} - \frac{1}{8}\text{Subst}\left(\int \frac{-300-60x}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right) + \frac{75}{2}\\
&= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{75}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{5}}\right.\\
&= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{75}{2}\text{Subst}\left(\int \frac{1}{-5+}\right.\\
&= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5}}{\right.}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 71, normalized size = 0.88

$$\frac{1}{2}\left(\sqrt{x^4+5}\,(x^4+20) - 15\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right) - \frac{5\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{x^4}{5}\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[5 + x^4]*(20 + x^4) - 15*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2 -
(5*Sqrt[5]*Hypergeometric2F1[-3/2, -1/2, 1/2, -1/5*x^4])/x^2
```

fricas [A] time = 0.68, size = 78, normalized size = 0.96

$$\frac{15\sqrt{5}x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-15x^2\log\left(-x^2+\sqrt{x^4+5}\right)-10x^2+\left(x^6+x^4+20x^2-10\right)\sqrt{x^4+5}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(15*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^2*log(-x^2 +
 sqrt(x^4 + 5)) - 10*x^2 + (x^6 + x^4 + 20*x^2 - 10)*sqrt(x^4 + 5))/x^2
```

giac [A] time = 0.26, size = 102, normalized size = 1.26

$$\frac{1}{2}\sqrt{x^4+5}\left(\left(x^2+1\right)x^2+20\right)+\frac{15}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{50}{\left(x^2-\sqrt{x^4+5}\right)^2-5}-\frac{15}{2}\log\left(-x^2+\sqrt{x^4-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 + 20) + 15/2*sqrt(5)*log(-(x^2 + sqrt(5) -
 sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 50/((x^2 - sqrt(x^4 + 5)
)^2 - 5) - 15/2*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.02, size = 75, normalized size = 0.93

$$\frac{\sqrt{x^4+5}\,x^4}{2}+\frac{\sqrt{x^4+5}\,x^2}{2}+\frac{15\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{15\sqrt{5}\ \mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}-\frac{5\sqrt{x^4+5}}{x^2}+10\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^3,x)
```

```
[Out] 1/2*(x^4+5)^(1/2)*x^4+10*(x^4+5)^(1/2)-15/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)
^(1/2))+1/2*(x^4+5)^(1/2)*x^2+15/2*arcsinh(1/5*5^(1/2)*x^2)-5*(x^4+5)^(1/2)
/x^2
```

maxima [A] time = 1.33, size = 122, normalized size = 1.51

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{15}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{15}{2}\sqrt{x^4+5}-\frac{5\sqrt{x^4+5}}{x^2}+\frac{5\sqrt{x^4+5}}{2x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{15}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) + 15/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5)
+ sqrt(x^4 + 5))) + 15/2*sqrt(x^4 + 5) - 5*sqrt(x^4 + 5)/x^2 + 5/2*sqrt(x^4
 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/4*log(sqrt(x^4 + 5)/x^2 + 1) - 15/4*lo
g(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.76, size = 64, normalized size = 0.79

$$\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}+\sqrt{x^4+5}\,\left(\frac{x^4}{2}+\frac{x^2}{2}+10\right)-\frac{5\,\sqrt{x^4+5}}{x^2}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,15\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^3,x)
```

```
[Out] (15*asinh((5^(1/2)*x^2)/5))/2 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/
5)*15i)/2 + (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 + 10) - (5*(x^4 + 5)^(1/2))/x^2
```

sympy [A] time = 11.39, size = 114, normalized size = 1.41

$$\frac{x^6}{2\sqrt{x^4+5}}-\frac{5x^2}{2\sqrt{x^4+5}}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{2}+\frac{15\sqrt{x^4+5}}{2}+\frac{15\sqrt{5}\log\left(x^4\right)}{4}-\frac{15\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}+\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**3,x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) - 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/2 +
 15*sqrt(x**4 + 5)/2 + 15*sqrt(5)*log(x**4)/4 - 15*sqrt(5)*log(sqrt(x**4/5
+ 1) + 1)/2 + 15*asinh(sqrt(5)*x**2/5)/2 - 25/(x**2*sqrt(x**4 + 5))
```

3.25 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=86

$$-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(2-3x^2)(x^4+5)^{3/2}}{4x^4}-\frac{3(15-2x^2)\sqrt{x^4+5}}{4x^2}$$

```
[Out] -1/4*(-3*x^2+2)*(x^4+5)^(3/2)/x^4+45/4*arcsinh(1/5*x^2*5^(1/2))-3/2*arctanh
(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-3/4*(-2*x^2+15)*(x^4+5)^(1/2)/x^2
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1252, 813, 844, 215, 266, 63, 207}

$$-\frac{(2-3x^2)(x^4+5)^{3/2}}{4x^4}-\frac{3(15-2x^2)\sqrt{x^4+5}}{4x^2}+\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-3*(15 - 2*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((2 - 3*x^2)*(5 + x^4)^(3/2))/(4*
x^4) + (45*ArcSinh[x^2/Sqrt[5]])/4 - (3*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[
5]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
```

```
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} - \frac{3}{16}\,\text{Subst}\left(\int \frac{(-60-8x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{3}{32}\,\text{Subst}\left(\int \frac{80+120x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{15}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{4}\,\text{Subst}\left(\int$$
$$= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{2}\,\text{Subst}\left(\int$$
$$= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\tanh^{-1}\left(\right.$$

Mathematica [C] time = 0.03, size = 60, normalized size = 0.70

$$\frac{1}{125}\left(x^4+5\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{x^4}{5}+1\right) - \frac{15\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{x^4}{5}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-15*Sqrt[5]*Hypergeometric2F1[-3/2, -1/2, 1/2, -1/5*x^4])/(2*x^2) + ((5 +
x^4)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + x^4/5])/125
```

fricas [A] time = 0.75, size = 82, normalized size = 0.95

$$\frac{6\sqrt{5}\,x^4\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 45\,x^4\log\left(-x^2+\sqrt{x^4+5}\right) - 30\,x^4 + \left(3\,x^6+4\,x^4-30\,x^2-10\right)\sqrt{x^4+5}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(6*sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 45*x^4*log(-x^2 +
sqrt(x^4 + 5)) - 30*x^4 + (3*x^6 + 4*x^4 - 30*x^2 - 10)*sqrt(x^4 + 5))/x^4
```

giac [B] time = 0.25, size = 146, normalized size = 1.70

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\frac{3}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{5\left(\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5x^2-5\right.}{\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 3/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4
+ 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 5*((x^2 - sqrt(x^4 + 5))^3 + 15*(x
^2 - sqrt(x^4 + 5))^2 + 5*x^2 - 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5)
)^2 - 5)^2 - 45/4*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.02, size = 73, normalized size = 0.85

$$\frac{3\sqrt{x^4+5}\,x^2}{4}+\frac{45\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}-\frac{3\sqrt{5}\,\ \mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}-\frac{15\sqrt{x^4+5}}{2x^2}-\frac{5\sqrt{x^4+5}}{2x^4}+\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^5,x)
```

```
[Out] (x^4+5)^(1/2)-3/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-5/2*(x^4+5)^(1/2)/
x^4+3/4*(x^4+5)^(1/2)*x^2+45/4*arcsinh(1/5*5^(1/2)*x^2)-15/2*(x^4+5)^(1/2)/
x^2
```

maxima [A] time = 1.37, size = 123, normalized size = 1.43

$$\frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\sqrt{x^4+5}-\frac{15\sqrt{x^4+5}}{2x^2}+\frac{15\sqrt{x^4+5}}{4x^2\left(\frac{x^4+5}{x^4}-1\right)}-\frac{5\sqrt{x^4+5}}{2x^4}+\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{45}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sqr
t(x^4 + 5) - 15/2*sqrt(x^4 + 5)/x^2 + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^
4 - 1)) - 5/2*sqrt(x^4 + 5)/x^4 + 45/8*log(sqrt(x^4 + 5)/x^2 + 1) - 45/8*lo
g(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.55, size = 71, normalized size = 0.83

$$\frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}+\sqrt{x^4+5}\,\left(\frac{3\,x^2}{4}+1\right)-\frac{15\,\sqrt{x^4+5}}{2\,x^2}-\frac{5\,\sqrt{x^4+5}}{2\,x^4}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,3\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^5,x)
```

```
[Out] (45*asinh((5^(1/2)*x^2)/5))/4 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/
5)*3i)/2 + (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (15*(x^4 + 5)^(1/2))/(2*x^2) -
 (5*(x^4 + 5)^(1/2))/(2*x^4)
```

sympy [**A**] time = 12.75, size = 133, normalized size = 1.55

$$\frac{3x^6}{4\sqrt{x^4+5}}-\frac{15x^2}{4\sqrt{x^4+5}}+\sqrt{x^4+5}+\frac{\sqrt{5}\log\left(x^4\right)}{2}-\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)-\frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{2}+\frac{45\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}-\frac{5\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**5,x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) - 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) + s
qrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1) - sqrt(5)*asinh(sqrt
(5)/x**2)/2 + 45*asinh(sqrt(5)*x**2/5)/4 - 5*sqrt(1 + 5/x**4)/(2*x**2) - 75
/(2*x**2*sqrt(x**4 + 5))
```

3.26 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=82

$$-\frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(4-9x^2)\sqrt{x^4+5}}{4x^2}-\frac{(9x^2+4)(x^4+5)^{3/2}}{12x^6}$$

```
[Out] -1/12*(9*x^2+4)*(x^4+5)^(3/2)/x^6+arcsinh(1/5*x^2*5^(1/2))-9/4*arctanh(1/5*
(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/4*(-9*x^2+4)*(x^4+5)^(1/2)/x^2
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1252, 811, 813, 844, 215, 266, 63, 207}

$$-\frac{(9x^2+4)(x^4+5)^{3/2}}{12x^6}-\frac{(4-9x^2)\sqrt{x^4+5}}{4x^2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] -((4 - 9*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((4 + 9*x^2)*(5 + x^4)^(3/2))/(12*x^
6) + ArcSinh[x^2/Sqrt[5]] - (9*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} - \frac{1}{40}\,\text{Subst}\left(\int \frac{(-40-90x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \frac{1}{80}\,\text{Subst}\left(\int \frac{900+80x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \frac{45}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right) +$$

$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{45}{8}\,\text{Subst}\left(\int \frac{1}{x\sqrt{5}\cdots}\right.$$

$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{45}{4}\,\text{Subst}\left(\int \frac{1}{-5+\cdots}\right.$$

$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+\cdots}}{\sqrt{5}\cdots}\right.$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.73

$$\frac{3}{250}\left(x^4+5\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{x^4}{5}+1\right) - \frac{5\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{x^4}{5}\right)}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] (-5*Sqrt[5]*Hypergeometric2F1[-3/2, -3/2, -1/2, -1/5*x^4])/(3*x^6) + (3*(5
+ x^4)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + x^4/5])/250
```

fricas [A] time = 0.70, size = 82, normalized size = 1.00

$$\frac{27\sqrt{5}\,x^6\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-12\,x^6\log\left(-x^2+\sqrt{x^4+5}\right)-16\,x^6+\left(18\,x^6-16\,x^4-45\,x^2-20\right)\sqrt{x^4+5}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(27*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 12*x^6*log(-x^2
+ sqrt(x^4 + 5)) - 16*x^6 + (18*x^6 - 16*x^4 - 45*x^2 - 20)*sqrt(x^4 + 5))/
x^6
```

giac [B] time = 0.27, size = 158, normalized size = 1.93

$$\frac{9}{4}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}+\frac{5\left(9\left(x^2-\sqrt{x^4+5}\right)^5+24\left(x^2-\sqrt{x^4+5}\right)^4-120\left(x^2-\sqrt{x^4+\right.}\right.}{6\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 9/4*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
+ 5))) + 3/2*sqrt(x^4 + 5) + 5/6*(9*(x^2 - sqrt(x^4 + 5))^5 + 24*(x^2 - sqr
t(x^4 + 5))^4 - 120*(x^2 - sqrt(x^4 + 5))^2 - 225*x^2 + 225*sqrt(x^4 + 5) +
 400)/((x^2 - sqrt(x^4 + 5))^2 - 5)^3 - log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.02, size = 73, normalized size = 0.89

$$\operatorname{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)-\frac{9\sqrt{5}\;\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{4}-\frac{4\sqrt{x^4+5}}{3x^2}-\frac{15\sqrt{x^4+5}}{4x^4}-\frac{5\sqrt{x^4+5}}{3x^6}+\frac{3\sqrt{x^4+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^7,x)
```

```
[Out] 3/2*(x^4+5)^(1/2)-9/4*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-15/4*(x^4+5)^(
1/2)/x^4+arcsinh(1/5*5^(1/2)*x^2)-4/3*(x^4+5)^(1/2)/x^2-5/3*(x^4+5)^(1/2)/x
^6
```

maxima [A] time = 1.35, size = 112, normalized size = 1.37

$$\frac{9}{8}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}-\frac{\sqrt{x^4+5}}{x^2}-\frac{15\sqrt{x^4+5}}{4\,x^4}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{3\,x^6}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/2
*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 - 15/4*sqrt(x^4 + 5)/x^4 - 1/3*(x^4 + 5)
```

```
^(3/2)/x^6 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1
)
```

mupad [B] time = 0.95, size = 82, normalized size = 1.00

$$\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{3\sqrt{x^4+5}}{2}+\sqrt{x^4+5}\,\left(\frac{2}{3\,x^2}-\frac{5}{3\,x^6}\right)-\frac{2\sqrt{x^4+5}}{x^2}-\frac{15\sqrt{x^4+5}}{4\,x^4}+\frac{\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,9\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^7,x)
```

```
[Out] asinh((5^(1/2)*x^2)/5) + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5)*9i)/
4 + (3*(x^4 + 5)^(1/2))/2 + (x^4 + 5)^(1/2)*(2/(3*x^2) - 5/(3*x^6)) - (2*(x
^4 + 5)^(1/2))/x^2 - (15*(x^4 + 5)^(1/2))/(4*x^4)
```

sympy [A] time = 12.56, size = 148, normalized size = 1.80

$$-\frac{x^2}{\sqrt{x^4+5}}-\frac{\sqrt{1+\frac{5}{x^4}}}{3}+\frac{3\sqrt{x^4+5}}{2}+\frac{3\sqrt{5}\log\left(x^4\right)}{4}-\frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}-\frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{4}+\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)-\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**7,x)
```

```
[Out] -x**2/sqrt(x**4 + 5) - sqrt(1 + 5/x**4)/3 + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*
log(x**4)/4 - 3*sqrt(5)*log(sqrt(x**4/5 + 1) + 1)/2 - 3*sqrt(5)*asinh(sqrt(
5)/x**2)/4 + asinh(sqrt(5)*x**2/5) - 15*sqrt(1 + 5/x**4)/(4*x**2) - 5/(x**2
*sqrt(x**4 + 5)) - 5*sqrt(1 + 5/x**4)/(3*x**4)
```

3.27 $\int x^4 (2+3x^2) (5+x^4)^{3/2} \, dx$

Optimal. Leaf size=235

$$\frac{200}{77}\sqrt{x^4+5}\,x+\frac{20}{13}\sqrt{x^4+5}\,x^3-\frac{300\sqrt{x^4+5}\,x}{13\left(x^2+\sqrt{5}\right)}-\frac{50\sqrt[4]{5}\left(231+26\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{1001\sqrt{x^4+5}}$$

```
[Out] 1/143*x^5*(33*x^2+26)*(x^4+5)^(3/2)+200/77*x*(x^4+5)^(1/2)+20/13*x^3*(x^4+5
)^(1/2)+10/1001*x^5*(77*x^2+78)*(x^4+5)^(1/2)-300/13*x*(x^4+5)^(1/2)/(x^2+5
^(1/2))+300/13*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(
1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^
(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-50/1001*5^(1/4)*(cos(2
*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin
(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(231+26*5^(1/2))*((x^4
+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1274, 1280, 1198, 220, 1196}

$$\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5+\frac{10\left(77x^2+78\right)\sqrt{x^4+5}\,x^5}{1001}+\frac{20}{13}\sqrt{x^4+5}\,x^3-\frac{300\sqrt{x^4+5}\,x}{13\left(x^2+\sqrt{5}\right)}+\frac{200}{77}\sqrt{x^4+5}\,x-\frac{50}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (200*x*Sqrt[5 + x^4])/77 + (20*x^3*Sqrt[5 + x^4])/13 - (300*x*Sqrt[5 + x^4]
)/(13*(Sqrt[5] + x^2)) + (10*x^5*(78 + 77*x^2)*Sqrt[5 + x^4])/1001 + (x^5*(
26 + 33*x^2)*(5 + x^4)^(3/2))/143 + (300*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(13*Sqrt[5 + x
^4]) - (50*5^(1/4)*(231 + 26*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[
5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(1001*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\int x^4\left(2+3x^2\right)\left(5+x^4\right)^{3/2}dx=\frac{1}{143}x^5\left(26+33x^2\right)\left(5+x^4\right)^{3/2}+\frac{30}{143}\int x^4\left(26+33x^2\right)\sqrt{5+x^4}\,dx$$

$$=\frac{10x^5\left(78+77x^2\right)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5\left(26+33x^2\right)\left(5+x^4\right)^{3/2}+\frac{100\int\frac{x^4\left(234+231x^2\right)}{\sqrt{5+x^4}}}{3003}$$

$$=\frac{20}{13}x^3\sqrt{5+x^4}+\frac{10x^5\left(78+77x^2\right)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5\left(26+33x^2\right)\left(5+x^4\right)^{3/2}-$$

$$=\frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4}+\frac{10x^5\left(78+77x^2\right)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5\left(26+33x^2\right.$$

$$=\frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4}+\frac{10x^5\left(78+77x^2\right)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5\left(26+33x^2\right.$$

$$=\frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4}-\frac{300x\sqrt{5+x^4}}{13\left(\sqrt{5}+x^2\right)}+\frac{10x^5\left(78+77x^2\right)\sqrt{5+x^4}}{1001}+$$

Mathematica [C] time = 0.04, size = 74, normalized size = 0.31

$$\frac{1}{143}x\left(-650\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)-825\sqrt{5}\,x^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)+\left(33x^2+26\right)\left(x^4+5\right)^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (x*((26 + 33*x^2)*(5 + x^4)^(5/2) - 650*Sqrt[5]*Hypergeometric2F1[-3/2, 1/4
, 5/4, -1/5*x^4] - 825*Sqrt[5]*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -1/5*x
^4]))/143
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(3\,x^{10}+2\,x^{8}+15\,x^{6}+10\,x^{4}\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
[Out] integral((3*x^10 + 2*x^8 + 15*x^6 + 10*x^4)*sqrt(x^4 + 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4, x)
```

maple [C] time = 0.02, size = 216, normalized size = 0.92

$$\frac{3\sqrt{x^4+5}\,x^{11}}{13}+\frac{2\sqrt{x^4+5}\,x^9}{11}+\frac{25\sqrt{x^4+5}\,x^7}{13}+\frac{130\sqrt{x^4+5}\,x^5}{77}+\frac{20\sqrt{x^4+5}\,x^3}{13}+\frac{200\sqrt{x^4+5}\,x}{77}-\frac{40\sqrt{5}\,\sqrt{-5i\sqrt{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)*(x^4+5)^(3/2),x)
[Out] 3/13*x^11*(x^4+5)^(1/2)+25/13*(x^4+5)^(1/2)*x^7+20/13*(x^4+5)^(1/2)*x^3-60/
13*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/
2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)-EllipticE(1/
5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))+2/11*x^9*(x^4+5)^(1/2)+130/77*(x^4+5)^(1/
2)*x^5+200/77*(x^4+5)^(1/2)*x-40/77*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)
*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/
2)*(I*5^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(x^4+5\right)}^{3/2}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
[Out] int(x^4*(x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

sympy [C] time = 3.97, size = 160, normalized size = 0.68

$$\frac{3\sqrt{5}\,x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{15}{4}\right)}+\frac{\sqrt{5}\,x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{13}{4}\right)}+\frac{15\sqrt{5}\,x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{5\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), x**4*exp_polar(I*p
i)/5)/(4*gamma(15/4)) + sqrt(5)*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,),
 x**4*exp_polar(I*pi)/5)/(2*gamma(13/4)) + 15*sqrt(5)*x**7*gamma(7/4)*hyper
((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(11/4)) + 5*sqrt(5)*
x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(2*gamma
(9/4))
```

3.28 $\int x^2 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=219

$$\frac{300}{77}\sqrt{x^4+5}\,x+\frac{40\sqrt{x^4+5}\,x}{3\left(x^2+\sqrt{5}\right)}+\frac{10\sqrt[4]{5}\left(154-45\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{231\sqrt{x^4+5}}-\frac{40\sqrt[4]{5}\left(x^2+\sqrt{\right.}}{}$$

```
[Out] 1/99*x^3*(27*x^2+22)*(x^4+5)^(3/2)+300/77*x*(x^4+5)^(1/2)+2/231*x^3*(135*x^
2+154)*(x^4+5)^(1/2)+40/3*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-40/3*5^(1/4)*(cos(2
*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin
(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))
^2)^(1/2)/(x^4+5)^(1/2)+10/231*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/
2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*
2^(1/2))*(154-45*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^
4+5)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1274, 1280, 1198, 220, 1196}

$$\frac{1}{99}\left(27x^2+22\right)\left(x^4+5\right)^{3/2}x^3+\frac{2}{231}\left(135x^2+154\right)\sqrt{x^4+5}\,x^3+\frac{40\sqrt{x^4+5}\,x}{3\left(x^2+\sqrt{5}\right)}+\frac{300}{77}\sqrt{x^4+5}\,x+\frac{10\sqrt[4]{5}\left(154-45\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (300*x*Sqrt[5 + x^4])/77 + (40*x*Sqrt[5 + x^4])/(3*(Sqrt[5] + x^2)) + (2*x^
3*(154 + 135*x^2)*Sqrt[5 + x^4])/231 + (x^3*(22 + 27*x^2)*(5 + x^4)^(3/2))/
99 - (40*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
E[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4]) + (10*5^(1/4)*(154 - 45*Sqrt
[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x
/5^(1/4)], 1/2])/(231*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\int x^2\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}+\frac{10}{33}\int x^2\left(22+27x^2\right)\sqrt{5+x^4}\,dx$$

$$= \frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}+\frac{20}{231}\int\frac{x^2\left(154\right.}{\sqrt{5}}$$

$$= \frac{300}{77}x\sqrt{5+x^4}+\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}-$$

$$= \frac{300}{77}x\sqrt{5+x^4}+\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}-$$

$$= \frac{300}{77}x\sqrt{5+x^4}+\frac{40x\sqrt{5+x^4}}{3\left(\sqrt{5}+x^2\right)}+\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27\right.$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.31

$$\frac{1}{33}x\left(-225\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+110\sqrt{5}\,x^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)+9\left(x^4+5\right)^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (x*(9*(5 + x^4)^(5/2) - 225*Sqrt[5]*Hypergeometric2F1[-3/2, 1/4, 5/4, -1/5*
x^4] + 110*Sqrt[5]*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -1/5*x^4]))/33
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(3\,x^8+2\,x^6+15\,x^4+10\,x^2\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^8 + 2*x^6 + 15*x^4 + 10*x^2)*sqrt(x^4 + 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2, x)
```

maple [C] time = 0.01, size = 204, normalized size = 0.93

$$\frac{3\sqrt{x^4+5}\,x^9}{11}+\frac{2\sqrt{x^4+5}\,x^7}{9}+\frac{195\sqrt{x^4+5}\,x^5}{77}+\frac{22\sqrt{x^4+5}\,x^3}{9}+\frac{300\sqrt{x^4+5}\,x}{77}-\frac{60\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}}}{77\sqrt{i\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 3/11*(x^4+5)^(1/2)*x^9+195/77*(x^4+5)^(1/2)*x^5+300/77*(x^4+5)^(1/2)*x-60/7
7*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25
)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)+2/9*(x^4
+5)^(1/2)*x^7+22/9*(x^4+5)^(1/2)*x^3+8/3*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*
x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/
2)*(I*5^(1/2))^(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(x^4+5\right)}^{3/2}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] int(x^2*(x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

sympy [C] time = 3.65, size = 160, normalized size = 0.73

$$\frac{3\sqrt{5}\,x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{\sqrt{5}\,x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{11}{4}\right)}+\frac{15\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{5\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1}{2\Gamma(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(13/4)) + sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x*
*4*exp_polar(I*pi)/5)/(2*gamma(11/4)) + 15*sqrt(5)*x**5*gamma(5/4)*hyper((-
1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(9/4)) + 5*sqrt(5)*x**3*
gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(7/4)
)
```

3.29 $\int \left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=197

$$\frac{1}{21}x\left(7x^2+6\right)\left(x^4+5\right)^{3/2}+\frac{2}{7}x\left(7x^2+10\right)\sqrt{x^4+5}+\frac{20x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{10\sqrt[4]{5}\left(7+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\right.}{7\sqrt{x^4+5}}$$

```
[Out] 1/21*x*(7*x^2+6)*(x^4+5)^(3/2)+2/7*x*(7*x^2+10)*(x^4+5)^(1/2)+20*x*(x^4+5)^
(1/2)/(x^2+5^(1/2))-20*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2
*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))
*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+10/7*5^(1/4)*(
cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Elliptic
F(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(7+2*5^(1/2))*((x
^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1177, 1198, 220, 1196}

$$\frac{1}{21}x\left(7x^2+6\right)\left(x^4+5\right)^{3/2}+\frac{2}{7}x\left(7x^2+10\right)\sqrt{x^4+5}+\frac{20x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{10\sqrt[4]{5}\left(7+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\right.}{7\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (20*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (2*x*(10 + 7*x^2)*Sqrt[5 + x^4])/7 +
 (x*(6 + 7*x^2)*(5 + x^4)^(3/2))/21 - (20*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
 x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]
 + (10*5^(1/4)*(7 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^
2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int \left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}+\frac{1}{21}\int \left(180+210x^2\right)\sqrt{5+x^4}\,dx$$
$$=\frac{2}{7}x\left(10+7x^2\right)\sqrt{5+x^4}+\frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}+\frac{1}{315}\int \frac{9000+6300x^2}{\sqrt{5+x^4}}\,dx$$
$$=\frac{2}{7}x\left(10+7x^2\right)\sqrt{5+x^4}+\frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}-\left(20\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx+\frac{1}{7}\Big($$
$$=\frac{20x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{2}{7}x\left(10+7x^2\right)\sqrt{5+x^4}+\frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}-\frac{20\sqrt[4]{5}\left(\sqrt{5}+\right.}{}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.25

$$5\sqrt{5}\,x\left(2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+x^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)*(5 + x^4)^(3/2),x]
[Out] 5*Sqrt[5]*x*(2*Hypergeometric2F1[-3/2, 1/4, 5/4, -1/5*x^4] + x^2*Hypergeome
tric2F1[-3/2, 3/4, 7/4, -1/5*x^4])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(3\,x^6+2\,x^4+15\,x^2+10\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

maple [C] time = 0.01, size = 192, normalized size = 0.97

$$\frac{\sqrt{x^4+5}\,x^7}{3}+\frac{2\sqrt{x^4+5}\,x^5}{7}+\frac{11\sqrt{x^4+5}\,x^3}{3}+\frac{30\sqrt{x^4+5}\,x}{7}+\frac{8\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\mathrm{EllipticF}\left(\frac{\sqrt{5}}{}\right.}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 1/3*(x^4+5)^(1/2)*x^7+11/3*(x^4+5)^(1/2)*x^3+4*I/(I*5^(1/2))^(1/2)*(-5*I*5^
(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5
*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I
))+2/7*(x^4+5)^(1/2)*x^5+30/7*(x^4+5)^(1/2)*x+8/7*5^(1/2)/(I*5^(1/2))^(1/2)
*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*Ellip
ticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x^4+5\right)}^{3/2}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] int((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

sympy [C] time = 3.63, size = 158, normalized size = 0.80

$$\frac{3\sqrt{5}\,x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{15\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{5\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1}{2\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**
4*exp_polar(I*pi)/5)/(2*gamma(9/4)) + 15*sqrt(5)*x**3*gamma(3/4)*hyper((-1/
2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(7/4)) + 5*sqrt(5)*x*gamma
(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(5/4))
```

3.30 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=199

$$-\frac{(14-3x^2)(x^4+5)^{3/2}}{7x}+\frac{6}{35}x(14x^2+25)\sqrt{x^4+5}+\frac{24x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{6\sqrt[4]{5}\,(14+5\sqrt{5})(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\,F(2\,\mathrm{ta}}{7\sqrt{x^4+5}}$$

```
[Out] -1/7*(-3*x^2+14)*(x^4+5)^(3/2)/x+6/35*x*(14*x^2+25)*(x^4+5)^(1/2)+24*x*(x^4
+5)^(1/2)/(x^2+5^(1/2))-24*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/c
os(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1
/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+6/7*5^(1/4
)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ellip
ticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(14+5*5^(1/2))
*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1272, 1177, 1198, 220, 1196}

$$-\frac{(14-3x^2)(x^4+5)^{3/2}}{7x}+\frac{6}{35}x(14x^2+25)\sqrt{x^4+5}+\frac{24x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{6\sqrt[4]{5}\,(14+5\sqrt{5})(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\,F(2\,\mathrm{ta}}{7\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^2,x]
```

```
[Out] (24*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (6*x*(25 + 14*x^2)*Sqrt[5 + x^4])/35
 - ((14 - 3*x^2)*(5 + x^4)^(3/2))/(7*x) - (24*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[
(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 +
x^4] + (6*5^(1/4)*(14 + 5*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5]
+ x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(5+x^4\right)^{3/2}}{x^2}\,dx = -\frac{\left(14-3x^2\right)\left(5+x^4\right)^{3/2}}{7x} - \frac{6}{7}\int \left(-15-14x^2\right)\sqrt{5+x^4}\,dx$$

$$= \frac{6}{35}x\left(25+14x^2\right)\sqrt{5+x^4} - \frac{\left(14-3x^2\right)\left(5+x^4\right)^{3/2}}{7x} - \frac{2}{35}\int \frac{-750-420x^2}{\sqrt{5+x^4}}\,dx$$

$$= \frac{6}{35}x\left(25+14x^2\right)\sqrt{5+x^4} - \frac{\left(14-3x^2\right)\left(5+x^4\right)^{3/2}}{7x} - \left(24\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \ldots$$

$$= \frac{24x\sqrt{5+x^4}}{\sqrt{5}+x^2} + \frac{6}{35}x\left(25+14x^2\right)\sqrt{5+x^4} - \frac{\left(14-3x^2\right)\left(5+x^4\right)^{3/2}}{7x} - \frac{24\sqrt[4]{5}\left(\sqrt{5}\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.27

$$15\sqrt{5}\,x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right) - \frac{10\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^2,x]
```

```
[Out] (-10*Sqrt[5]*Hypergeometric2F1[-3/2, -1/4, 3/4, -1/5*x^4])/x + 15*Sqrt[5]*x
*Hypergeometric2F1[-3/2, 1/4, 5/4, -1/5*x^4]
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,x^6+2\,x^4+15\,x^2+10\right)\sqrt{x^4+5}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2, x)
```

maple [C] time = 0.02, size = 192, normalized size = 0.96

$$\frac{3\sqrt{x^4+5}\,x^5}{7}+\frac{2\sqrt{x^4+5}\,x^3}{5}+\frac{45\sqrt{x^4+5}\,x}{7}+\frac{12\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\mathrm{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^2,x)
```

```
[Out] 3/7*(x^4+5)^(1/2)*x^5+45/7*(x^4+5)^(1/2)*x+12/7*5^(1/2)/(I*5^(1/2))^(1/2)*(
-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*Ellipti
cF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)-10*(x^4+5)^(1/2)/x+2/5*(x^4+5)^(1/2)*
x^3+24/5*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+2
5)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)-Ellipt
icE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2, x)
```

mupad [B] time = 0.53, size = 48, normalized size = 0.24

$$15\,\sqrt{5}\,x\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{x^4}{5}\right)+\frac{2\left(x^4+5\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{5}{4};\ -\frac{1}{4};\ -\frac{5}{x^4}\right)}{5\,x\left(\frac{5}{x^4}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^2,x)
```

```
[Out] 15*5^(1/2)*x*hypergeom([-3/2, 1/4], 5/4, -x^4/5) + (2*(x^4 + 5)^(3/2)*hyper
geom([-3/2, -5/4], -1/4, -5/x^4))/(5*x*(5/x^4 + 1)^(3/2))
```

sympy [C] time = 4.26, size = 160, normalized size = 0.80

$$\frac{3\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)}+\frac{15\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{5\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(-\frac{1}{2}\right.}{2x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**2,x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*
exp_polar(I*pi)/5)/(2*gamma(7/4)) + 15*sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/
4), (5/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(5/4)) + 5*sqrt(5)*gamma(-1/4)*
hyper((-1/2, -1/4), (3/4,), x**4*exp_polar(I*pi)/5)/(2*x*gamma(3/4))
```

3.31 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4} dx$

Optimal. Leaf size=201

$$-\frac{2(27-2x^2)\sqrt{x^4+5}}{3x}+\frac{36x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{2\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}-\frac{36\sqrt[4]{5}\left(x^2+\right.}{}$$

```
[Out] -1/15*(-9*x^2+10)*(x^4+5)^(3/2)/x^3-2/3*(-2*x^2+27)*(x^4+5)^(1/2)/x+36*x*(x
^4+5)^(1/2)/(x^2+5^(1/2))-36*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)
/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^
(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+2/3*5^(1
/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ell
ipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(27+2*5^(1/2
))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1272, 1198, 220, 1196}

$$-\frac{(10-9x^2)(x^4+5)^{3/2}}{15x^3}-\frac{2(27-2x^2)\sqrt{x^4+5}}{3x}+\frac{36x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{2\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\right.}{3\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^4,x]
```

```
[Out] (-2*(27 - 2*x^2)*Sqrt[5 + x^4])/(3*x) + (36*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2
) - ((10 - 9*x^2)*(5 + x^4)^(3/2))/(15*x^3) - (36*5^(1/4)*(Sqrt[5] + x^2)*S
qrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[
5 + x^4] + (2*5^(1/4)*(27 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt
[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = -\frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} - \frac{2}{5}\int \frac{(-45-10x^2)\sqrt{5+x^4}}{x^2}\,dx$$

$$= -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} - \frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} + \frac{4}{15}\int \frac{50+135x^2}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} - \frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} - (36\sqrt{5})\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \frac{1}{3}($$

$$= -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} + \frac{36x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} - \frac{36\sqrt[4]{5}(\sqrt{5}+}{}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.27

$$-\frac{5\sqrt{5}\left(2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{x^4}{5}\right)+9x^2\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^4,x]
[Out] (-5*Sqrt[5]*(2*Hypergeometric2F1[-3/2, -3/4, 1/4, -1/5*x^4] + 9*x^2*Hyperge
ometric2F1[-3/2, -1/4, 3/4, -1/5*x^4]))/(3*x^3)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(3x^6+2x^4+15x^2+10)\sqrt{x^4+5}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="fricas")
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4+5)^{\frac{3}{2}}(3x^2+2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4, x)
```

maple [C] time = 0.02, size = 192, normalized size = 0.96

$$\frac{3\sqrt{x^4+5}\,x^3}{5}+\frac{2\sqrt{x^4+5}\,x}{3}+\frac{8\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\operatorname{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{15\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{15\sqrt{x^4+5}}{x}-\frac{10\sqrt{x^4}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^4,x)
```

```
[Out] -15*(x^4+5)^(1/2)/x+3/5*(x^4+5)^(1/2)*x^3+36/5*I/(I*5^(1/2))^(1/2)*(-5*I*5^
(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5
*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I
))-10/3*(x^4+5)^(1/2)/x^3+2/3*(x^4+5)^(1/2)*x+8/15*5^(1/2)/(I*5^(1/2))^(1/2
)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*Elli
pticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4+5\right)^{3/2}\,\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^4,x)
```

```
[Out] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^4, x)
```

sympy [C] time = 4.12, size = 163, normalized size = 0.81

$$\frac{3\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}+\frac{15\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4x\Gamma\left(\frac{3}{4}\right)}+\frac{5\sqrt{5}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\\\end{matrix}\right.}{2x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**4,x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp
_polar(I*pi)/5)/(2*gamma(5/4)) + 15*sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4),
 (3/4,), x**4*exp_polar(I*pi)/5)/(4*x*gamma(3/4)) + 5*sqrt(5)*gamma(-3/4)*h
yper((-3/4, -1/2), (1/4,), x**4*exp_polar(I*pi)/5)/(2*x**3*gamma(1/4))
```

3.32 $\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{1}{3}\sqrt{x^4+5}\,x^4+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{3}{8}\sqrt{x^4+5}\,x^6-\frac{5}{48}\left(27x^2+32\right)\sqrt{x^4+5}$$

```
[Out] 225/16*arcsinh(1/5*x^2*5^(1/2))+1/3*x^4*(x^4+5)^(1/2)+3/8*x^6*(x^4+5)^(1/2)
-5/48*(27*x^2+32)*(x^4+5)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1252, 833, 780, 215}

$$\frac{3}{8}\sqrt{x^4+5}\,x^6+\frac{1}{3}\sqrt{x^4+5}\,x^4-\frac{5}{48}\left(27x^2+32\right)\sqrt{x^4+5}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x^4*Sqrt[5 + x^4])/3 + (3*x^6*Sqrt[5 + x^4])/8 - (5*(32 + 27*x^2)*Sqrt[5 +
 x^4])/48 + (225*ArcSinh[x^2/Sqrt[5]])/16
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3(2+3x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{8}x^6\sqrt{5+x^4} + \frac{1}{8}\operatorname{Subst}\left(\int \frac{x^2(-45+8x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{3}x^4\sqrt{5+x^4} + \frac{3}{8}x^6\sqrt{5+x^4} + \frac{1}{24}\operatorname{Subst}\left(\int \frac{(-80-135x)x}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{3}x^4\sqrt{5+x^4} + \frac{3}{8}x^6\sqrt{5+x^4} - \frac{5}{48}\left(32+27x^2\right)\sqrt{5+x^4} + \frac{225}{16}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{3}x^4\sqrt{5+x^4} + \frac{3}{8}x^6\sqrt{5+x^4} - \frac{5}{48}\left(32+27x^2\right)\sqrt{5+x^4} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.66

$$\frac{1}{48}\left(675\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\sqrt{x^4+5}\left(18x^6+16x^4-135x^2-160\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(-160 - 135*x^2 + 16*x^4 + 18*x^6) + 675*ArcSinh[x^2/Sqrt[5]
])/48
```

fricas [A] time = 0.80, size = 43, normalized size = 0.64

$$\frac{1}{48}\left(18x^6+16x^4-135x^2-160\right)\sqrt{x^4+5}-\frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 - 135*x^2 - 160)*sqrt(x^4 + 5) - 225/16*log(-x^2 + sq
rt(x^4 + 5))
```

giac [A] time = 0.19, size = 46, normalized size = 0.69

$$\frac{1}{48}\sqrt{x^4+5}\left(\left(2\left(9x^2+8\right)x^2-135\right)x^2-160\right)-\frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 - 135)*x^2 - 160) - 225/16*log(-x^2
+ sqrt(x^4 + 5))
```

maple [A] time = 0.02, size = 51, normalized size = 0.76

$$\frac{3\sqrt{x^4+5}\,x^6}{8}-\frac{45\sqrt{x^4+5}\,x^2}{16}+\frac{225\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}+\frac{\sqrt{x^4+5}\,\left(x^4-10\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 3/8*(x^4+5)^(1/2)*x^6-45/16*(x^4+5)^(1/2)*x^2+225/16*arcsinh(1/5*5^(1/2)*x^
2)+1/3*(x^4+5)^(1/2)*(x^4-10)
```

maxima [A] time = 1.31, size = 104, normalized size = 1.55

$$\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}-5\sqrt{x^4+5}-\frac{75\left(\frac{5\sqrt{x^4+5}}{x^2}-\frac{3\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) - 5*sqrt(x^4 + 5) - 75/16*(5*sqrt(x^4 + 5)/x^2 - 3*(x^4
 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*log(sqrt(
x^4 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.59, size = 38, normalized size = 0.57

$$\frac{225\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}-\sqrt{x^4+5}\,\left(-\frac{3\,x^6}{8}-\frac{x^4}{3}+\frac{45\,x^2}{16}+\frac{10}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] (225*asinh((5^(1/2)*x^2)/5))/16 - (x^4 + 5)^(1/2)*((45*x^2)/16 - x^4/3 - (3
*x^6)/8 + 10/3)
```

sympy [A] time = 7.07, size = 85, normalized size = 1.27

$$\frac{3x^{10}}{8\sqrt{x^4+5}}-\frac{15x^6}{16\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{3}-\frac{225x^2}{16\sqrt{x^4+5}}-\frac{10\sqrt{x^4+5}}{3}+\frac{225\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) - 15*x**6/(16*sqrt(x**4 + 5)) + x**4*sqrt(x**4 +
 5)/3 - 225*x**2/(16*sqrt(x**4 + 5)) - 10*sqrt(x**4 + 5)/3 + 225*asinh(sqrt
(5)*x**2/5)/16
```

3.33 $\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}} \, dx$

Optimal. Leaf size=51

$$\frac{1}{2}\sqrt{x^4+5}\,x^4-\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{1}{2}\left(10-x^2\right)\sqrt{x^4+5}$$

```
[Out] -5/2*arcsinh(1/5*x^2*5^(1/2))+1/2*x^4*(x^4+5)^(1/2)-1/2*(-x^2+10)*(x^4+5)^(
1/2)
```

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1252, 833, 780, 215}

$$\frac{1}{2}\sqrt{x^4+5}\,x^4-\frac{1}{2}\left(10-x^2\right)\sqrt{x^4+5}-\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x^4*Sqrt[5 + x^4])/2 - ((10 - x^2)*Sqrt[5 + x^4])/2 - (5*ArcSinh[x^2/Sqrt[
5]])/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{x^5 \left(2+3x^2\right)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(2+3x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}x^4\sqrt{5+x^4} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{x(-30+6x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}x^4\sqrt{5+x^4} - \frac{1}{2}\left(10-x^2\right)\sqrt{5+x^4} - \frac{5}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}x^4\sqrt{5+x^4} - \frac{1}{2}\left(10-x^2\right)\sqrt{5+x^4} - \frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.69

$$\frac{1}{2}\left(\sqrt{x^4+5}\left(x^4+x^2-10\right) - 5\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(-10 + x^2 + x^4) - 5*ArcSinh[x^2/Sqrt[5]])/2
```

fricas [A] time = 0.70, size = 34, normalized size = 0.67

$$\frac{1}{2}\left(x^4+x^2-10\right)\sqrt{x^4+5} + \frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/2*(x^4 + x^2 - 10)*sqrt(x^4 + 5) + 5/2*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.22, size = 37, normalized size = 0.73

$$\frac{1}{2}\sqrt{x^4+5}\left(\left(x^2+1\right)x^2-10\right) + \frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 - 10) + 5/2*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{\sqrt{x^4+5}\,x^2}{2} - \frac{5\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} + \frac{\sqrt{x^4+5}\,\left(x^4-10\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5)^(1/2),x)
[Out] 1/2*(x^4+5)^(1/2)*(x^4-10)+1/2*(x^4+5)^(1/2)*x^2-5/2*arcsinh(1/5*5^(1/2)*x^
2)
```

maxima [A] time = 1.22, size = 76, normalized size = 1.49

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}} - \frac{15}{2}\sqrt{x^4+5} + \frac{5\sqrt{x^4+5}}{2\,x^2\left(\frac{x^4+5}{x^4}-1\right)} - \frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) + \frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) - 15/2*sqrt(x^4 + 5) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5
)/x^4 - 1)) - 5/4*log(sqrt(x^4 + 5)/x^2 + 1) + 5/4*log(sqrt(x^4 + 5)/x^2 -
1)
```

mupad [B] time = 0.31, size = 32, normalized size = 0.63

$$\sqrt{x^4+5}\,\left(\frac{x^4}{2}+\frac{x^2}{2}-5\right)-\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 - 5) - (5*asinh((5^(1/2)*x^2)/5))/2
```

sympy [A] time = 5.52, size = 66, normalized size = 1.29

$$\frac{x^6}{2\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{5x^2}{2\sqrt{x^4+5}}-5\sqrt{x^4+5}-\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) + x**4*sqrt(x**4 + 5)/2 + 5*x**2/(2*sqrt(x**4 + 5))
 - 5*sqrt(x**4 + 5) - 5*asinh(sqrt(5)*x**2/5)/2
```

3.34 $\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}} \, dx$

Optimal. Leaf size=35

$$\frac{1}{4}\left(3x^2+4\right)\sqrt{x^4+5}-\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -15/4*arcsinh(1/5*x^2*5^(1/2))+1/4*(3*x^2+4)*(x^4+5)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1252, 780, 215}

$$\frac{1}{4}\left(3x^2+4\right)\sqrt{x^4+5}-\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 - (15*ArcSinh[x^2/Sqrt[5]])/4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(2+3x^2\right)}{\sqrt{5+x^4}} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(2+3x)}{\sqrt{5+x^2}} \, dx, x, x^2\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}-\frac{15}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}} \, dx, x, x^2\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}-\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.97

$$\frac{1}{4}\left(\left(3x^2+4\right)\sqrt{x^4+5}-15\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4] - 15*ArcSinh[x^2/Sqrt[5]])/4
```

fricas [A] time = 0.51, size = 33, normalized size = 0.94

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.22, size = 33, normalized size = 0.94

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$\frac{3\sqrt{x^4+5}\,x^2}{4}-\frac{15\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}+\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5)^(1/2),x)
[Out] 3/4*(x^4+5)^(1/2)*x^2-15/4*arcsinh(1/5*5^(1/2)*x^2)+(x^4+5)^(1/2)
```

maxima [B] time = 1.16, size = 65, normalized size = 1.86

$$\sqrt{x^4+5}+\frac{15\sqrt{x^4+5}}{4x^2\left(\frac{x^4+5}{x^4}-1\right)}-\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) - 15/8*log(sqr
t(x^4 + 5)/x^2 + 1) + 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.49, size = 27, normalized size = 0.77

$$\sqrt{x^4+5}\,\left(\frac{3x^2}{4}+1\right)-\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
[Out] (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (15*asinh((5^(1/2)*x^2)/5))/4
```

sympy [A] time = 4.04, size = 53, normalized size = 1.51

$$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{15x^2}{4\sqrt{x^4+5}}+\sqrt{x^4+5}-\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) - 1
5*asinh(sqrt(5)*x**2/5)/4
```

3.35 $\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=24

$$\frac{3\sqrt{x^4+5}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] arcsinh(1/5*x^2*5^(1/2))+3/2*(x^4+5)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1248, 641, 215}

$$\frac{3\sqrt{x^4+5}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(2+3x^2\right)}{\sqrt{5+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= \frac{3\sqrt{5+x^4}}{2}+\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= \frac{3\sqrt{5+x^4}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$\frac{3\sqrt{x^4+5}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]
```

fricas [A] time = 0.66, size = 26, normalized size = 1.08

$$\frac{3}{2}\sqrt{x^4+5} - \log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

giac [A] time = 0.20, size = 26, normalized size = 1.08

$$\frac{3}{2}\sqrt{x^4+5} - \log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
[Out] 3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 20, normalized size = 0.83

$$\operatorname{arcsinh}\left(\frac{\sqrt{5}\, x^2}{5}\right) + \frac{3\sqrt{x^4+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5)^(1/2),x)
[Out] arcsinh(1/5*5^(1/2)*x^2)+3/2*(x^4+5)^(1/2)
```

maxima [B] time = 1.12, size = 42, normalized size = 1.75

$$\frac{3}{2}\sqrt{x^4+5} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
[Out] 3/2*sqrt(x^4 + 5) + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/
x^2 - 1)
```

mupad [B] time = 0.29, size = 19, normalized size = 0.79

$$\operatorname{asinh}\left(\frac{\sqrt{5}\, x^2}{5}\right) + \frac{3\sqrt{x^4+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
[Out] asinh((5^(1/2)*x^2)/5) + (3*(x^4 + 5)^(1/2))/2
```

sympy [A] time = 2.13, size = 22, normalized size = 0.92

$$\frac{3\sqrt{x^4+5}}{2} + \operatorname{asinh}\left(\frac{\sqrt{5}\, x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(x**4 + 5)/2 + asinh(sqrt(5)*x**2/5)
```

3.36 $\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=38

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))-1/5*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1252, 844, 215, 266, 63, 207}

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
```

```
[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{2+3x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) + \text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5]
```

fricas [A] time = 0.70, size = 41, normalized size = 1.08

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - \frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 3/2*log(-x^2 + sqrt(x^4 +
 5))
```

giac [B] time = 0.20, size = 61, normalized size = 1.61

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) - \frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/5*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
+ 5))) - 3/2*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 30, normalized size = 0.79

$$\frac{3\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} - \frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5)^(1/2),x)
```

```
[Out] 3/2*arcsinh(1/5*5^(1/2)*x^2)-1/5*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

maxima [B] time = 1.17, size = 67, normalized size = 1.76

$$\frac{1}{10}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/
4*log(sqrt(x^4 + 5)/x^2 + 1) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.61, size = 30, normalized size = 0.79

$$\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x*(x^4 + 5)^(1/2)),x)
```

```
[Out] (3*asinh((5^(1/2)*x^2)/5))/2 - (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))
/5
```

sympy [A] time = 6.01, size = 31, normalized size = 0.82

$$-\frac{\sqrt{5}\,\mathrm{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{5}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5)**(1/2),x)
```

```
[Out] -sqrt(5)*asinh(sqrt(5)/x**2)/5 + 3*asinh(sqrt(5)*x**2/5)/2
```

3.37 $\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=42

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}-\frac{\sqrt{x^4+5}}{5x^2}$$

```
[Out] -3/10*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/5*(x^4+5)^(1/2)/x^2
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1252, 807, 266, 63, 207}

$$-\frac{\sqrt{x^4+5}}{5x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
```

```
[Out] -Sqrt[5 + x^4]/(5*x^2) - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x^2\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\
&= -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\
&= -\frac{\sqrt{5+x^4}}{5x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}-\frac{\sqrt{x^4+5}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
```

```
[Out] -1/5*Sqrt[5 + x^4]/x^2 - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])
```

fricas [A] time = 0.77, size = 47, normalized size = 1.12

$$\frac{3\sqrt{5}\,x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-2\,x^2-2\sqrt{x^4+5}}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2 - 2*sqrt(x^
4 + 5))/x^2
```

giac [B] time = 0.22, size = 66, normalized size = 1.57

$$\frac{3}{10}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{2}{\left(x^2-\sqrt{x^4+5}\right)^2-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 3/10*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + 2/((x^2 - sqrt(x^4 + 5))^2 - 5)
```

maple [A] time = 0.01, size = 31, normalized size = 0.74

$$-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}-\frac{\sqrt{x^4+5}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5)^(1/2),x)
```

```
[Out] -3/10*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/5*(x^4+5)^(1/2)/x^2
```

maxima [A] time = 1.26, size = 47, normalized size = 1.12

$$\frac{3}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{\sqrt{x^4+5}}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 1/
5*sqrt(x^4 + 5)/x^2
```

mupad [B] time = 0.33, size = 31, normalized size = 0.74

$$-\frac{3\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{10}-\frac{\sqrt{x^4+5}}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^3*(x^4 + 5)^(1/2)),x)
```

```
[Out] - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/10 - (x^4 + 5)^(1/2)/(5*x^
2)
```

sympy [A] time = 3.60, size = 31, normalized size = 0.74

$$-\frac{\sqrt{1+\frac{5}{x^4}}}{5}-\frac{3\sqrt{5}\,\mathrm{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5)**(1/2),x)
```

```
[Out] -sqrt(1 + 5/x**4)/5 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/10
```

3.38 $\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}-\frac{3\sqrt{x^4+5}}{10x^2}$$

```
[Out] 1/50*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/10*(x^4+5)^(1/2)/x^4-3/10
*(x^4+5)^(1/2)/x^2
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1252, 835, 807, 266, 63, 207}

$$-\frac{3\sqrt{x^4+5}}{10x^2}-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

```
[Out] -Sqrt[5 + x^4]/(10*x^4) - (3*Sqrt[5 + x^4])/(10*x^2) + ArcTanh[Sqrt[5 + x^4
]/Sqrt[5]]/(10*Sqrt[5])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
```

```
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x^3\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{1}{20}\operatorname{Subst}\left(\int \frac{-30+2x}{x^2\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{20}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.84

$$\frac{\sqrt{5}\,x^4\tanh^{-1}\left(\sqrt{\frac{x^4}{5}+1}\right)-5\left(3x^2+1\right)\sqrt{x^4+5}}{50x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

```
[Out] (-5*(1 + 3*x^2)*Sqrt[5 + x^4] + Sqrt[5]*x^4*ArcTanh[Sqrt[1 + x^4/5]])/(50*x
^4)
```

fricas [A] time = 0.58, size = 50, normalized size = 0.86

$$\frac{\sqrt{5}\,x^4\log\left(\frac{\sqrt{5}+\sqrt{x^4+5}}{x^2}\right)-15\,x^4-5\,\sqrt{x^4+5}\left(3\,x^2+1\right)}{50\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/50*(sqrt(5)*x^4*log((sqrt(5) + sqrt(x^4 + 5))/x^2) - 15*x^4 - 5*sqrt(x^4
+ 5)*(3*x^2 + 1))/x^4
```

giac [B] time = 0.22, size = 114, normalized size = 1.97

$$-\frac{1}{50}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5x^2-5\sqrt{x^4+5}-75}{5\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/50*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^
4 + 5))) + 1/5*((x^2 - sqrt(x^4 + 5))^3 + 15*(x^2 - sqrt(x^4 + 5))^2 + 5*x^
2 - 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5))^2 - 5)^2
```

maple [A] time = 0.02, size = 43, normalized size = 0.74

$$\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}-\frac{3\sqrt{x^4+5}}{10x^2}-\frac{\sqrt{x^4+5}}{10x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^5/(x^4+5)^(1/2),x)
```

```
[Out] -1/10*(x^4+5)^(1/2)/x^4+1/50*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-3/10*(x
^4+5)^(1/2)/x^2
```

maxima [A] time = 1.12, size = 59, normalized size = 1.02

$$-\frac{1}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{10\,x^2}-\frac{\sqrt{x^4+5}}{10\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) -
3/10*sqrt(x^4 + 5)/x^2 - 1/10*sqrt(x^4 + 5)/x^4
```

mupad [B] time = 0.69, size = 43, normalized size = 0.74

$$\frac{\sqrt{5}\ \operatorname{atanh}\left(\frac{\sqrt{5}\ \sqrt{x^4+5}}{5}\right)}{50}-\frac{3\sqrt{x^4+5}}{10\,x^2}-\frac{\sqrt{x^4+5}}{10\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^5*(x^4 + 5)^(1/2)),x)
```

```
[Out] (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/50 - (3*(x^4 + 5)^(1/2))/(10*x
^2) - (x^4 + 5)^(1/2)/(10*x^4)
```

sympy [A] time = 14.33, size = 88, normalized size = 1.52

$$\frac{\sqrt{5}\left(-\frac{\log\left(\sqrt{\frac{x^4}{5}+1}-1\right)}{4}+\frac{\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{4}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}+1\right)}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}-1\right)}\right)}{25}-\frac{3\sqrt{5}\sqrt{5x^4+25}}{50x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**5/(x**4+5)**(1/2),x)
```

```
[Out] sqrt(5)*(-log(sqrt(x**4/5 + 1) - 1)/4 + log(sqrt(x**4/5 + 1) + 1)/4 - 1/(4*
(sqrt(x**4/5 + 1) + 1)) - 1/(4*(sqrt(x**4/5 + 1) - 1)))/25 - 3*sqrt(5)*sqrt
(5*x**4 + 25)/(50*x**2)
```

3.39 $\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=185

$$\frac{2}{3}\sqrt{x^4+5}\,x+\frac{3}{5}\sqrt{x^4+5}\,x^3-\frac{9\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}-\frac{\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{6\sqrt{x^4+5}}+\frac{9\sqrt[4]{5}\left(x^2\right.}{}$$

```
[Out] 2/3*x*(x^4+5)^(1/2)+3/5*x^3*(x^4+5)^(1/2)-9*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+9
*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4))
)*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5
)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-1/6*5^(1/4)*(cos(2*arctan(1/5*x*5^(3
/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^
(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(27+2*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)
^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1280, 1198, 220, 1196}

$$\frac{3}{5}\sqrt{x^4+5}\,x^3-\frac{9\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{2}{3}\sqrt{x^4+5}\,x-\frac{\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{6\sqrt{x^4+5}}+\frac{9\sqrt[4]{5}\left(x^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (2*x*Sqrt[5 + x^4])/3 + (3*x^3*Sqrt[5 + x^4])/5 - (9*x*Sqrt[5 + x^4])/(Sqrt
[5] + x^2) + (9*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*E
llipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] - (5^(1/4)*(27 + 2*Sqrt[5
])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5
^(1/4)], 1/2])/(6*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(2+3x^2\right)}{\sqrt{5+x^4}}\,dx &= \frac{3}{5}x^3\sqrt{5+x^4}-\frac{1}{5}\int \frac{x^2\left(45-10x^2\right)}{\sqrt{5+x^4}}\,dx\\
&= \frac{2}{3}x\sqrt{5+x^4}+\frac{3}{5}x^3\sqrt{5+x^4}+\frac{1}{15}\int \frac{-50-135x^2}{\sqrt{5+x^4}}\,dx\\
&= \frac{2}{3}x\sqrt{5+x^4}+\frac{3}{5}x^3\sqrt{5+x^4}+\left(9\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{3}\left(10+27\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\
&= \frac{2}{3}x\sqrt{5+x^4}+\frac{3}{5}x^3\sqrt{5+x^4}-\frac{9x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{9\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 74, normalized size = 0.40

$$\frac{1}{15}x\left(-10\sqrt{5}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)-9\sqrt{5}\,x^2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)+\left(9x^2+10\right)\sqrt{x^4+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x*((10 + 9*x^2)*Sqrt[5 + x^4] - 10*Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4
, -1/5*x^4] - 9*Sqrt[5]*x^2*Hypergeometric2F1[1/2, 3/4, 7/4, -1/5*x^4]))/15
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{3\,x^6+2\,x^4}{\sqrt{x^4+5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)/sqrt(x^4 + 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^2+2\right)x^4}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5), x)
```

maple [C] time = 0.02, size = 168, normalized size = 0.91

$$\frac{3\sqrt{x^4+5}\,x^3}{5}+\frac{2\sqrt{x^4+5}\,x}{3}-\frac{2\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\operatorname{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{15\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{9i\sqrt{-5i\sqrt{5}\,x^2+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 3/5*(x^4+5)^(1/2)*x^3-9/5*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(
5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/2)*(I*5^(1/2))^
(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))+2/3*(x^4+5)^(1/2)*
x-2/15*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x
^2+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(3x^2+2)x^4}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^4\,(3x^2+2)}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] int((x^4*(3*x^2 + 2))/(x^4 + 5)^(1/2), x)
```

sympy [C] time = 2.55, size = 75, normalized size = 0.41

$$\frac{3\sqrt{5}\,x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5
)/(20*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4
*exp_polar(I*pi)/5)/(10*gamma(9/4))
```

3.40 $\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=166

$$\sqrt{x^4+5}\,x+\frac{2\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{\sqrt[4]{5}\left(2-\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{x^4+5}}-\frac{2\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(\right.}{\sqrt{x^4+5}}$$

```
[Out] x*(x^4+5)^(1/2)+2*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-2*5^(1/4)*(cos(2*arctan(1/5
*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1
/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(
x^4+5)^(1/2)+1/2*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arcta
n(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(2-5^
(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1280, 1198, 220, 1196}

$$\frac{2\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\sqrt{x^4+5}\,x+\frac{\sqrt[4]{5}\left(2-\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{x^4+5}}-\frac{2\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(\right.}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] x*Sqrt[5 + x^4] + (2*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (2*5^(1/4)*(Sqrt[5]
 + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/
2])/Sqrt[5 + x^4] + (5^(1/4)*(2 - Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(
Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
```

```
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = x\sqrt{5+x^4} - \frac{1}{3}\int \frac{15-6x^2}{\sqrt{5+x^4}}\,dx$$
$$= x\sqrt{5+x^4} - \left(2\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx - \left(5-2\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$
$$= x\sqrt{5+x^4} + \frac{2x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{2\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{\sqrt[4]{5}\left(2-\sqrt{\right.}}{}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.40

$$-\sqrt{5}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right) + \frac{2x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)}{3\sqrt{5}} + \sqrt{x^4+5}\,x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] x*Sqrt[5 + x^4] - Sqrt[5]*x*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4] + (2
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -1/5*x^4])/(3*Sqrt[5])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{3\,x^4+2\,x^2}{\sqrt{x^4+5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] integral((3*x^4 + 2*x^2)/sqrt(x^4 + 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^2+2\right)x^2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5), x)
```

maple [C] time = 0.01, size = 155, normalized size = 0.93

$$\sqrt{x^4+5}\;x - \frac{\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\;\mathrm{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}} + \frac{2i\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] (x^4+5)^(1/2)*x-1/5*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(
5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(
1/2)*x,I)+2/5*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*
x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)-E
llipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^2+2\right)x^2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(3\,x^2+2\right)}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] int((x^2*(3*x^2 + 2))/(x^4 + 5)^(1/2), x)
```

sympy [C] time = 2.36, size = 75, normalized size = 0.45

$$\frac{3\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)
/(20*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*e
xp_polar(I*pi)/5)/(10*gamma(7/4))
```

3.41 $\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=155

$$\frac{3\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right.\right.}{\sqrt{x^4+5}}$$

```
[Out] 3*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-3*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^
(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1
/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/1
0*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ellip
ticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2+3*5^(1/2))*
((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1198, 220, 1196}

$$\frac{3\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right.\right.}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/Sqrt[5 + x^4],x]
```

```
[Out] (3*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (3*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]
+ ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellipt
icF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = -\left((3\sqrt{5})\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx\right) + \left(2+3\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= \frac{3x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{3\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x}{(\sqrt{5}+x}}}{2\sqrt[4]{5}\sqrt{5+}}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.31

$$\frac{x\left(2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)+x^2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/Sqrt[5 + x^4],x]
```

```
[Out] (x*(2*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4] + x^2*Hypergeometric2F1[1/
2, 3/4, 7/4, -1/5*x^4]))/Sqrt[5]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{3\,x^2+2}{\sqrt{x^4+5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

maple [C] time = 0.01, size = 146, normalized size = 0.94

$$\frac{2\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\mathrm{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3i\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\left(-\mathrm{Ellipti}\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 3/5*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1
/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)-EllipticE(1
/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))+2/25*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(
```

```
1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5
^(1/2)*(I*5^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^4 + 5)^(1/2),x)
```

```
[Out] int((3*x^2 + 2)/(x^4 + 5)^(1/2), x)
```

sympy [C] time = 1.71, size = 73, normalized size = 0.47

$$\frac{3\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)
/(20*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_
polar(I*pi)/5)/(10*gamma(5/4))
```

3.42 $\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=173

$$-\frac{2\sqrt{x^4+5}}{5x}+\frac{2\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}+\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{x^4+5}}-\frac{2\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ta}\right.}{5^{3/4}\sqrt{x^4+5}}$$

```
[Out] -2/5*(x^4+5)^(1/2)/x+2/5*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-2/5*5^(1/4)*(cos(2*a
rctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2
*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2
)^(1/2)/(x^4+5)^(1/2)+1/10*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arc
tan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^
2+5^(1/2))*(2+3*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(1/4)/(x^4+5)^(1
/2)
```

Rubi [A] time = 0.06, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1282, 1198, 220, 1196}

$$\frac{2\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}-\frac{2\sqrt{x^4+5}}{5x}+\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{x^4+5}}-\frac{2\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{tan}\right.}{5^{3/4}\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^2*Sqrt[5 + x^4]),x]
```

```
[Out] (-2*Sqrt[5 + x^4])/(5*x) + (2*x*Sqrt[5 + x^4])/(5*(Sqrt[5] + x^2)) - (2*(Sq
rt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)
], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) + ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5
 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(3/4)*
Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = -\frac{2\sqrt{5+x^4}}{5x} - \frac{1}{5}\int \frac{-15-2x^2}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{2\sqrt{5+x^4}}{5x} - \frac{2\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{\sqrt{5}} + \frac{1}{5}\left(15+2\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{2\sqrt{5+x^4}}{5x} + \frac{2x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)} - \frac{2\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}} + \frac{\left(2+3\sqrt{5}\right)(}{}$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.31

$$\frac{3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{\sqrt{5}} - \frac{2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{\sqrt{5}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^2*Sqrt[5 + x^4]),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1/2, 3/4, -1/5*x^4])/(Sqrt[5]*x) + (3*x*Hyperge
ometric2F1[1/4, 1/2, 5/4, -1/5*x^4])/Sqrt[5]
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^6+5\,x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/(x^6 + 5*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)
```

maple [C] time = 0.02, size = 158, normalized size = 0.91

$$\frac{3\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\mathrm{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5}, i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}} - \frac{2\sqrt{x^4+5}}{5x} + \frac{2i\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^2/(x^4+5)^(1/2),x)
```

```
[Out] 3/25*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2
+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)-2/5*(
x^4+5)^(1/2)/x+2/25*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^
(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*
x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)
```

mupad [B] time = 0.50, size = 48, normalized size = 0.28

$$\frac{3\,\sqrt{5}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{x^4}{5}\right)}{5}-\frac{2\,\sqrt{\frac{5}{x^4}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{5}{x^4}\right)}{3\,x\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^2*(x^4 + 5)^(1/2)),x)
```

```
[Out] (3*5^(1/2)*x*hypergeom([1/4, 1/2], 5/4, -x^4/5))/5 - (2*(5/x^4 + 1)^(1/2)*h
ypergeom([1/2, 3/4], 7/4, -5/x^4))/(3*x*(x^4 + 5)^(1/2))
```

sympy [C] time = 1.82, size = 75, normalized size = 0.43

$$\frac{3\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi)/5)/(2
0*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_pol
ar(I*pi)/5)/(10*x*gamma(3/4))
```

3.43 $\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=189

$$-\frac{3\sqrt{x^4+5}}{5x}-\frac{2\sqrt{x^4+5}}{15x^3}+\frac{3\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}-\frac{\left(2-9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{30\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\left(x^2+\sqrt{5}\right)\sqrt{}}{}$$

```
[Out] -2/15*(x^4+5)^(1/2)/x^3-3/5*(x^4+5)^(1/2)/x+3/5*x*(x^4+5)^(1/2)/(x^2+5^(1/2
))-3/5*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^
(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(
(x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-1/150*(cos(2*arctan(1/5*x*5^(3
/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^
(3/4))),1/2*2^(1/2))*(2-9*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^
(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1282, 1198, 220, 1196}

$$\frac{3\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}-\frac{3\sqrt{x^4+5}}{5x}-\frac{2\sqrt{x^4+5}}{15x^3}-\frac{\left(2-9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{30\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{}{(x}}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^4*Sqrt[5 + x^4]),x]
```

```
[Out] (-2*Sqrt[5 + x^4])/(15*x^3) - (3*Sqrt[5 + x^4])/(5*x) + (3*x*Sqrt[5 + x^4])
/(5*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]
*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) - ((2 - 9*Sqr
t[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[
x/5^(1/4)], 1/2])/(30*5^(1/4)*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx &= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{1}{15}\int \frac{-45+2x^2}{x^2\sqrt{5+x^4}}\,dx\\ &= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}+\frac{1}{75}\int \frac{-10+45x^2}{\sqrt{5+x^4}}\,dx\\ &= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}-\frac{3\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{\sqrt{5}}+\frac{1}{15}\left(-2+9\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\ &= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}+\frac{3x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)}-\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}}-\frac{(2}{}\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.29

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{x^4}{5}\right)+9x^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{3\sqrt{5}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^4*Sqrt[5 + x^4]),x]
[Out] -1/3*(2*Hypergeometric2F1[-3/4, 1/2, 1/4, -1/5*x^4] + 9*x^2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -1/5*x^4])/(Sqrt[5]*x^3)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^8+5\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/(x^8 + 5*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

maple [C] time = 0.02, size = 170, normalized size = 0.90

$$-\frac{2\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\operatorname{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{375\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{3\sqrt{x^4+5}}{5x}-\frac{2\sqrt{x^4+5}}{15x^3}+\frac{3i\sqrt{-5i\sqrt{5}\,x^2+25}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^4/(x^4+5)^(1/2),x)
```

```
[Out] -3/5*(x^4+5)^(1/2)/x+3/25*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(
5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/2)*(I*5^(1/2))^
(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))-2/15*(x^4+5)^(1/2)
/x^3-2/375*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/
2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^2+2}{x^4\,\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^4*(x^4 + 5)^(1/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^4*(x^4 + 5)^(1/2)), x)
```

sympy [C] time = 2.08, size = 80, normalized size = 0.42

$$\frac{3\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20x\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**4/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(I*pi)/5)/(2
0*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4*exp_p
olar(I*pi)/5)/(10*x**3*gamma(1/4))
```

3.44 $\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(3x^2+2)\,x^4}{2\sqrt{x^4+5}}+\frac{1}{4}\left(9x^2+8\right)\sqrt{x^4+5}$$

```
[Out] -45/4*arcsinh(1/5*x^2*5^(1/2))-1/2*x^4*(3*x^2+2)/(x^4+5)^(1/2)+1/4*(9*x^2+8
)*(x^4+5)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1252, 819, 780, 215}

$$-\frac{(3x^2+2)\,x^4}{2\sqrt{x^4+5}}+\frac{1}{4}\left(9x^2+8\right)\sqrt{x^4+5}-\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(x^4*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + ((8 + 9*x^2)*Sqrt[5 + x^4])/4 - (45*
ArcSinh[x^2/Sqrt[5]])/4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{x^7\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(2+3x)}{\left(5+x^2\right)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{x^4\left(2+3x^2\right)}{2\sqrt{5+x^4}} + \frac{1}{10}\text{Subst}\left(\int \frac{x(20+45x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{x^4\left(2+3x^2\right)}{2\sqrt{5+x^4}} + \frac{1}{4}\left(8+9x^2\right)\sqrt{5+x^4} - \frac{45}{4}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{x^4\left(2+3x^2\right)}{2\sqrt{5+x^4}} + \frac{1}{4}\left(8+9x^2\right)\sqrt{5+x^4} - \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.88

$$\frac{3x^6+4x^4+45x^2-45\sqrt{x^4+5}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+40}{4\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (40 + 45*x^2 + 4*x^4 + 3*x^6 - 45*Sqrt[5 + x^4]*ArcSinh[x^2/Sqrt[5]])/(4*Sq
rt[5 + x^4])
```

fricas [A] time = 0.55, size = 62, normalized size = 1.07

$$\frac{30\,x^4+45\left(x^4+5\right)\log\left(-x^2+\sqrt{x^4+5}\right)+\left(3\,x^6+4\,x^4+45\,x^2+40\right)\sqrt{x^4+5}+150}{4\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(30*x^4 + 45*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + (3*x^6 + 4*x^4 + 45*
x^2 + 40)*sqrt(x^4 + 5) + 150)/(x^4 + 5)
```

giac [A] time = 0.21, size = 45, normalized size = 0.78

$$\frac{\left(\left(3\,x^2+4\right)x^2+45\right)x^2+40}{4\,\sqrt{x^4+5}} + \frac{45}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((3*x^2 + 4)*x^2 + 45)*x^2 + 40)/sqrt(x^4 + 5) + 45/4*log(-x^2 + sqrt(
x^4 + 5))
```

maple [A] time = 0.02, size = 50, normalized size = 0.86

$$\frac{3x^6}{4\sqrt{x^4+5}} + \frac{45x^2}{4\sqrt{x^4+5}} - \frac{45\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4} + \frac{x^4+10}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] 3/4*x^6/(x^4+5)^(1/2)+45/4*x^2/(x^4+5)^(1/2)-45/4*arcsinh(1/5*5^(1/2)*x^2)+
1/(x^4+5)^(1/2)*(x^4+10)
```

maxima [A] time = 1.21, size = 89, normalized size = 1.53

$$\sqrt{x^4+5} - \frac{15\left(\frac{3\left(x^4+5\right)}{x^4} - 2\right)}{4\left(\frac{\sqrt{x^4+5}}{x^2} - \frac{\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)} + \frac{5}{\sqrt{x^4+5}} - \frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2} + 1\right) + \frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 + 5) - 15/4*(3*(x^4 + 5)/x^4 - 2)/(sqrt(x^4 + 5)/x^2 - (x^4 + 5)^(
3/2)/x^6) + 5/sqrt(x^4 + 5) - 45/8*log(sqrt(x^4 + 5)/x^2 + 1) + 45/8*log(sq
rt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 1.11, size = 97, normalized size = 1.67

$$\sqrt{x^4+5}\,\left(\frac{3\,x^2}{4} + 1\right) - \frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4} + \frac{\sqrt{5}\,\left(10 + \sqrt{5}\,15\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(-x^2 + \sqrt{5}\,1\mathrm{i}\right)} - \frac{\sqrt{5}\,\left(-10 + \sqrt{5}\,15\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(x^2 + \sqrt{5}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (45*asinh((5^(1/2)*x^2)/5))/4 + (5^(1/2)*
(5^(1/2)*15i + 10)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) - (5^(1/2)*(
5^(1/2)*15i - 10)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))
```

sympy [A] time = 14.28, size = 66, normalized size = 1.14

$$\frac{3x^6}{4\sqrt{x^4+5}} + \frac{x^4}{\sqrt{x^4+5}} + \frac{45x^2}{4\sqrt{x^4+5}} - \frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4} + \frac{10}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + x**4/sqrt(x**4 + 5) + 45*x**2/(4*sqrt(x**4 + 5)
) - 45*asinh(sqrt(5)*x**2/5)/4 + 10/sqrt(x**4 + 5)
```

3.45 $\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=45

$$3\sqrt{x^4+5}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(3x^2+2)\,x^2}{2\sqrt{x^4+5}}$$

[Out] arcsinh(1/5*x^2*5^(1/2))-1/2*x^2*(3*x^2+2)/(x^4+5)^(1/2)+3*(x^4+5)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{1252, 819, 641, 215\}$

$$-\frac{(3x^2+2)\,x^2}{2\sqrt{x^4+5}}+3\sqrt{x^4+5}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]

[Out] -(x^2*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + 3*Sqrt[5 + x^4] + ArcSinh[x^2/Sqrt[5]]

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) || !ILtQ[m + 2*p + 3, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{x^5\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(2+3x)}{\left(5+x^2\right)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{x^2\left(2+3x^2\right)}{2\sqrt{5+x^4}} + \frac{1}{10}\operatorname{Subst}\left(\int \frac{10+30x}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{x^2\left(2+3x^2\right)}{2\sqrt{5+x^4}} + 3\sqrt{5+x^4} + \operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{x^2\left(2+3x^2\right)}{2\sqrt{5+x^4}} + 3\sqrt{5+x^4} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Mathematica [**A**] time = 0.02, size = 46, normalized size = 1.02

$$\frac{3x^4-2x^2+2\sqrt{x^4+5}\,\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+30}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (30 - 2*x^2 + 3*x^4 + 2*Sqrt[5 + x^4]*ArcSinh[x^2/Sqrt[5]])/(2*Sqrt[5 + x^4
])
```

fricas [**A**] time = 0.67, size = 58, normalized size = 1.29

$$-\frac{2\,x^4+2\left(x^4+5\right)\log\left(-x^2+\sqrt{x^4+5}\right)-\left(3\,x^4-2\,x^2+30\right)\sqrt{x^4+5}+10}{2\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^4 + 2*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) - (3*x^4 - 2*x^2 + 30)*
sqrt(x^4 + 5) + 10)/(x^4 + 5)
```

giac [**A**] time = 0.23, size = 39, normalized size = 0.87

$$\frac{\left(3\,x^2-2\right)x^2+30}{2\,\sqrt{x^4+5}} - \log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((3*x^2 - 2)*x^2 + 30)/sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

maple [**A**] time = 0.01, size = 37, normalized size = 0.82

$$-\frac{x^2}{\sqrt{x^4+5}} + \operatorname{arcsinh}\left(\frac{\sqrt{5}\,x^2}{5}\right) + \frac{\frac{3x^4}{2}+15}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5)^(3/2),x)
```

[Out] 3/2/(x^4+5)^(1/2)*(x^4+10)-1/(x^4+5)^(1/2)*x^2+arcsinh(1/5*5^(1/2)*x^2)

maxima [A] time = 1.41, size = 63, normalized size = 1.40

$$-\frac{x^2}{\sqrt{x^4+5}} + \frac{3}{2}\sqrt{x^4+5} + \frac{15}{2\sqrt{x^4+5}} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")

[Out] -x^2/sqrt(x^4 + 5) + 3/2*sqrt(x^4 + 5) + 15/2/sqrt(x^4 + 5) + 1/2*log(sqrt(
x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.89, size = 89, normalized size = 1.98

$$\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right) + \frac{3\sqrt{x^4+5}}{2} - \frac{\sqrt{5}\,\left(-15+\sqrt{5}\,2\mathrm{i}\right)\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)} + \frac{\sqrt{5}\,\left(15+\sqrt{5}\,2\mathrm{i}\right)\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)

[Out] asinh((5^(1/2)*x^2)/5) + (3*(x^4 + 5)^(1/2))/2 - (5^(1/2)*(5^(1/2)*2i - 15)
*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) + (5^(1/2)*(5^(1/2)*2i + 15)*(
x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))

sympy [A] time = 12.34, size = 48, normalized size = 1.07

$$\frac{3x^4}{2\sqrt{x^4+5}} - \frac{x^2}{\sqrt{x^4+5}} + \operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right) + \frac{15}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(3*x**2+2)/(x**4+5)**(3/2),x)

[Out] 3*x**4/(2*sqrt(x**4 + 5)) - x**2/sqrt(x**4 + 5) + asinh(sqrt(5)*x**2/5) + 1
5/sqrt(x**4 + 5)

3.46 $\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}} \, dx$

Optimal. Leaf size=35

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{-3x^2-2}{2\sqrt{x^4+5}}$$

```
[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))+1/2*(-3*x^2-2)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1252, 778, 215}

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3x^2+2}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(2 + 3*x^2)/(2*Sqrt[5 + x^4]) + (3*ArcSinh[x^2/Sqrt[5]])/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{\left(5+x^2\right)^{3/2}} \, dx, x, x^2\right)\\
&= -\frac{2+3x^2}{2\sqrt{5+x^4}}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}} \, dx, x, x^2\right)\\
&= -\frac{2+3x^2}{2\sqrt{5+x^4}}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.17

$$\frac{-3x^2+3\sqrt{x^4+5}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-2}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (-2 - 3*x^2 + 3*Sqrt[5 + x^4]*ArcSinh[x^2/Sqrt[5]])/(2*Sqrt[5 + x^4])
```

fricas [A] time = 0.77, size = 52, normalized size = 1.49

$$-\frac{3 x^{4} + 3 \left(x^{4} + 5\right) \log\left(-x^{2} + \sqrt{x^{4} + 5}\right) + \sqrt{x^{4} + 5} \left(3 x^{2} + 2\right) + 15}{2 \left(x^{4} + 5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^4 + 3*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + sqrt(x^4 + 5)*(3*x^2
+ 2) + 15)/(x^4 + 5)
```

giac [A] time = 0.25, size = 33, normalized size = 0.94

$$-\frac{3 x^{2} + 2}{2 \sqrt{x^{4} + 5}} - \frac{3}{2} \log\left(-x^{2} + \sqrt{x^{4} + 5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*x^2 + 2)/sqrt(x^4 + 5) - 3/2*log(-x^2 + sqrt(x^4 + 5))
```

maple [A] time = 0.01, size = 34, normalized size = 0.97

$$-\frac{3x^{2}}{2\sqrt{x^{4}+5}} + \frac{3 \operatorname{arcsinh}\left(\frac{\sqrt{5}\, x^{2}}{5}\right)}{2} - \frac{1}{\sqrt{x^{4}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] -3/2/(x^4+5)^(1/2)*x^2+3/2*arcsinh(1/5*5^(1/2)*x^2)-1/(x^4+5)^(1/2)
```

maxima [A] time = 1.23, size = 54, normalized size = 1.54

$$-\frac{3 x^{2}}{2 \sqrt{x^{4} + 5}} - \frac{1}{\sqrt{x^{4} + 5}} + \frac{3}{4} \log\left(\frac{\sqrt{x^{4} + 5}}{x^{2}} + 1\right) - \frac{3}{4} \log\left(\frac{\sqrt{x^{4} + 5}}{x^{2}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*x^2/sqrt(x^4 + 5) - 1/sqrt(x^4 + 5) + 3/4*log(sqrt(x^4 + 5)/x^2 + 1) -
 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

mupad [B] time = 0.84, size = 82, normalized size = 2.34

$$\frac{3 \operatorname{asinh}\left(\frac{\sqrt{5}\, x^{2}}{5}\right)}{2} - \frac{\sqrt{5}\, \left(2 + \sqrt{5}\, 3\mathrm{i}\right) \sqrt{x^{4} + 5}\, 1\mathrm{i}}{20 \left(-x^{2} + \sqrt{5}\, 1\mathrm{i}\right)} + \frac{\sqrt{5}\, \left(-2 + \sqrt{5}\, 3\mathrm{i}\right) \sqrt{x^{4} + 5}\, 1\mathrm{i}}{20 \left(x^{2} + \sqrt{5}\, 1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] (3*asinh((5^(1/2)*x^2)/5))/2 - (5^(1/2)*(5^(1/2)*3i + 2)*(x^4 + 5)^(1/2)*1i
)/(20*(5^(1/2)*1i - x^2)) + (5^(1/2)*(5^(1/2)*3i - 2)*(x^4 + 5)^(1/2)*1i)/(
20*(5^(1/2)*1i + x^2))
```

sympy [A] time = 10.68, size = 39, normalized size = 1.11

$$-\frac{3x^2}{2\sqrt{x^4+5}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] -3*x**2/(2*sqrt(x**4 + 5)) + 3*asinh(sqrt(5)*x**2/5)/2 - 1/sqrt(x**4 + 5)
```

3.47 $\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=20

$$\frac{2x^2-15}{10\sqrt{x^4+5}}$$

[Out] 1/10*(2*x^2-15)/(x^4+5)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1248, 637}

$$-\frac{15-2x^2}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

[In] Int[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]

[Out] -(15 - 2*x^2)/(10*Sqrt[5 + x^4])

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{\left(5+x^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{15-2x^2}{10\sqrt{5+x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{2x^2-15}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]

[Out] (-15 + 2*x^2)/(10*Sqrt[5 + x^4])

fricas [A] time = 0.72, size = 31, normalized size = 1.55

$$\frac{2\,x^4+\sqrt{x^4+5}\left(2\,x^2-15\right)+10}{10\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*(2*x^4 + sqrt(x^4 + 5)*(2*x^2 - 15) + 10)/(x^4 + 5)
```

giac [A] time = 0.24, size = 16, normalized size = 0.80

$$\frac{2\,x^2-15}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/10*(2*x^2 - 15)/sqrt(x^4 + 5)
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{2x^2-15}{10\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] 1/10*(2*x^2-15)/(x^4+5)^(1/2)
```

maxima [A] time = 1.49, size = 22, normalized size = 1.10

$$\frac{x^2}{5\,\sqrt{x^4+5}}-\frac{3}{2\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*x^2/sqrt(x^4 + 5) - 3/2/sqrt(x^4 + 5)
```

mupad [B] time = 0.16, size = 16, normalized size = 0.80

$$\frac{2\,x^2-15}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] (2*x^2 - 15)/(10*(x^4 + 5)^(1/2))
```

sympy [B] time = 7.86, size = 31, normalized size = 1.55

$$\frac{\sqrt{5}\,x^2}{5\sqrt{5x^4+25}}-\frac{3}{2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] sqrt(5)*x**2/(5*sqrt(5*x**4 + 25)) - 3/(2*sqrt(x**4 + 5))
```

3.48 $\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

```
[Out] -1/25*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+1/10*(3*x^2+2)/(x^4+5)^(1/
2)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1252, 823, 12, 266, 63, 207}

$$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*Sqrt[5 + x^4]) - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(5*Sqrt[5])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x\left(5+x^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} - \frac{1}{50}\operatorname{Subst}\left(\int -\frac{10}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} + \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} + \frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} + \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{5\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{1}{50}\left(\frac{5\left(3x^2+2\right)}{\sqrt{x^4+5}} - 2\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

```
[Out] ((5*(2 + 3*x^2))/Sqrt[5 + x^4] - 2*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/
50
```

fricas [A] time = 0.54, size = 61, normalized size = 1.33

$$\frac{15\,x^4 + 2\,\sqrt{5}\left(x^4+5\right)\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) + 5\,\sqrt{x^4+5}\left(3\,x^2+2\right) + 75}{50\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50*(15*x^4 + 2*sqrt(5)*(x^4 + 5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) + 5*
sqrt(x^4 + 5)*(3*x^2 + 2) + 75)/(x^4 + 5)
```

giac [A] time = 0.25, size = 61, normalized size = 1.33

$$\frac{1}{25}\,\sqrt{5}\,\log\left(x^2+\sqrt{5}-\sqrt{x^4+5}\right) - \frac{1}{25}\,\sqrt{5}\,\log\left(-x^2+\sqrt{5}+\sqrt{x^4+5}\right) + \frac{3\,x^2+2}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/25*sqrt(5)*log(x^2 + sqrt(5) - sqrt(x^4 + 5)) - 1/25*sqrt(5)*log(-x^2 + s
qrt(5) + sqrt(x^4 + 5)) + 1/10*(3*x^2 + 2)/sqrt(x^4 + 5)
```

maple [A] time = 0.02, size = 40, normalized size = 0.87

$$\frac{3x^2}{10\sqrt{x^4+5}} - \frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{25} + \frac{1}{5\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5)^(3/2),x)
```

```
[Out] 3/10/(x^4+5)^(1/2)*x^2+1/5/(x^4+5)^(1/2)-1/25*5^(1/2)*arctanh(5^(1/2)/(x^4+
5)^(1/2))
```

maxima [A] time = 1.41, size = 56, normalized size = 1.22

$$\frac{3\,x^2}{10\,\sqrt{x^4+5}} + \frac{1}{50}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{1}{5\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/10*x^2/sqrt(x^4 + 5) + 1/50*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(
5) + sqrt(x^4 + 5))) + 1/5/sqrt(x^4 + 5)
```

mupad [B] time = 0.48, size = 40, normalized size = 0.87

$$\frac{1}{5\,\sqrt{x^4+5}} - \frac{\sqrt{5}\ \operatorname{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{25} + \frac{3\,x^2}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x*(x^4 + 5)^(3/2)),x)
```

```
[Out] 1/(5*(x^4 + 5)^(1/2)) - (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/25 + (
3*x^2)/(10*(x^4 + 5)^(1/2))
```

sympy [B] time = 19.62, size = 212, normalized size = 4.61

$$\frac{2x^4\log\left(x^4\right)}{20\sqrt{5}\,x^4+100\sqrt{5}} - \frac{4x^4\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}\,x^4+100\sqrt{5}} - \frac{2x^4\log(5)}{20\sqrt{5}\,x^4+100\sqrt{5}} + \frac{3x^2}{10\sqrt{x^4+5}} + \frac{4\sqrt{5}\sqrt{x^4+5}}{20\sqrt{5}\,x^4+100\sqrt{5}} + \frac{10\log(}{20\sqrt{5}\,x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5)**(3/2),x)
```

```
[Out] 2*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 4*x**4*log(sqrt(x**4/5 +
 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 2*x**4*log(5)/(20*sqrt(5)*x**4 +
 100*sqrt(5)) + 3*x**2/(10*sqrt(x**4 + 5)) + 4*sqrt(5)*sqrt(x**4 + 5)/(20*s
qrt(5)*x**4 + 100*sqrt(5)) + 10*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) -
 20*log(sqrt(x**4/5 + 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 10*log(5)/(
20*sqrt(5)*x**4 + 100*sqrt(5))
```

3.49 $\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}+\frac{3x^2+2}{10x^2\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x^2}$$

```
[Out] -3/50*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+1/10*(3*x^2+2)/x^2/(x^4+5)
^(1/2)-2/25*(x^4+5)^(1/2)/x^2
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1252, 823, 807, 266, 63, 207}

$$\frac{3x^2+2}{10x^2\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x^2*Sqrt[5 + x^4]) - (2*Sqrt[5 + x^4])/(25*x^2) - (3*ArcTan
h[Sqrt[5 + x^4]/Sqrt[5]])/(10*Sqrt[5])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
```

```
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x^2\left(5+x^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{1}{50}\operatorname{Subst}\left(\int \frac{-20-15x}{x^2\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}+\frac{3}{10}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}+\frac{3}{20}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}+\frac{3}{10}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.69

$$\frac{15x^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{x^4}{5}+1\right)-4x^4-10}{50x^2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

```
[Out] (-10 - 4*x^4 + 15*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + x^4/5])/(50*x^2*S
qrt[5 + x^4])
```

fricas [A] time = 0.59, size = 77, normalized size = 1.18

$$-\frac{4\,x^6-3\,\sqrt{5}\left(x^6+5\,x^2\right)\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)+20\,x^2+\left(4\,x^4-15\,x^2+10\right)\sqrt{x^4+5}}{50\left(x^6+5\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/50*(4*x^6 - 3*sqrt(5)*(x^6 + 5*x^2)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2)
+ 20*x^2 + (4*x^4 - 15*x^2 + 10)*sqrt(x^4 + 5))/(x^6 + 5*x^2)
```

giac [A] time = 0.24, size = 82, normalized size = 1.26

$$\frac{3}{50}\,\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)-\frac{2\,x^2-15}{50\,\sqrt{x^4+5}}+\frac{2}{5\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 3/50*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) - 1/50*(2*x^2 - 15)/sqrt(x^4 + 5) + 2/5/((x^2 - sqrt(x^4 + 5))^2 -
5)
```

maple [A] time = 0.01, size = 47, normalized size = 0.72

$$-\frac{3\sqrt{5}\ \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}-\frac{2x^4+5}{25\sqrt{x^4+5}\ x^2}+\frac{3}{10\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5)^(3/2),x)
```

```
[Out] 3/10/(x^4+5)^(1/2)-3/50*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/25/x^2*(2*
x^4+5)/(x^4+5)^(1/2)
```

maxima [A] time = 1.10, size = 68, normalized size = 1.05

$$-\frac{x^2}{25\,\sqrt{x^4+5}}+\frac{3}{100}\,\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{10\,\sqrt{x^4+5}}-\frac{\sqrt{x^4+5}}{25\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/25*x^2/sqrt(x^4 + 5) + 3/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqr
t(5) + sqrt(x^4 + 5))) + 3/10/sqrt(x^4 + 5) - 1/25*sqrt(x^4 + 5)/x^2
```

mupad [B] time = 0.54, size = 47, normalized size = 0.72

$$\frac{3}{10\,\sqrt{x^4+5}}-\frac{3\,\sqrt{5}\,\operatorname{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{50}-\frac{2\,x^4+5}{25\,x^2\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^3*(x^4 + 5)^(3/2)),x)
```

```
[Out] 3/(10*(x^4 + 5)^(1/2)) - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/50
- (2*x^4 + 5)/(25*x^2*(x^4 + 5)^(1/2))
```

sympy [B] time = 12.98, size = 228, normalized size = 3.51

$$\frac{3x^4\log\left(x^4\right)}{20\sqrt{5}\,x^4+100\sqrt{5}}-\frac{6x^4\log\left(\sqrt{\frac{x^4}{5}+1}\;+1\right)}{20\sqrt{5}\,x^4+100\sqrt{5}}-\frac{3x^4\log\left(5\right)}{20\sqrt{5}\,x^4+100\sqrt{5}}+\frac{6\sqrt{5}\,\sqrt{x^4+5}}{20\sqrt{5}\,x^4+100\sqrt{5}}+\frac{15\log\left(x^4\right)}{20\sqrt{5}\,x^4+100\sqrt{5}}-\frac{30\log}{20\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 6*x**4*log(sqrt(x**4/5 +
 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 3*x**4*log(5)/(20*sqrt(5)*x**4 +
 100*sqrt(5)) + 6*sqrt(5)*sqrt(x**4 + 5)/(20*sqrt(5)*x**4 + 100*sqrt(5)) +
15*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 30*log(sqrt(x**4/5 + 1) + 1)
/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 15*log(5)/(20*sqrt(5)*x**4 + 100*sqrt(5)
) - 2/(25*sqrt(1 + 5/x**4)) - 1/(5*x**4*sqrt(1 + 5/x**4))
```

3.50 $\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=196

$$-\frac{1}{5}\sqrt{x^4+5}\,x+\frac{9\sqrt{x^4+5}\,x}{2\left(x^2+\sqrt{5}\right)}+\frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{5}\sqrt{x^4+5}}-\frac{9\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E}{2\sqrt{x^4+5}}$$

```
[Out] -1/10*x^3*(-2*x^2+15)/(x^4+5)^(1/2)-1/5*x*(x^4+5)^(1/2)+9/2*x*(x^4+5)^(1/2)
/(x^2+5^(1/2))-9/2*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arc
tan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^
2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/20*(cos(2*arctan
(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arct
an(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2+9*5^(1/2))*((x^4+5)/(x^2+5
^(1/2))^2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1276, 1280, 1198, 220, 1196}

$$-\frac{\left(15-2x^2\right)x^3}{10\sqrt{x^4+5}}+\frac{9\sqrt{x^4+5}\,x}{2\left(x^2+\sqrt{5}\right)}-\frac{1}{5}\sqrt{x^4+5}\,x+\frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{5}\sqrt{x^4+5}}-\frac{9\sqrt[4]{5}\left(x^2+\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(x^3*(15 - 2*x^2))/(10*Sqrt[5 + x^4]) - (x*Sqrt[5 + x^4])/5 + (9*x*Sqrt[5
+ x^4])/(2*(Sqrt[5] + x^2)) - (9*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sq
rt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(2*Sqrt[5 + x^4]) + ((
2 + 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[
2*ArcTan[x/5^(1/4)], 1/2])/(4*5^(1/4)*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*(a*e - c*d*x^2))/(4*a
*c*(p + 1)), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx &= -\frac{x^3\left(15-2x^2\right)}{10\sqrt{5+x^4}}+\frac{1}{10}\int \frac{x^2\left(45-6x^2\right)}{\sqrt{5+x^4}}\,dx\\
&= -\frac{x^3\left(15-2x^2\right)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}-\frac{1}{30}\int \frac{-30-135x^2}{\sqrt{5+x^4}}\,dx\\
&= -\frac{x^3\left(15-2x^2\right)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}-\frac{1}{2}\left(9\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{2}\left(-2-9\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\
&= -\frac{x^3\left(15-2x^2\right)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}+\frac{9x\sqrt{5+x^4}}{2\left(\sqrt{5}+x^2\right)}-\frac{9\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\ldots\right.\right.}{2\sqrt{5+x^4}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.36

$$
\frac{x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{\sqrt{5}}-\frac{3x^3\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^4}{5}\right)}{\sqrt{5}}+\frac{\left(3x^2-1\right)x}{\sqrt{x^4+5}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(-1 + 3*x^2))/Sqrt[5 + x^4] + (x*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x
^4])/Sqrt[5] - (3*x^3*Hypergeometric2F1[3/4, 3/2, 7/4, -1/5*x^4])/Sqrt[5]
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(3\,x^6+2\,x^4\right)\sqrt{x^4+5}}{x^8+10\,x^4+25},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)*sqrt(x^4 + 5)/(x^8 + 10*x^4 + 25), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)x^4}{(x^4+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2), x)
```

maple [C] time = 0.02, size = 168, normalized size = 0.86

$$-\frac{3x^3}{2\sqrt{x^4+5}}-\frac{x}{\sqrt{x^4+5}}+\frac{\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\operatorname{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{9i\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] -3/2*x^3/(x^4+5)^(1/2)+9/10*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)
*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/2)*(I*5^(1/2)
)^(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))-x/(x^4+5)^(1/2)+
1/25*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2
+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)x^4}{(x^4+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,(3x^2+2)}{(x^4+5)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] int((x^4*(3*x^2 + 2))/(x^4 + 5)^(3/2), x)
```

sympy [C] time = 5.59, size = 75, normalized size = 0.38

$$\frac{3\sqrt{5}\,x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5
)/(100*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**
4*exp_polar(I*pi)/5)/(50*gamma(9/4))
```

3.51 $\int \frac{x^2\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}-\frac{\left(15-2x^2\right)x}{10\sqrt{x^4+5}}-\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\,5^{3/4}\sqrt{x^4+5}}+\frac{\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\,t\right.}{5^{3/4}\sqrt{x^4+5}}$$

```
[Out] -1/10*x*(-2*x^2+15)/(x^4+5)^(1/2)-1/5*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+1/5*5^(
1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*El
lipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x
^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-1/20*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1
/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2
*2^(1/2))*(2-3*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(1/
4)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1276, 1198, 220, 1196}

$$-\frac{\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}-\frac{\left(15-2x^2\right)x}{10\sqrt{x^4+5}}-\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\,5^{3/4}\sqrt{x^4+5}}+\frac{\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\,t\right.}{5^{3/4}\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(x*(15 - 2*x^2))/(10*Sqrt[5 + x^4]) - (x*Sqrt[5 + x^4])/(5*(Sqrt[5] + x^2)
) + ((Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x
/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) - ((2 - 3*Sqrt[5])*(Sqrt[5] + x^2)
*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(4*
5^(3/4)*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*(a*e - c*d*x^2))/(4*a
*c*(p + 1)), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rubi steps

$$\int \frac{x^2\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx = -\frac{x\left(15-2x^2\right)}{10\sqrt{5+x^4}} + \frac{1}{10}\int \frac{15-2x^2}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{x\left(15-2x^2\right)}{10\sqrt{5+x^4}} + \frac{\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{\sqrt{5}} + \frac{1}{10}\left(15-2\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{x\left(15-2x^2\right)}{10\sqrt{5+x^4}} - \frac{x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)} + \frac{\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}} - \frac{\left(2-3\sqrt{5}\right)}{}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.38

$$\frac{1}{150}x\left(45\sqrt{5}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)+4\sqrt{5}\,x^2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^4}{5}\right)-\frac{225}{\sqrt{x^4+5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(-225/Sqrt[5 + x^4] + 45*Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x
^4] + 4*Sqrt[5]*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -1/5*x^4]))/150
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,x^4+2\,x^2\right)\sqrt{x^4+5}}{x^8+10\,x^4+25},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)*sqrt(x^4 + 5)/(x^8 + 10*x^4 + 25), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^2+2\right)x^2}{\left(x^4+5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2), x)
```

maple [C] time = 0.02, size = 168, normalized size = 0.95

$$\frac{x^3}{5\sqrt{x^4+5}}-\frac{3x}{2\sqrt{x^4+5}}+\frac{3\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\operatorname{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{50\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{i\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] -3/2/(x^4+5)^(1/2)*x+3/50*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(
1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/2)*(I*5^(1
/2))^(1/2)*x,I)+1/5/(x^4+5)^(1/2)*x^3-1/25*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2
)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(
1/2)*(I*5^(1/2))^(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)x^2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(3x^2+2)}{(x^4+5)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] int((x^2*(3*x^2 + 2))/(x^4 + 5)^(3/2), x)
```

sympy [C] time = 5.09, size = 75, normalized size = 0.42

$$\frac{3\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi)/5)
/(100*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*
exp_polar(I*pi)/5)/(50*gamma(7/4))
```

3.52 $\int \frac{2+3x^2}{\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{3\sqrt{x^4+5}\,x}{10\left(x^2+\sqrt{5}\right)}+\frac{\left(3x^2+2\right)x}{10\sqrt{x^4+5}}+\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\sqrt[4]{5}\,\sqrt{x^4+5}}+\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,}{2\,5^{3/4}\sqrt{x^4}}$$

```
[Out] 1/10*x*(3*x^2+2)/(x^4+5)^(1/2)-3/10*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+3/10*5^(1
/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ell
ipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^
2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/100*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1
/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2
*2^(1/2))*(2-3*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(3/
4)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1179, 1198, 220, 1196}

$$-\frac{3\sqrt{x^4+5}\,x}{10\left(x^2+\sqrt{5}\right)}+\frac{\left(3x^2+2\right)x}{10\sqrt{x^4+5}}+\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\sqrt[4]{5}\,\sqrt{x^4+5}}+\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,}{2\,5^{3/4}\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(2 + 3*x^2))/(10*Sqrt[5 + x^4]) - (3*x*Sqrt[5 + x^4])/(10*(Sqrt[5] + x^2
)) + (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTa
n[x/5^(1/4)], 1/2])/(2*5^(3/4)*Sqrt[5 + x^4]) + ((2 - 3*Sqrt[5])*(Sqrt[5] +
 x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2]
)/(20*5^(1/4)*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{2+3x^2}{\left(5+x^4\right)^{3/2}}\,dx = \frac{x\left(2+3x^2\right)}{10\sqrt{5+x^4}} - \frac{1}{10}\int \frac{-2+3x^2}{\sqrt{5+x^4}}\,dx$$

$$= \frac{x\left(2+3x^2\right)}{10\sqrt{5+x^4}} + \frac{3\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{2\sqrt{5}} - \frac{1}{10}\left(-2+3\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= \frac{x\left(2+3x^2\right)}{10\sqrt{5+x^4}} - \frac{3x\sqrt{5+x^4}}{10\left(\sqrt{5}+x^2\right)} + \frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)\middle|\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{5+x^4}} + \frac{\left(2-3\sqrt{5}\right)\left(\sqrt{5}\right.}{}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.37

$$\frac{1}{25}x\left(\sqrt{5}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)+\sqrt{5}\,x^2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^4}{5}\right)+\frac{5}{\sqrt{x^4+5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(5/Sqrt[5 + x^4] + Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4] +
Sqrt[5]*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -1/5*x^4]))/25
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^8+10\,x^4+25},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/(x^8 + 10*x^4 + 25), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

maple [C] time = 0.01, size = 168, normalized size = 0.93

$$\frac{3x^3}{10\sqrt{x^4+5}}+\frac{x}{5\sqrt{x^4+5}}+\frac{\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\operatorname{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{3i\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] 3/10/(x^4+5)^(1/2)*x^3-3/50*I/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)
*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*5^(1/2)*(I*5^(1/2)
)^(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I))+1/5/(x^4+5)^(1/2
)*x+1/125*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2
)*x^2+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^2+2}{{\left(x^4+5\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^4 + 5)^(3/2),x)
```

```
[Out] int((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

sympy [C] time = 5.07, size = 73, normalized size = 0.41

$$\frac{3\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*exp_polar(I*pi)/5)
/(100*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp
_polar(I*pi)/5)/(50*gamma(5/4))
```

3.53 $\int \frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=196

$$-\frac{3\sqrt{x^4+5}}{25x}+\frac{3\sqrt{x^4+5}\,x}{25\left(x^2+\sqrt{5}\right)}+\frac{3x^2+2}{10\sqrt{x^4+5}\,x}+\frac{3\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\,5^{3/4}\sqrt{x^4+5}}-\frac{3\left(x^2+\sqrt{5}\right)\cdots}{\cdots}$$

```
[Out] 1/10*(3*x^2+2)/x/(x^4+5)^(1/2)-3/25*(x^4+5)^(1/2)/x+3/25*x*(x^4+5)^(1/2)/(x
^2+5^(1/2))-3/25*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arcta
n(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+
5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+3/100*5^(1/4)*(cos(2
*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin
(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(2+5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(
x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1278, 1282, 1198, 220, 1196}

$$\frac{3\sqrt{x^4+5}\,x}{25\left(x^2+\sqrt{5}\right)}-\frac{3\sqrt{x^4+5}}{25x}+\frac{3x^2+2}{10\sqrt{x^4+5}\,x}+\frac{3\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\,5^{3/4}\sqrt{x^4+5}}-\frac{3\left(x^2+\sqrt{5}\right)\cdots}{5\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^2*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x*Sqrt[5 + x^4]) - (3*Sqrt[5 + x^4])/(25*x) + (3*x*Sqrt[5 +
 x^4])/(25*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] +
x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5*5^(3/4)*Sqrt[5 + x^4]) + (3
*(2 + Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[
2*ArcTan[x/5^(1/4)], 1/2])/(20*5^(3/4)*Sqrt[5 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1278

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> -Simp[((f*x)^(m + 1)*(a + c*x^4)^(p + 1)*(d + e*x^2))/(4*a*f*(p
+ 1)), x] + Dist[1/(4*a*(p + 1)), Int[(f*x)^m*(a + c*x^4)^(p + 1)*Simp[d*(m
 + 4*(p + 1) + 1) + e*(m + 2*(2*p + 3) + 1)*x^2, x], x], x] /; FreeQ[{a, c,
 d, e, f, m}, x] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ
[m])
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx = \frac{2+3x^2}{10x\sqrt{5+x^4}} - \frac{1}{10}\int \frac{-6-3x^2}{x^2\sqrt{5+x^4}}\,dx$$

$$= \frac{2+3x^2}{10x\sqrt{5+x^4}} - \frac{3\sqrt{5+x^4}}{25x} + \frac{1}{50}\int \frac{15+6x^2}{\sqrt{5+x^4}}\,dx$$

$$= \frac{2+3x^2}{10x\sqrt{5+x^4}} - \frac{3\sqrt{5+x^4}}{25x} - \frac{3\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{5\sqrt{5}} + \frac{1}{50}\left(3\left(5+2\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= \frac{2+3x^2}{10x\sqrt{5+x^4}} - \frac{3\sqrt{5+x^4}}{25x} + \frac{3x\sqrt{5+x^4}}{25\left(\sqrt{5}+x^2\right)} - \frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5\,5^{3/4}\sqrt{5+x^4}}$$

Mathematica [C] time = 0.05, size = 71, normalized size = 0.36

$$\frac{3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{10\sqrt{5}} - \frac{2\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{5\sqrt{5}\,x} + \frac{3x}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^2*(5 + x^4)^(3/2)),x]
```

```
[Out] (3*x)/(10*Sqrt[5 + x^4]) - (2*Hypergeometric2F1[-1/4, 3/2, 3/4, -1/5*x^4])/
(5*Sqrt[5]*x) + (3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4])/(10*Sqrt[5
])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^{10}+10\,x^6+25\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/(x^10 + 10*x^6 + 25*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2), x)
```

maple [C] time = 0.02, size = 180, normalized size = 0.92

$$-\frac{x^3}{25\sqrt{x^4+5}}+\frac{3x}{10\sqrt{x^4+5}}+\frac{3\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\operatorname{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x}+\frac{3i\sqrt{-5i}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^2/(x^4+5)^(3/2),x)
```

```
[Out] 3/10/(x^4+5)^(1/2)*x+3/250*5^(1/2)/(I*5^(1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^
(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*5^(1/2)*(I*5^(
1/2))^(1/2)*x,I)-2/25*(x^4+5)^(1/2)/x-1/25/(x^4+5)^(1/2)*x^3+3/125*I/(I*5^(
1/2))^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^
(1/2)*(EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(
I*5^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2), x)
```

mupad [B] time = 0.46, size = 48, normalized size = 0.24

$$\frac{3\,\sqrt{5}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{x^4}{5}\right)}{25}-\frac{2\left(\frac{5}{x^4}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{5}{x^4}\right)}{7\,x\left(x^4+5\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^2*(x^4 + 5)^(3/2)),x)
```

```
[Out] (3*5^(1/2)*x*hypergeom([1/4, 3/2], 5/4, -x^4/5))/25 - (2*(5/x^4 + 1)^(3/2)*
hypergeom([3/2, 7/4], 11/4, -5/x^4))/(7*x*(x^4 + 5)^(3/2))
```

sympy [C] time = 7.20, size = 75, normalized size = 0.38

$$\frac{3\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi)/5)/(1
00*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_po
lar(I*pi)/5)/(50*x*gamma(3/4))
```

3.54 $\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{9\sqrt{x^4+5}}{50x}-\frac{\sqrt{x^4+5}}{15x^3}+\frac{9\sqrt{x^4+5}\,x}{50\left(x^2+\sqrt{5}\right)}+\frac{\left(27-2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{60\,5^{3/4}\sqrt{x^4+5}}-\frac{9\left(x^2+\sqrt{5}\right)\sqrt{\frac{}{(x}}}{10}$$

```
[Out] 1/10*(3*x^2+2)/x^3/(x^4+5)^(1/2)-1/15*(x^4+5)^(1/2)/x^3-9/50*(x^4+5)^(1/2)/
x+9/50*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-9/50*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/
4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(
3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(
1/2)+1/300*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4
)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(27-2*5^(1/2))*(x^2
+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(1/4)/(x^4+5)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1278, 1282, 1198, 220, 1196}

$$\frac{9\sqrt{x^4+5}\,x}{50\left(x^2+\sqrt{5}\right)}-\frac{9\sqrt{x^4+5}}{50x}-\frac{\sqrt{x^4+5}}{15x^3}+\frac{3x^2+2}{10\sqrt{x^4+5}\,x^3}+\frac{\left(27-2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{60\,5^{3/4}\sqrt{x^4+5}}-\frac{9(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^4*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x^3*Sqrt[5 + x^4]) - Sqrt[5 + x^4]/(15*x^3) - (9*Sqrt[5 + x
^4])/(50*x) + (9*x*Sqrt[5 + x^4])/(50*(Sqrt[5] + x^2)) - (9*(Sqrt[5] + x^2)
*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(10
*5^(3/4)*Sqrt[5 + x^4]) + ((27 - 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/
(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(60*5^(3/4)*Sqrt[5
+ x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1278

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> -Simp[((f*x)^(m + 1)*(a + c*x^4)^(p + 1)*(d + e*x^2))/(4*a*f*(p
+ 1)), x] + Dist[1/(4*a*(p + 1)), Int[(f*x)^m*(a + c*x^4)^(p + 1)*Simp[d*(m
 + 4*(p + 1) + 1) + e*(m + 2*(2*p + 3) + 1)*x^2, x], x], x] /; FreeQ[{a, c,
 d, e, f, m}, x] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ
[m])
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx &= \frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{1}{10}\int \frac{-10-9x^2}{x^4\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{\sqrt{5+x^4}}{15x^3}+\frac{1}{150}\int \frac{135-10x^2}{x^2\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{\sqrt{5+x^4}}{15x^3}-\frac{9\sqrt{5+x^4}}{50x}-\frac{1}{750}\int \frac{50-135x^2}{\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{\sqrt{5+x^4}}{15x^3}-\frac{9\sqrt{5+x^4}}{50x}-\frac{9\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{10\sqrt{5}}-\frac{1}{150}\left(10-27\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\\
&= \frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{\sqrt{5+x^4}}{15x^3}-\frac{9\sqrt{5+x^4}}{50x}+\frac{9x\sqrt{5+x^4}}{50\left(\sqrt{5}+x^2\right)}-\frac{9\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\right.}{10\,5^{3/4}\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.25

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{x^4}{5}\right)+9x^2\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{15\sqrt{5}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^4*(5 + x^4)^(3/2)),x]
```

```
[Out] -1/15*(2*Hypergeometric2F1[-3/4, 3/2, 1/4, -1/5*x^4] + 9*x^2*Hypergeometric
2F1[-1/4, 3/2, 3/4, -1/5*x^4])/(Sqrt[5]*x^3)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^{12}+10\,x^8+25\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/(x^12 + 10*x^8 + 25*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4), x)
```

maple [C] time = 0.02, size = 192, normalized size = 0.90

$$-\frac{3x^3}{50\sqrt{x^4+5}}-\frac{x}{25\sqrt{x^4+5}}-\frac{\sqrt{5}\,\sqrt{-5i\sqrt{5}\,x^2+25}\,\sqrt{5i\sqrt{5}\,x^2+25}\,\operatorname{EllipticF}\left(\frac{\sqrt{5}\,\sqrt{i\sqrt{5}}\,x}{5},i\right)}{375\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{3\sqrt{x^4+5}}{25x}-\frac{2\sqrt{x^4+5}}{75x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^4/(x^4+5)^(3/2),x)
[Out] -3/25*(x^4+5)^(1/2)/x-3/50/(x^4+5)^(1/2)*x^3+9/250*I/(I*5^(1/2))^(1/2)*(-5*
I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2)*(EllipticF
(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)-EllipticE(1/5*5^(1/2)*(I*5^(1/2))^(1/2)
*x,I))-2/75*(x^4+5)^(1/2)/x^3-1/25/(x^4+5)^(1/2)*x-1/375*5^(1/2)/(I*5^(1/2)
)^(1/2)*(-5*I*5^(1/2)*x^2+25)^(1/2)*(5*I*5^(1/2)*x^2+25)^(1/2)/(x^4+5)^(1/2
)*EllipticF(1/5*5^(1/2)*(I*5^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{x^4\,{\left(x^4+5\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^4*(x^4 + 5)^(3/2)),x)
[Out] int((3*x^2 + 2)/(x^4*(x^4 + 5)^(3/2)), x)
```

sympy [C] time = 8.15, size = 80, normalized size = 0.37

$$\frac{3\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100x\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**4/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_polar(I*pi)/5)/(1
00*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), x**4*exp_
polar(I*pi)/5)/(50*x**3*gamma(1/4))
```

3.55 $\int (fx)^m \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=269

$$\frac{(d+10e)(fx)^{m+21}}{f^{21}(m+21)}+\frac{5(2d+9e)(fx)^{m+19}}{f^{19}(m+19)}+\frac{15(3d+8e)(fx)^{m+17}}{f^{17}(m+17)}+\frac{30(4d+7e)(fx)^{m+15}}{f^{15}(m+15)}+\frac{42(5d+6e)(fx)^{m+13}}{f^{13}(m+13)}+\frac{42(}{}$$

```
[Out] d*(f*x)^(1+m)/f/(1+m)+(10*d+e)*(f*x)^(3+m)/f^3/(3+m)+5*(9*d+2*e)*(f*x)^(5+m
)/f^5/(5+m)+15*(8*d+3*e)*(f*x)^(7+m)/f^7/(7+m)+30*(7*d+4*e)*(f*x)^(9+m)/f^9
/(9+m)+42*(6*d+5*e)*(f*x)^(11+m)/f^11/(11+m)+42*(5*d+6*e)*(f*x)^(13+m)/f^13
/(13+m)+30*(4*d+7*e)*(f*x)^(15+m)/f^15/(15+m)+15*(3*d+8*e)*(f*x)^(17+m)/f^1
7/(17+m)+5*(2*d+9*e)*(f*x)^(19+m)/f^19/(19+m)+(d+10*e)*(f*x)^(21+m)/f^21/(2
1+m)+e*(f*x)^(23+m)/f^23/(23+m)
```

Rubi [A] time = 0.16, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {28, 448}

$$\frac{(10d+e)(fx)^{m+3}}{f^3(m+3)}+\frac{5(9d+2e)(fx)^{m+5}}{f^5(m+5)}+\frac{15(8d+3e)(fx)^{m+7}}{f^7(m+7)}+\frac{30(7d+4e)(fx)^{m+9}}{f^9(m+9)}+\frac{42(6d+5e)(fx)^{m+11}}{f^{11}(m+11)}+\frac{42(5d}{f^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*(f*x)^(1 + m))/(f*(1 + m)) + ((10*d + e)*(f*x)^(3 + m))/(f^3*(3 + m)) +
(5*(9*d + 2*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (15*(8*d + 3*e)*(f*x)^(7 + m)
)/(f^7*(7 + m)) + (30*(7*d + 4*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (42*(6*d +
 5*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (42*(5*d + 6*e)*(f*x)^(13 + m))/(f^
13*(13 + m)) + (30*(4*d + 7*e)*(f*x)^(15 + m))/(f^15*(15 + m)) + (15*(3*d +
 8*e)*(f*x)^(17 + m))/(f^17*(17 + m)) + (5*(2*d + 9*e)*(f*x)^(19 + m))/(f^1
9*(19 + m)) + ((d + 10*e)*(f*x)^(21 + m))/(f^21*(21 + m)) + (e*(f*x)^(23 +
m))/(f^23*(23 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (fx)^m \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 \, dx &= \int (fx)^m \left(1+x^2\right)^{10} \left(d+ex^2\right) \, dx\\ &= \int \left(d(fx)^m+\frac{(10d+e)(fx)^{2+m}}{f^2}+\frac{5(9d+2e)(fx)^{4+m}}{f^4}+\frac{15(8d+3e)(fx)^{6}}{f^6}\right.\\ &= \frac{d(fx)^{1+m}}{f(1+m)}+\frac{(10d+e)(fx)^{3+m}}{f^3(3+m)}+\frac{5(9d+2e)(fx)^{5+m}}{f^5(5+m)}+\frac{15(8d+3e)(fx)^{7+}}{f^7(7+m)}\end{aligned}$$

Mathematica [A] time = 0.56, size = 189, normalized size = 0.70

$$x(fx)^m\left(\frac{x^{20}(d+10e)}{m+21}+\frac{5x^{18}(2d+9e)}{m+19}+\frac{15x^{16}(3d+8e)}{m+17}+\frac{30x^{14}(4d+7e)}{m+15}+\frac{42x^{12}(5d+6e)}{m+13}+\frac{42x^{10}(6d+5e)}{m+11}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x*(f*x)^m*(d/(1 + m) + ((10*d + e)*x^2)/(3 + m) + (5*(9*d + 2*e)*x^4)/(5 +
m) + (15*(8*d + 3*e)*x^6)/(7 + m) + (30*(7*d + 4*e)*x^8)/(9 + m) + (42*(6*d
 + 5*e)*x^10)/(11 + m) + (42*(5*d + 6*e)*x^12)/(13 + m) + (30*(4*d + 7*e)*x
^14)/(15 + m) + (15*(3*d + 8*e)*x^16)/(17 + m) + (5*(2*d + 9*e)*x^18)/(19 +
 m) + ((d + 10*e)*x^20)/(21 + m) + (e*x^22)/(23 + m))
```

fricas [B] time = 0.73, size = 1571, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] ((e*m^11 + 121*e*m^10 + 6435*e*m^9 + 197835*e*m^8 + 3889578*e*m^7 + 5106901
8*e*m^6 + 453714470*e*m^5 + 2702025590*e*m^4 + 10431670821*e*m^3 + 24372200
061*e*m^2 + 29985521895*e*m + 13749310575*e)*x^23 + ((d + 10*e)*m^11 + 123*
(d + 10*e)*m^10 + 6635*(d + 10*e)*m^9 + 206505*(d + 10*e)*m^8 + 4103178*(d
+ 10*e)*m^7 + 54362574*(d + 10*e)*m^6 + 486687830*(d + 10*e)*m^5 + 29170139
70*(d + 10*e)*m^4 + 11320966021*(d + 10*e)*m^3 + 26560342503*(d + 10*e)*m^2
 + 32778930735*(d + 10*e)*m + 15058768725*d + 150587687250*e)*x^21 + 5*((2*
d + 9*e)*m^11 + 125*(2*d + 9*e)*m^10 + 6843*(2*d + 9*e)*m^9 + 215823*(2*d +
 9*e)*m^8 + 4339146*(2*d + 9*e)*m^7 + 58085538*(2*d + 9*e)*m^6 + 524676662*
(2*d + 9*e)*m^5 + 3168601822*(2*d + 9*e)*m^4 + 12374824773*(2*d + 9*e)*m^3
+ 29178958257*(2*d + 9*e)*m^2 + 36145916415*(2*d + 9*e)*m + 33287804550*d +
 149795120475*e)*x^19 + 15*((3*d + 8*e)*m^11 + 127*(3*d + 8*e)*m^10 + 7059*
(3*d + 8*e)*m^9 + 225837*(3*d + 8*e)*m^8 + 4600554*(3*d + 8*e)*m^7 + 623198
94*(3*d + 8*e)*m^6 + 568863686*(3*d + 8*e)*m^5 + 3466775738*(3*d + 8*e)*m^4
 + 13643071845*(3*d + 8*e)*m^3 + 32368407579*(3*d + 8*e)*m^2 + 40283194455*
(3*d + 8*e)*m + 55806025275*d + 148816067400*e)*x^17 + 30*((4*d + 7*e)*m^11
 + 129*(4*d + 7*e)*m^10 + 7283*(4*d + 7*e)*m^9 + 236595*(4*d + 7*e)*m^8 + 4
890858*(4*d + 7*e)*m^7 + 67166442*(4*d + 7*e)*m^6 + 620805254*(4*d + 7*e)*m
^5 + 3825379590*(4*d + 7*e)*m^4 + 15197565541*(4*d + 7*e)*m^3 + 36337145829
*(4*d + 7*e)*m^2 + 45488935863*(4*d + 7*e)*m + 84329104860*d + 147575933505
*e)*x^15 + 42*((5*d + 6*e)*m^11 + 131*(5*d + 6*e)*m^10 + 7515*(5*d + 6*e)*m
^9 + 248145*(5*d + 6*e)*m^8 + 5213898*(5*d + 6*e)*m^7 + 72748638*(5*d + 6*e
)*m^6 + 682569590*(5*d + 6*e)*m^5 + 4264053730*(5*d + 6*e)*m^4 + 1714556090
1*(5*d + 6*e)*m^3 + 41408337231*(5*d + 6*e)*m^2 + 52237739295*(5*d + 6*e)*m
 + 121628516625*d + 145954219950*e)*x^13 + 42*((6*d + 5*e)*m^11 + 133*(6*d
+ 5*e)*m^10 + 7755*(6*d + 5*e)*m^9 + 260535*(6*d + 5*e)*m^8 + 5573898*(6*d
+ 5*e)*m^7 + 79216434*(6*d + 5*e)*m^6 + 756921110*(6*d + 5*e)*m^5 + 4811326
190*(6*d + 5*e)*m^4 + 19653671301*(6*d + 5*e)*m^3 + 48110244633*(6*d + 5*e)
*m^2 + 61333432335*(6*d + 5*e)*m + 172491350850*d + 143742792375*e)*x^11 +
30*((7*d + 4*e)*m^11 + 135*(7*d + 4*e)*m^10 + 8003*(7*d + 4*e)*m^9 + 273813
*(7*d + 4*e)*m^8 + 5975466*(7*d + 4*e)*m^7 + 86750118*(7*d + 4*e)*m^6 + 847
550822*(7*d + 4*e)*m^5 + 5509501002*(7*d + 4*e)*m^4 + 22992750373*(7*d + 4*
e)*m^3 + 57365875587*(7*d + 4*e)*m^2 + 74253243015*(7*d + 4*e)*m + 24595988
9175*d + 140548508100*e)*x^9 + 15*((8*d + 3*e)*m^11 + 137*(8*d + 3*e)*m^10
+ 8259*(8*d + 3*e)*m^9 + 288027*(8*d + 3*e)*m^8 + 6423594*(8*d + 3*e)*m^7 +
 95564154*(8*d + 3*e)*m^6 + 959352806*(8*d + 3*e)*m^5 + 6421988758*(8*d + 3
*e)*m^4 + 27624338085*(8*d + 3*e)*m^3 + 70930262349*(8*d + 3*e)*m^2 + 94034
286855*(8*d + 3*e)*m + 361410449400*d + 135528918525*e)*x^7 + 5*((9*d + 2*e
```

```
)*m^11 + 139*(9*d + 2*e)*m^10 + 8523*(9*d + 2*e)*m^9 + 303225*(9*d + 2*e)*m
^8 + 6923658*(9*d + 2*e)*m^7 + 105911022*(9*d + 2*e)*m^6 + 1098746774*(9*d
+ 2*e)*m^5 + 7643724530*(9*d + 2*e)*m^4 + 34359636741*(9*d + 2*e)*m^3 + 925
02445239*(9*d + 2*e)*m^2 + 128033897103*(9*d + 2*e)*m + 569221457805*d + 12
6493657290*e)*x^5 + ((10*d + e)*m^11 + 141*(10*d + e)*m^10 + 8795*(10*d + e
)*m^9 + 319455*(10*d + e)*m^8 + 7481418*(10*d + e)*m^7 + 118085058*(10*d +
e)*m^6 + 1274046710*(10*d + e)*m^5 + 9315318270*(10*d + e)*m^4 + 4463230458
1*(10*d + e)*m^3 + 130403715201*(10*d + e)*m^2 + 199334977695*(10*d + e)*m
+ 1054113810750*d + 105411381075*e)*x^3 + (d*m^11 + 143*d*m^10 + 9075*d*m^9
 + 336765*d*m^8 + 8103018*d*m^7 + 132426294*d*m^6 + 1495875590*d*m^5 + 1164
1582810*d*m^4 + 60936676581*d*m^3 + 203363952363*d*m^2 + 387182170935*d*m +
 316234143225*d)*x)*(f*x)^m/(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 843
9783*m^8 + 140529312*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m
^4 + 264300628944*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)
```

giac [B] time = 0.46, size = 3752, normalized size = 13.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] ((f*x)^m*m^11*x^23*e + 121*(f*x)^m*m^10*x^23*e + (f*x)^m*d*m^11*x^21 + 10*(
f*x)^m*m^11*x^21*e + 6435*(f*x)^m*m^9*x^23*e + 123*(f*x)^m*d*m^10*x^21 + 12
30*(f*x)^m*m^10*x^21*e + 197835*(f*x)^m*m^8*x^23*e + 10*(f*x)^m*d*m^11*x^19
 + 6635*(f*x)^m*d*m^9*x^21 + 45*(f*x)^m*m^11*x^19*e + 66350*(f*x)^m*m^9*x^2
1*e + 3889578*(f*x)^m*m^7*x^23*e + 1250*(f*x)^m*d*m^10*x^19 + 206505*(f*x)^
m*d*m^8*x^21 + 5625*(f*x)^m*m^10*x^19*e + 2065050*(f*x)^m*m^8*x^21*e + 5106
9018*(f*x)^m*m^6*x^23*e + 45*(f*x)^m*d*m^11*x^17 + 68430*(f*x)^m*d*m^9*x^19
 + 4103178*(f*x)^m*d*m^7*x^21 + 120*(f*x)^m*m^11*x^17*e + 307935*(f*x)^m*m^
9*x^19*e + 41031780*(f*x)^m*m^7*x^21*e + 453714470*(f*x)^m*m^5*x^23*e + 571
5*(f*x)^m*d*m^10*x^17 + 2158230*(f*x)^m*d*m^8*x^19 + 54362574*(f*x)^m*d*m^6
*x^21 + 15240*(f*x)^m*m^10*x^17*e + 9712035*(f*x)^m*m^8*x^19*e + 543625740*
(f*x)^m*m^6*x^21*e + 2702025590*(f*x)^m*m^4*x^23*e + 120*(f*x)^m*d*m^11*x^1
5 + 317655*(f*x)^m*d*m^9*x^17 + 43391460*(f*x)^m*d*m^7*x^19 + 486687830*(f*
x)^m*d*m^5*x^21 + 210*(f*x)^m*m^11*x^15*e + 847080*(f*x)^m*m^9*x^17*e + 195
261570*(f*x)^m*m^7*x^19*e + 4866878300*(f*x)^m*m^5*x^21*e + 10431670821*(f*
x)^m*m^3*x^23*e + 15480*(f*x)^m*d*m^10*x^15 + 10162665*(f*x)^m*d*m^8*x^17 +
 580855380*(f*x)^m*d*m^6*x^19 + 2917013970*(f*x)^m*d*m^4*x^21 + 27090*(f*x)
^m*m^10*x^15*e + 27100440*(f*x)^m*m^8*x^17*e + 2613849210*(f*x)^m*m^6*x^19*
e + 29170139700*(f*x)^m*m^4*x^21*e + 24372200061*(f*x)^m*m^2*x^23*e + 210*(
f*x)^m*d*m^11*x^13 + 873960*(f*x)^m*d*m^9*x^15 + 207024930*(f*x)^m*d*m^7*x^
17 + 5246766620*(f*x)^m*d*m^5*x^19 + 11320966021*(f*x)^m*d*m^3*x^21 + 252*(
f*x)^m*m^11*x^13*e + 1529430*(f*x)^m*m^9*x^15*e + 552066480*(f*x)^m*m^7*x^1
7*e + 23610449790*(f*x)^m*m^5*x^19*e + 113209660210*(f*x)^m*m^3*x^21*e + 29
985521895*(f*x)^m*m*x^23*e + 27510*(f*x)^m*d*m^10*x^13 + 28391400*(f*x)^m*d
*m^8*x^15 + 2804395230*(f*x)^m*d*m^6*x^17 + 31686018220*(f*x)^m*d*m^4*x^19
+ 26560342503*(f*x)^m*d*m^2*x^21 + 33012*(f*x)^m*m^10*x^13*e + 49684950*(f*
x)^m*m^8*x^15*e + 7478387280*(f*x)^m*m^6*x^17*e + 142587081990*(f*x)^m*m^4*
x^19*e + 265603425030*(f*x)^m*m^2*x^21*e + 13749310575*(f*x)^m*x^23*e + 252
*(f*x)^m*d*m^11*x^11 + 1578150*(f*x)^m*d*m^9*x^13 + 586902960*(f*x)^m*d*m^7
*x^15 + 25598865870*(f*x)^m*d*m^5*x^17 + 123748247730*(f*x)^m*d*m^3*x^19 +
32778930735*(f*x)^m*d*m*x^21 + 210*(f*x)^m*m^11*x^11*e + 1893780*(f*x)^m*m^
9*x^13*e + 1027080180*(f*x)^m*m^7*x^15*e + 68263642320*(f*x)^m*m^5*x^17*e +
 556867114785*(f*x)^m*m^3*x^19*e + 327789307350*(f*x)^m*m*x^21*e + 33516*(f
*x)^m*d*m^10*x^11 + 52110450*(f*x)^m*d*m^8*x^13 + 8059973040*(f*x)^m*d*m^6*
x^15 + 156004908210*(f*x)^m*d*m^4*x^17 + 291789582570*(f*x)^m*d*m^2*x^19 +
15058768725*(f*x)^m*d*x^21 + 27930*(f*x)^m*m^10*x^11*e + 62532540*(f*x)^m*m
^8*x^13*e + 14104952820*(f*x)^m*m^6*x^15*e + 416013088560*(f*x)^m*m^4*x^17*
e + 1313053121565*(f*x)^m*m^2*x^19*e + 150587687250*(f*x)^m*x^21*e + 210*(f
```

```
*x)^m*d*m^11*x^9 + 1954260*(f*x)^m*d*m^9*x^11 + 1094918580*(f*x)^m*d*m^7*x^
13 + 74496630480*(f*x)^m*d*m^5*x^15 + 613938233025*(f*x)^m*d*m^3*x^17 + 361
459164150*(f*x)^m*d*m*x^19 + 120*(f*x)^m*m^11*x^9*e + 1628550*(f*x)^m*m^9*x
^11*e + 1313902296*(f*x)^m*m^7*x^13*e + 130369103340*(f*x)^m*m^5*x^15*e + 1
637168621400*(f*x)^m*m^3*x^17*e + 162656238675*(f*x)^m*m*x^19*e + 28350*(f
*x)^m*d*m^10*x^9 + 65654820*(f*x)^m*d*m^8*x^11 + 15277213980*(f*x)^m*d*m^6*
x^13 + 459045550800*(f*x)^m*d*m^4*x^15 + 1456578341055*(f*x)^m*d*m^2*x^17 +
 166439022750*(f*x)^m*d*x^19 + 16200*(f*x)^m*m^10*x^9*e + 54712350*(f*x)^m*
m^8*x^11*e + 18332656776*(f*x)^m*m^6*x^13*e + 803329713900*(f*x)^m*m^4*x^15
*e + 3884208909480*(f*x)^m*m^2*x^17*e + 748975602375*(f*x)^m*x^19*e + 120*(
f*x)^m*d*m^11*x^7 + 1680630*(f*x)^m*d*m^9*x^9 + 1404622296*(f*x)^m*d*m^7*x^
11 + 143339613900*(f*x)^m*d*m^5*x^13 + 1823707864920*(f*x)^m*d*m^3*x^15 + 1
812743750475*(f*x)^m*d*m*x^17 + 45*(f*x)^m*m^11*x^7*e + 960360*(f*x)^m*m^9*
x^9*e + 1170518580*(f*x)^m*m^7*x^11*e + 172007536680*(f*x)^m*m^5*x^13*e + 3
191488763610*(f*x)^m*m^3*x^15*e + 4833983334600*(f*x)^m*m*x^17*e + 16440*(f
*x)^m*d*m^10*x^7 + 57500730*(f*x)^m*d*m^8*x^9 + 19962541368*(f*x)^m*d*m^6*x
^11 + 895451283300*(f*x)^m*d*m^4*x^13 + 4360457499480*(f*x)^m*d*m^2*x^15 +
837090379125*(f*x)^m*d*x^17 + 6165*(f*x)^m*m^10*x^7*e + 32857560*(f*x)^m*m^
8*x^9*e + 16635451140*(f*x)^m*m^6*x^11*e + 1074541539960*(f*x)^m*m^4*x^13*e
 + 7630800624090*(f*x)^m*m^2*x^15*e + 2232241011000*(f*x)^m*x^17*e + 45*(f*
x)^m*d*m^11*x^5 + 991080*(f*x)^m*d*m^9*x^7 + 1254847860*(f*x)^m*d*m^7*x^9 +
 190744119720*(f*x)^m*d*m^5*x^11 + 3600567789210*(f*x)^m*d*m^3*x^13 + 54586
72303560*(f*x)^m*d*m*x^15 + 10*(f*x)^m*m^11*x^5*e + 371655*(f*x)^m*m^9*x^7*
e + 717055920*(f*x)^m*m^7*x^9*e + 158953433100*(f*x)^m*m^5*x^11*e + 4320681
347052*(f*x)^m*m^3*x^13*e + 9552676531230*(f*x)^m*m*x^15*e + 6255*(f*x)^m*d
*m^10*x^5 + 34563240*(f*x)^m*d*m^8*x^7 + 18217524780*(f*x)^m*d*m^6*x^9 + 12
12454199880*(f*x)^m*d*m^4*x^11 + 8695750818510*(f*x)^m*d*m^2*x^13 + 2529873
145800*(f*x)^m*d*x^15 + 1390*(f*x)^m*m^10*x^5*e + 12961215*(f*x)^m*m^8*x^7*
e + 10410014160*(f*x)^m*m^6*x^9*e + 1010378499900*(f*x)^m*m^4*x^11*e + 1043
4900982212*(f*x)^m*m^2*x^13*e + 4427278005150*(f*x)^m*x^15*e + 10*(f*x)^m*d
*m^11*x^3 + 383535*(f*x)^m*d*m^9*x^5 + 770831280*(f*x)^m*d*m^7*x^7 + 177985
672620*(f*x)^m*d*m^5*x^9 + 4952725167852*(f*x)^m*d*m^3*x^11 + 1096992525195
0*(f*x)^m*d*m*x^13 + (f*x)^m*m^11*x^3*e + 85230*(f*x)^m*m^9*x^5*e + 2890617
30*(f*x)^m*m^7*x^7*e + 101706098640*(f*x)^m*m^5*x^9*e + 4127270973210*(f*x)
^m*m^3*x^11*e + 13163910302340*(f*x)^m*m*x^13*e + 1410*(f*x)^m*d*m^10*x^3 +
 13645125*(f*x)^m*d*m^8*x^5 + 11467698480*(f*x)^m*d*m^6*x^7 + 1156995210420
*(f*x)^m*d*m^4*x^9 + 12123781647516*(f*x)^m*d*m^2*x^11 + 5108397698250*(f*x
)^m*d*x^13 + 141*(f*x)^m*m^10*x^3*e + 3032250*(f*x)^m*m^8*x^5*e + 430038693
0*(f*x)^m*m^6*x^7*e + 661140120240*(f*x)^m*m^4*x^9*e + 10103151372930*(f*x)
^m*m^2*x^11*e + 6130077237900*(f*x)^m*x^13*e + (f*x)^m*d*m^11*x + 87950*(f*
x)^m*d*m^9*x^3 + 311564610*(f*x)^m*d*m^7*x^5 + 115122336720*(f*x)^m*d*m^5*x
^7 + 4828477578330*(f*x)^m*d*m^3*x^9 + 15456024948420*(f*x)^m*d*m*x^11 + 87
95*(f*x)^m*m^9*x^3*e + 69236580*(f*x)^m*m^7*x^5*e + 43170876270*(f*x)^m*m^5
*x^7*e + 2759130044760*(f*x)^m*m^3*x^9*e + 12880020790350*(f*x)^m*m*x^11*e
+ 143*(f*x)^m*d*m^10*x + 3194550*(f*x)^m*d*m^8*x^3 + 4765995990*(f*x)^m*d*m
^6*x^5 + 770638650960*(f*x)^m*d*m^4*x^7 + 12046833873270*(f*x)^m*d*m^2*x^9
+ 7244636735700*(f*x)^m*d*x^11 + 319455*(f*x)^m*m^8*x^3*e + 1059110220*(f*x
)^m*m^6*x^5*e + 288989494110*(f*x)^m*m^4*x^7*e + 6883905070440*(f*x)^m*m^2*
x^9*e + 6037197279750*(f*x)^m*x^11*e + 9075*(f*x)^m*d*m^9*x + 74814180*(f*x
)^m*d*m^7*x^3 + 49443604830*(f*x)^m*d*m^5*x^5 + 3314920570200*(f*x)^m*d*m^3
*x^7 + 15593181033150*(f*x)^m*d*m*x^9 + 7481418*(f*x)^m*m^7*x^3*e + 1098746
7740*(f*x)^m*m^5*x^5*e + 1243095213825*(f*x)^m*m^3*x^7*e + 8910389161800*(f
*x)^m*m*x^9*e + 336765*(f*x)^m*d*m^8*x + 1180850580*(f*x)^m*d*m^6*x^3 + 343
967603850*(f*x)^m*d*m^4*x^5 + 8511631481880*(f*x)^m*d*m^2*x^7 + 73787966752
50*(f*x)^m*d*x^9 + 118085058*(f*x)^m*m^6*x^3*e + 76437245300*(f*x)^m*m^4*x^
5*e + 3191861805705*(f*x)^m*m^2*x^7*e + 4216455243000*(f*x)^m*x^9*e + 81030
18*(f*x)^m*d*m^7*x + 12740467100*(f*x)^m*d*m^5*x^3 + 1546183653345*(f*x)^m*
d*m^3*x^5 + 11284114422600*(f*x)^m*d*m*x^7 + 1274046710*(f*x)^m*m^5*x^3*e +
 343596367410*(f*x)^m*m^3*x^5*e + 4231542908475*(f*x)^m*m*x^7*e + 132426294
```

```
*(f*x)^m*d*m^6*x + 93153182700*(f*x)^m*d*m^4*x^3 + 4162610035755*(f*x)^m*d*
m^2*x^5 + 5421156741000*(f*x)^m*d*x^7 + 9315318270*(f*x)^m*m^4*x^3*e + 9250
24452390*(f*x)^m*m^2*x^5*e + 2032933777875*(f*x)^m*x^7*e + 1495875590*(f*x)
^m*d*m^5*x + 446323045810*(f*x)^m*d*m^3*x^3 + 5761525369635*(f*x)^m*d*m*x^5
 + 44632304581*(f*x)^m*m^3*x^3*e + 1280338971030*(f*x)^m*m*x^5*e + 11641582
810*(f*x)^m*d*m^4*x + 1304037152010*(f*x)^m*d*m^2*x^3 + 2846107289025*(f*x)
^m*d*x^5 + 130403715201*(f*x)^m*m^2*x^3*e + 632468286450*(f*x)^m*x^5*e + 60
936676581*(f*x)^m*d*m^3*x + 1993349776950*(f*x)^m*d*m*x^3 + 199334977695*(f
*x)^m*m*x^3*e + 203363952363*(f*x)^m*d*m^2*x + 1054113810750*(f*x)^m*d*x^3
+ 105411381075*(f*x)^m*x^3*e + 387182170935*(f*x)^m*d*m*x + 316234143225*(f
*x)^m*d*x)/(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529
312*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944
*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)
```

maple [B] time = 0.03, size = 2295, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] (f*x)^m*(e*m^11*x^22+121*e*m^10*x^22+d*m^11*x^20+10*e*m^11*x^20+6435*e*m^9*
x^22+123*d*m^10*x^20+1230*e*m^10*x^20+197835*e*m^8*x^22+10*d*m^11*x^18+6635
*d*m^9*x^20+45*e*m^11*x^18+66350*e*m^9*x^20+3889578*e*m^7*x^22+1250*d*m^10*
x^18+206505*d*m^8*x^20+5625*e*m^10*x^18+2065050*e*m^8*x^20+51069018*e*m^6*x
^22+45*d*m^11*x^16+68430*d*m^9*x^18+4103178*d*m^7*x^20+120*e*m^11*x^16+3079
35*e*m^9*x^18+41031780*e*m^7*x^20+453714470*e*m^5*x^22+5715*d*m^10*x^16+215
8230*d*m^8*x^18+54362574*d*m^6*x^20+15240*e*m^10*x^16+9712035*e*m^8*x^18+54
3625740*e*m^6*x^20+2702025590*e*m^4*x^22+120*d*m^11*x^14+317655*d*m^9*x^16+
43391460*d*m^7*x^18+486687830*d*m^5*x^20+210*e*m^11*x^14+847080*e*m^9*x^16+
195261570*e*m^7*x^18+4866878300*e*m^5*x^20+10431670821*e*m^3*x^22+15480*d*m
^10*x^14+10162665*d*m^8*x^16+580855380*d*m^6*x^18+2917013970*d*m^4*x^20+270
90*e*m^10*x^14+27100440*e*m^8*x^16+2613849210*e*m^6*x^18+29170139700*e*m^4*
x^20+24372200061*e*m^2*x^22+210*d*m^11*x^12+873960*d*m^9*x^14+207024930*d*m
^7*x^16+5246766620*d*m^5*x^18+11320966021*d*m^3*x^20+252*e*m^11*x^12+152943
0*e*m^9*x^14+552066480*e*m^7*x^16+23610449790*e*m^5*x^18+113209660210*e*m^3
*x^20+29985521895*e*m*x^22+27510*d*m^10*x^12+28391400*d*m^8*x^14+2804395230
*d*m^6*x^16+31686018220*d*m^4*x^18+26560342503*d*m^2*x^20+33012*e*m^10*x^12
+49684950*e*m^8*x^14+7478387280*e*m^6*x^16+142587081990*e*m^4*x^18+26560342
5030*e*m^2*x^20+13749310575*e*x^22+252*d*m^11*x^10+1578150*d*m^9*x^12+58690
2960*d*m^7*x^14+25598865870*d*m^5*x^16+123748247730*d*m^3*x^18+32778930735*
d*m*x^20+210*e*m^11*x^10+1893780*e*m^9*x^12+1027080180*e*m^7*x^14+682636423
20*e*m^5*x^16+556867114785*e*m^3*x^18+327789307350*e*m*x^20+33516*d*m^10*x^
10+52110450*d*m^8*x^12+8059973040*d*m^6*x^14+156004908210*d*m^4*x^16+291789
582570*d*m^2*x^18+15058768725*d*x^20+27930*e*m^10*x^10+62532540*e*m^8*x^12+
14104952820*e*m^6*x^14+416013088560*e*m^4*x^16+1313053121565*e*m^2*x^18+150
587687250*e*x^20+210*d*m^11*x^8+1954260*d*m^9*x^10+1094918580*d*m^7*x^12+74
496630480*d*m^5*x^14+613938233025*d*m^3*x^16+361459164150*d*m*x^18+120*e*m^
11*x^8+1628550*e*m^9*x^10+1313902296*e*m^7*x^12+130369103340*e*m^5*x^14+163
7168621400*e*m^3*x^16+1626566238675*e*m*x^18+28350*d*m^10*x^8+65654820*d*m^
8*x^10+15277213980*d*m^6*x^12+459045550800*d*m^4*x^14+1456578341055*d*m^2*x
^16+166439022750*d*x^18+16200*e*m^10*x^8+54712350*e*m^8*x^10+18332656776*e*
m^6*x^12+803329713900*e*m^4*x^14+3884208909480*e*m^2*x^16+748975602375*e*x^
18+120*d*m^11*x^6+1680630*d*m^9*x^8+1404622296*d*m^7*x^10+143339613900*d*m^
5*x^12+1823707864920*d*m^3*x^14+1812743750475*d*m*x^16+45*e*m^11*x^6+960360
*e*m^9*x^8+1170518580*e*m^7*x^10+172007536680*e*m^5*x^12+3191488763610*e*m^
3*x^14+4833983334600*e*m*x^16+16440*d*m^10*x^6+57500730*d*m^8*x^8+199625413
68*d*m^6*x^10+895451283300*d*m^4*x^12+4360457499480*d*m^2*x^14+837090379125
*d*x^16+6165*e*m^10*x^6+32857560*e*m^8*x^8+16635451140*e*m^6*x^10+107454153
9960*e*m^4*x^12+7630800624090*e*m^2*x^14+2232241011000*e*x^16+45*d*m^11*x^4
```

```
+991080*d*m^9*x^6+1254847860*d*m^7*x^8+190744119720*d*m^5*x^10+360056778921
0*d*m^3*x^12+5458672303560*d*m*x^14+10*e*m^11*x^4+371655*e*m^9*x^6+71705592
0*e*m^7*x^8+158953433100*e*m^5*x^10+4320681347052*e*m^3*x^12+9552676531230*
e*m*x^14+6255*d*m^10*x^4+34563240*d*m^8*x^6+18217524780*d*m^6*x^8+121245419
9880*d*m^4*x^10+8695750818510*d*m^2*x^12+2529873145800*d*x^14+1390*e*m^10*x
^4+12961215*e*m^8*x^6+10410014160*e*m^6*x^8+1010378499900*e*m^4*x^10+104349
00982212*e*m^2*x^12+4427278005150*e*x^14+10*d*m^11*x^2+383535*d*m^9*x^4+770
831280*d*m^7*x^6+177985672620*d*m^5*x^8+4952725167852*d*m^3*x^10+1096992525
1950*d*m*x^12+e*m^11*x^2+85230*e*m^9*x^4+289061730*e*m^7*x^6+101706098640*e
*m^5*x^8+4127270973210*e*m^3*x^10+13163910302340*e*m*x^12+1410*d*m^10*x^2+1
3645125*d*m^8*x^4+11467698480*d*m^6*x^6+1156995210420*d*m^4*x^8+12123781647
516*d*m^2*x^10+5108397698250*d*x^12+141*e*m^10*x^2+3032250*e*m^8*x^4+430038
6930*e*m^6*x^6+661140120240*e*m^4*x^8+10103151372930*e*m^2*x^10+61300772379
00*e*x^12+d*m^11+87950*d*m^9*x^2+311564610*d*m^7*x^4+115122336720*d*m^5*x^6
+4828477578330*d*m^3*x^8+15456024948420*d*m*x^10+8795*e*m^9*x^2+69236580*e*
m^7*x^4+43170876270*e*m^5*x^6+2759130044760*e*m^3*x^8+12880020790350*e*m*x^
10+143*d*m^10+3194550*d*m^8*x^2+4765995990*d*m^6*x^4+770638650960*d*m^4*x^6
+12046833873270*d*m^2*x^8+7244636735700*d*x^10+319455*e*m^8*x^2+1059110220*
e*m^6*x^4+288989494110*e*m^4*x^6+6883905070440*e*m^2*x^8+6037197279750*e*x^
10+9075*d*m^9+74814180*d*m^7*x^2+49443604830*d*m^5*x^4+3314920570200*d*m^3*
x^6+15593181033150*d*m*x^8+7481418*e*m^7*x^2+10987467740*e*m^5*x^4+12430952
13825*e*m^3*x^6+8910389161800*e*m*x^8+336765*d*m^8+1180850580*d*m^6*x^2+343
967603850*d*m^4*x^4+8511631481880*d*m^2*x^6+7378796675250*d*x^8+118085058*e
*m^6*x^2+76437245300*e*m^4*x^4+3191861805705*e*m^2*x^6+4216455243000*e*x^8+
8103018*d*m^7+12740467100*d*m^5*x^2+1546183653345*d*m^3*x^4+11284114422600*
d*m*x^6+1274046710*e*m^5*x^2+343596367410*e*m^3*x^4+4231542908475*e*m*x^6+1
32426294*d*m^6+93153182700*d*m^4*x^2+4162610035755*d*m^2*x^4+5421156741000*
d*x^6+9315318270*e*m^4*x^2+925024452390*e*m^2*x^4+2032933777875*e*x^6+14958
75590*d*m^5+446323045810*d*m^3*x^2+5761525369635*d*m*x^4+44632304581*e*m^3*
x^2+1280338971030*e*m*x^4+11641582810*d*m^4+1304037152010*d*m^2*x^2+2846107
289025*d*x^4+130403715201*e*m^2*x^2+632468286450*e*x^4+60936676581*d*m^3+19
93349776950*d*m*x^2+199334977695*e*m*x^2+203363952363*d*m^2+1054113810750*d
*x^2+105411381075*e*x^2+387182170935*d*m+316234143225*d)*x/(1+m)/(3+m)/(5+m
)/(7+m)/(9+m)/(11+m)/(13+m)/(15+m)/(17+m)/(19+m)/(21+m)/(23+m)
```

maxima [A] time = 0.89, size = 372, normalized size = 1.38

$$\frac{ef^m x^{23} x^m}{m+23} + \frac{df^m x^{21} x^m}{m+21} + \frac{10\,ef^m x^{21} x^m}{m+21} + \frac{10\,df^m x^{19} x^m}{m+19} + \frac{45\,ef^m x^{19} x^m}{m+19} + \frac{45\,df^m x^{17} x^m}{m+17} + \frac{120\,ef^m x^{17} x^m}{m+17} + \frac{120\,df^m x}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] e*f^m*x^23*x^m/(m + 23) + d*f^m*x^21*x^m/(m + 21) + 10*e*f^m*x^21*x^m/(m +
21) + 10*d*f^m*x^19*x^m/(m + 19) + 45*e*f^m*x^19*x^m/(m + 19) + 45*d*f^m*x^
17*x^m/(m + 17) + 120*e*f^m*x^17*x^m/(m + 17) + 120*d*f^m*x^15*x^m/(m + 15)
 + 210*e*f^m*x^15*x^m/(m + 15) + 210*d*f^m*x^13*x^m/(m + 13) + 252*e*f^m*x^
13*x^m/(m + 13) + 252*d*f^m*x^11*x^m/(m + 11) + 210*e*f^m*x^11*x^m/(m + 11)
 + 210*d*f^m*x^9*x^m/(m + 9) + 120*e*f^m*x^9*x^m/(m + 9) + 120*d*f^m*x^7*x^
m/(m + 7) + 45*e*f^m*x^7*x^m/(m + 7) + 45*d*f^m*x^5*x^m/(m + 5) + 10*e*f^m*
x^5*x^m/(m + 5) + 10*d*f^m*x^3*x^m/(m + 3) + e*f^m*x^3*x^m/(m + 3) + (f*x)^
(m + 1)*d/(f*(m + 1))
```

mupad [B] time = 1.78, size = 1539, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] (d*x*(f*x)^m*(387182170935*m + 203363952363*m^2 + 60936676581*m^3 + 1164158
2810*m^4 + 1495875590*m^5 + 132426294*m^6 + 8103018*m^7 + 336765*m^8 + 9075
*m^9 + 143*m^10 + m^11 + 316234143225))/(703416314160*m + 590546123298*m^2
+ 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 1
40529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316
234143225) + (e*x^23*(f*x)^m*(29985521895*m + 24372200061*m^2 + 10431670821
*m^3 + 2702025590*m^4 + 453714470*m^5 + 51069018*m^6 + 3889578*m^7 + 197835
*m^8 + 6435*m^9 + 121*m^10 + m^11 + 13749310575))/(703416314160*m + 5905461
23298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 16283018
84*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 +
m^12 + 316234143225) + (30*x^15*(f*x)^m*(4*d + 7*e)*(45488935863*m + 363371
45829*m^2 + 15197565541*m^3 + 3825379590*m^4 + 620805254*m^5 + 67166442*m^6
 + 4890858*m^7 + 236595*m^8 + 7283*m^9 + 129*m^10 + m^11 + 21082276215))/(7
03416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 131
37458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 +
9218*m^10 + 144*m^11 + m^12 + 316234143225) + (42*x^13*(f*x)^m*(5*d + 6*e)*
(52237739295*m + 41408337231*m^2 + 17145560901*m^3 + 4264053730*m^4 + 68256
9590*m^5 + 72748638*m^6 + 5213898*m^7 + 248145*m^8 + 7515*m^9 + 131*m^10 +
m^11 + 24325703325))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3
+ 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439
783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (30*x^
9*(f*x)^m*(7*d + 4*e)*(74253243015*m + 57365875587*m^2 + 22992750373*m^3 +
5509501002*m^4 + 847550822*m^5 + 86750118*m^6 + 5975466*m^7 + 273813*m^8 +
8003*m^9 + 135*m^10 + m^11 + 35137127025))/(703416314160*m + 590546123298*m
^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6
+ 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 +
316234143225) + (x^3*(f*x)^m*(10*d + e)*(199334977695*m + 130403715201*m^2
+ 44632304581*m^3 + 9315318270*m^4 + 1274046710*m^5 + 118085058*m^6 + 74814
18*m^7 + 319455*m^8 + 8795*m^9 + 141*m^10 + m^11 + 105411381075))/(70341631
4160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 1313745840
0*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^
10 + 144*m^11 + m^12 + 316234143225) + (5*x^19*(f*x)^m*(2*d + 9*e)*(3614591
6415*m + 29178958257*m^2 + 12374824773*m^3 + 3168601822*m^4 + 524676662*m^5
 + 58085538*m^6 + 4339146*m^7 + 215823*m^8 + 6843*m^9 + 125*m^10 + m^11 + 1
6643902275))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725782
59391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8
+ 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (42*x^11*(f*x)
^m*(6*d + 5*e)*(61333432335*m + 48110244633*m^2 + 19653671301*m^3 + 4811326
190*m^4 + 756921110*m^5 + 79216434*m^6 + 5573898*m^7 + 260535*m^8 + 7755*m^
9 + 133*m^10 + m^11 + 28748558475))/(703416314160*m + 590546123298*m^2 + 26
4300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 14052
9312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 3162341
43225) + (15*x^7*(f*x)^m*(8*d + 3*e)*(94034286855*m + 70930262349*m^2 + 276
24338085*m^3 + 6421988758*m^4 + 959352806*m^5 + 95564154*m^6 + 6423594*m^7
+ 288027*m^8 + 8259*m^9 + 137*m^10 + m^11 + 45176306175))/(703416314160*m +
 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 +
1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144
*m^11 + m^12 + 316234143225) + (5*x^5*(f*x)^m*(9*d + 2*e)*(128033897103*m +
 92502445239*m^2 + 34359636741*m^3 + 7643724530*m^4 + 1098746774*m^5 + 1059
11022*m^6 + 6923658*m^7 + 303225*m^8 + 8523*m^9 + 139*m^10 + m^11 + 6324682
8645))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*
m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 3458
40*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (15*x^17*(f*x)^m*(3*
d + 8*e)*(40283194455*m + 32368407579*m^2 + 13643071845*m^3 + 3466775738*m^
4 + 568863686*m^5 + 62319894*m^6 + 4600554*m^7 + 225837*m^8 + 7059*m^9 + 12
7*m^10 + m^11 + 18602008425))/(703416314160*m + 590546123298*m^2 + 26430062
8944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m
^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)
 + (x^21*(f*x)^m*(d + 10*e)*(32778930735*m + 26560342503*m^2 + 11320966021*
```

```
m^3 + 2917013970*m^4 + 486687830*m^5 + 54362574*m^6 + 4103178*m^7 + 206505*
m^8 + 6635*m^9 + 123*m^10 + m^11 + 15058768725))/(703416314160*m + 59054612
3298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 162830188
4*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m
^12 + 316234143225)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(x**4+2*x**2+1)**5,x)

[Out] Timed out
```

3.56 $\int x^5 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=63

$$\frac{1}{26} \left(x^2+1\right)^{13} (d-3e) - \frac{1}{24} \left(x^2+1\right)^{12} (2d-3e) + \frac{1}{22} \left(x^2+1\right)^{11} (d-e) + \frac{1}{28} e \left(x^2+1\right)^{14}$$

```
[Out] 1/22*(d-e)*(x^2+1)^11-1/24*(2*d-3*e)*(x^2+1)^12+1/26*(d-3*e)*(x^2+1)^13+1/2
8*e*(x^2+1)^14
```

Rubi [A] time = 0.20, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {28, 446, 76}

$$\frac{1}{26} \left(x^2+1\right)^{13} (d-3e) - \frac{1}{24} \left(x^2+1\right)^{12} (2d-3e) + \frac{1}{22} \left(x^2+1\right)^{11} (d-e) + \frac{1}{28} e \left(x^2+1\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] ((d - e)*(1 + x^2)^11)/22 - ((2*d - 3*e)*(1 + x^2)^12)/24 + ((d - 3*e)*(1 +
 x^2)^13)/26 + (e*(1 + x^2)^14)/28
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx &= \int x^5 \left(1+x^2\right)^{10} \left(d+ex^2\right) \, dx \\
&= \frac{1}{2} \text{Subst}\left(\int x^2(1+x)^{10}(d+ex) \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left((d-e)(1+x)^{10} + (-2d+3e)(1+x)^{11} + (d-3e)(1+x)^{12} + e(1\right.\right. \\
&= \frac{1}{22}(d-e) \left(1+x^2\right)^{11} - \frac{1}{24}(2d-3e) \left(1+x^2\right)^{12} + \frac{1}{26}(d-3e) \left(1+x^2\right)^{13} + \frac{1}{28}e
\end{aligned}$$

Mathematica [B] time = 0.02, size = 153, normalized size = 2.43

$$\frac{1}{26}x^{26}(d+10e)+\frac{5}{24}x^{24}(2d+9e)+\frac{15}{22}x^{22}(3d+8e)+\frac{3}{2}x^{20}(4d+7e)+\frac{7}{3}x^{18}(5d+6e)+\frac{21}{8}x^{16}(6d+5e)+\frac{15}{7}x^{14}(7d+4e)+\frac{5}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^6)/6 + ((10*d + e)*x^8)/8 + ((9*d + 2*e)*x^10)/2 + (5*(8*d + 3*e)*x^12
)/4 + (15*(7*d + 4*e)*x^14)/7 + (21*(6*d + 5*e)*x^16)/8 + (7*(5*d + 6*e)*x^
18)/3 + (3*(4*d + 7*e)*x^20)/2 + (15*(3*d + 8*e)*x^22)/22 + (5*(2*d + 9*e)*
x^24)/24 + ((d + 10*e)*x^26)/26 + (e*x^28)/28
```

fricas [B] time = 0.40, size = 132, normalized size = 2.10

$$\frac{1}{28}x^{28}e+\frac{5}{13}x^{26}e+\frac{1}{26}x^{26}d+\frac{15}{8}x^{24}e+\frac{5}{12}x^{24}d+\frac{60}{11}x^{22}e+\frac{45}{22}x^{22}d+\frac{21}{2}x^{20}e+6x^{20}d+14x^{18}e+\frac{35}{3}x^{18}d+\frac{105}{8}x^{16}e+\frac{63}{4}x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/28*x^28*e + 5/13*x^26*e + 1/26*x^26*d + 15/8*x^24*e + 5/12*x^24*d + 60/11
*x^22*e + 45/22*x^22*d + 21/2*x^20*e + 6*x^20*d + 14*x^18*e + 35/3*x^18*d +
 105/8*x^16*e + 63/4*x^16*d + 60/7*x^14*e + 15*x^14*d + 15/4*x^12*e + 10*x^
12*d + x^10*e + 9/2*x^10*d + 1/8*x^8*e + 5/4*x^8*d + 1/6*x^6*d
```

giac [B] time = 0.28, size = 143, normalized size = 2.27

$$\frac{1}{28}x^{28}e+\frac{1}{26}dx^{26}+\frac{5}{13}x^{26}e+\frac{5}{12}dx^{24}+\frac{15}{8}x^{24}e+\frac{45}{22}dx^{22}+\frac{60}{11}x^{22}e+6dx^{20}+\frac{21}{2}x^{20}e+\frac{35}{3}dx^{18}+14x^{18}e+\frac{63}{4}dx^{16}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/28*x^28*e + 1/26*d*x^26 + 5/13*x^26*e + 5/12*d*x^24 + 15/8*x^24*e + 45/22
*d*x^22 + 60/11*x^22*e + 6*d*x^20 + 21/2*x^20*e + 35/3*d*x^18 + 14*x^18*e +
 63/4*d*x^16 + 105/8*x^16*e + 15*d*x^14 + 60/7*x^14*e + 10*d*x^12 + 15/4*x^
12*e + 9/2*d*x^10 + x^10*e + 5/4*d*x^8 + 1/8*x^8*e + 1/6*d*x^6
```

maple [B] time = 0.00, size = 130, normalized size = 2.06

$$\frac{e\,x^{28}}{28}+\frac{(d+10e)\,x^{26}}{26}+\frac{(10d+45e)\,x^{24}}{24}+\frac{(45d+120e)\,x^{22}}{22}+\frac{(120d+210e)\,x^{20}}{20}+\frac{(210d+252e)\,x^{18}}{18}+\frac{(252d+210e}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/28*e*x^28+1/26*(d+10*e)*x^26+1/24*(10*d+45*e)*x^24+1/22*(45*d+120*e)*x^22
+1/20*(120*d+210*e)*x^20+1/18*(210*d+252*e)*x^18+1/16*(252*d+210*e)*x^16+1/
14*(210*d+120*e)*x^14+1/12*(120*d+45*e)*x^12+1/10*(45*d+10*e)*x^10+1/8*(10*
d+e)*x^8+1/6*d*x^6
```

maxima [B] time = 0.59, size = 129, normalized size = 2.05

$$\frac{1}{28}ex^{28}+\frac{1}{26}(d+10e)x^{26}+\frac{5}{24}(2d+9e)x^{24}+\frac{15}{22}(3d+8e)x^{22}+\frac{3}{2}(4d+7e)x^{20}+\frac{7}{3}(5d+6e)x^{18}+\frac{21}{8}(6d+5e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/28*e*x^28 + 1/26*(d + 10*e)*x^26 + 5/24*(2*d + 9*e)*x^24 + 15/22*(3*d + 8
*e)*x^22 + 3/2*(4*d + 7*e)*x^20 + 7/3*(5*d + 6*e)*x^18 + 21/8*(6*d + 5*e)*x
^16 + 15/7*(7*d + 4*e)*x^14 + 5/4*(8*d + 3*e)*x^12 + 1/2*(9*d + 2*e)*x^10 +
 1/8*(10*d + e)*x^8 + 1/6*d*x^6
```

mupad [B] time = 0.09, size = 121, normalized size = 1.92

$$\frac{e\,x^{28}}{28}+\left(\frac{d}{26}+\frac{5\,e}{13}\right)\,x^{26}+\left(\frac{5\,d}{12}+\frac{15\,e}{8}\right)\,x^{24}+\left(\frac{45\,d}{22}+\frac{60\,e}{11}\right)\,x^{22}+\left(6\,d+\frac{21\,e}{2}\right)\,x^{20}+\left(\frac{35\,d}{3}+14\,e\right)\,x^{18}+\left(\frac{63\,d}{4}+\frac{105\,e}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^8*((5*d)/4 + e/8) + x^12*(10*d + (15*e)/4) + x^20*(6*d + (21*e)/2) + x^24
*((5*d)/12 + (15*e)/8) + x^18*((35*d)/3 + 14*e) + x^26*(d/26 + (5*e)/13) +
x^14*(15*d + (60*e)/7) + x^22*((45*d)/22 + (60*e)/11) + x^16*((63*d)/4 + (1
05*e)/8) + (d*x^6)/6 + (e*x^28)/28 + x^10*((9*d)/2 + e)
```

sympy [B] time = 0.10, size = 134, normalized size = 2.13

$$\frac{dx^{6}}{6}+\frac{ex^{28}}{28}+x^{26}\left(\frac{d}{26}+\frac{5e}{13}\right)+x^{24}\left(\frac{5d}{12}+\frac{15e}{8}\right)+x^{22}\left(\frac{45d}{22}+\frac{60e}{11}\right)+x^{20}\left(6d+\frac{21e}{2}\right)+x^{18}\left(\frac{35d}{3}+14e\right)+x^{16}\left(\frac{63d}{4}+\frac{105}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**6/6 + e*x**28/28 + x**26*(d/26 + 5*e/13) + x**24*(5*d/12 + 15*e/8) + x
**22*(45*d/22 + 60*e/11) + x**20*(6*d + 21*e/2) + x**18*(35*d/3 + 14*e) + x
**16*(63*d/4 + 105*e/8) + x**14*(15*d + 60*e/7) + x**12*(10*d + 15*e/4) + x
**10*(9*d/2 + e) + x**8*(5*d/4 + e/8)
```

3.57 $\int x^4 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=153

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d+4e)+$$

```
[Out] 1/5*d*x^5+1/7*(10*d+e)*x^7+5/9*(9*d+2*e)*x^9+15/11*(8*d+3*e)*x^11+30/13*(7*
d+4*e)*x^13+14/5*(6*d+5*e)*x^15+42/17*(5*d+6*e)*x^17+30/19*(4*d+7*e)*x^19+5
/7*(3*d+8*e)*x^21+5/23*(2*d+9*e)*x^23+1/25*(d+10*e)*x^25+1/27*e*x^27
```

Rubi [A] time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{28, 448\}$

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d+4e)+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^
11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*
e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d +
 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 \, dx &= \int x^4 \left(1+x^2\right)^{10}\left(d+ex^2\right) \, dx\\ &= \int \left(dx^4+(10d+e)x^6+5(9d+2e)x^8+15(8d+3e)x^{10}+30(7d+4e)x^{12}\right.\\ &= \frac{dx^5}{5}+\frac{1}{7}(10d+e)x^7+\frac{5}{9}(9d+2e)x^9+\frac{15}{11}(8d+3e)x^{11}+\frac{30}{13}(7d+4e)x^{13}\end{aligned}$$

Mathematica [A] time = 0.02, size = 153, normalized size = 1.00

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d+4e)+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^
11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*
e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d +
 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

fricas [A] time = 0.45, size = 133, normalized size = 0.87

$$\frac{1}{27}x^{27}e+\frac{2}{5}x^{25}e+\frac{1}{25}x^{25}d+\frac{45}{23}x^{23}e+\frac{10}{23}x^{23}d+\frac{40}{7}x^{21}e+\frac{15}{7}x^{21}d+\frac{210}{19}x^{19}e+\frac{120}{19}x^{19}d+\frac{252}{17}x^{17}e+\frac{210}{17}x^{17}d+14x^{15}e+\frac{84}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/27*x^27*e + 2/5*x^25*e + 1/25*x^25*d + 45/23*x^23*e + 10/23*x^23*d + 40/7
*x^21*e + 15/7*x^21*d + 210/19*x^19*e + 120/19*x^19*d + 252/17*x^17*e + 210
/17*x^17*d + 14*x^15*e + 84/5*x^15*d + 120/13*x^13*e + 210/13*x^13*d + 45/1
1*x^11*e + 120/11*x^11*d + 10/9*x^9*e + 5*x^9*d + 1/7*x^7*e + 10/7*x^7*d +
1/5*x^5*d
```

giac [A] time = 0.30, size = 144, normalized size = 0.94

$$\frac{1}{27}x^{27}e+\frac{1}{25}dx^{25}+\frac{2}{5}x^{25}e+\frac{10}{23}dx^{23}+\frac{45}{23}x^{23}e+\frac{15}{7}dx^{21}+\frac{40}{7}x^{21}e+\frac{120}{19}dx^{19}+\frac{210}{19}x^{19}e+\frac{210}{17}dx^{17}+\frac{252}{17}x^{17}e+\frac{84}{5}dx^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/27*x^27*e + 1/25*d*x^25 + 2/5*x^25*e + 10/23*d*x^23 + 45/23*x^23*e + 15/7
*d*x^21 + 40/7*x^21*e + 120/19*d*x^19 + 210/19*x^19*e + 210/17*d*x^17 + 252
/17*x^17*e + 84/5*d*x^15 + 14*x^15*e + 210/13*d*x^13 + 120/13*x^13*e + 120/
11*d*x^11 + 45/11*x^11*e + 5*d*x^9 + 10/9*x^9*e + 10/7*d*x^7 + 1/7*x^7*e +
1/5*d*x^5
```

maple [A] time = 0.00, size = 130, normalized size = 0.85

$$\frac{e\,x^{27}}{27}+\frac{(d+10e)\,x^{25}}{25}+\frac{(10d+45e)\,x^{23}}{23}+\frac{(45d+120e)\,x^{21}}{21}+\frac{(120d+210e)\,x^{19}}{19}+\frac{(210d+252e)\,x^{17}}{17}+\frac{(252d+210e)\,x}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/27*e*x^27+1/25*(d+10*e)*x^25+1/23*(10*d+45*e)*x^23+1/21*(45*d+120*e)*x^21
+1/19*(120*d+210*e)*x^19+1/17*(210*d+252*e)*x^17+1/15*(252*d+210*e)*x^15+1/
13*(210*d+120*e)*x^13+1/11*(120*d+45*e)*x^11+1/9*(45*d+10*e)*x^9+1/7*(10*d+
e)*x^7+1/5*d*x^5
```

maxima [A] time = 0.67, size = 129, normalized size = 0.84

$$\frac{1}{27}ex^{27}+\frac{1}{25}(d+10e)x^{25}+\frac{5}{23}(2d+9e)x^{23}+\frac{5}{7}(3d+8e)x^{21}+\frac{30}{19}(4d+7e)x^{19}+\frac{42}{17}(5d+6e)x^{17}+\frac{14}{5}(6d+5e)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/27*e*x^27 + 1/25*(d + 10*e)*x^25 + 5/23*(2*d + 9*e)*x^23 + 5/7*(3*d + 8*e
)*x^21 + 30/19*(4*d + 7*e)*x^19 + 42/17*(5*d + 6*e)*x^17 + 14/5*(6*d + 5*e)
*x^15 + 30/13*(7*d + 4*e)*x^13 + 15/11*(8*d + 3*e)*x^11 + 5/9*(9*d + 2*e)*x
^9 + 1/7*(10*d + e)*x^7 + 1/5*d*x^5
```

mupad [B] time = 0.12, size = 123, normalized size = 0.80

$$\frac{e\,x^{27}}{27}+\left(\frac{d}{25}+\frac{2\,e}{5}\right)\,x^{25}+\left(\frac{10\,d}{23}+\frac{45\,e}{23}\right)\,x^{23}+\left(\frac{15\,d}{7}+\frac{40\,e}{7}\right)\,x^{21}+\left(\frac{120\,d}{19}+\frac{210\,e}{19}\right)\,x^{19}+\left(\frac{210\,d}{17}+\frac{252\,e}{17}\right)\,x^{17}+\left(\frac{84}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^7*((10*d)/7 + e/7) + x^9*(5*d + (10*e)/9) + x^25*(d/25 + (2*e)/5) + x^21*
((15*d)/7 + (40*e)/7) + x^15*((84*d)/5 + 14*e) + x^23*((10*d)/23 + (45*e)/2
3) + x^11*((120*d)/11 + (45*e)/11) + x^13*((210*d)/13 + (120*e)/13) + x^19*
((120*d)/19 + (210*e)/19) + x^17*((210*d)/17 + (252*e)/17) + (d*x^5)/5 + (e
*x^27)/27
```

sympy [A] time = 0.10, size = 141, normalized size = 0.92

$$\frac{dx^5}{5}+\frac{ex^{27}}{27}+x^{25}\left(\frac{d}{25}+\frac{2e}{5}\right)+x^{23}\left(\frac{10d}{23}+\frac{45e}{23}\right)+x^{21}\left(\frac{15d}{7}+\frac{40e}{7}\right)+x^{19}\left(\frac{120d}{19}+\frac{210e}{19}\right)+x^{17}\left(\frac{210d}{17}+\frac{252e}{17}\right)+x^{15}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**5/5 + e*x**27/27 + x**25*(d/25 + 2*e/5) + x**23*(10*d/23 + 45*e/23) +
x**21*(15*d/7 + 40*e/7) + x**19*(120*d/19 + 210*e/19) + x**17*(210*d/17 + 2
52*e/17) + x**15*(84*d/5 + 14*e) + x**13*(210*d/13 + 120*e/13) + x**11*(120
*d/11 + 45*e/11) + x**9*(5*d + 10*e/9) + x**7*(10*d/7 + e/7)
```

3.58 $\int x^3 \left(d+e x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=45

$$\frac{1}{24} \left(x^2+1\right)^{12} (d-2 e)-\frac{1}{22} \left(x^2+1\right)^{11} (d-e)+\frac{1}{26} e \left(x^2+1\right)^{13}$$

```
[Out] -1/22*(d-e)*(x^2+1)^11+1/24*(d-2*e)*(x^2+1)^12+1/26*e*(x^2+1)^13
```

Rubi [A] time = 0.12, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {28, 446, 76}

$$\frac{1}{24} \left(x^2+1\right)^{12} (d-2 e)-\frac{1}{22} \left(x^2+1\right)^{11} (d-e)+\frac{1}{26} e \left(x^2+1\right)^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] -((d - e)*(1 + x^2)^11)/22 + ((d - 2*e)*(1 + x^2)^12)/24 + (e*(1 + x^2)^13)
/26
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int x^3 \left(d+e x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx &= \int x^3 \left(1+x^2\right)^{10} \left(d+e x^2\right) \, dx \\ &= \frac{1}{2} \text{Subst}\left(\int x (1+x)^{10} (d+e x) \, dx,x,x^2\right) \\ &= \frac{1}{2} \text{Subst}\left(\int \left((-d+e) (1+x)^{10}+(d-2 e) (1+x)^{11}+e (1+x)^{12}\right) \, dx,x,x^2\right) \\ &= -\frac{1}{22} (d-e) \left(1+x^2\right)^{11}+\frac{1}{24} (d-2 e) \left(1+x^2\right)^{12}+\frac{1}{26} e \left(1+x^2\right)^{13} \end{aligned}$$

Mathematica [B] time = 0.02, size = 151, normalized size = 3.36

$$\frac{1}{24} x^{24} (d+10 e)+\frac{5}{22} x^{22} (2 d+9 e)+\frac{3}{4} x^{20} (3 d+8 e)+\frac{5}{3} x^{18} (4 d+7 e)+\frac{21}{8} x^{16} (5 d+6 e)+3 x^{14} (6 d+5 e)+\frac{5}{2} x^{12} (7 d+4 e)+\frac{3}{2} x^{10} (8 d$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^4)/4 + ((10*d + e)*x^6)/6 + (5*(9*d + 2*e)*x^8)/8 + (3*(8*d + 3*e)*x^1
0)/2 + (5*(7*d + 4*e)*x^12)/2 + 3*(6*d + 5*e)*x^14 + (21*(5*d + 6*e)*x^16)/
8 + (5*(4*d + 7*e)*x^18)/3 + (3*(3*d + 8*e)*x^20)/4 + (5*(2*d + 9*e)*x^22)/
22 + ((d + 10*e)*x^24)/24 + (e*x^26)/26
```

fricas [B] time = 0.65, size = 133, normalized size = 2.96

$$\frac{1}{26}x^{26}e+\frac{5}{12}x^{24}e+\frac{1}{24}x^{24}d+\frac{45}{22}x^{22}e+\frac{5}{11}x^{22}d+6x^{20}e+\frac{9}{4}x^{20}d+\frac{35}{3}x^{18}e+\frac{20}{3}x^{18}d+\frac{63}{4}x^{16}e+\frac{105}{8}x^{16}d+15x^{14}e+18x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/26*x^26*e + 5/12*x^24*e + 1/24*x^24*d + 45/22*x^22*e + 5/11*x^22*d + 6*x^
20*e + 9/4*x^20*d + 35/3*x^18*e + 20/3*x^18*d + 63/4*x^16*e + 105/8*x^16*d
+ 15*x^14*e + 18*x^14*d + 10*x^12*e + 35/2*x^12*d + 9/2*x^10*e + 12*x^10*d
+ 5/4*x^8*e + 45/8*x^8*d + 1/6*x^6*e + 5/3*x^6*d + 1/4*x^4*d
```

giac [B] time = 0.40, size = 144, normalized size = 3.20

$$\frac{1}{26}x^{26}e+\frac{1}{24}dx^{24}+\frac{5}{12}x^{24}e+\frac{5}{11}dx^{22}+\frac{45}{22}x^{22}e+\frac{9}{4}dx^{20}+6x^{20}e+\frac{20}{3}dx^{18}+\frac{35}{3}x^{18}e+\frac{105}{8}dx^{16}+\frac{63}{4}x^{16}e+18dx^{14}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/26*x^26*e + 1/24*d*x^24 + 5/12*x^24*e + 5/11*d*x^22 + 45/22*x^22*e + 9/4*
d*x^20 + 6*x^20*e + 20/3*d*x^18 + 35/3*x^18*e + 105/8*d*x^16 + 63/4*x^16*e
+ 18*d*x^14 + 15*x^14*e + 35/2*d*x^12 + 10*x^12*e + 12*d*x^10 + 9/2*x^10*e
+ 45/8*d*x^8 + 5/4*x^8*e + 5/3*d*x^6 + 1/6*x^6*e + 1/4*d*x^4
```

maple [B] time = 0.00, size = 130, normalized size = 2.89

$$\frac{e\,x^{26}}{26}+\frac{(d+10e)\,x^{24}}{24}+\frac{(10d+45e)\,x^{22}}{22}+\frac{(45d+120e)\,x^{20}}{20}+\frac{(120d+210e)\,x^{18}}{18}+\frac{(210d+252e)\,x^{16}}{16}+\frac{(252d+210e}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/26*e*x^26+1/24*(d+10*e)*x^24+1/22*(10*d+45*e)*x^22+1/20*(45*d+120*e)*x^20
+1/18*(120*d+210*e)*x^18+1/16*(210*d+252*e)*x^16+1/14*(252*d+210*e)*x^14+1/
12*(210*d+120*e)*x^12+1/10*(120*d+45*e)*x^10+1/8*(45*d+10*e)*x^8+1/6*(10*d+
e)*x^6+1/4*d*x^4
```

maxima [B] time = 0.65, size = 129, normalized size = 2.87

$$\frac{1}{26}ex^{26}+\frac{1}{24}(d+10e)x^{24}+\frac{5}{22}(2d+9e)x^{22}+\frac{3}{4}(3d+8e)x^{20}+\frac{5}{3}(4d+7e)x^{18}+\frac{21}{8}(5d+6e)x^{16}+3(6d+5e)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/26*e*x^26 + 1/24*(d + 10*e)*x^24 + 5/22*(2*d + 9*e)*x^22 + 3/4*(3*d + 8*e
)*x^20 + 5/3*(4*d + 7*e)*x^18 + 21/8*(5*d + 6*e)*x^16 + 3*(6*d + 5*e)*x^14
+ 5/2*(7*d + 4*e)*x^12 + 3/2*(8*d + 3*e)*x^10 + 5/8*(9*d + 2*e)*x^8 + 1/6*(
10*d + e)*x^6 + 1/4*d*x^4
```

mupad [B] time = 0.08, size = 123, normalized size = 2.73

$$\frac{e\,x^{26}}{26}+\left(\frac{d}{24}+\frac{5\,e}{12}\right)\,x^{24}+\left(\frac{5\,d}{11}+\frac{45\,e}{22}\right)\,x^{22}+\left(\frac{9\,d}{4}+6\,e\right)\,x^{20}+\left(\frac{20\,d}{3}+\frac{35\,e}{3}\right)\,x^{18}+\left(\frac{105\,d}{8}+\frac{63\,e}{4}\right)\,x^{16}+(18\,d+15\,e)\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^6*((5*d)/3 + e/6) + x^10*(12*d + (9*e)/2) + x^20*((9*d)/4 + 6*e) + x^14*(
18*d + 15*e) + x^12*((35*d)/2 + 10*e) + x^24*(d/24 + (5*e)/12) + x^8*((45*d
)/8 + (5*e)/4) + x^18*((20*d)/3 + (35*e)/3) + x^22*((5*d)/11 + (45*e)/22) +
 x^16*((105*d)/8 + (63*e)/4) + (d*x^4)/4 + (e*x^26)/26
```

sympy [B] time = 0.10, size = 136, normalized size = 3.02

$$\frac{dx^4}{4}+\frac{ex^{26}}{26}+x^{24}\left(\frac{d}{24}+\frac{5e}{12}\right)+x^{22}\left(\frac{5d}{11}+\frac{45e}{22}\right)+x^{20}\left(\frac{9d}{4}+6e\right)+x^{18}\left(\frac{20d}{3}+\frac{35e}{3}\right)+x^{16}\left(\frac{105d}{8}+\frac{63e}{4}\right)+x^{14}\left(18d+15e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**4/4 + e*x**26/26 + x**24*(d/24 + 5*e/12) + x**22*(5*d/11 + 45*e/22) +
x**20*(9*d/4 + 6*e) + x**18*(20*d/3 + 35*e/3) + x**16*(105*d/8 + 63*e/4) +
x**14*(18*d + 15*e) + x**12*(35*d/2 + 10*e) + x**10*(12*d + 9*e/2) + x**8*(
45*d/8 + 5*e/4) + x**6*(5*d/3 + e/6)
```

3.59 $\int x^2 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=153

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d+4e)+$$

```
[Out] 1/3*d*x^3+1/5*(10*d+e)*x^5+5/7*(9*d+2*e)*x^7+5/3*(8*d+3*e)*x^9+30/11*(7*d+4
*e)*x^11+42/13*(6*d+5*e)*x^13+14/5*(5*d+6*e)*x^15+30/17*(4*d+7*e)*x^17+15/1
9*(3*d+8*e)*x^19+5/21*(2*d+9*e)*x^21+1/23*(d+10*e)*x^23+1/25*e*x^25
```

Rubi [A] time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {28, 448}

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d+4e)+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9
)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)
*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d +
9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx &= \int x^2 \left(1+x^2\right)^{10} \left(d+ex^2\right) \, dx \\
&= \int \left(dx^2+(10d+e)x^4+5(9d+2e)x^6+15(8d+3e)x^8+30(7d+4e)x^{10}+\right. \\
&= \frac{dx^3}{3}+\frac{1}{5}(10d+e)x^5+\frac{5}{7}(9d+2e)x^7+\frac{5}{3}(8d+3e)x^9+\frac{30}{11}(7d+4e)x^{11}+
\end{aligned}$$

Mathematica [A] time = 0.02, size = 153, normalized size = 1.00

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d+4e)+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9
)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)
*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d +
9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

fricas [A] time = 0.54, size = 133, normalized size = 0.87

$$\frac{1}{25}x^{25}e+\frac{10}{23}x^{23}e+\frac{1}{23}x^{23}d+\frac{15}{7}x^{21}e+\frac{10}{21}x^{21}d+\frac{120}{19}x^{19}e+\frac{45}{19}x^{19}d+\frac{210}{17}x^{17}e+\frac{120}{17}x^{17}d+\frac{84}{5}x^{15}e+14x^{15}d+\frac{210}{13}x^{13}e+\frac{25}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*e + 10/23*x^23*e + 1/23*x^23*d + 15/7*x^21*e + 10/21*x^21*d + 120
/19*x^19*e + 45/19*x^19*d + 210/17*x^17*e + 120/17*x^17*d + 84/5*x^15*e + 1
4*x^15*d + 210/13*x^13*e + 252/13*x^13*d + 120/11*x^11*e + 210/11*x^11*d +
5*x^9*e + 40/3*x^9*d + 10/7*x^7*e + 45/7*x^7*d + 1/5*x^5*e + 2*x^5*d + 1/3*
x^3*d
```

giac [A] time = 0.29, size = 144, normalized size = 0.94

$$\frac{1}{25}x^{25}e+\frac{1}{23}dx^{23}+\frac{10}{23}x^{23}e+\frac{10}{21}dx^{21}+\frac{15}{7}x^{21}e+\frac{45}{19}dx^{19}+\frac{120}{19}x^{19}e+\frac{120}{17}dx^{17}+\frac{210}{17}x^{17}e+14dx^{15}+\frac{84}{5}x^{15}e+\frac{252}{13}dx^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/25*x^25*e + 1/23*d*x^23 + 10/23*x^23*e + 10/21*d*x^21 + 15/7*x^21*e + 45/
19*d*x^19 + 120/19*x^19*e + 120/17*d*x^17 + 210/17*x^17*e + 14*d*x^15 + 84/
5*x^15*e + 252/13*d*x^13 + 210/13*x^13*e + 210/11*d*x^11 + 120/11*x^11*e +
40/3*d*x^9 + 5*x^9*e + 45/7*d*x^7 + 10/7*x^7*e + 2*d*x^5 + 1/5*x^5*e + 1/3*
d*x^3
```

maple [A] time = 0.00, size = 130, normalized size = 0.85

$$\frac{e\,x^{25}}{25}+\frac{(d+10e)\,x^{23}}{23}+\frac{(10d+45e)\,x^{21}}{21}+\frac{(45d+120e)\,x^{19}}{19}+\frac{(120d+210e)\,x^{17}}{17}+\frac{(210d+252e)\,x^{15}}{15}+\frac{(252d+210e)\,x^{1}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/25*e*x^25+1/23*(d+10*e)*x^23+1/21*(10*d+45*e)*x^21+1/19*(45*d+120*e)*x^19
+1/17*(120*d+210*e)*x^17+1/15*(210*d+252*e)*x^15+1/13*(252*d+210*e)*x^13+1/
11*(210*d+120*e)*x^11+1/9*(120*d+45*e)*x^9+1/7*(45*d+10*e)*x^7+1/5*(10*d+e)
*x^5+1/3*d*x^3
```

maxima [A] time = 0.50, size = 129, normalized size = 0.84

$$\frac{1}{25}ex^{25}+\frac{1}{23}(d+10e)x^{23}+\frac{5}{21}(2d+9e)x^{21}+\frac{15}{19}(3d+8e)x^{19}+\frac{30}{17}(4d+7e)x^{17}+\frac{14}{5}(5d+6e)x^{15}+\frac{42}{13}(6d+5e)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/25*e*x^25 + 1/23*(d + 10*e)*x^23 + 5/21*(2*d + 9*e)*x^21 + 15/19*(3*d + 8
*e)*x^19 + 30/17*(4*d + 7*e)*x^17 + 14/5*(5*d + 6*e)*x^15 + 42/13*(6*d + 5*
e)*x^13 + 30/11*(7*d + 4*e)*x^11 + 5/3*(8*d + 3*e)*x^9 + 5/7*(9*d + 2*e)*x^
7 + 1/5*(10*d + e)*x^5 + 1/3*d*x^3
```

mupad [B] time = 0.08, size = 123, normalized size = 0.80

$$\frac{e\,x^{25}}{25}+\left(\frac{d}{23}+\frac{10\,e}{23}\right)\,x^{23}+\left(\frac{10\,d}{21}+\frac{15\,e}{7}\right)\,x^{21}+\left(\frac{45\,d}{19}+\frac{120\,e}{19}\right)\,x^{19}+\left(\frac{120\,d}{17}+\frac{210\,e}{17}\right)\,x^{17}+\left(14\,d+\frac{84\,e}{5}\right)\,x^{15}+\left(\frac{25}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^5*(2*d + e/5) + x^9*((40*d)/3 + 5*e) + x^21*((10*d)/21 + (15*e)/7) + x^7*
((45*d)/7 + (10*e)/7) + x^23*(d/23 + (10*e)/23) + x^15*(14*d + (84*e)/5) +
x^19*((45*d)/19 + (120*e)/19) + x^11*((210*d)/11 + (120*e)/11) + x^17*((120
*d)/17 + (210*e)/17) + x^13*((252*d)/13 + (210*e)/13) + (d*x^3)/3 + (e*x^25
)/25
```

sympy [A] time = 0.10, size = 139, normalized size = 0.91

$$\frac{dx^3}{3}+\frac{ex^{25}}{25}+x^{23}\left(\frac{d}{23}+\frac{10e}{23}\right)+x^{21}\left(\frac{10d}{21}+\frac{15e}{7}\right)+x^{19}\left(\frac{45d}{19}+\frac{120e}{19}\right)+x^{17}\left(\frac{120d}{17}+\frac{210e}{17}\right)+x^{15}\left(14d+\frac{84e}{5}\right)+x^{13}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**3/3 + e*x**25/25 + x**23*(d/23 + 10*e/23) + x**21*(10*d/21 + 15*e/7) +
 x**19*(45*d/19 + 120*e/19) + x**17*(120*d/17 + 210*e/17) + x**15*(14*d + 8
4*e/5) + x**13*(252*d/13 + 210*e/13) + x**11*(210*d/11 + 120*e/11) + x**9*(
40*d/3 + 5*e) + x**7*(45*d/7 + 10*e/7) + x**5*(2*d + e/5)
```

3.60 $\int x\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=29

$$\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{24}e\left(x^2+1\right)^{12}$$

```
[Out] 1/22*(d-e)*(x^2+1)^11+1/24*e*(x^2+1)^12
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 444, 43}

$$\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{24}e\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] ((d - e)*(1 + x^2)^11)/22 + (e*(1 + x^2)^12)/24
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x\left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int (1+x)^{10}(d+ex)\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left((d-e)(1+x)^{10}+e(1+x)^{11}\right)\,dx,x,x^2\right)\\ &= \frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}e\left(1+x^2\right)^{12}\end{aligned}$$

Mathematica [B] time = 0.01, size = 149, normalized size = 5.14

$$\frac{1}{22}x^{22}(d+10e)+\frac{1}{4}x^{20}(2d+9e)+\frac{5}{6}x^{18}(3d+8e)+\frac{15}{8}x^{16}(4d+7e)+3x^{14}(5d+6e)+\frac{7}{2}x^{12}(6d+5e)+3x^{10}(7d+4e)+\frac{15}{8}x^8(8d+$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^2)/2 + ((10*d + e)*x^4)/4 + (5*(9*d + 2*e)*x^6)/6 + (15*(8*d + 3*e)*x^
8)/8 + 3*(7*d + 4*e)*x^10 + (7*(6*d + 5*e)*x^12)/2 + 3*(5*d + 6*e)*x^14 + (
15*(4*d + 7*e)*x^16)/8 + (5*(3*d + 8*e)*x^18)/6 + ((2*d + 9*e)*x^20)/4 + ((
d + 10*e)*x^22)/22 + (e*x^24)/24
```

fricas [B] time = 0.64, size = 133, normalized size = 4.59

$$\frac{1}{24}x^{24}e+\frac{5}{11}x^{22}e+\frac{1}{22}x^{22}d+\frac{9}{4}x^{20}e+\frac{1}{2}x^{20}d+\frac{20}{3}x^{18}e+\frac{5}{2}x^{18}d+\frac{105}{8}x^{16}e+\frac{15}{2}x^{16}d+18x^{14}e+15x^{14}d+\frac{35}{2}x^{12}e+21x^{12}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*e + 5/11*x^22*e + 1/22*x^22*d + 9/4*x^20*e + 1/2*x^20*d + 20/3*x^
18*e + 5/2*x^18*d + 105/8*x^16*e + 15/2*x^16*d + 18*x^14*e + 15*x^14*d + 35
/2*x^12*e + 21*x^12*d + 12*x^10*e + 21*x^10*d + 45/8*x^8*e + 15*x^8*d + 5/3
*x^6*e + 15/2*x^6*d + 1/4*x^4*e + 5/2*x^4*d + 1/2*x^2*d
```

giac [B] time = 0.31, size = 144, normalized size = 4.97

$$\frac{1}{24}x^{24}e+\frac{1}{22}dx^{22}+\frac{5}{11}x^{22}e+\frac{1}{2}dx^{20}+\frac{9}{4}x^{20}e+\frac{5}{2}dx^{18}+\frac{20}{3}x^{18}e+\frac{15}{2}dx^{16}+\frac{105}{8}x^{16}e+15\,dx^{14}+18\,x^{14}e+21\,dx^{12}+\frac{35}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/24*x^24*e + 1/22*d*x^22 + 5/11*x^22*e + 1/2*d*x^20 + 9/4*x^20*e + 5/2*d*x
^18 + 20/3*x^18*e + 15/2*d*x^16 + 105/8*x^16*e + 15*d*x^14 + 18*x^14*e + 21
*d*x^12 + 35/2*x^12*e + 21*d*x^10 + 12*x^10*e + 15*d*x^8 + 45/8*x^8*e + 15/
2*d*x^6 + 5/3*x^6*e + 5/2*d*x^4 + 1/4*x^4*e + 1/2*d*x^2
```

maple [B] time = 0.00, size = 130, normalized size = 4.48

$$\frac{e\,x^{24}}{24}+\frac{(d+10e)\,x^{22}}{22}+\frac{(10d+45e)\,x^{20}}{20}+\frac{(45d+120e)\,x^{18}}{18}+\frac{(120d+210e)\,x^{16}}{16}+\frac{(210d+252e)\,x^{14}}{14}+\frac{(252d+210e}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/24*e*x^24+1/22*(d+10*e)*x^22+1/20*(10*d+45*e)*x^20+1/18*(45*d+120*e)*x^18
+1/16*(120*d+210*e)*x^16+1/14*(210*d+252*e)*x^14+1/12*(252*d+210*e)*x^12+1/
10*(210*d+120*e)*x^10+1/8*(120*d+45*e)*x^8+1/6*(45*d+10*e)*x^6+1/4*(10*d+e)
*x^4+1/2*d*x^2
```

maxima [B] time = 0.65, size = 129, normalized size = 4.45

$$\frac{1}{24}\,ex^{24}+\frac{1}{22}\,(d+10\,e)x^{22}+\frac{1}{4}\,(2\,d+9\,e)x^{20}+\frac{5}{6}\,(3\,d+8\,e)x^{18}+\frac{15}{8}\,(4\,d+7\,e)x^{16}+3\,(5\,d+6\,e)x^{14}+\frac{7}{2}\,(6\,d+5\,e)x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/24*e*x^24 + 1/22*(d + 10*e)*x^22 + 1/4*(2*d + 9*e)*x^20 + 5/6*(3*d + 8*e)
*x^18 + 15/8*(4*d + 7*e)*x^16 + 3*(5*d + 6*e)*x^14 + 7/2*(6*d + 5*e)*x^12 +
 3*(7*d + 4*e)*x^10 + 15/8*(8*d + 3*e)*x^8 + 5/6*(9*d + 2*e)*x^6 + 1/4*(10*
d + e)*x^4 + 1/2*d*x^2
```

mupad [B] time = 0.08, size = 123, normalized size = 4.24

$$\frac{e\,x^{24}}{24}+\left(\frac{d}{22}+\frac{5\,e}{11}\right)\,x^{22}+\left(\frac{d}{2}+\frac{9\,e}{4}\right)\,x^{20}+\left(\frac{5\,d}{2}+\frac{20\,e}{3}\right)\,x^{18}+\left(\frac{15\,d}{2}+\frac{105\,e}{8}\right)\,x^{16}+(15\,d+18\,e)\,x^{14}+\left(21\,d+\frac{35\,e}{2}\right)\,x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^4*((5*d)/2 + e/4) + x^6*((15*d)/2 + (5*e)/3) + x^20*(d/2 + (9*e)/4) + x^1
0*(21*d + 12*e) + x^14*(15*d + 18*e) + x^18*((5*d)/2 + (20*e)/3) + x^22*(d/
22 + (5*e)/11) + x^12*(21*d + (35*e)/2) + x^8*(15*d + (45*e)/8) + x^16*((15
*d)/2 + (105*e)/8) + (d*x^2)/2 + (e*x^24)/24
```

sympy [B] time = 0.10, size = 133, normalized size = 4.59

$$\frac{dx^2}{2}+\frac{ex^{24}}{24}+x^{22}\left(\frac{d}{22}+\frac{5e}{11}\right)+x^{20}\left(\frac{d}{2}+\frac{9e}{4}\right)+x^{18}\left(\frac{5d}{2}+\frac{20e}{3}\right)+x^{16}\left(\frac{15d}{2}+\frac{105e}{8}\right)+x^{14}\left(15d+18e\right)+x^{12}\left(21d+\frac{35e}{2}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**2/2 + e*x**24/24 + x**22*(d/22 + 5*e/11) + x**20*(d/2 + 9*e/4) + x**18
*(5*d/2 + 20*e/3) + x**16*(15*d/2 + 105*e/8) + x**14*(15*d + 18*e) + x**12*
(21*d + 35*e/2) + x**10*(21*d + 12*e) + x**8*(15*d + 45*e/8) + x**6*(15*d/2
 + 5*e/3) + x**4*(5*d/2 + e/4)
```

3.61 $\int \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=143

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+4e)+\frac{15}{7}$$

```
[Out] d*x+1/3*(10*d+e)*x^3+(9*d+2*e)*x^5+15/7*(8*d+3*e)*x^7+10/3*(7*d+4*e)*x^9+42
/11*(6*d+5*e)*x^11+42/13*(5*d+6*e)*x^13+2*(4*d+7*e)*x^15+15/17*(3*d+8*e)*x^
17+5/19*(2*d+9*e)*x^19+1/21*(d+10*e)*x^21+1/23*e*x^23
```

Rubi [A] time = 0.07, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {28, 373}

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+4e)+\frac{15}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(
7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2
*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((
d + 10*e)*x^21)/21 + (e*x^23)/23
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int \left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\ &= \int \left(d+(10d+e)x^2+5(9d+2e)x^4+15(8d+3e)x^6+30(7d+4e)x^8+42(6\right.\\ &= dx+\frac{1}{3}(10d+e)x^3+(9d+2e)x^5+\frac{15}{7}(8d+3e)x^7+\frac{10}{3}(7d+4e)x^9+\frac{42}{11}(6d\end{aligned}$$

Mathematica [A] time = 0.02, size = 143, normalized size = 1.00

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+4e)+\frac{15}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(
7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2
```

```
*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((
d + 10*e)*x^21)/21 + (e*x^23)/23
```

fricas [A] time = 0.54, size = 130, normalized size = 0.91

$$\frac{1}{23}x^{23}e+\frac{10}{21}x^{21}e+\frac{1}{21}x^{21}d+\frac{45}{19}x^{19}e+\frac{10}{19}x^{19}d+\frac{120}{17}x^{17}e+\frac{45}{17}x^{17}d+14x^{15}e+8x^{15}d+\frac{252}{13}x^{13}e+\frac{210}{13}x^{13}d+\frac{210}{11}x^{11}e+\frac{252}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*e + 10/21*x^21*e + 1/21*x^21*d + 45/19*x^19*e + 10/19*x^19*d + 12
0/17*x^17*e + 45/17*x^17*d + 14*x^15*e + 8*x^15*d + 252/13*x^13*e + 210/13*
x^13*d + 210/11*x^11*e + 252/11*x^11*d + 40/3*x^9*e + 70/3*x^9*d + 45/7*x^7
*e + 120/7*x^7*d + 2*x^5*e + 9*x^5*d + 1/3*x^3*e + 10/3*x^3*d + x*d
```

giac [A] time = 0.36, size = 141, normalized size = 0.99

$$\frac{1}{23}x^{23}e+\frac{1}{21}dx^{21}+\frac{10}{21}x^{21}e+\frac{10}{19}dx^{19}+\frac{45}{19}x^{19}e+\frac{45}{17}dx^{17}+\frac{120}{17}x^{17}e+8dx^{15}+14x^{15}e+\frac{210}{13}dx^{13}+\frac{252}{13}x^{13}e+\frac{252}{11}dx^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/23*x^23*e + 1/21*d*x^21 + 10/21*x^21*e + 10/19*d*x^19 + 45/19*x^19*e + 45
/17*d*x^17 + 120/17*x^17*e + 8*d*x^15 + 14*x^15*e + 210/13*d*x^13 + 252/13*
x^13*e + 252/11*d*x^11 + 210/11*x^11*e + 70/3*d*x^9 + 40/3*x^9*e + 120/7*d*
x^7 + 45/7*x^7*e + 9*d*x^5 + 2*x^5*e + 10/3*d*x^3 + 1/3*x^3*e + d*x
```

maple [A] time = 0.00, size = 127, normalized size = 0.89

$$\frac{e x^{23}}{23}+\frac{(d+10e)x^{21}}{21}+\frac{(10d+45e)x^{19}}{19}+\frac{(45d+120e)x^{17}}{17}+\frac{(120d+210e)x^{15}}{15}+\frac{(210d+252e)x^{13}}{13}+\frac{(252d+210e)x}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/23*e*x^23+1/21*(d+10*e)*x^21+1/19*(10*d+45*e)*x^19+1/17*(45*d+120*e)*x^17
+1/15*(120*d+210*e)*x^15+1/13*(210*d+252*e)*x^13+1/11*(252*d+210*e)*x^11+1/
9*(210*d+120*e)*x^9+1/7*(120*d+45*e)*x^7+1/5*(45*d+10*e)*x^5+1/3*(10*d+e)*x
^3+d*x
```

maxima [A] time = 0.58, size = 125, normalized size = 0.87

$$\frac{1}{23}ex^{23}+\frac{1}{21}(d+10e)x^{21}+\frac{5}{19}(2d+9e)x^{19}+\frac{15}{17}(3d+8e)x^{17}+2(4d+7e)x^{15}+\frac{42}{13}(5d+6e)x^{13}+\frac{42}{11}(6d+5e)x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/23*e*x^23 + 1/21*(d + 10*e)*x^21 + 5/19*(2*d + 9*e)*x^19 + 15/17*(3*d + 8
*e)*x^17 + 2*(4*d + 7*e)*x^15 + 42/13*(5*d + 6*e)*x^13 + 42/11*(6*d + 5*e)*
x^11 + 10/3*(7*d + 4*e)*x^9 + 15/7*(8*d + 3*e)*x^7 + (9*d + 2*e)*x^5 + 1/3*
(10*d + e)*x^3 + d*x
```

mupad [B] time = 0.08, size = 120, normalized size = 0.84

$$\frac{e x^{23}}{23}+\left(\frac{d}{21}+\frac{10e}{21}\right)x^{21}+\left(\frac{10d}{19}+\frac{45e}{19}\right)x^{19}+\left(\frac{45d}{17}+\frac{120e}{17}\right)x^{17}+(8d+14e)x^{15}+\left(\frac{210d}{13}+\frac{252e}{13}\right)x^{13}+\left(\frac{252d}{11}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^5*(9*d + 2*e) + x^3*((10*d)/3 + e/3) + x^15*(8*d + 14*e) + x^21*(d/21 + (
10*e)/21) + x^19*((10*d)/19 + (45*e)/19) + x^9*((70*d)/3 + (40*e)/3) + x^7*
((120*d)/7 + (45*e)/7) + x^17*((45*d)/17 + (120*e)/17) + x^11*((252*d)/11 +
 (210*e)/11) + x^13*((210*d)/13 + (252*e)/13) + d*x + (e*x^23)/23
```

sympy [A] time = 0.10, size = 134, normalized size = 0.94

$$dx+\frac{ex^{23}}{23}+x^{21}\left(\frac{d}{21}+\frac{10e}{21}\right)+x^{19}\left(\frac{10d}{19}+\frac{45e}{19}\right)+x^{17}\left(\frac{45d}{17}+\frac{120e}{17}\right)+x^{15}\left(8d+14e\right)+x^{13}\left(\frac{210d}{13}+\frac{252e}{13}\right)+x^{11}\left(\frac{252}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x + e*x**23/23 + x**21*(d/21 + 10*e/21) + x**19*(10*d/19 + 45*e/19) + x**
17*(45*d/17 + 120*e/17) + x**15*(8*d + 14*e) + x**13*(210*d/13 + 252*e/13)
+ x**11*(252*d/11 + 210*e/11) + x**9*(70*d/3 + 40*e/3) + x**7*(120*d/7 + 45
*e/7) + x**5*(9*d + 2*e) + x**3*(10*d/3 + e/3)
```

3.62 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx$

Optimal. Leaf size=93

$$\frac{dx^{20}}{20}+\frac{5dx^{18}}{9}+\frac{45dx^{16}}{16}+\frac{60dx^{14}}{7}+\frac{35dx^{12}}{2}+\frac{126dx^{10}}{5}+\frac{105dx^8}{4}+20dx^6+\frac{45dx^4}{4}+5dx^2+d\log(x)+\frac{1}{22}e\left(x^2+1\right)^{11}$$

```
[Out] 5*d*x^2+45/4*d*x^4+20*d*x^6+105/4*d*x^8+126/5*d*x^10+35/2*d*x^12+60/7*d*x^1
4+45/16*d*x^16+5/9*d*x^18+1/20*d*x^20+1/22*e*(x^2+1)^11+d*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = $\{28, 446, 80, 43\}$

$$\frac{dx^{20}}{20}+\frac{5dx^{18}}{9}+\frac{45dx^{16}}{16}+\frac{60dx^{14}}{7}+\frac{35dx^{12}}{2}+\frac{126dx^{10}}{5}+\frac{105dx^8}{4}+20dx^6+\frac{45dx^4}{4}+5dx^2+d\log(x)+\frac{1}{22}e\left(x^2+1\right)^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] 5*d*x^2 + (45*d*x^4)/4 + 20*d*x^6 + (105*d*x^8)/4 + (126*d*x^10)/5 + (35*d*
x^12)/2 + (60*d*x^14)/7 + (45*d*x^16)/16 + (5*d*x^18)/9 + (d*x^20)/20 + (e*
(1 + x^2)^11)/22 + d*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx = \int \frac{(1+x^2)^{10}(d+ex^2)}{x}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{10}(d+ex)}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{22}e(1+x^2)^{11} + \frac{1}{2}d\,\text{Subst}\left(\int \frac{(1+x)^{10}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{22}e(1+x^2)^{11} + \frac{1}{2}d\,\text{Subst}\left(\int \left(10+\frac{1}{x}+45x+120x^2+210x^3+252x^4+2\right.\right.$$
$$= 5dx^2 + \frac{45dx^4}{4} + 20dx^6 + \frac{105dx^8}{4} + \frac{126dx^{10}}{5} + \frac{35dx^{12}}{2} + \frac{60dx^{14}}{7} + \frac{45dx^{16}}{16}$$

Mathematica [A] time = 0.03, size = 149, normalized size = 1.60

$$\frac{1}{20}x^{20}(d+10e)+\frac{5}{18}x^{18}(2d+9e)+\frac{15}{16}x^{16}(3d+8e)+\frac{15}{7}x^{14}(4d+7e)+\frac{7}{2}x^{12}(5d+6e)+\frac{21}{5}x^{10}(6d+5e)+\frac{15}{4}x^{8}(7d+4e)+\frac{5}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] ((10*d + e)*x^2)/2 + (5*(9*d + 2*e)*x^4)/4 + (5*(8*d + 3*e)*x^6)/2 + (15*(7
*d + 4*e)*x^8)/4 + (21*(6*d + 5*e)*x^10)/5 + (7*(5*d + 6*e)*x^12)/2 + (15*(
4*d + 7*e)*x^14)/7 + (15*(3*d + 8*e)*x^16)/16 + (5*(2*d + 9*e)*x^18)/18 + (
(d + 10*e)*x^20)/20 + (e*x^22)/22 + d*Log[x]
```

fricas [A] time = 0.64, size = 127, normalized size = 1.37

$$\frac{1}{22}ex^{22}+\frac{1}{20}(d+10e)x^{20}+\frac{5}{18}(2d+9e)x^{18}+\frac{15}{16}(3d+8e)x^{16}+\frac{15}{7}(4d+7e)x^{14}+\frac{7}{2}(5d+6e)x^{12}+\frac{21}{5}(6d+5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/16*(3*d + 8
*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12 + 21/5*(6*d + 5*e)*
x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4 + 1
/2*(10*d + e)*x^2 + d*log(x)
```

giac [A] time = 0.27, size = 145, normalized size = 1.56

$$\frac{1}{22}x^{22}e+\frac{1}{20}dx^{20}+\frac{1}{2}x^{20}e+\frac{5}{9}dx^{18}+\frac{5}{2}x^{18}e+\frac{45}{16}dx^{16}+\frac{15}{2}x^{16}e+\frac{60}{7}dx^{14}+15x^{14}e+\frac{35}{2}dx^{12}+21x^{12}e+\frac{126}{5}dx^{10}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*x^22*e + 1/20*d*x^20 + 1/2*x^20*e + 5/9*d*x^18 + 5/2*x^18*e + 45/16*d*
x^16 + 15/2*x^16*e + 60/7*d*x^14 + 15*x^14*e + 35/2*d*x^12 + 21*x^12*e + 12
6/5*d*x^10 + 21*x^10*e + 105/4*d*x^8 + 15*x^8*e + 20*d*x^6 + 15/2*x^6*e + 4
5/4*d*x^4 + 5/2*x^4*e + 5*d*x^2 + 1/2*x^2*e + 1/2*d*log(x^2)
```

maple [A] time = 0.00, size = 132, normalized size = 1.42

$$\frac{e\,x^{22}}{22}+\frac{d\,x^{20}}{20}+\frac{e\,x^{20}}{2}+\frac{5d\,x^{18}}{9}+\frac{5e\,x^{18}}{2}+\frac{45d\,x^{16}}{16}+\frac{15e\,x^{16}}{2}+\frac{60d\,x^{14}}{7}+15e\,x^{14}+\frac{35d\,x^{12}}{2}+21e\,x^{12}+\frac{126d\,x^{10}}{5}+21e\,x^{10}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x,x)
```

```
[Out] 1/22*e*x^22+1/20*d*x^20+1/2*e*x^20+5/9*d*x^18+5/2*e*x^18+45/16*d*x^16+15/2*
x^16*e+60/7*d*x^14+15*e*x^14+35/2*d*x^12+21*e*x^12+126/5*d*x^10+21*e*x^10+1
05/4*d*x^8+15*e*x^8+20*d*x^6+15/2*e*x^6+45/4*d*x^4+5/2*e*x^4+5*d*x^2+1/2*e*
x^2+d*ln(x)
```

maxima [A] time = 0.48, size = 130, normalized size = 1.40

$$\frac{1}{22}\,ex^{22}+\frac{1}{20}\,(d+10\,e)x^{20}+\frac{5}{18}\,(2\,d+9\,e)x^{18}+\frac{15}{16}\,(3\,d+8\,e)x^{16}+\frac{15}{7}\,(4\,d+7\,e)x^{14}+\frac{7}{2}\,(5\,d+6\,e)x^{12}+\frac{21}{5}\,(6\,d+5\,e)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/16*(3*d + 8
*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12 + 21/5*(6*d + 5*e)*
x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4 + 1
/2*(10*d + e)*x^2 + 1/2*d*log(x^2)
```

mupad [B] time = 0.13, size = 121, normalized size = 1.30

$$x^2\,\left(5\,d+\frac{e}{2}\right)+x^{18}\,\left(\frac{5\,d}{9}+\frac{5\,e}{2}\right)+x^6\,\left(20\,d+\frac{15\,e}{2}\right)+x^{20}\,\left(\frac{d}{20}+\frac{e}{2}\right)+x^4\,\left(\frac{45\,d}{4}+\frac{5\,e}{2}\right)+x^{12}\,\left(\frac{35\,d}{2}+21\,e\right)+x^{16}\,\left(\frac{45\,d}{16}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x,x)
```

```
[Out] x^2*(5*d + e/2) + x^18*((5*d)/9 + (5*e)/2) + x^6*(20*d + (15*e)/2) + x^20*(
d/20 + e/2) + x^4*((45*d)/4 + (5*e)/2) + x^12*((35*d)/2 + 21*e) + x^16*((45
*d)/16 + (15*e)/2) + x^14*((60*d)/7 + 15*e) + x^8*((105*d)/4 + 15*e) + x^10
*((126*d)/5 + 21*e) + (e*x^22)/22 + d*log(x)
```

sympy [A] time = 0.33, size = 131, normalized size = 1.41

$$d\log{(x)}+\frac{ex^{22}}{22}+x^{20}\left(\frac{d}{20}+\frac{e}{2}\right)+x^{18}\left(\frac{5d}{9}+\frac{5e}{2}\right)+x^{16}\left(\frac{45d}{16}+\frac{15e}{2}\right)+x^{14}\left(\frac{60d}{7}+15e\right)+x^{12}\left(\frac{35d}{2}+21e\right)+x^{10}\left(\frac{126d}{5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] d*log(x) + e*x**22/22 + x**20*(d/20 + e/2) + x**18*(5*d/9 + 5*e/2) + x**16*
(45*d/16 + 15*e/2) + x**14*(60*d/7 + 15*e) + x**12*(35*d/2 + 21*e) + x**10*
(126*d/5 + 21*e) + x**8*(105*d/4 + 15*e) + x**6*(20*d + 15*e/2) + x**4*(45*
d/4 + 5*e/2) + x**2*(5*d + e/2)
```

3.63 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx$

Optimal. Leaf size=141

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e)+3x^5($$

```
[Out] -d/x+(10*d+e)*x+5/3*(9*d+2*e)*x^3+3*(8*d+3*e)*x^5+30/7*(7*d+4*e)*x^7+14/3*(
6*d+5*e)*x^9+42/11*(5*d+6*e)*x^11+30/13*(4*d+7*e)*x^13+(3*d+8*e)*x^15+5/17*
(2*d+9*e)*x^17+1/19*(d+10*e)*x^19+1/21*e*x^21
```

Rubi [A] time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {28, 448}

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e)+3x^5($$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*
d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(
4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10
*e)*x^19)/19 + (e*x^21)/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5}{x^2}\,dx &= \int \frac{\left(1+x^2\right)^{10}\left(d+ex^2\right)}{x^2}\,dx\\ &= \int \left(10d\left(1+\frac{e}{10d}\right)+\frac{d}{x^2}+5(9d+2e)x^2+15(8d+3e)x^4+30(7d+4e)x^6+\right.\\ &= -\frac{d}{x}+(10d+e)x+\frac{5}{3}(9d+2e)x^3+3(8d+3e)x^5+\frac{30}{7}(7d+4e)x^7+\frac{14}{3}(6d+\end{aligned}$$

Mathematica [A] time = 0.03, size = 141, normalized size = 1.00

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e)+3x^5($$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*
d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(
4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10
*e)*x^19)/19 + (e*x^21)/21
```

fricas [A] time = 0.56, size = 131, normalized size = 0.93

$$\frac{46189\,ex^{22} + 51051\,(d + 10\,e)x^{20} + 285285\,(2\,d + 9\,e)x^{18} + 969969\,(3\,d + 8\,e)x^{16} + 2238390\,(4\,d + 7\,e)x^{14} + 3703}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/969969*(46189*e*x^22 + 51051*(d + 10*e)*x^20 + 285285*(2*d + 9*e)*x^18 +
969969*(3*d + 8*e)*x^16 + 2238390*(4*d + 7*e)*x^14 + 3703518*(5*d + 6*e)*x^
12 + 4526522*(6*d + 5*e)*x^10 + 4157010*(7*d + 4*e)*x^8 + 2909907*(8*d + 3*
e)*x^6 + 1616615*(9*d + 2*e)*x^4 + 969969*(10*d + e)*x^2 - 969969*d)/x
```

giac [A] time = 0.36, size = 139, normalized size = 0.99

$$\frac{1}{21}\,x^{21}e+\frac{1}{19}\,dx^{19}+\frac{10}{19}\,x^{19}e+\frac{10}{17}\,dx^{17}+\frac{45}{17}\,x^{17}e+3\,dx^{15}+8\,x^{15}e+\frac{120}{13}\,dx^{13}+\frac{210}{13}\,x^{13}e+\frac{210}{11}\,dx^{11}+\frac{252}{11}\,x^{11}e+28\,dx^{9}+\frac{7}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*x^21*e + 1/19*d*x^19 + 10/19*x^19*e + 10/17*d*x^17 + 45/17*x^17*e + 3*
d*x^15 + 8*x^15*e + 120/13*d*x^13 + 210/13*x^13*e + 210/11*d*x^11 + 252/11*
x^11*e + 28*d*x^9 + 70/3*x^9*e + 30*d*x^7 + 120/7*x^7*e + 24*d*x^5 + 9*x^5*
e + 15*d*x^3 + 10/3*x^3*e + 10*d*x + x*e - d/x
```

maple [A] time = 0.00, size = 129, normalized size = 0.91

$$\frac{e\,x^{21}}{21}+\frac{d\,x^{19}}{19}+\frac{10e\,x^{19}}{19}+\frac{10d\,x^{17}}{17}+\frac{45e\,x^{17}}{17}+3d\,x^{15}+8e\,x^{15}+\frac{120d\,x^{13}}{13}+\frac{210e\,x^{13}}{13}+\frac{210d\,x^{11}}{11}+\frac{252e\,x^{11}}{11}+28d\,x^{9}+\frac{70e\,x^{9}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x)
```

```
[Out] 1/21*e*x^21+1/19*x^19*d+10/19*x^19*e+10/17*x^17*d+45/17*x^17*e+3*x^15*d+8*x
^15*e+120/13*x^13*d+210/13*x^13*e+210/11*x^11*d+252/11*x^11*e+28*x^9*d+70/3
*x^9*e+30*x^7*d+120/7*x^7*e+24*d*x^5+9*x^5*e+15*d*x^3+10/3*x^3*e+10*d*x+e*x
-d/x
```

maxima [A] time = 0.50, size = 125, normalized size = 0.89

$$\frac{1}{21}\,ex^{21}+\frac{1}{19}\,(d+10\,e)x^{19}+\frac{5}{17}\,(2\,d+9\,e)x^{17}+(3\,d+8\,e)x^{15}+\frac{30}{13}\,(4\,d+7\,e)x^{13}+\frac{42}{11}\,(5\,d+6\,e)x^{11}+\frac{14}{3}\,(6\,d+5\,e)x^{9}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*e*x^21 + 1/19*(d + 10*e)*x^19 + 5/17*(2*d + 9*e)*x^17 + (3*d + 8*e)*x^
15 + 30/13*(4*d + 7*e)*x^13 + 42/11*(5*d + 6*e)*x^11 + 14/3*(6*d + 5*e)*x^9
 + 30/7*(7*d + 4*e)*x^7 + 3*(8*d + 3*e)*x^5 + 5/3*(9*d + 2*e)*x^3 + (10*d +
 e)*x - d/x
```

mupad [B] time = 0.08, size = 119, normalized size = 0.84

$$x^{15}\,\left(3\,d+8\,e\right)+x^3\,\left(15\,d+\frac{10\,e}{3}\right)+x^5\,\left(24\,d+9\,e\right)+x^{19}\,\left(\frac{d}{19}+\frac{10\,e}{19}\right)+x^{17}\,\left(\frac{10\,d}{17}+\frac{45\,e}{17}\right)+x^9\,\left(28\,d+\frac{70\,e}{3}\right)+x^7\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x^2,x)
```

```
[Out] x^15*(3*d + 8*e) + x^3*(15*d + (10*e)/3) + x^5*(24*d + 9*e) + x^19*(d/19 +
(10*e)/19) + x^17*((10*d)/17 + (45*e)/17) + x^9*(28*d + (70*e)/3) + x^7*(30
*d + (120*e)/7) + x^13*((120*d)/13 + (210*e)/13) + x^11*((210*d)/11 + (252*
e)/11) + x*(10*d + e) - d/x + (e*x^21)/21
```

sympy [A] time = 0.32, size = 124, normalized size = 0.88

$$-\frac{d}{x}+\frac{ex^{21}}{21}+x^{19}\left(\frac{d}{19}+\frac{10e}{19}\right)+x^{17}\left(\frac{10d}{17}+\frac{45e}{17}\right)+x^{15}\left(3d+8e\right)+x^{13}\left(\frac{120d}{13}+\frac{210e}{13}\right)+x^{11}\left(\frac{210d}{11}+\frac{252e}{11}\right)+x^9\left(28d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] -d/x + e*x**21/21 + x**19*(d/19 + 10*e/19) + x**17*(10*d/17 + 45*e/17) + x*
*15*(3*d + 8*e) + x**13*(120*d/13 + 210*e/13) + x**11*(210*d/11 + 252*e/11)
 + x**9*(28*d + 70*e/3) + x**7*(30*d + 120*e/7) + x**5*(24*d + 9*e) + x**3*
(15*d + 10*e/3) + x*(10*d + e)
```

3.64 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3} dx$

Optimal. Leaf size=147

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^{8}(6d+5e)+5x^6(7d+4e)+\frac{15}{4}x^4(8d$$

```
[Out] -1/2*d/x^2+5/2*(9*d+2*e)*x^2+15/4*(8*d+3*e)*x^4+5*(7*d+4*e)*x^6+21/4*(6*d+5
*e)*x^8+21/5*(5*d+6*e)*x^10+5/2*(4*d+7*e)*x^12+15/14*(3*d+8*e)*x^14+5/16*(2
*d+9*e)*x^16+1/18*(d+10*e)*x^18+1/20*e*x^20+(10*d+e)*ln(x)
```

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {28, 446, 76}

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^{8}(6d+5e)+5x^6(7d+4e)+\frac{15}{4}x^4(8d$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -d/(2*x^2) + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)
*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^
12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^
18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx = \int \frac{(1+x^2)^{10}(d+ex^2)}{x^3}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{(1+x)^{10}(d+ex)}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(5(9d+2e)+\frac{d}{x^2}+\frac{10d+e}{x}+15(8d+3e)x+30(7d+4e)x^2+\right.\right.$$
$$= -\frac{d}{2x^2}+\frac{5}{2}(9d+2e)x^2+\frac{15}{4}(8d+3e)x^4+5(7d+4e)x^6+\frac{21}{4}(6d+5e)x^8+\frac{2}{5}$$

Mathematica [A] time = 0.03, size = 147, normalized size = 1.00

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^8(6d+5e)+5x^6(7d+4e)+\frac{15}{4}x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/2*d/x^2 + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)
*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^
12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^
18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]
```

fricas [A] time = 0.76, size = 133, normalized size = 0.90

$$\frac{252\,ex^{22}+280\,(d+10\,e)x^{20}+1575\,(2\,d+9\,e)x^{18}+5400\,(3\,d+8\,e)x^{16}+12600\,(4\,d+7\,e)x^{14}+21168\,(5\,d+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*e*x^22 + 280*(d + 10*e)*x^20 + 1575*(2*d + 9*e)*x^18 + 5400*(3*
d + 8*e)*x^16 + 12600*(4*d + 7*e)*x^14 + 21168*(5*d + 6*e)*x^12 + 26460*(6*
d + 5*e)*x^10 + 25200*(7*d + 4*e)*x^8 + 18900*(8*d + 3*e)*x^6 + 12600*(9*d
+ 2*e)*x^4 + 5040*(10*d + e)*x^2*log(x) - 2520*d)/x^2
```

giac [A] time = 0.26, size = 156, normalized size = 1.06

$$\frac{1}{20}x^{20}e+\frac{1}{18}dx^{18}+\frac{5}{9}x^{18}e+\frac{5}{8}dx^{16}+\frac{45}{16}x^{16}e+\frac{45}{14}dx^{14}+\frac{60}{7}x^{14}e+10\,dx^{12}+\frac{35}{2}x^{12}e+21\,dx^{10}+\frac{126}{5}x^{10}e+\frac{63}{2}dx^8+\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*x^20*e + 1/18*d*x^18 + 5/9*x^18*e + 5/8*d*x^16 + 45/16*x^16*e + 45/14*
d*x^14 + 60/7*x^14*e + 10*d*x^12 + 35/2*x^12*e + 21*d*x^10 + 126/5*x^10*e +
 63/2*d*x^8 + 105/4*x^8*e + 35*d*x^6 + 20*x^6*e + 30*d*x^4 + 45/4*x^4*e + 4
5/2*d*x^2 + 5*x^2*e + 1/2*(10*d + e)*log(x^2) - 1/2*(10*d*x^2 + x^2*e + d)/
x^2
```

maple [A] time = 0.01, size = 131, normalized size = 0.89

$$\frac{e\,x^{20}}{20}+\frac{d\,x^{18}}{18}+\frac{5e\,x^{18}}{9}+\frac{5d\,x^{16}}{8}+\frac{45e\,x^{16}}{16}+\frac{45d\,x^{14}}{14}+\frac{60e\,x^{14}}{7}+10d\,x^{12}+\frac{35e\,x^{12}}{2}+21d\,x^{10}+\frac{126e\,x^{10}}{5}+\frac{63d\,x^8}{2}+\frac{105e\,x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x)
```

```
[Out] 1/20*e*x^20+1/18*d*x^18+5/9*e*x^18+5/8*d*x^16+45/16*e*x^16+45/14*d*x^14+60/
7*e*x^14+10*d*x^12+35/2*e*x^12+21*d*x^10+126/5*e*x^10+63/2*d*x^8+105/4*e*x^
8+35*d*x^6+20*e*x^6+30*d*x^4+45/4*e*x^4+45/2*d*x^2+5*e*x^2+10*d*ln(x)+ln(x)
*e-1/2*d/x^2
```

maxima [A] time = 0.70, size = 130, normalized size = 0.88

$$\frac{1}{20}\,ex^{20}+\frac{1}{18}\,(d+10\,e)x^{18}+\frac{5}{16}\,(2\,d+9\,e)x^{16}+\frac{15}{14}\,(3\,d+8\,e)x^{14}+\frac{5}{2}\,(4\,d+7\,e)x^{12}+\frac{21}{5}\,(5\,d+6\,e)x^{10}+\frac{21}{4}\,(6\,d+5\,e)x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*e*x^20 + 1/18*(d + 10*e)*x^18 + 5/16*(2*d + 9*e)*x^16 + 15/14*(3*d + 8
*e)*x^14 + 5/2*(4*d + 7*e)*x^12 + 21/5*(5*d + 6*e)*x^10 + 21/4*(6*d + 5*e)*
x^8 + 5*(7*d + 4*e)*x^6 + 15/4*(8*d + 3*e)*x^4 + 5/2*(9*d + 2*e)*x^2 + 1/2*
(10*d + e)*log(x^2) - 1/2*d/x^2
```

mupad [B] time = 0.08, size = 120, normalized size = 0.82

$$x^{18}\left(\frac{d}{18}+\frac{5\,e}{9}\right)+x^2\left(\frac{45\,d}{2}+5\,e\right)+x^{12}\left(10\,d+\frac{35\,e}{2}\right)+x^6\,(35\,d+20\,e)+x^4\left(30\,d+\frac{45\,e}{4}\right)+x^{16}\left(\frac{5\,d}{8}+\frac{45\,e}{16}\right)+x^{14}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x^3,x)
```

```
[Out] x^18*(d/18 + (5*e)/9) + x^2*((45*d)/2 + 5*e) + x^12*(10*d + (35*e)/2) + x^6
*(35*d + 20*e) + x^4*(30*d + (45*e)/4) + x^16*((5*d)/8 + (45*e)/16) + x^14*
((45*d)/14 + (60*e)/7) + x^10*(21*d + (126*e)/5) + x^8*((63*d)/2 + (105*e)/
4) - d/(2*x^2) + (e*x^20)/20 + log(x)*(10*d + e)
```

sympy [A] time = 0.39, size = 131, normalized size = 0.89

$$-\frac{d}{2x^2}+\frac{ex^{20}}{20}+x^{18}\left(\frac{d}{18}+\frac{5e}{9}\right)+x^{16}\left(\frac{5d}{8}+\frac{45e}{16}\right)+x^{14}\left(\frac{45d}{14}+\frac{60e}{7}\right)+x^{12}\left(10d+\frac{35e}{2}\right)+x^{10}\left(21d+\frac{126e}{5}\right)+x^8\left(\frac{63d}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] -d/(2*x**2) + e*x**20/20 + x**18*(d/18 + 5*e/9) + x**16*(5*d/8 + 45*e/16) +
 x**14*(45*d/14 + 60*e/7) + x**12*(10*d + 35*e/2) + x**10*(21*d + 126*e/5)
+ x**8*(63*d/2 + 105*e/4) + x**6*(35*d + 20*e) + x**4*(30*d + 45*e/4) + x**
2*(45*d/2 + 5*e) + (10*d + e)*log(x)
```

3.65 $\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=203

$$\frac{(fx)^{m+23}}{f^{23}(m+23)}+\frac{11(fx)^{m+21}}{f^{21}(m+21)}+\frac{55(fx)^{m+19}}{f^{19}(m+19)}+\frac{165(fx)^{m+17}}{f^{17}(m+17)}+\frac{330(fx)^{m+15}}{f^{15}(m+15)}+\frac{462(fx)^{m+13}}{f^{13}(m+13)}+\frac{462(fx)^{m+11}}{f^{11}(m+11)}+\frac{330(fx)}{f^9(m+}$$

```
[Out] (f*x)^(1+m)/f/(1+m)+11*(f*x)^(3+m)/f^3/(3+m)+55*(f*x)^(5+m)/f^5/(5+m)+165*(
f*x)^(7+m)/f^7/(7+m)+330*(f*x)^(9+m)/f^9/(9+m)+462*(f*x)^(11+m)/f^11/(11+m)
+462*(f*x)^(13+m)/f^13/(13+m)+330*(f*x)^(15+m)/f^15/(15+m)+165*(f*x)^(17+m)
/f^17/(17+m)+55*(f*x)^(19+m)/f^19/(19+m)+11*(f*x)^(21+m)/f^21/(21+m)+(f*x)^
(23+m)/f^23/(23+m)
```

Rubi [A] time = 0.07, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {28, 270}

$$\frac{11(fx)^{m+3}}{f^3(m+3)}+\frac{55(fx)^{m+5}}{f^5(m+5)}+\frac{165(fx)^{m+7}}{f^7(m+7)}+\frac{330(fx)^{m+9}}{f^9(m+9)}+\frac{462(fx)^{m+11}}{f^{11}(m+11)}+\frac{462(fx)^{m+13}}{f^{13}(m+13)}+\frac{330(fx)^{m+15}}{f^{15}(m+15)}+\frac{165(fx)^{m+17}}{f^{17}(m+17)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (f*x)^(1 + m)/(f*(1 + m)) + (11*(f*x)^(3 + m))/(f^3*(3 + m)) + (55*(f*x)^(5
 + m))/(f^5*(5 + m)) + (165*(f*x)^(7 + m))/(f^7*(7 + m)) + (330*(f*x)^(9 +
m))/(f^9*(9 + m)) + (462*(f*x)^(11 + m))/(f^11*(11 + m)) + (462*(f*x)^(13 +
 m))/(f^13*(13 + m)) + (330*(f*x)^(15 + m))/(f^15*(15 + m)) + (165*(f*x)^(1
7 + m))/(f^17*(17 + m)) + (55*(f*x)^(19 + m))/(f^19*(19 + m)) + (11*(f*x)^(
21 + m))/(f^21*(21 + m)) + (f*x)^(23 + m)/(f^23*(23 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx &= \int (fx)^m \left(1+x^2\right)^{11} \, dx\\
&= \int \left((fx)^m+\frac{11(fx)^{2+m}}{f^2}+\frac{55(fx)^{4+m}}{f^4}+\frac{165(fx)^{6+m}}{f^6}+\frac{330(fx)^{8+m}}{f^8}+\frac{4}{}\right.\\
&= \frac{(fx)^{1+m}}{f(1+m)}+\frac{11(fx)^{3+m}}{f^3(3+m)}+\frac{55(fx)^{5+m}}{f^5(5+m)}+\frac{165(fx)^{7+m}}{f^7(7+m)}+\frac{330(fx)^{9+m}}{f^9(9+m)}+\frac{4}{f}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 122, normalized size = 0.60

$$x\left(\frac{x^{22}}{m+23}+\frac{11x^{20}}{m+21}+\frac{55x^{18}}{m+19}+\frac{165x^{16}}{m+17}+\frac{330x^{14}}{m+15}+\frac{462x^{12}}{m+13}+\frac{462x^{10}}{m+11}+\frac{330x^8}{m+9}+\frac{165x^6}{m+7}+\frac{55x^4}{m+5}+\frac{11x^2}{m+3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x*(f*x)^m*((1 + m)^(-1) + (11*x^2)/(3 + m) + (55*x^4)/(5 + m) + (165*x^6)/(
7 + m) + (330*x^8)/(9 + m) + (462*x^10)/(11 + m) + (462*x^12)/(13 + m) + (3
30*x^14)/(15 + m) + (165*x^16)/(17 + m) + (55*x^18)/(19 + m) + (11*x^20)/(2
1 + m) + x^22/(23 + m))
```

fricas [B] time = 0.83, size = 759, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] ((m^11 + 121*m^10 + 6435*m^9 + 197835*m^8 + 3889578*m^7 + 51069018*m^6 + 45
3714470*m^5 + 2702025590*m^4 + 10431670821*m^3 + 24372200061*m^2 + 29985521
895*m + 13749310575)*x^23 + 11*(m^11 + 123*m^10 + 6635*m^9 + 206505*m^8 + 4
103178*m^7 + 54362574*m^6 + 486687830*m^5 + 2917013970*m^4 + 11320966021*m^
3 + 26560342503*m^2 + 32778930735*m + 15058768725)*x^21 + 55*(m^11 + 125*m^
10 + 6843*m^9 + 215823*m^8 + 4339146*m^7 + 58085538*m^6 + 524676662*m^5 + 3
168601822*m^4 + 12374824773*m^3 + 29178958257*m^2 + 36145916415*m + 1664390
2275)*x^19 + 165*(m^11 + 127*m^10 + 7059*m^9 + 225837*m^8 + 4600554*m^7 + 6
2319894*m^6 + 568863686*m^5 + 3466775738*m^4 + 13643071845*m^3 + 3236840757
9*m^2 + 40283194455*m + 18602008425)*x^17 + 330*(m^11 + 129*m^10 + 7283*m^9
 + 236595*m^8 + 4890858*m^7 + 67166442*m^6 + 620805254*m^5 + 3825379590*m^4
 + 15197565541*m^3 + 36337145829*m^2 + 45488935863*m + 21082276215)*x^15 +
462*(m^11 + 131*m^10 + 7515*m^9 + 248145*m^8 + 5213898*m^7 + 72748638*m^6 +
 682569590*m^5 + 4264053730*m^4 + 17145560901*m^3 + 41408337231*m^2 + 52237
739295*m + 24325703325)*x^13 + 462*(m^11 + 133*m^10 + 7755*m^9 + 260535*m^8
 + 5573898*m^7 + 79216434*m^6 + 756921110*m^5 + 4811326190*m^4 + 1965367130
1*m^3 + 48110244633*m^2 + 61333432335*m + 28748558475)*x^11 + 330*(m^11 + 1
35*m^10 + 8003*m^9 + 273813*m^8 + 5975466*m^7 + 86750118*m^6 + 847550822*m^
5 + 5509501002*m^4 + 22992750373*m^3 + 57365875587*m^2 + 74253243015*m + 35
137127025)*x^9 + 165*(m^11 + 137*m^10 + 8259*m^9 + 288027*m^8 + 6423594*m^7
 + 95564154*m^6 + 959352806*m^5 + 6421988758*m^4 + 27624338085*m^3 + 709302
62349*m^2 + 94034286855*m + 45176306175)*x^7 + 55*(m^11 + 139*m^10 + 8523*m
^9 + 303225*m^8 + 6923658*m^7 + 105911022*m^6 + 1098746774*m^5 + 7643724530
*m^4 + 34359636741*m^3 + 92502445239*m^2 + 128033897103*m + 63246828645)*x^
5 + 11*(m^11 + 141*m^10 + 8795*m^9 + 319455*m^8 + 7481418*m^7 + 118085058*m
^6 + 1274046710*m^5 + 9315318270*m^4 + 44632304581*m^3 + 130403715201*m^2 +
 199334977695*m + 105411381075)*x^3 + (m^11 + 143*m^10 + 9075*m^9 + 336765*
m^8 + 8103018*m^7 + 132426294*m^6 + 1495875590*m^5 + 11641582810*m^4 + 6093
6676581*m^3 + 203363952363*m^2 + 387182170935*m + 316234143225)*x)*(f*x)^m/
(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529312*m^7 + 1
628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944*m^3 + 5905
46123298*m^2 + 703416314160*m + 316234143225)
```

giac [B] time = 0.62, size = 1848, normalized size = 9.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] ((f*x)^m*m^11*x^23 + 121*(f*x)^m*m^10*x^23 + 11*(f*x)^m*m^11*x^21 + 6435*(f
*x)^m*m^9*x^23 + 1353*(f*x)^m*m^10*x^21 + 197835*(f*x)^m*m^8*x^23 + 55*(f*x
)^m*m^11*x^19 + 72985*(f*x)^m*m^9*x^21 + 3889578*(f*x)^m*m^7*x^23 + 6875*(f
*x)^m*m^10*x^19 + 2271555*(f*x)^m*m^8*x^21 + 51069018*(f*x)^m*m^6*x^23 + 16
```

```
5*(f*x)^m*m^11*x^17 + 376365*(f*x)^m*m^9*x^19 + 45134958*(f*x)^m*m^7*x^21 +
 453714470*(f*x)^m*m^5*x^23 + 20955*(f*x)^m*m^10*x^17 + 11870265*(f*x)^m*m^
8*x^19 + 597988314*(f*x)^m*m^6*x^21 + 2702025590*(f*x)^m*m^4*x^23 + 330*(f*
x)^m*m^11*x^15 + 1164735*(f*x)^m*m^9*x^17 + 238653030*(f*x)^m*m^7*x^19 + 53
53566130*(f*x)^m*m^5*x^21 + 10431670821*(f*x)^m*m^3*x^23 + 42570*(f*x)^m*m^
10*x^15 + 37263105*(f*x)^m*m^8*x^17 + 3194704590*(f*x)^m*m^6*x^19 + 3208715
3670*(f*x)^m*m^4*x^21 + 24372200061*(f*x)^m*m^2*x^23 + 462*(f*x)^m*m^11*x^1
3 + 2403390*(f*x)^m*m^9*x^15 + 759091410*(f*x)^m*m^7*x^17 + 28857216410*(f*
x)^m*m^5*x^19 + 124530626231*(f*x)^m*m^3*x^21 + 29985521895*(f*x)^m*m*x^23
 + 60522*(f*x)^m*m^10*x^13 + 78076350*(f*x)^m*m^8*x^15 + 10282782510*(f*x)^m
*m^6*x^17 + 174273100210*(f*x)^m*m^4*x^19 + 292163767533*(f*x)^m*m^2*x^21 +
 13749310575*(f*x)^m*x^23 + 462*(f*x)^m*m^11*x^11 + 3471930*(f*x)^m*m^9*x^1
3 + 1613983140*(f*x)^m*m^7*x^15 + 93862508190*(f*x)^m*m^5*x^17 + 6806153625
15*(f*x)^m*m^3*x^19 + 360568238085*(f*x)^m*m*x^21 + 61446*(f*x)^m*m^10*x^11
 + 114642990*(f*x)^m*m^8*x^13 + 22164925860*(f*x)^m*m^6*x^15 + 572017996770
*(f*x)^m*m^4*x^17 + 1604842704135*(f*x)^m*m^2*x^19 + 165646455975*(f*x)^m*x
^21 + 330*(f*x)^m*m^11*x^9 + 3582810*(f*x)^m*m^9*x^11 + 2408820876*(f*x)^m*
m^7*x^13 + 204865733820*(f*x)^m*m^5*x^15 + 2251106854425*(f*x)^m*m^3*x^17 +
 1988025402825*(f*x)^m*m*x^19 + 44550*(f*x)^m*m^10*x^9 + 120367170*(f*x)^m*
m^8*x^11 + 33609870756*(f*x)^m*m^6*x^13 + 1262375264700*(f*x)^m*m^4*x^15 +
5340787250535*(f*x)^m*m^2*x^17 + 915414625125*(f*x)^m*x^19 + 165*(f*x)^m*m^
11*x^7 + 2640990*(f*x)^m*m^9*x^9 + 2575140876*(f*x)^m*m^7*x^11 + 3153471505
80*(f*x)^m*m^5*x^13 + 5015196628530*(f*x)^m*m^3*x^15 + 6646727085075*(f*x)^
m*m*x^17 + 22605*(f*x)^m*m^10*x^7 + 90358290*(f*x)^m*m^8*x^9 + 36597992508*
(f*x)^m*m^6*x^11 + 1969992823260*(f*x)^m*m^4*x^13 + 11991258123570*(f*x)^m*
m^2*x^15 + 3069331390125*(f*x)^m*x^17 + 55*(f*x)^m*m^11*x^5 + 1362735*(f*x)
^m*m^9*x^7 + 1971903780*(f*x)^m*m^7*x^9 + 349697552820*(f*x)^m*m^5*x^11 + 7
921249136262*(f*x)^m*m^3*x^13 + 15011348834790*(f*x)^m*m*x^15 + 7645*(f*x)^
m*m^10*x^5 + 47524455*(f*x)^m*m^8*x^7 + 28627538940*(f*x)^m*m^6*x^9 + 22228
32699780*(f*x)^m*m^4*x^11 + 19130651800722*(f*x)^m*m^2*x^13 + 6957151150950
*(f*x)^m*x^15 + 11*(f*x)^m*m^11*x^3 + 468765*(f*x)^m*m^9*x^5 + 1059893010*(
f*x)^m*m^7*x^7 + 279691771260*(f*x)^m*m^5*x^9 + 9079996141062*(f*x)^m*m^3*x
^11 + 24133835554290*(f*x)^m*m*x^13 + 1551*(f*x)^m*m^10*x^3 + 16677375*(f*x
)^m*m^8*x^5 + 15768085410*(f*x)^m*m^6*x^7 + 1818135330660*(f*x)^m*m^4*x^9 +
 22226933020446*(f*x)^m*m^2*x^11 + 11238474936150*(f*x)^m*x^13 + (f*x)^m*m^
11*x + 96745*(f*x)^m*m^9*x^3 + 380801190*(f*x)^m*m^7*x^5 + 158293212990*(f*
x)^m*m^5*x^7 + 7587607623090*(f*x)^m*m^3*x^9 + 28336045738770*(f*x)^m*m*x^1
1 + 143*(f*x)^m*m^10*x + 3514005*(f*x)^m*m^8*x^3 + 5825106210*(f*x)^m*m^6*x
^5 + 1059628145070*(f*x)^m*m^4*x^7 + 18930738943710*(f*x)^m*m^2*x^9 + 13281
834015450*(f*x)^m*x^11 + 9075*(f*x)^m*m^9*x + 82295598*(f*x)^m*m^7*x^3 + 60
431072570*(f*x)^m*m^5*x^5 + 4558015784025*(f*x)^m*m^3*x^7 + 24503570194950*
(f*x)^m*m*x^9 + 336765*(f*x)^m*m^8*x + 1298935638*(f*x)^m*m^6*x^3 + 4204048
49150*(f*x)^m*m^4*x^5 + 11703493287585*(f*x)^m*m^2*x^7 + 11595251918250*(f*
x)^m*x^9 + 8103018*(f*x)^m*m^7*x + 14014513810*(f*x)^m*m^5*x^3 + 1889780020
755*(f*x)^m*m^3*x^5 + 15515657331075*(f*x)^m*m*x^7 + 132426294*(f*x)^m*m^6*
x + 102468500970*(f*x)^m*m^4*x^3 + 5087634488145*(f*x)^m*m^2*x^5 + 74540905
18875*(f*x)^m*x^7 + 1495875590*(f*x)^m*m^5*x + 490955350391*(f*x)^m*m^3*x^3
 + 7041864340665*(f*x)^m*m*x^5 + 11641582810*(f*x)^m*m^4*x + 1434440867211*
(f*x)^m*m^2*x^3 + 3478575575475*(f*x)^m*x^5 + 60936676581*(f*x)^m*m^3*x + 2
192684754645*(f*x)^m*m*x^3 + 203363952363*(f*x)^m*m^2*x + 1159525191825*(f*
x)^m*x^3 + 387182170935*(f*x)^m*m*x + 316234143225*(f*x)^m*x)/(m^12 + 144*m
^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529312*m^7 + 1628301884*m^6
 + 13137458400*m^5 + 72578259391*m^4 + 264300628944*m^3 + 590546123298*m^2
+ 703416314160*m + 316234143225)
```

maple [B] time = 0.01, size = 1121, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] (f*x)^m*(m^11*x^22+121*m^10*x^22+11*m^11*x^20+6435*m^9*x^22+1353*m^10*x^20+
197835*m^8*x^22+55*m^11*x^18+72985*m^9*x^20+3889578*m^7*x^22+6875*m^10*x^18
+2271555*m^8*x^20+51069018*m^6*x^22+165*m^11*x^16+376365*m^9*x^18+45134958*
m^7*x^20+453714470*m^5*x^22+20955*m^10*x^16+11870265*m^8*x^18+597988314*m^6
*x^20+2702025590*m^4*x^22+330*m^11*x^14+1164735*m^9*x^16+238653030*m^7*x^18
+5353566130*m^5*x^20+10431670821*m^3*x^22+42570*m^10*x^14+37263105*m^8*x^16
+3194704590*m^6*x^18+32087153670*m^4*x^20+24372200061*m^2*x^22+462*m^11*x^1
2+2403390*m^9*x^14+759091410*m^7*x^16+28857216410*m^5*x^18+124530626231*m^3
*x^20+29985521895*m*x^22+60522*m^10*x^12+78076350*m^8*x^14+10282782510*m^6*
x^16+174273100210*m^4*x^18+292163767533*m^2*x^20+13749310575*x^22+462*m^11*
x^10+3471930*m^9*x^12+1613983140*m^7*x^14+93862508190*m^5*x^16+680615362515
*m^3*x^18+360568238085*m*x^20+61446*m^10*x^10+114642990*m^8*x^12+2216492586
0*m^6*x^14+572017996770*m^4*x^16+1604842704135*m^2*x^18+165646455975*x^20+3
30*m^11*x^8+3582810*m^9*x^10+2408820876*m^7*x^12+204865733820*m^5*x^14+2251
106854425*m^3*x^16+1988025402825*m*x^18+44550*m^10*x^8+120367170*m^8*x^10+3
3609870756*m^6*x^12+1262375264700*m^4*x^14+5340787250535*m^2*x^16+915414625
125*x^18+165*m^11*x^6+2640990*m^9*x^8+2575140876*m^7*x^10+315347150580*m^5*
x^12+5015196628530*m^3*x^14+6646727085075*m*x^16+22605*m^10*x^6+90358290*m^
8*x^8+36597992508*m^6*x^10+1969992823260*m^4*x^12+11991258123570*m^2*x^14+3
069331390125*x^16+55*m^11*x^4+1362735*m^9*x^6+1971903780*m^7*x^8+3496975528
20*m^5*x^10+7921249136262*m^3*x^12+15011348834790*m*x^14+7645*m^10*x^4+4752
4455*m^8*x^6+28627538940*m^6*x^8+2222832699780*m^4*x^10+19130651800722*m^2*
x^12+6957151150950*x^14+11*m^11*x^2+468765*m^9*x^4+1059893010*m^7*x^6+27969
1771260*m^5*x^8+9079996141062*m^3*x^10+24133835554290*m*x^12+1551*m^10*x^2+
16677375*m^8*x^4+15768085410*m^6*x^6+1818135330660*m^4*x^8+22226933020446*m
^2*x^10+11238474936150*x^12+m^11+96745*m^9*x^2+380801190*m^7*x^4+1582932129
90*m^5*x^6+7587607623090*m^3*x^8+28336045738770*m*x^10+143*m^10+3514005*m^8
*x^2+5825106210*m^6*x^4+1059628145070*m^4*x^6+18930738943710*m^2*x^8+132818
34015450*x^10+9075*m^9+82295598*m^7*x^2+60431072570*m^5*x^4+4558015784025*m
^3*x^6+24503570194950*m*x^8+336765*m^8+1298935638*m^6*x^2+420404849150*m^4*
x^4+11703493287585*m^2*x^6+11595251918250*x^8+8103018*m^7+14014513810*m^5*x
^2+1889780020755*m^3*x^4+15515657331075*m*x^6+132426294*m^6+102468500970*m^
4*x^2+5087634488145*m^2*x^4+7454090518875*x^6+1495875590*m^5+490955350391*m
^3*x^2+7041864340665*m*x^4+11641582810*m^4+1434440867211*m^2*x^2+3478575575
475*x^4+60936676581*m^3+2192684754645*m*x^2+203363952363*m^2+1159525191825*
x^2+387182170935*m+316234143225)*x/(m+1)/(m+3)/(m+5)/(m+7)/(m+9)/(m+11)/(m+
13)/(m+15)/(m+17)/(m+19)/(m+21)/(m+23)
```

maxima [A] time = 0.97, size = 192, normalized size = 0.95

$$\frac{f^m x^{23} x^m}{m+23}+\frac{11\, f^m x^{21} x^m}{m+21}+\frac{55\, f^m x^{19} x^m}{m+19}+\frac{165\, f^m x^{17} x^m}{m+17}+\frac{330\, f^m x^{15} x^m}{m+15}+\frac{462\, f^m x^{13} x^m}{m+13}+\frac{462\, f^m x^{11} x^m}{m+11}+\frac{330\, f^m x^9 x^m}{m+9}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] f^m*x^23*x^m/(m + 23) + 11*f^m*x^21*x^m/(m + 21) + 55*f^m*x^19*x^m/(m + 19)
 + 165*f^m*x^17*x^m/(m + 17) + 330*f^m*x^15*x^m/(m + 15) + 462*f^m*x^13*x^m
/(m + 13) + 462*f^m*x^11*x^m/(m + 11) + 330*f^m*x^9*x^m/(m + 9) + 165*f^m*x
^7*x^m/(m + 7) + 55*f^m*x^5*x^m/(m + 5) + 11*f^m*x^3*x^m/(m + 3) + (f*x)^(m
 + 1)/(f*(m + 1))
```

mupad [B] time = 1.25, size = 1483, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)*(f*x)^m*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] (x^3*(f*x)^m*(2192684754645*m + 1434440867211*m^2 + 490955350391*m^3 + 1024
68500970*m^4 + 14014513810*m^5 + 1298935638*m^6 + 82295598*m^7 + 3514005*m^
8 + 96745*m^9 + 1551*m^10 + 11*m^11 + 1159525191825))/(703416314160*m + 590
546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628
301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^1
1 + m^12 + 316234143225) + (x^19*(f*x)^m*(1988025402825*m + 1604842704135*m
^2 + 680615362515*m^3 + 174273100210*m^4 + 28857216410*m^5 + 3194704590*m^6
 + 238653030*m^7 + 11870265*m^8 + 376365*m^9 + 6875*m^10 + 55*m^11 + 915414
625125))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 7257825939
1*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 34
5840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^11*(f*x)^m*(283
36045738770*m + 22226933020446*m^2 + 9079996141062*m^3 + 2222832699780*m^4
+ 349697552820*m^5 + 36597992508*m^6 + 2575140876*m^7 + 120367170*m^8 + 358
2810*m^9 + 61446*m^10 + 462*m^11 + 13281834015450))/(703416314160*m + 59054
6123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 162830
1884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11
+ m^12 + 316234143225) + (x^21*(f*x)^m*(360568238085*m + 292163767533*m^2 +
 124530626231*m^3 + 32087153670*m^4 + 5353566130*m^5 + 597988314*m^6 + 4513
4958*m^7 + 2271555*m^8 + 72985*m^9 + 1353*m^10 + 11*m^11 + 165646455975))/(
703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13
137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 +
 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^5*(f*x)^m*(7041864340665*
m + 5087634488145*m^2 + 1889780020755*m^3 + 420404849150*m^4 + 60431072570*
m^5 + 5825106210*m^6 + 380801190*m^7 + 16677375*m^8 + 468765*m^9 + 7645*m^1
0 + 55*m^11 + 3478575575475))/(703416314160*m + 590546123298*m^2 + 26430062
8944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m
^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)
 + (x^17*(f*x)^m*(6646727085075*m + 5340787250535*m^2 + 2251106854425*m^3 +
 572017996770*m^4 + 93862508190*m^5 + 10282782510*m^6 + 759091410*m^7 + 372
63105*m^8 + 1164735*m^9 + 20955*m^10 + 165*m^11 + 3069331390125))/(70341631
4160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 1313745840
0*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^
10 + 144*m^11 + m^12 + 316234143225) + (x*(f*x)^m*(387182170935*m + 2033639
52363*m^2 + 60936676581*m^3 + 11641582810*m^4 + 1495875590*m^5 + 132426294*
m^6 + 8103018*m^7 + 336765*m^8 + 9075*m^9 + 143*m^10 + m^11 + 316234143225)
)/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 +
 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^
9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^23*(f*x)^m*(2998552189
5*m + 24372200061*m^2 + 10431670821*m^3 + 2702025590*m^4 + 453714470*m^5 +
51069018*m^6 + 3889578*m^7 + 197835*m^8 + 6435*m^9 + 121*m^10 + m^11 + 1374
9310575))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725782593
91*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 3
45840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^7*(f*x)^m*(155
15657331075*m + 11703493287585*m^2 + 4558015784025*m^3 + 1059628145070*m^4
+ 158293212990*m^5 + 15768085410*m^6 + 1059893010*m^7 + 47524455*m^8 + 1362
735*m^9 + 22605*m^10 + 165*m^11 + 7454090518875))/(703416314160*m + 5905461
23298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 16283018
84*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 +
m^12 + 316234143225) + (x^15*(f*x)^m*(15011348834790*m + 11991258123570*m^2
 + 5015196628530*m^3 + 1262375264700*m^4 + 204865733820*m^5 + 22164925860*m
^6 + 1613983140*m^7 + 78076350*m^8 + 2403390*m^9 + 42570*m^10 + 330*m^11 +
6957151150950))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725
78259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m
^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^9*(f*x)^
m*(24503570194950*m + 18930738943710*m^2 + 7587607623090*m^3 + 181813533066
0*m^4 + 279691771260*m^5 + 28627538940*m^6 + 1971903780*m^7 + 90358290*m^8
+ 2640990*m^9 + 44550*m^10 + 330*m^11 + 11595251918250))/(703416314160*m +
590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1
```

```
628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*
m^11 + m^12 + 316234143225) + (x^13*(f*x)^m*(24133835554290*m + 19130651800
722*m^2 + 7921249136262*m^3 + 1969992823260*m^4 + 315347150580*m^5 + 336098
70756*m^6 + 2408820876*m^7 + 114642990*m^8 + 3471930*m^9 + 60522*m^10 + 462
*m^11 + 11238474936150))/(703416314160*m + 590546123298*m^2 + 264300628944*
m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 +
8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] Timed out
```

3.66 $\int x^5 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=34

$$\frac{1}{28}\left(x^2+1\right)^{14}-\frac{1}{13}\left(x^2+1\right)^{13}+\frac{1}{24}\left(x^2+1\right)^{12}$$

```
[Out] 1/24*(x^2+1)^12-1/13*(x^2+1)^13+1/28*(x^2+1)^14
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 266, 43}

$$\frac{1}{28}\left(x^2+1\right)^{14}-\frac{1}{13}\left(x^2+1\right)^{13}+\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24 - (1 + x^2)^13/13 + (1 + x^2)^14/28
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5\left(1+x^2\right)\left(1+2x^2+x^4\right)^5 \, dx &= \int x^5\left(1+x^2\right)^{11} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int x^2(1+x)^{11} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left((1+x)^{11}-2(1+x)^{12}+(1+x)^{13}\right) \, dx, x, x^2\right) \\
&= \frac{1}{24}\left(1+x^2\right)^{12}-\frac{1}{13}\left(1+x^2\right)^{13}+\frac{1}{28}\left(1+x^2\right)^{14}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 85, normalized size = 2.50

$$\frac{x^{28}}{28}+\frac{11x^{26}}{26}+\frac{55x^{24}}{24}+\frac{15x^{22}}{2}+\frac{33x^{20}}{2}+\frac{77x^{18}}{3}+\frac{231x^{16}}{8}+\frac{165x^{14}}{7}+\frac{55x^{12}}{4}+\frac{11x^{10}}{2}+\frac{11x^8}{8}+\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)/
8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26 +
 x^28/28
```

fricas [B] time = 0.49, size = 61, normalized size = 1.79

$$\frac{1}{28}x^{28}+\frac{11}{26}x^{26}+\frac{55}{24}x^{24}+\frac{15}{2}x^{22}+\frac{33}{2}x^{20}+\frac{77}{3}x^{18}+\frac{231}{8}x^{16}+\frac{165}{7}x^{14}+\frac{55}{4}x^{12}+\frac{11}{2}x^{10}+\frac{11}{8}x^{8}+\frac{1}{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

giac [B] time = 0.34, size = 61, normalized size = 1.79

$$\frac{1}{28}\,x^{28}+\frac{11}{26}\,x^{26}+\frac{55}{24}\,x^{24}+\frac{15}{2}\,x^{22}+\frac{33}{2}\,x^{20}+\frac{77}{3}\,x^{18}+\frac{231}{8}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{55}{4}\,x^{12}+\frac{11}{2}\,x^{10}+\frac{11}{8}\,x^{8}+\frac{1}{6}\,x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

maple [B] time = 0.00, size = 62, normalized size = 1.82

$$\frac{1}{28}x^{28}+\frac{11}{26}x^{26}+\frac{55}{24}x^{24}+\frac{15}{2}x^{22}+\frac{33}{2}x^{20}+\frac{77}{3}x^{18}+\frac{231}{8}x^{16}+\frac{165}{7}x^{14}+\frac{55}{4}x^{12}+\frac{11}{2}x^{10}+\frac{11}{8}x^{8}+\frac{1}{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/28*x^28+11/26*x^26+55/24*x^24+15/2*x^22+33/2*x^20+77/3*x^18+231/8*x^16+16
5/7*x^14+55/4*x^12+11/2*x^10+11/8*x^8+1/6*x^6
```

maxima [B] time = 0.64, size = 61, normalized size = 1.79

$$\frac{1}{28}\,x^{28}+\frac{11}{26}\,x^{26}+\frac{55}{24}\,x^{24}+\frac{15}{2}\,x^{22}+\frac{33}{2}\,x^{20}+\frac{77}{3}\,x^{18}+\frac{231}{8}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{55}{4}\,x^{12}+\frac{11}{2}\,x^{10}+\frac{11}{8}\,x^{8}+\frac{1}{6}\,x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

mupad [B] time = 0.06, size = 61, normalized size = 1.79

$$\frac{x^{28}}{28}+\frac{11\,x^{26}}{26}+\frac{55\,x^{24}}{24}+\frac{15\,x^{22}}{2}+\frac{33\,x^{20}}{2}+\frac{77\,x^{18}}{3}+\frac{231\,x^{16}}{8}+\frac{165\,x^{14}}{7}+\frac{55\,x^{12}}{4}+\frac{11\,x^{10}}{2}+\frac{11\,x^{8}}{8}+\frac{x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)/
8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26 +
 x^28/28
```

sympy [B] time = 0.07, size = 76, normalized size = 2.24

$$\frac{x^{28}}{28}+\frac{11x^{26}}{26}+\frac{55x^{24}}{24}+\frac{15x^{22}}{2}+\frac{33x^{20}}{2}+\frac{77x^{18}}{3}+\frac{231x^{16}}{8}+\frac{165x^{14}}{7}+\frac{55x^{12}}{4}+\frac{11x^{10}}{2}+\frac{11x^{8}}{8}+\frac{x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**28/28 + 11*x**26/26 + 55*x**24/24 + 15*x**22/2 + 33*x**20/2 + 77*x**18/3
 + 231*x**16/8 + 165*x**14/7 + 55*x**12/4 + 11*x**10/2 + 11*x**8/8 + x**6/6
```

3.67 $\int x^4 (1+x^2) (1+2x^2+x^4)^5 \, dx$

Optimal. Leaf size=83

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

```
[Out] 1/5*x^5+11/7*x^7+55/9*x^9+15*x^11+330/13*x^13+154/5*x^15+462/17*x^17+330/19
*x^19+55/7*x^21+55/23*x^23+11/25*x^25+1/27*x^27
```

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {28, 270}

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 (1+x^2) (1+2x^2+x^4)^5 \, dx &= \int x^4 (1+x^2)^{11} \, dx\\ &= \int (x^4+11x^6+55x^8+165x^{10}+330x^{12}+462x^{14}+462x^{16}+330x^{18}+165x\\ &= \frac{x^5}{5}+\frac{11x^7}{7}+\frac{55x^9}{9}+15x^{11}+\frac{330x^{13}}{13}+\frac{154x^{15}}{5}+\frac{462x^{17}}{17}+\frac{330x^{19}}{19}+\frac{55x^{21}}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 83, normalized size = 1.00

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

fricas [A] time = 0.60, size = 61, normalized size = 0.73

$$\frac{1}{27}x^{27}+\frac{11}{25}x^{25}+\frac{55}{23}x^{23}+\frac{55}{7}x^{21}+\frac{330}{19}x^{19}+\frac{462}{17}x^{17}+\frac{154}{5}x^{15}+\frac{330}{13}x^{13}+15x^{11}+\frac{55}{9}x^{9}+\frac{11}{7}x^{7}+\frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

giac [A] time = 0.41, size = 61, normalized size = 0.73

$$\frac{1}{27}x^{27}+\frac{11}{25}x^{25}+\frac{55}{23}x^{23}+\frac{55}{7}x^{21}+\frac{330}{19}x^{19}+\frac{462}{17}x^{17}+\frac{154}{5}x^{15}+\frac{330}{13}x^{13}+15x^{11}+\frac{55}{9}x^{9}+\frac{11}{7}x^{7}+\frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

maple [A] time = 0.00, size = 62, normalized size = 0.75

$$\frac{1}{27}x^{27}+\frac{11}{25}x^{25}+\frac{55}{23}x^{23}+\frac{55}{7}x^{21}+\frac{330}{19}x^{19}+\frac{462}{17}x^{17}+\frac{154}{5}x^{15}+\frac{330}{13}x^{13}+15x^{11}+\frac{55}{9}x^{9}+\frac{11}{7}x^{7}+\frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/5*x^5+11/7*x^7+55/9*x^9+15*x^11+330/13*x^13+154/5*x^15+462/17*x^17+330/19
*x^19+55/7*x^21+55/23*x^23+11/25*x^25+1/27*x^27
```

maxima [A] time = 0.76, size = 61, normalized size = 0.73

$$\frac{1}{27}x^{27}+\frac{11}{25}x^{25}+\frac{55}{23}x^{23}+\frac{55}{7}x^{21}+\frac{330}{19}x^{19}+\frac{462}{17}x^{17}+\frac{154}{5}x^{15}+\frac{330}{13}x^{13}+15x^{11}+\frac{55}{9}x^{9}+\frac{11}{7}x^{7}+\frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

mupad [B] time = 0.06, size = 61, normalized size = 0.73

$$\frac{x^{27}}{27}+\frac{11\,x^{25}}{25}+\frac{55\,x^{23}}{23}+\frac{55\,x^{21}}{7}+\frac{330\,x^{19}}{19}+\frac{462\,x^{17}}{17}+\frac{154\,x^{15}}{5}+\frac{330\,x^{13}}{13}+15\,x^{11}+\frac{55\,x^9}{9}+\frac{11\,x^7}{7}+\frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

sympy [A] time = 0.07, size = 75, normalized size = 0.90

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**27/27 + 11*x**25/25 + 55*x**23/23 + 55*x**21/7 + 330*x**19/19 + 462*x**1
7/17 + 154*x**15/5 + 330*x**13/13 + 15*x**11 + 55*x**9/9 + 11*x**7/7 + x**5
/5
```

3.68 $\int x^3 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=23

$$\frac{1}{26}\left(x^2+1\right)^{13}-\frac{1}{24}\left(x^2+1\right)^{12}$$

```
[Out] -1/24*(x^2+1)^12+1/26*(x^2+1)^13
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 266, 43}

$$\frac{1}{26}\left(x^2+1\right)^{13}-\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] -(1 + x^2)^12/24 + (1 + x^2)^13/26
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx &= \int x^3 \left(1+x^2\right)^{11} \, dx\\
&= \frac{1}{2}\text{Subst}\left(\int x(1+x)^{11} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-(1+x)^{11}+(1+x)^{12}\right) \, dx, x, x^2\right)\\
&= -\frac{1}{24}\left(1+x^2\right)^{12}+\frac{1}{26}\left(1+x^2\right)^{13}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 83, normalized size = 3.61

$$\frac{x^{26}}{26}+\frac{11x^{24}}{24}+\frac{5x^{22}}{2}+\frac{33x^{20}}{4}+\frac{55x^{18}}{3}+\frac{231x^{16}}{8}+33x^{14}+\frac{55x^{12}}{2}+\frac{33x^{10}}{2}+\frac{55x^8}{8}+\frac{11x^6}{6}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (23
1*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/26
```

fricas [B] time = 0.49, size = 61, normalized size = 2.65

$$\frac{1}{26}x^{26}+\frac{11}{24}x^{24}+\frac{5}{2}x^{22}+\frac{33}{4}x^{20}+\frac{55}{3}x^{18}+\frac{231}{8}x^{16}+33x^{14}+\frac{55}{2}x^{12}+\frac{33}{2}x^{10}+\frac{55}{8}x^{8}+\frac{11}{6}x^{6}+\frac{1}{4}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

giac [B] time = 0.36, size = 61, normalized size = 2.65

$$\frac{1}{26}\,x^{26}+\frac{11}{24}\,x^{24}+\frac{5}{2}\,x^{22}+\frac{33}{4}\,x^{20}+\frac{55}{3}\,x^{18}+\frac{231}{8}\,x^{16}+33\,x^{14}+\frac{55}{2}\,x^{12}+\frac{33}{2}\,x^{10}+\frac{55}{8}\,x^{8}+\frac{11}{6}\,x^{6}+\frac{1}{4}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

maple [B] time = 0.00, size = 62, normalized size = 2.70

$$\frac{1}{26}x^{26}+\frac{11}{24}x^{24}+\frac{5}{2}x^{22}+\frac{33}{4}x^{20}+\frac{55}{3}x^{18}+\frac{231}{8}x^{16}+33x^{14}+\frac{55}{2}x^{12}+\frac{33}{2}x^{10}+\frac{55}{8}x^{8}+\frac{11}{6}x^{6}+\frac{1}{4}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/26*x^26+11/24*x^24+5/2*x^22+33/4*x^20+55/3*x^18+231/8*x^16+33*x^14+55/2*x
^12+33/2*x^10+55/8*x^8+11/6*x^6+1/4*x^4
```

maxima [B] time = 0.56, size = 61, normalized size = 2.65

$$\frac{1}{26}\,x^{26}+\frac{11}{24}\,x^{24}+\frac{5}{2}\,x^{22}+\frac{33}{4}\,x^{20}+\frac{55}{3}\,x^{18}+\frac{231}{8}\,x^{16}+33\,x^{14}+\frac{55}{2}\,x^{12}+\frac{33}{2}\,x^{10}+\frac{55}{8}\,x^{8}+\frac{11}{6}\,x^{6}+\frac{1}{4}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

mupad [B] time = 0.06, size = 61, normalized size = 2.65

$$\frac{x^{26}}{26}+\frac{11\,x^{24}}{24}+\frac{5\,x^{22}}{2}+\frac{33\,x^{20}}{4}+\frac{55\,x^{18}}{3}+\frac{231\,x^{16}}{8}+33\,x^{14}+\frac{55\,x^{12}}{2}+\frac{33\,x^{10}}{2}+\frac{55\,x^{8}}{8}+\frac{11\,x^{6}}{6}+\frac{x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (23
1*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/26
```

sympy [B] time = 0.07, size = 75, normalized size = 3.26

$$\frac{x^{26}}{26} + \frac{11x^{24}}{24} + \frac{5x^{22}}{2} + \frac{33x^{20}}{4} + \frac{55x^{18}}{3} + \frac{231x^{16}}{8} + 33x^{14} + \frac{55x^{12}}{2} + \frac{33x^{10}}{2} + \frac{55x^{8}}{8} + \frac{11x^{6}}{6} + \frac{x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**26/26 + 11*x**24/24 + 5*x**22/2 + 33*x**20/4 + 55*x**18/3 + 231*x**16/8
+ 33*x**14 + 55*x**12/2 + 33*x**10/2 + 55*x**8/8 + 11*x**6/6 + x**4/4
```

3.69 $\int x^2 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=83

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

```
[Out] 1/3*x^3+11/5*x^5+55/7*x^7+55/3*x^9+30*x^11+462/13*x^13+154/5*x^15+330/17*x^
17+165/19*x^19+55/21*x^21+11/23*x^23+1/25*x^25
```

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {28, 270}

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx &= \int x^2 \left(1+x^2\right)^{11} \, dx\\ &= \int \left(x^2+11x^4+55x^6+165x^8+330x^{10}+462x^{12}+462x^{14}+330x^{16}+165x^1\right.\\ &= \frac{x^3}{3}+\frac{11x^5}{5}+\frac{55x^7}{7}+\frac{55x^9}{3}+30x^{11}+\frac{462x^{13}}{13}+\frac{154x^{15}}{5}+\frac{330x^{17}}{17}+\frac{165x^{19}}{19}+\end{aligned}$$

Mathematica [A] time = 0.00, size = 83, normalized size = 1.00

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

fricas [A] time = 0.55, size = 61, normalized size = 0.73

$$\frac{1}{25}x^{25}+\frac{11}{23}x^{23}+\frac{55}{21}x^{21}+\frac{165}{19}x^{19}+\frac{330}{17}x^{17}+\frac{154}{5}x^{15}+\frac{462}{13}x^{13}+30x^{11}+\frac{55}{3}x^{9}+\frac{55}{7}x^{7}+\frac{11}{5}x^{5}+\frac{1}{3}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

giac [A] time = 0.30, size = 61, normalized size = 0.73

$$\frac{1}{25}\,x^{25}+\frac{11}{23}\,x^{23}+\frac{55}{21}\,x^{21}+\frac{165}{19}\,x^{19}+\frac{330}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{462}{13}\,x^{13}+30\,x^{11}+\frac{55}{3}\,x^{9}+\frac{55}{7}\,x^{7}+\frac{11}{5}\,x^{5}+\frac{1}{3}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

maple [A] time = 0.00, size = 62, normalized size = 0.75

$$\frac{1}{25}x^{25}+\frac{11}{23}x^{23}+\frac{55}{21}x^{21}+\frac{165}{19}x^{19}+\frac{330}{17}x^{17}+\frac{154}{5}x^{15}+\frac{462}{13}x^{13}+30x^{11}+\frac{55}{3}x^{9}+\frac{55}{7}x^{7}+\frac{11}{5}x^{5}+\frac{1}{3}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/3*x^3+11/5*x^5+55/7*x^7+55/3*x^9+30*x^11+462/13*x^13+154/5*x^15+330/17*x^
17+165/19*x^19+55/21*x^21+11/23*x^23+1/25*x^25
```

maxima [A] time = 0.88, size = 61, normalized size = 0.73

$$\frac{1}{25}\,x^{25}+\frac{11}{23}\,x^{23}+\frac{55}{21}\,x^{21}+\frac{165}{19}\,x^{19}+\frac{330}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{462}{13}\,x^{13}+30\,x^{11}+\frac{55}{3}\,x^{9}+\frac{55}{7}\,x^{7}+\frac{11}{5}\,x^{5}+\frac{1}{3}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

mupad [B] time = 0.06, size = 61, normalized size = 0.73

$$\frac{x^{25}}{25}+\frac{11\,x^{23}}{23}+\frac{55\,x^{21}}{21}+\frac{165\,x^{19}}{19}+\frac{330\,x^{17}}{17}+\frac{154\,x^{15}}{5}+\frac{462\,x^{13}}{13}+30\,x^{11}+\frac{55\,x^{9}}{3}+\frac{55\,x^{7}}{7}+\frac{11\,x^{5}}{5}+\frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

sympy [A] time = 0.07, size = 75, normalized size = 0.90

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^{9}}{3}+\frac{55x^{7}}{7}+\frac{11x^{5}}{5}+\frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**25/25 + 11*x**23/23 + 55*x**21/21 + 165*x**19/19 + 330*x**17/17 + 154*x*
*15/5 + 462*x**13/13 + 30*x**11 + 55*x**9/3 + 55*x**7/7 + 11*x**5/5 + x**3/
3
```

3.70 $\int x\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=11

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

```
[Out] 1/24*(x^2+1)^12
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {28, 261}

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx&=\int x\left(1+x^2\right)^{11}\,dx\\&=\frac{1}{24}\left(1+x^2\right)^{12}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24
```

fricas [B] time = 0.58, size = 61, normalized size = 5.55

$$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+\frac{165}{8}x^{16}+33x^{14}+\frac{77}{2}x^{12}+33x^{10}+\frac{165}{8}x^{8}+\frac{55}{6}x^{6}+\frac{11}{4}x^{4}+\frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*
x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

giac [B] time = 0.31, size = 76, normalized size = 6.91

$$\frac{1}{24}\left(x^4+2x^2\right)^6+\frac{1}{4}\left(x^4+2x^2\right)^5+\frac{5}{8}\left(x^4+2x^2\right)^4+\frac{1}{4}x^4+\frac{5}{6}\left(x^4+2x^2\right)^3+\frac{5}{8}\left(x^4+2x^2\right)^2+\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/24*(x^4 + 2*x^2)^6 + 1/4*(x^4 + 2*x^2)^5 + 5/8*(x^4 + 2*x^2)^4 + 1/4*x^4
+ 5/6*(x^4 + 2*x^2)^3 + 5/8*(x^4 + 2*x^2)^2 + 1/2*x^2
```

maple [B] time = 0.00, size = 62, normalized size = 5.64

$$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+\frac{165}{8}x^{16}+33x^{14}+\frac{77}{2}x^{12}+33x^{10}+\frac{165}{8}x^{8}+\frac{55}{6}x^{6}+\frac{11}{4}x^{4}+\frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/24*x^24+1/2*x^22+11/4*x^20+55/6*x^18+165/8*x^16+33*x^14+77/2*x^12+33*x^10
+165/8*x^8+55/6*x^6+11/4*x^4+1/2*x^2
```

maxima [B] time = 0.78, size = 61, normalized size = 5.55

$$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+\frac{165}{8}x^{16}+33x^{14}+\frac{77}{2}x^{12}+33x^{10}+\frac{165}{8}x^{8}+\frac{55}{6}x^{6}+\frac{11}{4}x^{4}+\frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*
x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

mupad [B] time = 0.06, size = 61, normalized size = 5.55

$$\frac{x^{24}}{24}+\frac{x^{22}}{2}+\frac{11x^{20}}{4}+\frac{55x^{18}}{6}+\frac{165x^{16}}{8}+33x^{14}+\frac{77x^{12}}{2}+33x^{10}+\frac{165x^{8}}{8}+\frac{55x^{6}}{6}+\frac{11x^{4}}{4}+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^2/2 + (11*x^4)/4 + (55*x^6)/6 + (165*x^8)/8 + 33*x^10 + (77*x^12)/2 + 33*
x^14 + (165*x^16)/8 + (55*x^18)/6 + (11*x^20)/4 + x^22/2 + x^24/24
```

sympy [B] time = 0.07, size = 71, normalized size = 6.45

$$\frac{x^{24}}{24}+\frac{x^{22}}{2}+\frac{11x^{20}}{4}+\frac{55x^{18}}{6}+\frac{165x^{16}}{8}+33x^{14}+\frac{77x^{12}}{2}+33x^{10}+\frac{165x^{8}}{8}+\frac{55x^{6}}{6}+\frac{11x^{4}}{4}+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**24/24 + x**22/2 + 11*x**20/4 + 55*x**18/6 + 165*x**16/8 + 33*x**14 + 77*
x**12/2 + 33*x**10 + 165*x**8/8 + 55*x**6/6 + 11*x**4/4 + x**2/2
```

3.71 $\int (1+x^2)(1+2x^2+x^4)^5\,dx$

Optimal. Leaf size=73

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

```
[Out] x+11/3*x^3+11*x^5+165/7*x^7+110/3*x^9+42*x^11+462/13*x^13+22*x^15+165/17*x^
17+55/19*x^19+11/21*x^21+1/23*x^23
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {28, 194}

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (1+x^2)(1+2x^2+x^4)^5\,dx &= \int (1+x^2)^{11}\,dx\\ &= \int (1+11x^2+55x^4+165x^6+330x^8+462x^{10}+462x^{12}+330x^{14}+165x^{16}\\ &= x+\frac{11x^3}{3}+11x^5+\frac{165x^7}{7}+\frac{110x^9}{3}+42x^{11}+\frac{462x^{13}}{13}+22x^{15}+\frac{165x^{17}}{17}+\frac{55}{1}\end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.00

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

fricas [A] time = 0.56, size = 57, normalized size = 0.78

$$\frac{1}{23}x^{23}+\frac{11}{21}x^{21}+\frac{55}{19}x^{19}+\frac{165}{17}x^{17}+22x^{15}+\frac{462}{13}x^{13}+42x^{11}+\frac{110}{3}x^9+\frac{165}{7}x^7+11x^5+\frac{11}{3}x^3+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

giac [A] time = 0.27, size = 57, normalized size = 0.78

$$\frac{1}{23}\,x^{23}+\frac{11}{21}\,x^{21}+\frac{55}{19}\,x^{19}+\frac{165}{17}\,x^{17}+22\,x^{15}+\frac{462}{13}\,x^{13}+42\,x^{11}+\frac{110}{3}\,x^{9}+\frac{165}{7}\,x^{7}+11\,x^{5}+\frac{11}{3}\,x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

maple [A] time = 0.00, size = 58, normalized size = 0.79

$$\frac{1}{23}x^{23}+\frac{11}{21}x^{21}+\frac{55}{19}x^{19}+\frac{165}{17}x^{17}+22x^{15}+\frac{462}{13}x^{13}+42x^{11}+\frac{110}{3}x^{9}+\frac{165}{7}x^{7}+11x^{5}+\frac{11}{3}x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] x+11/3*x^3+11*x^5+165/7*x^7+110/3*x^9+42*x^11+462/13*x^13+22*x^15+165/17*x^
17+55/19*x^19+11/21*x^21+1/23*x^23
```

maxima [A] time = 0.98, size = 57, normalized size = 0.78

$$\frac{1}{23}\,x^{23}+\frac{11}{21}\,x^{21}+\frac{55}{19}\,x^{19}+\frac{165}{17}\,x^{17}+22\,x^{15}+\frac{462}{13}\,x^{13}+42\,x^{11}+\frac{110}{3}\,x^{9}+\frac{165}{7}\,x^{7}+11\,x^{5}+\frac{11}{3}\,x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

mupad [B] time = 0.06, size = 57, normalized size = 0.78

$$\frac{x^{23}}{23}+\frac{11\,x^{21}}{21}+\frac{55\,x^{19}}{19}+\frac{165\,x^{17}}{17}+22\,x^{15}+\frac{462\,x^{13}}{13}+42\,x^{11}+\frac{110\,x^9}{3}+\frac{165\,x^7}{7}+11\,x^5+\frac{11\,x^3}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

sympy [A] time = 0.07, size = 68, normalized size = 0.93

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^{9}}{3}+\frac{165x^{7}}{7}+11x^{5}+\frac{11x^{3}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**23/23 + 11*x**21/21 + 55*x**19/19 + 165*x**17/17 + 22*x**15 + 462*x**13/
13 + 42*x**11 + 110*x**9/3 + 165*x**7/7 + 11*x**5 + 11*x**3/3 + x
```

3.72 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx$

Optimal. Leaf size=80

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

```
[Out] 11/2*x^2+55/4*x^4+55/2*x^6+165/4*x^8+231/5*x^10+77/2*x^12+165/7*x^14+165/16
*x^16+55/18*x^18+11/20*x^20+1/22*x^22+ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 266, 43}

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^1
2)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22
 + Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx &= \int \frac{(1+x^2)^{11}}{x}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{11}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(11+\frac{1}{x}+55x+165x^2+330x^3+462x^4+462x^5+330x^6+165\right.\right.\\
&= \frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{1}}{18}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 80, normalized size = 1.00

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^{8}}{4}+\frac{55x^{6}}{2}+\frac{55x^{4}}{4}+\frac{11x^{2}}{2}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^1
2)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22
 + Log[x]
```

fricas [A] time = 0.49, size = 58, normalized size = 0.72

$$\frac{1}{22}x^{22}+\frac{11}{20}x^{20}+\frac{55}{18}x^{18}+\frac{165}{16}x^{16}+\frac{165}{7}x^{14}+\frac{77}{2}x^{12}+\frac{231}{5}x^{10}+\frac{165}{4}x^{8}+\frac{55}{2}x^{6}+\frac{55}{4}x^{4}+\frac{11}{2}x^{2}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + log(x)
```

giac [A] time = 0.23, size = 62, normalized size = 0.78

$$\frac{1}{22}x^{22}+\frac{11}{20}x^{20}+\frac{55}{18}x^{18}+\frac{165}{16}x^{16}+\frac{165}{7}x^{14}+\frac{77}{2}x^{12}+\frac{231}{5}x^{10}+\frac{165}{4}x^{8}+\frac{55}{2}x^{6}+\frac{55}{4}x^{4}+\frac{11}{2}x^{2}+\frac{1}{2}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + 1/2*log(x^2)
```

maple [A] time = 0.00, size = 59, normalized size = 0.74

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^{8}}{4}+\frac{55x^{6}}{2}+\frac{55x^{4}}{4}+\frac{11x^{2}}{2}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x,x)
```

```
[Out] 11/2*x^2+55/4*x^4+55/2*x^6+165/4*x^8+231/5*x^10+77/2*x^12+165/7*x^14+165/16
*x^16+55/18*x^18+11/20*x^20+1/22*x^22+ln(x)
```

maxima [A] time = 0.82, size = 62, normalized size = 0.78

$$\frac{1}{22}x^{22}+\frac{11}{20}x^{20}+\frac{55}{18}x^{18}+\frac{165}{16}x^{16}+\frac{165}{7}x^{14}+\frac{77}{2}x^{12}+\frac{231}{5}x^{10}+\frac{165}{4}x^{8}+\frac{55}{2}x^{6}+\frac{55}{4}x^{4}+\frac{11}{2}x^{2}+\frac{1}{2}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + 1/2*log(x^2)
```

mupad [B] time = 0.06, size = 58, normalized size = 0.72

$$\ln(x)+\frac{11\,x^2}{2}+\frac{55\,x^4}{4}+\frac{55\,x^6}{2}+\frac{165\,x^8}{4}+\frac{231\,x^{10}}{5}+\frac{77\,x^{12}}{2}+\frac{165\,x^{14}}{7}+\frac{165\,x^{16}}{16}+\frac{55\,x^{18}}{18}+\frac{11\,x^{20}}{20}+\frac{x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x,x)
```

```
[Out] log(x) + (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5
+ (77*x^12)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20
+ x^22/22
```

sympy [**A**] time = 0.11, size = 75, normalized size = 0.94

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^{8}}{4}+\frac{55x^{6}}{2}+\frac{55x^{4}}{4}+\frac{11x^{2}}{2}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] x**22/22 + 11*x**20/20 + 55*x**18/18 + 165*x**16/16 + 165*x**14/7 + 77*x**1
2/2 + 231*x**10/5 + 165*x**8/4 + 55*x**6/2 + 55*x**4/4 + 11*x**2/2 + log(x)
```

3.73 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx$

Optimal. Leaf size=73

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

```
[Out] -1/x+11*x+55/3*x^3+33*x^5+330/7*x^7+154/3*x^9+42*x^11+330/13*x^13+11*x^15+5
5/17*x^17+11/19*x^19+1/21*x^21
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {28, 270}

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
+ (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^2\right)\left(1+2x^2+x^4\right)^5}{x^2}\,dx &= \int \frac{\left(1+x^2\right)^{11}}{x^2}\,dx\\
&= \int \left(11+\frac{1}{x^2}+55x^2+165x^4+330x^6+462x^8+462x^{10}+330x^{12}+165x^{14}+55\right.\\
&= -\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.00

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
+ (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

fricas [A] time = 0.64, size = 62, normalized size = 0.85

$$\frac{4199\,x^{22} + 51051\,x^{20} + 285285\,x^{18} + 969969\,x^{16} + 2238390\,x^{14} + 3703518\,x^{12} + 4526522\,x^{10} + 4157010\,x^{8} + \cdots}{88179\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/88179*(4199*x^22 + 51051*x^20 + 285285*x^18 + 969969*x^16 + 2238390*x^14
+ 3703518*x^12 + 4526522*x^10 + 4157010*x^8 + 2909907*x^6 + 1616615*x^4 + 9
69969*x^2 - 88179)/x
```

giac [A] time = 0.30, size = 59, normalized size = 0.81

$$\frac{1}{21}\,x^{21} + \frac{11}{19}\,x^{19} + \frac{55}{17}\,x^{17} + 11\,x^{15} + \frac{330}{13}\,x^{13} + 42\,x^{11} + \frac{154}{3}\,x^{9} + \frac{330}{7}\,x^{7} + 33\,x^{5} + \frac{55}{3}\,x^{3} + 11\,x - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 154
/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

maple [A] time = 0.00, size = 60, normalized size = 0.82

$$\frac{x^{21}}{21} + \frac{11x^{19}}{19} + \frac{55x^{17}}{17} + 11x^{15} + \frac{330x^{13}}{13} + 42x^{11} + \frac{154x^{9}}{3} + \frac{330x^{7}}{7} + 33x^{5} + \frac{55x^{3}}{3} + 11x - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x^2,x)
```

```
[Out] -1/x+11*x+55/3*x^3+33*x^5+330/7*x^7+154/3*x^9+42*x^11+330/13*x^13+11*x^15+5
5/17*x^17+11/19*x^19+1/21*x^21
```

maxima [A] time = 0.83, size = 59, normalized size = 0.81

$$\frac{1}{21}\,x^{21} + \frac{11}{19}\,x^{19} + \frac{55}{17}\,x^{17} + 11\,x^{15} + \frac{330}{13}\,x^{13} + 42\,x^{11} + \frac{154}{3}\,x^{9} + \frac{330}{7}\,x^{7} + 33\,x^{5} + \frac{55}{3}\,x^{3} + 11\,x - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 154
/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

mupad [B] time = 0.06, size = 59, normalized size = 0.81

$$11\,x - \frac{1}{x} + \frac{55\,x^{3}}{3} + 33\,x^{5} + \frac{330\,x^{7}}{7} + \frac{154\,x^{9}}{3} + 42\,x^{11} + \frac{330\,x^{13}}{13} + 11\,x^{15} + \frac{55\,x^{17}}{17} + \frac{11\,x^{19}}{19} + \frac{x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x^2,x)
```

```
[Out] 11*x - 1/x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11 + (3
30*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

sympy [A] time = 0.10, size = 66, normalized size = 0.90

$$\frac{x^{21}}{21} + \frac{11x^{19}}{19} + \frac{55x^{17}}{17} + 11x^{15} + \frac{330x^{13}}{13} + 42x^{11} + \frac{154x^{9}}{3} + \frac{330x^{7}}{7} + 33x^{5} + \frac{55x^{3}}{3} + 11x - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] x**21/21 + 11*x**19/19 + 55*x**17/17 + 11*x**15 + 330*x**13/13 + 42*x**11 +
 154*x**9/3 + 330*x**7/7 + 33*x**5 + 55*x**3/3 + 11*x - 1/x
```

3.74 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} \, dx$

Optimal. Leaf size=80

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

```
[Out] -1/2/x^2+55/2*x^2+165/4*x^4+55*x^6+231/4*x^8+231/5*x^10+55/2*x^12+165/14*x^
14+55/16*x^16+11/18*x^18+1/20*x^20+11*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 266, 43}

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5
 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11
*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} \, dx &= \int \frac{(1+x^2)^{11}}{x^3} \, dx \\
&= \frac{1}{2} \text{Subst}\left(\int \frac{(1+x)^{11}}{x^2} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(55+\frac{1}{x^2}+\frac{11}{x}+165x+330x^2+462x^3+462x^4+330x^5+165\right.\right. \\
&= -\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 80, normalized size = 1.00

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/2*1/x^2 + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5
 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11
*Log[x]
```

fricas [A] time = 0.67, size = 64, normalized size = 0.80

$$\frac{252\,x^{22}+3080\,x^{20}+17325\,x^{18}+59400\,x^{16}+138600\,x^{14}+232848\,x^{12}+291060\,x^{10}+277200\,x^{8}+207900\,x^{6}+}{5040\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*x^22 + 3080*x^20 + 17325*x^18 + 59400*x^16 + 138600*x^14 + 2328
48*x^12 + 291060*x^10 + 277200*x^8 + 207900*x^6 + 138600*x^4 + 55440*x^2*lo
g(x) - 2520)/x^2
```

giac [A] time = 0.36, size = 69, normalized size = 0.86

$$\frac{1}{20}\,x^{20}+\frac{11}{18}\,x^{18}+\frac{55}{16}\,x^{16}+\frac{165}{14}\,x^{14}+\frac{55}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{231}{4}\,x^{8}+55\,x^{6}+\frac{165}{4}\,x^{4}+\frac{55}{2}\,x^{2}-\frac{11\,x^{2}+1}{2\,x^{2}}+\frac{11}{2}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 231/5*x^10
+ 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2*(11*x^2 + 1)/x^2 + 11/2*l
og(x^2)
```

maple [A] time = 0.00, size = 61, normalized size = 0.76

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}+11\ln(x)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x^3,x)
```

```
[Out] -1/2/x^2+55/2*x^2+165/4*x^4+55*x^6+231/4*x^8+231/5*x^10+55/2*x^12+165/14*x^
14+55/16*x^16+11/18*x^18+1/20*x^20+11*ln(x)
```

maxima [A] time = 0.74, size = 62, normalized size = 0.78

$$\frac{1}{20}\,x^{20}+\frac{11}{18}\,x^{18}+\frac{55}{16}\,x^{16}+\frac{165}{14}\,x^{14}+\frac{55}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{231}{4}\,x^{8}+55\,x^{6}+\frac{165}{4}\,x^{4}+\frac{55}{2}\,x^{2}-\frac{1}{2\,x^{2}}+\frac{11}{2}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 231/5*x^10
+ 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2/x^2 + 11/2*log(x^2)
```

mupad [B] time = 0.06, size = 60, normalized size = 0.75

$$11\ \ln(x)-\frac{1}{2\,x^2}+\frac{55\,x^2}{2}+\frac{165\,x^4}{4}+55\,x^6+\frac{231\,x^8}{4}+\frac{231\,x^{10}}{5}+\frac{55\,x^{12}}{2}+\frac{165\,x^{14}}{14}+\frac{55\,x^{16}}{16}+\frac{11\,x^{18}}{18}+\frac{x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x^3,x)
```

```
[Out] 11*log(x) - 1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (
231*x^10)/5 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x
^20/20
```

sympy [A] time = 0.11, size = 75, normalized size = 0.94

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}+11\log{(x)}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] x**20/20 + 11*x**18/18 + 55*x**16/16 + 165*x**14/14 + 55*x**12/2 + 231*x**1
0/5 + 231*x**8/4 + 55*x**6 + 165*x**4/4 + 55*x**2/2 + 11*log(x) - 1/(2*x**2
)
```

3.75 $\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=145

$$\frac{x\left(a+bx^2\right)(bd-ae)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-a*e+b*d)*x*(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/3*e*x^3*(b*x^2+a)/b/((b*x^
2+a)^2)^(1/2)-(-a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2
)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1250, 459, 321, 205}

$$\frac{x\left(a+bx^2\right)(bd-ae)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((b*d - a*e)*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (e*x^3*
(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(b*d - a*e)*(
a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{x^2\left(d+ex^2\right)}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\left(-3b^2d+3abe\right)\left(ab+b^2x^2\right)\right)\int \frac{x^2}{ab+b^2x^2}\,dx}{3b^2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(bd-ae)x\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a\left(-3b^2d+3abe\right)\left(ab+b^2x\right.\right.}{3b^3\sqrt{a^2+2abx^2+}}$$
$$= \frac{(bd-ae)x\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}\,(bd-ae)\left(a+bx^2\right)\tan^{-1}}{b^{5/2}\sqrt{a^2+2abx^2+b^2x}}$$

Mathematica [A] time = 0.08, size = 80, normalized size = 0.55

$$\frac{\left(a+bx^2\right)\left(\sqrt{b}\,x\left(-3ae+3bd+bex^2\right)+3\sqrt{a}\,(ae-bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x*(3*b*d - 3*a*e + b*e*x^2) + 3*Sqrt[a]*(-(b*d) + a*e
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.77, size = 129, normalized size = 0.89

$$\left[\frac{2\,bex^3-3\,(bd-ae)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,(bd-ae)x}{6\,b^2},\ \frac{bex^3-3\,(bd-ae)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\,(bd-a}{3\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*e*x^3 - 3*(b*d - a*e)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
a)/(b*x^2 + a)) + 6*(b*d - a*e)*x)/b^2, 1/3*(b*e*x^3 - 3*(b*d - a*e)*sqrt(a
/b)*arctan(b*x*sqrt(a/b)/a) + 3*(b*d - a*e)*x)/b^2]
```

giac [A] time = 0.43, size = 101, normalized size = 0.70

$$-\frac{\left(abd\mathrm{sgn}\left(bx^2+a\right)-a^2e\mathrm{sgn}\left(bx^2+a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{b^2x^3e\mathrm{sgn}\left(bx^2+a\right)+3\,b^2dx\mathrm{sgn}\left(bx^2+a\right)-3\,abxe\mathrm{sg}}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b*d*sgn(b*x^2 + a) - a^2*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*b^2) + 1/3*(b^2*x^3*e*sgn(b*x^2 + a) + 3*b^2*d*x*sgn(b*x^2 + a) - 3*a*
b*x*e*sgn(b*x^2 + a))/b^3
```

maple [A] time = 0.04, size = 90, normalized size = 0.62

$$\frac{\left(b\,x^2+a\right)\left(\sqrt{ab}\,be\,x^3+3a^2e\arctan\left(\frac{bx}{\sqrt{ab}}\right)-3abd\arctan\left(\frac{bx}{\sqrt{ab}}\right)-3\sqrt{ab}\,aex+3\sqrt{ab}\,bdx\right)}{3\sqrt{\left(b\,x^2+a\right)^2}\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*((a*b)^(1/2)*x^3*b*e+3*arctan(b*x/(a*b)^(1/2))*a^2*e-3*arctan
(b*x/(a*b)^(1/2))*a*b*d-3*(a*b)^(1/2)*x*a*e+3*(a*b)^(1/2)*x*b*d)/((b*x^2+a)
^2)^(1/2)/b^2/(a*b)^(1/2)
```

maxima [A] time = 1.52, size = 54, normalized size = 0.37

$$-\frac{\left(abd-a^{2}e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{bex^{3}+3\,(bd-ae)x}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(a*b*d - a^2*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*e*x^3 + 3*(
b*d - a*e)*x)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2}\,\left(e\,x^{2}+d\right)}{\sqrt{{\left(b\,x^{2}+a\right)}^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2))/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((x^2*(d + e*x^2))/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.36, size = 90, normalized size = 0.62

$$x\left(-\frac{ae}{b^{2}}+\frac{d}{b}\right)-\frac{\sqrt{-\frac{a}{b^{5}}}\,(ae-bd)\log\left(-b^{2}\sqrt{-\frac{a}{b^{5}}}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^{5}}}\,(ae-bd)\log\left(b^{2}\sqrt{-\frac{a}{b^{5}}}+x\right)}{2}+\frac{ex^{3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] x*(-a*e/b**2 + d/b) - sqrt(-a/b**5)*(a*e - b*d)*log(-b**2*sqrt(-a/b**5) + x
)/2 + sqrt(-a/b**5)*(a*e - b*d)*log(b**2*sqrt(-a/b**5) + x)/2 + e*x**3/(3*b
)
```

3.76 $\int \frac{x\left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}} \, dx$

Optimal. Leaf size=83

$$\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}$$

```
[Out] 1/2*(-a*e+b*d)*(b*x^2+a)*ln(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/2*e*((b*x^2+
a)^2)^(1/2)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1247, 640, 608, 31}

$$\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*b^2) + ((b*d - a*e)*(a + b*x^2)*Log[
a + b*x^2])/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)$$
$$=\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}+\frac{(bd-ae)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)}{2b}$$
$$=\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}+\frac{\left((bd-ae)\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^2\right)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$
$$=\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}+\frac{(bd-ae)\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.61

$$\frac{\left(a+bx^2\right)\left((bd-ae)\log\left(a+bx^2\right)+bex^2\right)}{2b^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] ((a + b*x^2)*(b*e*x^2 + (b*d - a*e)*Log[a + b*x^2]))/(2*b^2*Sqrt[(a + b*x^2
)^2])
```

fricas [A] time = 0.80, size = 29, normalized size = 0.35

$$\frac{bex^2+(bd-ae)\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(b*e*x^2 + (b*d - a*e)*log(b*x^2 + a))/b^2
```

giac [A] time = 0.38, size = 42, normalized size = 0.51

$$\frac{1}{2}\left(\frac{x^2e}{b}+\frac{(bd-ae)\log\left(\left|bx^2+a\right|\right)}{b^2}\right)\operatorname{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(x^2*e/b + (b*d - a*e)*log(abs(b*x^2 + a))/b^2)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 55, normalized size = 0.66

$$-\frac{\left(b\,x^2+a\right)\left(-be\,x^2+ae\ln\left(b\,x^2+a\right)-bd\ln\left(b\,x^2+a\right)\right)}{2\sqrt{\left(b\,x^2+a\right)^2}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x)
[Out] -1/2*(b*x^2+a)*(-x^2*e*b+ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d)/((b*x^2+a)^2)^(1/
2)/b^2
```

maxima [A] time = 0.78, size = 31, normalized size = 0.37

$$\frac{ex^2}{2\,b} + \frac{(bd - ae)\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^2/b + 1/2*(b*d - a*e)*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.88, size = 103, normalized size = 1.24

$$\frac{e\sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{2\,b^2} - \frac{a\,b\,e\,\ln\left(a\,b + \sqrt{\left(b\,x^2 + a\right)^2}\,\sqrt{b^2} + b^2\,x^2\right)}{2\,\left(b^2\right)^{3/2}} + \frac{b^2\,d\,\ln\left(b^2\,x^2 + a\,b\right)\,\mathrm{sign}\left(2\,b^2\,x^2 + 2\,a\,b\right)}{2\,\left(b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2))/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] (e*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*b^2) - (a*b*e*log(a*b + ((a + b*x^
2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(3/2)) + (b^2*d*log(a*b + b^2*
x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(3/2))
```

sympy [A] time = 0.28, size = 27, normalized size = 0.33

$$\frac{ex^2}{2b} - \frac{(ae - bd)\log\left(a + bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] e*x**2/(2*b) - (a*e - b*d)*log(a + b*x**2)/(2*b**2)
```

3.77 $\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=97

$$\frac{ex\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] e*x*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)+(-a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a
^(1/2))/b^(3/2)/a^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1148, 388, 205}

$$\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(a + b
*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1148

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{d+ex^2}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{ex\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(\left(-b^2d+abe\right)\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{b^2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{ex\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 69, normalized size = 0.71

$$-\frac{\left(a+bx^2\right)\left((ae-bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\sqrt{a}\,\sqrt{b}\,ex\right)}{\sqrt{a}\,b^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*e*x) + (-(b*d) + a*e)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]]))/(Sqrt[a]*b^(3/2)*Sqrt[(a + b*x^2)^2]))
```

fricas [A] time = 0.83, size = 98, normalized size = 1.01

$$\left[\frac{2\,abex+\sqrt{-ab}\,(bd-ae)\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab^2},\ \frac{abex+\sqrt{ab}\,(bd-ae)\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*e*x + sqrt(-a*b)*(b*d - a*e)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (a*b*e*x + sqrt(a*b)*(b*d - a*e)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

giac [A] time = 0.26, size = 59, normalized size = 0.61

$$\frac{xe\mathrm{sgn}\left(bx^2+a\right)}{b}+\frac{\left(bd\mathrm{sgn}\left(bx^2+a\right)-ae\mathrm{sgn}\left(bx^2+a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] x*e*sgn(b*x^2 + a)/b + (b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*b)
```

maple [A] time = 0.01, size = 62, normalized size = 0.64

$$\frac{\left(b\,x^2+a\right)\left(-ae\arctan\left(\frac{bx}{\sqrt{ab}}\right)+bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\sqrt{ab}\;ex\right)}{\sqrt{\left(b\,x^2+a\right)^2}\;\sqrt{ab}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] (b*x^2+a)*(e*x*(a*b)^(1/2)-arctan(1/(a*b)^(1/2)*b*x)*a*e+arctan(1/(a*b)^(1/
2)*b*x)*b*d)/((b*x^2+a)^2)^(1/2)/b/(a*b)^(1/2)
```

maxima [A] time = 1.51, size = 33, normalized size = 0.34

$$\frac{ex}{b}+\frac{(bd-ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] e*x/b + (b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2 + d}{\sqrt{\left(b\,x^2 + a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d + e*x^2)/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.32, size = 82, normalized size = 0.85

$$\frac{\sqrt{-\frac{1}{ab^3}}\,(ae - bd)\log\left(-ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^3}}\,(ae - bd)\log\left(ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} + \frac{ex}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*e - b*d)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/(
a*b**3))*(a*e - b*d)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + e*x/b
```

3.78 $\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{d\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] d*(b*x^2+a)*ln(x)/a/((b*x^2+a)^2)^(1/2)-1/2*(-a*e+b*d)*(b*x^2+a)*ln(b*x^2+a
)/a/b/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 446, 72}

$$\frac{d\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (d*(a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*(
a + b*x^2)*Log[a + b*x^2])/(2*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{d+ex^2}{x\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{d+ex}{x\left(ab+b^2x\right)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \left(\frac{d}{abx}+\frac{-bd+ae}{ab(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{d\left(a+bx^2\right)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)\left(a+bx^2\right)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.59

$$\frac{\left(a+bx^2\right)\left((ae-bd)\log\left(a+bx^2\right)+2bd\log(x)\right)}{2ab\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(2*b*d*Log[x] + (-(b*d) + a*e)*Log[a + b*x^2]))/(2*a*b*Sqrt[(a
 + b*x^2)^2])
```

fricas [A] time = 0.72, size = 33, normalized size = 0.36

$$\frac{2\,bd\log(x)-(bd-ae)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*log(x) - (b*d - a*e)*log(b*x^2 + a))/(a*b)
```

giac [A] time = 0.38, size = 61, normalized size = 0.66

$$\frac{d\log\left(x^2\right)\text{sgn}\left(bx^2+a\right)}{2\,a} - \frac{\left(bd\text{sgn}\left(bx^2+a\right)-ae\text{sgn}\left(bx^2+a\right)\right)\log\left(\left|bx^2+a\right|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d*log(x^2)*sgn(b*x^2 + a)/a - 1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 +
 a))*log(abs(b*x^2 + a))/(a*b)
```

maple [A] time = 0.01, size = 57, normalized size = 0.62

$$\frac{\left(b\,x^2+a\right)\left(ae\ln\left(b\,x^2+a\right)+2bd\ln(x)-bd\ln\left(b\,x^2+a\right)\right)}{2\sqrt{\left(b\,x^2+a\right)^2}\;ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*(2*d*ln(x)*b+a*e*ln(b*x^2+a)-b*d*ln(b*x^2+a))/((b*x^2+a)^2)^(
1/2)/a/b
```

maxima [A] time = 0.73, size = 35, normalized size = 0.38

$$\frac{d\log\left(x^2\right)}{2\,a} - \frac{(bd-ae)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*d*log(x^2)/a - 1/2*(b*d - a*e)*log(b*x^2 + a)/(a*b)
```

mupad [B] time = 0.77, size = 83, normalized size = 0.90

$$\frac{e\,\ln\left(b^2\,x^2+a\,b\right)\,\mathrm{sign}\left(2\,b^2\,x^2+2\,a\,b\right)}{2\,\sqrt{b^2}} - \frac{d\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}} - \frac{d\,\ln\left(\sqrt{\left(b\,x^2+a\right)^2}\,\sqrt{a^2}+a^2+a\,b\,x^2\right)}{2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x*((a + b*x^2)^2)^(1/2)),x)
[Out] (e*log(a*b + b^2*x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(1/2)) - (d*log(1/x
^2))/(2*(a^2)^(1/2)) - (d*log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) + a^2 + a*b
*x^2))/(2*(a^2)^(1/2))
```

sympy [A] time = 0.71, size = 26, normalized size = 0.28

$$\frac{d\log\left(x\right)}{a} + \frac{(ae-bd)\log\left(\frac{a}{b}+x^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x/((b*x**2+a)**2)**(1/2),x)
[Out] d*log(x)/a + (a*e - b*d)*log(a/b + x**2)/(2*a*b)
```

3.79 $\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=101

$$-\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -d*(b*x^2+a)/a/x/((b*x^2+a)^2)^(1/2)-(-a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/
a^(1/2))/a^(3/2)/b^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 453, 205}

$$-\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((d*(a + b*x^2))/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - ((b*d - a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{d+ex^2}{x^2\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\left(b^2d-abe\right)\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{ab\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.71

$$\frac{\left(a+bx^2\right)\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(aex-bdx)-\sqrt{a}\,\sqrt{b}\,d\right)}{a^{3/2}\sqrt{b}\,x\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*d) + (-(b*d*x) + a*e*x)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]]))/(a^(3/2)*Sqrt[b]*x*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.69, size = 105, normalized size = 1.04

$$\left[\frac{\sqrt{-ab}\,(bd-ae)x\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,abd}{2\,a^2bx}, -\frac{\sqrt{ab}\,(bd-ae)x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+abd}{a^2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a*b)*(b*d - a*e)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*a*b*d)/(a^2*b*x), -(sqrt(a*b)*(b*d - a*e)*x*arctan(sqrt(a*b)*x/a) + a
*b*d)/(a^2*b*x)]
```

giac [A] time = 0.34, size = 62, normalized size = 0.61

$$-\frac{\left(bd\mathrm{sgn}\left(bx^2+a\right)-ae\mathrm{sgn}\left(bx^2+a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} - \frac{d\mathrm{sgn}\left(bx^2+a\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a) - d*sgn(b*x^2 + a)/(a*x)
```

maple [A] time = 0.01, size = 67, normalized size = 0.66

$$-\frac{\left(b\,x^2+a\right)\left(-aex\arctan\left(\frac{bx}{\sqrt{ab}}\right)+bdx\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\sqrt{ab}\,d\right)}{\sqrt{\left(b\,x^2+a\right)^2}\,\sqrt{ab}\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -(b*x^2+a)*(-arctan(1/(a*b)^(1/2)*b*x)*x*a*e+arctan(1/(a*b)^(1/2)*b*x)*x*b*
d+d*(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a/x/(a*b)^(1/2)
```

maxima [A] time = 1.22, size = 37, normalized size = 0.37

$$-\frac{(bd-ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - d/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{x^2\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x^2*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int((d + e*x^2)/(x^2*((a + b*x^2)^2)^(1/2)), x)
```

sympy [A] time = 0.37, size = 82, normalized size = 0.81

$$-\frac{\sqrt{-\frac{1}{a^3b}}\,(ae-bd)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b}}\,(ae-bd)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-1/(a**3*b))*(a*e - b*d)*log(-a**2*sqrt(-1/(a**3*b)) + x)/2 + sqrt(-1
/(a**3*b))*(a*e - b*d)*log(a**2*sqrt(-1/(a**3*b)) + x)/2 - d/(a*x)
```

3.80 $\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=137

$$-\frac{\log(x)\left(a+bx^2\right)(bd-ae)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/2*d*(b*x^2+a)/a/x^2/((b*x^2+a)^2)^(1/2)-(-a*e+b*d)*(b*x^2+a)*ln(x)/a^2/(
(b*x^2+a)^2)^(1/2)+1/2*(-a*e+b*d)*(b*x^2+a)*ln(b*x^2+a)/a^2/((b*x^2+a)^2)^(
1/2)
```

Rubi [A] time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 446, 77}

$$-\frac{\log(x)\left(a+bx^2\right)(bd-ae)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(d*(a + b*x^2))/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*(
a + b*x^2)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(a
+ b*x^2)*Log[a + b*x^2])/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{d+ex^2}{x^3\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{d+ex}{x^2\left(ab+b^2x\right)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \left(\frac{d}{abx^2}+\frac{-bd+ae}{a^2bx}+\frac{bd-ae}{a^2(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{d\left(a+bx^2\right)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)\left(a+bx^2\right)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)\left(a+bx^2\right)\log}{2a^2\sqrt{a^2+2abx^2+}}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.51

$$\frac{\left(a+bx^2\right)\left(2x^2\log(x)(ae-bd)+x^2(bd-ae)\log\left(a+bx^2\right)-ad\right)}{2a^2x^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(-(a*d) + 2*(-(b*d) + a*e)*x^2*Log[x] + (b*d - a*e)*x^2*Log[a
+ b*x^2]))/(2*a^2*x^2*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.90, size = 48, normalized size = 0.35

$$\frac{(bd-ae)x^2\log\left(bx^2+a\right)-2\,(bd-ae)x^2\log(x)-ad}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((b*d - a*e)*x^2*log(b*x^2 + a) - 2*(b*d - a*e)*x^2*log(x) - a*d)/(a^2*
x^2)
```

giac [A] time = 0.39, size = 131, normalized size = 0.96

$$-\frac{\left(bd\text{sgn}\left(bx^2+a\right)-ae\text{sgn}\left(bx^2+a\right)\right)\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2d\text{sgn}\left(bx^2+a\right)-abe\text{sgn}\left(bx^2+a\right)\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b}+\frac{bdx^2\text{sgr}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*log(x^2)/a^2 + 1/2*(b^2*d*sg
n(b*x^2 + a) - a*b*e*sgn(b*x^2 + a))*log(abs(b*x^2 + a))/(a^2*b) + 1/2*(b*d
*x^2*sgn(b*x^2 + a) - a*x^2*e*sgn(b*x^2 + a) - a*d*sgn(b*x^2 + a))/(a^2*x^2
)
```

maple [A] time = 0.01, size = 79, normalized size = 0.58

$$\frac{\left(b\,x^2+a\right)\left(2ae\,x^2\ln(x)-ae\,x^2\ln\left(b\,x^2+a\right)-2bd\,x^2\ln(x)+bd\,x^2\ln\left(b\,x^2+a\right)-ad\right)}{2\sqrt{\left(b\,x^2+a\right)^2}\;a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*(2*ln(x)*x^2*a*e-2*ln(x)*x^2*b*d-ln(b*x^2+a)*x^2*a*e+ln(b*x^2
+a)*x^2*b*d-a*d)/((b*x^2+a)^2)^(1/2)/x^2/a^2
```

maxima [A] time = 0.78, size = 48, normalized size = 0.35

$$\frac{(bd-ae)\log\left(bx^2+a\right)}{2\,a^2}-\frac{(bd-ae)\log\left(x^2\right)}{2\,a^2}-\frac{d}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*d - a*e)*log(b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*log(x^2)/a^2 - 1/2*d/(
a*x^2)
```

mupad [B] time = 0.81, size = 125, normalized size = 0.91

$$\frac{a\,b\,d\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^2}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}\right)}{2\,{\left(a^2\right)}^{3/2}}-\frac{e\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}}-\frac{d\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,a^2\,x^2}-\frac{e\,\ln\left(\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{a^2}+a^2+a\,b\,x^2\right)}{2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x^3*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] (a*b*d*atanh((a^2 + a*b*x^2)/((a^2)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)
)))/(2*(a^2)^(3/2)) - (e*log(1/x^2))/(2*(a^2)^(1/2)) - (d*(a^2 + b^2*x^4 +
2*a*b*x^2)^(1/2))/(2*a^2*x^2) - (e*log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) +
a^2 + a*b*x^2))/(2*(a^2)^(1/2))
```

sympy [A] time = 0.73, size = 41, normalized size = 0.30

$$-\frac{d}{2ax^2}+\frac{(ae-bd)\log(x)}{a^2}-\frac{(ae-bd)\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -d/(2*a*x**2) + (a*e - b*d)*log(x)/a**2 - (a*e - b*d)*log(a/b + x**2)/(2*a*
*2)
```

3.81 $\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{x(bd-5ae)}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/8*(-5*a*e+b*d)*x/a/b^2/((b*x^2+a)^2)^(1/2)-1/4*(-a*e+b*d)*x/b^2/(b*x^2+a)
/((b*x^2+a)^2)^(1/2)+1/8*(3*a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^
(3/2)/b^(5/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1250, 455, 385, 205}

$$\frac{x(bd-5ae)}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((b*d - 5*a*e)*x)/(8*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*
x)/(4*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d + 3*a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
```

```
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^2\right)}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{x^2\left(d+ex^2\right)}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(bd-ae)x}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ab+b^2x^2\right)\int \frac{-b(bd-ae)-4b^2ex^2}{\left(ab+b^2x^2\right)^2}\,dx}{4b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left((bd+3ae)\left(\right.\right.}{8ab^2\sqrt{}}$$

$$= \frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd+3ae)\left(a\right.}{8a^{3/2}b^{5/2}\sqrt{}}$$

Mathematica [A] time = 0.06, size = 108, normalized size = 0.71

$$\frac{\left(a+bx^2\right)^2(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\sqrt{a}\,\sqrt{b}\,x\left(3a^2e+ab\left(d+5ex^2\right)-b^2dx^2\right)}{8a^{3/2}b^{5/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[b]*x*(3*a^2*e - b^2*d*x^2 + a*b*(d + 5*e*x^2))) + (b*d + 3*
a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*(a + b*x
^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.75, size = 300, normalized size = 1.96

$$\left[\frac{2\left(ab^3d-5\,a^2b^2e\right)x^3-\left(\left(b^3d+3\,ab^2e\right)x^4+a^2bd+3\,a^3e+2\left(ab^2d+3\,a^2be\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\left(\right.}{16\left(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(2*(a*b^3*d - 5*a^2*b^2*e)*x^3 - ((b^3*d + 3*a*b^2*e)*x^4 + a^2*b*d +
 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) - 2*(a^2*b^2*d + 3*a^3*b*e)*x)/(a^2*b^5*x^4 + 2*a^3*b
^4*x^2 + a^4*b^3), 1/8*((a*b^3*d - 5*a^2*b^2*e)*x^3 + ((b^3*d + 3*a*b^2*e)*
x^4 + a^2*b*d + 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) - (a^2*b^2*d + 3*a^3*b*e)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4
*b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 188, normalized size = 1.23

$$-\frac{\left(-3a\,b^2e\,x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)-b^3d\,x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)-6a^2be\,x^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)-2a\,b^2d\,x^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)+5\sqrt{ab}\,abe\,x}{8\sqrt{ab}\,\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/8*(-3*arctan(1/(a*b)^(1/2)*b*x)*x^4*a*b^2*e-arctan(1/(a*b)^(1/2)*b*x)*x^
4*b^3*d+5*(a*b)^(1/2)*x^3*a*b*e-(a*b)^(1/2)*x^3*b^2*d-6*arctan(1/(a*b)^(1/2
)*b*x)*x^2*a^2*b*e-2*arctan(1/(a*b)^(1/2)*b*x)*x^2*a*b^2*d+3*(a*b)^(1/2)*x*
a^2*e+(a*b)^(1/2)*x*a*b*d-3*arctan(1/(a*b)^(1/2)*b*x)*a^3*e-arctan(1/(a*b)^
(1/2)*b*x)*a^2*b*d)*(b*x^2+a)/(a*b)^(1/2)/a/b^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.49, size = 125, normalized size = 0.82

$$-\frac{1}{8}\,e\left(\frac{5\,bx^3+3\,ax}{b^4x^4+2\,ab^3x^2+a^2b^2}-\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}\right)+\frac{1}{8}\,d\left(\frac{bx^3-ax}{ab^3x^4+2\,a^2b^2x^2+a^3b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/8*e*((5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) - 3*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^2)) + 1/8*d*((b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x
^2 + a^3*b) + arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(e\,x^2+d\right)}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((x^2*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(d + e x^{2}\right)}{\left(\left(a + b x^{2}\right)^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

3.82 $\int \frac{x\left(d+ex^2\right)}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{bd-ae}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/2*e/b^2/((b*x^2+a)^2)^(1/2)+1/4*(a*e-b*d)/b^2/(b*x^2+a)/((b*x^2+a)^2)^(1
/2)
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1247, 640, 607}

$$-\frac{bd-ae}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -e/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*b^2*(a + b*x^2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(d+ex^2\right)}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+ex}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)\operatorname{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^2\right)}{2b}\\
&= -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{bd-ae}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.58

$$\frac{-ae-b\left(d+2ex^2\right)}{4b^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] (-(a*e) - b*(d + 2*e*x^2))/(4*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.69, size = 42, normalized size = 0.55

$$-\frac{2\,bex^2+bd+ae}{4\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] -1/4*(2*b*e*x^2 + b*d + a*e)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.51, size = 40, normalized size = 0.52

$$-\frac{2\,bx^2e+bd+ae}{4\left(bx^2+a\right)^2b^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] -1/4*(2*b*x^2*e + b*d + a*e)/((b*x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

maple [A] time = 0.01, size = 38, normalized size = 0.49

$$-\frac{\left(b\,x^2+a\right)\left(2be\,x^2+ae+bd\right)}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
[Out] -1/4*(b*x^2+a)*(2*b*e*x^2+a*e+b*d)/b^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 0.64, size = 65, normalized size = 0.84

$$-\frac{\left(2\,bx^2+a\right)e}{4\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}-\frac{d}{4\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] -1/4*(2*b*x^2 + a)*e/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) - 1/4*d/(b^3*x^4 + 2
*a*b^2*x^2 + a^2*b)
```

mupad [B] time = 0.18, size = 48, normalized size = 0.62

$$-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(2\,b\,e\,x^2+a\,e+b\,d\right)}{4\,b^2\,\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] -((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a*e + b*d + 2*b*e*x^2))/(4*b^2*(a + b*
x^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(d+ex^2\right)}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

3.83 $\int \frac{d+ex^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=156

$$\frac{x(ae+3bd)}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x(bd-ae)}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/8*(a*e+3*b*d)*x/a^2/b/((b*x^2+a)^2)^(1/2)+1/4*(-a*e+b*d)*x/a/b/(b*x^2+a)/
((b*x^2+a)^2)^(1/2)+1/8*(a*e+3*b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(
5/2)/b^(3/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1148, 385, 199, 205}

$$\frac{x(ae+3bd)}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x(bd-ae)}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((3*b*d + a*e)*x)/(8*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*
x)/(4*a*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d + a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1148

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-ae)x}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left((3bd+ae)\left(ab+b^2x^2\right)\right)\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)x}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left((3bd+ae)\left(\right.\right.}{8a^2b\sqrt{}}$$

$$= \frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)x}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3bd+ae)\left(\right.}{8a^{5/2}b^{3/2}\sqrt{}}$$

Mathematica [A] time = 0.05, size = 108, normalized size = 0.69

$$\frac{\sqrt{a}\sqrt{b}\,x\left(a^2(-e)+ab\left(5d+ex^2\right)+3b^2dx^2\right)+\left(a+bx^2\right)^2(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-(a^2*e) + 3*b^2*d*x^2 + a*b*(5*d + e*x^2)) + (3*b*d +
a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*(a + b*x
^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.75, size = 301, normalized size = 1.93

$$\left[\frac{2\left(3\,ab^3d+a^2b^2e\right)x^3-\left(\left(3\,b^3d+ab^2e\right)x^4+3\,a^2bd+a^3e+2\left(3\,ab^2d+a^2be\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\,(}{16\left(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a*b^3*d + a^2*b^2*e)*x^3 - ((3*b^3*d + a*b^2*e)*x^4 + 3*a^2*b*d
 + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^2*d - a^3*b*e)*x)/(a^3*b^4*x^4 + 2*a^4*b
^3*x^2 + a^5*b^2), 1/8*((3*a*b^3*d + a^2*b^2*e)*x^3 + ((3*b^3*d + a*b^2*e)*
x^4 + 3*a^2*b*d + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) + (5*a^2*b^2*d - a^3*b*e)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5
*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 186, normalized size = 1.19

$$\frac{\left(a\,b^2e\,x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)+3b^3d\,x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)+2a^2be\,x^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)+6a\,b^2d\,x^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\sqrt{ab}\ abe\,x^3+\right.}{8\sqrt{ab}\ \left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\ a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(a*b^2*e*x^4*arctan(1/(a*b)^(1/2)*b*x)+3*b^3*d*x^4*arctan(1/(a*b)^(1/2)
*b*x)+(a*b)^(1/2)*a*b*e*x^3+3*(a*b)^(1/2)*b^2*d*x^3+2*a^2*b*e*x^2*arctan(1/
(a*b)^(1/2)*b*x)+6*a*b^2*d*x^2*arctan(1/(a*b)^(1/2)*b*x)-(a*b)^(1/2)*a^2*e*
x+5*(a*b)^(1/2)*a*b*d*x+a^3*e*arctan(1/(a*b)^(1/2)*b*x)+3*a^2*b*d*arctan(1/
(a*b)^(1/2)*b*x))*(b*x^2+a)/(a*b)^(1/2)/b/a^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.67, size = 124, normalized size = 0.79

$$\frac{1}{8}\,d\left(\frac{3\,bx^3+5\,ax}{a^2b^2x^4+2\,a^3bx^2+a^4}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}\right)+\frac{1}{8}\,e\left(\frac{bx^3-ax}{ab^3x^4+2\,a^2b^2x^2+a^3b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*d*((3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2)) + 1/8*e*((b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^
2 + a^3*b) + arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d + e*x^2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

3.84 $\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)\log\left(a+bx\right.}{2a^3\sqrt{a^2+2abx^2+b^2x}}$$

```
[Out] 1/2*d/a^2/((b*x^2+a)^2)^(1/2)+1/4*(-a*e+b*d)/a/b/(b*x^2+a)/((b*x^2+a)^2)^(1
/2)+d*(b*x^2+a)*ln(x)/a^3/((b*x^2+a)^2)^(1/2)-1/2*d*(b*x^2+a)*ln(b*x^2+a)/a
^3/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 446, 77}

$$\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)\log\left(a+bx\right.}{2a^3\sqrt{a^2+2abx^2+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] d/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*d - a*e)/(4*a*b*(a + b*x^2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(a + b*x^2)*Log[x])/(a^3*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2)*Log[a + b*x^2])/(2*a^3*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{x\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{d+ex}{x\left(ab+b^2x\right)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{d}{a^3b^3x}+\frac{-bd+ae}{ab^3(a+bx)^3}-\frac{d}{a^2b^2(a+bx)^2}-\frac{d}{a^3b^2(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\left(a+bx^2\right)1}{a^3\sqrt{a^2+2abx^2}}$$

Mathematica [A] time = 0.04, size = 92, normalized size = 0.57

$$\frac{a\left(a^2(-e)+3abd+2b^2dx^2\right)+4bd\log(x)\left(a+bx^2\right)^2-2bd\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^3b\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(3*a*b*d - a^2*e + 2*b^2*d*x^2) + 4*b*d*(a + b*x^2)^2*Log[x] - 2*b*d*(a
+ b*x^2)^2*Log[a + b*x^2])/(4*a^3*b*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.66, size = 119, normalized size = 0.74

$$\frac{2\,ab^2dx^2+3\,a^2bd-a^3e-2\left(b^3dx^4+2\,ab^2dx^2+a^2bd\right)\log\left(bx^2+a\right)+4\left(b^3dx^4+2\,ab^2dx^2+a^2bd\right)\log(x)}{4\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b^2*d*x^2 + 3*a^2*b*d - a^3*e - 2*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a^2
*b*d)*log(b*x^2 + a) + 4*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a^2*b*d)*log(x))/(a^3
*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

giac [A] time = 0.53, size = 96, normalized size = 0.60

$$-\frac{d\log\left(\left|bx^2+a\right|\right)}{2\,a^3\text{sgn}\left(bx^2+a\right)}+\frac{d\log\left(|x|\right)}{a^3\text{sgn}\left(bx^2+a\right)}+\frac{2\,ab^2dx^2+3\,a^2bd-a^3e}{4\left(bx^2+a\right)^2a^3b\text{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*d*log(abs(b*x^2 + a))/(a^3*sgn(b*x^2 + a)) + d*log(abs(x))/(a^3*sgn(b*
x^2 + a)) + 1/4*(2*a*b^2*d*x^2 + 3*a^2*b*d - a^3*e)/((b*x^2 + a)^2*a^3*b*sg
n(b*x^2 + a))
```

maple [A] time = 0.02, size = 133, normalized size = 0.83

$$\frac{\left(4b^3d\,x^4\ln(x)-2b^3d\,x^4\ln\left(b\,x^2+a\right)+8a\,b^2d\,x^2\ln(x)-4a\,b^2d\,x^2\ln\left(b\,x^2+a\right)+2a\,b^2d\,x^2+4a^2bd\ln(x)-2a^2bd\right.}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/4*(4*ln(x)*x^4*b^3*d-2*ln(b*x^2+a)*x^4*b^3*d+8*ln(x)*x^2*a*b^2*d-4*ln(b*x
^2+a)*x^2*a*b^2*d+2*b^2*d*x^2*a+4*ln(x)*a^2*b*d-2*ln(b*x^2+a)*a^2*b*d-a^3*e
+3*a^2*b*d)*(b*x^2+a)/b/a^3/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 0.89, size = 88, normalized size = 0.55

$$\frac{1}{4}\,d\left(\frac{2\,bx^2+3\,a}{a^2b^2x^4+2\,a^3bx^2+a^4}-\frac{2\,\log\left(bx^2+a\right)}{a^3}+\frac{4\,\log(x)}{a^3}\right)-\frac{e}{4\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*d*((2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 2*log(b*x^2 + a)
/a^3 + 4*log(x)/a^3) - 1/4*e/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{x\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x^2)/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x*((a + b*x**2)**2)**(3/2)), x)
```

3.85 $\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{x(bd-ae)}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3\left(a+bx^2\right)(5bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(7bd-3ae)}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+\right.}{a^3x\sqrt{a^2+2a}}$$

```
[Out] -1/8*(-3*a*e+7*b*d)*x/a^3/((b*x^2+a)^2)^(1/2)-1/4*(-a*e+b*d)*x/a^2/(b*x^2+a
)/((b*x^2+a)^2)^(1/2)-d*(b*x^2+a)/a^3/x/((b*x^2+a)^2)^(1/2)-3/8*(-a*e+5*b*d
)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1250, 456, 453, 205}

$$-\frac{x(7bd-3ae)}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3\left(a+bx^2\right)(5bd-ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+\right.}{a^3x\sqrt{a^2+2a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -((7*b*d - 3*a*e)*x)/(8*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)
*x)/(4*a^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))/(
a^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*(5*b*d - a*e)*(a + b*x^2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
```

```
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{x^2\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{-\frac{4d}{ab}+\frac{3(bd-ae)x^2}{a^2b}}{x^2\left(ab+b^2x^2\right)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2\left(ab+\right.\right.}{8\sqrt{}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\left(\right.}{a^3x\sqrt{a^2+}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\left(\right.}{a^3x\sqrt{a^2+}}$$

Mathematica [A] time = 0.07, size = 124, normalized size = 0.65

$$\frac{\sqrt{a}\sqrt{b}\left(a^2\left(5ex^2-8d\right)+ab\left(3ex^4-25dx^2\right)-15b^2dx^4\right)+3x\left(a+bx^2\right)^2(ae-5bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\,x\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*(-15*b^2*d*x^4 + a^2*(-8*d + 5*e*x^2) + a*b*(-25*d*x^2 + 3
*e*x^4)) + 3*(-5*b*d + a*e)*x*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(7/2)*Sqrt[b]*x*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.67, size = 334, normalized size = 1.76

$$\left[-\frac{16\,a^3bd+6\left(5\,ab^3d-a^2b^2e\right)x^4+10\left(5\,a^2b^2d-a^3be\right)x^2-3\left(\left(5\,b^3d-ab^2e\right)x^5+2\left(5\,ab^2d-a^2be\right)x^3+\left(5\,a^2b\right.\right.}{16\left(a^4b^3x^5+2\,a^5b^2x^3+a^6bx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*(16*a^3*b*d + 6*(5*a*b^3*d - a^2*b^2*e)*x^4 + 10*(5*a^2*b^2*d - a^3*
b*e)*x^2 - 3*((5*b^3*d - a*b^2*e)*x^5 + 2*(5*a*b^2*d - a^2*b*e)*x^3 + (5*a^
2*b*d - a^3*e)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))
/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), -1/8*(8*a^3*b*d + 3*(5*a*b^3*d -
a^2*b^2*e)*x^4 + 5*(5*a^2*b^2*d - a^3*b*e)*x^2 + 3*((5*b^3*d - a*b^2*e)*x^5
 + 2*(5*a*b^2*d - a^2*b*e)*x^3 + (5*a^2*b*d - a^3*e)*x)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 206, normalized size = 1.08

$$\frac{\left(3a\,b^2e\,x^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)-15b^3d\,x^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)+6a^2be\,x^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)-30a\,b^2d\,x^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)+3\sqrt{ab}\;abe\right.}{8\sqrt{ab}\;\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(3*arctan(1/(a*b)^(1/2)*b*x)*x^5*a*b^2*e-15*arctan(1/(a*b)^(1/2)*b*x)*x
^5*b^3*d+3*(a*b)^(1/2)*x^4*a*b*e-15*(a*b)^(1/2)*x^4*b^2*d+6*arctan(1/(a*b)^
(1/2)*b*x)*x^3*a^2*b*e-30*arctan(1/(a*b)^(1/2)*b*x)*x^3*a*b^2*d+5*(a*b)^(1/
2)*x^2*a^2*e-25*(a*b)^(1/2)*x^2*a*b*d+3*arctan(1/(a*b)^(1/2)*b*x)*x*a^3*e-1
5*arctan(1/(a*b)^(1/2)*b*x)*x*a^2*b*d-8*(a*b)^(1/2)*a^2*d)*(b*x^2+a)/(a*b)^
(1/2)/x/a^3/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.40, size = 134, normalized size = 0.71

$$-\frac{1}{8}\,d\left(\frac{15\,b^2x^4+25\,abx^2+8\,a^2}{a^3b^2x^5+2\,a^4bx^3+a^5x}+\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}\right)+\frac{1}{8}\,e\left(\frac{3\,bx^3+5\,ax}{a^2b^2x^4+2\,a^3bx^2+a^4}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/8*d*((15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*
x) + 15*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)) + 1/8*e*((3*b*x^3 + 5*a*x)
/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2
))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{x^2\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x^2)/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{x^2\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

3.86 $\int \frac{d+ex^2}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\log(x)\left(a+bx^2\right)(3bd-ae)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3bd-ae)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{2a^3\sqrt{a^2}}$$

```
[Out] 1/2*(a*e-2*b*d)/a^3/((b*x^2+a)^2)^(1/2)+1/4*(a*e-b*d)/a^2/(b*x^2+a)/((b*x^2
+a)^2)^(1/2)-1/2*d*(b*x^2+a)/a^3/x^2/((b*x^2+a)^2)^(1/2)-(-a*e+3*b*d)*(b*x^
2+a)*ln(x)/a^4/((b*x^2+a)^2)^(1/2)+1/2*(-a*e+3*b*d)*(b*x^2+a)*ln(b*x^2+a)/a
^4/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 446, 77}

$$-\frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\log(x)\left(a+bx^2\right)(3bd-ae)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3bd-}{2a^4\sqrt{a^2+2a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -(2*b*d - a*e)/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*a^2
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))/(2*a^3*x^2*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((3*b*d - a*e)*(a + b*x^2)*Log[x])/(a^4*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d - a*e)*(a + b*x^2)*Log[a + b*x^2
])/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{x^3\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{d+ex}{x^2\left(ab+b^2x\right)^3}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{d}{a^3b^3x^2}+\frac{-3bd+ae}{a^4b^3x}+\frac{bd-ae}{a^2b^2(a+bx)^3}+\frac{2bd-ae}{a^3b^2(a+bx)^2}+\frac{3bd-ae}{a^4b^2(a+b}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a\right.}{2a^3x^2\sqrt{a^2+}}$$

Mathematica [A] time = 0.07, size = 130, normalized size = 0.58

$$\frac{a\left(a^2\left(3ex^2-2d\right)+ab\left(2ex^4-9dx^2\right)-6b^2dx^4\right)+4x^2\log(x)\left(a+bx^2\right)^2(ae-3bd)+2x^2\left(a+bx^2\right)^2(3bd-ae)\log}{4a^4x^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(-6*b^2*d*x^4 + a^2*(-2*d + 3*e*x^2) + a*b*(-9*d*x^2 + 2*e*x^4)) + 4*(-3
*b*d + a*e)*x^2*(a + b*x^2)^2*Log[x] + 2*(3*b*d - a*e)*x^2*(a + b*x^2)^2*Lo
g[a + b*x^2])/(4*a^4*x^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.65, size = 205, normalized size = 0.92

$$-\frac{2\left(3\,ab^2d-a^2be\right)x^4+2\,a^3d+3\left(3\,a^2bd-a^3e\right)x^2-2\left(\left(3\,b^3d-ab^2e\right)x^6+2\left(3\,ab^2d-a^2be\right)x^4+\left(3\,a^2bd-a^3e\right)x^2\right)}{4\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/4*(2*(3*a*b^2*d - a^2*b*e)*x^4 + 2*a^3*d + 3*(3*a^2*b*d - a^3*e)*x^2 - 2
*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2*b*e)*x^4 + (3*a^2*b*d - a^3*
e)*x^2)*log(b*x^2 + a) + 4*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2*b*
e)*x^4 + (3*a^2*b*d - a^3*e)*x^2)*log(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*
x^2)
```

giac [A] time = 0.40, size = 144, normalized size = 0.65

$$-\frac{(3\,bd-ae)\log\left(|x|\right)}{a^4\text{sgn}\left(bx^2+a\right)}+\frac{\left(3\,b^2d-abe\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^4b\text{sgn}\left(bx^2+a\right)}-\frac{2\left(3\,ab^2d-a^2be\right)x^4+2\,a^3d+3\left(3\,a^2bd-a^3e\right)x^2}{4\left(bx^2+a\right)^2a^4x^2\text{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(3*b*d - a*e)*log(abs(x))/(a^4*sgn(b*x^2 + a)) + 1/2*(3*b^2*d - a*b*e)*log
(abs(b*x^2 + a))/(a^4*b*sgn(b*x^2 + a)) - 1/4*(2*(3*a*b^2*d - a^2*b*e)*x^4
```

```
+ 2*a^3*d + 3*(3*a^2*b*d - a^3*e)*x^2)/((b*x^2 + a)^2*a^4*x^2*sgn(b*x^2 + a
))
```

maple [A] time = 0.02, size = 249, normalized size = 1.12

$$\frac{\left(4a\,b^2e\,x^6\ln(x)-2a\,b^2e\,x^6\ln\left(b\,x^2+a\right)-12b^3d\,x^6\ln(x)+6b^3d\,x^6\ln\left(b\,x^2+a\right)+8a^2be\,x^4\ln(x)-4a^2be\,x^4\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/4*(4*ln(x)*x^6*a*b^2*e-12*ln(x)*x^6*b^3*d-2*ln(b*x^2+a)*x^6*a*b^2*e+6*ln(
b*x^2+a)*x^6*b^3*d+8*ln(x)*x^4*a^2*b*e-24*ln(x)*x^4*a*b^2*d-4*ln(b*x^2+a)*x
^4*a^2*b*e+12*ln(b*x^2+a)*x^4*a*b^2*d+2*x^4*a^2*b*e-6*x^4*a*b^2*d+4*ln(x)*x
^2*a^3*e-12*ln(x)*x^2*a^2*b*d-2*ln(b*x^2+a)*x^2*a^3*e+6*ln(b*x^2+a)*x^2*a^2
*b*d+3*x^2*a^3*e-9*x^2*a^2*b*d-2*a^3*d)*(b*x^2+a)/x^2/a^4/((b*x^2+a)^2)^(3/
2)
```

maxima [A] time = 0.84, size = 138, normalized size = 0.62

$$-\frac{1}{4}\,d\left(\frac{6\,b^2x^4+9\,abx^2+2\,a^2}{a^3b^2x^6+2\,a^4bx^4+a^5x^2}-\frac{6\,b\log\left(bx^2+a\right)}{a^4}+\frac{12\,b\log(x)}{a^4}\right)+\frac{1}{4}\,e\left(\frac{2\,bx^2+3\,a}{a^2b^2x^4+2\,a^3bx^2+a^4}-\frac{2\,\log\left(bx^2+a\right)}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/4*d*((6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^
2) - 6*b*log(b*x^2 + a)/a^4 + 12*b*log(x)/a^4) + 1/4*e*((2*b*x^2 + 3*a)/(a^
2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 2*log(b*x^2 + a)/a^3 + 4*log(x)/a^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{x^3\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x^2)/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^{2}}{x^{3} \left(\left(a + bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

3.87 $\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=400

$$\frac{10a^2b^2\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+7}(ae+bd)}{f^7(m+7)\left(a+bx^2\right)}+\frac{b^5e\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+13}}{f^{13}(m+13)\left(a+bx^2\right)}+\frac{b^4\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+11}(}{f^{11}(m+11)\left(a+bx^2\right)}$$

```
[Out] a^5*d*(f*x)^(1+m)*((b*x^2+a)^2)^(1/2)/f/(1+m)/(b*x^2+a)+a^4*(a*e+5*b*d)*(f*
x)^(3+m)*((b*x^2+a)^2)^(1/2)/f^3/(3+m)/(b*x^2+a)+5*a^3*b*(a*e+2*b*d)*(f*x)^
(5+m)*((b*x^2+a)^2)^(1/2)/f^5/(5+m)/(b*x^2+a)+10*a^2*b^2*(a*e+b*d)*(f*x)^(7
+m)*((b*x^2+a)^2)^(1/2)/f^7/(7+m)/(b*x^2+a)+5*a*b^3*(2*a*e+b*d)*(f*x)^(9+m)
*((b*x^2+a)^2)^(1/2)/f^9/(9+m)/(b*x^2+a)+b^4*(5*a*e+b*d)*(f*x)^(11+m)*((b*x
^2+a)^2)^(1/2)/f^11/(11+m)/(b*x^2+a)+b^5*e*(f*x)^(13+m)*((b*x^2+a)^2)^(1/2)
/f^13/(13+m)/(b*x^2+a)
```

Rubi [A] time = 0.24, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1250, 448}

$$\frac{a^4\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+3}(ae+5bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{5a^3b\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}(ae+2bd)}{f^5(m+5)\left(a+bx^2\right)}+\frac{10a^2b^2\sqrt{a^2+2abx^2+b^2}}{f^7(m+7)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2
)) + (a^4*(5*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3
*(3 + m)*(a + b*x^2)) + (5*a^3*b*(2*b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (10*a^2*b^2*(b*d + a*e)*(f*x
)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (5*a
*b^3*(b*d + 2*a*e)*(f*x)^(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 +
 m)*(a + b*x^2)) + (b^4*(b*d + 5*a*e)*(f*x)^(11 + m)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(f^11*(11 + m)*(a + b*x^2)) + (b^5*e*(f*x)^(13 + m)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(f^13*(13 + m)*(a + b*x^2))
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int (fx)^m\left(ab+b^2x^2\right)^5\left(d+ex^2\right)dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int \left(a^5b^5d(fx)^m + \frac{a^4b^5(5bd+ae)(fx)^{2+m}}{f^2} + \frac{5a^3b^6}{\ldots}\right.}{\ldots}$$

$$= \frac{a^5d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)} + \frac{a^4(5bd+ae)(fx)^{3+m}\sqrt{a^2+\ldots}}{f^3(3+m)\left(a+b\ldots\right.}$$

Mathematica [A] time = 0.22, size = 160, normalized size = 0.40

$$\frac{x\sqrt{\left(a+bx^2\right)^2}\,(fx)^m\left(\frac{a^5d}{m+1}+\frac{a^4x^2(ae+5bd)}{m+3}+\frac{5a^3bx^4(ae+2bd)}{m+5}+\frac{10a^2b^2x^6(ae+bd)}{m+7}+\frac{b^4x^{10}(5ae+bd)}{m+11}+\frac{5ab^3x^8(2ae+bd)}{m+9}+\frac{b^5ex^{12}}{m+13}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x*(f*x)^m*Sqrt[(a + b*x^2)^2]*((a^5*d)/(1 + m) + (a^4*(5*b*d + a*e)*x^2)/(
3 + m) + (5*a^3*b*(2*b*d + a*e)*x^4)/(5 + m) + (10*a^2*b^2*(b*d + a*e)*x^6)
/(7 + m) + (5*a*b^3*(b*d + 2*a*e)*x^8)/(9 + m) + (b^4*(b*d + 5*a*e)*x^10)/(
11 + m) + (b^5*e*x^12)/(13 + m)))/(a + b*x^2)
```

fricas [B] time = 0.75, size = 853, normalized size = 2.13

$$\frac{\left(\left(b^5em^6+36\,b^5em^5+505\,b^5em^4+3480\,b^5em^3+12139\,b^5em^2+19524\,b^5em+10395\,b^5e\right)x^{13}+\left(\left(b^5d+5\,ab^4\ldots\right.\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fri
cas")
```

```
[Out] ((b^5*e*m^6 + 36*b^5*e*m^5 + 505*b^5*e*m^4 + 3480*b^5*e*m^3 + 12139*b^5*e*m
^2 + 19524*b^5*e*m + 10395*b^5*e)*x^13 + ((b^5*d + 5*a*b^4*e)*m^6 + 12285*b
^5*d + 61425*a*b^4*e + 38*(b^5*d + 5*a*b^4*e)*m^5 + 555*(b^5*d + 5*a*b^4*e)
*m^4 + 3940*(b^5*d + 5*a*b^4*e)*m^3 + 14039*(b^5*d + 5*a*b^4*e)*m^2 + 22902
*(b^5*d + 5*a*b^4*e)*m)*x^11 + 5*((a*b^4*d + 2*a^2*b^3*e)*m^6 + 15015*a*b^4
*d + 30030*a^2*b^3*e + 40*(a*b^4*d + 2*a^2*b^3*e)*m^5 + 613*(a*b^4*d + 2*a^
2*b^3*e)*m^4 + 4528*(a*b^4*d + 2*a^2*b^3*e)*m^3 + 16627*(a*b^4*d + 2*a^2*b^
3*e)*m^2 + 27688*(a*b^4*d + 2*a^2*b^3*e)*m)*x^9 + 10*((a^2*b^3*d + a^3*b^2*
e)*m^6 + 19305*a^2*b^3*d + 19305*a^3*b^2*e + 42*(a^2*b^3*d + a^3*b^2*e)*m^5
 + 679*(a^2*b^3*d + a^3*b^2*e)*m^4 + 5292*(a^2*b^3*d + a^3*b^2*e)*m^3 + 203
35*(a^2*b^3*d + a^3*b^2*e)*m^2 + 34986*(a^2*b^3*d + a^3*b^2*e)*m)*x^7 + 5*(
(2*a^3*b^2*d + a^4*b*e)*m^6 + 54054*a^3*b^2*d + 27027*a^4*b*e + 44*(2*a^3*b
^2*d + a^4*b*e)*m^5 + 753*(2*a^3*b^2*d + a^4*b*e)*m^4 + 6280*(2*a^3*b^2*d +
 a^4*b*e)*m^3 + 25979*(2*a^3*b^2*d + a^4*b*e)*m^2 + 47436*(2*a^3*b^2*d + a^
4*b*e)*m)*x^5 + ((5*a^4*b*d + a^5*e)*m^6 + 225225*a^4*b*d + 45045*a^5*e + 4
6*(5*a^4*b*d + a^5*e)*m^5 + 835*(5*a^4*b*d + a^5*e)*m^4 + 7540*(5*a^4*b*d +
 a^5*e)*m^3 + 34759*(5*a^4*b*d + a^5*e)*m^2 + 73054*(5*a^4*b*d + a^5*e)*m)*
x^3 + (a^5*d*m^6 + 48*a^5*d*m^5 + 925*a^5*d*m^4 + 9120*a^5*d*m^3 + 48259*a^
5*d*m^2 + 129072*a^5*d*m + 135135*a^5*d)*x)*(f*x)^m/(m^7 + 49*m^6 + 973*m^5
 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

giac [B] time = 0.69, size = 2213, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b^5*m^6*x^13*e*sgn(b*x^2 + a) + 36*(f*x)^m*b^5*m^5*x^13*e*sgn(b*x^
2 + a) + (f*x)^m*b^5*d*m^6*x^11*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*m^6*x^11*e
*sgn(b*x^2 + a) + 505*(f*x)^m*b^5*m^4*x^13*e*sgn(b*x^2 + a) + 38*(f*x)^m*b^
5*d*m^5*x^11*sgn(b*x^2 + a) + 190*(f*x)^m*a*b^4*m^5*x^11*e*sgn(b*x^2 + a) +
 3480*(f*x)^m*b^5*m^3*x^13*e*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*d*m^6*x^9*sgn
(b*x^2 + a) + 555*(f*x)^m*b^5*d*m^4*x^11*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^
3*m^6*x^9*e*sgn(b*x^2 + a) + 2775*(f*x)^m*a*b^4*m^4*x^11*e*sgn(b*x^2 + a) +
 12139*(f*x)^m*b^5*m^2*x^13*e*sgn(b*x^2 + a) + 200*(f*x)^m*a*b^4*d*m^5*x^9*
sgn(b*x^2 + a) + 3940*(f*x)^m*b^5*d*m^3*x^11*sgn(b*x^2 + a) + 400*(f*x)^m*a
^2*b^3*m^5*x^9*e*sgn(b*x^2 + a) + 19700*(f*x)^m*a*b^4*m^3*x^11*e*sgn(b*x^2
+ a) + 19524*(f*x)^m*b^5*m*x^13*e*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^3*d*m^6
*x^7*sgn(b*x^2 + a) + 3065*(f*x)^m*a*b^4*d*m^4*x^9*sgn(b*x^2 + a) + 14039*(
f*x)^m*b^5*d*m^2*x^11*sgn(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*m^6*x^7*e*sgn(b*x
^2 + a) + 6130*(f*x)^m*a^2*b^3*m^4*x^9*e*sgn(b*x^2 + a) + 70195*(f*x)^m*a*b
^4*m^2*x^11*e*sgn(b*x^2 + a) + 10395*(f*x)^m*b^5*x^13*e*sgn(b*x^2 + a) + 42
0*(f*x)^m*a^2*b^3*d*m^5*x^7*sgn(b*x^2 + a) + 22640*(f*x)^m*a*b^4*d*m^3*x^9*
sgn(b*x^2 + a) + 22902*(f*x)^m*b^5*d*m*x^11*sgn(b*x^2 + a) + 420*(f*x)^m*a^
3*b^2*m^5*x^7*e*sgn(b*x^2 + a) + 45280*(f*x)^m*a^2*b^3*m^3*x^9*e*sgn(b*x^2
+ a) + 114510*(f*x)^m*a*b^4*m*x^11*e*sgn(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*d*
m^6*x^5*sgn(b*x^2 + a) + 6790*(f*x)^m*a^2*b^3*d*m^4*x^7*sgn(b*x^2 + a) + 83
135*(f*x)^m*a*b^4*d*m^2*x^9*sgn(b*x^2 + a) + 12285*(f*x)^m*b^5*d*x^11*sgn(b
*x^2 + a) + 5*(f*x)^m*a^4*b*m^6*x^5*e*sgn(b*x^2 + a) + 6790*(f*x)^m*a^3*b^2
*m^4*x^7*e*sgn(b*x^2 + a) + 166270*(f*x)^m*a^2*b^3*m^2*x^9*e*sgn(b*x^2 + a)
 + 61425*(f*x)^m*a*b^4*x^11*e*sgn(b*x^2 + a) + 440*(f*x)^m*a^3*b^2*d*m^5*x^
5*sgn(b*x^2 + a) + 52920*(f*x)^m*a^2*b^3*d*m^3*x^7*sgn(b*x^2 + a) + 138440*
(f*x)^m*a*b^4*d*m*x^9*sgn(b*x^2 + a) + 220*(f*x)^m*a^4*b*m^5*x^5*e*sgn(b*x^
2 + a) + 52920*(f*x)^m*a^3*b^2*m^3*x^7*e*sgn(b*x^2 + a) + 276880*(f*x)^m*a^
2*b^3*m*x^9*e*sgn(b*x^2 + a) + 5*(f*x)^m*a^4*b*d*m^6*x^3*sgn(b*x^2 + a) + 7
530*(f*x)^m*a^3*b^2*d*m^4*x^5*sgn(b*x^2 + a) + 203350*(f*x)^m*a^2*b^3*d*m^2
*x^7*sgn(b*x^2 + a) + 75075*(f*x)^m*a*b^4*d*x^9*sgn(b*x^2 + a) + (f*x)^m*a^
5*m^6*x^3*e*sgn(b*x^2 + a) + 3765*(f*x)^m*a^4*b*m^4*x^5*e*sgn(b*x^2 + a) +
203350*(f*x)^m*a^3*b^2*m^2*x^7*e*sgn(b*x^2 + a) + 150150*(f*x)^m*a^2*b^3*x^
9*e*sgn(b*x^2 + a) + 230*(f*x)^m*a^4*b*d*m^5*x^3*sgn(b*x^2 + a) + 62800*(f*
x)^m*a^3*b^2*d*m^3*x^5*sgn(b*x^2 + a) + 349860*(f*x)^m*a^2*b^3*d*m*x^7*sgn(
b*x^2 + a) + 46*(f*x)^m*a^5*m^5*x^3*e*sgn(b*x^2 + a) + 31400*(f*x)^m*a^4*b*
m^3*x^5*e*sgn(b*x^2 + a) + 349860*(f*x)^m*a^3*b^2*m*x^7*e*sgn(b*x^2 + a) +
(f*x)^m*a^5*d*m^6*x*sgn(b*x^2 + a) + 4175*(f*x)^m*a^4*b*d*m^4*x^3*sgn(b*x^2
 + a) + 259790*(f*x)^m*a^3*b^2*d*m^2*x^5*sgn(b*x^2 + a) + 193050*(f*x)^m*a^
2*b^3*d*x^7*sgn(b*x^2 + a) + 835*(f*x)^m*a^5*m^4*x^3*e*sgn(b*x^2 + a) + 129
895*(f*x)^m*a^4*b*m^2*x^5*e*sgn(b*x^2 + a) + 193050*(f*x)^m*a^3*b^2*x^7*e*s
gn(b*x^2 + a) + 48*(f*x)^m*a^5*d*m^5*x*sgn(b*x^2 + a) + 37700*(f*x)^m*a^4*b
*d*m^3*x^3*sgn(b*x^2 + a) + 474360*(f*x)^m*a^3*b^2*d*m*x^5*sgn(b*x^2 + a) +
 7540*(f*x)^m*a^5*m^3*x^3*e*sgn(b*x^2 + a) + 237180*(f*x)^m*a^4*b*m*x^5*e*s
gn(b*x^2 + a) + 925*(f*x)^m*a^5*d*m^4*x*sgn(b*x^2 + a) + 173795*(f*x)^m*a^4
*b*d*m^2*x^3*sgn(b*x^2 + a) + 270270*(f*x)^m*a^3*b^2*d*x^5*sgn(b*x^2 + a) +
 34759*(f*x)^m*a^5*m^2*x^3*e*sgn(b*x^2 + a) + 135135*(f*x)^m*a^4*b*x^5*e*sg
n(b*x^2 + a) + 9120*(f*x)^m*a^5*d*m^3*x*sgn(b*x^2 + a) + 365270*(f*x)^m*a^4
*b*d*m*x^3*sgn(b*x^2 + a) + 73054*(f*x)^m*a^5*m*x^3*e*sgn(b*x^2 + a) + 4825
9*(f*x)^m*a^5*d*m^2*x*sgn(b*x^2 + a) + 225225*(f*x)^m*a^4*b*d*x^3*sgn(b*x^2
 + a) + 45045*(f*x)^m*a^5*x^3*e*sgn(b*x^2 + a) + 129072*(f*x)^m*a^5*d*m*x*s
gn(b*x^2 + a) + 135135*(f*x)^m*a^5*d*x*sgn(b*x^2 + a))/(m^7 + 49*m^6 + 973*
m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

maple [B] time = 0.01, size = 1099, normalized size = 2.75

$$\frac{\left(b^5 e\,m^6 x^{12} + 36 b^5 e\,m^5 x^{12} + 5 a\,b^4 e\,m^6 x^{10} + b^5 d\,m^6 x^{10} + 505 b^5 e\,m^4 x^{12} + 190 a\,b^4 e\,m^5 x^{10} + 38 b^5 d\,m^5 x^{10} + 348\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] x*(b^5*e*m^6*x^12+36*b^5*e*m^5*x^12+5*a*b^4*e*m^6*x^10+b^5*d*m^6*x^10+505*b
^5*e*m^4*x^12+190*a*b^4*e*m^5*x^10+38*b^5*d*m^5*x^10+3480*b^5*e*m^3*x^12+10
*a^2*b^3*e*m^6*x^8+5*a*b^4*d*m^6*x^8+2775*a*b^4*e*m^4*x^10+555*b^5*d*m^4*x^
10+12139*b^5*e*m^2*x^12+400*a^2*b^3*e*m^5*x^8+200*a*b^4*d*m^5*x^8+19700*a*b
^4*e*m^3*x^10+3940*b^5*d*m^3*x^10+19524*b^5*e*m*x^12+10*a^3*b^2*e*m^6*x^6+1
0*a^2*b^3*d*m^6*x^6+6130*a^2*b^3*e*m^4*x^8+3065*a*b^4*d*m^4*x^8+70195*a*b^4
*e*m^2*x^10+14039*b^5*d*m^2*x^10+10395*b^5*e*x^12+420*a^3*b^2*e*m^5*x^6+420
*a^2*b^3*d*m^5*x^6+45280*a^2*b^3*e*m^3*x^8+22640*a*b^4*d*m^3*x^8+114510*a*b
^4*e*m*x^10+22902*b^5*d*m*x^10+5*a^4*b*e*m^6*x^4+10*a^3*b^2*d*m^6*x^4+6790*
a^3*b^2*e*m^4*x^6+6790*a^2*b^3*d*m^4*x^6+166270*a^2*b^3*e*m^2*x^8+83135*a*b
^4*d*m^2*x^8+61425*a*b^4*e*x^10+12285*b^5*d*x^10+220*a^4*b*e*m^5*x^4+440*a^
3*b^2*d*m^5*x^4+52920*a^3*b^2*e*m^3*x^6+52920*a^2*b^3*d*m^3*x^6+276880*a^2*
b^3*e*m*x^8+138440*a*b^4*d*m*x^8+a^5*e*m^6*x^2+5*a^4*b*d*m^6*x^2+3765*a^4*b
*e*m^4*x^4+7530*a^3*b^2*d*m^4*x^4+203350*a^3*b^2*e*m^2*x^6+203350*a^2*b^3*d
*m^2*x^6+150150*a^2*b^3*e*x^8+75075*a*b^4*d*x^8+46*a^5*e*m^5*x^2+230*a^4*b*
d*m^5*x^2+31400*a^4*b*e*m^3*x^4+62800*a^3*b^2*d*m^3*x^4+349860*a^3*b^2*e*m*
x^6+349860*a^2*b^3*d*m*x^6+a^5*d*m^6+835*a^5*e*m^4*x^2+4175*a^4*b*d*m^4*x^2
+129895*a^4*b*e*m^2*x^4+259790*a^3*b^2*d*m^2*x^4+193050*a^3*b^2*e*x^6+19305
0*a^2*b^3*d*x^6+48*a^5*d*m^5+7540*a^5*e*m^3*x^2+37700*a^4*b*d*m^3*x^2+23718
0*a^4*b*e*m*x^4+474360*a^3*b^2*d*m*x^4+925*a^5*d*m^4+34759*a^5*e*m^2*x^2+17
3795*a^4*b*d*m^2*x^2+135135*a^4*b*e*x^4+270270*a^3*b^2*d*x^4+9120*a^5*d*m^3
+73054*a^5*e*m*x^2+365270*a^4*b*d*m*x^2+48259*a^5*d*m^2+45045*a^5*e*x^2+225
225*a^4*b*d*x^2+129072*a^5*d*m+135135*a^5*d)*(f*x)^m*((b*x^2+a)^2)^(5/2)/(m
+13)/(m+11)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)/(b*x^2+a)^5
```

maxima [A] time = 0.82, size = 491, normalized size = 1.23

$$\frac{\left(\left(m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945\right)b^5 f^m x^{11} + 5\left(m^5 + 27\,m^4 + 262\,m^3 + 1122\,m^2 + 2041\,m + \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="max
ima")
```

```
[Out] ((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*f^m*x^11 + 5*(m^5 +
27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*f^m*x^9 + 10*(m^5 + 29*m
^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*b^3*f^m*x^7 + 10*(m^5 + 31*m^4
 + 350*m^3 + 1730*m^2 + 3489*m + 2079)*a^3*b^2*f^m*x^5 + 5*(m^5 + 33*m^4 +
406*m^3 + 2262*m^2 + 5353*m + 3465)*a^4*b*f^m*x^3 + (m^5 + 35*m^4 + 470*m^3
 + 3010*m^2 + 9129*m + 10395)*a^5*f^m*x)*d*x^m/(m^6 + 36*m^5 + 505*m^4 + 34
80*m^3 + 12139*m^2 + 19524*m + 10395) + ((m^5 + 35*m^4 + 470*m^3 + 3010*m^2
 + 9129*m + 10395)*b^5*f^m*x^13 + 5*(m^5 + 37*m^4 + 518*m^3 + 3422*m^2 + 10
617*m + 12285)*a*b^4*f^m*x^11 + 10*(m^5 + 39*m^4 + 574*m^3 + 3954*m^2 + 126
73*m + 15015)*a^2*b^3*f^m*x^9 + 10*(m^5 + 41*m^4 + 638*m^3 + 4654*m^2 + 156
81*m + 19305)*a^3*b^2*f^m*x^7 + 5*(m^5 + 43*m^4 + 710*m^3 + 5570*m^2 + 2040
9*m + 27027)*a^4*b*f^m*x^5 + (m^5 + 45*m^4 + 790*m^3 + 6750*m^2 + 28009*m +
 45045)*a^5*f^m*x^3)*e*x^m/(m^6 + 48*m^5 + 925*m^4 + 9120*m^3 + 48259*m^2 +
 129072*m + 135135)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m\left(d+ex^2\right)\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(5/2), x)
```

3.88 $\int (fx)^m \left(d+ex^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=276

$$\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+7}(3ae+bd)}{f^7(m+7)\left(a+bx^2\right)}+\frac{3ab\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}(ae+bd)}{f^5(m+5)\left(a+bx^2\right)}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\,(}{f^3(m+3)\left(a\right.}$$

```
[Out] a^3*d*(f*x)^(1+m)*((b*x^2+a)^2)^(1/2)/f/(1+m)/(b*x^2+a)+a^2*(a*e+3*b*d)*(f*
x)^(3+m)*((b*x^2+a)^2)^(1/2)/f^3/(3+m)/(b*x^2+a)+3*a*b*(a*e+b*d)*(f*x)^(5+m
)*((b*x^2+a)^2)^(1/2)/f^5/(5+m)/(b*x^2+a)+b^2*(3*a*e+b*d)*(f*x)^(7+m)*((b*x
^2+a)^2)^(1/2)/f^7/(7+m)/(b*x^2+a)+b^3*e*(f*x)^(9+m)*((b*x^2+a)^2)^(1/2)/f^
9/(9+m)/(b*x^2+a)
```

Rubi [A] time = 0.15, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1250, 448}

$$\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+3}(ae+3bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{3ab\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}(ae+bd)}{f^5(m+5)\left(a+bx^2\right)}+\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\,(}{f^7(m+7)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2
)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3
*(3 + m)*(a + b*x^2)) + (3*a*b*(b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (b^2*(b*d + 3*a*e)*(f*x)^(7 + m)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (b^3*e*(f*x)^
(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 + m)*(a + b*x^2))
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int (fx)^m\left(d+ex^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (fx)^m\left(ab+b^2x^2\right)^3\left(d+ex^2\right)dx}{b^2\left(ab+b^2x^2\right)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^3b^3d(fx)^m+\frac{a^2b^3(3bd+ae)(fx)^{2+m}}{f^2}+\frac{3ab^4(}{}\right.}{b^2\left(ab+b^2x^2\right)}\\ &= \frac{a^3d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)}+\frac{a^2(3bd+ae)(fx)^{3+m}\sqrt{a^2+}}{f^3(3+m)\left(a+b\right.}\end{aligned}$$

Mathematica [A] time = 0.11, size = 112, normalized size = 0.41

$$\frac{x\left(\left(a+bx^2\right)^2\right)^{3/2}(fx)^m\left(\frac{a^3d}{m+1}+\frac{a^2x^2(ae+3bd)}{m+3}+\frac{b^2x^6(3ae+bd)}{m+7}+\frac{3abx^4(ae+bd)}{m+5}+\frac{b^3ex^8}{m+9}\right)}{\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*((a + b*x^2)^2)^(3/2)*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2)
/(3 + m) + (3*a*b*(b*d + a*e)*x^4)/(5 + m) + (b^2*(b*d + 3*a*e)*x^6)/(7 + m
) + (b^3*e*x^8)/(9 + m)))/(a + b*x^2)^3
```

fricas [A] time = 1.03, size = 381, normalized size = 1.38

$$\frac{\left(\left(b^3em^4+16\,b^3em^3+86\,b^3em^2+176\,b^3em+105\,b^3e\right)x^9+\left(\left(b^3d+3\,ab^2e\right)m^4+135\,b^3d+405\,ab^2e+18\left(b^3d+3\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fri
cas")
```

```
[Out] ((b^3*e*m^4 + 16*b^3*e*m^3 + 86*b^3*e*m^2 + 176*b^3*e*m + 105*b^3*e)*x^9 +
((b^3*d + 3*a*b^2*e)*m^4 + 135*b^3*d + 405*a*b^2*e + 18*(b^3*d + 3*a*b^2*e)
*m^3 + 104*(b^3*d + 3*a*b^2*e)*m^2 + 222*(b^3*d + 3*a*b^2*e)*m)*x^7 + 3*((a
*b^2*d + a^2*b*e)*m^4 + 189*a*b^2*d + 189*a^2*b*e + 20*(a*b^2*d + a^2*b*e)*
m^3 + 130*(a*b^2*d + a^2*b*e)*m^2 + 300*(a*b^2*d + a^2*b*e)*m)*x^5 + ((3*a^
2*b*d + a^3*e)*m^4 + 945*a^2*b*d + 315*a^3*e + 22*(3*a^2*b*d + a^3*e)*m^3 +
 164*(3*a^2*b*d + a^3*e)*m^2 + 458*(3*a^2*b*d + a^3*e)*m)*x^3 + (a^3*d*m^4
+ 24*a^3*d*m^3 + 206*a^3*d*m^2 + 744*a^3*d*m + 945*a^3*d)*x)*(f*x)^m/(m^5 +
 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

giac [B] time = 0.52, size = 1013, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b^3*m^4*x^9*e*sgn(b*x^2 + a) + 16*(f*x)^m*b^3*m^3*x^9*e*sgn(b*x^2
+ a) + (f*x)^m*b^3*d*m^4*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*m^4*x^7*e*sgn
(b*x^2 + a) + 86*(f*x)^m*b^3*m^2*x^9*e*sgn(b*x^2 + a) + 18*(f*x)^m*b^3*d*m^
3*x^7*sgn(b*x^2 + a) + 54*(f*x)^m*a*b^2*m^3*x^7*e*sgn(b*x^2 + a) + 176*(f*x
)^m*b^3*m*x^9*e*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*d*m^4*x^5*sgn(b*x^2 + a) +
 104*(f*x)^m*b^3*d*m^2*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a^2*b*m^4*x^5*e*sgn(b
*x^2 + a) + 312*(f*x)^m*a*b^2*m^2*x^7*e*sgn(b*x^2 + a) + 105*(f*x)^m*b^3*x^
9*e*sgn(b*x^2 + a) + 60*(f*x)^m*a*b^2*d*m^3*x^5*sgn(b*x^2 + a) + 222*(f*x)^
m*b^3*d*m*x^7*sgn(b*x^2 + a) + 60*(f*x)^m*a^2*b*m^3*x^5*e*sgn(b*x^2 + a) +
666*(f*x)^m*a*b^2*m*x^7*e*sgn(b*x^2 + a) + 3*(f*x)^m*a^2*b*d*m^4*x^3*sgn(b*
x^2 + a) + 390*(f*x)^m*a*b^2*d*m^2*x^5*sgn(b*x^2 + a) + 135*(f*x)^m*b^3*d*x
^7*sgn(b*x^2 + a) + (f*x)^m*a^3*m^4*x^3*e*sgn(b*x^2 + a) + 390*(f*x)^m*a^2*
b*m^2*x^5*e*sgn(b*x^2 + a) + 405*(f*x)^m*a*b^2*x^7*e*sgn(b*x^2 + a) + 66*(f
*x)^m*a^2*b*d*m^3*x^3*sgn(b*x^2 + a) + 900*(f*x)^m*a*b^2*d*m*x^5*sgn(b*x^2
+ a) + 22*(f*x)^m*a^3*m^3*x^3*e*sgn(b*x^2 + a) + 900*(f*x)^m*a^2*b*m*x^5*e*
sgn(b*x^2 + a) + (f*x)^m*a^3*d*m^4*x*sgn(b*x^2 + a) + 492*(f*x)^m*a^2*b*d*m
^2*x^3*sgn(b*x^2 + a) + 567*(f*x)^m*a*b^2*d*x^5*sgn(b*x^2 + a) + 164*(f*x)^
m*a^3*m^2*x^3*e*sgn(b*x^2 + a) + 567*(f*x)^m*a^2*b*x^5*e*sgn(b*x^2 + a) + 2
```

```
4*(f*x)^m*a^3*d*m^3*x*sgn(b*x^2 + a) + 1374*(f*x)^m*a^2*b*d*m*x^3*sgn(b*x^2
 + a) + 458*(f*x)^m*a^3*m*x^3*e*sgn(b*x^2 + a) + 206*(f*x)^m*a^3*d*m^2*x*sg
n(b*x^2 + a) + 945*(f*x)^m*a^2*b*d*x^3*sgn(b*x^2 + a) + 315*(f*x)^m*a^3*x^3
*e*sgn(b*x^2 + a) + 744*(f*x)^m*a^3*d*m*x*sgn(b*x^2 + a) + 945*(f*x)^m*a^3*
d*x*sgn(b*x^2 + a))/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

maple [B] time = 0.01, size = 495, normalized size = 1.79

$$\frac{\left(b^3 e\, m^4 x^8 + 16 b^3 e\, m^3 x^8 + 3 a\, b^2 e\, m^4 x^6 + b^3 d\, m^4 x^6 + 86 b^3 e\, m^2 x^8 + 54 a\, b^2 e\, m^3 x^6 + 18 b^3 d\, m^3 x^6 + 176 b^3 e m\, x^8 + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] x*(b^3*e*m^4*x^8+16*b^3*e*m^3*x^8+3*a*b^2*e*m^4*x^6+b^3*d*m^4*x^6+86*b^3*e*
m^2*x^8+54*a*b^2*e*m^3*x^6+18*b^3*d*m^3*x^6+176*b^3*e*m*x^8+3*a^2*b*e*m^4*x
^4+3*a*b^2*d*m^4*x^4+312*a*b^2*e*m^2*x^6+104*b^3*d*m^2*x^6+105*b^3*e*x^8+60
*a^2*b*e*m^3*x^4+60*a*b^2*d*m^3*x^4+666*a*b^2*e*m*x^6+222*b^3*d*m*x^6+a^3*e
*m^4*x^2+3*a^2*b*d*m^4*x^2+390*a^2*b*e*m^2*x^4+390*a*b^2*d*m^2*x^4+405*a*b^
2*e*x^6+135*b^3*d*x^6+22*a^3*e*m^3*x^2+66*a^2*b*d*m^3*x^2+900*a^2*b*e*m*x^4
+900*a*b^2*d*m*x^4+a^3*d*m^4+164*a^3*e*m^2*x^2+492*a^2*b*d*m^2*x^2+567*a^2*
b*e*x^4+567*a*b^2*d*x^4+24*a^3*d*m^3+458*a^3*e*m*x^2+1374*a^2*b*d*m*x^2+206
*a^3*d*m^2+315*a^3*e*x^2+945*a^2*b*d*x^2+744*a^3*d*m+945*a^3*d)*(f*x)^m*((b
*x^2+a)^2)^(3/2)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)/(b*x^2+a)^3
```

maxima [A] time = 0.96, size = 243, normalized size = 0.88

$$\frac{\left(\left(m^3 + 9\,m^2 + 23\,m + 15\right) b^3 f^m x^7 + 3\left(m^3 + 11\,m^2 + 31\,m + 21\right) a b^2 f^m x^5 + 3\left(m^3 + 13\,m^2 + 47\,m + 35\right) a^2 b f^m \cdots\right.}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="max
ima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*f^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a*b^2
*f^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*f^m*x^3 + (m^3 + 15*m^2 + 71*
m + 105)*a^3*f^m*x)*d*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) + ((m^3 + 1
5*m^2 + 71*m + 105)*b^3*f^m*x^9 + 3*(m^3 + 17*m^2 + 87*m + 135)*a*b^2*f^m*x
^7 + 3*(m^3 + 19*m^2 + 111*m + 189)*a^2*b*f^m*x^5 + (m^3 + 21*m^2 + 143*m +
 315)*a^3*f^m*x^3)*e*x^m/(m^4 + 24*m^3 + 206*m^2 + 744*m + 945)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(d + ex^{2}\right) \left(\left(a + bx^{2}\right)^{2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(3/2), x)
```

3.89 $\int (fx)^m \left(d+ex^2\right) \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+3}(ae+bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{ad\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}+\frac{be\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}}{f^5(m+5)\left(a+bx^2\right)}$$

```
[Out] a*d*(f*x)^(1+m)*((b*x^2+a)^2)^(1/2)/f/(1+m)/(b*x^2+a)+(a*e+b*d)*(f*x)^(3+m)
*((b*x^2+a)^2)^(1/2)/f^3/(3+m)/(b*x^2+a)+b*e*(f*x)^(5+m)*((b*x^2+a)^2)^(1/2
)/f^5/(5+m)/(b*x^2+a)
```

Rubi [A] time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1250, 448}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+3}(ae+bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{ad\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}+\frac{be\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}}{f^5(m+5)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2))
 + ((b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3*(3 + m)
*(a + b*x^2)) + (b*e*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^5*(5
 + m)*(a + b*x^2))
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int (fx)^m \left(d+ex^2\right) \sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (fx)^m \left(ab+b^2x^2\right)\left(d+ex^2\right) dx}{ab+b^2x^2}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(abd(fx)^m+\frac{b(bd+ae)(fx)^{2+m}}{f^2}+\frac{b^2e(fx)^{4+m}}{f^4}\right) dx}{ab+b^2x^2}\\
&= \frac{ad(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)}+\frac{(bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b}}{f^3(3+m)\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 86, normalized size = 0.56

$$\frac{x\sqrt{\left(a+bx^2\right)^2}\,(fx)^m \left(a(m+5)\left(d(m+3)+e(m+1)x^2\right)+b(m+1)x^2\left(d(m+5)+e(m+3)x^2\right)\right)}{(m+1)(m+3)(m+5)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[(a + b*x^2)^2]*(a*(5 + m)*(d*(3 + m) + e*(1 + m)*x^2) + b*(
1 + m)*x^2*(d*(5 + m) + e*(3 + m)*x^2)))/((1 + m)*(3 + m)*(5 + m)*(a + b*x^
2))
```

fricas [A] time = 0.86, size = 94, normalized size = 0.61

$$\frac{\left(\left(bem^{2}+4\,bem+3\,be\right)x^{5}+\left((bd+ae)m^{2}+5\,bd+5\,ae+6\,(bd+ae)m\right)x^{3}+\left(adm^{2}+8\,adm+15\,ad\right)x\right)\left(fx\right)^{m}}{m^{3}+9\,m^{2}+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] ((b*e*m^2 + 4*b*e*m + 3*b*e)*x^5 + ((b*d + a*e)*m^2 + 5*b*d + 5*a*e + 6*(b*
d + a*e)*m)*x^3 + (a*d*m^2 + 8*a*d*m + 15*a*d)*x)*(f*x)^m/(m^3 + 9*m^2 + 23
*m + 15)
```

giac [B] time = 0.32, size = 269, normalized size = 1.76

$$\left(fx\right)^{m}bm^{2}x^{5}e\mathrm{sgn}\left(bx^{2}+a\right)+4\left(fx\right)^{m}bmx^{5}e\mathrm{sgn}\left(bx^{2}+a\right)+\left(fx\right)^{m}bdm^{2}x^{3}\mathrm{sgn}\left(bx^{2}+a\right)+\left(fx\right)^{m}am^{2}x^{3}e\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b*m^2*x^5*e*sgn(b*x^2 + a) + 4*(f*x)^m*b*m*x^5*e*sgn(b*x^2 + a) +
(f*x)^m*b*d*m^2*x^3*sgn(b*x^2 + a) + (f*x)^m*a*m^2*x^3*e*sgn(b*x^2 + a) + 3
*(f*x)^m*b*x^5*e*sgn(b*x^2 + a) + 6*(f*x)^m*b*d*m*x^3*sgn(b*x^2 + a) + 6*(f
*x)^m*a*m*x^3*e*sgn(b*x^2 + a) + (f*x)^m*a*d*m^2*x*sgn(b*x^2 + a) + 5*(f*x)
^m*b*d*x^3*sgn(b*x^2 + a) + 5*(f*x)^m*a*x^3*e*sgn(b*x^2 + a) + 8*(f*x)^m*a*
d*m*x*sgn(b*x^2 + a) + 15*(f*x)^m*a*d*x*sgn(b*x^2 + a))/(m^3 + 9*m^2 + 23*m
 + 15)
```

maple [A] time = 0.01, size = 131, normalized size = 0.86

$$\frac{\left(be\,m^{2}x^{4}+4bem\,x^{4}+ae\,m^{2}x^{2}+bd\,m^{2}x^{2}+3be\,x^{4}+6aem\,x^{2}+6bdm\,x^{2}+ad\,m^{2}+5ae\,x^{2}+5bd\,x^{2}+8adm+1}{(m+5)\,(m+3)\,(m+1)\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

```
[Out] x*(b*e*m^2*x^4+4*b*e*m*x^4+a*e*m^2*x^2+b*d*m^2*x^2+3*b*e*x^4+6*a*e*m*x^2+6*
b*d*m*x^2+a*d*m^2+5*a*e*x^2+5*b*d*x^2+8*a*d*m+15*a*d)*(f*x)^m*((b*x^2+a)^2)
^(1/2)/(m+5)/(m+3)/(m+1)/(b*x^2+a)
```

maxima [A] time = 0.81, size = 75, normalized size = 0.49

$$\frac{\left(bf^{m}(m+1)x^{3}+af^{m}(m+3)x\right)dx^{m}}{m^{2}+4\,m+3}+\frac{\left(bf^{m}(m+3)x^{5}+af^{m}(m+5)x^{3}\right)ex^{m}}{m^{2}+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] (b*f^m*(m + 1)*x^3 + a*f^m*(m + 3)*x)*d*x^m/(m^2 + 4*m + 3) + (b*f^m*(m + 3
)*x^5 + a*f^m*(m + 5)*x^3)*e*x^m/(m^2 + 8*m + 15)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (f\,x)^m\,\left(e\,x^2+d\right)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right) \sqrt{\left(a+bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*sqrt((a + b*x**2)**2), x)
```

3.90 $\int \frac{(fx)^m \left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=134

$$\frac{\left(a+bx^2\right)(fx)^{m+1}(bd-ae)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\left(a+bx^2\right)(fx)^{m+1}}{bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] e*(f*x)^(1+m)*(b*x^2+a)/b/f/(1+m)/((b*x^2+a)^2)^(1/2)+(-a*e+b*d)*(f*x)^(1+m
)*(b*x^2+a)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/b/f/(1+m)/((b*
x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1250, 459, 364}

$$\frac{\left(a+bx^2\right)(fx)^{m+1}(bd-ae)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\left(a+bx^2\right)(fx)^{m+1}}{bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*(f*x)^(1 + m)*(a + b*x^2))/(b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + ((b*d - a*e)*(f*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((b*x^2)/a)])/(a*b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(fx)^m \left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{(fx)^m\left(d+ex^2\right)}{ab+b^2x^2}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{e(fx)^{1+m}\left(a+bx^2\right)}{bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\left(-b^2d(1+m)+abe(1+m)\right)\left(ab+b^2x^2\right)\right)\int \frac{(fx)^{\prime}}{ab+b^2}}{b^2(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{e(fx)^{1+m}\left(a+bx^2\right)}{bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)(fx)^{1+m}\left(a+bx^2\right){}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right.}{abf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.07, size = 78, normalized size = 0.58

$$-\frac{x\left(a+bx^2\right)(fx)^m\left((ae-bd)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)-ae\right)}{ab(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((x*(f*x)^m*(a + b*x^2)*(-(a*e) + (-(b*d) + a*e)*Hypergeometric2F1[1, (1 +
 m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)*Sqrt[(a + b*x^2)^2]))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex^2+d\right)\left(fx\right)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)\left(fx\right)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)\left(fx\right)^m}{\sqrt{b^2x^4+2ab\,x^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)\left(fx\right)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f\,x\right)^m\,\left(e\,x^2+d\right)}{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
```

```
[Out] int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m\left(d+ex^2\right)}{\sqrt{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/sqrt((a + b*x**2)**2), x)
```

3.91 $\int \frac{(fx)^m\left(d+ex^2\right)}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=154

$$\frac{(fx)^{m+1}(bd-ae)}{4abf\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(fx)^{m+1}(ae(m+1)+bd(3-m))\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{4a^3bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/4*(-a*e+b*d)*(f*x)^(1+m)/a/b/f/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1/4*(b*d*(3-
m)+a*e*(1+m))*(f*x)^(1+m)*(b*x^2+a)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-b
*x^2/a)/a^3/b/f/(1+m)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1250, 457, 364}

$$\frac{\left(a+bx^2\right)(fx)^{m+1}(ae(m+1)+bd(3-m))\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{4a^3bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(fx)^{m+1}(bd-ae)}{4abf\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((b*d - a*e)*(f*x)^(1 + m))/(4*a*b*f*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) + ((b*d*(3 - m) + a*e*(1 + m))*(f*x)^(1 + m)*(a + b*x^2)*Hypergeomet
ric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(4*a^3*b*f*(1 + m)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(fx)^m \left(d+ex^2\right)}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(fx)^m\left(d+ex^2\right)}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-ae)(fx)^{1+m}}{4abf\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left((bd(3-m)+ae(1+m))\left(ab+b^2x^2\right)\right)}{4a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-ae)(fx)^{1+m}}{4abf\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd(3-m)+ae(1+m))(fx)^{1+m}\left(a+b\right.}{4a^3bf(1+m)\sqrt{a^2+2a}}$$

Mathematica [A] time = 0.07, size = 101, normalized size = 0.66

$$\frac{x\left(a+bx^2\right)(fx)^m\left((bd-ae)\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+ae\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^3b(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*(a + b*x^2)*(a*e*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b
*x^2)/a)] + (b*d - a*e)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2
)/a)]))/(a^3*b*(1 + m)*Sqrt[(a + b*x^2)^2])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b^2x^4+2\,abx^2+a^2}\left(ex^2+d\right)\left(fx\right)^m}{b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(e*x^2 + d)*(f*x)^m/(b^4*x^8 + 4*a
*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)\left(fx\right)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)\left(fx\right)^m}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)(fx)^m}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f\,x)^m\,(e\,x^2+d)}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(d+ex^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

3.92 $\int x\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=34

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b(p+1)}$$

```
[Out] 1/4*(b^2*x^4+2*a*b*x^2+a^2)^(1+p)/b/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1247, 629}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2 + 2*a*b*x^2 + b^2*x^4)^(1 + p)/(4*b*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx &= \frac{1}{2}\text{Subst}\left(\int (a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)\\ &= \frac{\left(a^2+2abx^2+b^2x^4\right)^{1+p}}{4b(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.74

$$\frac{\left(\left(a+bx^2\right)^2\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)^2)^(1 + p)/(4*b*(1 + p))
```

fricas [A] time = 0.63, size = 47, normalized size = 1.38

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\left(bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(b*p + b)
```

giac [A] time = 0.29, size = 32, normalized size = 0.94

$$\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{p+1}}{4\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)^(p + 1)/(b*(p + 1))
```

maple [A] time = 0.00, size = 40, normalized size = 1.18

$$\frac{\left(b\,x^2 + a\right)^2\left(b^2x^4 + 2ab\,x^2 + a^2\right)^p}{4\left(p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/4*(b*x^2+a)^2/b/(1+p)*(b^2*x^4+2*a*b*x^2+a^2)^p
```

maxima [B] time = 0.72, size = 86, normalized size = 2.53

$$\frac{\left(bx^2 + a\right)\left(bx^2 + a\right)^{2\,p} a}{2\,b\left(2\,p+1\right)} + \frac{\left(b^2\left(2\,p+1\right)x^4 + 2\,abpx^2 - a^2\right)\left(bx^2 + a\right)^{2\,p}}{4\left(2\,p^2 + 3\,p + 1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)*a/(b*(2*p + 1)) + 1/4*(b^2*(2*p + 1)*x^4
+ 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*p + 1)*b)
```

mupad [B] time = 0.14, size = 59, normalized size = 1.74

$$\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^p\left(\frac{a^2}{4\,b\,\left(p+1\right)} + \frac{a\,x^2}{2\,\left(p+1\right)} + \frac{b\,x^4}{4\,\left(p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*(a^2/(4*b*(p + 1)) + (a*x^2)/(2*(p + 1)) + (b
*x^4)/(4*(p + 1)))
```

sympy [A] time = 9.60, size = 165, normalized size = 4.85

$$\begin{cases} \frac{x^2}{2a} & \text{for}\: b = 0 \wedge p = -1 \\ \frac{ax^2\left(a^2\right)^p}{2} & \text{for}\: b = 0 \\ \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + x\right)}{2b} + \frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + x\right)}{2b} & \text{for}\: p = -1 \\ \frac{a^2\left(a^2+2abx^2+b^2x^4\right)^p}{4bp+4b} + \frac{2abx^2\left(a^2+2abx^2+b^2x^4\right)^p}{4bp+4b} + \frac{b^2x^4\left(a^2+2abx^2+b^2x^4\right)^p}{4bp+4b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a*x**2*(a**2)**p/2, Eq(b, 0)
), (log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b) + log(I*sqrt(a)*sqrt(1/b) + x)/(2*b
), Eq(p, -1)), (a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + 2*a
*b*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + b**2*x**4*(a**2
+ 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b), True))
```

3.93 $\int x^3 \left(a+bx^2\right) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=86

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}$$

```
[Out] -1/4*a*(b*x^2+a)^2*(b^2*x^4+2*a*b*x^2+a^2)^p/b^2/(1+p)+1/2*(b*x^2+a)^3*(b^2
*x^4+2*a*b*x^2+a^2)^p/b^2/(3+2*p)
```

Rubi [A] time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1249, 770, 21, 43}

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -(a*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p)) + ((a + b*
x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(3 + 2*p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 770

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPa
rt[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(f + g*x)*(b/2 + c*x)^(
2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 1249

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b
^2 - 4*a*c, 0] &&  !IntegerQ[p] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int x^3\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{1}{2}\text{Subst}\left(\int x(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)$$
$$=\frac{1}{2}\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x(a+bx)\left(ab+\right.\right.$$
$$=\frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x\left(ab+b^2x\right)^{1+2p}\right.}{2b}$$
$$=\frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^{1+2p}}{b}+\right.\right.}{2b}$$
$$=-\frac{a\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(1+p)}+\frac{\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x\right.}{2b^2(3+2p)}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.52

$$\frac{\left(\left(a+bx^2\right)^2\right)^{p+1}\left(2b(p+1)x^2-a\right)}{4b^2(p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (((a + b*x^2)^2)^(1 + p)*(-a + 2*b*(1 + p)*x^2))/(4*b^2*(1 + p)*(3 + 2*p))
```

fricas [A] time = 0.52, size = 92, normalized size = 1.07

$$\frac{\left(2\left(b^3p+b^3\right)x^6+2\,a^2bpx^2+\left(4\,ab^2p+3\,ab^2\right)x^4-a^3\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\left(2\,b^2p^2+5\,b^2p+3\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^3*p + b^3)*x^6 + 2*a^2*b*p*x^2 + (4*a*b^2*p + 3*a*b^2)*x^4 - a^3)
*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b^2*p^2 + 5*b^2*p + 3*b^2)
```

giac [B] time = 0.43, size = 196, normalized size = 2.28

$$\frac{2\left(b^2x^4+2\,abx^2+a^2\right)^pb^3px^6+2\left(b^2x^4+2\,abx^2+a^2\right)^pb^3x^6+4\left(b^2x^4+2\,abx^2+a^2\right)^pab^2px^4+3\left(b^2x^4+2\,abx\right.}{4\left(2\,b^2p^2+5\,b^2p+3\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a
^2)^p*b^3*x^6 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p*x^4 + 3*(b^2*x^4 +
2*a*b*x^2 + a^2)^p*a*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b*p*x^2
- (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3)/(2*b^2*p^2 + 5*b^2*p + 3*b^2)
```

maple [A] time = 0.01, size = 62, normalized size = 0.72

$$-\frac{\left(-2x^2pb-2b\,x^2+a\right)\left(b\,x^2+a\right)^2\left(b^2x^4+2ab\,x^2+a^2\right)^p}{4\left(2p^2+5p+3\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-2*b*p*x^2-2*b*x^2+a)*(b*x^2+a)^2/b^2/(2*p^
2+5*p+3)
```

maxima [A] time = 0.74, size = 135, normalized size = 1.57

$$\frac{\left(b^2\left(2\,p+1\right)x^4+2\,abpx^2-a^2\right)\left(bx^2+a\right)^{2\,p}a}{4\left(2\,p^2+3\,p+1\right)b^2}+\frac{\left(\left(2\,p^2+3\,p+1\right)b^3x^6+\left(2\,p^2+p\right)ab^2x^4-2\,a^2bpx^2+a^3\right)\left(bx^2+a\right)^{2\,p}}{2\left(4\,p^3+12\,p^2+11\,p+3\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)*a/((2*p^2 + 3
*p + 1)*b^2) + 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a
^2*b*p*x^2 + a^3)*(b*x^2 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^2)
```

mupad [B] time = 0.17, size = 108, normalized size = 1.26

$$\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,\left(\frac{b\,x^6\,\left(p+1\right)}{2\,\left(2\,p^2+5\,p+3\right)}-\frac{a^3}{4\,b^2\,\left(2\,p^2+5\,p+3\right)}+\frac{a\,x^4\,\left(4\,p+3\right)}{4\,\left(2\,p^2+5\,p+3\right)}+\frac{a^2\,p\,x^2}{2\,b\,\left(2\,p^2+5\,p+3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((b*x^6*(p + 1))/(2*(5*p + 2*p^2 + 3)) - a^3/
(4*b^2*(5*p + 2*p^2 + 3)) + (a*x^4*(4*p + 3))/(4*(5*p + 2*p^2 + 3)) + (a^2*
p*x^2)/(2*b*(5*p + 2*p^2 + 3)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{ax^4\left(a^2\right)^p}{4} \\ \int \frac{x^3\left(a+bx^2\right)}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx \\ -\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2}+\frac{x^2}{2b} \\ -\frac{a^3\left(a^2+2abx^2+b^2x^4\right)^p}{8b^2p^2+20b^2p+12b^2}+\frac{2a^2bpx^2\left(a^2+2abx^2+b^2x^4\right)^p}{8b^2p^2+20b^2p+12b^2}+\frac{4ab^2px^4\left(a^2+2abx^2+b^2x^4\right)^p}{8b^2p^2+20b^2p+12b^2}+\frac{3ab^2x^4\left(a^2+2abx^2+b^2x^4\right)^p}{8b^2p^2+20b^2p+12b^2}+\frac{2b^3px^6\left(a^2+2abx^2+b^2x^4\right)^p}{8b^2p^2+20b^2p+12b^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((a*x**4*(a**2)**p/4, Eq(b, 0)), (Integral(x**3*(a + b*x**2)/((a +
 b*x**2)**2)**(3/2), x), Eq(p, -3/2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2
*b**2) - a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)),
(-a**3*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**
2) + 2*a**2*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b
**2*p + 12*b**2) + 4*a*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b*
*2*p**2 + 20*b**2*p + 12*b**2) + 3*a*b**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x*
*4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*b**3*p*x**6*(a**2 + 2*a*b*x*
*2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*b**3*x**6*(a**2
+ 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2), True))
```

3.94 $\int x^5 \left(a+bx^2\right) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=128

$$\frac{\left(a+bx^2\right)^4 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+2)}-\frac{a\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{b^3(2p+3)}+\frac{a^2\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+1)}$$

```
[Out] 1/4*a^2*(b*x^2+a)^2*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(1+p)-a*(b*x^2+a)^3*(b^2*
x^4+2*a*b*x^2+a^2)^p/b^3/(3+2*p)+1/4*(b*x^2+a)^4*(b^2*x^4+2*a*b*x^2+a^2)^p/
b^3/(2+p)
```

Rubi [A] time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = $\{1249, 770, 21, 43\}$

$$\frac{\left(a+bx^2\right)^4 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+2)}-\frac{a\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{b^3(2p+3)}+\frac{a^2\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(1 + p)) - (a*(a +
 b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^3*(3 + 2*p)) + ((a + b*x^2)^4*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(2 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 770

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPa
rt[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(f + g*x)*(b/2 + c*x)^(
2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 1249

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b
^2 - 4*a*c, 0] &&  !IntegerQ[p] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int x^5 \left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \frac{1}{2}\,\text{Subst}\left(\int x^2(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)$$
$$= \frac{1}{2}\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^2(a+bx)\left(ab+b\right.\right.$$
$$= \frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^{1+2p}dx\right.}{2b}$$
$$= \frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^{1+2p}}{b^2}-\frac{2a(ab}{}\right.\right.}{2b}$$
$$= \frac{a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(1+p)} - \frac{a\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)}{b^3(3+2p)}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.53

$$\frac{\left(\left(a+bx^2\right)^2\right)^{p+1}\left(a^2-2ab(p+1)x^2+b^2\left(2p^2+5p+3\right)x^4\right)}{4b^3(p+1)(p+2)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (((a + b*x^2)^2)^(1 + p)*(a^2 - 2*a*b*(1 + p)*x^2 + b^2*(3 + 5*p + 2*p^2)*x
^4))/(4*b^3*(1 + p)*(2 + p)*(3 + 2*p))
```

fricas [A] time = 0.69, size = 140, normalized size = 1.09

$$\frac{\left(\left(2\,b^4p^2+5\,b^4p+3\,b^4\right)x^8-2\,a^3bpx^2+4\left(ab^3p^2+2\,ab^3p+ab^3\right)x^6+\left(2\,a^2b^2p^2+a^2b^2p\right)x^4+a^4\right)\left(b^2x^4+2\,abx^2+\right.}{4\left(2\,b^3p^3+9\,b^3p^2+13\,b^3p+6\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^4*p^2 + 5*b^4*p + 3*b^4)*x^8 - 2*a^3*b*p*x^2 + 4*(a*b^3*p^2 + 2*a
*b^3*p + a*b^3)*x^6 + (2*a^2*b^2*p^2 + a^2*b^2*p)*x^4 + a^4)*(b^2*x^4 + 2*a
*b*x^2 + a^2)^p/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

giac [B] time = 0.38, size = 331, normalized size = 2.59

$$\frac{2\left(b^2x^4+2\,abx^2+a^2\right)^pb^4p^2x^8+5\left(b^2x^4+2\,abx^2+a^2\right)^pb^4px^8+4\left(b^2x^4+2\,abx^2+a^2\right)^pab^3p^2x^6+3\left(b^2x^4+2\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p^2*x^8 + 5*(b^2*x^4 + 2*a*b*x^2 +
 a^2)^p*b^4*p*x^8 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p^2*x^6 + 3*(b^2*
x^4 + 2*a*b*x^2 + a^2)^p*b^4*x^8 + 8*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p*
x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p^2*x^4 + 4*(b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a*b^3*x^6 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p*x^4 - 2*(b
^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3*b*p*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^4
)/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

maple [A] time = 0.01, size = 99, normalized size = 0.77

$$\frac{\left(b\,x^{2}+a\right)^{2}\left(2b^{2}p^{2}x^{4}+5b^{2}p\,x^{4}+3b^{2}x^{4}-2abp\,x^{2}-2ab\,x^{2}+a^{2}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{p}}{4\left(2p^{3}+9p^{2}+13p+6\right)b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/4*(b*x^2+a)^2*(2*b^2*p^2*x^4+5*b^2*p*x^4+3*b^2*x^4-2*a*b*p*x^2-2*a*b*x^2+
a^2)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(2*p^3+9*p^2+13*p+6)
```

maxima [A] time = 0.70, size = 196, normalized size = 1.53

$$\frac{\left(\left(2\,p^{2}+3\,p+1\right)b^{3}x^{6}+\left(2\,p^{2}+p\right)ab^{2}x^{4}-2\,a^{2}bpx^{2}+a^{3}\right)\left(bx^{2}+a\right)^{2\,p}a}{2\left(4\,p^{3}+12\,p^{2}+11\,p+3\right)b^{3}}+\frac{\left(\left(4\,p^{3}+12\,p^{2}+11\,p+3\right)b^{4}x^{8}+2\left(2\,p\right.\right.}{4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + a^
3)*(b*x^2 + a)^(2*p)*a/((4*p^3 + 12*p^2 + 11*p + 3)*b^3) + 1/4*((4*p^3 + 12
*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^6 - 3*(2*p^2 + p)*
a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4)*(b*x^2 + a)^(2*p)/((4*p^4 + 20*p^3 + 3
5*p^2 + 25*p + 6)*b^3)
```

mupad [B] time = 0.20, size = 169, normalized size = 1.32

$$\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{p}\left(\frac{a^{4}}{4\,b^{3}\,\left(2\,p^{3}+9\,p^{2}+13\,p+6\right)}+\frac{a\,x^{6}\,\left(p+1\right)^{2}}{2\,p^{3}+9\,p^{2}+13\,p+6}+\frac{b\,x^{8}\,\left(2\,p^{2}+5\,p+3\right)}{4\,\left(2\,p^{3}+9\,p^{2}+13\,p+6\right)}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*(a^4/(4*b^3*(13*p + 9*p^2 + 2*p^3 + 6)) + (a*
x^6*(p + 1)^2)/(13*p + 9*p^2 + 2*p^3 + 6) + (b*x^8*(5*p + 2*p^2 + 3))/(4*(1
3*p + 9*p^2 + 2*p^3 + 6)) - (a^3*p*x^2)/(2*b^2*(13*p + 9*p^2 + 2*p^3 + 6))
+ (a^2*p*x^4*(2*p + 1))/(4*b*(13*p + 9*p^2 + 2*p^3 + 6)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{ax^{6}\left(a^{2}\right)^{p}}{6} \\ \frac{2a^{2}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2a^{2}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{3a^{2}}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{}{4a^{2}b^{3}+8} \\ \int\frac{x^{5}\left(a+bx^{2}\right)}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\,dx \\ \frac{a^{2}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^{3}}+\frac{a^{2}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^{3}}-\frac{ax^{2}}{2b^{2}}+\frac{x^{4}}{4b} \\ \frac{a^{4}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}-\frac{2a^{3}bpx^{2}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}+\frac{2a^{2}b^{2}p^{2}x^{4}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}+\frac{a^{2}b^{2}px^{4}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+36b^{3}p^{2}+52b^{3}p+24b^{3}}+\frac{4ab^{3}p^{2}x^{6}\left(a^{2}+\right.}{8b^{3}p^{3}+36b^{3}p} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((a*x**6*(a**2)**p/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b)
+ x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqr
t(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b*
*3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x
)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sq
rt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a*
*2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/
b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqr
t(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -2))
, (Integral(x**5*(a + b*x**2)/((a + b*x**2)**2)**(3/2), x), Eq(p, -3/2)), (
a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b)
+ x)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (a**4*(a**2 + 2*a
*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3)
- 2*a**3*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3
*p**2 + 52*b**3*p + 24*b**3) + 2*a**2*b**2*p**2*x**4*(a**2 + 2*a*b*x**2 + b
**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + a**2*b**2
*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52
*b**3*p + 24*b**3) + 4*a*b**3*p**2*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/
(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + 8*a*b**3*p*x**6*(a**2
+ 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b
**3) + 4*a*b**3*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b
**3*p**2 + 52*b**3*p + 24*b**3) + 2*b**4*p**2*x**8*(a**2 + 2*a*b*x**2 + b**
2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + 5*b**4*p*x*
*8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3
*p + 24*b**3) + 3*b**4*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3
 + 36*b**3*p**2 + 52*b**3*p + 24*b**3), True))
```

3.95 $\int x^3 (A + Bx^2) (a + bx^2 + cx^4)^3 \, dx$

Optimal. Leaf size=166

$$\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2x^6(aB+3Ab)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{3}{8}ax^8\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3\right.$$

```
[Out] 1/4*a^3*A*x^4+1/6*a^2*(3*A*b+B*a)*x^6+3/8*a*(a*b*B+A*(a*c+b^2))*x^8+1/10*(3
*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^10+1/12*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*
b^3)*x^12+3/14*c*(A*b*c+B*a*c+B*b^2)*x^14+1/16*c^2*(A*c+3*B*b)*x^16+1/18*B*
c^3*x^18
```

Rubi [A] time = 0.39, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1251, 765}

$$\frac{1}{6}a^2x^6(aB+3Ab)+\frac{1}{4}a^3Ax^4+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{1}{10}x^{10}\left(A\left(6a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/
14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18
```

Rule 765

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3 \, dx &= \frac{1}{2}\text{Subst}\left(\int x(A+Bx)\left(a+bx+cx^2\right)^3 \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^3Ax+a^2(3Ab+aB)x^2+3a\left(abB+A\left(b^2+ac\right)\right)x^3+\left(3\right.\right.\right.\\ &= \frac{1}{4}a^3Ax^4+\frac{1}{6}a^2(3Ab+aB)x^6+\frac{3}{8}a\left(abB+A\left(b^2+ac\right)\right)x^8+\frac{1}{10}\left(3aB\left(b\right.\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 166, normalized size = 1.00

$$\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2x^6(aB+3Ab)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{3}{8}ax^8\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/
14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18
```

fricas [A] time = 0.63, size = 193, normalized size = 1.16

$$\frac{1}{18}x^{18}c^3B+\frac{3}{16}x^{16}c^2bB+\frac{1}{16}x^{16}c^3A+\frac{3}{14}x^{14}cb^2B+\frac{3}{14}x^{14}c^2aB+\frac{3}{14}x^{14}c^2bA+\frac{1}{12}x^{12}b^3B+\frac{1}{2}x^{12}cbaB+\frac{1}{4}x^{12}cb^2A+\frac{1}{4}x^{12}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*c^3*B + 3/16*x^16*c^2*b*B + 1/16*x^16*c^3*A + 3/14*x^14*c*b^2*B +
 3/14*x^14*c^2*a*B + 3/14*x^14*c^2*b*A + 1/12*x^12*b^3*B + 1/2*x^12*c*b*a*B
 + 1/4*x^12*c*b^2*A + 1/4*x^12*c^2*a*A + 3/10*x^10*b^2*a*B + 3/10*x^10*c*a^
2*B + 1/10*x^10*b^3*A + 3/5*x^10*c*b*a*A + 3/8*x^8*b*a^2*B + 3/8*x^8*b^2*a*
A + 3/8*x^8*c*a^2*A + 1/6*x^6*a^3*B + 1/2*x^6*b*a^2*A + 1/4*x^4*a^3*A
```

giac [A] time = 0.34, size = 193, normalized size = 1.16

$$\frac{1}{18}Bc^3x^{18}+\frac{3}{16}Bbc^2x^{16}+\frac{1}{16}Ac^3x^{16}+\frac{3}{14}Bb^2cx^{14}+\frac{3}{14}Bac^2x^{14}+\frac{3}{14}Abc^2x^{14}+\frac{1}{12}Bb^3x^{12}+\frac{1}{2}Babcx^{12}+\frac{1}{4}Ab^2cx^{12}+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/18*B*c^3*x^18 + 3/16*B*b*c^2*x^16 + 1/16*A*c^3*x^16 + 3/14*B*b^2*c*x^14 +
 3/14*B*a*c^2*x^14 + 3/14*A*b*c^2*x^14 + 1/12*B*b^3*x^12 + 1/2*B*a*b*c*x^12
 + 1/4*A*b^2*c*x^12 + 1/4*A*a*c^2*x^12 + 3/10*B*a*b^2*x^10 + 1/10*A*b^3*x^1
0 + 3/10*B*a^2*c*x^10 + 3/5*A*a*b*c*x^10 + 3/8*B*a^2*b*x^8 + 3/8*A*a*b^2*x^
8 + 3/8*A*a^2*c*x^8 + 1/6*B*a^3*x^6 + 1/2*A*a^2*b*x^6 + 1/4*A*a^3*x^4
```

maple [A] time = 0.00, size = 226, normalized size = 1.36

$$\frac{B\,c^3x^{18}}{18}+\frac{\left(A\,c^3+3Bb\,c^2\right)x^{16}}{16}+\frac{\left(3Ab\,c^2+\left(a\,c^2+2b^2c+\left(2ac+b^2\right)c\right)B\right)x^{14}}{14}+\frac{\left(\left(a\,c^2+2b^2c+\left(2ac+b^2\right)c\right)A+(4\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/18*B*c^3*x^18+1/16*(A*c^3+3*B*b*c^2)*x^16+1/14*(3*A*b*c^2+B*(a*c^2+2*b^2*
c+c*(2*a*c+b^2)))*x^14+1/12*(A*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+B*(4*a*b*c+b*(
2*a*c+b^2)))*x^12+1/10*(A*(4*a*b*c+b*(2*a*c+b^2))+B*(a*(2*a*c+b^2)+2*b^2*a+
c*a^2))*x^10+1/8*(A*(a*(2*a*c+b^2)+2*b^2*a+c*a^2)+3*B*a^2*b)*x^8+1/6*(3*A*a
^2*b+B*a^3)*x^6+1/4*a^3*A*x^4
```

maxima [A] time = 0.60, size = 166, normalized size = 1.00

$$\frac{1}{18}Bc^3x^{18}+\frac{1}{16}\left(3\,Bbc^2+Ac^3\right)x^{16}+\frac{3}{14}\left(Bb^2c+(Ba+Ab)c^2\right)x^{14}+\frac{1}{12}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^{12}+\frac{1}{10}\left(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*B*c^3*x^18 + 1/16*(3*B*b*c^2 + A*c^3)*x^16 + 3/14*(B*b^2*c + (B*a + A*
b)*c^2)*x^14 + 1/12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^12 + 1/10
```

```
*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^10 + 3/8*(B*a^2*b + A*a*b^2
+ A*a^2*c)*x^8 + 1/4*A*a^3*x^4 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6
```

mupad [B] time = 0.08, size = 169, normalized size = 1.02

$$x^{10}\left(\frac{3\,B\,c\,a^2}{10}+\frac{3\,B\,a\,b^2}{10}+\frac{3\,A\,c\,a\,b}{5}+\frac{A\,b^3}{10}\right)+x^{12}\left(\frac{B\,b^3}{12}+\frac{A\,b^2\,c}{4}+\frac{B\,a\,b\,c}{2}+\frac{A\,a\,c^2}{4}\right)+x^6\left(\frac{B\,a^3}{6}+\frac{A\,b\,a^2}{2}\right)+x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^10*((A*b^3)/10 + (3*B*a*b^2)/10 + (3*B*a^2*c)/10 + (3*A*a*b*c)/5) + x^12*
((B*b^3)/12 + (A*a*c^2)/4 + (A*b^2*c)/4 + (B*a*b*c)/2) + x^6*((B*a^3)/6 + (
A*a^2*b)/2) + x^16*((A*c^3)/16 + (3*B*b*c^2)/16) + x^8*((3*A*a*b^2)/8 + (3*
A*a^2*c)/8 + (3*B*a^2*b)/8) + x^14*((3*A*b*c^2)/14 + (3*B*a*c^2)/14 + (3*B*
b^2*c)/14) + (A*a^3*x^4)/4 + (B*c^3*x^18)/18
```

sympy [A] time = 0.10, size = 202, normalized size = 1.22

$$\frac{Aa^3x^4}{4}+\frac{Bc^3x^{18}}{18}+x^{16}\left(\frac{Ac^3}{16}+\frac{3Bbc^2}{16}\right)+x^{14}\left(\frac{3Abc^2}{14}+\frac{3Bac^2}{14}+\frac{3Bb^2c}{14}\right)+x^{12}\left(\frac{Aac^2}{4}+\frac{Ab^2c}{4}+\frac{Babc}{2}+\frac{Bb^3}{12}\right)+x^1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**4/4 + B*c**3*x**18/18 + x**16*(A*c**3/16 + 3*B*b*c**2/16) + x**14
*(3*A*b*c**2/14 + 3*B*a*c**2/14 + 3*B*b**2*c/14) + x**12*(A*a*c**2/4 + A*b*
*2*c/4 + B*a*b*c/2 + B*b**3/12) + x**10*(3*A*a*b*c/5 + A*b**3/10 + 3*B*a**2
*c/10 + 3*B*a*b**2/10) + x**8*(3*A*a**2*c/8 + 3*A*a*b**2/8 + 3*B*a**2*b/8)
+ x**6*(A*a**2*b/2 + B*a**3/6)
```

3.96 $\int x^2 \left(A + Bx^2\right) \left(a + bx^2 + cx^4\right)^3 \, dx$

Optimal. Leaf size=166

$$\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2x^5(aB+3Ab)+\frac{3}{13}cx^{13}\left(aBc+Abc+b^2B\right)+\frac{3}{7}ax^7\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{11}x^{11}\left(3aAc^2+6abBc+3Ab\right.$$

```
[Out] 1/3*a^3*A*x^3+1/5*a^2*(3*A*b+B*a)*x^5+3/7*a*(a*b*B+A*(a*c+b^2))*x^7+1/9*(3*
a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^9+1/11*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^
3)*x^11+3/13*c*(A*b*c+B*a*c+B*b^2)*x^13+1/15*c^2*(A*c+3*B*b)*x^15+1/17*B*c^
3*x^17
```

Rubi [A] time = 0.15, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1261}

$$\frac{1}{5}a^2x^5(aB+3Ab)+\frac{1}{3}a^3Ax^3+\frac{1}{11}x^{11}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{13}cx^{13}\left(aBc+Abc+b^2B\right)+\frac{1}{9}x^9\left(A\left(6abc+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^2*
c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)/13
 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3Ax^2+a^2(3Ab+aB)x^4+3a\left(abB+A\left(b^2+ac\right)\right)x^6+\left(3aB\left(b^2+ac\right.\right.\right.\\ &= \frac{1}{3}a^3Ax^3+\frac{1}{5}a^2(3Ab+aB)x^5+\frac{3}{7}a\left(abB+A\left(b^2+ac\right)\right)x^7+\frac{1}{9}\left(3aB\left(b^2+\right.\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 166, normalized size = 1.00

$$\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2x^5(aB+3Ab)+\frac{3}{13}cx^{13}\left(aBc+Abc+b^2B\right)+\frac{3}{7}ax^7\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{11}x^{11}\left(3aAc^2+6abBc+3Ab\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^2*
c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)/13
 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17
```

fricas [A] time = 0.58, size = 193, normalized size = 1.16

$$\frac{1}{17}x^{17}c^3B+\frac{1}{5}x^{15}c^2bB+\frac{1}{15}x^{15}c^3A+\frac{3}{13}x^{13}cb^2B+\frac{3}{13}x^{13}c^2aB+\frac{3}{13}x^{13}c^2bA+\frac{1}{11}x^{11}b^3B+\frac{6}{11}x^{11}cbaB+\frac{3}{11}x^{11}cb^2A+\frac{3}{11}x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*c^3*B + 1/5*x^15*c^2*b*B + 1/15*x^15*c^3*A + 3/13*x^13*c*b^2*B +
3/13*x^13*c^2*a*B + 3/13*x^13*c^2*b*A + 1/11*x^11*b^3*B + 6/11*x^11*c*b*a*B
 + 3/11*x^11*c*b^2*A + 3/11*x^11*c^2*a*A + 1/3*x^9*b^2*a*B + 1/3*x^9*c*a^2*
B + 1/9*x^9*b^3*A + 2/3*x^9*c*b*a*A + 3/7*x^7*b*a^2*B + 3/7*x^7*b^2*a*A + 3
/7*x^7*c*a^2*A + 1/5*x^5*a^3*B + 3/5*x^5*b*a^2*A + 1/3*x^3*a^3*A
```

giac [A] time = 0.27, size = 193, normalized size = 1.16

$$\frac{1}{17}Bc^3x^{17}+\frac{1}{5}Bbc^2x^{15}+\frac{1}{15}Ac^3x^{15}+\frac{3}{13}Bb^2cx^{13}+\frac{3}{13}Bac^2x^{13}+\frac{3}{13}Abc^2x^{13}+\frac{1}{11}Bb^3x^{11}+\frac{6}{11}Babcx^{11}+\frac{3}{11}Ab^2cx^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/17*B*c^3*x^17 + 1/5*B*b*c^2*x^15 + 1/15*A*c^3*x^15 + 3/13*B*b^2*c*x^13 +
3/13*B*a*c^2*x^13 + 3/13*A*b*c^2*x^13 + 1/11*B*b^3*x^11 + 6/11*B*a*b*c*x^11
 + 3/11*A*b^2*c*x^11 + 3/11*A*a*c^2*x^11 + 1/3*B*a*b^2*x^9 + 1/9*A*b^3*x^9
+ 1/3*B*a^2*c*x^9 + 2/3*A*a*b*c*x^9 + 3/7*B*a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 3
/7*A*a^2*c*x^7 + 1/5*B*a^3*x^5 + 3/5*A*a^2*b*x^5 + 1/3*A*a^3*x^3
```

maple [A] time = 0.00, size = 226, normalized size = 1.36

$$\frac{B\,c^3x^{17}}{17}+\frac{\left(A\,c^3+3Bb\,c^2\right)x^{15}}{15}+\frac{\left(3Ab\,c^2+\left(a\,c^2+2b^2c+\left(2ac+b^2\right)c\right)B\right)x^{13}}{13}+\frac{\left(\left(a\,c^2+2b^2c+\left(2ac+b^2\right)c\right)A+\right.}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/17*B*c^3*x^17+1/15*(A*c^3+3*B*b*c^2)*x^15+1/13*(3*A*b*c^2+(a*c^2+2*b^2*c+
(2*a*c+b^2)*c)*B)*x^13+1/11*((a*c^2+2*b^2*c+(2*a*c+b^2)*c)*A+(4*a*b*c+(2*a*
c+b^2)*b)*B)*x^11+1/9*((4*a*b*c+(2*a*c+b^2)*b)*A+(a^2*c+2*a*b^2+(2*a*c+b^2)
*a)*B)*x^9+1/7*(3*B*a^2*b+(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*A)*x^7+1/5*(3*A*a^2
*b+B*a^3)*x^5+1/3*a^3*A*x^3
```

maxima [A] time = 0.73, size = 166, normalized size = 1.00

$$\frac{1}{17}Bc^3x^{17}+\frac{1}{15}\left(3\,Bbc^2+Ac^3\right)x^{15}+\frac{3}{13}\left(Bb^2c+(Ba+Ab)c^2\right)x^{13}+\frac{1}{11}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^{11}+\frac{1}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/17*B*c^3*x^17 + 1/15*(3*B*b*c^2 + A*c^3)*x^15 + 3/13*(B*b^2*c + (B*a + A*
b)*c^2)*x^13 + 1/11*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^11 + 1/9*
(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^9 + 3/7*(B*a^2*b + A*a*b^2 +
A*a^2*c)*x^7 + 1/3*A*a^3*x^3 + 1/5*(B*a^3 + 3*A*a^2*b)*x^5
```

mupad [B] time = 0.10, size = 169, normalized size = 1.02

$$x^9\left(\frac{B\,c\,a^2}{3}+\frac{B\,a\,b^2}{3}+\frac{2\,A\,c\,a\,b}{3}+\frac{A\,b^3}{9}\right)+x^{11}\left(\frac{B\,b^3}{11}+\frac{3\,A\,b^2\,c}{11}+\frac{6\,B\,a\,b\,c}{11}+\frac{3\,A\,a\,c^2}{11}\right)+x^5\left(\frac{B\,a^3}{5}+\frac{3\,A\,b\,a^2}{5}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^9*((A*b^3)/9 + (B*a*b^2)/3 + (B*a^2*c)/3 + (2*A*a*b*c)/3) + x^11*((B*b^3)
/11 + (3*A*a*c^2)/11 + (3*A*b^2*c)/11 + (6*B*a*b*c)/11) + x^5*((B*a^3)/5 +
(3*A*a^2*b)/5) + x^15*((A*c^3)/15 + (B*b*c^2)/5) + x^7*((3*A*a*b^2)/7 + (3*
A*a^2*c)/7 + (3*B*a^2*b)/7) + x^13*((3*A*b*c^2)/13 + (3*B*a*c^2)/13 + (3*B*
b^2*c)/13) + (A*a^3*x^3)/3 + (B*c^3*x^17)/17
```

sympy [A] time = 0.10, size = 204, normalized size = 1.23

$$\frac{Aa^3x^3}{3}+\frac{Bc^3x^{17}}{17}+x^{15}\left(\frac{Ac^3}{15}+\frac{Bbc^2}{5}\right)+x^{13}\left(\frac{3Abc^2}{13}+\frac{3Bac^2}{13}+\frac{3Bb^2c}{13}\right)+x^{11}\left(\frac{3Aac^2}{11}+\frac{3Ab^2c}{11}+\frac{6Babc}{11}+\frac{Bb^3}{11}\right)+x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**3/3 + B*c**3*x**17/17 + x**15*(A*c**3/15 + B*b*c**2/5) + x**13*(3
*A*b*c**2/13 + 3*B*a*c**2/13 + 3*B*b**2*c/13) + x**11*(3*A*a*c**2/11 + 3*A*
b**2*c/11 + 6*B*a*b*c/11 + B*b**3/11) + x**9*(2*A*a*b*c/3 + A*b**3/9 + B*a*
*2*c/3 + B*a*b**2/3) + x**7*(3*A*a**2*c/7 + 3*A*a*b**2/7 + 3*B*a**2*b/7) +
x**5*(3*A*a**2*b/5 + B*a**3/5)
```

3.97 $\int x\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=166

$$\frac{1}{2}a^3Ax^2+\frac{1}{4}a^2x^4(aB+3Ab)+\frac{1}{4}cx^{12}\left(aBc+Abc+b^2B\right)+\frac{1}{2}ax^6\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{10}x^{10}\left(3aAc^2+6abBc+3A\right.$$

```
[Out] 1/2*a^3*A*x^2+1/4*a^2*(3*A*b+B*a)*x^4+1/2*a*(a*b*B+A*(a*c+b^2))*x^6+1/8*(3*
a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^8+1/10*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^
3)*x^10+1/4*c*(A*b*c+B*a*c+B*b^2)*x^12+1/14*c^2*(A*c+3*B*b)*x^14+1/16*B*c^3
*x^16
```

Rubi [A] time = 0.29, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1247, 631}

$$\frac{1}{4}a^2x^4(aB+3Ab)+\frac{1}{2}a^3Ax^2+\frac{1}{10}x^{10}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{4}cx^{12}\left(aBc+Abc+b^2B\right)+\frac{1}{8}x^8\left(A\left(6abc\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^2)/2 + (a^2*(3*A*b + a*B)*x^4)/4 + (a*(a*b*B + A*(b^2 + a*c))*x^6)
/2 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^8)/8 + ((b^3*B + 3*A*b^2*c
+ 6*a*b*B*c + 3*a*A*c^2)*x^10)/10 + (c*(b^2*B + A*b*c + a*B*c)*x^12)/4 + (c
^2*(3*b*B + A*c)*x^14)/14 + (B*c^3*x^16)/16
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx &= \frac{1}{2}\text{Subst}\left(\int (A+Bx)\left(a+bx+cx^2\right)^3\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^3A+a^2(3Ab+aB)x+3a\left(abB+A\left(b^2+ac\right)\right)x^2+\left(3aB\right.\right.\right.\\ &= \frac{1}{2}a^3Ax^2+\frac{1}{4}a^2(3Ab+aB)x^4+\frac{1}{2}a\left(abB+A\left(b^2+ac\right)\right)x^6+\frac{1}{8}\left(3aB\left(b^2\right.\right.\end{aligned}$$

Mathematica [A] time = 0.06, size = 154, normalized size = 0.93

$$\frac{1}{560}x^2\left(280a^3A+140a^2x^2(aB+3Ab)+140cx^{10}\left(aBc+Abc+b^2B\right)+280ax^4\left(A\left(ac+b^2\right)+abB\right)+56x^8\left(3a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(280*a^3*A + 140*a^2*(3*A*b + a*B)*x^2 + 280*a*(a*b*B + A*(b^2 + a*c))
*x^4 + 70*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6 + 56*(b^3*B + 3*A*b^2
*c + 6*a*b*B*c + 3*a*A*c^2)*x^8 + 140*c*(b^2*B + A*b*c + a*B*c)*x^10 + 40*c
^2*(3*b*B + A*c)*x^12 + 35*B*c^3*x^14))/560
```

fricas [A] time = 0.54, size = 193, normalized size = 1.16

$$\frac{1}{16}x^{16}c^3B+\frac{3}{14}x^{14}c^2bB+\frac{1}{14}x^{14}c^3A+\frac{1}{4}x^{12}cb^2B+\frac{1}{4}x^{12}c^2aB+\frac{1}{4}x^{12}c^2bA+\frac{1}{10}x^{10}b^3B+\frac{3}{5}x^{10}cbaB+\frac{3}{10}x^{10}cb^2A+\frac{3}{10}x^{10}c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*c^3*B + 3/14*x^14*c^2*b*B + 1/14*x^14*c^3*A + 1/4*x^12*c*b^2*B +
1/4*x^12*c^2*a*B + 1/4*x^12*c^2*b*A + 1/10*x^10*b^3*B + 3/5*x^10*c*b*a*B +
3/10*x^10*c*b^2*A + 3/10*x^10*c^2*a*A + 3/8*x^8*b^2*a*B + 3/8*x^8*c*a^2*B +
 1/8*x^8*b^3*A + 3/4*x^8*c*b*a*A + 1/2*x^6*b*a^2*B + 1/2*x^6*b^2*a*A + 1/2*
x^6*c*a^2*A + 1/4*x^4*a^3*B + 3/4*x^4*b*a^2*A + 1/2*x^2*a^3*A
```

giac [A] time = 0.29, size = 193, normalized size = 1.16

$$\frac{1}{16}Bc^3x^{16}+\frac{3}{14}Bbc^2x^{14}+\frac{1}{14}Ac^3x^{14}+\frac{1}{4}Bb^2cx^{12}+\frac{1}{4}Bac^2x^{12}+\frac{1}{4}Abc^2x^{12}+\frac{1}{10}Bb^3x^{10}+\frac{3}{5}Babcx^{10}+\frac{3}{10}Ab^2cx^{10}+\frac{3}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*B*c^3*x^16 + 3/14*B*b*c^2*x^14 + 1/14*A*c^3*x^14 + 1/4*B*b^2*c*x^12 +
1/4*B*a*c^2*x^12 + 1/4*A*b*c^2*x^12 + 1/10*B*b^3*x^10 + 3/5*B*a*b*c*x^10 +
3/10*A*b^2*c*x^10 + 3/10*A*a*c^2*x^10 + 3/8*B*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3
/8*B*a^2*c*x^8 + 3/4*A*a*b*c*x^8 + 1/2*B*a^2*b*x^6 + 1/2*A*a*b^2*x^6 + 1/2*
A*a^2*c*x^6 + 1/4*B*a^3*x^4 + 3/4*A*a^2*b*x^4 + 1/2*A*a^3*x^2
```

maple [A] time = 0.00, size = 226, normalized size = 1.36

$$\frac{Bc^3x^{16}}{16}+\frac{\left(Ac^3+3Bbc^2\right)x^{14}}{14}+\frac{\left(3Abc^2+\left(ac^2+2b^2c+\left(2ac+b^2\right)c\right)B\right)x^{12}}{12}+\frac{\left(\left(ac^2+2b^2c+\left(2ac+b^2\right)c\right)A+\left(4\right.\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/16*B*c^3*x^16+1/14*(A*c^3+3*B*b*c^2)*x^14+1/12*(3*A*b*c^2+(a*c^2+2*b^2*c+
(2*a*c+b^2)*c)*B)*x^12+1/10*((a*c^2+2*b^2*c+(2*a*c+b^2)*c)*A+(4*a*b*c+(2*a*
c+b^2)*b)*B)*x^10+1/8*((4*a*b*c+(2*a*c+b^2)*b)*A+(a^2*c+2*a*b^2+(2*a*c+b^2)
*a)*B)*x^8+1/6*(3*B*a^2*b+(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*A)*x^6+1/4*(3*A*a^2
*b+B*a^3)*x^4+1/2*a^3*A*x^2
```

maxima [A] time = 0.77, size = 166, normalized size = 1.00

$$\frac{1}{16}Bc^3x^{16}+\frac{1}{14}\left(3Bbc^2+Ac^3\right)x^{14}+\frac{1}{4}\left(Bb^2c+(Ba+Ab)c^2\right)x^{12}+\frac{1}{10}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^{10}+\frac{1}{8}\left(3B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*B*c^3*x^16 + 1/14*(3*B*b*c^2 + A*c^3)*x^14 + 1/4*(B*b^2*c + (B*a + A*b
)*c^2)*x^12 + 1/10*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^10 + 1/8*(
3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^8 + 1/2*(B*a^2*b + A*a*b^2 + A
*a^2*c)*x^6 + 1/2*A*a^3*x^2 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

mupad [B] time = 0.05, size = 169, normalized size = 1.02

$$x^8\left(\frac{3\,B\,c\,a^2}{8}+\frac{3\,B\,a\,b^2}{8}+\frac{3\,A\,c\,a\,b}{4}+\frac{A\,b^3}{8}\right)+x^{10}\left(\frac{B\,b^3}{10}+\frac{3\,A\,b^2\,c}{10}+\frac{3\,B\,a\,b\,c}{5}+\frac{3\,A\,a\,c^2}{10}\right)+x^4\left(\frac{B\,a^3}{4}+\frac{3\,A\,b\,a^2}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^8*((A*b^3)/8 + (3*B*a*b^2)/8 + (3*B*a^2*c)/8 + (3*A*a*b*c)/4) + x^10*((B*
b^3)/10 + (3*A*a*c^2)/10 + (3*A*b^2*c)/10 + (3*B*a*b*c)/5) + x^4*((B*a^3)/4
 + (3*A*a^2*b)/4) + x^14*((A*c^3)/14 + (3*B*b*c^2)/14) + x^6*((A*a*b^2)/2 +
 (A*a^2*c)/2 + (B*a^2*b)/2) + x^12*((A*b*c^2)/4 + (B*a*c^2)/4 + (B*b^2*c)/4
) + (A*a^3*x^2)/2 + (B*c^3*x^16)/16
```

sympy [A] time = 0.10, size = 199, normalized size = 1.20

$$\frac{Aa^3x^2}{2}+\frac{Bc^3x^{16}}{16}+x^{14}\left(\frac{Ac^3}{14}+\frac{3Bbc^2}{14}\right)+x^{12}\left(\frac{Abc^2}{4}+\frac{Bac^2}{4}+\frac{Bb^2c}{4}\right)+x^{10}\left(\frac{3Aac^2}{10}+\frac{3Ab^2c}{10}+\frac{3Babc}{5}+\frac{Bb^3}{10}\right)+x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**2/2 + B*c**3*x**16/16 + x**14*(A*c**3/14 + 3*B*b*c**2/14) + x**12
*(A*b*c**2/4 + B*a*c**2/4 + B*b**2*c/4) + x**10*(3*A*a*c**2/10 + 3*A*b**2*c
/10 + 3*B*a*b*c/5 + B*b**3/10) + x**8*(3*A*a*b*c/4 + A*b**3/8 + 3*B*a**2*c/
8 + 3*B*a*b**2/8) + x**6*(A*a**2*c/2 + A*a*b**2/2 + B*a**2*b/2) + x**4*(3*A
*a**2*b/4 + B*a**3/4)
```

3.98 $\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=161

$$a^3Ax+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{3}{5}ax^5\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+\right.$$

```
[Out] a^3*A*x+1/3*a^2*(3*A*b+B*a)*x^3+3/5*a*(a*b*B+A*(a*c+b^2))*x^5+1/7*(3*a*B*(a
*c+b^2)+A*(6*a*b*c+b^3))*x^7+1/9*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^9+
3/11*c*(A*b*c+B*a*c+B*b^2)*x^11+1/13*c^2*(A*c+3*B*b)*x^13+1/15*B*c^3*x^15
```

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$\frac{1}{3}a^2x^3(aB+3Ab)+a^3Ax+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{1}{7}x^7\left(A\left(6abc+b^3\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c + 6*
a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c^2*
(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3A+a^2(3Ab+aB)x^2+3a\left(abB+A\left(b^2+ac\right)\right)x^4+\left(3aB\left(b^2+ac\right)+A\right.\right.\\ &= a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{5}a\left(abB+A\left(b^2+ac\right)\right)x^5+\frac{1}{7}\left(3aB\left(b^2+ac\right)+\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 161, normalized size = 1.00

$$a^3Ax+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{3}{5}ax^5\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c + 6*
a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c^2*
(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

fricas [A] time = 0.60, size = 189, normalized size = 1.17

$$\frac{1}{15}x^{15}c^3B+\frac{3}{13}x^{13}c^2bB+\frac{1}{13}x^{13}c^3A+\frac{3}{11}x^{11}cb^2B+\frac{3}{11}x^{11}c^2aB+\frac{3}{11}x^{11}c^2bA+\frac{1}{9}x^9b^3B+\frac{2}{3}x^9cbaB+\frac{1}{3}x^9cb^2A+\frac{1}{3}x^9c^2aA-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*c^3*B + 3/13*x^13*c^2*b*B + 1/13*x^13*c^3*A + 3/11*x^11*c*b^2*B +
 3/11*x^11*c^2*a*B + 3/11*x^11*c^2*b*A + 1/9*x^9*b^3*B + 2/3*x^9*c*b*a*B +
1/3*x^9*c*b^2*A + 1/3*x^9*c^2*a*A + 3/7*x^7*b^2*a*B + 3/7*x^7*c*a^2*B + 1/7
*x^7*b^3*A + 6/7*x^7*c*b*a*A + 3/5*x^5*b*a^2*B + 3/5*x^5*b^2*a*A + 3/5*x^5*
c*a^2*A + 1/3*x^3*a^3*B + x^3*b*a^2*A + x*a^3*A
```

giac [A] time = 0.26, size = 189, normalized size = 1.17

$$\frac{1}{15}Bc^3x^{15}+\frac{3}{13}Bbc^2x^{13}+\frac{1}{13}Ac^3x^{13}+\frac{3}{11}Bb^2cx^{11}+\frac{3}{11}Bac^2x^{11}+\frac{3}{11}Abc^2x^{11}+\frac{1}{9}Bb^3x^9+\frac{2}{3}Babcx^9+\frac{1}{3}Ab^2cx^9+\frac{1}{3}A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/15*B*c^3*x^15 + 3/13*B*b*c^2*x^13 + 1/13*A*c^3*x^13 + 3/11*B*b^2*c*x^11 +
 3/11*B*a*c^2*x^11 + 3/11*A*b*c^2*x^11 + 1/9*B*b^3*x^9 + 2/3*B*a*b*c*x^9 +
1/3*A*b^2*c*x^9 + 1/3*A*a*c^2*x^9 + 3/7*B*a*b^2*x^7 + 1/7*A*b^3*x^7 + 3/7*B
*a^2*c*x^7 + 6/7*A*a*b*c*x^7 + 3/5*B*a^2*b*x^5 + 3/5*A*a*b^2*x^5 + 3/5*A*a^
2*c*x^5 + 1/3*B*a^3*x^3 + A*a^2*b*x^3 + A*a^3*x
```

maple [A] time = 0.00, size = 223, normalized size = 1.39

$$\frac{B\,c^3x^{15}}{15}+\frac{\left(A\,c^3+3Bb\,c^2\right)x^{13}}{13}+\frac{\left(3Ab\,c^2+\left(a\,c^2+2b^2c+\left(2ac+b^2\right)c\right)B\right)x^{11}}{11}+\frac{\left(\left(a\,c^2+2b^2c+\left(2ac+b^2\right)c\right)A+\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/15*B*c^3*x^15+1/13*(A*c^3+3*B*b*c^2)*x^13+1/11*(3*A*b*c^2+(a*c^2+2*b^2*c+
(2*a*c+b^2)*c)*B)*x^11+1/9*((a*c^2+2*b^2*c+(2*a*c+b^2)*c)*A+(4*a*b*c+(2*a*c
+b^2)*b)*B)*x^9+1/7*((4*a*b*c+(2*a*c+b^2)*b)*A+(a^2*c+2*a*b^2+(2*a*c+b^2)*a
)*B)*x^7+1/5*(3*B*a^2*b+(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*A)*x^5+1/3*(3*A*a^2*b
+B*a^3)*x^3+a^3*A*x
```

maxima [A] time = 0.73, size = 163, normalized size = 1.01

$$\frac{1}{15}Bc^3x^{15}+\frac{1}{13}\left(3\,Bbc^2+Ac^3\right)x^{13}+\frac{3}{11}\left(Bb^2c+(Ba+Ab)c^2\right)x^{11}+\frac{1}{9}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^9+\frac{1}{7}\left(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/15*B*c^3*x^15 + 1/13*(3*B*b*c^2 + A*c^3)*x^13 + 3/11*(B*b^2*c + (B*a + A*
b)*c^2)*x^11 + 1/9*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^9 + 1/7*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^7 + 3/5*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^5 + A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

mupad [B] time = 0.05, size = 165, normalized size = 1.02

$$x^7\left(\frac{3\,B\,c\,a^2}{7}+\frac{3\,B\,a\,b^2}{7}+\frac{6\,A\,c\,a\,b}{7}+\frac{A\,b^3}{7}\right)+x^9\left(\frac{B\,b^3}{9}+\frac{A\,b^2\,c}{3}+\frac{2\,B\,a\,b\,c}{3}+\frac{A\,a\,c^2}{3}\right)+x^3\left(\frac{B\,a^3}{3}+A\,b\,a^2\right)+x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^7*((A*b^3)/7 + (3*B*a*b^2)/7 + (3*B*a^2*c)/7 + (6*A*a*b*c)/7) + x^9*((B*b
^3)/9 + (A*a*c^2)/3 + (A*b^2*c)/3 + (2*B*a*b*c)/3) + x^3*((B*a^3)/3 + A*a^2
*b) + x^13*((A*c^3)/13 + (3*B*b*c^2)/13) + x^5*((3*A*a*b^2)/5 + (3*A*a^2*c)
/5 + (3*B*a^2*b)/5) + x^11*((3*A*b*c^2)/11 + (3*B*a*c^2)/11 + (3*B*b^2*c)/1
1) + (B*c^3*x^15)/15 + A*a^3*x
```

sympy [A] time = 0.10, size = 199, normalized size = 1.24

$$Aa^3x+\frac{Bc^3x^{15}}{15}+x^{13}\left(\frac{Ac^3}{13}+\frac{3Bbc^2}{13}\right)+x^{11}\left(\frac{3Abc^2}{11}+\frac{3Bac^2}{11}+\frac{3Bb^2c}{11}\right)+x^9\left(\frac{Aac^2}{3}+\frac{Ab^2c}{3}+\frac{2Babc}{3}+\frac{Bb^3}{9}\right)+x^7\left(\frac{6A\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x + B*c**3*x**15/15 + x**13*(A*c**3/13 + 3*B*b*c**2/13) + x**11*(3*A
*b*c**2/11 + 3*B*a*c**2/11 + 3*B*b**2*c/11) + x**9*(A*a*c**2/3 + A*b**2*c/3
 + 2*B*a*b*c/3 + B*b**3/9) + x**7*(6*A*a*b*c/7 + A*b**3/7 + 3*B*a**2*c/7 +
3*B*a*b**2/7) + x**5*(3*A*a**2*c/5 + 3*A*a*b**2/5 + 3*B*a**2*b/5) + x**3*(A
*a**2*b + B*a**3/3)
```

3.99 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x} dx$

Optimal. Leaf size=162

$$a^3 A \log(x)+\frac{1}{2}a^2x^2(aB+3Ab)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{3}{4}ax^4\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{8}x^8\left(3aAc^2+6abBc+\right.$$

```
[Out] 1/2*a^2*(3*A*b+B*a)*x^2+3/4*a*(a*b*B+A*(a*c+b^2))*x^4+1/6*(3*a*B*(a*c+b^2)+
A*(6*a*b*c+b^3))*x^6+1/8*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^8+3/10*c*(
A*b*c+B*a*c+B*b^2)*x^10+1/12*c^2*(A*c+3*B*b)*x^12+1/14*B*c^3*x^14+a^3*A*ln(
x)
```

Rubi [A] time = 0.23, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1251, 765}

$$\frac{1}{2}a^2x^2(aB+3Ab)+a^3A\log(x)+\frac{1}{8}x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{1}{6}x^6\left(A\left(6ab\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(
b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c +
3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A
*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]
```

Rule 765

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^2(3Ab+aB)+\frac{a^3A}{x}+3a\left(abB+A\left(b^2+ac\right)\right)x+\left(3aB\left(b^2\right.\right.\right.\right.\\ &= \frac{1}{2}a^2(3Ab+aB)x^2+\frac{3}{4}a\left(abB+A\left(b^2+ac\right)\right)x^4+\frac{1}{6}\left(3aB\left(b^2+ac\right)+A\left(b\right.\right.\end{aligned}$$

Mathematica [A] time = 0.06, size = 162, normalized size = 1.00

$$a^3 A \log(x)+\frac{1}{2}a^2x^2(aB+3Ab)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{3}{4}ax^4\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{8}x^8\left(3aAc^2+6abBc+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(
b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c +
3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A
*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]
```

fricas [A] time = 0.62, size = 164, normalized size = 1.01

$$\frac{1}{14}Bc^3x^{14}+\frac{1}{12}\left(3Bbc^2+Ac^3\right)x^{12}+\frac{3}{10}\left(Bb^2c+(Ba+Ab)c^2\right)x^{10}+\frac{1}{8}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^8+\frac{1}{6}\left(3Ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A*
b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^4 + A*a^3*log(x) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

giac [A] time = 0.34, size = 193, normalized size = 1.19

$$\frac{1}{14}Bc^3x^{14}+\frac{1}{4}Bbc^2x^{12}+\frac{1}{12}Ac^3x^{12}+\frac{3}{10}Bb^2cx^{10}+\frac{3}{10}Bac^2x^{10}+\frac{3}{10}Abc^2x^{10}+\frac{1}{8}Bb^3x^8+\frac{3}{4}Babcx^8+\frac{3}{8}Ab^2cx^8+\frac{3}{8}Aac^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/14*B*c^3*x^14 + 1/4*B*b*c^2*x^12 + 1/12*A*c^3*x^12 + 3/10*B*b^2*c*x^10 +
3/10*B*a*c^2*x^10 + 3/10*A*b*c^2*x^10 + 1/8*B*b^3*x^8 + 3/4*B*a*b*c*x^8 + 3
/8*A*b^2*c*x^8 + 3/8*A*a*c^2*x^8 + 1/2*B*a*b^2*x^6 + 1/6*A*b^3*x^6 + 1/2*B*
a^2*c*x^6 + A*a*b*c*x^6 + 3/4*B*a^2*b*x^4 + 3/4*A*a*b^2*x^4 + 3/4*A*a^2*c*x
^4 + 1/2*B*a^3*x^2 + 3/2*A*a^2*b*x^2 + 1/2*A*a^3*log(x^2)
```

maple [A] time = 0.00, size = 191, normalized size = 1.18

$$\frac{Bc^3x^{14}}{14}+\frac{Ac^3x^{12}}{12}+\frac{Bbc^2x^{12}}{4}+\frac{3Abc^2x^{10}}{10}+\frac{3Bac^2x^{10}}{10}+\frac{3Bb^2cx^{10}}{10}+\frac{3Aac^2x^8}{8}+\frac{3Ab^2cx^8}{8}+\frac{3Babcx^8}{4}+\frac{Bb^3x^8}{8}+Aabc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x)
```

```
[Out] 1/14*B*c^3*x^14+1/12*A*x^12*c^3+1/4*B*x^12*b*c^2+3/10*A*x^10*b*c^2+3/10*B*x
^10*a*c^2+3/10*B*x^10*b^2*c+3/8*A*x^8*a*c^2+3/8*A*x^8*b^2*c+3/4*B*x^8*a*b*c
+1/8*B*x^8*b^3+A*x^6*a*b*c+1/6*A*x^6*b^3+1/2*B*x^6*a^2*c+1/2*B*x^6*a*b^2+3/
4*A*x^4*a^2*c+3/4*A*x^4*a*b^2+3/4*B*x^4*a^2*b+3/2*A*x^2*a^2*b+1/2*B*x^2*a^3
+a^3*A*ln(x)
```

maxima [A] time = 0.79, size = 167, normalized size = 1.03

$$\frac{1}{14}Bc^3x^{14}+\frac{1}{12}\left(3Bbc^2+Ac^3\right)x^{12}+\frac{3}{10}\left(Bb^2c+(Ba+Ab)c^2\right)x^{10}+\frac{1}{8}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^8+\frac{1}{6}\left(3Ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A*
b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*(3
```

```
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^4 + 1/2*A*a^3*log(x^2) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

mupad [B] time = 0.10, size = 166, normalized size = 1.02

$$x^6\left(\frac{B\,c\,a^2}{2}+\frac{B\,a\,b^2}{2}+A\,c\,a\,b+\frac{A\,b^3}{6}\right)+x^8\left(\frac{B\,b^3}{8}+\frac{3\,A\,b^2\,c}{8}+\frac{3\,B\,a\,b\,c}{4}+\frac{3\,A\,a\,c^2}{8}\right)+x^2\left(\frac{B\,a^3}{2}+\frac{3\,A\,b\,a^2}{2}\right)+x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x)
```

```
[Out] x^6*((A*b^3)/6 + (B*a*b^2)/2 + (B*a^2*c)/2 + A*a*b*c) + x^8*((B*b^3)/8 + (3
*A*a*c^2)/8 + (3*A*b^2*c)/8 + (3*B*a*b*c)/4) + x^2*((B*a^3)/2 + (3*A*a^2*b)
/2) + x^12*((A*c^3)/12 + (B*b*c^2)/4) + x^4*((3*A*a*b^2)/4 + (3*A*a^2*c)/4
+ (3*B*a^2*b)/4) + x^10*((3*A*b*c^2)/10 + (3*B*a*c^2)/10 + (3*B*b^2*c)/10)
+ (B*c^3*x^14)/14 + A*a^3*log(x)
```

sympy [A] time = 0.31, size = 199, normalized size = 1.23

$$Aa^3\log(x)+\frac{Bc^3x^{14}}{14}+x^{12}\left(\frac{Ac^3}{12}+\frac{Bbc^2}{4}\right)+x^{10}\left(\frac{3Abc^2}{10}+\frac{3Bac^2}{10}+\frac{3Bb^2c}{10}\right)+x^8\left(\frac{3Aac^2}{8}+\frac{3Ab^2c}{8}+\frac{3Babc}{4}+\frac{Bb}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] A*a**3*log(x) + B*c**3*x**14/14 + x**12*(A*c**3/12 + B*b*c**2/4) + x**10*(3
*A*b*c**2/10 + 3*B*a*c**2/10 + 3*B*b**2*c/10) + x**8*(3*A*a*c**2/8 + 3*A*b*
*2*c/8 + 3*B*a*b*c/4 + B*b**3/8) + x**6*(A*a*b*c + A*b**3/6 + B*a**2*c/2 +
B*a*b**2/2) + x**4*(3*A*a**2*c/4 + 3*A*a*b**2/4 + 3*B*a**2*b/4) + x**2*(3*A
*a**2*b/2 + B*a**3/2)
```

3.100 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2} dx$

Optimal. Leaf size=156

$$-\frac{a^3A}{x}+a^2x(aB+3Ab)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+ax^3\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)$$

```
[Out] -a^3*A/x+a^2*(3*A*b+B*a)*x+a*(a*b*B+A*(a*c+b^2))*x^3+1/5*(3*a*B*(a*c+b^2)+A
*(6*a*b*c+b^3))*x^5+1/7*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^7+1/3*c*(A*
b*c+B*a*c+B*b^2)*x^9+1/11*c^2*(A*c+3*B*b)*x^11+1/13*B*c^3*x^13
```

Rubi [A] time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1261}

$$a^2x(aB+3Ab)-\frac{a^3A}{x}+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+\frac{1}{5}x^5\left(A\left(6abc+b^3\right)+3aB\right.$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a*
B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c
 + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B + A*
c)*x^11)/11 + (B*c^3*x^13)/13
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x^2}\,dx &= \int \left(a^2(3Ab+aB)+\frac{a^3A}{x^2}+3a\left(abB+A\left(b^2+ac\right)\right)x^2+\left(3aB\left(b^2+ac\right)+A\left(\right.\right.\right.\\ &= -\frac{a^3A}{x}+a^2(3Ab+aB)x+a\left(abB+A\left(b^2+ac\right)\right)x^3+\frac{1}{5}\left(3aB\left(b^2+ac\right)+A\left(\right.\right.\end{aligned}$$

Mathematica [A] time = 0.08, size = 156, normalized size = 1.00

$$-\frac{a^3A}{x}+a^2x(aB+3Ab)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+ax^3\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a*
B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c
 + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B + A*
c)*x^11)/11 + (B*c^3*x^13)/13
```

fricas [A] time = 0.57, size = 168, normalized size = 1.08

$$1155\,Bc^3x^{14}+1365\left(3\,Bbc^2+Ac^3\right)x^{12}+5005\left(Bb^2c+(Ba+Ab)c^2\right)x^{10}+2145\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*B*c^3*x^14 + 1365*(3*B*b*c^2 + A*c^3)*x^12 + 5005*(B*b^2*c +
(B*a + A*b)*c^2)*x^10 + 2145*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^
8 + 3003*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 15015*(B*a^2*b +
 A*a*b^2 + A*a^2*c)*x^4 - 15015*A*a^3 + 15015*(B*a^3 + 3*A*a^2*b)*x^2)/x
```

giac [A] time = 0.31, size = 185, normalized size = 1.19

$$\frac{1}{13}Bc^3x^{13}+\frac{3}{11}Bbc^2x^{11}+\frac{1}{11}Ac^3x^{11}+\frac{1}{3}Bb^2cx^9+\frac{1}{3}Bac^2x^9+\frac{1}{3}Abc^2x^9+\frac{1}{7}Bb^3x^7+\frac{6}{7}Babcx^7+\frac{3}{7}Ab^2cx^7+\frac{3}{7}Aac^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/13*B*c^3*x^13 + 3/11*B*b*c^2*x^11 + 1/11*A*c^3*x^11 + 1/3*B*b^2*c*x^9 + 1
/3*B*a*c^2*x^9 + 1/3*A*b*c^2*x^9 + 1/7*B*b^3*x^7 + 6/7*B*a*b*c*x^7 + 3/7*A*
b^2*c*x^7 + 3/7*A*a*c^2*x^7 + 3/5*B*a*b^2*x^5 + 1/5*A*b^3*x^5 + 3/5*B*a^2*c
*x^5 + 6/5*A*a*b*c*x^5 + B*a^2*b*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3 + B*a^3*x
+ 3*A*a^2*b*x - A*a^3/x
```

maple [A] time = 0.00, size = 186, normalized size = 1.19

$$\frac{B\,c^3x^{13}}{13}+\frac{A\,c^3x^{11}}{11}+\frac{3Bb\,c^2x^{11}}{11}+\frac{Ab\,c^2x^9}{3}+\frac{Ba\,c^2x^9}{3}+\frac{B\,b^2c\,x^9}{3}+\frac{3Aa\,c^2x^7}{7}+\frac{3A\,b^2c\,x^7}{7}+\frac{6Babc\,x^7}{7}+\frac{B\,b^3x^7}{7}+\frac{6Aabc}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x)
```

```
[Out] 1/13*B*c^3*x^13+1/11*A*x^11*c^3+3/11*B*x^11*b*c^2+1/3*A*x^9*b*c^2+1/3*B*x^9
*a*c^2+1/3*B*x^9*b^2*c+3/7*A*x^7*a*c^2+3/7*A*x^7*b^2*c+6/7*B*x^7*a*b*c+1/7*
B*x^7*b^3+6/5*A*x^5*a*b*c+1/5*A*x^5*b^3+3/5*B*x^5*a^2*c+3/5*B*x^5*a*b^2+A*x
^3*a^2*c+A*x^3*a*b^2+B*x^3*a^2*b+3*A*a^2*b*x+B*a^3*x-a^3*A/x
```

maxima [A] time = 0.71, size = 162, normalized size = 1.04

$$\frac{1}{13}Bc^3x^{13}+\frac{1}{11}\left(3\,Bbc^2+Ac^3\right)x^{11}+\frac{1}{3}\left(Bb^2c+(Ba+Ab)c^2\right)x^9+\frac{1}{7}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^7+\frac{1}{5}\left(3\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + (B*a + A*b
)*c^2)*x^9 + 1/7*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^7 + 1/5*(3*B
*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^5 + (B*a^2*b + A*a*b^2 + A*a^2*c)
*x^3 - A*a^3/x + (B*a^3 + 3*A*a^2*b)*x
```

mupad [B] time = 0.05, size = 163, normalized size = 1.04

$$x^5\left(\frac{3\,B\,c\,a^2}{5}+\frac{3\,B\,a\,b^2}{5}+\frac{6\,A\,c\,a\,b}{5}+\frac{A\,b^3}{5}\right)+x^7\left(\frac{B\,b^3}{7}+\frac{3\,A\,b^2\,c}{7}+\frac{6\,B\,a\,b\,c}{7}+\frac{3\,A\,a\,c^2}{7}\right)+x\,\left(B\,a^3+3\,A\,b\,a^2\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x)
```

```
[Out] x^5*((A*b^3)/5 + (3*B*a*b^2)/5 + (3*B*a^2*c)/5 + (6*A*a*b*c)/5) + x^7*((B*b
^3)/7 + (3*A*a*c^2)/7 + (3*A*b^2*c)/7 + (6*B*a*b*c)/7) + x*(B*a^3 + 3*A*a^2
*b) + x^11*((A*c^3)/11 + (3*B*b*c^2)/11) + x^3*(A*a*b^2 + A*a^2*c + B*a^2*b
```

```
) + x^9*((A*b*c^2)/3 + (B*a*c^2)/3 + (B*b^2*c)/3) - (A*a^3)/x + (B*c^3*x^13
)/13
```

sympy [A] time = 0.31, size = 185, normalized size = 1.19

$$-\frac{Aa^3}{x}+\frac{Bc^3x^{13}}{13}+x^{11}\left(\frac{Ac^3}{11}+\frac{3Bbc^2}{11}\right)+x^9\left(\frac{Abc^2}{3}+\frac{Bac^2}{3}+\frac{Bb^2c}{3}\right)+x^7\left(\frac{3Aac^2}{7}+\frac{3Ab^2c}{7}+\frac{6Babc}{7}+\frac{Bb^3}{7}\right)+x^5\left(\frac{6Aa}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -A*a**3/x + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) + x**9*(A*b
*c**2/3 + B*a*c**2/3 + B*b**2*c/3) + x**7*(3*A*a*c**2/7 + 3*A*b**2*c/7 + 6*
B*a*b*c/7 + B*b**3/7) + x**5*(6*A*a*b*c/5 + A*b**3/5 + 3*B*a**2*c/5 + 3*B*a
*b**2/5) + x**3*(A*a**2*c + A*a*b**2 + B*a**2*b) + x*(3*A*a**2*b + B*a**3)
```

3.101 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3} dx$

Optimal. Leaf size=162

$$-\frac{a^3A}{2x^2}+a^2\log(x)(aB+3Ab)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{3}{2}ax^2\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab\right.$$

```
[Out] -1/2*a^3*A/x^2+3/2*a*(a*b*B+A*(a*c+b^2))*x^2+1/4*(3*a*B*(a*c+b^2)+A*(6*a*b*
c+b^3))*x^4+1/6*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^6+3/8*c*(A*b*c+B*a*
c+B*b^2)*x^8+1/10*c^2*(A*c+3*B*b)*x^10+1/12*B*c^3*x^12+a^2*(3*A*b+B*a)*ln(x
)
```

Rubi [A] time = 0.23, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1251, 765}

$$a^2\log(x)(aB+3Ab)-\frac{a^3A}{2x^2}+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{1}{4}x^4\left(A\left(6abc+b^3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] -(a^3*A)/(2*x^2) + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*c
) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2
)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/1
0 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]
```

Rule 765

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(3a\left(abB+A\left(b^2+ac\right)\right)+\frac{a^3A}{x^2}+\frac{a^2(3Ab+aB)}{x}+\left(3aB\left(b^2\right.\right.\right.\right.\\ &= -\frac{a^3A}{2x^2}+\frac{3}{2}a\left(abB+A\left(b^2+ac\right)\right)x^2+\frac{1}{4}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x\end{aligned}$$

Mathematica [A] time = 0.07, size = 162, normalized size = 1.00

$$-\frac{a^3A}{2x^2}+a^2\log(x)(aB+3Ab)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{3}{2}ax^2\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] -1/2*(a^3*A)/x^2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*c
) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2
)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/1
0 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]
```

fricas [A] time = 0.75, size = 170, normalized size = 1.05

$$\frac{10\,Bc^{3}x^{14}+12\left(3\,Bbc^{2}+Ac^{3}\right)x^{12}+45\left(Bb^{2}c+(Ba+Ab)c^{2}\right)x^{10}+20\left(Bb^{3}+3\,Aac^{2}+3\left(2\,Bab+Ab^{2}\right)c\right)x^{8}+30}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(10*B*c^3*x^14 + 12*(3*B*b*c^2 + A*c^3)*x^12 + 45*(B*b^2*c + (B*a + A
*b)*c^2)*x^10 + 20*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 30*(3*
B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 180*(B*a^2*b + A*a*b^2 + A*a
^2*c)*x^4 - 60*A*a^3 + 120*(B*a^3 + 3*A*a^2*b)*x^2*log(x))/x^2
```

giac [A] time = 0.40, size = 212, normalized size = 1.31

$$\frac{1}{12}\,Bc^{3}x^{12}+\frac{3}{10}\,Bbc^{2}x^{10}+\frac{1}{10}\,Ac^{3}x^{10}+\frac{3}{8}\,Bb^{2}cx^{8}+\frac{3}{8}\,Bac^{2}x^{8}+\frac{3}{8}\,Abc^{2}x^{8}+\frac{1}{6}\,Bb^{3}x^{6}+Babcx^{6}+\frac{1}{2}\,Ab^{2}cx^{6}+\frac{1}{2}\,Aac^{2}x^{6}+\frac{3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/12*B*c^3*x^12 + 3/10*B*b*c^2*x^10 + 1/10*A*c^3*x^10 + 3/8*B*b^2*c*x^8 + 3
/8*B*a*c^2*x^8 + 3/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + B*a*b*c*x^6 + 1/2*A*b^2*
c*x^6 + 1/2*A*a*c^2*x^6 + 3/4*B*a*b^2*x^4 + 1/4*A*b^3*x^4 + 3/4*B*a^2*c*x^4
 + 3/2*A*a*b*c*x^4 + 3/2*B*a^2*b*x^2 + 3/2*A*a*b^2*x^2 + 3/2*A*a^2*c*x^2 +
1/2*(B*a^3 + 3*A*a^2*b)*log(x^2) - 1/2*(B*a^3*x^2 + 3*A*a^2*b*x^2 + A*a^3)/
x^2
```

maple [A] time = 0.01, size = 190, normalized size = 1.17

$$\frac{B\,c^{3}x^{12}}{12}+\frac{A\,c^{3}x^{10}}{10}+\frac{3Bb\,c^{2}x^{10}}{10}+\frac{3Ab\,c^{2}x^{8}}{8}+\frac{3Ba\,c^{2}x^{8}}{8}+\frac{3B\,b^{2}c\,x^{8}}{8}+\frac{Aa\,c^{2}x^{6}}{2}+\frac{A\,b^{2}c\,x^{6}}{2}+Babc\,x^{6}+\frac{B\,b^{3}x^{6}}{6}+\frac{3Aabc\,x^{4}}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x)
```

```
[Out] 1/12*B*c^3*x^12+1/10*A*x^10*c^3+3/10*B*x^10*b*c^2+3/8*A*x^8*b*c^2+3/8*B*x^8
*a*c^2+3/8*B*x^8*b^2*c+1/2*A*x^6*a*c^2+1/2*A*x^6*b^2*c+B*x^6*a*b*c+1/6*B*x^
6*b^3+3/2*A*x^4*a*b*c+1/4*A*x^4*b^3+3/4*B*x^4*a^2*c+3/4*B*x^4*a*b^2+3/2*A*x
^2*a^2*c+3/2*A*x^2*a*b^2+3/2*B*x^2*a^2*b+3*A*ln(x)*a^2*b+B*ln(x)*a^3-1/2*a^
3*A/x^2
```

maxima [A] time = 0.79, size = 167, normalized size = 1.03

$$\frac{1}{12}\,Bc^{3}x^{12}+\frac{1}{10}\left(3\,Bbc^{2}+Ac^{3}\right)x^{10}+\frac{3}{8}\left(Bb^{2}c+(Ba+Ab)c^{2}\right)x^{8}+\frac{1}{6}\left(Bb^{3}+3\,Aac^{2}+3\left(2\,Bab+Ab^{2}\right)c\right)x^{6}+\frac{1}{4}\left(3\,Bab^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + (B*a + A*b
)*c^2)*x^8 + 1/6*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^6 + 1/4*(3*B
*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^4 + 3/2*(B*a^2*b + A*a*b^2 + A*a^
2*c)*x^2 - 1/2*A*a^3/x^2 + 1/2*(B*a^3 + 3*A*a^2*b)*log(x^2)
```

mupad [B] time = 0.06, size = 166, normalized size = 1.02

$$x^4\left(\frac{3\,B\,c\,a^2}{4}+\frac{3\,B\,a\,b^2}{4}+\frac{3\,A\,c\,a\,b}{2}+\frac{A\,b^3}{4}\right)+x^6\left(\frac{B\,b^3}{6}+\frac{A\,b^2\,c}{2}+B\,a\,b\,c+\frac{A\,a\,c^2}{2}\right)+x^{10}\left(\frac{A\,c^3}{10}+\frac{3\,B\,b\,c^2}{10}\right)+\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x)
```

```
[Out] x^4*((A*b^3)/4 + (3*B*a*b^2)/4 + (3*B*a^2*c)/4 + (3*A*a*b*c)/2) + x^6*((B*b
^3)/6 + (A*a*c^2)/2 + (A*b^2*c)/2 + B*a*b*c) + x^10*((A*c^3)/10 + (3*B*b*c^
2)/10) + log(x)*(B*a^3 + 3*A*a^2*b) + x^2*((3*A*a*b^2)/2 + (3*A*a^2*c)/2 +
(3*B*a^2*b)/2) + x^8*((3*A*b*c^2)/8 + (3*B*a*c^2)/8 + (3*B*b^2*c)/8) - (A*a
^3)/(2*x^2) + (B*c^3*x^12)/12
```

sympy [A] time = 0.40, size = 197, normalized size = 1.22

$$-\frac{Aa^3}{2x^2}+\frac{Bc^3x^{12}}{12}+a^2\left(3Ab+Ba\right)\log\left(x\right)+x^{10}\left(\frac{Ac^3}{10}+\frac{3Bbc^2}{10}\right)+x^8\left(\frac{3Abc^2}{8}+\frac{3Bac^2}{8}+\frac{3Bb^2c}{8}\right)+x^6\left(\frac{Aac^2}{2}+\frac{Ab^2c}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -A*a**3/(2*x**2) + B*c**3*x**12/12 + a**2*(3*A*b + B*a)*log(x) + x**10*(A*c
**3/10 + 3*B*b*c**2/10) + x**8*(3*A*b*c**2/8 + 3*B*a*c**2/8 + 3*B*b**2*c/8)
 + x**6*(A*a*c**2/2 + A*b**2*c/2 + B*a*b*c + B*b**3/6) + x**4*(3*A*a*b*c/2
+ A*b**3/4 + 3*B*a**2*c/4 + 3*B*a*b**2/4) + x**2*(3*A*a**2*c/2 + 3*A*a*b**2
/2 + 3*B*a**2*b/2)
```

3.102 $\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4} dx$

Optimal. Leaf size=133

$$\frac{(-aBc-Abc+b^2B)\log(a+bx^2+cx^4)}{4c^3}+\frac{(2aAc^2-3abBc-Ab^2c+b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}-\frac{x^2(bB-Ac)}{2c^2}+\frac{Bx^4}{4c}$$

```
[Out] -1/2*(-A*c+B*b)*x^2/c^2+1/4*B*x^4/c+1/4*(-A*b*c-B*a*c+B*b^2)*ln(c*x^4+b*x^2
+a)/c^3+1/2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)*arctanh((2*c*x^2+b)/(-4*a*c
+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 800, 634, 618, 206, 628}

$$\frac{(-aBc-Abc+b^2B)\log(a+bx^2+cx^4)}{4c^3}+\frac{(2aAc^2-3abBc-Ab^2c+b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}-\frac{x^2(bB-Ac)}{2c^2}+\frac{Bx^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*B - A*c)*x^2)/(2*c^2) + (B*x^4)/(4*c) + ((b^3*B - A*b^2*c - 3*a*b*B*c
+ 2*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*
a*c]) + ((b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{bB-Ac}{c^2}+\frac{Bx}{c}+\frac{a(bB-Ac)+\left(b^2B-Abc-aBc\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)\\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\text{Subst}\left(\int \frac{a(bB-Ac)+\left(b^2B-Abc-aBc\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^2}\\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\left(b^2B-Abc-aBc\right)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3}-\frac{\left(b^3B-Ab^2c\right.}{}\\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\left(b^2B-Abc-aBc\right)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{\left(b^3B-Ab^2c-3abBc\right.}{}\\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\left(b^3B-Ab^2c-3abBc+2aAc^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2B-Ab\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 126, normalized size = 0.95

$$\frac{\left(-aBc-Abc+b^2B\right)\log\left(a+bx^2+cx^4\right)+\frac{2\left(-2aAc^2+3abBc+Ab^2c+b^3(-B)\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+2cx^2(Ac-bB)+Bc^2x^4}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(-(b*B) + A*c)*x^2 + B*c^2*x^4 + (2*(-(b^3*B) + A*b^2*c + 3*a*b*B*c -
2*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (
b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

fricas [A] time = 0.81, size = 421, normalized size = 3.17

$$\left[\frac{\left(Bb^2c^2-4\,Bac^3\right)x^4-2\left(Bb^3c+4\,Aac^3-\left(4\,Bab+Ab^2\right)c^2\right)x^2+\left(Bb^3+2\,Aac^2-\left(3\,Bab+Ab^2\right)c\right)\sqrt{b^2-4\,ac}\,\mathrm{l}}{4\left(b^2c^3-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((B*b^2*c^2 - 4*B*a*c^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b
^2)*c^2)*x^2 + (B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))
/(c*x^4 + b*x^2 + a)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*b^2 + A*b^3
)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((B*b^2*c^2 - 4*B*a*c
```

```
^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b^2)*c^2)*x^2 + 2*(B*b^3 +
2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*
b^2 + A*b^3)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

giac [A] time = 1.87, size = 126, normalized size = 0.95

$$\frac{Bcx^4 - 2Bbx^2 + 2Acx^2}{4c^2} + \frac{\left(Bb^2 - Bac - Abc\right)\log\left(cx^4 + bx^2 + a\right)}{4c^3} - \frac{\left(Bb^3 - 3Babc - Ab^2c + 2Aac^2\right)\arctan\left(\frac{2cx^2 \cdots}{\sqrt{-b^2+ \cdots}}\right.}{2\sqrt{-b^2 + 4ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/4*(B*b^2 - B*a*c - A*b*c)*log
(c*x^4 + b*x^2 + a)/c^3 - 1/2*(B*b^3 - 3*B*a*b*c - A*b^2*c + 2*A*a*c^2)*arc
tan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

maple [B] time = 0.01, size = 261, normalized size = 1.96

$$\frac{Bx^4}{4c} - \frac{Aa\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c} + \frac{Ab^2\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^2} + \frac{Ax^2}{2c} + \frac{3Bab\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^2} - \frac{Bb^3\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^3} - \frac{Bbx^2 \cdots}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*B*x^4/c+1/2/c*A*x^2-1/2/c^2*B*x^2*b-1/4/c^2*ln(c*x^4+b*x^2+a)*A*b-1/4/c
^2*ln(c*x^4+b*x^2+a)*a*B+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2*B-1/c/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*A+3/2/c^2/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*B+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^2+b)/(4*a*c-b^2)^(1/2))*b^3*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.46, size = 1343, normalized size = 10.10

$$x^2\left(\frac{A}{2c} - \frac{Bb}{2c^2}\right) + \frac{Bx^4}{4c} - \frac{\ln\left(cx^4 + bx^2 + a\right)\left(8Ba^2c^2 - 10Bab^2c + 8Aabc^2 + 2Bb^4 - 2Ab^3c\right)}{2\left(16ac^4 - 4b^2c^3\right)} + \frac{\mathrm{atan}\left(\frac{2c^4\left(4ac- \cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2*(A/(2*c) - (B*b)/(2*c^2)) + (B*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2*
B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*(16*a*c^4
 - 4*b^2*c^3)) + (atan((2*c^4*(4*a*c - b^2)*(x^2*(((((6*A*b^2*c^4 - 6*B*b^3
*c^3 - 4*A*a*c^5 + 10*B*a*b*c^4)/c^4 - (4*b*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*
A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(B*b^3 + 2*A
*a*c^2 - A*b^2*c - 3*B*a*b*c))/(8*c^3*(4*a*c - b^2)^(1/2)) - (b*(B*b^3 + 2*
A*a*c^2 - A*b^2*c - 3*B*a*b*c)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b
*c^2 - 10*B*a*b^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a -
 (b*((((6*A*b^2*c^4 - 6*B*b^3*c^3 - 4*A*a*c^5 + 10*B*a*b*c^4)/c^4 - (4*b*c^
2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c
^4 - 4*b^2*c^3))*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*
b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (B^2*b^5 + A^2*b^3*c^2 - 2*A*B*b^4*c -
 A*B*a^2*c^3 - A^2*a*b*c^3 - 3*B^2*a*b^3*c + 2*B^2*a^2*b*c^2 + 4*A*B*a*b^2*
c^2)/c^4 + (b*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)^2)/(2*c^4*(4*a*c -
b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + ((((8*B*a^2*c^4 + 8*A*a*b*c^4 - 8*B*a*
b^2*c^3)/c^4 - (8*a*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 -
10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a
*b*c))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*
a*b*c)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(c
*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a - (b*((((8*B*a^2*c^4 + 8*A*
a*b*c^4 - 8*B*a*b^2*c^3)/c^4 - (8*a*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c
+ 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*B*b^4 + 8*B*a^2*c
^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) -
(B^2*a*b^4 + B^2*a^3*c^2 + A^2*a*b^2*c^2 - 2*B^2*a^2*b^2*c - 2*A*B*a*b^3*c
+ 2*A*B*a^2*b*c^2)/c^4 + (a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)^2)/(c
^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2)))))/(B^2*b^6 + 4*A^2*a^2*c^4 +
A^2*b^4*c^2 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2
*c^3 + 10*A*B*a*b^3*c^2 - 12*A*B*a^2*b*c^3))*(B*b^3 + 2*A*a*c^2 - A*b^2*c -
 3*B*a*b*c))/(2*c^3*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 43.89, size = 620, normalized size = 4.66

$$\frac{Bx^4}{4c}+x^2\left(\frac{A}{2c}-\frac{Bb}{2c^2}\right)+\left(-\frac{\sqrt{-4ac+b^2}\left(-2Aac^2+Ab^2c+3Babc-Bb^3\right)}{4c^3\left(4ac-b^2\right)}-\frac{Abc+Bac-Bb^2}{4c^3}\right)\log\left(x^2+\frac{Aabc+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**4/(4*c) + x**2*(A/(2*c) - B*b/(2*c**2)) + (-sqrt(-4*a*c + b**2)*(-2*A*
a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c +
B*a*c - B*b**2)/(4*c**3))*log(x**2 + (A*a*b*c + 2*B*a**2*c - B*a*b**2 + 8*a
*c**3*(-sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(
4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3)) - 2*b**2*c**2*(
-sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*
(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3)))/(-2*A*a*c**2 + A*b**2
*c + 3*B*a*b*c - B*b**3)) + (sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c +
3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c
**3))*log(x**2 + (A*a*b*c + 2*B*a**2*c - B*a*b**2 + 8*a*c**3*(sqrt(-4*a*c +
 b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)
) - (A*b*c + B*a*c - B*b**2)/(4*c**3)) - 2*b**2*c**2*(sqrt(-4*a*c + b**2)*(
-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b
*c + B*a*c - B*b**2)/(4*c**3)))/(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**
3))
```

3.103 $\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=97

$$-\frac{\left(-2aBc-Abc+b^2B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{Bx^2}{2c}$$

```
[Out] 1/2*B*x^2/c-1/4*(-A*c+B*b)*ln(c*x^4+b*x^2+a)/c^2-1/2*(-A*b*c-2*B*a*c+B*b^2)
*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 773, 634, 618, 206, 628}

$$-\frac{\left(-2aBc-Abc+b^2B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{Bx^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x^2)/(2*c) - ((b^2*B - A*b*c - 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - ((b*B - A*c)*Log[a + b*x^2 + c*x^4])/
(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{Bx^2}{2c}+\frac{\operatorname{Subst}\left(\int \frac{-aB+(-bB+Ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c}\\
&= \frac{Bx^2}{2c}-\frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2}+\frac{\left(b^2B-Abc-2aBc\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,d
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{Bx^2}{2c}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{\left(b^2B-Abc-2aBc\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b
\end{aligned}
$$

$$
= \frac{Bx^2}{2c}-\frac{\left(b^2B-Abc-2aBc\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}
$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.96

$$
\frac{\frac{2\left(-2aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+(Ac-bB)\log\left(a+bx^2+cx^4\right)+2Bcx^2}{4c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*B*c*x^2 + (2*(b^2*B - A*b*c - 2*a*B*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*B) + A*c)*Log[a + b*x^2 + c*x^4])/(4*c^2
)
```

fricas [A] time = 0.84, size = 312, normalized size = 3.22

$$
\left[\frac{2\left(Bb^2c-4\,Bac^2\right)x^2-\left(Bb^2-(2\,Ba+Ab)c\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(Bb^3+4\,A
}{4\left(b^2c^2-4\,ac^3\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 - (B*b^2 - (2*B*a + A*b)*c)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*log(
c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 -
 2*(B*b^2 - (2*B*a + A*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(
-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*lo
g(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 1.81, size = 91, normalized size = 0.94

$$
\frac{Bx^2}{2\,c}-\frac{(Bb-Ac)\log\left(cx^4+bx^2+a\right)}{4\,c^2}+\frac{\left(Bb^2-2\,Bac-Abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c - 1/4*(B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(B*b^2 - 2*B
*a*c - A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
c^2)
```

maple [A] time = 0.00, size = 175, normalized size = 1.80

$$-\frac{Ab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c}-\frac{Ba\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c}+\frac{B\,b^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^2}+\frac{B\,x^2}{2c}+\frac{A\ln\left(c\,x^4+b\,x^2+a\right)}{4c}-\frac{Bb\ln\left(c\,x^4+\right.}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*B*x^2/c+1/4/c*ln(c*x^4+b*x^2+a)*A-1/4/c^2*ln(c*x^4+b*x^2+a)*b*B-1/c/(4*
a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*B-1/2/c/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b+1/2/c^2/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.65, size = 979, normalized size = 10.09

$$\frac{B\,x^2}{2\,c}+\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,B\,b^3-2\,A\,b^2\,c-8\,B\,a\,b\,c+8\,A\,a\,c^2\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}-\frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\left(\frac{\left(\frac{8\,A\,a\,c^3-8\,B\,a\,b\,c^2}{c^2}-\frac{8\,a\,c^2\,\left(2\,B\,b^3-2\,A\,b^2\,c\cdots\right.}{16\,a\,c^3-4\cdots}\right.}{8\,c^2\,\sqrt{4\cdots}}\cdots\right)}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (B*x^2)/(2*c) + (log(a + b*x^2 + c*x^4)*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c -
8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (atan((2*c^2*(4*a*c - b^2)*(((((8*
A*a*c^3 - 8*B*a*b*c^2)/c^2 - (8*a*c^2*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*
B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(A*b*c - B*b^2 + 2*B*a*c))/(8*c^2*(4*a*c
- b^2)^(1/2)) - (a*(A*b*c - B*b^2 + 2*B*a*c)*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2
```

```
*c - 8*B*a*b*c))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a + x^2*((((
(6*A*b*c^3 - 6*B*b^2*c^2 + 4*B*a*c^3)/c^2 - (4*b*c^2*(2*B*b^3 + 8*A*a*c^2 -
 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(A*b*c - B*b^2 + 2*B*a*c))
/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(A*b*c - B*b^2 + 2*B*a*c)*(2*B*b^3 + 8*A*
a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^
2)))/a + (b*((((6*A*b*c^3 - 6*B*b^2*c^2 + 4*B*a*c^3)/c^2 - (4*b*c^2*(2*B*b^
3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*B*b^3 +
8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (B^2*b^3 +
 A^2*b*c^2 + A*B*a*c^2 - 2*A*B*b^2*c - B^2*a*b*c)/c^2 + (b*(A*b*c - B*b^2 +
 2*B*a*c)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((((8*
A*a*c^3 - 8*B*a*b*c^2)/c^2 - (8*a*c^2*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*
B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*
b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (A^2*a*c^2 + B^2*a*b^2 - 2*A*B*a*b*c)/c^
2 + (a*(A*b*c - B*b^2 + 2*B*a*c)^2)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2
)^(1/2))))/(B^2*b^4 + A^2*b^2*c^2 + 4*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b
^2*c + 4*A*B*a*b*c^2))*(A*b*c - B*b^2 + 2*B*a*c))/(2*c^2*(4*a*c - b^2)^(1/2
))
```

sympy [B] time = 10.55, size = 434, normalized size = 4.47

$$\frac{Bx^2}{2c} + \left(-\frac{-Ac+Bb}{4c^2} - \frac{\sqrt{-4ac+b^2}\left(Abc+2Bac-Bb^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2 + \frac{2Aac - Bab - 8ac^2\left(-\frac{-Ac+Bb}{4c^2} - \frac{\sqrt{-4ac+b^2}\left(Abc\right.}{4c^2\left(4ac\right.}\right.}{Abc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**2/(2*c) + (-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a
*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c**
2*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c +
 b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*
c - B*b**2)) + (-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a
*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c**
2*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c +
 b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*
c - B*b**2))
```

3.104 $\int \frac{x\left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=71

$$\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] 1/4*B*ln(c*x^4+b*x^2+a)/c+1/2*(-2*A*c+B*b)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/c/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1247, 634, 618, 206, 628}

$$\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*
a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{a+bx+cx^2}\,dx,x,x^2\right)$$
$$= \frac{B\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c} + \frac{(-bB+2Ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}$$
$$= \frac{B\log\left(a+bx^2+cx^4\right)}{4c} - \frac{(-bB+2Ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c}$$
$$= \frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 1.00

$$\frac{B\log\left(a+bx^2+cx^4\right) - \frac{2(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-2*(b*B - 2*A*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c] + B*Log[a + b*x^2 + c*x^4])/(4*c)
```

fricas [A] time = 0.57, size = 219, normalized size = 3.08

$$\left[-\frac{(Bb-2\,Ac)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right) - \left(Bb^2-4\,Bac\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c-4\,ac^2\right)}, \frac{2\,(Bb-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((B*b - 2*A*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (B*b^2 - 4*B
*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*(B*b - 2*A*c)*sqrt(
-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*
b^2 - 4*B*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 1.71, size = 67, normalized size = 0.94

$$\frac{B\log\left(cx^4+bx^2+a\right)}{4\,c} - \frac{(Bb-2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*B*log(c*x^4 + b*x^2 + a)/c - 1/2*(B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

maple [A] time = 0.00, size = 98, normalized size = 1.38

$$\frac{A\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{Bb\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,c} + \frac{B\ln\left(c\,x^4+b\,x^2+a\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*B*ln(c*x^4+b*x^2+a)/c+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2
)^(1/2))*A-1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*b/
c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.50, size = 606, normalized size = 8.54

$$-\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,B\,b^2-8\,B\,a\,c\right)}{2\,\left(16\,a\,c^2-4\,b^2\,c\right)}-\frac{\operatorname{atan}\left(\frac{2\,\left(4\,a\,c-b^2\right)\,\left(x^2\,\left(\frac{\frac{(2\,A\,c-B\,b)\,\left(6\,B\,b\,c-4\,A\,c^2+\frac{4\,b\,c^2\,\left(2\,B\,b^2-8\,B\,a\,c\right)}{16\,a\,c^2-4\,b^2\,c}\right)}{8\,c\,\sqrt{4\,a\,c-b^2}}+\frac{b\,c\,\left(2\,B\,b^2-8\,B\,a\,c\right)\,(2\,A\,c-B\,b)}{2\,\left(16\,a\,c^2-4\,b^2\,c\right)\,\sqrt{4\,a\,c-b^2}}\cdots}{a}+\cdots\right)\cdots\right)}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] - (log(a + b*x^2 + c*x^4)*(2*B*b^2 - 8*B*a*c))/(2*(16*a*c^2 - 4*b^2*c)) - (
atan((2*(4*a*c - b^2)*(x^2*((((2*A*c - B*b)*(6*B*b*c - 4*A*c^2 + (4*b*c^2*(
2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (b*c
*(2*B*b^2 - 8*B*a*c)*(2*A*c - B*b))/(2*(16*a*c^2 - 4*b^2*c)*(4*a*c - b^2)^(
1/2)))/a + (b*(B^2*b - A*B*c - (b*(2*A*c - B*b)^2)/(2*(4*a*c - b^2)) + ((2*
B*b^2 - 8*B*a*c)*(6*B*b*c - 4*A*c^2 + (4*b*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c
^2 - 4*b^2*c)))/(2*(16*a*c^2 - 4*b^2*c))))/(2*a*(4*a*c - b^2)^(1/2))) + (((
8*B*a*c + (8*a*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c))*(2*A*c - B*b)
)/(8*c*(4*a*c - b^2)^(1/2)) + (a*c*(2*B*b^2 - 8*B*a*c)*(2*A*c - B*b))/((16*
a*c^2 - 4*b^2*c)*(4*a*c - b^2)^(1/2)))/a + (b*(B^2*a + ((2*B*b^2 - 8*B*a*c)
*(8*B*a*c + (8*a*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c)))/(2*(16*a*c
^2 - 4*b^2*c)) - (a*(2*A*c - B*b)^2)/(4*a*c - b^2)))/(2*a*(4*a*c - b^2)^(1/
2))))/(4*A^2*c^2 + B^2*b^2 - 4*A*B*b*c))*(2*A*c - B*b))/(2*c*(4*a*c - b^2)^
(1/2))
```

sympy [B] time = 3.59, size = 287, normalized size = 4.04

$$\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-Ab+2Ba-8ac\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}\right)+2b^2\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac\cdots}}{4c\left(4ac-b^2\right)}\cdots\right)\cdots}{-2Ac+Bb}\cdots\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] (B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)))*log(x**
2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*
c*(4*a*c - b**2))) + 2*b**2*(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(
4*c*(4*a*c - b**2))))/(-2*A*c + B*b)) + (B/(4*c) + (-2*A*c + B*b)*sqrt(-4*a
*c + b**2)/(4*c*(4*a*c - b**2)))*log(x**2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c)
+ (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))) + 2*b**2*(B/(4*c
) + (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))))/(-2*A*c + B*b
))
```

3.105 $\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=78

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}$$

```
[Out] A*ln(x)/a-1/4*A*ln(c*x^4+b*x^2+a)/a+1/2*(A*b-2*B*a)*arctanh((2*c*x^2+b)/(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 800, 634, 618, 206, 628}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*
a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{A}{ax}+\frac{-Ab+aB-Acx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)\\ &= \frac{A\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-Ab+aB-Acx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a}\\ &= \frac{A\log(x)}{a}-\frac{A\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}+\frac{(-Ab+2aB)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x\right.}{4a}\\ &= \frac{A\log(x)}{a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}-\frac{(-Ab+2aB)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right.}{2a}\\ &= \frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{A\log(x)}{a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}\end{aligned}$$

Mathematica [A] time = 0.10, size = 128, normalized size = 1.64

$$\frac{-\left(A\left(\sqrt{b^2-4ac}+b\right)-2aB\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+\left(A\left(b-\sqrt{b^2-4ac}\right)-2aB\right)\log\left(\sqrt{b^2-4ac}+b\right.}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*A*Sqrt[b^2 - 4*a*c]*Log[x] - (-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*Log[b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (-2*a*B + A*(b - Sqrt[b^2 - 4*a*c]))*Log[b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

fricas [A] time = 0.80, size = 249, normalized size = 3.19

$$\left[-\frac{(2\,Ba-Ab)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(Ab^2-4\,Aac\right)\log\left(cx^4+bx^2+a\right)-4\left(Ab\right.}{4\left(ab^2-4\,a^2c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*a - A*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (A*b^2 - 4*A
*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c
), -1/4*(2*(2*B*a - A*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2
 + 4*a*c)/(b^2 - 4*a*c)) + (A*b^2 - 4*A*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*
b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [**A**] time = 1.61, size = 78, normalized size = 1.00

$$-\frac{A\log\left(cx^4+bx^2+a\right)}{4\,a}+\frac{A\log\left(x^2\right)}{2\,a}+\frac{(2\,Ba-Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*A*log(c*x^4 + b*x^2 + a)/a + 1/2*A*log(x^2)/a + 1/2*(2*B*a - A*b)*arct
an((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

maple [**A**] time = 0.01, size = 105, normalized size = 1.35

$$-\frac{Ab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a}+\frac{B\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{A\ln(x)}{a}-\frac{A\ln\left(c\,x^4+b\,x^2+a\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] A*ln(x)/a-1/4*A*ln(c*x^4+b*x^2+a)/a-1/2/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*A*b+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*B
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 4.48, size = 2424, normalized size = 31.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (A*log(x))/a - (log((A*B^2*c^2 + ((A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^
2)))^(1/2))*(B^2*a*c^2 + ((A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/
2))*(4*A*b^2*c^2 + 2*c^2*x^2*(5*A*b*c - 4*B*b^2 + 10*B*a*c) - 4*B*a*b*c^2 +
 (b*c^2*(A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x
^2 - 10*a*c*x^2))/a))/(4*a) - 4*A*B*b*c^2 - B*c^2*x^2*(5*A*c + B*b)))/(4*a)
 + B^3*c^2*x^2)*(A*B^2*c^2 + ((A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))
^(1/2))*(B^2*a*c^2 + ((A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*
(4*A*b^2*c^2 + 2*c^2*x^2*(5*A*b*c - 4*B*b^2 + 10*B*a*c) - 4*B*a*b*c^2 + (b*
c^2*(A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 -
 10*a*c*x^2))/a))/(4*a) - 4*A*B*b*c^2 - B*c^2*x^2*(5*A*c + B*b)))/(4*a) + B
^3*c^2*x^2))*(2*A*b^2 - 8*A*a*c))/(2*(4*a*b^2 - 16*a^2*c)) - (atan((2*(4*a*
c - b^2)^(3/2)*(3*A*b^3 - B*a*b^2 + B*a^2*c - 8*A*a*b*c)*(A*B^2*c^2 + ((2*A
*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2
*c^2*(2*A*b^2 - 8*A*a*c))/(4*a*b^2 - 16*a^2*c)))/(2*(4*a*b^2 - 16*a^2*c)) +
 B^2*a*c^2 - 4*A*B*b*c^2))/(2*(4*a*b^2 - 16*a^2*c)) - ((A*b - 2*B*a)*(((A*b
```

```
 - 2*B*a)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/(4
*a*b^2 - 16*a^2*c)))/(4*a*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*b^2 - 8*A*a*
c)*(A*b - 2*B*a))/(2*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))/(4*a*(4*a*
c - b^2)^(1/2)) - (b^2*c^2*(2*A*b^2 - 8*A*a*c)*(A*b - 2*B*a)^2)/(8*a*(4*a*b
^2 - 16*a^2*c)*(4*a*c - b^2))))/(c^2*(A^2*b^2*c^2 + 4*B^2*a^2*c^2 - 4*A*B*a
*b*c^2)*(6*A^2*b^2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)) - (16*a^3*x^2*(((3*A*
b^3 - B*a*b^2 + B*a^2*c - 8*A*a*b*c)*(((2*A*b^2 - 8*A*a*c)*(B^2*b*c^2 + 5*A
*B*c^3 - ((2*A*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^
3))/(2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(2*(
4*a*b^2 - 16*a^2*c))))/(2*(4*a*b^2 - 16*a^2*c)) - B^3*c^2 + ((A*b - 2*B*a)*
(((A*b - 2*B*a)*(((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3))/(2*(4*a*b^
2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(4*a*(4*a*c - b^2)
^(1/2)) + ((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a))/(8*
a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))/(4*a*(4*a*c - b^2)^(1/2)) + (
(2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a)^2)/(32*a^2*(4*a
*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(8*a^3*c^2*(6*A^2*b^2 - B^2*a^2 - 25*A^2*
a*c + A*B*a*b)) + ((((12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a)^3)/(64*a^3*(4*
a*c - b^2)^(3/2)) - ((2*A*b^2 - 8*A*a*c)*(((A*b - 2*B*a)*(((2*A*b^2 - 8*A*a
*c)*(12*b^3*c^2 - 40*a*b*c^3))/(2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*
A*b*c^3 + 20*B*a*c^3))/(4*a*(4*a*c - b^2)^(1/2)) + ((2*A*b^2 - 8*A*a*c)*(12
*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a))/(8*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^
2)^(1/2))))/(2*(4*a*b^2 - 16*a^2*c)) + ((A*b - 2*B*a)*(B^2*b*c^2 + 5*A*B*c^
3 - ((2*A*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3))/(
2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(2*(4*a*b
^2 - 16*a^2*c))))/(4*a*(4*a*c - b^2)^(1/2)))*(3*A*b^4 + 10*A*a^2*c^2 - B*a*
b^3 - 14*A*a*b^2*c + 3*B*a^2*b*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(6*A^2*b^
2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)))*(4*a*c - b^2)^(3/2))/(A^2*b^2*c^2 + 4
*B^2*a^2*c^2 - 4*A*B*a*b*c^2) + (2*(4*a*c - b^2)*(((2*A*b^2 - 8*A*a*c)*(((A
*b - 2*B*a)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/
(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*b^2 - 8*A*
a*c)*(A*b - 2*B*a))/(2*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(4*a*
b^2 - 16*a^2*c)) + ((A*b - 2*B*a)*(((2*A*b^2 - 8*A*a*c)*(4*A*b^2*c^2 - 4*B*
a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/(4*a*b^2 - 16*a^2*c)))/(2*(4*a*
b^2 - 16*a^2*c)) + B^2*a*c^2 - 4*A*B*b*c^2))/(4*a*(4*a*c - b^2)^(1/2)) - (b
^2*c^2*(A*b - 2*B*a)^3)/(16*a^2*(4*a*c - b^2)^(3/2)))*(3*A*b^4 + 10*A*a^2*c
^2 - B*a*b^3 - 14*A*a*b^2*c + 3*B*a^2*b*c))/(c^2*(A^2*b^2*c^2 + 4*B^2*a^2*c
^2 - 4*A*B*a*b*c^2)*(6*A^2*b^2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)))*(A*b - 2
*B*a))/(2*a*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.106 $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=112

$$-\frac{\left(-2aAc-abB+Ab^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

```
[Out] -1/2*A/a/x^2-(A*b-B*a)*ln(x)/a^2+1/4*(A*b-B*a)*ln(c*x^4+b*x^2+a)/a^2-1/2*(-
2*A*a*c+A*b^2-B*a*b)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^
2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 800, 634, 618, 206, 628}

$$-\frac{\left(-2aAc-abB+Ab^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -A/(2*a*x^2) - ((A*b^2 - a*b*B - 2*a*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B
)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aB}{a^2x}+\frac{-abB+A\left(b^2-ac\right)+(Ab-aB)cx}{a^2\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)$$
$$= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{-abB+A\left(b^2-ac\right)+(Ab-aB)cx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2}$$
$$= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2}+\frac{\left(-abB+A\right.}{}$$
$$= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(-abB+A\left(b^2-2a\right.\right.}{}$$
$$= -\frac{A}{2ax^2}+\frac{\left(abB-A\left(b^2-2ac\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)}{}$$

Mathematica [A] time = 0.22, size = 186, normalized size = 1.66

$$\frac{\frac{\left(A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-aB\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(A\left(b\sqrt{b^2-4ac}+2ac-b^2\right)+aB\left(b-\sqrt{b^2-4ac}\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^2 + 4*(-(A*b) + a*B)*Log[x] + ((-(a*B*(b + Sqrt[b^2 - 4*a*c]))
+ A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/Sqrt[b^2 - 4*a*c] + ((a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c + b
*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c
])/(4*a^2)
```

fricas [A] time = 1.01, size = 385, normalized size = 3.44

$$\left[\frac{\left(Bab-Ab^2+2\,Aac\right)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-2\,Aab^2+8\,Aa^2c-\left(Bab^2-Ab^3\right.}{4\left(a^2b^2-4\,a^3c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((B*a*b - A*b^2 + 2*A*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*
c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
 - 2*A*a*b^2 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(
```

```
c*x^4 + b*x^2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(x))/
((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(B*a*b - A*b^2 + 2*A*a*c)*sqrt(-b^2 + 4*a
*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*A*a*b^2
 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(c*x^4 + b*x^
2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(x))/((a^2*b^2 -
4*a^3*c)*x^2)]
```

giac [A] time = 1.87, size = 124, normalized size = 1.11

$$-\frac{(Ba-Ab)\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{(Ba-Ab)\log\left(x^2\right)}{2\,a^2}-\frac{\left(Bab-Ab^2+2\,Aac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^2}-\frac{Bax^2-Abx^2+A}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(B*a - A*b)*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*log(x^2)/a^2
- 1/2*(B*a*b - A*b^2 + 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(s
qrt(-b^2 + 4*a*c)*a^2) - 1/2*(B*a*x^2 - A*b*x^2 + A*a)/(a^2*x^2)
```

maple [A] time = 0.01, size = 191, normalized size = 1.71

$$-\frac{Ac\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a}+\frac{A\,b^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^2}-\frac{Bb\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a}-\frac{Ab\ln(x)}{a^2}+\frac{Ab\ln\left(c\,x^4+b\,x^2+a\right)}{4a^2}+\frac{B\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*A/a/x^2-1/a^2*ln(x)*A*b+1/a*ln(x)*B+1/4/a^2*ln(c*x^4+b*x^2+a)*A*b-1/4/
a*ln(c*x^4+b*x^2+a)*B-1/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^
(1/2))*A*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*
A*b^2-1/2/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.85, size = 3729, normalized size = 33.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - A/(2*a*x^2) - (log(x)*(A*b - B*a))/a^2 - (log(((A^3*c^5*x^2)/a^3 - (((((4
*b*c^2*(A*a*c - A*b^2 + B*a*b))/a - (2*c^3*x^2*(A*b^2 + 10*A*a*c - 5*B*a*b)
)/a + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(B*a - A*b + a^2*(-(2*A*a*c - A
*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/a^2)*(B*a - A*b + a^2*(-(2*A*a
*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(4*a^2) + (A*c^3*(A*a*c
- 4*A*b^2 + 4*B*a*b))/a^2 - (A*c^4*x^2*(6*A*b - 5*B*a))/a^2)*(B*a - A*b + a
```

```
^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(4*a^2) + (A^
2*c^4*(A*b - B*a))/a^3)*((((((2*c^3*x^2*(A*b^2 + 10*A*a*c - 5*B*a*b))/a - (
4*b*c^2*(A*a*c - A*b^2 + B*a*b))/a + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*
(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))
/a^2)*(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(
1/2)))/(4*a^2) + (A*c^3*(A*a*c - 4*A*b^2 + 4*B*a*b))/a^2 - (A*c^4*x^2*(6*A*
b - 5*B*a))/a^2)*(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c
 - b^2)))^(1/2)))/(4*a^2) + (A^3*c^5*x^2)/a^3 + (A^2*c^4*(A*b - B*a))/a^3))
*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2))
- (atan((16*a^6*x^2*((((((5*A*B*a^2*c^4 - 6*A^2*a*b*c^4)/a^3 - (((20*A*a^3*
c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c
^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2
*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2
*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2
*b^2)) - (A^3*c^5)/a^3 + (((((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c
^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c
 - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(
4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*
b^2 + B*a*b)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^5*(4*a*c -
 b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a
*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*b^2 + B*a*
b)^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(32*a^7*(4*a*c - b^2)*(
16*a^3*c - 4*a^2*b^2)))*(3*A*b^4 + A*a^2*c^2 - 3*B*a*b^3 - 9*A*a*b^2*c + 8*
B*a^2*b*c))/(8*a^3*c^2*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2*
b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)) + (((((5*A*B*a^2*c^4
 - 6*A^2*a*b*c^4)/a^3 - (((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)
/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c -
8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*
c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a
^2*(4*a*c - b^2)^(1/2)) - (((((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*
c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*
c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/
(4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A
*b^2 + B*a*b)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^5*(4*a*c
- b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*
A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*
A*a*c - A*b^2 + B*a*b)^3)/(64*a^9*(4*a*c - b^2)^(3/2)))*(6*A*b^5 - 20*B*a^3
*c^2 - 6*B*a*b^4 - 30*A*a*b^3*c + 26*A*a^2*b*c^2 + 28*B*a^2*b^2*c))/(16*a^3
*c^2*(4*a*c - b^2)^(1/2)*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^
2*b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)))*(4*a*c - b^2)^(3/
2))/(4*A^2*a^2*c^4 + A^2*b^4*c^2 + B^2*a^2*b^2*c^2 - 4*A^2*a*b^2*c^3 - 2*A*
B*a*b^3*c^2 + 4*A*B*a^2*b*c^3) + (a^3*(4*a*c - b^2)*((((((4*A*a^3*b*c^3 - 4
*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b^2*c^2*(2*A*b^3 - 2*B*a*b^2 +
 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*A*a*c - A*b^2 + B*a*b))
/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*a*c - A*b^2 + B*a*b)*(2*A*b^3
- 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a*(4*a*c - b^2)^(1/2)*(16*a^3*c -
4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c -
4*a^2*b^2)) + (((A^2*a^2*c^4 - 4*A^2*a*b^2*c^3 + 4*A*B*a^2*b*c^3)/a^3 + (((
4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b^2*c^2*(2*A*
b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*A*b^3
- 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c
- A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(2*A*a*c - A*b^2 +
 B*a*b)^3)/(16*a^5*(4*a*c - b^2)^(3/2)))*(6*A*b^5 - 20*B*a^3*c^2 - 6*B*a*b^
4 - 30*A*a*b^3*c + 26*A*a^2*b*c^2 + 28*B*a^2*b^2*c))/(c^2*(4*A^2*a^2*c^4 +
A^2*b^4*c^2 + B^2*a^2*b^2*c^2 - 4*A^2*a*b^2*c^3 - 2*A*B*a*b^3*c^2 + 4*A*B*a
^2*b*c^3)*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2*b^2 + 12*A*B*
a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)) - (2*a^3*(4*a*c - b^2)^(3/2)*((A^
3*b*c^4 - A^2*B*a*c^4)/a^3 - (((A^2*a^2*c^4 - 4*A^2*a*b^2*c^3 + 4*A*B*a^2*b
*c^3)/a^3 + (((4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*
```

```
a*b^2*c^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*
b^2))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b
^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b
^2)) + (((((4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b
^2*c^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2
))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*a
*c - A*b^2 + B*a*b)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a*(4*
a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2
*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*a*c - A*b^2 + B*a*b)^2*(2*A*b^3 - 2*B
*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^3*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2
)))*(3*A*b^4 + A*a^2*c^2 - 3*B*a*b^3 - 9*A*a*b^2*c + 8*B*a^2*b*c))/(c^2*(4*
A^2*a^2*c^4 + A^2*b^4*c^2 + B^2*a^2*b^2*c^2 - 4*A^2*a*b^2*c^3 - 2*A*B*a*b^3
*c^2 + 4*A*B*a^2*b*c^3)*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2
*b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)))*(2*A*a*c - A*b^2 +
 B*a*b))/(2*a^2*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.107 $\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=261

$$\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(-A*c+B*b)*x/c^2+1/3*B*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*(b^2*B-A*b*c-a*B*c+(-2*A*a*c^2+A*b^2*c+3*B*a*b*c-B*b^3)/(-4*a
*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^
(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*B-A*b*c-a*B*c+(2*A*a*c^2-A
*b^2*c-3*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] time = 1.49, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1279, 1166, 205}

$$\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + ((b^2*B - A*b*c - a*B*c - (b^3*B -
 A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + ((b^2*B - A*b*c - a*B*c + (b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx = \frac{Bx^3}{3c} - \frac{\int \frac{x^2\left(3aB+3(bB-Ac)x^2\right)}{a+bx^2+cx^4}\,dx}{3c}$$

$$= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\int \frac{3a(bB-Ac)+3\left(b^2B-Abc-aBc\right)x^2}{a+bx^2+cx^4}\,dx}{3c^2}$$

$$= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2} + \ldots$$

$$= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \ldots$$

Mathematica [A] time = 0.40, size = 327, normalized size = 1.25

$$\frac{\left(-Abc\sqrt{b^2-4ac}-2aAc^2+b^2B\sqrt{b^2-4ac}-aBc\sqrt{b^2-4ac}+3abBc+Ab^2c+b^3(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-(b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + ((-(b^3*B) + A*b^2*c + 3*a*b*B*c -
 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 - 4*a*c] - a*B*c*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B - A*
b^2*c - 3*a*b*B*c + 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 -
4*a*c] - a*B*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

fricas [B] time = 2.89, size = 5140, normalized size = 19.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*c*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c
^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*
c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^
3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2
+ 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*
A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2
*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b
^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4 + (5*A^3
*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a^2*b^2 -
3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*b^4)*c)*x
 + sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A
^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 + A^3*b^4)
*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 +
 3*A*B^2*b^6)*c - (B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^2 + A*b^3
```

```
)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a
*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3
+ A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2
*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2
*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(B^2*b^5 - (4
*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*
B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6
- 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*
b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 +
 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 +
 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^
2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5
 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2
- (5*B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2
*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3
*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*
b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*
b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c
)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*
a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*
a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2
*b^3 - 3*A^2*B^2*a*b^4)*c)*x - sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^
2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2
 + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2
*B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c - (B*b^4*c^5 + 4*(2*B*a^2 + A*a
*b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2
*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B
^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^
2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*
b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a
*c^11)))*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*
B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*
sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c
^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b
^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*
b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4
*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*
sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b +
 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c - (b^2*c^5 - 4*a*
c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b
^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 +
A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A
^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(
3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*
log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^
2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2
- (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*b^4)*c)*x + sqrt(1/2)*(B^3*b
^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*
B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2*
b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c + (
B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*sqrt((B^4*b^
8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^
4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*
(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (
11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A
*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*
b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b
^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 +
4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*
```

```
b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 +
 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6
*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11))
)/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A
^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*
A*B*b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a
^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2
*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*
b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^
5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c
^11)))/(b^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4
 + (5*A^3*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a
^2*b^2 - 3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*
b^4)*c)*x - sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^
2*b + 5*A^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 +
 A^3*b^4)*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^
3*a*b^5 + 3*A*B^2*b^6)*c + (B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^
2 + A*b^3)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*
b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3
*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*
a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6
)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(B^2
*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*
c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4
*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A
*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*
a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*
a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^
7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*(B*b - A*c)*x)/c^2
```

giac [B] time = 3.79, size = 4391, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (
2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4
*a*c)*a^2*c^4)*B*c^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3
 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
```

```
*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5
 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2
- 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(c) - 2*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a
*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a
*c)*a^2*b*c^4)*B*abs(c) - (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*
c)*a*b*c^6)*A + (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*
a*b^2*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c
^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 +
 a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b
^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 3
2*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 +
 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 -
 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*c^2 - 2*(sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4
*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)
*A*abs(c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4
+ sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4
```

```
*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*B*abs(c) - (2*b^5*c^5 - 12*a*b
^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 -
4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*A + (2*b^6*c^4 - 14*a*b^4*c^5 + 2
4*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*
c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 -
 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 -
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a
*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c^
3 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^
5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/3*(B*c^2*x^3
 - 3*B*b*c*x + 3*A*c^2*x)/c^3
```

maple [B] time = 0.05, size = 825, normalized size = 3.16

$$\frac{\sqrt{2}\,Aa\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,Aa\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,Ab^2\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*B*x^3/c+1/c*A*x-1/c^2*b*B*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+1/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*A*b^2+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B-1/2/c^2*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b^2*B-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B+1/2/c^2/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*B-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+1/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*A*b^2-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B+1/2/c^2*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B-
3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B+1/2/c^2/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b^3*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bcx^3-3\,(Bb-Ac)x}{3\,c^2}-\frac{-\int\frac{Bab-Aac+\left(Bb^2-(Ba+Ab)c\right)x^2}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(B*c*x^3 - 3*(B*b - A*c)*x)/c^2 - integrate(-(B*a*b - A*a*c + (B*b^2 -
(B*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

mupad [B] time = 1.59, size = 10177, normalized size = 38.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x*(A/c - (B*b)/c^2) - atan(((((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3
 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5
*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2
+ A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 -
A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3
- 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(
-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2
*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*
B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*
(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c -
b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2
- 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a
*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*
c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 +
 A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2
) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A
^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 -
 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-
(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((16*A*a^2*c^5 - 4*A*a*b^2*c^4
+ 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^
2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B
^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A
^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A
*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2
- 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^
2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3
*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c
^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2
*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^
3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(
16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(B^2*b^6 + 2*A^2*a^2*c^4
 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*
b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2
*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^
2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^
2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*
```

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B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 -
 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((16*A*a^2*c^5 -
 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16
*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A
*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a
*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a
^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16
*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*
(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c +
 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 +
 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 -
 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4
*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6
+ 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2
*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^
3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*
B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a
^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*
b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^
3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*
A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a
*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((16
*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*
b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)
^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5
*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2
) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a
*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^
(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2
*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^
2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2
*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2
*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) +
16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2
*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9
*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*
A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^
(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*
c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2)
 - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)
^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*
b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2) + (2*(B^3*a^4*c - B^3*a^3*b^2 + A*B^2*a^2*b^3 + A^2*B*a^3*c^2 + A^3*a^
2*b*c^2 - 2*A^2*B*a^2*b^2*c))/c^3))*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4
*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4
 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c
- b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(
4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(
1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8
```

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*a*b^2*c^6)))^(1/2)*2i - atan(((((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*
c^3 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*
b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c
^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4
 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c
^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*
c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16
*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 -
B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b
^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16
*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c
^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2
*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c
 - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +
 b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c
^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^
2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b
^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^
2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4
+ A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^
3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c
*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((16*A*a^2*c^5 - 4*A*a*b^2*c
^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-
(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 2
5*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4
*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 1
2*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 3
6*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c
^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3
)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 +
 A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*
b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2
)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*
b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*
b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B
*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(B^2*b^6 + 2*A^2*a^2*
c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2
*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(
B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25
*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12
*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36
*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^
2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((16*A*a^2*c^
5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 -
 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) -
2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B
^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^
2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^
2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) +
 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^
3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*
c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2
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*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^
3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^
3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*
b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b
^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*
b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b
*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2
*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^
2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2
*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2
*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) +
16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + ((
(16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*
(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)
^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*
b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(
1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2
)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*
B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2)
- 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) -
 B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*
A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*
B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
 + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) +
 (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c
+ 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 -
10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^
3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b
^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1
/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B
*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))
)^(1/2) + (2*(B^3*a^4*c - B^3*a^3*b^2 + A*B^2*a^2*b^3 + A^2*B*a^3*c^2 + A^3
*a^2*b*c^2 - 2*A^2*B*a^2*b^2*c))/c^3))*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(
-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*
c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a
*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*
(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3
)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5
- 8*a*b^2*c^6)))^(1/2)*2i + (B*x^3)/(3*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.108 $\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=208

$$-\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}$$

```
[Out] B*x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*B-A*c+(
A*b*c+2*B*a*c-B*b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*B-
A*c+(-A*b*c-2*B*a*c+B*b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1279, 1166, 205}

$$-\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((b*B - A*c - (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - ((b*B - A*c + (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{Bx}{c} - \frac{\int \frac{aB+(bB-Ac)x^2}{a+bx^2+cx^4}\,dx}{c}$$
$$= \frac{Bx}{c} - \frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} - \frac{\left(bB-Ac+\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-}$$
$$= \frac{Bx}{c} - \frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(bB-Ac+\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{}{\sqrt{b}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.16, size = 251, normalized size = 1.21

$$-\frac{\left(-Ac\sqrt{b^2-4ac}+bB\sqrt{b^2-4ac}+2aBc+Abc+b^2(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(-Ac\sqrt{b^2-4ac}+bB\sqrt{b^2-4ac}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((-(b^2*B) + A*b*c + 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2*B - A*b*
c - 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 0.88, size = 2632, normalized size = 12.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*
B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A
^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 +
2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2
 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b
^2)*c)*x + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b +
A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2
*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)
*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3
*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*
B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A
^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2
*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
 sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^
2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B
*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*
B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A
*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*
c)*x - sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*
B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*
```

```
B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3
 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3
)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*
a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B
^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^
4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqr
t(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c
 - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*
c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*
b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3
*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x
 + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^
2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*
b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (
B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)
/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b
+ 2*A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a
 + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*
b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/
2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c - (
b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3
+ (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)
*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3
 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x - s
qrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c
^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b +
A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*
a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^
2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*
A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2
*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2
+ 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + 2*B*x)/c
```

giac [B] time = 3.50, size = 3179, normalized size = 15.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] B*x/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b
^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2
*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 - 2*(sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*
a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
```

```
*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2
*c^4)*B*abs(c) - (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 - 12*a*b^3*c^
5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^
3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 -
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c
)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sq
rt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16
*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*((2*b^4*c^3 - 16
*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c
^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c
)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a
^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(
b^2 - 4*a*c)*a*b*c^3)*B*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*
b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 +
2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*abs(c) - (2*b^4*c^5
- 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2
 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*
a*b*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))
/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b
^2*c^5 - 4*a^2*c^6)*c^2)
```

maple [B] time = 0.03, size = 560, normalized size = 2.69

$$\frac{\sqrt{2}\, Ab \arctanh\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, Ab \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, Ba \arctanh\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+}\right.}}\right.}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] B*x/c-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*A*b+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*a*B-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+1/2*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*A+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/2/c*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*b*B+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B-1/2/c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bx}{c}+\frac{-\int \frac{(Bb-Ac)x^2+Ba}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*x/c + integrate(-((B*b - A*c)*x^2 + B*a)/(c*x^4 + b*x^2 + a), x)/c
```

mupad [B] time = 1.26, size = 6366, normalized size = 30.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (B*x)/c - atan(((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a
*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b
^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3
- 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A
*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c -
b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2
*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 1
2*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(B^2*b^4 - 2*A^2*a*c^3
 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^
2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^
2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 -
```

```
7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*
B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c + (2*x*(4*b
^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4
*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^
2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c
 - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)
^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*
a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(B^2*b^4
- 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c +
 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*
A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2
*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/
c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a
*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*
B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2
) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^
2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3
*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-
(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1
/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*
B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B
*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2)
 + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*B*a^2*c^3 - 4*B*a
*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4
*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*
B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5
 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)
^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c +
B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c -
b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) + (2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*
b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*
c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B
*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(A^3*a*c^2
 - B^3*a^2*b + A*B^2*a*b^2 + A*B^2*a^2*c - 2*A^2*B*a*b*c))/c))*(-(B^2*b^5 +
 A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3
)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^
2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i - atan(((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a
*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b
^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3
- 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A
*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c -
```

```
b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2
*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 1
2*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(B^2*b^4 - 2*A^2*a*c^3
 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^
2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^
2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 -
 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*
B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c + (2*x*(4*b
^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4
*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^
2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c
 - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)
^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*
a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(B^2*b^4
 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c +
 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*
A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2
*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/
c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a
*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*
B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2
) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^
2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3
*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-
(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1
/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*
B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B
*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2)
 + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*B*a^2*c^3 - 4*B*a
*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4
*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*
B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5
 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)
^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c -
B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c -
b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) + (2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*
b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*
c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B
*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(A^3*a*c^2
 - B^3*a^2*b + A*B^2*a*b^2 + A*B^2*a^2*c - 2*A^2*B*a*b*c))/c))*(-(B^2*b^5 +
 A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3
)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^
2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^
```

```
2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.109 $\int \frac{A+Bx^2}{a+bx^2+cx^4} dx$

Optimal. Leaf size=172

$$\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] 1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(B+(2*A*c-B*b)/(
-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(
x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B+(-2*A*c+B*b)/(-4*a*c+b^2
)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1166, 205}

$$\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((B +
(b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^2}{a+bx^2+cx^4} dx = \frac{1}{2}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx+\frac{1}{2}\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4}$$

$$=\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.09, size = 173, normalized size = 1.01

$$\frac{\frac{\left(B\sqrt{b^2-4ac}+2Ac-bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((b*B - 2*A*c + B*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqr
t[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

fricas [B] time = 0.87, size = 1569, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*
sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x + sqr
t(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3
 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c
 + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*
c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(B^2*a*b -
 (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a
^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*
c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2
 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^
3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt(
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c -
4*a^2*c^2))) + 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*
c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A
^4*c^2)*x + sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*
c - (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 -
 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A
*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*s
qrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 -
 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)
)*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - sqrt(1/2)*(A*B^2*a
*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c - (4*(2*B*a^3 - A*a^2*b)*c^2
 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2
*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4
*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3
)))/(a*b^2*c - 4*a^2*c^2)))
```

giac [B] time = 2.42, size = 1400, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*B)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2
- 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2
+ a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*
c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b
^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*A + 2*(2*a*b^2*c^2 - 8*a^2*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*B)*arcta
n(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*
a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

maple [B] time = 0.02, size = 328, normalized size = 1.91

$$-\frac{\sqrt{2}\, Ac \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, Ac \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, Bb \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b}\cdots\right.}}\right.}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A-1/2*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1
/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-c/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*A+1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.00, size = 4109, normalized size = 23.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((((-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*
(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c -
 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3
*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*
c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2
*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)
 - 4*A*b^2*c^2 + 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4
*A*B*b*c^2))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^
2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b
*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i +
((-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c
 - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a
*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*
(8*b^3*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^
2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 -
 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))
^(1/2) - 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c
^2))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4
*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A
*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((-(B^2*
a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^
3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/
(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2 - 32*a*b*c^
3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a
*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B
*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*A*b^2*c^2 +
 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(
B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b
^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2
*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - ((-(B^2*a*b^3 + B^2
*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) +
16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*
c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*(8*b^3*c^2 - 32*a*b
*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(
4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*
A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 16*A*a*c^3
) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(B^2*a*b^3 +
 B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2
) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*
a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 2*A^2*B*c^2 + 2*B^3*a*c - 2*A*
B^2*b*c))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c
*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c
```

```
- 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*2i - ata
n((((-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*
a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*
B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2
- 32*a*b*c^3)*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A
^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*
b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*
A*b^2*c^2 + 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*
b*c^2))*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(
-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c -
4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i + ((-(B
^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^
2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*
c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*(8*b^
3*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3
*c + A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^
2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2
) - 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*
(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c
- b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*
b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((-(B^2*a*b^3
 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3)^(1
/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(1
6*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2 - 32*a*b*c^3)*(-
(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c -
b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^
2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*A*b^2*c^2 + 16*A
*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(B^2*a
*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3
)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(
8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - ((-(B^2*a*b^3 - B^2*a*(-
(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*
B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 -
 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*(8*b^3*c^2 - 32*a*b*c^3)
*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c
 - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a
*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 16*A*a*c^3) + x
*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(B^2*a*b^3 - B^2*
a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 1
6*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c
^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 2*A^2*B*c^2 + 2*B^3*a*c - 2*A*B^2*b
*c))*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4
*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A
*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*2i
```

sympy [A] time = 16.96, size = 314, normalized size = 1.83

$$\mathrm{RootSum}\left(t^4\left(256a^3c^3-128a^2b^2c^2+16ab^4c\right)+t^2\left(-16A^2abc^2+4A^2b^3c+64ABa^2c^2-16ABab^2c-16B^2a^2b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**3 - 128*a**2*b**2*c**2 + 16*a*b**4*c) + _t**2*(-
16*A**2*a*b*c**2 + 4*A**2*b**3*c + 64*A*B*a**2*c**2 - 16*A*B*a*b**2*c - 16*
B**2*a**2*b*c + 4*B**2*a*b**3) + A**4*c**2 - 2*A**3*B*b*c + 2*A**2*B**2*a*c
 + A**2*B**2*b**2 - 2*A*B**3*a*b + B**4*a**2, Lambda(_t, _t*log(x + (-32*_t
**3*A*a**2*b*c**2 + 8*_t**3*A*a*b**3*c + 64*_t**3*B*a**3*c**2 - 16*_t**3*B*
a**2*b**2*c - 4*_t*A**3*a*c**2 + 2*_t*A**3*b**2*c - 6*_t*A**2*B*a*b*c + 12*
```

```
_t*A*B**2*a**2*c - 2*_t*B**3*a**2*b)/(-A**4*c**2 + A**3*B*b*c - A*B**3*a*b
+ B**4*a**2))))
```

3.110 $\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=189

$$-\frac{\sqrt{c}\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}$$

```
[Out] -A/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
A+(A*b-2*B*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/
2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A+(-A*b+2
*B*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1281, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{A}{ax} - \frac{\int \frac{Ab-aB+Acx^2}{a+bx^2+cx^4}\,dx}{a}$$

$$= -\frac{A}{ax} - \frac{\left(c\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a} - \frac{\left(c\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,d}{2a}$$

$$= -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [**A**] time = 0.29, size = 206, normalized size = 1.09

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}+b\right)-2aB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}-b\right)+2aB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{2A}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((2*A)/x + (Sqrt[2]*Sqrt[c]*(-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*a*B + A*(-b + Sqrt[b^2 - 4*a
*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a
```

fricas [**B**] time = 0.99, size = 2914, normalized size = 15.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 -
3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^
2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*
B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A
^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2
*a*b^2 - A^3*B*b^3)*c)*x + sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2
*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*
a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c - (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^
5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^
2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*
A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*
a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4
*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^
2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c))
)/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^
2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A
*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*
(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2
 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*
B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x - sqrt(1/2)*(B^3*a^3*b^2 - 3*
A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (
```

```
4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c - (B*a^4*b^3
 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 -
 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2
- 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt
(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2
 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b
^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)
/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(B^2*a^2
*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4*a^4*c
)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2
- 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*
c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x + sqrt(1
/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3
 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*
a*b^3)*c + (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^
2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A
^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*
b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*
A^2*a*b)*c - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*
a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*
a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/
2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c
 - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^
4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (
A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3
*B*b^3)*c)*x - sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A
^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A
^2*B*a^2*b^2 - 5*A^3*a*b^3)*c + (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2
*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b
+ A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*
b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B
^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A
^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
 4*a^4*c))) - 2*A)/(a*x)
```

giac [B] time = 3.57, size = 2805, normalized size = 14.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -A/(a*x) - 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*a^2 + 2*(sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*
c)*a^2*b*c^2)*A*abs(a) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4
```

```
 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 16*a^3*b^2*c^2
 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 32*a^4*c^3 + 2*(b^2
- 4*a*c)*a^2*b^2*c - 8*(b^2 - 4*a*c)*a^3*c^2)*B*abs(a) + (2*a^2*b^4*c^2 - 8
*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2
*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2
*(b^2 - 4*a*c)*a^2*b^2*c^2)*A - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b*c^2)*B)*
arctan(2*sqrt(1/2)*x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4
 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a
^4*c^3)*abs(a)*abs(c)) + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A
*a^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c +
 8*(b^2 - 4*a*c)*a^2*b*c^2)*A*abs(a) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*a^2*b^4*c + 16*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
16*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*a^4
*c^3 - 2*(b^2 - 4*a*c)*a^2*b^2*c + 8*(b^2 - 4*a*c)*a^3*c^2)*B*abs(a) + (2*a
^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*A - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^
3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*
a^3*b*c^2)*B)*arctan(2*sqrt(1/2)*x/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*
c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3
*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(c))
```

maple [B] time = 0.03, size = 353, normalized size = 1.87

$$\frac{\sqrt{2}\, A b c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, A b c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}-\frac{\sqrt{2}\, B c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -A/a/x+1/2/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*A*b-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B-1/2/a*c*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*A+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-c/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((A*c*x^2 - B*a + A*b)/(c*x^4 + b*x^2 + a), x)/a - A/(a*x)
```

mupad [B] time = 1.35, size = 6335, normalized size = 33.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - atan(((x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b
*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a
^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c
- A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A
*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^
2 - 8*a^4*b^2*c)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2
*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1
/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3
)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
 - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2*c^2))*(-(A
^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c
- b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-
(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*
a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^
2*c)))^(1/2)*1i + (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4
*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/
2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^
2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b
*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*B*a^6*c^3 + x*(32*a^6*b*c^3 - 8*a^5*
b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*
a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c
 - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*
A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c
^2 - 8*a^4*b^2*c)))^(1/2) - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c^2 - 4*B*a^5*b^2*
c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2
*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c -
A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B
*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b
```

```
^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b
^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*
c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12
*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c
^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(
-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^
2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a
*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2) - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*
B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2)
 + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*
a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c
^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^
2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*
a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A
*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b
*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2
*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*B*a^6*c^3 + x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2
)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^
2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A
^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c
^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3
)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 -
 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*
a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*A^3*a^3*c^4 + 2*A*B^2*a^4*c^3 -
 2*A^2*B*a^3*b*c^3))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^
(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7
*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^
2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^
4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - atan(((x*(4*A^2*a^4*c^4 - 4*B^2*
a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 -
 A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*
B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2)
 - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) +
12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(x*(32*a^
6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)
^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2
 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^
2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^
3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 -
 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(
4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16
*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3
*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b
^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i + (x*(4*A^2*a^4*c^
4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2
*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1
/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3
)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
*(16*B*a^6*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3
- A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A
*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2
) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) +
```

```
12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*A*a
^5*b*c^3 + 4*A*a^4*b^3*c^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 - A
^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*
a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) -
 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12
*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*
A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2
*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c -
b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4
*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*
c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*
c)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 - A^2
*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*
b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4
*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A
*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*B*a^6*c^
3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a
^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2
) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^
3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^
(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) -
 (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) +
 (-(A^2*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4
*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a
*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*
(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)*(16*B*a^6*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b
^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^
2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a
*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c
- b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)
))^(1/2) - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5
+ B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^
3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c
- b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b
^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^
(1/2) + 2*A^3*a^3*c^4 + 2*A*B^2*a^4*c^3 - 2*A^2*B*a^3*b*c^3))*(-(A^2*b^5 +
B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)
^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c -
b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2
)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1
/2)*2i - A/(a*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.111 $\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)} dx$

Optimal. Leaf size=271

$$-\frac{\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(-b\right.\right.}{\sqrt{2}a^2\sqrt{b^2-}}$$

```
[Out] -1/3*A/a/x^3+(A*b-B*a)/a^2/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2))*c^(1/2)*(a*B*(b+(-4*a*c+b^2)^(1/2))-A*(b^2-2*a*c+b*(-4*a*c+b^2
)^(1/2)))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*a
rctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(b-(-4*a
*c+b^2)^(1/2))-A*(b^2-2*a*c-b*(-4*a*c+b^2)^(1/2)))/a^2*2^(1/2)/(-4*a*c+b^2)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1281, 1166, 205}

$$-\frac{\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(-b\right.\right.}{\sqrt{2}a^2\sqrt{b^2-}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[c]*(a*B*(b + Sqrt[b^2 - 4*a*c])
- A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) - A*(b^2 - 2*a*c - b*Sqrt[b
^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}dx = -\frac{A}{3ax^3}-\frac{\int \frac{3(Ab-aB)+3Acx^2}{x^2\left(a+bx^2+cx^4\right)}dx}{3a}$$

$$= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{\int \frac{3\left(Ab^2-abB-aAc\right)+3(Ab-aB)cx^2}{a+bx^2+cx^4}dx}{3a^2}$$

$$= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{\left(c\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(b^2-2ac-b\sqrt{b^2-4ac}\right)\right)\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}}}{2a^2\sqrt{b^2-4ac}}$$

$$= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}-\frac{\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\tan^{-1}}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.32, size = 267, normalized size = 0.99

$$\frac{-\frac{3\sqrt{2}\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}+2ac-b^2\right)+aB\left(b-\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{\sqrt{b^2-}}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^3 + (6*A*b - 6*a*B)/x - (3*Sqrt[2]*Sqrt[c]*(a*B*(b + Sqrt[b^2 -
 4*a*c]) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (3*Sqrt[2]*Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c +
b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

fricas [B] time = 2.32, size = 5442, normalized size = 20.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*
B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c +
(a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6
 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b +
 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^
3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A
^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))
/(a^5*b^2 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4
- (B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3
 + (B^4*a^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x + s
qrt(1/2)*(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a
^4*c^4 + (4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b
 - 25*A*B^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b
^3 - 18*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*c - (B*a^6*b^4 - A*
a^5*b^5 + 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt
```

```
((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b
^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B
^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2
*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*
B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*
A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*
a^2*b^2 + 5*A^2*a*b^3)*c + (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*
a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^
2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b +
 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b
^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)
*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqr
t(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (
3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c + (a^5*b^2 - 4*a^6*c)*sqrt((B^
4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 +
 A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a
^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4
)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^
2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*
(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b +
 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^
2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x - sqrt(1/2)*(B^3*a^3*b^5 - 3*A*
B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A
^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2
*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A
^2*B*a^2*b^5 - 8*A^3*a*b^6)*c - (B*a^6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A*a
^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*
b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^
2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*
A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 -
 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/
(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B
*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c + (
a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6
- 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b +
3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3
*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^
2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/
(a^5*b^2 - 4*a^6*c))) + 3*sqrt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^
4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2
+ 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5
+ 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^
5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*
B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A
*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^1
0*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*
b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a
*b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^
3*B*b^5)*c^2)*x + sqrt(1/2)*(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7
- A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)
*c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)
*c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*
c + (B*a^6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7
*A*a^6*b^3)*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*
A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^
4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a
^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^
2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt
(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3
```

```
*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*sqrt((B^4
*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 +
A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^
6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)
*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2
*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sq
rt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5
*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c - (a^5*b^2
- 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*
B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^
3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b
^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^
3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2
 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4 - (B^4*a^
4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3 + (B^4*a
^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x - sqrt(1/2)*
(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a^4*c^4 +
(4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b - 25*A*B
^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b^3 - 18*A
*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*c + (B*a^6*b^4 - A*a^5*b^5 +
 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt((B^4*a^4
*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*
a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 -
8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2
 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5
 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4
 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 +
 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 +
 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5
 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B
^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*
B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10
*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 6*(B*a - A*b)*x^2 - 2*A*a)/(a^2*
x^3)
```

giac [B] time = 3.35, size = 2870, normalized size = 10.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a^2*b^2*
c^3 - 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32
*a^3*c^4 + 24*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c
 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 -
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 3*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c
)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4
*a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*A - (sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*
```

```
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 +
16*a^2*b^3*c^2 + 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 - 32*a^3*b*c^3 - 12*a^2*b^2*c^3 + 16*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2
*(b^2 - 4*a*c)*a*b^2*c^2 + 4*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(2*sqrt(1/2)*x
/sqrt((a^2*b + sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c -
 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c))
+ 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
5*c + 2*b^6*c + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^4*c^2 - 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^3 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 48*a^2*b^
2*c^3 + 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 -
32*a^3*c^4 - 24*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^5 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4
*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 +
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a
*c)*b^4*c + 10*(b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 -
 4*a*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b*c^3)*A - (sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c -
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2
- 16*a^2*b^3*c^2 - 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^3 + 32*a^3*b*c^3 + 12*a^2*b^2*c^3 - 16*a^3*c^4 + sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2 +
 2*(b^2 - 4*a*c)*a*b^2*c^2 - 4*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(2*sqrt(1/2)
*x/sqrt((a^2*b - sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c
 - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)
) - 1/3*(3*B*a*x^2 - 3*A*b*x^2 + A*a)/(a^2*x^3)
```

maple [B] time = 0.03, size = 611, normalized size = 2.25

$$\frac{\sqrt{2}\, A c^2 \arctanh\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\, \sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, a}+\frac{\sqrt{2}\, A c^2 \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\, \sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, a}-\frac{\sqrt{2}\, A b^2 c \arctanh\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+\cdots}\right.}}\right.}{2\sqrt{-4ac+b^2}\, \sqrt{\left(-b+\sqrt{-4ac+\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3*A/a/x^3+1/a^2/x*A*b-1/a/x*B-1/2/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b+1/a*c^
2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*A*b^2+1/2/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1/2/a*c/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B+1/2/a^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b+1/a*c
^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*A*b^2-1/2/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1/2/a*c/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\int \frac{(Ba-Ab)cx^2+Bab-Ab^2+Aac}{cx^4+bx^2+a}\,dx}{a^2} - \frac{3\,(Ba-Ab)x^2+Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(-((B*a - A*b)*c*x^2 + B*a*b - A*b^2 + A*a*c)/(c*x^4 + b*x^2 + a),
 x)/a^2 - 1/3*(3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)
```

mupad [B] time = 2.19, size = 10101, normalized size = 37.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - (A/(3*a) - (x^2*(A*b - B*a))/a^2)/x^3 - atan((((-(A^2*b^7 + B^2*a^2*b^5 +
 A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
6*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2
*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^
2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) +
 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 1
6*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10
*b^3*c^2)*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A
*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*
a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*
a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^
3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*
A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B
*a^10*b*c^3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*
A^2*a^8*c^5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2
*a^8*b^2*c^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*
b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 +
 A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2
) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 1
2*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 -
```

```
3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1
/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^
4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i - ((-(A^2*b^7 + B^2*a^2*b^5 + A^2*
b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B
*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*
b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b
^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A
*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7
*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*
c^2)*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*
b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b
^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b
*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1
/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a
*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10
*b*c^3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a
^8*c^5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*
b^2*c^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 +
 A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
6*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2
*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^
2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) +
 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 1
6*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i)/(((-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(
-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*
c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2
 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*
(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^
2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2
 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*
(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 +
 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-
(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3
 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) -
 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*
(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^
3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*A^2*a^8*c^
5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c
^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*
b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B
*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*
b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b
^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A
*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7
*c^2 - 8*a^6*b^2*c)))^(1/2) + ((-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c
- b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*
A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a
^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c
 - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c
+ 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*
b^2*c)))^(1/2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b
^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*
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a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*
a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*
a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c
 - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A*
a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a^8*c^5 - 4*B^
2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A
*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4
*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3
 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 -
B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(
4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b
^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8
*a^6*b^2*c)))^(1/2) + 2*B^3*a^8*c^4 + 2*A^2*B*a^7*c^5 - 2*A^3*a^6*b*c^5 - 4
*A*B^2*a^7*b*c^4 + 2*A^2*B*a^6*b^2*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4
*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*
(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^
4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c
^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*
a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^
2 - 8*a^6*b^2*c)))^(1/2)*2i - atan((((-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(
4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4
*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^
3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 +
 B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*
b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 -
8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-
(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 2
5*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4
*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 -
7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 3
6*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-
(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3
+ 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*A^2*a^8*c^5
- 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3
 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^
4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2
*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a
^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*
c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2
*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B
*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c
^2 - 8*a^6*b^2*c)))^(1/2)*1i - ((-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9
*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*
a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c
 - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6
*b^2*c)))^(1/2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*
b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2
*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c
- b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2
*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B
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*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*
c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A
*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a^8*c^5 - 4*B
^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*
A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(
4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4
*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^
3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 +
 B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*
b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 -
8*a^6*b^2*c)))^(1/2)*1i)/(((-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^
2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2
)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*
a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c
*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b
^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*
A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*
c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b^7 +
 B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*
b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2
)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*
b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*
b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b
^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A*a^8*
b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*A^2*a^8*c^5 - 4*B^2*a^
9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A*B*a
^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9
*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*
a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c
 - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6
*b^2*c)))^(1/2) + ((-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1
/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c
- 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*
c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(
1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*
b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/
2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b^7 + B^2*a^2
*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2
- A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c +
12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2
+ 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(
1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b
^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A*a^8*b^4*c^2
- 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a^8*c^5 - 4*B^2*a^9*c^4 +
2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A*B*a^7*b^3*c
^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^
3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b
^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-
(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B
```

```
*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c))
)^(1/2) + 2*B^3*a^8*c^4 + 2*A^2*B*a^7*c^5 - 2*A^3*a^6*b*c^5 - 4*A*B^2*a^7*b
*c^4 + 2*A^2*B*a^6*b^2*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c -
b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b
^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^
2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3
*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c -
 b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c -
4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^
2*c)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.112 $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=212

$$-\frac{(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3(b^2-4ac)^{3/2}}+\frac{x^2(-6aBc-Abc+2b^2B)}{2c^2(b^2-4ac)}-\frac{x^4(x^2(-2aBc-A}{2c(b^2-4}$$

```
[Out] 1/2*(-A*b*c-6*B*a*c+2*B*b^2)*x^2/c^2/(-4*a*c+b^2)-1/2*x^4*(a*(-2*A*c+B*b)+(
-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(6*A*a*b*c^2-
A*b^3*c+12*B*a^2*c^2-12*B*a*b^2*c+2*B*b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-1/4*(-A*c+2*B*b)*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A] time = 0.38, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 818, 773, 634, 618, 206, 628}

$$-\frac{(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3(b^2-4ac)^{3/2}}+\frac{x^2(-6aBc-Abc+2b^2B)}{2c^2(b^2-4ac)}-\frac{x^4(x^2(-2aBc-A}{2c(b^2-4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*b^2*B - A*b*c - 6*a*B*c)*x^2)/(2*c^2*(b^2 - 4*a*c)) - (x^4*(a*(b*B - 2*
A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2)*ArcT
anh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(3/2)) - ((2*b*B
 - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3(A+Bx)}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$

$$=-\frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\operatorname{Subst}\left(\int \frac{x\left(2a(bB-2Ac)+\left(2b^2B-Abc-6aBc\right)x\right.}{a+bx+cx^2}\right.}{2c\left(b^2-4ac\right)}$$

$$=\frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\operatorname{Subst}\left(\int -\right.}{}$$

$$=\frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(2bB-Ac)}{}$$

$$=\frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(2bB-Ac)}{}$$

$$=\frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(2b^4B-Ab\right.}{}$$

Mathematica [A] time = 0.28, size = 208, normalized size = 0.98

$$\frac{-\frac{2\left(a^2c\left(2c\left(A+Bx^2\right)-3bB\right)+ab\left(-bc\left(A+4Bx^2\right)+3Ac^2x^2+b^2B\right)+b^3x^2(bB-Ac)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2\left(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+(}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*B*c*x^2 - (2*(b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a
*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) - (2*(2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2
)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (-2*b*B
+ A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

fricas [B] time = 1.04, size = 1323, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c
^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*b
+ A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2 +
7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + (2*B*a*b^4 + (2*B*b^4*c + 6*(
2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2*b
)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*x^
2 - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^
2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2
*(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^5 - 16*A*a^3*c^3 + (2*B*b^5*c - 16*A*
a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3 - (16*B*a*b^3 + A*b^4)*c^2)*x^4 + 8*(
4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6 - 16*A*a^2*b*c^3 + 8*(4*B*a^2*b^2 + A
*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*x^2 - (16*B*a^2*b^3 + A*a*b^4)*c)*log
(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2),
 -1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c
^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*b
+ A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2 +
7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + 2*(2*B*a*b^4 + (2*B*b^4*c + 6
*(2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2
*b)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*
x^2 - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^
5 - 16*A*a^3*c^3 + (2*B*b^5*c - 16*A*a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3
- (16*B*a*b^3 + A*b^4)*c^2)*x^4 + 8*(4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6
- 16*A*a^2*b*c^3 + 8*(4*B*a^2*b^2 + A*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*
x^2 - (16*B*a^2*b^3 + A*a*b^4)*c)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^
2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

giac [A] time = 1.65, size = 239, normalized size = 1.13

$$\frac{Bx^2}{2\,c^2}+\frac{\left(2\,Bb^4-12\,Bab^2c-Ab^3c+12\,Ba^2c^2+6\,Aabc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(b^2c^3-4\,ac^4\right)\sqrt{-b^2+4\,ac}}+\frac{2\,Bb^3x^4-8\,Babcx^4-Ab^2cx^4+4\,Aac}{4\,\left(cx^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c^2 + 1/2*(2*B*b^4 - 12*B*a*b^2*c - A*b^3*c + 12*B*a^2*c^2 + 6*A*
a*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt
(-b^2 + 4*a*c)) + 1/4*(2*B*b^3*x^4 - 8*B*a*b*c*x^4 - A*b^2*c*x^4 + 4*A*a*c^
2*x^4 + A*b^3*x^2 - 4*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + A*a*b^2)/((
```

```
c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/4*(2*B*b - A*c)*log(c*x^4 + b*x
^2 + a)/c^3
```

maple [B] time = 0.02, size = 689, normalized size = 3.25

$$\frac{3Aab\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)c}-\frac{A\,b^3x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)c^2}+\frac{B\,a^2x^2}{\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)c}-\frac{2Ba}{\left(c\,x^4+b\,x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*B*x^2/c^2+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*A*b-1/2/c^2/(c*x^4+b*
x^2+a)/(4*a*c-b^2)*x^2*A*b^3+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*B-2/c^
2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^2*B+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^
2)*x^2*b^4*B+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*A-1/2/c^2/(c*x^4+b*x^2+a)*
a/(4*a*c-b^2)*A*b^2-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b*B+1/2/c^3/(c*
x^4+b*x^2+a)*a/(4*a*c-b^2)*b^3*B+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*A-1/4/
c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*A*b^2-2/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)
*a*b*B+1/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3*B-3/c/(4*a*c-b^2)^(3/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*a*b-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*B+6/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*B*a*b^2+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*b^3*A-1/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*b^4*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.83, size = 2282, normalized size = 10.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c*(4*a*c - b^2)) + (x^2*(
B*b^4 + 2*B*a^2*c^2 - A*b^3*c + 3*A*a*b*c^2 - 4*B*a*b^2*c))/(2*c*(4*a*c - b
^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + (B*x^2)/(2*c^2) + (log(a + b*x^2 + c*
x^4)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 -
 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(256*a^3*c^6 -
 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)) - (atan(((8*a*c^5*(4*a*c - b^
2)^3 - 2*b^2*c^4*(4*a*c - b^2)^3)*(x^2*(((((24*B*a^2*c^5 - 6*A*b^3*c^4 + 12
*B*b^4*c^3 + 28*A*a*b*c^5 - 56*B*a*b^2*c^4)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c
^6 - 32*a*b*c^7)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A
*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(4
*a*c^5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5
)))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c))/(8*c^3
*(4*a*c - b^2)^(3/2)) + ((8*b^3*c^6 - 32*a*b*c^7)*(2*B*b^4 + 12*B*a^2*c^2 -
 A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c
```

```
 - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192
*B*a^2*b^3*c^2))/(16*c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4)*(256*a^3*c
^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*(
(4*B^2*b^5 + A^2*b^3*c^2 - 4*A*B*b^4*c - 6*A*B*a^2*c^3 - 5*A^2*a*b*c^3 - 20
*B^2*a*b^3*c + 12*B^2*a^2*b*c^2 + 20*A*B*a*b^2*c^2)/(4*a*c^5 - b^2*c^4) + (
((24*B*a^2*c^5 - 6*A*b^3*c^4 + 12*B*b^4*c^3 + 28*A*a*b*c^5 - 56*B*a*b^2*c^4
)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 - 32*a*b*c^7)*(4*B*b^7 + 128*A*a^3*c^4
- 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^
2*c^3 + 192*B*a^2*b^3*c^2))/(2*(4*a*c^5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3
 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c -
 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B
*a^2*b^3*c^2))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5
)) - (((b^3*c^6)/2 - 2*a*b*c^7)*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b
*c^2 - 12*B*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^3*(4*a*c^5 - b^2*c^4))))/(2*a*(4
*a*c - b^2)^(3/2))) + ((((8*A*a*c^4 - 16*B*a*b*c^3)/c^4 - (8*a*c^2*(4*B*b^7
 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*
c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(256*a^3*c^6 - 4*b^6*c^3 + 48*
a*b^4*c^4 - 192*a^2*b^2*c^5))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c
^2 - 12*B*a*b^2*c))/(8*c^3*(4*a*c - b^2)^(3/2)) - (a*(2*B*b^4 + 12*B*a^2*c^
2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^
6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 +
192*B*a^2*b^3*c^2))/(c*(4*a*c - b^2)^(3/2)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*
b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*((((8*A*a*c^4 - 16*B*a*
b*c^3)/c^4 - (8*a*c^2*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c +
 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/
(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5))*(4*B*b^7 + 128*
A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 9
6*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^
4*c^4 - 192*a^2*b^2*c^5)) - (A^2*a*c^2 + 4*B^2*a*b^2 - 4*A*B*a*b*c)/c^4 + (
a*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)^2)/(c^4*(
4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(4*B^2*b^8 + A^2*b^6*c^2 + 14
4*B^2*a^4*c^4 - 4*A*B*b^7*c + 36*A^2*a^2*b^2*c^4 + 192*B^2*a^2*b^4*c^2 - 28
8*B^2*a^3*b^2*c^3 - 48*B^2*a*b^6*c - 12*A^2*a*b^4*c^3 - 168*A*B*a^2*b^3*c^3
 + 48*A*B*a*b^5*c^2 + 144*A*B*a^3*b*c^4))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c
 + 6*A*a*b*c^2 - 12*B*a*b^2*c))/(2*c^3*(4*a*c - b^2)^(3/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.113 $\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{x^2\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(4aAc^2-6abBc+b^3B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] -1/2*x^2*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+
b*x^2+a)+1/2*(4*A*a*c^2-6*B*a*b*c+B*b^3)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(
1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/4*B*ln(c*x^4+b*x^2+a)/c^2
```

Rubi [A] time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 818, 634, 618, 206, 628}

$$\frac{\left(4aAc^2-6abBc+b^3B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}-\frac{x^2\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x^2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((b^3*B - 6*a*b*B*c + 4*a*A*c^2)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[a + b*x^2 + c*
x^4])/(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
```

```
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$

$$=-\frac{x^2\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\text{Subst}\left(\int \frac{a(bB-2Ac)+B\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c\left(b^2-4ac\right)}$$

$$=-\frac{x^2\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}-\frac{\left(b^3B-\right.}{}$$

$$=-\frac{x^2\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{\left(b^3B-6abBc+\right.}{}$$

$$=-\frac{x^2\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^3B-6abBc+4aAc^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right.}{2c^2\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.19, size = 160, normalized size = 1.09

$$\frac{-\frac{2\left(2a^2Bc+a\left(bc\left(A+3Bx^2\right)-2Ac^2x^2+b^2(-B)\right)+b^2x^2(Ac-bB)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2\left(4aAc^2-6abBc+b^3B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*
(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(b^3*B - 6*a*b*B*
c + 4*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/
2) + B*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

fricas [B] time = 0.86, size = 849, normalized size = 5.78

$$\left[\frac{2\,Bab^4 + 8\left(2\,Ba^3 + Aa^2b\right)c^2 + 2\left(Bb^5 - 8\,Aa^2c^3 + 6\left(2\,Ba^2b + Aab^2\right)c^2 - \left(7\,Bab^3 + Ab^4\right)c\right)x^2 - \left(Bab^3 - 6\,Ba\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*c^3 + 6*(2
*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 - (B*a*b^3 - 6*B*a^2*b
*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B*b^4 - 6*B*a
*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b
^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(6*B
*a^2*b^2 + A*a*b^3)*c + (B*a*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c
- 8*B*a*b^2*c^2 + 16*B*a^2*c^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2
)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*
c^4)*x^2), 1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*
c^3 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 + 2*(B*a*b^3
 - 6*B*a^2*b*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B
*b^4 - 6*B*a*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2
+ b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(6*B*a^2*b^2 + A*a*b^3)*c + (B*a
*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c - 8*B*a*b^2*c^2 + 16*B*a^2*c
^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a
))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

giac [A] time = 1.62, size = 194, normalized size = 1.32

$$-\frac{\left(Bb^3 - 6\,Babc + 4\,Aac^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^2c^2 - 4\,ac^3\right)\sqrt{-b^2 + 4\,ac}} + \frac{B\log\left(cx^4 + bx^2 + a\right)}{4\,c^2} - \frac{Bb^2cx^4 - 4\,Bac^2x^4 - Bb^3x^2 + 2\,Babcx^2}{4\left(cx^4 + bx^2 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^3 - 6*B*a*b*c + 4*A*a*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a)/
c^2 - 1/4*(B*b^2*c*x^4 - 4*B*a*c^2*x^4 - B*b^3*x^2 + 2*B*a*b*c*x^2 + 2*A*b^
2*c*x^2 - 4*A*a*c^2*x^2 - B*a*b^2 + 2*A*a*b*c)/((c*x^4 + b*x^2 + a)*(b^2*c^
2 - 4*a*c^3))
```

maple [B] time = 0.01, size = 286, normalized size = 1.95

$$\frac{2Aa\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}} - \frac{3Bab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c} + \frac{B\,b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(4ac-b^2\right)^{\frac{3}{2}}c^2} + \frac{Ba\ln\left(c\,x^4 + b\,x^2 + a\right)}{\left(4ac-b^2\right)c} - \frac{B\,b^2\ln\left(c\,x^4 + \right.}{4\left(4ac - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-1/c^2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(4*a*c-b^2)*x^2+a*(A*b*c+2*
B*a*c-B*b^2)/c^2/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)/c*ln(c*x^4+b*x^
2+a)*a*B-1/4/(4*a*c-b^2)/c^2*ln(c*x^4+b*x^2+a)*b^2*B+2/(4*a*c-b^2)^(3/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*A-3/(4*a*c-b^2)^(3/2)/c*arctan((2*c*x
```

```
^2+b)/(4*a*c-b^2)^(1/2))*a*b*B+1/2/(4*a*c-b^2)^(3/2)/c^2*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*b^3*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.22, size = 1527, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^2*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c^2*(4*a*c - b^2)) -
(a*(A*b*c - B*b^2 + 2*B*a*c))/(2*c^2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) -
(log(a + b*x^2 + c*x^4)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*
b^2*c^2))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) -
(atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4*a*c - b^2)^3)*((((8*B*a + (8
*a*c^2*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(256*a^
3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(B*b^3 + 4*A*a*c^2 - 6
*B*a*b*c))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c)
*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/((4*a*c - b^2
)^(3/2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*
a*c - b^2)) - x^2*(((((6*B*b^3*c^2 + 8*A*a*c^4 - 28*B*a*b*c^3)/(4*a*c^3 - b
^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c
 + 96*B*a^2*b^2*c^2))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*
a*b^4*c^3 - 192*a^2*b^2*c^4)))*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c))/(8*c^2*(4*a
*c - b^2)^(3/2)) + ((8*b^3*c^4 - 32*a*b*c^5)*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c
)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(16*c^2*(4*a
*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3
 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) + (b*((B^2*b^3 + 2*A*B*a*c^2 - 5*B^
2*a*b*c)/(4*a*c^3 - b^2*c^2) + (((6*B*b^3*c^2 + 8*A*a*c^4 - 28*B*a*b*c^3)/(
4*a*c^3 - b^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*B*b^6 - 128*B*a^3*c^3 - 2
4*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^
6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a
*b^4*c + 96*B*a^2*b^2*c^2))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 19
2*a^2*b^2*c^4)) - (((b^3*c^4)/2 - 2*a*b*c^5)*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c
)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3 - b^2*c^2))))/(2*a*(4*a*c - b^2)^(3/2)))
 + (b*(((8*B*a + (8*a*c^2*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^
2*b^2*c^2))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*
B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(2*(256*a^3*c^5 -
 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) + (B^2*a)/c^2 - (a*(B*b^3 + 4
*A*a*c^2 - 6*B*a*b*c)^2)/(c^2*(4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2)))
)/(B^2*b^6 + 16*A^2*a^2*c^4 + 36*B^2*a^2*b^2*c^2 - 12*B^2*a*b^4*c + 8*A*B*a
*b^3*c^2 - 48*A*B*a^2*b*c^3))*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c))/(2*c^2*(4*a*
c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.114 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] 1/2*(-a*(-2*A*c+B*b)-(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^
2+a)-(A*b-2*B*a)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1251, 777, 618, 206}

$$-\frac{x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(2*c*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3 \left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2} dx = \frac{1}{2} \text{Subst}\left(\int \frac{x(A+Bx)}{\left(a+bx+cx^2\right)^2} dx, x, x^2\right)$$

$$= -\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(Ab-2aB)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(Ab-2aB)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+2\right.}{b^2-4ac}$$

$$= -\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 1.04

$$\frac{-2ac\left(A+Bx^2\right)+abB+bx^2(bB-Ac)}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} - \frac{(Ab-2aB)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*
x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-
b^2 + 4*a*c)^(3/2)
```

fricas [B] time = 0.68, size = 538, normalized size = 5.03

$$\left[-\frac{Bab^3 + 8\,Aa^2c^2 + \left(Bb^4 + 4\left(2\,Ba^2 + Aab\right)c^2 - \left(6\,Bab^2 + Ab^3\right)c\right)x^2 - \left((2\,Ba - Ab)c^2x^4 + \left(2\,Bab - Ab^2\right)cx^2 \right.}{2\left(ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + \left(b^4c^2 - 8\,ab^2c^3 + 16\,a^2c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(B*a*b^3 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^
2 + A*b^3)*c)*x^2 - ((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B
*a^2 - A*a*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c
 + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(2*B*a^2*b + A
*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(B*a*b^3
 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^2 + A*b^3)*c)*
x^2 - 2*((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B*a^2 - A*a*b
)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)) - 2*(2*B*a^2*b + A*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*x^2)]
```

giac [A] time = 1.72, size = 120, normalized size = 1.12

$$-\frac{(2\,Ba - Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}} - \frac{Bb^2x^2 - 2\,Bacx^2 - Abcx^2 + Bab - 2\,Aac}{2\left(cx^4+bx^2+a\right)\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -(2*B*a - A*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt
(-b^2 + 4*a*c)) - 1/2*(B*b^2*x^2 - 2*B*a*c*x^2 - A*b*c*x^2 + B*a*b - 2*A*a*
c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
```

maple [A] time = 0.01, size = 158, normalized size = 1.48

$$-\frac{Ab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{2Ba\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{-\frac{\left(Abc+2aBc-b^2B\right)x^2}{\left(4ac-b^2\right)c}-\frac{(2Ac-bB)a}{\left(4ac-b^2\right)c}}{2c\,x^4+2b\,x^2+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^2-a*(2*A*c-B*b)/(4*a*c-b^2)/c)/
(c*x^4+b*x^2+a)-1/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A
*b+2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.32, size = 283, normalized size = 2.64

$$-\frac{\frac{x^2\left(-B\,b^2+A\,c\,b+2\,B\,a\,c\right)}{2\,c\left(4\,a\,c-b^2\right)}+\frac{a\,(2\,A\,c-B\,b)}{2\,c\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}-\frac{\mathrm{atan}\left(\frac{\left(4\,a\,c-b^2\right)^4\left(x^2\left(\frac{(A\,b-2\,B\,a)\left(A\,b\,c^2-2\,B\,a\,c^2\right)}{a\left(4\,a\,c-b^2\right)^{7/2}}+\frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)(A\,b-2\,B\,a)^2\left(b^3-4\,a\,b\,c\right)}{2\,a\left(4\,a\,c-b^2\right)^{13/2}}\right)-\frac{2\,c^2\,(A\,b-2\,B\,a)^2\,(}{\left(4\,a\,c-b^2\right)^{1}}\right.}{2\,A^2\,b^2\,c^2-8\,A\,B\,a\,b\,c^2+8\,B^2\,a^2\,c^2}\right.}{\left(4\,a\,c-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^2*(A*b*c - B*b^2 + 2*B*a*c))/(2*c*(4*a*c - b^2)) + (a*(2*A*c - B*b))/
(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - (atan(((4*a*c - b^2)^4*(x^2*(((A
*b - 2*B*a)*(A*b*c^2 - 2*B*a*c^2))/(a*(4*a*c - b^2)^(7/2)) + ((2*b^3*c^2 -
8*a*b*c^3)*(A*b - 2*B*a)^2*(b^3 - 4*a*b*c))/(2*a*(4*a*c - b^2)^(13/2))) - (
2*c^2*(A*b - 2*B*a)^2*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(11/2)))/(2*A^2*b^2*c^
2 + 8*B^2*a^2*c^2 - 8*A*B*a*b*c^2))*(A*b - 2*B*a))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 5.38, size = 394, normalized size = 3.68

$$-\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-Ab+2Ba)\log\left(x^2+\frac{-Ab^2+2Bab-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-Ab+2Ba)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-Ab+2Ba)-b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-A}{-2Abc+4Bac}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a)*log(x**2 + (-A*b**2 + 2*B*a*b -
16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) + 8*a*b**2*c*sqrt(-1
/(4*a*c - b**2)**3)*(-A*b + 2*B*a) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(-A*b
+ 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 + sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*
a)*log(x**2 + (-A*b**2 + 2*B*a*b + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*
(-A*b + 2*B*a) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) + b**
4*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 + (-2*
A*a*c + B*a*b + x**2*(-A*b*c - 2*B*a*c + B*b**2))/(8*a**2*c**2 - 2*a*b**2*c
 + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c))
```

3.115 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=94

$$-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2aB-\left(x^2(bB-2Ac)\right)+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[Out] 1/2*(-A*b+2*a*B+(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-(-2*A*c+B*b)
*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1247, 638, 618, 206}

$$-\frac{-2aB+x^2(-(bB-2Ac))+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$
$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{(bB-2Ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}$$
$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{(bB-2Ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$
$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Mathematica [A] time = 0.07, size = 101, normalized size = 1.07

$$\frac{\frac{2(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{B(2a+bx^2)-A(b+2cx^2)}{a+bx^2+cx^4}}{2(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((B*(2*a + b*x^2) - A*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + (2*(b*B - 2*A*c)
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*
a*c))
```

fricas [B] time = 0.69, size = 474, normalized size = 5.04

$$\left[\frac{2\,Bab^2 - Ab^3 + \left(Bb^3 + 8\,Aac^2 - 2\left(2\,Bab + Ab^2\right)c\right)x^2 + \left(\left(Bbc - 2\,Ac^2\right)x^4 + Bab - 2\,Aac + \left(Bb^2 - 2\,Abc\right)x^2\right)}{2\left(ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + \left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)x^4 + \left(b^5\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B*a*b + A*b^2)*c)*x^2 +
 ((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2 - 2*A*b*c)*x^2)*sqrt(b^2
 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2
 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^2 - A*a*b)*c)/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2), 1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B
*a*b + A*b^2)*c)*x^2 - 2*((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2
- 2*A*b*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)
/(b^2 - 4*a*c)) - 4*(2*B*a^2 - A*a*b)*c)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)
*x^2)]
```

giac [A] time = 1.37, size = 102, normalized size = 1.09

$$\frac{(Bb-2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} + \frac{Bbx^2 - 2\,Acx^2 + 2\,Ba - Ab}{2\left(cx^4+bx^2+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(
-b^2 + 4*a*c)) + 1/2*(B*b*x^2 - 2*A*c*x^2 + 2*B*a - A*b)/((c*x^4 + b*x^2 +
a)*(b^2 - 4*a*c))
```

maple [A] time = 0.01, size = 127, normalized size = 1.35

$$\frac{2Ac\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}-\frac{Bb\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{Ab-2Ba+(2Ac-bB)\,x^2}{2\left(4ac-b^2\right)\left(c\,x^4+b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*((2*A*c-B*b)*x^2+A*b-2*a*B)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)^(
3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*c-1/(4*a*c-b^2)^(3/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.30, size = 264, normalized size = 2.81

$$\frac{\frac{A\,b-2\,B\,a}{2\left(4\,a\,c-b^2\right)}+\frac{x^2\,(2\,A\,c-B\,b)}{2\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\mathrm{atan}\left(\frac{\left(x^2\left(\frac{(2\,A\,c-B\,b)\left(2\,A\,c^3-B\,b\,c^2\right)}{a\left(4\,a\,c-b^2\right)^{7/2}}+\frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)(2\,A\,c-B\,b)^2\left(b^3-4\,a\,b\,c\right)}{2\,a\left(4\,a\,c-b^2\right)^{13/2}}\right)-\frac{2\,c^2\,(2\,A\,c-B\,b)^2\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{11/2}}\right)\left(4\,a\,c-b^2\right)^4}{8\,A^2\,c^4-8\,A\,B\,b\,c^3+2\,B^2\,b^2\,c^2}\right)\,(2}{\left(4\,a\,c-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((A*b - 2*B*a)/(2*(4*a*c - b^2)) + (x^2*(2*A*c - B*b))/(2*(4*a*c - b^2)))/(
a + b*x^2 + c*x^4) + (atan(((x^2*(((2*A*c - B*b)*(2*A*c^3 - B*b*c^2))/(a*(4
*a*c - b^2)^(7/2)) + ((2*b^3*c^2 - 8*a*b*c^3)*(2*A*c - B*b)^2*(b^3 - 4*a*b*
c))/(2*a*(4*a*c - b^2)^(13/2))) - (2*c^2*(2*A*c - B*b)^2*(b^3 - 4*a*b*c))/(
4*a*c - b^2)^(11/2))*(4*a*c - b^2)^4)/(8*A^2*c^4 + 2*B^2*b^2*c^2 - 8*A*B*b*
c^3))*(2*A*c - B*b))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 3.36, size = 374, normalized size = 3.98

$$\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-2Ac+Bb)\log\left(x^2+\frac{-2Abc+Bb^2-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-2Ac+Bb)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-2Ac+Bb)-b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(-2Ac-}{-4Ac^2+2Bbc}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b)*log(x**2 + (-2*A*b*c + B*b**2 - 1
6*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) + 8*a*b**2*c*sqrt(-1/
(4*a*c - b**2)**3)*(-2*A*c + B*b) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c
 + B*b))/(-4*A*c**2 + 2*B*b*c))/2 - sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*
b)*log(x**2 + (-2*A*b*c + B*b**2 + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*
(-2*A*c + B*b) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) + b**
4*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b))/(-4*A*c**2 + 2*B*b*c))/2 + (A*
b - 2*B*a + x**2*(2*A*c - B*b))/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b
**2*c) + x**2*(8*a*b*c - 2*b**3))
```

3.116 $\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=150

$$\frac{\left(4a^2Bc+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}-\frac{-A\left(b^2-2ac\right)-\left(cx^2(Ab-2aB)\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] 1/2*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a
)+1/2*(4*a^2*B*c+A*(-6*a*b*c+b^3))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/
a^2/(-4*a*c+b^2)^(3/2)+A*ln(x)/a^2-1/4*A*ln(c*x^4+b*x^2+a)/a^2
```

Rubi [A] time = 0.33, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{\left(4a^2Bc+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c))*ArcTanh[(b + 2*c*x^2)/Sqrt
[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b
*x^2 + c*x^4])/(4*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-A\left(b^2-4ac\right)-(Ab-2aB)cx}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{A\left(-b^2+4ac\right)}{ax} + \frac{2a^2Bc+A\left(b^3-5abc\right)+Ac}{a\left(a+bx+cx^2\right)}\right.\right.}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{2a^2Bc+A\left(b^3-5abc\right)+Ac(b}{a+bx+cx^2}\right.}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{A\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{(4a^2}{}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(4a^2Bc+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right.}{2a^2\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.33, size = 243, normalized size = 1.62

$$\frac{-\frac{\left(4a^2Bc+A\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(4a^2Bc+A\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\right)\log\left(\sqrt{b^2-4ac}\right.}{\left(b^2-4ac\right)^{3/2}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + 4*A*Log[x] - ((4*a^2*B*c + A*(b^3 - 6*a*b*c + b^2*Sqrt[b
^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/(b^2 - 4*a*c)^(3/2) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*
a*c] + 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2
 - 4*a*c)^(3/2))/(4*a^2)
```

fricas [B] time = 1.43, size = 1014, normalized size = 6.76

$$\left[-\frac{2\,Ba^2b^3 - 2\,Aab^4 - 16\,Aa^3c^2 - 2\left(4\left(2\,Ba^3 - Aa^2b\right)c^2 - \left(2\,Ba^2b^2 - Aab^3\right)c\right)x^2 - \left(Aab^3 + \left(Ab^3c + 2\left(2\,Ba^2 - 3\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c^
2 - (2*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A
*a*b)*c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3
*A*a^2*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (
2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^3*b - 3*A*a
^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*
c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*log(c
*x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c -
8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*
x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c
^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), -1/4*(2
*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B
*a^2*b^2 - A*a*b^3)*c)*x^2 - 2*(A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A*a*b)*
c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3*A*a^2
*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - 4*(2*B*a^3*b - 3*A*a^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^
3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c
 + 16*A*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c
 + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8
*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c
+ 16*a^4*b*c^2)*x^2)]
```

giac [A] time = 1.71, size = 201, normalized size = 1.34

$$-\frac{\left(Ab^3 + 4\,Ba^2c - 6\,Aabc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2b^2 - 4\,a^3c\right)\sqrt{-b^2 + 4\,ac}} - \frac{A\log\left(cx^4 + bx^2 + a\right)}{4\,a^2} + \frac{A\log\left(x^2\right)}{2\,a^2} + \frac{Ab^2cx^4 - 4\,Aac^2x^4 + Ab^3x^2}{4\,(cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/4*A*log(c*x^4 + b*x^2 + a)/
a^2 + 1/2*A*log(x^2)/a^2 + 1/4*(A*b^2*c*x^4 - 4*A*a*c^2*x^4 + A*b^3*x^2 - 4
*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + 3*A*a*b^2 - 8*A*a^2*c)/((c*x^4 +
 b*x^2 + a)*(a^2*b^2 - 4*a^3*c))
```

maple [B] time = 0.02, size = 361, normalized size = 2.41

$$-\frac{Abc\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}+\frac{Bc\,x^2}{\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)}-\frac{3Abc\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a}+\frac{A\,b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(4ac-b^2\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] A*ln(x)/a^2-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*A*b+1/(c*x^4+b*x^2+a)*c
/(4*a*c-b^2)*x^2*B+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*c-1/2/a/(c*x^4+b*x^2+a)/
(4*a*c-b^2)*A*b^2+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*B-1/a/(4*a*c-b^2)*c*ln(
c*x^4+b*x^2+a)*A+1/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*A*b^2-3/a/(4*a*c-b^2
)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b*c+1/2/a^2/(4*a*c-b^2)^(3/
2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^3+2/(4*a*c-b^2)^(3/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 7.88, size = 7119, normalized size = 47.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((2*A*a*c - A*b^2 + B*a*b)/(2*a*(4*a*c - b^2)) - (c*x^2*(A*b - 2*B*a))/(2*a
*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + (A*log(x))/a^2 - (log((((A + a^2*(-(
A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((A + a^2*(
-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((4*b*c^2*
(A*b^3 + 2*B*a^2*c - 5*A*a*b*c))/(a*(4*a*c - b^2)) - (b*c^2*(A + a^2*(-(A*b
^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^
2 - 10*a*c*x^2))/a^2 + (2*c^3*x^2*(A*b^3 + 8*B*a*b^2 - 20*B*a^2*c - 10*A*a*
b*c))/(a*(4*a*c - b^2))))/(4*a^2) + (c^3*(A*b - 2*B*a)*(4*A*b^3 + 2*B*a^2*c
 - 17*A*a*b*c))/(a^2*(4*a*c - b^2)^2) - (2*c^4*x^2*(A*b - 2*B*a)*(10*A*a*c
 - 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) + (c^5*x^2*(A*b - 2*B*a
)^3)/(a^3*(4*a*c - b^2)^3) - (A*c^4*(A*b - 2*B*a)^2)/(a^3*(4*a*c - b^2)^2))
*(((A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2
))*(((A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1
/2))*((4*b*c^2*(A*b^3 + 2*B*a^2*c - 5*A*a*b*c))/(a*(4*a*c - b^2)) - (b*c^2*
(A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*
(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 + (2*c^3*x^2*(A*b^3 + 8*B*a*b^2 - 20*B*
a^2*c - 10*A*a*b*c))/(a*(4*a*c - b^2))))/(4*a^2) + (c^3*(A*b - 2*B*a)*(4*A*
b^3 + 2*B*a^2*c - 17*A*a*b*c))/(a^2*(4*a*c - b^2)^2) - (2*c^4*x^2*(A*b - 2*
B*a)*(10*A*a*c - 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) + (c^5*x
^2*(A*b - 2*B*a)^3)/(a^3*(4*a*c - b^2)^3) - (A*c^4*(A*b - 2*B*a)^2)/(a^3*(4
*a*c - b^2)^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2
))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (atan((
```

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x^2*((((A^3*b^3*c^5 - 8*B^3*a^3*c^5 + 12*A*B^2*a^2*b*c^5 - 6*A^2*B*a*b^2*c^
5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((44*A^2*a^2*b
^3*c^5 - 4*B^2*a^3*b^3*c^4 + 160*A*B*a^4*c^6 - 6*A^2*a*b^5*c^4 - 80*A^2*a^3
*b*c^6 + 16*B^2*a^4*b*c^5 + 14*A*B*a^2*b^4*c^4 - 96*A*B*a^3*b^2*c^5)/(a^3*b
^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (((640*B*a^6*c^6 + 320*A
*a^5*b*c^6 - 2*A*a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*
a^3*b^6*c^3 + 168*B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c
 + 96*A*a^2*b^2*c^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1
056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c
^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^
2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^
3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*
a^3*b^4*c + 192*a^4*b^2*c^2)) + (((((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A*
a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168
*B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +
48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c
^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 -
 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*
B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3 + 4*B*a^2*c - 6
*A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560
*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6
*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2
)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3
+ 4*B*a^2*c - 6*A*a*b*c)^2*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a
^2*b^2*c^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b
^5*c^4 - 2688*a^6*b^3*c^5))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 1
92*a^4*b^2*c^2)))*(3*A*b^5 - 2*B*a^3*c^2 - 21*A*a*b^3*c + 33*A*a^2*b*c^2 +
2*B*a^2*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4
*B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12*
A*B*a^3*b*c^2)) + (((((((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A*a^2*b^7*c^3
+ 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168*B*a^4*b^4*c
^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560*a^
7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^
3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^
6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*
A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2
*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560*a^7*b*c^6 +
 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(
8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2
*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6
 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^
5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (((44*A^2*a^2*b^3*c^5 - 4*B^2*a^
3*b^3*c^4 + 160*A*B*a^4*c^6 - 6*A^2*a*b^5*c^4 - 80*A^2*a^3*b*c^6 + 16*B^2*a
^4*b*c^5 + 14*A*B*a^2*b^4*c^4 - 96*A*B*a^3*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A*
a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168
*B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +
48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c
^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 -
 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 -
128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c
^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*
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a^2*(4*a*c - b^2)^(3/2)) + ((A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^3*(2560*a^7*b*c
^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5
))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2)))*(3*A*b^6 - 40*A*a^3*c^3 - 27*A*a*b^4*c + 2*B*a^2*b^3*c - 6*B*a
^3*b*c^2 + 69*A*a^2*b^2*c^2))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(400*A^2*a^3*c
^3 - 6*A^2*b^6 + 4*B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A
*B*a^2*b^3*c - 12*A*B*a^3*b*c^2)))*(16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a
^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^
2*c^2*(4*a*c - b^2)^(9/2)))/(A^2*b^6*c^2 + 16*B^2*a^4*c^4 + 36*A^2*a^2*b^2*
c^4 - 12*A^2*a*b^4*c^3 + 8*A*B*a^2*b^3*c^3 - 48*A*B*a^3*b*c^4) - (((((((4*A
*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*a^3*b^4*c^3 + 80*A*a^4*b^2*c^4 + 8*B*a
^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5
*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a
^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c
^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*
a^2*(4*a*c - b^2)^(3/2)) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^
4)*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c
 + 96*A*a^2*b^2*c^2))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A
*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 25
6*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((4*B^2*a^4*c^4 + 17*A^2*a^
2*b^2*c^4 - 4*A^2*a*b^4*c^3 + 8*A*B*a^2*b^3*c^3 - 36*A*B*a^3*b*c^4)/(a^3*b^
4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*A*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*
a^3*b^4*c^3 + 80*A*a^4*b^2*c^4 + 8*B*a^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8
*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 -
 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2
 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))
)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^
6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*
A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 +
64*a^6*b^2*c^4)*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^3)/(64*a^6*(4*a*c - b^2)^(9
/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))*(16*a^6*b^6*(4*a*c - b^2)^(9/2)
 - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 76
8*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*A*b^6 - 40*A*a^3*c^3 - 27*A*a*b^4*c +
 2*B*a^2*b^3*c - 6*B*a^3*b*c^2 + 69*A*a^2*b^2*c^2))/(8*a^3*c^2*(4*a*c - b^2
)^(7/2)*(A^2*b^6*c^2 + 16*B^2*a^4*c^4 + 36*A^2*a^2*b^2*c^4 - 12*A^2*a*b^4*c
^3 + 8*A*B*a^2*b^3*c^3 - 48*A*B*a^3*b*c^4)*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4
*B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12*
A*B*a^3*b*c^2)) + ((16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c -
b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b
^2)^(9/2))*((A^3*b^2*c^4 + 4*A*B^2*a^2*c^4 - 4*A^2*B*a*b*c^4)/(a^3*b^4 + 16
*a^5*c^2 - 8*a^4*b^2*c) + (((4*B^2*a^4*c^4 + 17*A^2*a^2*b^2*c^4 - 4*A^2*a*b
^4*c^3 + 8*A*B*a^2*b^3*c^3 - 36*A*B*a^3*b*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^
4*b^2*c) + (((4*A*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*a^3*b^4*c^3 + 80*A*a^
4*b^2*c^4 + 8*B*a^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4
*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A
*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^
2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^
3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*
a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96
*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c
^2)) - (((((4*A*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*a^3*b^4*c^3 + 80*A*a^4*
b^2*c^4 + 8*B*a^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b
^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a
*b^4*c + 96*A*a^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*
b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c -
6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3
+ 64*a^6*b^2*c^4)*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3
- 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^4 + 1
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6*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*
b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - (
(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(A*b^3 + 4*B*a^2*c - 6*A*
a*b*c)^2*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(32*a
^4*(4*a*c - b^2)^3*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^
5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(3*A*b^5 - 2*B*a^3*c^2 - 21*A*a*b
^3*c + 33*A*a^2*b*c^2 + 2*B*a^2*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(A^2*b^6
*c^2 + 16*B^2*a^4*c^4 + 36*A^2*a^2*b^2*c^4 - 12*A^2*a*b^4*c^3 + 8*A*B*a^2*b
^3*c^3 - 48*A*B*a^3*b*c^4)*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4*B^2*a^4*c^2 - 2
91*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12*A*B*a^3*b*c^2)))
*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(2*a^2*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.117 $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=223

$$\frac{(2Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{-6aAc-abB+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}+\frac{\left(abB\left(b^2-6ac\right)-2A\left(6a^2c^2-6a\right.\right.}{2a^3\left(b^2-4\right.}$$

```
[Out] 1/2*(6*A*a*c-2*A*b^2+B*a*b)/a^2/(-4*a*c+b^2)/x^2+1/2*(-a*b*B+A*(-2*a*c+b^2)
+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2+a)+1/2*(a*b*B*(-6*a*c+b
^2)-2*A*(6*a^2*c^2-6*a*b^2*c+b^4))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/
a^3/(-4*a*c+b^2)^(3/2)-(2*A*b-B*a)*ln(x)/a^3+1/4*(2*A*b-B*a)*ln(c*x^4+b*x^2
+a)/a^3
```

Rubi [A] time = 0.42, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{\left(abB\left(b^2-6ac\right)-2A\left(6a^2c^2-6ab^2c+b^4\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}-\frac{-6aAc-abB+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}+\frac{(2Ab-aB)\log\left(a+bx\right.}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(2*A*b^2 - a*b*B - 6*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x^2) - (a*b*B - A*(b^2 -
2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) +
 ((a*b*B*(b^2 - 6*a*c) - 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2))*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*B)*Log
[x])/a^3 + ((2*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2Ab^2+abB+6aAc-2(Ab-2aB)cx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^{\cdots}\right.}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int\left(\frac{-2Ab^2+abB+6aAc}{ax^2}+\frac{(-2Ab+aB)\left(-b^2+4a\cdots\right.}{a^2x}\cdots\right.\right.}{\cdots}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aB)\log(x)}{a^3} - \cdots$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aB)\log(x)}{a^3} + \cdots$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aB)\log(x)}{a^3} + \cdots$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} + \frac{\left(abB\left(b^2-6ac\right)-2A\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.56, size = 379, normalized size = 1.70

$$\frac{\left(2A\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)+aB\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}+6abc-b^3\right)\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(2A\left(-6a^2c^2+6ab^2c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*A)/x^2 - (2*a*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^2
*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(-2*A*b + a
*B)*Log[x] + ((a*B*(-b^3 + 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2
 - 4*a*c]) + 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a
*b*c*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c
)^(3/2) + ((a*B*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4
*a*c]) + 2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*
c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^3)
```

fricas [B] time = 3.38, size = 1635, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3*c^3 + 2*(2
*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2*(B*a^2*b^4
 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5*A*a^2*b^3)*
c)*x^2 + ((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b^3 - 2*A*b^4)
*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 - 2*A*a*b^3)*c
)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2*A*a^2*b^2)*c
)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^
2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((16*(B*a^3 - 2*A*a^2*b)*c
^3 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5
- 2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x
^4 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2
*A*a^2*b^3)*c)*x^2)*log(c*x^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*A*a^2*b)*c^3
 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 -
2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4
 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A
*a^2*b^3)*c)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6
*c^2)*x^2), -1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3
*c^3 + 2*(2*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2
*(B*a^2*b^4 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5*
A*a^2*b^3)*c)*x^2 + 2*((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b
^3 - 2*A*b^4)*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 -
 2*A*a*b^3)*c)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2
*A*a^2*b^2)*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*
a*c)/(b^2 - 4*a*c)) + ((16*(B*a^3 - 2*A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b
^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2
*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5
 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*log(c*x
^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b^3
)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2*A
*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5 +
 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*log(x))/(
```

```
(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*
a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

giac [A] time = 1.65, size = 250, normalized size = 1.12

$$-\frac{\left(Bab^3 - 2\,Ab^4 - 6\,Ba^2bc + 12\,Aab^2c - 12\,Aa^2c^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^3b^2 - 4\,a^4c\right)\sqrt{-b^2+4\,ac}} + \frac{Babcx^4 - 2\,Ab^2cx^4 + 6\,Aac^2x^4 + Bab^2x^2}{2\left(cx^6 + bx^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^3 - 2*A*b^4 - 6*B*a^2*b*c + 12*A*a*b^2*c - 12*A*a^2*c^2)*arctan
((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c))
 + 1/2*(B*a*b*c*x^4 - 2*A*b^2*c*x^4 + 6*A*a*c^2*x^4 + B*a*b^2*x^2 - 2*A*b^3
*x^2 - 2*B*a^2*c*x^2 + 7*A*a*b*c*x^2 - A*a*b^2 + 4*A*a^2*c)/((c*x^6 + b*x^4
 + a*x^2)*(a^2*b^2 - 4*a^3*c)) - 1/4*(B*a - 2*A*b)*log(c*x^4 + b*x^2 + a)/a
^3 + 1/2*(B*a - 2*A*b)*log(x^2)/a^3
```

maple [B] time = 0.02, size = 622, normalized size = 2.79

$$-\frac{A\,c^2x^2}{\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a} + \frac{A\,b^2c\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a^2} - \frac{Bbc\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a} - \frac{6A\,c^2\arctan\left(\right.}{\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2*A/a^2/x^2-2/a^3*ln(x)*A*b+1/a^2*ln(x)*B-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c
-b^2)*x^2*A+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*A*b^2-1/2/a/(c*x^4+b*
x^2+a)*c/(4*a*c-b^2)*x^2*b*B-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*b*c+1/2/a^
2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*b^3+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B*c-1/2/a
/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B*b^2+2/a^2/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*A*
b-1/2/a^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*A*b^3-1/a/(4*a*c-b^2)*c*ln(c*x^4+b*
x^2+a)*B+1/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*B-6/a/(4*a*c-b^2)^(3/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*c^2+6/a^2/(4*a*c-b^2)^(3/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2*c-1/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2))*A*b^4-3/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*b*B*c+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-
b^2)^(1/2))*B*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 9.09, size = 10034, normalized size = 45.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] (log(((c^4*(2*A*b - B*a)*(6*A*a*c - 2*A*b^2 + B*a*b)^2)/(a^6*(4*a*c - b^2)^
2) - ((((B*a - 2*A*b + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2
*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((b*c^2*(B*a - 2*A*b + a^
3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/(a^6*
(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (4*b*c^2*(2*
A*b^4 + 6*A*a^2*c^2 - B*a*b^3 - 10*A*a*b^2*c + 5*B*a^2*b*c))/(a^2*(4*a*c -
b^2)) + (2*c^3*x^2*(2*A*b^4 - 60*A*a^2*c^2 - B*a*b^3 + 4*A*a*b^2*c + 10*B*a
^2*b*c))/(a^2*(4*a*c - b^2))))/(4*a^3) + (c^3*(36*A^2*a^3*c^3 - 16*A^2*b^6
- 4*B^2*a^2*b^4 + 16*A*B*a*b^5 - 216*A^2*a^2*b^2*c^2 + 116*A^2*a*b^4*c + 17
*B^2*a^3*b^2*c - 92*A*B*a^2*b^3*c + 108*A*B*a^3*b*c^2))/(a^4*(4*a*c - b^2)^
2) - (2*c^4*x^2*(12*A^2*b^5 + 3*B^2*a^2*b^3 - 12*A*B*a*b^4 - 60*A*B*a^3*c^2
 - 82*A^2*a*b^3*c - 10*B^2*a^3*b*c + 138*A^2*a^2*b*c^2 + 61*A*B*a^2*b^2*c))
/(a^4*(4*a*c - b^2)^2))*(B*a - 2*A*b + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*
b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3)
+ (c^5*x^2*(6*A*a*c - 2*A*b^2 + B*a*b)^3)/(a^6*(4*a*c - b^2)^3))*((c^4*(2*A
*b - B*a)*(6*A*a*c - 2*A*b^2 + B*a*b)^2)/(a^6*(4*a*c - b^2)^2) - ((((2*A*b
- B*a + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*
c)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((4*b*c^2*(2*A*b^4 + 6*A*a^2*c^2 - B*a*b
^3 - 10*A*a*b^2*c + 5*B*a^2*b*c))/(a^2*(4*a*c - b^2)) - (b*c^2*(2*A*b - B*a
 + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/
(a^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (2*c^3*
x^2*(2*A*b^4 - 60*A*a^2*c^2 - B*a*b^3 + 4*A*a*b^2*c + 10*B*a^2*b*c))/(a^2*(
4*a*c - b^2))))/(4*a^3) - (c^3*(36*A^2*a^3*c^3 - 16*A^2*b^6 - 4*B^2*a^2*b^4
 + 16*A*B*a*b^5 - 216*A^2*a^2*b^2*c^2 + 116*A^2*a*b^4*c + 17*B^2*a^3*b^2*c
- 92*A*B*a^2*b^3*c + 108*A*B*a^3*b*c^2))/(a^4*(4*a*c - b^2)^2) + (2*c^4*x^2
*(12*A^2*b^5 + 3*B^2*a^2*b^3 - 12*A*B*a*b^4 - 60*A*B*a^3*c^2 - 82*A^2*a*b^3
*c - 10*B^2*a^3*b*c + 138*A^2*a^2*b*c^2 + 61*A*B*a^2*b^2*c))/(a^4*(4*a*c -
b^2)^2))*(2*A*b - B*a + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^
2*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3) + (c^5*x^2*(6*A
*a*c - 2*A*b^2 + B*a*b)^3)/(a^6*(4*a*c - b^2)^3)))*(4*A*b^7 + 128*B*a^4*c^3
 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*
b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 1
92*a^5*b^2*c^2)) - (log(x)*(2*A*b - B*a))/a^3 - (A/(2*a) - (x^2*(2*A*b^3 -
B*a*b^2 + 2*B*a^2*c - 7*A*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(6*A*a*c -
 2*A*b^2 + B*a*b))/(2*a^2*(4*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) + (atan((
x^2*((((216*A^3*a^3*c^8 - 8*A^3*b^6*c^5 - 216*A^3*a^2*b^2*c^7 + B^3*a^3*b^3
*c^5 + 72*A^3*a*b^4*c^6 + 12*A^2*B*a*b^5*c^5 + 108*A^2*B*a^3*b*c^7 - 6*A*B^
2*a^2*b^4*c^5 + 18*A*B^2*a^3*b^2*c^6 - 72*A^2*B*a^2*b^3*c^6)/(a^6*b^6 - 64*
a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((24*A^2*a^2*b^7*c^4 - 260*A^2*
a^3*b^5*c^5 + 932*A^2*a^4*b^3*c^6 + 6*B^2*a^4*b^5*c^4 - 44*B^2*a^5*b^3*c^5
+ 480*A*B*a^6*c^7 - 1104*A^2*a^5*b*c^7 + 80*B^2*a^6*b*c^6 - 24*A*B*a^3*b^6*
c^4 + 218*A*B*a^4*b^4*c^5 - 608*A*B*a^5*b^2*c^6)/(a^6*b^6 - 64*a^9*c^3 - 12
*a^7*b^4*c + 48*a^8*b^2*c^2) + (((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^
4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B
*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3*c^5)/(a^6*b^6 - 64*a^9*c^3
- 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184
*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*A*b^7 + 128*B*a^4*c^
3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2
*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*
a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(
4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B
*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6
*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b
^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 -
96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2
*c^2)) - (((((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5
*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*
a^6*b^5*c^4 + 192*B*a^7*b^3*c^5)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*
```

```
a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*
a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A
*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*
b^2*c^2))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*
b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c
^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - (
(2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 26
88*a^9*b^3*c^5)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*
b*c)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3
 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c - b
^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6
 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2
 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((25
60*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*
a^9*b^3*c^5)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c
)^2*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 +
 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(32*a^6*(4*a*c - b
^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 2
56*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(6*A*a^3*c^3 - 6*A*b^6 + 3*B
*a*b^5 + 42*A*a*b^4*c - 21*B*a^2*b^3*c + 33*B*a^3*b*c^2 - 72*A*a^2*b^2*c^2)
)/(8*a^3*c^2*(4*a*c - b^2)^3*(36*A^2*a^4*c^4 - 24*A^2*b^8 - 6*B^2*a^2*b^6 +
 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b^4*c^2 + 1528*A^2*a^3*b^2*c
^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72*B^2*a^3*b^4*c + 1158*A*B*a^
3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c^3)) + (((((((1920*A*a^8*c^
7 - 320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^
5 - 1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3
*c^5)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*
b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*
c^5)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3
 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^6 - 64*
a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*
b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c
 + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((2560*a^10*b*c^6 + 12*a^6*b
^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(2*A*b^4 +
12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)*(4*A*b^7 + 128*B*a^4*c
^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^
2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9
*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4
*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c
 - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2)
)/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) + (((24*A^
2*a^2*b^7*c^4 - 260*A^2*a^3*b^5*c^5 + 932*A^2*a^4*b^3*c^6 + 6*B^2*a^4*b^5*c
^4 - 44*B^2*a^5*b^3*c^5 + 480*A*B*a^6*c^7 - 1104*A^2*a^5*b*c^7 + 80*B^2*a^6
*b*c^6 - 24*A*B*a^3*b^6*c^4 + 218*A*B*a^4*b^4*c^5 - 608*A*B*a^5*b^2*c^6)/(a
^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((1920*A*a^8*c^7 -
320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^5 -
1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3*c^5
)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^
6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)
*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24
*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^6 - 64*a^9*
c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*
c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c
- 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))
/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 +
 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)
^(3/2)) + ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b
^5*c^4 - 2688*a^9*b^3*c^5)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c
 + 6*B*a^2*b*c)^3)/(64*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a
```

```
^7*b^4*c + 48*a^8*b^2*c^2)))*(768*A*b^7 + 5120*B*a^4*c^3 - 384*B*a*b^6 - 69
12*A*a*b^5*c - 12544*A*a^3*b*c^3 + 3456*B*a^2*b^4*c + 18432*A*a^2*b^3*c^2 -
 8832*B*a^3*b^2*c^2))/(1024*a^3*c^2*(4*a*c - b^2)^(7/2)*(36*A^2*a^4*c^4 - 2
4*A^2*b^8 - 6*B^2*a^2*b^6 + 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b
^4*c^2 + 1528*A^2*a^3*b^2*c^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72*
B^2*a^3*b^4*c + 1158*A*B*a^3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c
^3)))*(16*a^9*b^6*(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)^(9/2) -
 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))
)/(144*A^2*a^4*c^6 + 4*A^2*b^8*c^2 + 192*A^2*a^2*b^4*c^4 - 288*A^2*a^3*b^2*
c^5 + B^2*a^2*b^6*c^2 - 12*B^2*a^3*b^4*c^3 + 36*B^2*a^4*b^2*c^4 - 48*A^2*a*
b^6*c^3 + 48*A*B*a^2*b^5*c^3 - 168*A*B*a^3*b^3*c^4 - 4*A*B*a*b^7*c^2 + 144*
A*B*a^4*b*c^5) + (((((((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3
 - 184*A*a^6*b^3*c^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^
4)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3
+ 64*a^9*b^2*c^4)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256
*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(
a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c
 + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*
B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3
+ 64*a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^
2*b*c)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^
3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c -
 b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 -
 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 4
8*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a
^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2))
 - (((36*A^2*a^5*c^6 - 16*A^2*a^2*b^6*c^3 + 116*A^2*a^3*b^4*c^4 - 216*A^2*a
^4*b^2*c^5 - 4*B^2*a^4*b^4*c^3 + 17*B^2*a^5*b^2*c^4 + 16*A*B*a^3*b^5*c^3 -
92*A*B*a^4*b^3*c^4 + 108*A*B*a^5*b*c^5)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c
) - (((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3 - 184*A*a^6*b^3*
c^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^4)/(a^6*b^4 + 16*
a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)
*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24
*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^4 + 16*a^8*
c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^
2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3
+ 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 2
56*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*
a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((4*a^7*
b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^
3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^3)/(64*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^4 + 1
6*a^8*c^2 - 8*a^7*b^2*c)))*(16*a^9*b^6*(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*
(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2
*(4*a*c - b^2)^(9/2))*(768*A*b^7 + 5120*B*a^4*c^3 - 384*B*a*b^6 - 6912*A*a*
b^5*c - 12544*A*a^3*b*c^3 + 3456*B*a^2*b^4*c + 18432*A*a^2*b^3*c^2 - 8832*B
*a^3*b^2*c^2))/(1024*a^3*c^2*(4*a*c - b^2)^(7/2)*(144*A^2*a^4*c^6 + 4*A^2*b
^8*c^2 + 192*A^2*a^2*b^4*c^4 - 288*A^2*a^3*b^2*c^5 + B^2*a^2*b^6*c^2 - 12*B
^2*a^3*b^4*c^3 + 36*B^2*a^4*b^2*c^4 - 48*A^2*a*b^6*c^3 + 48*A*B*a^2*b^5*c^3
 - 168*A*B*a^3*b^3*c^4 - 4*A*B*a*b^7*c^2 + 144*A*B*a^4*b*c^5)*(36*A^2*a^4*c
^4 - 24*A^2*b^8 - 6*B^2*a^2*b^6 + 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2
*a^2*b^4*c^2 + 1528*A^2*a^3*b^2*c^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c
 + 72*B^2*a^3*b^4*c + 1158*A*B*a^3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a
^4*b*c^3)) + ((16*a^9*b^6*(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)
^(9/2) - 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2
)^(9/2))*((B^3*a^3*b^2*c^4 - 8*A^3*b^5*c^4 + 36*A^2*B*a^3*c^6 + 48*A^3*a*b^
3*c^5 - 72*A^3*a^2*b*c^6 + 12*A*B^2*a^3*b*c^5 + 12*A^2*B*a*b^4*c^4 - 6*A*B^
2*a^2*b^3*c^4 - 48*A^2*B*a^2*b^2*c^5)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)
- (((36*A^2*a^5*c^6 - 16*A^2*a^2*b^6*c^3 + 116*A^2*a^3*b^4*c^4 - 216*A^2*a^
4*b^2*c^5 - 4*B^2*a^4*b^4*c^3 + 17*B^2*a^5*b^2*c^4 + 16*A*B*a^3*b^5*c^3 - 9
```

```
2*A*B*a^4*b^3*c^4 + 108*A*B*a^5*b*c^5)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)
 - (((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3 - 184*A*a^6*b^3*c
^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^4)/(a^6*b^4 + 16*a
^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*
(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*
B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^4 + 16*a^8*c
^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2
)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 +
 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 25
6*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*
B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c
^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^
5*b^2*c^2)) - (((((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3 - 18
4*A*a^6*b^3*c^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^4)/(a
^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*
a^9*b^2*c^4)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^
3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b
^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 19
2*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2
*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*
a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c
)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 2
4*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c - b^2)
^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a
^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^
2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((4*a^7*b^6*c^2 - 32*a^8*
b^4*c^3 + 64*a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c
+ 6*B*a^2*b*c)^2*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*
A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(32*a
^6*(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^
6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(6*A*a^3*c^3 - 6*A*b^6 + 3*B*a*b^
5 + 42*A*a*b^4*c - 21*B*a^2*b^3*c + 33*B*a^3*b*c^2 - 72*A*a^2*b^2*c^2))/(8*
a^3*c^2*(4*a*c - b^2)^3*(144*A^2*a^4*c^6 + 4*A^2*b^8*c^2 + 192*A^2*a^2*b^4*
c^4 - 288*A^2*a^3*b^2*c^5 + B^2*a^2*b^6*c^2 - 12*B^2*a^3*b^4*c^3 + 36*B^2*a
^4*b^2*c^4 - 48*A^2*a*b^6*c^3 + 48*A*B*a^2*b^5*c^3 - 168*A*B*a^3*b^3*c^4 -
4*A*B*a*b^7*c^2 + 144*A*B*a^4*b*c^5)*(36*A^2*a^4*c^4 - 24*A^2*b^8 - 6*B^2*a
^2*b^6 + 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b^4*c^2 + 1528*A^2*a
^3*b^2*c^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72*B^2*a^3*b^4*c + 115
8*A*B*a^3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c^3)))*(2*A*b^4 + 12
*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(2*a^3*(4*a*c - b^2)^(3
/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.118 $\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=425

$$-\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2Bc^2+8aAbc^2-19\ldots}{\sqrt{b^2-\ldots}}\right.}{}$$

```
[Out] 1/2*(-A*b*c-10*B*a*c+3*B*b^2)*x/c^2/(-4*a*c+b^2)-1/2*(-2*A*c+B*b)*x^3/c/(-4
*a*c+b^2)-1/2*x^5*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*B-A*b^2*
c-13*a*b*B*c+6*a*A*c^2+(-8*A*a*b*c^2+A*b^3*c-20*B*a^2*c^2+19*B*a*b^2*c-3*B*
b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2)
)^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*B
-A*b^2*c-13*a*b*B*c+6*a*A*c^2+(8*A*a*b*c^2-A*b^3*c+20*B*a^2*c^2-19*B*a*b^2*
c+3*B*b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] time = 3.67, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1275, 1279, 1166, 205}

$$-\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2Bc^2+8aAbc^2-19\ldots}{\sqrt{b^2-\ldots}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((3*b^2*B - A*b*c - 10*a*B*c)*x)/(2*c^2*(b^2 - 4*a*c)) - ((b*B - 2*A*c)*x^3
)/(2*c*(b^2 - 4*a*c)) - (x^5*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*B - A*b^2*c - 13*a*b*B*c + 6*a*A*c^2 -
 (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2
]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*B - A*b^2*c
- 13*a*b*B*c + 6*a*A*c^2 + (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2
+ 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c
]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\int \frac{x^4\left(5(Ab-2aB)-3(bB-2Ac)x^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{(bB-2Ac)x^3}{2c\left(b^2-4ac\right)}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{x^2\left(-9a(bB-2Ac)-3\left(3b^2B-Abc-10aBc\right)x^2\right)}{a+bx^2+cx^4}\,d}{6c\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2B-Abc-10aBc\right)x}{2c^2\left(b^2-4ac\right)}-\frac{(bB-2Ac)x^3}{2c\left(b^2-4ac\right)}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\int \frac{-3a\left(3b^2B\right.}{}}{}$$

$$= \frac{\left(3b^2B-Abc-10aBc\right)x}{2c^2\left(b^2-4ac\right)}-\frac{(bB-2Ac)x^3}{2c\left(b^2-4ac\right)}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b^3B-A\right.}{}$$

$$= \frac{\left(3b^2B-Abc-10aBc\right)x}{2c^2\left(b^2-4ac\right)}-\frac{(bB-2Ac)x^3}{2c\left(b^2-4ac\right)}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b^3B-A\right.}{}$$

Mathematica [A] time = 1.20, size = 455, normalized size = 1.07

$$\frac{\frac{2\sqrt{c}\,x\left(-2a^2Bc+a\left(-bc\left(A+3Bx^2\right)+2Ac^2x^2+b^2B\right)+b^2x^2(bB-Ac)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{2}\left(2ac^2\left(3A\sqrt{b^2-4ac}-10aB\right)+b^2c\left(19aB-A\sqrt{b^2-4ac}\right)-abc\left(13B\sqrt{b^2-4ac}+8Ac\right)+\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*B*Sqrt[c]*x + (2*Sqrt[c]*x*(-2*a^2*B*c + b^2*(b*B - A*c)*x^2 + a*(b^2*B
+ 2*A*c^2*x^2 - b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
(Sqrt[2]*(-3*b^4*B + b^2*c*(19*a*B - A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(-10*a*
B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - a*b*c*(8*A
*c + 13*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3
```

```
*b^4*B - b^2*c*(19*a*B + A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(10*a*B + 3*A*Sqrt[
b^2 - 4*a*c]) + a*b*c*(8*A*c - 13*B*Sqrt[b^2 - 4*a*c]) + b^3*(-(A*c) + 3*B*
Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

fricas [B] time = 6.00, size = 7252, normalized size = 17.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(B*b^2*c - 4*B*a*c^2)*x^5 + 2*(3*B*b^3 + 2*A*a*c^2 - (11*B*a*b + A*b
^2)*c)*x^3 - sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (
b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4
 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 +
80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 1
2*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^
6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2
200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6
*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^
3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*
a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*
B^4*a^2*b^6 - 135*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*
a^2*b^2)*c^4 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 64
7*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3
 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3
*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x + 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2
*B*a^4 + A^3*a^3*b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*
b^2 + 11*A^3*a^2*b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A
^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 +
 153*A^2*B*a*b^6 + A^3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2
*B*b^8)*c^2 - 27*(17*B^3*a*b^8 + A*B^2*b^9)*c - (3*B*b^9*c^5 - 768*A*a^4*c^
10 + 128*(8*B*a^4*b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8
+ 24*(14*B*a^2*b^5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*
b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5
 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3
 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^
4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6
)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*
c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3
 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5
- 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2
*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4
+ 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4
- 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)
*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B
^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqr
t(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*
c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^
3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A
^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 + 48
*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B
^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b
+ 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^
2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*
a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*
```

```
b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c
^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 13
5*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (
2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3
- 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a
^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2
*B^2*a*b^6)*c)*x - 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3
*b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2
*b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 1
7*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6
 + A^3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27
*(17*B^3*a*b^8 + A*B^2*b^9)*c - (3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^
4*b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b
^5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2
*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4
+ 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4
- 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)
*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B
^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2
*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4
+ A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 +
 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^
2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3
*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3
*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B
^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B
^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^
6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b^2*c^
2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-
(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2
*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*
(35*B^2*a*b^5 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*
B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a
^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^
2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B
^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^1
0 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^
6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 135*A*B^3*a*b^7 +
324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (2500*B^4*a^5 + 2
500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c
^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B
*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x
 + 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3*b)*c^6 - 8*(500
*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2*b^3)*c^5 + (113
60*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4
 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6 + A^3*b^7)*c^3
+ 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27*(17*B^3*a*b^8 +
 A*B^2*b^9)*c + (3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^4*b + 5*A*a^3*b^
2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b^5 + A*a*b^6)*c^
7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^
2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3
*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3
*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B
^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B
^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqr
t(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*
a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 -
```

```
 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
- 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A
^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^
2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3
*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*
A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4
*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 +
(b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*
(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3
)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 +
2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt(
(81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*
b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3
*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*
B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2
*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c
^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 135*A*B^3*a*b^7 + 324*A^4*a^3*c^5
- 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b
- 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*
a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27
*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x - 1/2*sqrt(1/2)
*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3*b)*c^6 - 8*(500*B^3*a^5 + 930*A
*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2*b^3)*c^5 + (11360*B^3*a^4*b^2 +
 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4 - (8818*B^3*a^3
*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6 + A^3*b^7)*c^3 + 9*(329*B^3*a^2
*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27*(17*B^3*a*b^8 + A*B^2*b^9)*c +
(3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^4*b + 5*A*a^3*b^2)*c^9 - 192*(5*
B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b^5 + A*a*b^6)*c^7 - (52*B*a*b^7
+ A*b^8)*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A
^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^
2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3
*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*
A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*B^2*b^7 +
60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*
b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5
 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sq
rt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4
*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*
A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A
^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*
A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*
c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^
2*c^7 - 64*a^3*c^8))) + 2*(3*B*a*b^2 - (10*B*a^2 + A*a*b)*c)*x)/(a*b^2*c^2
- 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)
```

giac [B] time = 6.49, size = 5681, normalized size = 13.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] B*x/c^2 + 1/2*(B*b^3*x^3 - 3*B*a*b*c*x^3 - A*b^2*c*x^3 + 2*A*a*c^2*x^3 + B*
a*b^2*x - 2*B*a^2*c*x - A*a*b*c*x)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)
) + 1/16*((2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
```

```
c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^4)*(b^2*c^2 - 4
*a*c^3)^2*A - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3
)*(b^2*c^2 - 4*a*c^3)^2*B + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
5*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - 16*a^
2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 + 32*a^3*b*
c^7 - 2*(b^2 - 4*a*c)*a*b^3*c^5 + 8*(b^2 - 4*a*c)*a^2*b*c^6)*A*abs(-b^2*c^2
 + 4*a*c^3) - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^3*c^5 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4
*c^5 - 160*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^6 + 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*c^7 - 320*a^4*c^7 - 6*(b^2 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*
c)*a^2*b^2*c^5 - 80*(b^2 - 4*a*c)*a^3*c^6)*B*abs(-b^2*c^2 + 4*a*c^3) - (2*b
^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 256*a^3*b^2*c^10 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^2 - 4*a*c)*b^6*c^7 + 24*(b^2 - 4*
a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^9)*A + (6*b^9*c^6 - 86*a*b^7*c^
7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*
a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^
9)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*
b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c
```

```
^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*
b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^
9)*abs(-b^2*c^2 + 4*a*c^3)*abs(c)) - 1/16*((2*b^4*c^3 - 20*a*b^2*c^4 + 48*a
^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c +
10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*
(b^2 - 4*a*c)*a*c^4)*(b^2*c^2 - 4*a*c^3)^2*A - (6*b^5*c^2 - 50*a*b^3*c^3 +
104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b
^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2*B - 2*(sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 -
 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^3*c^6 + 16*a^2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^7 - 32*a^3*b*c^7 + 2*(b^2 - 4*a*c)*a*b^3*c^5 - 8*(b^2 -
 4*a*c)*a^2*b*c^6)*A*abs(-b^2*c^2 + 4*a*c^3) + 2*(3*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^4*c^4 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 6*a*b^6*c^
4 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^4*c^5 + 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 2
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 256*a^3*b^2*c^6 + 4
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 320*a^4*c^7 + 6*(b^2 -
4*a*c)*a*b^4*c^4 - 44*(b^2 - 4*a*c)*a^2*b^2*c^5 + 80*(b^2 - 4*a*c)*a^3*c^6)
*B*abs(-b^2*c^2 + 4*a*c^3) - (2*b^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 -
256*a^3*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^8*c^5 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^6*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c
^6 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c
^7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^
2 - 4*a*c)*b^6*c^7 + 24*(b^2 - 4*a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*
c^9)*A + (6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 64
0*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5
 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^
6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 19
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 3
1*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*
```

```
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^
2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3
*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 -
 4*a*b*c^3 - sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*
c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b
^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*
b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(-b^2*c^2 + 4*a*c^3)*abs(c))
```

maple [B] time = 0.04, size = 1507, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*A*b^2+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-
b^2)*x*B-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^3*B-3/2/(4*a*c-b^2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*a*A+3/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A-1/2/c^2/(c*x^4
+b*x^2+a)*a/(4*a*c-b^2)*x*b^2*B+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b*B
+1/2/c/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*A*b+1/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*A*b^2-3/4/c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B-1/4/c/(4*a*c-
b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*A*b^2+3/4/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B+
5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*B+5/(4*a*c-b^2)/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*B-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*B-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*a*b^2*B-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*A+2/(4*a*c-b
^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A*b+2/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A*b+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*b^4*B-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*A*b^3+13/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*B+3/4/c^2/(4*a*c-
b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*B-1/4/c/(4*a*c-b^2)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*a*b*B+B/c^2*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

[Out] 1/2*((B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*x^3 + (B*a*b^2 - (2*B*a^2 + A*a*b)*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + B*x/c^2 - 1/2*integrate((3*B*a*b^2 + (3*B*b^3 + 6*A*a*c^2 - (13*B*a*b + A*b^2)*c)*x^2 - (10*B*a^2 + A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)

mupad [B] time = 4.36, size = 16604, normalized size = 39.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)

[Out] (B*x)/c^2 - atan(((((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a *b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2 *b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3* b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c ^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^ 7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3* (-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 806 4*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2* a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a* c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6* c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840 *a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 102 4*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^1 2*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2 077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800 *B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(- (4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^ 9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B ^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4* b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2 *(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^ 3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5 *c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^ 2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B ^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2* a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a* c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^ 6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c ^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152* A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4 *a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a ^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/ 2)*1i - (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B*a*b^8 *c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4224*B*a ^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 -

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3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240
*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^
2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c -
 b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*
b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-
(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9
)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^1
2*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c
^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^
8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4
*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*
A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^
2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b
^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 38
40*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c
^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 -
30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a
*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) +
(x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A*B*b^7*
c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 - 114*B^
2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c^2 + 472
*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4*(-(4
*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a
^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9
*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^
5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9
)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^
2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 +
1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720
*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10
*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2
)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7
 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/(((
(10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B*a*b^8*c^3 + 192
*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4224*B*a^3*b^4*c^5
 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^
2*c^5)) - (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^
13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a
^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b
^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*
B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c
 + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(
1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 +
22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) +
44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24
*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*
a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^
2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4*(-(4*a*c
- b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^
7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2
+ 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A
^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/
2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5
*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*
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A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*
a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2
- 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^
(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 12
80*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*B^2*
b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2
*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 - 114*B^2*a*b^6*c
- 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c^2 + 472*A*B*a^3*b
*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4*(-(4*a*c - b^2
)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4
 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 106
56*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +
15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^
7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a
^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b
^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A
*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^
3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (216*A^3*a^4*c^
4 - 63*B^3*a^3*b^5 + 5*A^3*a^2*b^4*c^2 - 66*A^3*a^3*b^2*c^3 + 45*A*B^2*a^2*
b^6 + 600*A*B^2*a^5*c^3 + 573*B^3*a^4*b^3*c - 1300*B^3*a^5*b*c^2 - 402*A*B^
2*a^3*b^4*c - 30*A^2*B*a^2*b^5*c - 924*A^2*B*a^4*b*c^3 + 762*A*B^2*a^4*b^2*
c^2 + 339*A^2*B*a^3*b^3*c^2)/(4*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a
^2*b^2*c^5)) + (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*
B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4
224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^
4*c^4 - 48*a^2*b^2*c^5)) + (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^
11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5
*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3
- 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 +
 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(
4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A
*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c -
 b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^1
1 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^
4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a
^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*
B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c
 - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077
*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^
2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*
a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c
^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*
a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4
*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 15
2*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-
(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240
*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(
1/2) + (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A
*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 -
 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c^
2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b
^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 28
8*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*
a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5
*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c -
```

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b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 +
3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b
*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5
- 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B
*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*
b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2))
)*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*
b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6
- 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44
800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a
*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 2688
0*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a
^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2
) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*
c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6
 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^1
0)))^(1/2)*2i - ((x^3*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*(4*a*c
- b^2)) - (x*(2*B*a^2*c - B*a*b^2 + A*a*b*c))/(2*(4*a*c - b^2)))/(a*c^2 + c
^3*x^4 + b*c^2*x^2) - atan(((((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^
4*b*c^7 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^
2*b^6*c^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*
c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9
*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 15
04*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^
2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360
*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9
*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^
8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^
6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^
3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32
*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6
*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^
5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2)
 - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4
*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5
*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^
2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c -
 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/
2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22
400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2
)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44
*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a
*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^
5*b^2*c^10)))^(1/2) - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^
2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^
2*a^3*b^2*c^3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 +
86*A*B*a*b^5*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c
^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6
*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3
*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4
 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*
A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) +
 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*
A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)
```

^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B
*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^1
0*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^
2*c^10)))^(1/2)*1i - (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7
 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c
^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 1
2*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^
4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*
a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b
^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^
6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*
c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 +
 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*
B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(
4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*
a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 +
3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 -
 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*
B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^
6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 -
44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c
^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2
*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26
880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B
*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1
/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*
b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c
^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c
^10)))^(1/2) + (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c
^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b
^2*c^3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*
a*b^5*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(
-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^
12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 +
2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 4480
0*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(
-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b
^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*
B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4
*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)
+ 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^
2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 +
 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)
))^(1/2)*1i)/((((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B
*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 42
24*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4
*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*
a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5
*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3
+ 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 -
 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(
4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A
*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c -
 b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^1
1 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^

```
4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a
^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9
*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*
c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 207
7*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B
^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4
*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*
c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2
*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^
4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 1
52*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(
-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 24
0*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^
(1/2) - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*
A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3
- 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c
^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*B^2
*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c +
288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^
2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a
^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c
 - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3
- 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6
*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^
5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A
*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*
a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^
2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2
) - (216*A^3*a^4*c^4 - 63*B^3*a^3*b^5 + 5*A^3*a^2*b^4*c^2 - 66*A^3*a^3*b^2*
c^3 + 45*A*B^2*a^2*b^6 + 600*A*B^2*a^5*c^3 + 573*B^3*a^4*b^3*c - 1300*B^3*a
^5*b*c^2 - 402*A*B^2*a^3*b^4*c - 30*A^2*B*a^2*b^5*c - 924*A^2*B*a^4*b*c^3 +
 762*A*B^2*a^4*b^2*c^2 + 339*A^2*B*a^3*b^3*c^2)/(4*(64*a^3*c^6 - b^6*c^3 +
12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 102
4*A*a^4*b*c^7 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 73
6*B*a^2*b^6*c^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*B^2*b^13 + A^2*b^11*c
^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^
4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10
656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b
^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c
^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*
a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*
b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*
A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a
^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 19
2*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 -
 8*a*b^2*c^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)
^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A
^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a
^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +
 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^
11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^
9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^
4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c
 - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2
) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5
- 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6
```

```
144*a^5*b^2*c^10)))^(1/2) + (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 +
200*B^2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 -
718*B^2*a^3*b^2*c^3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*
c^3 + 86*A*B*a*b^5*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/
2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a
^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b
^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*
B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c
 - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(
1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 -
22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) +
44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24
*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*
a^5*b^2*c^10)))^(1/2)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c -
 b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5
+ 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 302
40*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)
^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*
B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c
 - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^
3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)
^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.119 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=336

$$-\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x(bB-2Ac)}{2c\left(b^2-4ac\right)}+\frac{\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*(-2*A*c+B*b)*x/c/(-4*a*c+b^2)-1/2*x^3*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4
*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(b^2*B+A*b*c-6*a*B*c+(-4*A*a*c^2-A*b^2*c+8*B*a*b*c-B*b^3)/(-4*a*c
+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*B+A*b*c-6*a*B*c
+(4*A*a*c^2+A*b^2*c-8*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^
2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.72, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*B - 2*A*c)*x)/(2*c*(b^2 - 4*a*c)) - (x^3*(A*b - 2*a*B - (b*B - 2*A*c)*
x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2*B + A*b*c - 6*a*B*c - (
b^3*B + A*b^2*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*B + A*b*c - 6*a*B*c + (b^3*B + A*b^2*c
 - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b
^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
```

```
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\int \frac{x^2\left(3(Ab-2aB)+(-bB+2Ac)x^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{(bB-2Ac)x}{2c\left(b^2-4ac\right)}-\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-a(bB-2Ac)+\left(-b^2B-Abc+6aBc\right)x^2}{a+bx^2+cx^4}\,dx}{2c\left(b^2-4ac\right)}$$

$$= -\frac{(bB-2Ac)x}{2c\left(b^2-4ac\right)}-\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab^2c-8a}{\sqrt{b^2-}}\right.}{4c\left(b^2-\right.}$$

$$= -\frac{(bB-2Ac)x}{2c\left(b^2-4ac\right)}-\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab^2c-8a}{\sqrt{b^2-}}\right.}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)}$$

Mathematica [A] time = 0.85, size = 362, normalized size = 1.08

$$\frac{\frac{2\sqrt{c}\left(2acx\left(A+Bx^2\right)-abBx+bx^3(Ac-bB)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b^2\left(B\sqrt{b^2-4ac}-Ac\right)+bc\left(A\sqrt{b^2-4ac}+8aB\right)-2ac\left(3B\sqrt{b^2-4ac}+2Ac\right)+b^3(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*(-(a*b*B*x) + b*(-(b*B) + A*c)*x^3 + 2*a*c*x*(A + B*x^2)))/((b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*B) + b*c*(8*a*B + A*Sqrt[
b^2 - 4*a*c]) + b^2*(-(A*c) + B*Sqrt[b^2 - 4*a*c]) - 2*a*c*(2*A*c + 3*B*Sqr
t[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*B + 2*a*c*
(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^2*(A*c + B*Sqrt[b^2 - 4*a*c]) + b*(-8*a
*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

fricas [B] time = 2.37, size = 4658, normalized size = 13.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(B*b^2 - (2*B*a + A*b)*c)*x^3 + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4
+ a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*A*B
*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2
*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c
^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a
^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(
b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4
*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4*A^
4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^2*b
 + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 +
 9*A^2*B^2*b^4)*c)*x + 1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2
*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b
+ 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A
*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 -
64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4*(6
*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b
)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2
- 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*
sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^
2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*
A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4
*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)
*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*
sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^
2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*
A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4
*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*
a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108
*B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*
a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x - 1/2*sqrt(1/2)*(B^3*b^7 - 1
7*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b
^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^
3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*
(3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2
*b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 -
2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*
b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (
60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c +
 (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4
*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A
^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 -
 64*a^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 +
 (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (
60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c -
 (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4
*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A
^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 -
 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b
 - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3
*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x +
1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3
```

```
 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12
*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^
5)*c^2 + (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*
a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*
sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 -
 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*
A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*
B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^
3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^
4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*
b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x
^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*
A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*
B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^
3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^
4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*
b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4
*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^
2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^
3 + 9*A^2*B^2*b^4)*c)*x - 1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*
a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3
*b + 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 1
8*A*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 + (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7
 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4
*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*
B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b
^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)
))*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a
*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a -
 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*
B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a
^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + 2*(B*a
*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c -
4*a*b*c^2)*x^2)
```

giac [B] time = 6.29, size = 4538, normalized size = 13.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^2*x^3 - 2*B*a*c*x^3 - A*b*c*x^3 + B*a*b*x - 2*A*a*c*x)/((c*x^4 +
b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)
^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^
3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*B
```

```
 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 -
 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(b^2*c - 4*a*c^2) + 2*(sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c
^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^
2*b*c^4)*B*abs(b^2*c - 4*a*c^2) - (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7
 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^
4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 64*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 2*(b^2
- 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*A - (2*b^8*c^4 - 32*a*b^6*c^
5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(
b^2 - 4*a*c)*a^2*b^2*c^6)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 +
 sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))
/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*
b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^
2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c)) - 1/16*((2*b^3*c^3 - 8*a*b
*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3
)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b
^2*c - 4*a*c^2)^2*B + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5
- 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 -
 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(b^2*c - 4*a*c^2) - 2*(sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt
```

```
(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^
4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 +
 8*(b^2 - 4*a*c)*a^2*b*c^4)*B*abs(b^2*c - 4*a*c^2) - (2*b^7*c^5 - 8*a*b^5*c
^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*A - (2*b^
8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*
c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*B)*arctan(2*sqrt(1/2)*x/sqrt((
b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b
^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*
a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a
^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c))
```

maple [B] time = 0.04, size = 1030, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^3-1/2*(2*A*c-B*b)/(4*a*c-b^2)*a
/c*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/(4*a*c-b^
2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A-1/4/(4*a*c-b^2)/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2-3/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*a*B+1/4/(4*a*c-b^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+2/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*B-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*b^3*B-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/(4*a*c-
b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A-1/4/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+3/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B-1/
4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b
```

```
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*a*b*B-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
c*x)*b^3*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((B*b^2 - (2*B*a + A*b)*c)*x^3 + (B*a*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*
c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + 1/2*integrate((
B*a*b - 2*A*a*c + (B*b^2 - (6*B*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b
^2*c - 4*a*c^2)
```

mupad [B] time = 5.18, size = 12396, normalized size = 36.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^3*(A*b*c - B*b^2 + 2*B*a*c))/(2*c*(4*a*c - b^2)) + (x*(2*A*a*c - B*a*
b))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((2048*A*a^4*c^6 - 32
*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536
*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*
c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*
c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^1
0*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504
*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c
 - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*
c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*
A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^
12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*
c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^
6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 +
 A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9
)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2
*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c
^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^
6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A
*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32
*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*
c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(B^2*b^6 + 8*A^2*a^
2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 1
6*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 9
6*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^
3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^
2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 1
92*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c
*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3
- 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6
144*a^5*b^2*c^8)))^(1/2)*1i - (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b
^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*
a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48
```

*a^2*b^2*c^3)) + (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^
4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840
*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c -
b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c
^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)
^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4
+ 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)
))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(
2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*
(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c
- 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2
*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9
*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5
+ 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*
b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c
^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7
- 6144*a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 7
2*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a
*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*
a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +
B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512
*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^
4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9
)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 12
8*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/
2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a
^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2
)*1i)/((((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c
^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3
*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(
B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*
c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5
+ 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072
*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^
2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^
4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^
8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 128
0*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 -
192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 -
8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 +
512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2
*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2
)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 -
128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^
(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 24
0*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(
1/2) - (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b
^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3
*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^1
1 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2
)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*
B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^
5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b
*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 153
6*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/
(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b
^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (3*A*B^2*a*b^5 - 21

```
6*B^3*a^4*c^2 - 5*B^3*a^2*b^4 - 24*A^2*B*a^3*c^3 + 3*A^3*a*b^3*c^2 + 4*A^3*
a^2*b*c^3 + 66*B^3*a^3*b^2*c - 51*A*B^2*a^2*b^3*c + 204*A*B^2*a^3*b*c^2 - 4
2*A^2*B*a^2*b^2*c^2 + 6*A^2*B*a*b^4*c)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^
2 + 48*a^2*b^2*c^3)) + (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2
 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5
*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3)) + (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +
 B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512
*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^
4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9
)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 12
8*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/
2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a
^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2
)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*
c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*
c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^
2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^
5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*
c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A
*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(
4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24
*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*
a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a
^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^
3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c
^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^
2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^
3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c
^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2)
 - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a
^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36
*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*
c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)))*(-(
B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*
c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5
 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072
*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^
2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^
4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^
8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 128
0*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i - atan(((((
2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A
*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/
(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((A^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)
^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*
a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^
6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6
 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*
B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*
(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c
^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c
^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^
2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*
(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*
b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4
 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) +
 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3
```

*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A
*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^
5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(B
^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^
2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B
*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) -
2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c
^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^
2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B
^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2
*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6
*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840
*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i - (((2048*A*a^4*c^6 - 32*A*a*b^
6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*
b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 1
2*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^
2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96
*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3
*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2
*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 19
2*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*
(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 -
 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61
44*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*
a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c -
b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) -
 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*
c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B
^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*
B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^
2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^
6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 384
0*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A
^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b
^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2
 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*
b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3
 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4
*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2
*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c
 - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2)*1i)/((((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1
024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^
3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c
^3)) - (x*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2
b^2(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2
*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^
3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1
/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*
B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) +
 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b
^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(1
6*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^
2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2
*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3

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 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(
4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^
2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c
 - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^
10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b
^2*c^8)))^(1/2) - (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^
3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 1
4*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))
*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4
*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*
c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3
072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768
*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4
*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a
*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 -
1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (3*A*B^2*
a*b^5 - 216*B^3*a^4*c^2 - 5*B^3*a^2*b^4 - 24*A^2*B*a^3*c^3 + 3*A^3*a*b^3*c^
2 + 4*A^3*a^2*b*c^3 + 66*B^3*a^3*b^2*c - 51*A*B^2*a^2*b^3*c + 204*A*B^2*a^3
*b*c^2 - 42*A^2*B*a^2*b^2*c^2 + 6*A^2*B*a*b^4*c)/(4*(b^6*c - 64*a^3*c^4 - 1
2*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*
a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192
*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3)) + (x*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B
^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*
c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 38
40*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c
 - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6
*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^
2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^
4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^
8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))
/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
 - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c
 + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^
2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c -
9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5
 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B
*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*
c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7
 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 -
72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*
a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8
*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 +
 B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512
*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^
4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9
)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 12
8*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/
2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a
^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2
)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(
-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b
^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4
- 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) +
768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*
b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*
B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5
 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.120 $\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=276

$$-\frac{x\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+b\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

```
[Out] -1/2*x*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan
(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*B-2*A*c+(4*A*b*c-4*B*a*
c-B*b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*B
-2*A*c+(-4*A*b*c+4*B*a*c+B*b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1275, 1166, 205}

$$-\frac{x\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + ((b*B - 2*A*c - (b^2*B - 4*A*b*c + 4*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 -
4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*B - 2*A*c + (b^2*B - 4*A*b*c + 4*
a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
```

```
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2} dx = -\frac{x \left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{Ab-2aB+(bB-2Ac)x^2}{a+bx^2+cx^4} dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x \left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{4\left(b^2-4ac\right)} +$$

$$= -\frac{x \left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} +$$

Mathematica [A] time = 0.66, size = 298, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2x\left(B\left(2a+bx^2\right)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(-2Ac\sqrt{b^2-4ac}+bB\sqrt{b^2-4ac}-4aBc+4Abc+b^2(-B)\right)\tan^{-1}}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (Sqrt[2]*(-(b^2*B) + 4*A*b*c - 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*
c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^
2*B - 4*A*b*c + 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*c*Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*
a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4
```

fricas [B] time = 1.26, size = 3467, normalized size = 12.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b - 2*A*c)*x^3 - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2
*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2
+ (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 4
8*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b
^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*
b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*a*b^3 -
 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3*a^2*b
+ A^3*B*b^3)*c)*x + 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*(2*A^2*
B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*b^2 - A
^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 - A^3*b^5
)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^2)*c^4
- 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*sqrt((B^4*a^
2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4
```

```
 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2
*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2
*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 -
12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 +
 48*a^3*b^2*c^3 - 64*a^4*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*
b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*
c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c -
12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*
a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3
*a^2*b + A^3*B*b^3)*c)*x - 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*
(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*
b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 -
A^3*b^5)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^
2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*sqrt(
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*
b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 +
(12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*
a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6
*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^
4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 +
 a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3
*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a
^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b
^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 -
 A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 1
2*A*B^3*a^2*b + A^3*B*b^3)*c)*x + 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^
5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2
*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a
*b^4 - A^3*b^5)*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A
*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c
)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 +
48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)
*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^
2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(
a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12
*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2
)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*
a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c
 - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*
a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)
))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^
2*b^2 - A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*
a^3 - 12*A*B^3*a^2*b + A^3*B*b^3)*c)*x - 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B
^2*a*b^5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b
+ 2*A^2*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*
A^2*B*a*b^4 - A^3*b^5)*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^
5*b + A*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a
*b^8)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4
*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A
^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2
*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6
*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) + 2*(2*B*a - A*b)*x)/(
(b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [B] time = 4.78, size = 3776, normalized size = 13.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b*x^3 - 2*A*c*x^3 + 2*B*a*x - A*b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a
*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^
3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2*
B - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3
 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4
*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*
a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^
4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c)
 - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a*b^2*c^4)*A
+ (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*
b*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2
 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c -
12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3
- 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*
abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*s
```

```
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)
^2*B + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2
- 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 -
16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3
*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a
*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a*b^2*c^4)
*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a
^2*b*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c
)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c
 - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c
^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*
c)*abs(c))
```

maple [B] time = 0.03, size = 733, normalized size = 2.66

$$\frac{\sqrt{2}\, A b c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{\left(4 a c-b^{2}\right) \sqrt{-4 a c+b^{2}}\, \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}+\frac{\sqrt{2}\, A b c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{\left(4 a c-b^{2}\right) \sqrt{-4 a c+b^{2}}\, \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}-\frac{\sqrt{2}\, B a c\, a}{\left(4 a c-b^{2}\right) \sqrt{-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (1/2*(2*A*c-B*b)/(4*a*c-b^2)*x^3+1/2*(A*b-2*B*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^
2+a)-1/2/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A+1/(4*a*c-b^2)*c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-
1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B-1/4/(4*a*c-b^2
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+1/2/(4*a*c-b^2)*c*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
```

```
(1/2)*c*x)*A+1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/4/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*a*B-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2
*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(Bb-2\,Ac)x^3+(2\,Ba-Ab)x}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}-\frac{-\int\frac{(Bb-2\,Ac)x^2-2\,Ba+Ab}{cx^4+bx^2+a}\,dx}{2\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 2*A*c)*x^3 + (2*B*a - A*b)*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 -
4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-((B*b - 2*A*c)*x^2 - 2*B*a
+ A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

mupad [B] time = 4.41, size = 9444, normalized size = 34.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x*(A*b - 2*B*a))/(2*(4*a*c - b^2)) + (x^3*(2*A*c - B*b))/(2*(4*a*c - b^2)
))/(a + b*x^2 + c*x^4) - atan(((((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b
^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*
b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*
b^4*c)) - (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^
2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 9
6*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*
c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*
B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4
*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*
c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c
 + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^
4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a
^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 -
12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 128
0*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) - (
x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2
 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-
(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c -
b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c
^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a
^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*
(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840
*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*1i - (((16*A*b^7*c^2 +
2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*
A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)
^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3
 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*
B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 3
```

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84*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 +
240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 +
a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^
3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c -
 b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^
5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 10
24*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^
2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c
^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c
^6 + a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B
^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2
*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*
b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768
*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*
c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3
 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/
2)*1i)/((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*B^2*b^4*c - 3*B^3*a*b^3*c + 8*A*B^
2*a^2*c^3 - 5*A^2*B*b^3*c^2 - 4*B^3*a^2*b*c^2 + 18*A*B^2*a*b^2*c^2 - 28*A^2
*B*a*b*c^3)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((16*A*
b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*
c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(B^2*a*b^9 - B^2*a*(-(4*
a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a
^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3
 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b
^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b
^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*
b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 +
768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*
(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*
A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^
3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*
a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*
a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144
*a^6*b^2*c^6 + a*b^12*c)))^(1/2) - (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2
*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(
1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 51
2*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5
*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*
B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a
^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^1
2*c)))^(1/2) + (((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*
a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B
*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(
B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b
^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^
2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^
5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(
4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*
a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*
c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
)*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*
c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b
^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B
^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/
(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 +
3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8
```

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*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c
^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2
*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) -
96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4
*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A
*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 -
24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6
144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^
9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3
+ 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B
*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 38
4*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 2
40*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a
*b^12*c)))^(1/2)*2i - atan(((((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*
c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4
*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
*c)) - (x*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c
*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B
^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5
 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a
*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^
6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2
 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c -
A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 -
 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*
b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*
A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a
^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) - (x*(
B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 +
2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^
2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2
)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2
+ 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*
b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(40
96*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^
5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*1i - (((16*A*b^7*c^2 + 204
8*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a
^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)
^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 +
512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a
^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*
A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240
*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b
^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c
^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^
2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c
^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*
A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 -
 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2
+ 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6
+ a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*
a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^
9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3
*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^
2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3
 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 -
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1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*
1i)/((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*B^2*b^4*c - 3*B^3*a*b^3*c + 8*A*B^2*a
^2*c^3 - 5*A^2*B*b^3*c^2 - 4*B^3*a^2*b*c^2 + 18*A*B^2*a*b^2*c^2 - 28*A^2*B*
a*b*c^3)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((16*A*b^7
*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2
 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(B^2*a*b^9 + B^2*a*(-(4*a*c
 - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*
b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 +
1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*
c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10
*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2
*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768
*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(
4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2
*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c
^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2
*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2
*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^
6*b^2*c^6 + a*b^12*c)))^(1/2) - (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^
3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2
) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A
^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^
5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a
^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*
b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c
)))^(1/2) + (((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3
*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^
3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(B^2
*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)
^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 +
 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b
*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(409
6*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5
*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3
 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(
-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c -
 b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*
c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*
a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32
*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 384
0*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8*A^
2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2
 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*
(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*
A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^
3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*
a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*
a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144
*a^6*b^2*c^6 + a*b^12*c)))^(1/2)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^
(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 5
12*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^
5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A
*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*
a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^
12*c)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.121 $\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=293

$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4al}{\sqrt{b^2-4}}\right.}{2\sqrt{2}}$$

```
[Out] 1/2*x*(A*b^2-a*b*B-2*a*A*c+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a
)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A*b-2
*a*B+(4*a*b*B+A*(-12*a*c+b^2))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*c^(1/2)*(A*b-2*a*B+(12*A*a*c-A*b^2-4*B*a*b)/(-4*a*c+b^2)^(1/2))
/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1178, 1166, 205}

$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4al}{\sqrt{b^2-4}}\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[c]*(A*b - 2*a*B + (4*a*b*B + A*(b^2 - 12*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a
*B - (A*b^2 + 4*a*b*B - 12*a*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^2}dx = \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-Ab^2-abB+6aAc-(Ab-2aB)cx^2}{a+bx^2+cx^4}dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(Ab-2aB-\frac{Ab^2+4abB-12aAc}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b}}}{4a\left(b^2-4ac\right)}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(2aB\left(2b-\sqrt{b^2-4ac}\right)+A\left(b^2-12a\right.\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.79, size = 304, normalized size = 1.04

$$\frac{\frac{2x\left(A\left(-2ac+b^2+bcx^2\right)-aB\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-2aB\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}+12\right.\right.}{(}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-2*a*B*(-2*b + Sqrt[b^2 - 4*a*c]) +
A*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[2]*Sqrt[c]*(-2*a*B*(2*b + Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 12*a*c + b*Sq
rt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

fricas [B] time = 3.33, size = 4885, normalized size = 16.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*a - A*b)*c*x^3 - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^
2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A
^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2
- 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*s
qrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4
+ 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^
4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*
a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4
 + A^3*B*b^5)*c)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B
*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A
^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 9
5*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 +
```

```
23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8
- 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*
A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a
*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b
^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(B^2*
a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B
^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 +
 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b
 + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B
^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(
4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^
2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))
)/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*
c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84
*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B
^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^5
 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^
2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4
*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*
a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144
*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^
3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6
*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a
^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3
 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^
3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3
*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*
(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*
a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c -
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*
B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 -
18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64
*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B
^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*
c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x
+ 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 +
864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2
*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2
- (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c +
(B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 +
64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt(
(B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81
*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b
^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^
2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^
4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^
```

```
4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*
a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B
*a^2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a
^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 +
 A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)
/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a
^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 -
65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*
B^2*a*b^4 + A^3*B*b^5)*c)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6
+ 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4
*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^
3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*
a^2*b^5 + 23*A^3*a*b^6)*c + (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 25
6*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5
*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 +
4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b
+ A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sq
rt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2
 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*
a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^
3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9
*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + 2*(B*a*b
 - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*
b^3 - 4*a^2*b*c)*x^2)
```

giac [B] time = 5.96, size = 4426, normalized size = 15.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*a*c*x^3 - A*b*c*x^3 + B*a*b*x - A*b^2*x + 2*A*a*c*x)/((c*x^4 + b*
x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*A - 2
*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b
^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 4
8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*
(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*A*abs(a*b^2 - 4*a^2*c
) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt
```

```
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3 + 2*(b^2 - 4*a*c)*a^2*
b^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*B*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2
- 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 -
 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A + 4*(2*a^3*b^6*c^2 - 16
*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*B)
*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2
- 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a
^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3
*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^
2 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*A - 2*(2*a*b
^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*
a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*B - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*
a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*(b^2 -
4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*A*abs(a*b^2 - 4*a^2*c) - 2*(
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^4*c + 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2
+ 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 32*a^4*b*c^3 - 2*(b^2 - 4*a*c)*a^2*b^3*c +
 8*(b^2 - 4*a*c)*a^3*b*c^2)*B*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2 - 40*a^
3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
```

```
rt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)
*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A + 4*(2*a^3*b^6*c^2 - 16*a^4*b^
4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4
*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2
*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*B)*arctan
(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^
2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6
- 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^
2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a
^2*c)*abs(c))
```

maple [B] time = 0.11, size = 1761, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4/(4*a*c-b^2)*(-4*a*c+b^2)^(1/2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/
c)*A-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*A*b+1/2/(4*
a*c-b^2)*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*B-12*c^3/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*a-8*c^2/(4*a*c-b^2)/(-4*a
*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+3/4*c/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)/a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^4-c^2/(4*a*c-b^2)/(4*
a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*A*b^3+2*c^2/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+3/
2*c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2+4*c^2/(4*a*c-b^2)/(-
4*a*c+b^2)^(1/2)*a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B+3*c/(4*a*c-b^2)/(-4
*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^3-1/4/(4*a*c-b^2)*(-4*
a*c+b^2)^(1/2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*A-1/4/(4*a*c-b^2)
/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*A*b+1/2/(4*a*c-b^2)*x/(x^2+1/2*
b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*B-12*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(4*a*
c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*a-8*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(4*
a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+3/4*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)
/a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^4+c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*A*b+3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*A*b^3-2*c^2/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B-3/2*
```

```
c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2+4*c^2/(4*a*c-b^2)/(
-4*a*c+b^2)^(1/2)*a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B+3*c/(4*a*c-b^2)
/(-4*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(2\,Ba-Ab)cx^3+\left(Bab-Ab^2+2\,Aac\right)x}{2\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}+\frac{-\int\frac{(2\,Ba-Ab)cx^2-Bab-Ab^2+6\,Aac}{cx^4+bx^2+a}\,dx}{2\left(ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*B*a - A*b)*c*x^3 + (B*a*b - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2*
c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate(-((
2*B*a - A*b)*c*x^2 - B*a*b - A*b^2 + 6*A*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^
2 - 4*a^2*c)
```

mupad [B] time = 4.84, size = 12349, normalized size = 42.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6
*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a
^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2)) - (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)
^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*
c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 51
2*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c
- b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*
c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2
)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
 + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5
)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*
c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 +
 A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*
B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^
4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*
b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a
^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4
 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4
096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7
*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8
*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40
*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^
2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(
1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2
*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5
- 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5
- 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*
a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a
^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3
 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*A*a^5*c^6 + 16
*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 -
```

```
5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c
^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(A^2*
b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c -
b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 +
 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B
*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a
^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 -
1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*
c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 -
16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c
^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^
(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c
^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512
*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c -
 b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c
^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)
^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
+ 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)
))^(1/2) - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^
2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c
 - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^
2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^
5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c
*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*
B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4
*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*
a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a
^8*b^2*c^5)))^(1/2)*1i)/((((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*
c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^
2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2
*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10
 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B
^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c -
9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4
 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B
*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*
c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4
 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5
*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b
^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b
^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 +
3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*
a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^
5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1
536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c
)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6
*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*A^2*a^2*c^
5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2
*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)
))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(
-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3
*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3
+ 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) -
3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*
b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*
B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2
```

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 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (((614
4*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*
A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 +
 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2)) + (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2
*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A
^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^
3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1
/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*
B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) -
 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b
^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1
024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^
2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(
4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 2
88*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a
^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^
2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 1
92*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b
*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6
 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6
144*a^8*b^2*c^5)))^(1/2) - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4
 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c
^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 +
A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B
*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4
 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b
^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^
6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4
+ 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 40
96*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*
b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*A^3*b^3*c^4 + 8*B^3*a^3*c^4 + 6*B^
3*a^2*b^2*c^3 - 36*A^3*a*b*c^5 + 72*A^2*B*a^2*c^5 - 3*A^2*B*b^4*c^3 + 3*A*B
^2*a*b^3*c^3 - 60*A*B^2*a^2*b*c^4 + 18*A^2*B*a*b^2*c^4)/(4*(a^2*b^6 - 64*a^
5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b
^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^
10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96
*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c
 - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c
^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A
*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^
9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c
^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i + atan(((((6144*A*a^5*c^6 + 16*A*a*b^8*c^
2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*
b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2
*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((A^2*b^2*(-(4*a*c
 - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2
) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^
4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 2
7*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 7
68*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5
*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*
b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 +
3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c
^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b
^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a
^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2
*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*
c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2
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) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*
a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 3
6*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8
*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x
*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^
2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^
11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2
+ 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^
2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^
2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2
+ 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)
^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2)*1i - (((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^
2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 19
2*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2)) + (x*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9
- A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*
b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2
- 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*
a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*
b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c -
 b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^
10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2
*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*
b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^
2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2
*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3
*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2
*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^
2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*
c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3
- 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 -
 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(
4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)
^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A
^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^
5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^
5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*
B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*A*a^5*c^6 +
16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4
- 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3
*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((A^2
*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c -
 b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3
- 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*
B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*
a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 +
 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8
*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a
^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 -
 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*
c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2
*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c
```

```
^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512
*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c -
 b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c
^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)
^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
+ 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)
))^(1/2) + (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^
2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2
*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2
*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5
*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*
(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B
*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*
a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a
^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^
8*b^2*c^5)))^(1/2) + (((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5
- 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^
7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^
2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 28
8*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^
4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2
*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 19
2*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*
(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 -
 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61
44*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3
- 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(
4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)
^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A
^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^
5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^
5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*
B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*A^2*a^2*c^5 + A^
2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a
*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A
^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c
 - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^
3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*
A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^
2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3
 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b
^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*A^3*b^3*c^
4 + 8*B^3*a^3*c^4 + 6*B^3*a^2*b^2*c^3 - 36*A^3*a*b*c^5 + 72*A^2*B*a^2*c^5 -
 3*A^2*B*b^4*c^3 + 3*A*B^2*a*b^3*c^3 - 60*A*B^2*a^2*b*c^4 + 18*A^2*B*a*b^2*
c^4)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*((A^2*b^2
*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2
)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 38
40*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^
6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*
b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 153
6*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/
(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b
^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i + ((x*(2*A*a*c - A
*b^2 + B*a*b))/(2*a*(4*a*c - b^2)) - (c*x^3*(A*b - 2*B*a))/(2*a*(4*a*c - b^
```

```
2)))/(a + b*x^2 + c*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

3.122 $\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=389

$$-\frac{-10aAc-abB+3Ab^2}{2a^2x\left(b^2-4ac\right)}+\frac{\sqrt{c}\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right.\right.}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*(10*A*a*c-3*A*b^2+B*a*b)/a^2/(-4*a*c+b^2)/x+1/2*(-a*b*B+A*(-2*a*c+b^2)+
(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(b^2-12*a*c+b*(-4*a*c+b^2)
^(1/2))-A*(3*b^3-16*a*b*c+3*b^2*(-4*a*c+b^2)^(1/2)-10*a*c*(-4*a*c+b^2)^(1/2
)))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(
x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*A*b^2-a*b*B-10*a
*A*c+(a*B*(-12*a*c+b^2)-A*(-16*a*b*c+3*b^3))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*
c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.22, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1277, 1281, 1166, 205}

$$-\frac{-10aAc-abB+3Ab^2}{2a^2x\left(b^2-4ac\right)}+\frac{\sqrt{c}\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right.\right.}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(3*A*b^2 - a*b*B - 10*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2 - 2
*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) + (S
qrt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) - A*(3*b^3 - 16*a*b*c + 3*
b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - a*b*B - 10*a*A*c + (a*B*(b^2 -
12*a*c) - A*(3*b^3 - 16*a*b*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
```

```
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-3Ab^2+abB+10aAc-3(Ab-2aB)cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x}-\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{\int \frac{aB\left(b^2-6ac\right)-A(3b \cdots}{\cdots}}{2 \cdots}$$

$$= -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x}-\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{\left(c\left(aB\left(b^2-12a \cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x}-\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(aB\left(b^2-12 \cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 1.04, size = 382, normalized size = 0.98

$$\frac{\frac{2x\left(aB\left(-2ac+b^2+bcx^2\right)-A\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)+aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\right)\tan^{-} \cdots}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \cdots}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*A)/x + (2*x*(a*B*(b^2 - 2*a*c + b*c*x^2) - A*(b^3 - 3*a*b*c + b^2*c*x^
2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(
a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(-3*b^3 + 16*a*b*c - 3*b^2*Sqr
t[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + (Sqrt[2]*Sqrt[c]*(a*B*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

fricas [B] time = 7.30, size = 7583, normalized size = 19.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^4 - 4*A*a*b^2 + 16*A*a^2*c + 2*
(B*a*b^2 - 3*A*b^3 - (2*B*a^2 - 11*A*a*b)*c)*x^2 - sqrt(1/2)*((a^2*b^2*c -
4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(
B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 +
5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 -
 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^
2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 -
 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^
3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2
*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^
2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*
b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 6
25*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b +
 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^
4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A
^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*
B*b^7)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b
^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 -
 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^
6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 +
 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^
7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*
a*b^9)*c - (B*a^6*b^9 - 3*A*a^5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b -
17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 4
9*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12
*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^
4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(2
7*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 10
17*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3
*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48
*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7
 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77
*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 1
2*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A
^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(
27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1
017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^
3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b
*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b
^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 +
77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c +
 (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 -
 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625
*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3
*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 +
 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*
a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c +
 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*
c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a
^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^
2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*
```

B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*B*b^7)*c^2)*x - 1/2*sqrt(1/2)*(B^3*
a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^
6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b
^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*
A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*
b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^
2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*a*b^9)*c - (B*a^6*b^9 - 3*A*a^5*b^1
0 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3
 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 -
 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6
 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*
A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A
^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*
b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*
a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt
(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3
 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^
3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^
6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44
*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*
A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5
*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4
*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^
5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/2)*((a^2*b^2
*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sq
rt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c
^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*
b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b
^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*
b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 -
44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 96
8*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a
^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A
^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^
6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^
4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B
^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 -
189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27
*A^3*B*b^7)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*
B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*
a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B
^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*
c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a
^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486
*A^3*a*b^9)*c + (B*a^6*b^9 - 3*A*a^5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10
*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^
5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4
 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 6
25*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 +
 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3
 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^
2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c
 + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^
2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2
 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*
c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^
4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 +

```
625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3
+ 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^
3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B
^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*
c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c
^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*
a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*
A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b
^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5
)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*
b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8
+ 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^
3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*
b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2
*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*
b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^
3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 16
74*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 -
 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*B*b^7)*c^2)*x - 1/2*sqrt(1/2)*
(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2
*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*
a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 1
2414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2
*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192
*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*a*b^9)*c + (B*a^6*b^9 - 3*A*a^
5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9
*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*
b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^
2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5
- 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b +
968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4
*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459
*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b
)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a
^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^
7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a
^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5
 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b +
 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^
4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 45
9*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))
)/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4
*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

giac [B] time = 6.16, size = 5408, normalized size = 13.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a*b*c*x^4 - 3*A*b^2*c*x^4 + 10*A*a*c^2*x^4 + B*a*b^2*x^2 - 3*A*b^3*x
^2 - 2*B*a^2*c*x^2 + 11*A*a*b*c*x^2 - 2*A*a*b^2 + 8*A*a^2*c)/((c*x^5 + b*x^
3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^
4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(
```

```
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 -
 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*A - (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a^2*b
^2 - 4*a^3*c)^2*B + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 -
37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 74*a^3
*b^5*c^2 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b*c^4 + 416*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a^2*b^5*c - 5
0*(b^2 - 4*a*c)*a^3*b^3*c^2 + 104*(b^2 - 4*a*c)*a^4*b*c^3)*A*abs(a^2*b^2 -
4*a^3*c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 - 14*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c - 2*a^3*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 +
 sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 28*a^4*b^4*c^2 - 96*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^3 - 48*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^5*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b^2*c^3 - 128*a^5*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
5*c^4 + 192*a^6*c^4 + 2*(b^2 - 4*a*c)*a^3*b^4*c - 20*(b^2 - 4*a*c)*a^4*b^2*
c^2 + 48*(b^2 - 4*a*c)*a^5*c^3)*B*abs(a^2*b^2 - 4*a^3*c) + (6*a^4*b^8*c^2 -
 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 +
 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4)*A - (2*a^5*b
^7*c^2 - 40*a^6*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 20*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32
*(b^2 - 4*a*c)*a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)*B)*arctan(2*sqrt(1
/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c + sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2
 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 -
 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2
 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*
a^3*c)*abs(c)) + 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2
```

- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*A - (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*B - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 74*a^3*b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 416*a^5*b*c^4 - 6*(b^2 - 4*a*c)*a^2*b^5*c + 50*(b^2 - 4*a*c)*a^3*b^3*c^2 - 104*(b^2 - 4*a*c)*a^4*b*c^3)*A*abs(a^2*b^2 - 4*a^3*c) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*a^3*b^6*c + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - 28*a^4*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 128*a^5*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^4 - 192*a^6*c^4 - 2*(b^2 - 4*a*c)*a^3*b^4*c + 20*(b^2 - 4*a*c)*a^4*b^2*c^2 - 48*(b^2 - 4*a*c)*a^5*c^3)*B*abs(a^2*b^2 - 4*a^3*c) + (6*a^4*b^8*c^2 - 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4)*A - (2*a^5*b^7*c^2 - 40*a^6*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32*(b^2 - 4*a*c)*a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c - sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs(c))

maple [B] time = 0.04, size = 1252, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -A/a^2/x-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*A+1/2/a^2/(c*x^4+b*x^2+a)*
c/(4*a*c-b^2)*x^3*A*b^2-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b*B-3/2/a/(
c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*b*c+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*
b^3+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*B*c-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x
*B*b^2+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A-3/4/a^2*c/(4*a*c-b^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*A*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*A*b^3+1/4/a*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-3*c^2/(4*a*c-b^2)
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2-5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*A+3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*A*b^3-1/4/a*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-3*c
^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1/4/a*c/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^4 - 2*A*a*b^2 + 8*A*a^2*c + (B*a*
b^2 - 3*A*b^3 - (2*B*a^2 - 11*A*a*b)*c)*x^2)/((a^2*b^2*c - 4*a^3*c^2)*x^5 +
 (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x) + 1/2*integrate((B*a*b^
2 - 3*A*b^3 + (10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^2 - (6*B*a^2 - 13*A*a*b)
*c)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

mupad [B] time = 5.38, size = 17591, normalized size = 45.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - (A/a - (x^2*(3*A*b^3 - B*a*b^2 + 2*B*a^2*c - 11*A*a*b*c))/(2*a^2*(4*a*c -
 b^2)) + (c*x^4*(10*A*a*c - 3*A*b^2 + B*a*b))/(2*a^2*(4*a*c - b^2)))/(a*x +
```

b*x^3 + c*x^5) - atan((((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 851968*A*a^14*b*c^8 + 192*A*a^8*b^13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^9*c^4 - 256000*A*a^11*b^7*c^5 + 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*c^7 - 64*B*a^9*b^12*c^2 + 1664*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102400*B*a^12*b^6*c^5 - 327680*B*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i + ((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3

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*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(
-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^
4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6
 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7
*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064
*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a
*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 15
2*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 +
 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*
a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^1
3*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983
040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) + 393216*B*a^15*c^8 - 851968*A*a^1
4*b*c^8 - 192*A*a^8*b^13*c^2 + 4672*A*a^9*b^11*c^3 - 47360*A*a^10*b^9*c^4 +
 256000*A*a^11*b^7*c^5 - 778240*A*a^12*b^5*c^6 + 1261568*A*a^13*b^3*c^7 + 6
4*B*a^9*b^12*c^2 - 1664*B*a^10*b^10*c^3 + 17920*B*a^11*b^8*c^4 - 102400*B*a
^12*b^6*c^5 + 327680*B*a^13*b^4*c^6 - 557056*B*a^14*b^2*c^7) + x*(204800*A^
2*a^12*c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*
c^4 + 30112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*
c^7 - 458752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 +
 4608*B^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 -
96*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520
*A*B*a^10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*(-(9*
A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12
 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 4
4800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^
2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3
840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*
b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)
^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4
*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*
A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240
*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(
1/2)*1i)/(((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2
) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2
*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1
/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2
*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c
+ 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c
*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 2
2400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^
2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 4
4*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*
a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a
^10*b^2*c^5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c -
 b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^
3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c
 - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c
^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*
A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5
- 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^
4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a
^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a
^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4
*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 -
 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^1
4*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 851968*A*a^14*b*c^8
 + 192*A*a^8*b^13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^9*c^4 - 256000
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*A*a^11*b^7*c^5 + 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*c^7 - 64*B*a^9
*b^12*c^2 + 1664*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102400*B*a^12*b^6
*c^5 - 327680*B*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*
c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 3
0112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 4
58752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B
^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*
a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^
10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*(-(9*A^2*b^1
3 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077
*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^
2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*
a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2
*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 -
 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/
2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 +
30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b
^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^
8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) -
((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*
a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c
^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*
a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c
^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^
2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c
- b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a
^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2
) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*
b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c
 + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^
5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(
1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*
A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)
^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*
B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11
*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^
3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3
- 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c
+ 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 -
24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 614
4*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11
*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6
- 1572864*a^15*b^3*c^7) + 393216*B*a^15*c^8 - 851968*A*a^14*b*c^8 - 192*A*a
^8*b^13*c^2 + 4672*A*a^9*b^11*c^3 - 47360*A*a^10*b^9*c^4 + 256000*A*a^11*b^
7*c^5 - 778240*A*a^12*b^5*c^6 + 1261568*A*a^13*b^3*c^7 + 64*B*a^9*b^12*c^2
- 1664*B*a^10*b^10*c^3 + 17920*B*a^11*b^8*c^4 - 102400*B*a^12*b^6*c^5 + 327
680*B*a^13*b^4*c^6 - 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 - 7372
8*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*A^2*a
^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752*A^2*
a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^10*b^
6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b^11*c
^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^5*c^6
 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*(-(9*A^2*b^13 + B^2*a^
2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b
^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*
c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)
^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c
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^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^
3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*
A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*
a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*
c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 12
80*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + 128000*A^3
*a^10*c^9 + 504*A^3*a^6*b^8*c^5 - 8112*A^3*a^7*b^6*c^6 + 48704*A^3*a^8*b^4*
c^7 - 129280*A^3*a^9*b^2*c^8 - 40*B^3*a^8*b^7*c^4 + 608*B^3*a^9*b^5*c^5 - 2
944*B^3*a^10*b^3*c^6 + 46080*A*B^2*a^11*c^8 + 4608*B^3*a^11*b*c^7 - 84480*A
^2*B*a^10*b*c^8 + 240*A*B^2*a^7*b^8*c^4 - 3792*A*B^2*a^8*b^6*c^5 + 21696*A*
B^2*a^9*b^4*c^6 - 52992*A*B^2*a^10*b^2*c^7 - 360*A^2*B*a^6*b^9*c^4 + 5736*A
^2*B*a^7*b^7*c^5 - 33888*A^2*B*a^8*b^5*c^6 + 87936*A^2*B*a^9*b^3*c^7))*(-(9
*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^1
2 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 -
44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b
^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 +
3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6
*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2
)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^
4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6
*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(
-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 24
0*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^
(1/2)*2i - atan(((((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A
^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 302
40*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)
^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 15
04*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b
^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2
*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c
^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*
c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10
*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6
 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 -
6144*a^10*b^2*c^5)))^(1/2)*(x*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^
2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b
^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*
b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 +
 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b
*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A
*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*
A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4
096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^
9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*
c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 98304
0*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 851968*A*a^14*
b*c^8 + 192*A*a^8*b^13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^9*c^4 - 2
56000*A*a^11*b^7*c^5 + 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*c^7 - 64*
B*a^9*b^12*c^2 + 1664*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102400*B*a^1
2*b^6*c^5 - 327680*B*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(204800*A^2*
a^12*c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^
4 + 30112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^
7 - 458752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4
608*B^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96
*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A
*B*a^10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*((9*A^2
```

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*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 -
2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 4480
0*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(
-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840
*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c
^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)
^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^
4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B
*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^
7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2
)*1i + (((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 +
6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4
*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
+ B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5
*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26
880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(
4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400
*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9
)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*
B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*
b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*
b^2*c^5)))^(1/2)*(x*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9
*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 3
0240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 +
1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a
*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B
^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6
*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*
a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^
10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c
^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4
- 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144
*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5
*c^6 - 1572864*a^15*b^3*c^7) + 393216*B*a^15*c^8 - 851968*A*a^14*b*c^8 - 19
2*A*a^8*b^13*c^2 + 4672*A*a^9*b^11*c^3 - 47360*A*a^10*b^9*c^4 + 256000*A*a^
11*b^7*c^5 - 778240*A*a^12*b^5*c^6 + 1261568*A*a^13*b^3*c^7 + 64*B*a^9*b^12
*c^2 - 1664*B*a^10*b^10*c^3 + 17920*B*a^11*b^8*c^4 - 102400*B*a^12*b^6*c^5
+ 327680*B*a^13*b^4*c^6 - 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 -
 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*
A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752
*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^
10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b
^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^
5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*((9*A^2*b^4*(-(4*
a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a
^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*
b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c -
b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b
^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^
2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1
548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*
A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(
4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)
^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2
- 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/((((
9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^
12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 +
```

44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*
b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 -
3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^
6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^
2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b
^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) -
6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 2
40*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))
^(1/2)*(x*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13
+ 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a
^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a
^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c -
26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(
-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 224
00*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)
^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*
A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^
6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^1
0*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11
*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 157
2864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 851968*A*a^14*b*c^8 + 192*A*a^8*b^
13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^9*c^4 - 256000*A*a^11*b^7*c^5
+ 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*c^7 - 64*B*a^9*b^12*c^2 + 166
4*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102400*B*a^12*b^6*c^5 - 327680*B
*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 - 73728*B^2
*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*A^2*a^8*b^
8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752*A^2*a^11*
b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^10*b^6*c^5
- 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b^11*c^3 +
2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^5*c^6 - 25
3952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*((9*A^2*b^4*(-(4*a*c - b^2)
^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2
+ 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 +
25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1
/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 1
5360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*
c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^
3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^
2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^
2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8
*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) - (((9*A^2*b^4*(-(
4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2
*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^
5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c
- b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6
*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*
B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) +
1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 3072
0*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(
-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^
2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*((9
*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^1
2 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 +
44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b
^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 -

```
3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6
*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2
)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^
4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6
*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(
-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 24
0*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^
(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*
a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3
*c^7) + 393216*B*a^15*c^8 - 851968*A*a^14*b*c^8 - 192*A*a^8*b^13*c^2 + 4672
*A*a^9*b^11*c^3 - 47360*A*a^10*b^9*c^4 + 256000*A*a^11*b^7*c^5 - 778240*A*a
^12*b^5*c^6 + 1261568*A*a^13*b^3*c^7 + 64*B*a^9*b^12*c^2 - 1664*B*a^10*b^10
*c^3 + 17920*B*a^11*b^8*c^4 - 102400*B*a^12*b^6*c^5 + 327680*B*a^13*b^4*c^6
 - 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 - 73728*B^2*a^13*c^8 + 1
44*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*A^2*a^8*b^8*c^5 - 14336
0*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752*A^2*a^11*b^2*c^8 + 16*
B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^10*b^6*c^5 - 25600*B^2*
a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*
b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^5*c^6 - 253952*A*B*a^11
*b^3*c^7 + 237568*A*B*a^12*b*c^8))*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B
^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*
a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^
2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2
*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*
c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*
a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8
064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^
2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) -
 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 38
40*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + 128000*A^3*a^10*c^9 + 504*A^3
*a^6*b^8*c^5 - 8112*A^3*a^7*b^6*c^6 + 48704*A^3*a^8*b^4*c^7 - 129280*A^3*a^
9*b^2*c^8 - 40*B^3*a^8*b^7*c^4 + 608*B^3*a^9*b^5*c^5 - 2944*B^3*a^10*b^3*c^
6 + 46080*A*B^2*a^11*c^8 + 4608*B^3*a^11*b*c^7 - 84480*A^2*B*a^10*b*c^8 + 2
40*A*B^2*a^7*b^8*c^4 - 3792*A*B^2*a^8*b^6*c^5 + 21696*A*B^2*a^9*b^4*c^6 - 5
2992*A*B^2*a^10*b^2*c^7 - 360*A^2*B*a^6*b^9*c^4 + 5736*A^2*B*a^7*b^7*c^5 -
33888*A^2*B*a^8*b^5*c^6 + 87936*A^2*B*a^9*b^3*c^7))*((9*A^2*b^4*(-(4*a*c -
b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9
*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^
5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9
)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4
 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*
b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*
B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^
6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c
- b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1
/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280
*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.123 $\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=522

$$-\frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3x\left(b^2-4ac\right)}-\frac{-14aAc-3abB+5Ab^2}{6a^2x^3\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16\right.\right.}{}$$

[Out] 1/6*(14*A*a*c-5*A*b^2+3*B*a*b)/a^2/(-4*a*c+b^2)/x^3+1/2*(-a*B*(-10*a*c+3*b^2)+A*(-19*a*b*c+5*b^3))/a^3/(-4*a*c+b^2)/x+1/2*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x^3/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(3*b^3-16*a*b*c+3*b^2*(-4*a*c+b^2)^(1/2)-10*a*c*(-4*a*c+b^2)^(1/2))-A*(5*b^4-29*a*b^2*c+28*a^2*c^2+5*(-4*a*c+b^2)^(1/2)*b^3-19*(-4*a*c+b^2)^(1/2)*a*b*c))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(3*b^3-16*a*b*c-3*b^2*(-4*a*c+b^2)^(1/2)+10*a*c*(-4*a*c+b^2)^(1/2))-A*(5*b^4-29*a*b^2*c+28*a^2*c^2-5*(-4*a*c+b^2)^(1/2)*b^3+19*(-4*a*c+b^2)^(1/2)*a*b*c))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)

Rubi [A] time = 1.36, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1277, 1281, 1166, 205}

$$-\frac{\sqrt{c}\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2+5b^3\sqrt{b^2-4ac}-29ab^2c-19abc\sqrt{b^2\cdots}\right.\right.}{2\sqrt{2}\,a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]

[Out] -(5*A*b^2 - 3*a*b*B - 14*a*A*c)/(6*a^2*(b^2 - 4*a*c)*x^3) - (a*B*(3*b^2 - 10*a*c) - A*(5*b^3 - 19*a*b*c))/(2*a^3*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*c] + 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-5Ab^2+3abB+14aAc-5(Ab-2aB)cx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} + \frac{\int \frac{-3\left(5Ab^3-3ab^2B-19\right.}{}}{}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)}{2a\left(b^2-4ac\right)x^3\left(\right.}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)}{2a\left(b^2-4ac\right)x^3\left(\right.}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)}{2a\left(b^2-4ac\right)x^3\left(\right.}$$

Mathematica [A] time = 1.20, size = 487, normalized size = 0.93

$$\frac{\frac{6x\left(A\left(2a^2c^2-4ab^2c-3abc^2x^2+b^4+b^3cx^2\right)+aB\left(3abc+2ac^2x^2-b^3-b^2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{2}\sqrt{c}\left(A\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)+aB\left(-3b^2\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*A)/x^3 + (24*A*b - 12*a*B)/x + (6*x*(a*B*(-b^3 + 3*a*b*c - b^2*c*x^2
 + 2*a*c^2*x^2) + A*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*a*b*c^2*x^
```

```
2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3
+ 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4
 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3
+ 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4
 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*c] + 19*a*b*c*Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

fricas [B] time = 18.85, size = 10190, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6 - 4*A*a^2
*b^2 + 16*A*a^3*c - 2*(9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*c^2 - (33*B*a^2*b -
62*A*a*b^2)*c)*x^4 - 4*(3*B*a^2*b^2 - 5*A*a*b^3 - 4*(3*B*a^3 - 5*A*a^2*b)*c
)*x^2 - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^
5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b
^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2
+ 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b
^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 -
 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*
B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 24
01*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^
5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*
a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^
3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 +
3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^
3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*
b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14
*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b
^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4*c^8 + 7203*(4*A^3*B
*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B^3*a^5*b + 43524*A^2
*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*c^6 + (5625*B^4*a^5*
b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 20190*A^3*B*a^2*b^5 - 1
2325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a^3*b^5 + 5560*A^2*B^
2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27*B^4*a^3*b^6 - 135*A
*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*x + 1/2*sqrt(1/2)*(2
7*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13 - 125*A^3*b^14 + 109
76*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b + 709*A^3*a^6*b^2)*c^6
 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*B*a^6*b^3 - 86495*A^
3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*b^4 + 224705*A^2*B*a
^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5 - 8820*A*B^2*a^5*b^6
 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330*B^3*a^5*b^7 - 17889
*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^10)*c^2 - (486*B^3*a
^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 2425*A^3*a*b^12)*c - (
3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12*b)*c^5 + 64*(34*B*a
^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*A*a^10*b^5)*c^3 + 28
*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 94*A*a^8*b^9)*c)*sqrt
((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a
*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^
6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^
2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7
*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 +
41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A
^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*
```

```
a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 +
4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c
^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9
*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 +
 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3
*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*
b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*
B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25
*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300
*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4
*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b
^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7
872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4
*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*
b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c +
48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a
^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8
+ 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*
B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198
*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c +
(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^
8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4
*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a
^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109
544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*
B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b
^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6
- 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*
A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10
)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6
- 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4*c^8 + 720
3*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B^3*a^5*b +
 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*c^6 + (562
5*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 20190*A^3*B*a
^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a^3*b^5 + 5
560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27*B^4*a^3*b
^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*x - 1/2*sq
rt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13 - 125*A^3*
b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b + 709*A^3*a^
6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*B*a^6*b^3
- 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*b^4 + 2247
05*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5 - 8820*A*B
^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330*B^3*a^5*b
^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^10)*c^2 -
(486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 2425*A^3*a*b
^12)*c - (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12*b)*c^5 +
64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*A*a^10*b^5
)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 94*A*a^8*b^
9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 15
00*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 18
6*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51
894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(12
75*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*
a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5
 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2
*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*
a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 -
 64*a^17*c^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A
```

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*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*
b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(
15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c
 + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 +
 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^
6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*
a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 766
86*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2
*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a
^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 +
 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^
2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^1
5*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3
*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*
A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5
*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3
*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a
*b^7)*c - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*
B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11
 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b +
 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6
*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2
- 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815
*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^
2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b
^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*
A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))
/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4
*c^8 + 7203*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B
^3*a^5*b + 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*
c^6 + (5625*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 201
90*A^3*B*a^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a
^3*b^5 + 5560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27
*B^4*a^3*b^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*
x + 1/2*sqrt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13
- 125*A^3*b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b +
709*A^3*a^6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*
B*a^6*b^3 - 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*
b^4 + 224705*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5
- 8820*A*B^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330
*B^3*a^5*b^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^
10)*c^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 24
25*A^3*a*b^12)*c + (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12
*b)*c^5 + 64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*
A*a^10*b^5)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 9
4*A*a^8*b^9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2
*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^
2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*
a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c
^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77
424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B
^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^
8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9
550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16
*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9
- 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 2
3*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)
*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c - (a^7*b^6 - 12
```

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*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3
*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*
A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 +
 (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5
*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 +
 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(
1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B
*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7
 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^
6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*
c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*
x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2
*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*
(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 2
10*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6
 + 45*A^2*a*b^7)*c - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)
*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^
3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3
*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A
^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^
4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b
^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22
508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459
*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b
^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a
^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((96
04*A^4*a^4*c^8 + 7203*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 -
 22500*A*B^3*a^5*b + 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4
*a^2*b^4)*c^6 + (5625*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3
*b^4 - 20190*A^3*B*a^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 33
51*A*B^3*a^3*b^5 + 5560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c
^4 + 7*(27*B^4*a^3*b^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*
b^9)*c^3)*x - 1/2*sqrt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2
*B*a*b^13 - 125*A^3*b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*
B*a^7*b + 709*A^3*a^6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 +
96044*A^2*B*a^6*b^3 - 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*
A*B^2*a^6*b^4 + 224705*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^
3*a^6*b^5 - 8820*A*B^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c
^3 + (3330*B^3*a^5*b^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*
A^3*a^2*b^10)*c^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2
*b^11 - 2425*A^3*a*b^12)*c + (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 -
 13*A*a^12*b)*c^5 + 64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11
*b^4 - 23*A*a^10*b^5)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a
^9*b^8 - 94*A*a^8*b^9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A
^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*
(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9
300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*
a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^
5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4
 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*
A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a
^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c
 + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 2
5*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a
^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^
2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c - (a^
7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 -
 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^
12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*
```

```
b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544
*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3
*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)
*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 2
8260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B
^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c
)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 1
2*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))))/((a^3*b^2*c - 4*a^4*c^2)*x^7
 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)
```

giac [B] time = 8.15, size = 6327, normalized size = 12.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^2*c*x^3 - A*b^3*c*x^3 - 2*B*a^2*c^2*x^3 + 3*A*a*b*c^2*x^3 + B*a
*b^3*x - A*b^4*x - 3*B*a^2*b*c*x + 4*A*a*b^2*c*x - 2*A*a^2*c^2*x)/((a^3*b^2
 - 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a
^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
+ 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*
c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*A - (6*a*b^4*c^2 - 44
*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 -
 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^
2*B + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c + 286*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 +
 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^2 + 128*a^4*b^6*c^2 -
496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 +
 110*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 + 992*a^6*b^2*c^4
- 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^5 - 448*a^7*c^5 + 10*(b^
2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4*a*c)*a^4*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5
*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c^4)*A*abs(a^3*b^2 - 4*a^4*c) - 2*(3*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*
c - 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 +
 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3
- 304*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 +
416*a^7*b*c^4 + 6*(b^2 - 4*a*c)*a^4*b^5*c - 50*(b^2 - 4*a*c)*a^5*b^3*c^2 +
104*(b^2 - 4*a*c)*a^6*b*c^3)*B*abs(a^3*b^2 - 4*a^4*c) + (10*a^6*b^9*c^2 - 1
38*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sq
```

```
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2 -
4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b
^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*A - (6*a^7*b^8*c^2 - 80*a^8*b^6*c^3 +
 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2 - 4*a*
c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqr
t((a^3*b^3 - 4*a^4*b*c + sqrt((a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*
c)*(a^3*b^2*c - 4*a^4*c^2)))/(a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b
^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^1
0*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*a
bs(c)) - 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b
*c^3)*(a^3*b^2 - 4*a^4*c)^2*A - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2
 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*B - 2*(5*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c + 10*a^
3*b^8*c + 286*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^6*c^2 - 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 + 572
*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 - 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^6*c^5 + 448*a^7*c^5 - 10*(b^2 - 4*a*c)*a^3*b^6*c + 88*
```

```
(b^2 - 4*a*c)*a^4*b^4*c^2 - 220*(b^2 - 4*a*c)*a^5*b^2*c^3 + 112*(b^2 - 4*a*
c)*a^6*c^4)*A*abs(a^3*b^2 - 4*a^4*c) + 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 6
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c + 6*a^4*b^7*c + 152*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^4*b^5*c^2 - 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^7*b*c^3 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 25*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 + 304*a^6*b^3*c^3 + 52*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 - 416*a^7*b*c^4 - 6*(b^2 - 4
*a*c)*a^4*b^5*c + 50*(b^2 - 4*a*c)*a^5*b^3*c^2 - 104*(b^2 - 4*a*c)*a^6*b*c^
3)*B*abs(a^3*b^2 - 4*a^4*c) + (10*a^6*b^9*c^2 - 138*a^7*b^7*c^3 + 680*a^8*b
^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2 - 4*a*c)*a^6*b^7*c^2 + 98*(b
^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b^3*c^4 + 224*(b^2 - 4*a*c)
*a^9*b*c^5)*A - (6*a^7*b^8*c^2 - 80*a^8*b^6*c^3 + 352*a^9*b^4*c^4 - 512*a^1
0*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7
*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^6
*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c
- 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2
 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2
+ 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^
3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c
^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c
^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c
^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2 - 4*a*c)*a^8*b^4*c^3 - 128*(b^2
- 4*a*c)*a^9*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c - s
qrt((a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)
))/(a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^
9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a
^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) - 1/3*(3*B*a*x^2 -
6*A*b*x^2 + A*a)/(a^3*x^3)
```

maple [B] time = 0.05, size = 1653, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*B*b^2+2/a^2/(c*x^4+b*x^2+a)/(4*a*
c-b^2)*x*A*b^2*c-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*B*c+3/2/a^2/(c*x^4+b
*x^2+a)*c^2/(4*a*c-b^2)*x^3*A*b-1/2/a^3/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*A
*b^3+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B-5/2/a*c^2/(4*a*c-b^2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*B+2/a^3/x*A*b+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/
```

```
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*A-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2+19/4/a
^2*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)*A+3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2+5
/4/a^3*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-5/4/a^3*c/(4*a*c-b^2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*c*x)*A*b^3-19/4/a^2*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/3
*A/a^2/x^3-1/a^2/x*B-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*
B-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^3+5/4/a^3
*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^4+5/4/a^3*c/(4*
a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^4-29/4/a^2*c^2/(4*a*c-b^
2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^3-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*B
-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*c^2+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2
)*x*B*b^3-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6 - 2*A*a^2*
b^2 + 8*A*a^3*c - (9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*c^2 - (33*B*a^2*b - 62*A
*a*b^2)*c)*x^4 - 2*(3*B*a^2*b^2 - 5*A*a*b^3 - 4*(3*B*a^3 - 5*A*a^2*b)*c)*x^
2)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*
a^5*c)*x^3) - 1/2*integrate((3*B*a*b^3 - 5*A*b^4 - 14*A*a^2*c^2 - ((10*B*a^
2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^2 - (13*B*a^2*b - 24*A*a*b^2
)*c)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

mupad [B] time = 5.70, size = 21554, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - atan((((-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1
/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 11692
8*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^
2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2)
 + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 4
4800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*
```

a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 268
80*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*
a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*
a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(
1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c -
b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(
1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^1
3*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4
*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9
*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*
A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*
A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9
)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 1
0656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B
^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c
- 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*
a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^
4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^
7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(
4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2
*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240
*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))
^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440
*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^
3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 -
82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2
867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672
*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*
a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B
^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^1
1*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A
^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B
^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 36556
8*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 1110
4*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1
469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9
))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) -
30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*
a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*
c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 207
7*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B
^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^
7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2
*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^
10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^
4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) +
51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9
)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) +
186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
- 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 -
6144*a^12*b^2*c^5)))^(1/2)*1i - ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*
b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 3576
7*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 21504
0*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4
*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 +
30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b
^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 -
213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c -

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b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*
a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b
^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) +
 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b
^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^1
0*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c
^9 - x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2
) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*
A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*
a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) +
 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 448
00*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^
8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880
*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^
3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^
6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/
2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^
2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/
2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*
c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c
^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6
144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*
b^5*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2
 + 7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1
536000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 1
92*B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B
*a^16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408
*A^2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10
*b^12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2
*a^13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B
^2*a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360
*B^2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480
*A*B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 53
0432*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8
 + 1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-
(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*
a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*
a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*
B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)
^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B
^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9
)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^
6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(
-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*
B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(
-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*
c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*A^2*b^15 +
 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 636
6*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 21974
4*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 -
 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25
*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*
c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^
2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*
a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*
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a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(
-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a
^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 2
40*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)
))^(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^
6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*
A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*
A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(
-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30
240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2
)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 2
13*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^
5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4
*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 3
0*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3
*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(
32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*
b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^
8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^
18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*
c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 +
468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2
719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*
B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*
a^18*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9
*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a
^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872
*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*
B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 4587
52*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105
824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 -
 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2
*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*
a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*
a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1
/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656
*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a
^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80
640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*
b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^
8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^
2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*
c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^1
2*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a
*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9
*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/
2) + ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2)
 - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A
^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a
^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) +
2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 4480
0*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8
*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*
B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3
*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6
*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2
) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2
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)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2
) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c
^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^
4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 - x*(-(25*A^2*b^15 + 9*B^
2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2
*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2
*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(
1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 1065
6*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*
a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 8
0640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2
*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b
^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b
^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a
*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^
12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^
9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1
/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^
17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c
^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82
816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867
200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*
a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^1
7*b^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408*A^2*a^16*c^10 - 204800*B^2*
a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b
^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*
a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*
a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B
^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A
*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469
440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*
(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*
A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4
*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3
*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B
^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*
a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 -
 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^
8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*
c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c
^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51
*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(
1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 18
6*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 2
4*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 61
44*a^12*b^2*c^5)))^(1/2) + 128000*B^3*a^15*c^9 - 1800*A^3*a^9*b^9*c^6 + 290
80*A^3*a^10*b^7*c^7 - 176032*A^3*a^11*b^5*c^8 + 473216*A^3*a^12*b^3*c^9 + 5
04*B^3*a^11*b^8*c^5 - 8112*B^3*a^12*b^6*c^6 + 48704*B^3*a^13*b^4*c^7 - 1292
80*B^3*a^14*b^2*c^8 + 250880*A^2*B*a^14*c^10 - 476672*A^3*a^13*b*c^10 - 442
880*A*B^2*a^14*b*c^9 - 1680*A*B^2*a^10*b^9*c^5 + 27176*A*B^2*a^11*b^7*c^6 -
 164448*A*B^2*a^12*b^5*c^7 + 441216*A*B^2*a^13*b^3*c^8 + 1400*A^2*B*a^9*b^1
0*c^5 - 21680*A^2*B*a^10*b^8*c^6 + 121648*A^2*B*a^11*b^6*c^7 - 275264*A^2*B
*a^12*b^4*c^8 + 121088*A^2*B*a^13*b^2*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13
 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*
c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c
^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B
^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*
b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-
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(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a
^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(
-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 1
19616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 1
65*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^
9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184
*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)
^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2
- 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i - a
tan((((-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2)
 - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A
^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a
^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) +
2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 4480
0*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8
*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*
B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3
*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6
*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2
) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2
)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2
) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c
^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^
4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9*B^
2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2
*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2
*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(
1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 1065
6*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*
a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 8
0640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2
*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b
^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b
^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a
*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^
12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^
9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1
/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^
17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c
^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82
816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867
200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*
a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^1
7*b^5*c^6 - 1261568*B*a^18*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B^2*
a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b
^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*
a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*
a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B
^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A
*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469
440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*
(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*
A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4
*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3
*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B
^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*
a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 -
 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^
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8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*
c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c
^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51
*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(
1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 18
6*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 2
4*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 61
44*a^12*b^2*c^5)))^(1/2)*1i - ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6
*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A
^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A
^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-
(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 302
40*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)
^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 21
3*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2
)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5
*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*
c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30
*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*
c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b
^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9
- x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) -
30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2
*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3
c^3(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 20
77*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*
B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c
^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^
2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b
^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b
^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2)
- 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^
9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2)
- 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
- 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4
- 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144
*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5
*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 +
7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536
000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*
B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^
16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408*A^
2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^
12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^
13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*
a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^
2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*
B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 53043
2*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 +
1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*
a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3
*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6
*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c
- b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2
*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1
/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*
a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c

^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4
*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a
b^5(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4
*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7
*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3
 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*A^2*b^15 + 9*
B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A
^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A
^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)
^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10
656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^
2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c -
 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a
^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4
*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7
*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4
*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*
b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*
a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^
(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(
-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2
*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2
*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4
*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240
*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9
)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*
B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b
^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*
(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A
*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*
(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*
(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6
*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 +
 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*
b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*c^8
 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 + 468
480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2719
744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*B*a
^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*a^1
8*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9*b^
14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a^12
*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872*A^
2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*B^2
*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 458752*
B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105824
*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 - 21
21728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2*a^
2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2
*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5
*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2)
 + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^
2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*
c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640
*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2
c^2(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c
^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c
^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c -

b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c
+ 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^
8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)
+ ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) -
30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*
a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*
c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 207
7*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B
^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^
7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2
*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^
10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^
4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) -
51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9
)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) -
186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
- 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 -
6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 - x*(-(25*A^2*b^15 + 9*B^2*a
^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^
2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^
5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2
) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B
^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4
c^2(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 8064
0*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*
c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*
c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c
- b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*
c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b
^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)
*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*
b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)
+ 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82816
*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867200
*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*a^1
4*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^17*b
^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408*A^2*a^16*c^10 - 204800*B^2*a^1
7*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b^10
*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*a^1
4*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*a^1
2*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B^2*
a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A*B*
a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469440
*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*(-(
25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B
*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^
7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-
(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*
a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7
*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 61
5*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b
*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2
+ 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5
- 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^
2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2
) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A

```
*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a
^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*
a^12*b^2*c^5)))^(1/2) + 128000*B^3*a^15*c^9 - 1800*A^3*a^9*b^9*c^6 + 29080*
A^3*a^10*b^7*c^7 - 176032*A^3*a^11*b^5*c^8 + 473216*A^3*a^12*b^3*c^9 + 504*
B^3*a^11*b^8*c^5 - 8112*B^3*a^12*b^6*c^6 + 48704*B^3*a^13*b^4*c^7 - 129280*
B^3*a^14*b^2*c^8 + 250880*A^2*B*a^14*c^10 - 476672*A^3*a^13*b*c^10 - 442880
*A*B^2*a^14*b*c^9 - 1680*A*B^2*a^10*b^9*c^5 + 27176*A*B^2*a^11*b^7*c^6 - 16
4448*A*B^2*a^12*b^5*c^7 + 441216*A*B^2*a^13*b^3*c^8 + 1400*A^2*B*a^9*b^10*c
^5 - 21680*A^2*B*a^10*b^8*c^6 + 121648*A^2*B*a^11*b^6*c^7 - 275264*A^2*B*a^
12*b^4*c^8 + 121088*A^2*B*a^13*b^2*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 +
25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2
 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5
+ 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*
a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7
*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*
a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*
b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4
*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 1196
16*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*
A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^
(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*
B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1
280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i - (A/(
3*a) - (x^2*(5*A*b - 3*B*a))/(3*a^2) + (x^4*(15*A*b^4 + 14*A*a^2*c^2 - 9*B*
a*b^3 - 62*A*a*b^2*c + 33*B*a^2*b*c))/(6*a^3*(4*a*c - b^2)) + (c*x^6*(5*A*b
^3 - 3*B*a*b^2 + 10*B*a^2*c - 19*A*a*b*c))/(2*a^3*(4*a*c - b^2)))/(a*x^3 +
b*x^5 + c*x^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.124 $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=365

$$-\frac{x^4\left(x^2\left(20a^2Bc^2+10aAbc^2-20ab^2Bc-Ab^3c+3b^4B\right)+a\left(16aAc^2-18abBc-Ab^2c+3b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^2\left(30a^2Bc^2+\right.}{}$$

```
[Out] 1/2*(7*A*a*b*c^2-A*b^3*c+30*B*a^2*c^2-21*B*a*b^2*c+3*B*b^4)*x^2/c^3/(-4*a*c
+b^2)^2-1/4*x^8*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/
(c*x^4+b*x^2+a)^2-1/4*x^4*(a*(16*A*a*c^2-A*b^2*c-18*B*a*b*c+3*B*b^3)+(10*A*
a*b*c^2-A*b^3*c+20*B*a^2*c^2-20*B*a*b^2*c+3*B*b^4)*x^2)/c^2/(-4*a*c+b^2)^2/
(c*x^4+b*x^2+a)-1/2*(-30*A*a^2*b*c^3+10*A*a*b^3*c^2-A*b^5*c-60*B*a^3*c^3+90
*B*a^2*b^2*c^2-30*B*a*b^4*c+3*B*b^6)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2)
)/c^4/(-4*a*c+b^2)^(5/2)-1/4*(-A*c+3*B*b)*ln(c*x^4+b*x^2+a)/c^4
```

Rubi [A] time = 1.45, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 818, 773, 634, 618, 206, 628}

$$-\frac{x^4\left(x^2\left(20a^2Bc^2+10aAbc^2-20ab^2Bc-Ab^3c+3b^4B\right)+a\left(16aAc^2-18abBc-Ab^2c+3b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^2\left(30a^2Bc^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((3*b^4*B - A*b^3*c - 21*a*b^2*B*c + 7*a*A*b*c^2 + 30*a^2*B*c^2)*x^2)/(2*c^
3*(b^2 - 4*a*c)^2) - (x^8*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)
)/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^4*(a*(3*b^3*B - A*b^2*c -
18*a*b*B*c + 16*a*A*c^2) + (3*b^4*B - A*b^3*c - 20*a*b^2*B*c + 10*a*A*b*c^2
 + 20*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((3*b^
6*B - A*b^5*c - 30*a*b^4*B*c + 10*a*A*b^3*c^2 + 90*a^2*b^2*B*c^2 - 30*a^2*A
*b*c^3 - 60*a^3*B*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^
2 - 4*a*c)^(5/2)) - ((3*b*B - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^{11}\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^5(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\text{Subst}\left(\int \frac{x^3\left(4a(bB-2Ac)+\left(3b^2B-Abc-10aBc\right)x\right)}{\left(a+bx+cx^2\right)^2}\right.}{4c\left(b^2-4ac\right)}$$

$$= -\frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^4\left(a\left(3b^3B-Ab^2c-18abBc+16aAc^2\right)\right.}{4c^2\left(b^2-\right.}$$

$$= \frac{\left(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2\right.\right.}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right.}$$

$$= \frac{\left(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2\right.\right.}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right.}$$

$$= \frac{\left(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2\right.\right.}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right.}$$

$$= \frac{\left(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2\right.\right.}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right.}$$

Mathematica [A] time = 0.66, size = 435, normalized size = 1.19

$$\frac{\frac{a^3c^2\left(2c\left(A+Bx^2\right)-5bB\right)+a^2bc\left(-bc\left(4A+9Bx^2\right)+5Ac^2x^2+5b^2B\right)+ab^3\left(bc\left(A+6Bx^2\right)-5Ac^2x^2+b^2(-B)\right)+b^5x^2(Ac-bB)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{2c\left(60a^3Bc^3+30a^2Abc^3-90a^2b^2B\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*B*c^2*x^2 + (b^7*B - b^6*c*(A + 6*B*x^2) + 4*a^3*c^4*(8*A + 9*B*x^2) - 3
*a^2*b^2*c^3*(13*A + 34*B*x^2) + a*b^4*c^2*(11*A + 48*B*x^2) + a*b^3*c^2*(6
1*a*B - 30*A*c*x^2) + 2*b^5*c*(-7*a*B + 2*A*c*x^2) + 2*a^2*b*c^3*(-39*a*B +
 25*A*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^5*(-(b*B) + A*c)*x
^2 + a^3*c^2*(-5*b*B + 2*c*(A + B*x^2)) + a*b^3*(-(b^2*B) - 5*A*c^2*x^2 + b
*c*(A + 6*B*x^2)) + a^2*b*c*(5*b^2*B + 5*A*c^2*x^2 - b*c*(4*A + 9*B*x^2)))/
((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(-3*b^6*B + A*b^5*c + 30*a*b^4
*B*c - 10*a*A*b^3*c^2 - 90*a^2*b^2*B*c^2 + 30*a^2*A*b*c^3 + 60*a^3*B*c^3)*A
rcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*(-3*b*B +
 A*c)*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

fricas [B] time = 1.27, size = 3196, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^6*c^3 - 12*B*a*b^4*c^4 + 48*B*a^2*b^2*c^5 - 64*B*a^3*c^6)*x^10
 - 5*B*a^2*b^7 - 96*A*a^5*c^4 + 4*(B*b^7*c^2 - 12*B*a*b^5*c^3 + 48*B*a^2*b^
```

```
3*c^4 - 64*B*a^3*b*c^5)*x^8 - 2*(2*B*b^8*c + 100*(2*B*a^4 + A*a^3*b)*c^5 -
(254*B*a^3*b^2 + 85*A*a^2*b^3)*c^4 + (123*B*a^2*b^4 + 23*A*a*b^5)*c^3 - 2*(
13*B*a*b^6 + A*b^7)*c^2)*x^6 - (5*B*b^9 + 128*A*a^4*c^5 + 4*(22*B*a^4*b + 3
*A*a^3*b^2)*c^4 - (314*B*a^3*b^3 + 87*A*a^2*b^4)*c^3 + (225*B*a^2*b^5 + 31*
A*a*b^6)*c^2 - (58*B*a*b^7 + 3*A*b^8)*c)*x^4 + 4*(58*B*a^5*b + 27*A*a^4*b^2
)*c^3 - (202*B*a^4*b^3 + 33*A*a^3*b^4)*c^2 - 2*(5*B*a*b^8 + 4*(30*B*a^5 + 3
1*A*a^4*b)*c^4 - (346*B*a^4*b^2 + 119*A*a^3*b^3)*c^3 + (235*B*a^3*b^4 + 34*
A*a^2*b^5)*c^2 - (59*B*a^2*b^6 + 3*A*a*b^7)*c)*x^2 - (3*B*a^2*b^6 + (3*B*b^
6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^5 + 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B
*a*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2)*c^4 +
10*(9*B*a^2*b^3 + A*a*b^4)*c^3 - (30*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 -
 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*(12*B*a^3*b^2 - A*a^2*b^3)*c^3 + 2*(15*B*a
^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a*b^6 + A*b^7)*c)*x^4 - 30*(2*B*a^5 + A*a^4
*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 - 30*(2*B*a^4*b +
 A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3 + A*a^2*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^
6)*c)*x^2 - (30*B*a^3*b^4 + A*a^2*b^5)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4
+ 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
 + a)) + (56*B*a^3*b^5 + 3*A*a^2*b^6)*c - (3*B*a^2*b^7 + 64*A*a^5*c^4 + (3*
B*b^7*c^2 + 64*A*a^3*c^6 - 48*(4*B*a^3*b + A*a^2*b^2)*c^5 + 12*(12*B*a^2*b^
3 + A*a*b^4)*c^4 - (36*B*a*b^5 + A*b^6)*c^3)*x^8 + 2*(3*B*b^8*c + 64*A*a^3*
b*c^5 - 48*(4*B*a^3*b^2 + A*a^2*b^3)*c^4 + 12*(12*B*a^2*b^4 + A*a*b^5)*c^3
- (36*B*a*b^6 + A*b^7)*c^2)*x^6 + (3*B*b^9 + 128*A*a^4*c^5 - 32*(12*B*a^4*b
 + A*a^3*b^2)*c^4 + 24*(4*B*a^3*b^3 - A*a^2*b^4)*c^3 + 2*(36*B*a^2*b^5 + 5*
A*a*b^6)*c^2 - (30*B*a*b^7 + A*b^8)*c)*x^4 - 48*(4*B*a^5*b + A*a^4*b^2)*c^3
 + 12*(12*B*a^4*b^3 + A*a^3*b^4)*c^2 + 2*(3*B*a*b^8 + 64*A*a^4*b*c^4 - 48*(
4*B*a^4*b^2 + A*a^3*b^3)*c^3 + 12*(12*B*a^3*b^4 + A*a^2*b^5)*c^2 - (36*B*a^
2*b^6 + A*a*b^7)*c)*x^2 - (36*B*a^3*b^5 + A*a^2*b^6)*c)*log(c*x^4 + b*x^2 +
 a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6*c^6
 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^8 + 2*(b^7*c^5 - 12*a*b^5*
c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^6 + (b^8*c^4 - 10*a*b^6*c^5 + 24*a^2
*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^4 + 2*(a*b^7*c^4 - 12*a^2*b^5*c^
5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x^2), 1/4*(2*(B*b^6*c^3 - 12*B*a*b^4*c^4
 + 48*B*a^2*b^2*c^5 - 64*B*a^3*c^6)*x^10 - 5*B*a^2*b^7 - 96*A*a^5*c^4 + 4*(
B*b^7*c^2 - 12*B*a*b^5*c^3 + 48*B*a^2*b^3*c^4 - 64*B*a^3*b*c^5)*x^8 - 2*(2*
B*b^8*c + 100*(2*B*a^4 + A*a^3*b)*c^5 - (254*B*a^3*b^2 + 85*A*a^2*b^3)*c^4
+ (123*B*a^2*b^4 + 23*A*a*b^5)*c^3 - 2*(13*B*a*b^6 + A*b^7)*c^2)*x^6 - (5*B
*b^9 + 128*A*a^4*c^5 + 4*(22*B*a^4*b + 3*A*a^3*b^2)*c^4 - (314*B*a^3*b^3 +
87*A*a^2*b^4)*c^3 + (225*B*a^2*b^5 + 31*A*a*b^6)*c^2 - (58*B*a*b^7 + 3*A*b^
8)*c)*x^4 + 4*(58*B*a^5*b + 27*A*a^4*b^2)*c^3 - (202*B*a^4*b^3 + 33*A*a^3*b
^4)*c^2 - 2*(5*B*a*b^8 + 4*(30*B*a^5 + 31*A*a^4*b)*c^4 - (346*B*a^4*b^2 + 1
19*A*a^3*b^3)*c^3 + (235*B*a^3*b^4 + 34*A*a^2*b^5)*c^2 - (59*B*a^2*b^6 + 3*
A*a*b^7)*c)*x^2 - 2*(3*B*a^2*b^6 + (3*B*b^6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^
5 + 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B*a*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B
*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2)*c^4 + 10*(9*B*a^2*b^3 + A*a*b^4)*c^3 -
(30*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 - 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*
(12*B*a^3*b^2 - A*a^2*b^3)*c^3 + 2*(15*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a
*b^6 + A*b^7)*c)*x^4 - 30*(2*B*a^5 + A*a^4*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3
*b^3)*c^2 + 2*(3*B*a*b^7 - 30*(2*B*a^4*b + A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3
 + A*a^2*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^6)*c)*x^2 - (30*B*a^3*b^4 + A*a^2
*b^5)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
 4*a*c)) + (56*B*a^3*b^5 + 3*A*a^2*b^6)*c - (3*B*a^2*b^7 + 64*A*a^5*c^4 + (
3*B*b^7*c^2 + 64*A*a^3*c^6 - 48*(4*B*a^3*b + A*a^2*b^2)*c^5 + 12*(12*B*a^2*
b^3 + A*a*b^4)*c^4 - (36*B*a*b^5 + A*b^6)*c^3)*x^8 + 2*(3*B*b^8*c + 64*A*a^
3*b*c^5 - 48*(4*B*a^3*b^2 + A*a^2*b^3)*c^4 + 12*(12*B*a^2*b^4 + A*a*b^5)*c^
3 - (36*B*a*b^6 + A*b^7)*c^2)*x^6 + (3*B*b^9 + 128*A*a^4*c^5 - 32*(12*B*a^4
*b + A*a^3*b^2)*c^4 + 24*(4*B*a^3*b^3 - A*a^2*b^4)*c^3 + 2*(36*B*a^2*b^5 +
5*A*a*b^6)*c^2 - (30*B*a*b^7 + A*b^8)*c)*x^4 - 48*(4*B*a^5*b + A*a^4*b^2)*c
^3 + 12*(12*B*a^4*b^3 + A*a^3*b^4)*c^2 + 2*(3*B*a*b^8 + 64*A*a^4*b*c^4 - 48
*(4*B*a^4*b^2 + A*a^3*b^3)*c^3 + 12*(12*B*a^3*b^4 + A*a^2*b^5)*c^2 - (36*B*
```

```
a^2*b^6 + A*a*b^7)*c)*x^2 - (36*B*a^3*b^5 + A*a^2*b^6)*c)*log(c*x^4 + b*x^2
 + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^8 + 2*(b^7*c^5 - 12*a*b^
5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^6 + (b^8*c^4 - 10*a*b^6*c^5 + 24*a
^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^4 + 2*(a*b^7*c^4 - 12*a^2*b^5*
c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x^2)]
```

giac [A] time = 5.88, size = 598, normalized size = 1.64

$$\frac{\left(3\,Bb^{6}-30\,Bab^{4}c-Ab^{5}c+90\,Ba^{2}b^{2}c^{2}+10\,Aab^{3}c^{2}-60\,Ba^{3}c^{3}-30\,Aa^{2}bc^{3}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\left(b^{4}c^{4}-8\,ab^{2}c^{5}+16\,a^{2}c^{6}\right)\sqrt{-b^{2}+4\,ac}}+\frac{Bx^{2}}{2\,c^{3}}+\frac{9\,Bb^{5}c^{2}x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*b^6 - 30*B*a*b^4*c - A*b^5*c + 90*B*a^2*b^2*c^2 + 10*A*a*b^3*c^2 -
 60*B*a^3*c^3 - 30*A*a^2*b*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) + 1/2*B*x^2/c^3 + 1
/8*(9*B*b^5*c^2*x^8 - 72*B*a*b^3*c^3*x^8 - 3*A*b^4*c^3*x^8 + 144*B*a^2*b*c^
4*x^8 + 24*A*a*b^2*c^4*x^8 - 48*A*a^2*c^5*x^8 + 6*B*b^6*c*x^6 - 48*B*a*b^4*
c^2*x^6 + 2*A*b^5*c^2*x^6 + 84*B*a^2*b^2*c^3*x^6 - 12*A*a*b^3*c^3*x^6 + 72*
B*a^3*c^4*x^6 + 4*A*a^2*b*c^4*x^6 - B*b^7*x^4 + 14*B*a*b^5*c*x^4 + 3*A*b^6*
c*x^4 - 82*B*a^2*b^3*c^2*x^4 - 20*A*a*b^4*c^2*x^4 + 204*B*a^3*b*c^3*x^4 + 2
2*A*a^2*b^2*c^3*x^4 - 32*A*a^3*c^4*x^4 - 2*B*a*b^6*x^2 + 8*B*a^2*b^4*c*x^2
+ 6*A*a*b^5*c*x^2 + 4*B*a^3*b^2*c^2*x^2 - 40*A*a^2*b^3*c^2*x^2 + 56*B*a^4*c
^3*x^2 + 28*A*a^3*b*c^3*x^2 - B*a^2*b^5 + 3*A*a^2*b^4*c + 28*B*a^4*b*c^2 -
18*A*a^3*b^2*c^2)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*(c*x^4 + b*x^2 + a)
^2) - 1/4*(3*B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^4
```

maple [B] time = 0.03, size = 2054, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*B*x^2/c^3+6/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*a*b^3-1/
2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))*b^5*A-2/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*A*a*
b^2-12/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*a^2*b+1/c^2/(c*x^
4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^5-5/4/c^4/(c*x^4+b*x^2+a)^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^7+7/c/(c*x^4+b*x^2+a)^2*a^4/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^2*B-21/4/c^2/(c*x^4+b*x^2+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+
b^4)*A*b^2-29/2/c^2/(c*x^4+b*x^2+a)^2*a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b-5/
4/c^4/(c*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^5+25/2/(c*x^4+b*
x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*a^2*b+3/2/c^4/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^6*B-30/c/
(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)
^(1/2))*a^3*B-3/2/c^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*b^
6+3/4/c^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^6+3/4/c^3/(c
*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^4+9/c^3/(c*x^4+b*x^2+a)^
2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^3+3/2/c^3/(c*x^4+b*x^2+a)^2*a/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^2*A*b^5-11/c^2/(c*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^2*A*b^3+11/4/c/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4
*A*a^2*b^2-19/4/c^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*a*b^
4-21/2/c/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*a^3*b-41/4/c^2/
(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*a^2*b^3+31/2/c/(c*x^4+b*
x^2+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b-71/2/c^2/(c*x^4+b*x^2+a)^2*
```

```
a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^2-5/2/c^4/(c*x^4+b*x^2+a)^2*a/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^2*B*b^6-15/2/c/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2
*c+b^4)*x^6*A*a*b^3+12/c^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6
*B*a*b^4-15/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+
b)/(4*a*c-b^2)^(1/2))*A*a^2*b+5/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*a*b^3+45/c^2/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*a^2*b^
2-15/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*B*a*b^4+19/c^3/(c*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^2*B*b^4+17/2/c^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B
*a*b^5-51/2/c/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*a^2*b^2+9/
(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*a^3+8/(c*x^4+b*x^2+a)^2/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*a^3+6/c/(c*x^4+b*x^2+a)^2*a^4/(16*a^2*c^2-
8*a*b^2*c+b^4)*A+1/4/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*A*b^4
+4/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*A*a^2-3/4/c^4/(16*a^2*c^2
-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*b^5
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 4.66, size = 4501, normalized size = 12.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^6*(18*B*a^3*c^3 - 3*B*b^6 + 2*A*b^5*c + 24*B*a*b^4*c - 15*A*a*b^3*c^2 +
 25*A*a^2*b*c^3 - 51*B*a^2*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (
a*(24*A*a^3*c^3 - 5*B*a*b^5 + 3*A*a*b^4*c + 36*B*a^2*b^3*c - 58*B*a^3*b*c^2
 - 21*A*a^2*b^2*c^2))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(14*B*a^4
*c^3 - 5*B*a*b^6 + 3*A*a*b^5*c + 31*A*a^3*b*c^3 + 38*B*a^2*b^4*c - 22*A*a^2
*b^3*c^2 - 71*B*a^3*b^2*c^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^4*(
5*B*b^7 - 32*A*a^3*c^4 - 3*A*b^6*c - 34*B*a*b^5*c + 19*A*a*b^4*c^2 + 42*B*a
^3*b*c^3 - 11*A*a^2*b^2*c^3 + 41*B*a^2*b^3*c^2))/(4*c*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)))/(a^2*c^3 + c^5*x^8 + x^4*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^6 + 2*
a*b*c^3*x^2) + (B*x^2)/(2*c^3) + (log(((a*(A*c - 3*B*b)^2)/c^6 - (((8*a*(A*
c - 3*B*b))/c^2 - (2*(2*a + b*x^2)*(A*c - 3*B*b + c^4*(-(60*B*a^3*c^3 - 3*B
*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*
b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/c^2 + (2*x^2*(60*B*a^3*c^3 - 9*B*
b^6 + 3*A*b^5*c + 78*B*a*b^4*c - 26*A*a*b^3*c^2 + 62*A*a^2*b*c^3 - 186*B*a^
2*b^2*c^2))/(c^2*(4*a*c - b^2)^2))*(A*c - 3*B*b + c^4*(-(60*B*a^3*c^3 - 3*B
*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*
b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/(4*c^4) + (x^2*(A*c - 3*B*b)*(30*
B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 27*B*a*b^4*c - 9*A*a*b^3*c^2 + 23*A*a^2*b*c
^3 - 69*B*a^2*b^2*c^2))/(c^6*(4*a*c - b^2)^2))*((a*(A*c - 3*B*b)^2)/c^6 + (
((2*(2*a + b*x^2)*(3*B*b - A*c + c^4*(-(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c +
30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2)^2/(c^8*(
4*a*c - b^2)^5))^(1/2)))/c^2 + (8*a*(A*c - 3*B*b))/c^2 + (2*x^2*(60*B*a^3*c
^3 - 9*B*b^6 + 3*A*b^5*c + 78*B*a*b^4*c - 26*A*a*b^3*c^2 + 62*A*a^2*b*c^3 -
 186*B*a^2*b^2*c^2))/(c^2*(4*a*c - b^2)^2))*(3*B*b - A*c + c^4*(-(60*B*a^3*
```

```
c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 -
90*B*a^2*b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/(4*c^4) + (x^2*(A*c - 3*
B*b)*(30*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 27*B*a*b^4*c - 9*A*a*b^3*c^2 + 23*
A*a^2*b*c^3 - 69*B*a^2*b^2*c^2))/(c^6*(4*a*c - b^2)^2)))*(6*B*b^11 + 2048*A
*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 -
 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^
7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(4096*a^5*c^9 - 4*b^10
*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8
)) - (atan(((32*a^2*c^8*(4*a*c - b^2)^5 + 2*b^4*c^6*(4*a*c - b^2)^5 - 16*a*
b^2*c^7*(4*a*c - b^2)^5)*(x^2*(((((6*A*b^5*c^5 + 120*B*a^3*c^7 - 18*B*b^6*c
^4 - 52*A*a*b^3*c^6 + 124*A*a^2*b*c^7 + 156*B*a*b^4*c^5 - 372*B*a^2*b^2*c^6
)/(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7) - ((8*b^5*c^8 - 64*a*b^3*c^9 + 128*a
^2*b*c^10)*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a
*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560
*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^
4))/(2*(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80
*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))*(60*B
*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c
^3 - 90*B*a^2*b^2*c^2))/(8*c^4*(4*a*c - b^2)^(5/2)) - ((8*b^5*c^8 - 64*a*b^
3*c^9 + 128*a^2*b*c^10)*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c -
10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2)*(6*B*b^11 + 2048*A*a^5*
c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*
A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2
 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(16*c^4*(4*a*c - b^2)^(5/2)*(1
6*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^
5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))/(a*(4*a*c - b^
2)^2) + (b*((((6*A*b^5*c^5 + 120*B*a^3*c^7 - 18*B*b^6*c^4 - 52*A*a*b^3*c^6
+ 124*A*a^2*b*c^7 + 156*B*a*b^4*c^5 - 372*B*a^2*b^2*c^6)/(16*a^2*c^8 + b^4*
c^6 - 8*a*b^2*c^7) - ((8*b^5*c^8 - 64*a*b^3*c^9 + 128*a^2*b*c^10)*(6*B*b^11
 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^
5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960
*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(16*a^2*c^8 +
 b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2
*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))*(6*B*b^11 + 2048*A*a^5*c^
6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*
a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 -
 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(4096*a^5*c^9 - 4*b^10*c^4 +
80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)) - (9
*B^2*b^7 + A^2*b^5*c^2 - 6*A*B*b^6*c + 207*B^2*a^2*b^3*c^2 + 30*A*B*a^3*c^4
 - 81*B^2*a*b^5*c - 9*A^2*a*b^3*c^3 + 23*A^2*a^2*b*c^4 - 90*B^2*a^3*b*c^3 -
 138*A*B*a^2*b^2*c^3 + 54*A*B*a*b^4*c^2)/(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^
7) + (((b^5*c^8)/2 - 4*a*b^3*c^9 + 8*a^2*b*c^10)*(60*B*a^3*c^3 - 3*B*b^6 +
A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2
)^2)/(c^8*(4*a*c - b^2)^5*(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7))))/(2*a*(4*a
*c - b^2)^(5/2))) + ((((8*A*a*c^5 - 24*B*a*b*c^4)/c^6 - (8*a*c^2*(6*B*b^11
+ 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5
*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*
B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(4096*a^5*c^9 - 4
*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^
2*c^8))*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 +
 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2))/(8*c^4*(4*a*c - b^2)^(5/2)) - (a*(60*B
*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c
^3 - 90*B*a^2*b^2*c^2)*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^
9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^
4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*
B*a^4*b^3*c^4))/(c^2*(4*a*c - b^2)^(5/2)*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*
b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))/(a*(4*a*
c - b^2)^2) + (b*((((8*A*a*c^5 - 24*B*a*b*c^4)/c^6 - (8*a*c^2*(6*B*b^11 + 2
048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*
```

```
c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a
^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(4096*a^5*c^9 - 4*b^
10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c
^8))*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c
^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4
*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2
*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4
*c^7 - 5120*a^4*b^2*c^8)) - (A^2*a*c^2 + 9*B^2*a*b^2 - 6*A*B*a*b*c)/c^6 + (
a*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*
a^2*b*c^3 - 90*B*a^2*b^2*c^2)^2)/(c^6*(4*a*c - b^2)^5)))/(2*a*(4*a*c - b^2)
^(5/2))))/(9*B^2*b^12 + A^2*b^10*c^2 + 3600*B^2*a^6*c^6 - 6*A*B*b^11*c + 16
0*A^2*a^2*b^6*c^4 - 600*A^2*a^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 + 1440*B^2*a^
2*b^8*c^2 - 5760*B^2*a^3*b^6*c^3 + 11700*B^2*a^4*b^4*c^4 - 10800*B^2*a^5*b^
2*c^5 - 180*B^2*a*b^10*c - 20*A^2*a*b^8*c^3 - 960*A*B*a^2*b^7*c^3 + 3720*A*
B*a^3*b^5*c^4 - 6600*A*B*a^4*b^3*c^5 + 120*A*B*a*b^9*c^2 + 3600*A*B*a^5*b*c
^6))*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30
*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2))/(2*c^4*(4*a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.125 $\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=254

$$\frac{\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{5/2}}-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B\right)+2a\left(\right.\right.}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^{4}\right.}$$

```
[Out] -1/4*x^6*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+
b*x^2+a)^2-1/4*x^2*(2*a*(6*A*a*c^2-7*B*a*b*c+B*b^3)+(6*A*a*b*c^2+16*B*a^2*c
^2-15*B*a*b^2*c+2*B*b^4)*x^2)/c^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/2*(-12*A
*a^2*c^3+30*B*a^2*b*c^2-10*B*a*b^3*c+B*b^5)*arctanh((2*c*x^2+b)/(-4*a*c+b^2
)^(1/2))/c^3/(-4*a*c+b^2)^(5/2)+1/4*B*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A] time = 0.40, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 818, 634, 618, 206, 628}

$$-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B\right)+2a\left(6aAc^2-7abBc+b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc\right.}{2c^3\left(b^2-4a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^6*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(4*c*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)^2) - (x^2*(2*a*(b^3*B - 7*a*b*B*c + 6*a*A*c^2) + (2*b^
4*B - 15*a*b^2*B*c + 6*a*A*b*c^2 + 16*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*c)
^2*(a + b*x^2 + c*x^4)) + ((b^5*B - 10*a*b^3*B*c + 30*a^2*b*B*c^2 - 12*a^2*
A*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(5/2)
) + (B*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3a(bB-2Ac)+2B\left(b^2-4ac\right)x\right)}{\left(a+bx+cx^2\right)^2}\,dx,\right.}{4c\left(b^2-4ac\right)}$$

$$= -\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^2\left(2a\left(b^3B-7abBc+6aAc^2\right)+\left(2b^4\right.\right.}{4c^2\left(b^2-4ac\right)^2}$$

$$= -\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^2\left(2a\left(b^3B-7abBc+6aAc^2\right)+\left(2b^4\right.\right.}{4c^2\left(b^2-4ac\right)^2}$$

$$= -\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^2\left(2a\left(b^3B-7abBc+6aAc^2\right)+\left(2b^4\right.\right.}{4c^2\left(b^2-4ac\right)^2}$$

$$= -\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^2\left(2a\left(b^3B-7abBc+6aAc^2\right)+\left(2b^4\right.\right.}{4c^2\left(b^2-4ac\right)^2}$$

Mathematica [A] time = 0.48, size = 354, normalized size = 1.39

$$\frac{-\frac{2c\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{2a^2bc^3\left(11A+25Bx^2\right)+4a^2c^3\left(8aB-5Acx^2\right)+b^4c\left(11aB-2Acx^2\right)-2ab^3c^2\left(4A+15Bx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-(b^6*B) + b^5*c*(A + 4*B*x^2) - 2*a*b^3*c^2*(4*A + 15*B*x^2) + 2*a^2*b*c
^3*(11*A + 25*B*x^2) + 4*a^2*c^3*(8*a*B - 5*A*c*x^2) + b^4*c*(11*a*B - 2*A*
c*x^2) + a*b^2*c^2*(-39*a*B + 16*A*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) + (2*a^3*B*c^2 + b^4*(b*B - A*c)*x^2 + a*b^2*(b^2*B + 4*A*c^2*x^2 - b
*c*(A + 5*B*x^2)) + a^2*c*(-4*b^2*B - 2*A*c^2*x^2 + b*c*(3*A + 5*B*x^2)))/(
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(b^5*B - 10*a*b^3*B*c + 30*a^2*
b*B*c^2 - 12*a^2*A*c^3)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4
*a*c)^(5/2) + B*c*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

fricas [B] time = 0.87, size = 2167, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^2*
b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^6
+ (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*c^3
 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(12*B
*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 +
 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 + 2*A*a
^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 - ((B*b^5*c^2 - 10*B*a*b^3*c^
3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 30*B*
a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c^3 -
 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a^3*c
^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c + 30
*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*
b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a
)) - (33*B*a^3*b^4 + A*a^2*b^5)*c + (B*a^2*b^6 - 12*B*a^3*b^4*c + 48*B*a^4*
b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2 - 12*B*a*b^4*c^3 + 48*B*a^2*b^2*c^4 - 6
4*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B*a*b^5*c^2 + 48*B*a^2*b^3*c^3 - 64*B*a^
3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c + 24*B*a^2*b^4*c^2 + 32*B*a^3*b^2*c^3
- 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 - 12*B*a^2*b^5*c + 48*B*a^3*b^3*c^2 - 64*
B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 4
8*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*
a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x
^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^
7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2
), 1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^
2*b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^
6 + (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*c
^3 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(12
*B*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*b^7
 + 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 + 2*A
*a^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 + 2*((B*b^5*c^2 - 10*B*a*b^
3*c^3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 3
0*B*a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c
^3 - 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a
^3*c^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c
+ 30*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x
^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (33*B*a^3*b^4 + A*a^2*b^5)*c +
(B*a^2*b^6 - 12*B*a^3*b^4*c + 48*B*a^4*b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2
- 12*B*a*b^4*c^3 + 48*B*a^2*b^2*c^4 - 64*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B
*a*b^5*c^2 + 48*B*a^2*b^3*c^3 - 64*B*a^3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c
 + 24*B*a^2*b^4*c^2 + 32*B*a^3*b^2*c^3 - 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 -
12*B*a^2*b^5*c + 48*B*a^3*b^3*c^2 - 64*B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2
+ a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5
*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^
```

```
2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c
^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2)]
```

giac [A] time = 6.31, size = 466, normalized size = 1.83

$$-\frac{\left(Bb^5-10\,Bab^3c+30\,Ba^2bc^2-12\,Aa^2c^3\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)\sqrt{-b^2+4\,ac}}+\frac{B\log\left(cx^4+bx^2+a\right)}{4\,c^3}-\frac{3\,Bb^4c^2x^8-24\,Bab^2c^3x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^5 - 10*B*a*b^3*c + 30*B*a^2*b*c^2 - 12*A*a^2*c^3)*arctan((2*c*x^2
 + b)/sqrt(-b^2 + 4*a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 +
 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a)/c^3 - 1/8*(3*B*b^4*c^2*x^8 - 24*B*a
*b^2*c^3*x^8 + 48*B*a^2*c^4*x^8 - 2*B*b^5*c*x^6 + 12*B*a*b^3*c^2*x^6 + 4*A*
b^4*c^2*x^6 - 4*B*a^2*b*c^3*x^6 - 32*A*a*b^2*c^3*x^6 + 40*A*a^2*c^4*x^6 - 3
*B*b^6*x^4 + 20*B*a*b^4*c*x^4 + 2*A*b^5*c*x^4 - 22*B*a^2*b^2*c^2*x^4 - 16*A
*a*b^3*c^2*x^4 + 32*B*a^3*c^3*x^4 - 4*A*a^2*b*c^3*x^4 - 6*B*a*b^5*x^2 + 40*
B*a^2*b^3*c*x^2 + 4*A*a*b^4*c*x^2 - 28*B*a^3*b*c^2*x^2 - 40*A*a^2*b^2*c^2*x
^2 + 24*A*a^3*c^3*x^2 - 3*B*a^2*b^4 + 18*B*a^3*b^2*c + 2*A*a^2*b^3*c - 20*A
*a^3*b*c^2)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.03, size = 723, normalized size = 2.85

$$\frac{6A\,a^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}-\frac{15B\,a^2b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}\;c}+\frac{5Ba\,b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}\;c^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-1/c^2*(10*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c-25*B*a^2*b*c^2+15*B*a*b^3*c
-2*B*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(2*A*a^2*b*c^3+8*A*a*b^3*c^2-A
*b^5*c+32*B*a^3*c^3+11*B*a^2*b^2*c^2-19*B*a*b^4*c+3*B*b^6)/c^3/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^4-a*(6*A*a^2*c^3-10*A*a*b^2*c^2+A*b^4*c-31*B*a^2*b*c^2+22*
B*a*b^3*c-3*B*b^5)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(10*A*a*b*c^2
-A*b^3*c+24*B*a^2*c^2-21*B*a*b^2*c+3*B*b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))
/(c*x^4+b*x^2+a)^2+4/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a^2*B-2
/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a*b^2*B+1/4/c^3/(16*a^2*c
^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*b^4*B+6/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*a^2-15/c/(16*a^2*c^2-
8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*
b*B+5/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(
4*a*c-b^2)^(1/2))*B*a*b^3-1/2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.15, size = 3062, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^4*(3*B*b^6 + 32*B*a^3*c^3 - A*b^5*c - 19*B*a*b^4*c + 8*A*a*b^3*c^2 + 2*
A*a^2*b*c^3 + 11*B*a^2*b^2*c^2))/(4*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (
x^6*(2*B*b^5 - 10*A*a^2*c^3 - A*b^4*c - 15*B*a*b^3*c + 8*A*a*b^2*c^2 + 25*B
*a^2*b*c^2))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*(24*B*a^3*c^2 + 3*
B*a*b^4 - A*a*b^3*c + 10*A*a^2*b*c^2 - 21*B*a^2*b^2*c))/(4*c^3*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) - (x^2*(6*A*a^3*c^3 - 3*B*a*b^5 + A*a*b^4*c + 22*B*a^2*
b^3*c - 31*B*a^3*b*c^2 - 10*A*a^2*b^2*c^2))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (log
(((B^2*a)/c^4 - ((B + c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2
*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*((8*B*a)/c - (2*(B + c^3*(-(B*b^5 -
 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/
2))*(2*a + b*x^2))/c + (2*x^2*(3*B*b^5 - 12*A*a^2*c^3 - 26*B*a*b^3*c + 62*B
*a^2*b*c^2))/(c*(4*a*c - b^2)^2)))/(4*c^3) + (B*x^2*(B*b^5 - 6*A*a^2*c^3 -
9*B*a*b^3*c + 23*B*a^2*b*c^2))/(c^4*(4*a*c - b^2)^2))*((B^2*a)/c^4 - ((B -
c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c
- b^2)^5))^(1/2))*((8*B*a)/c - (2*(B - c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a
*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*(2*a + b*x^2))/c +
 (2*x^2*(3*B*b^5 - 12*A*a^2*c^3 - 26*B*a*b^3*c + 62*B*a^2*b*c^2))/(c*(4*a*c
 - b^2)^2)))/(4*c^3) + (B*x^2*(B*b^5 - 6*A*a^2*c^3 - 9*B*a*b^3*c + 23*B*a^2
*b*c^2))/(c^4*(4*a*c - b^2)^2)))*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c
+ 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(2*(4096*a^
5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 51
20*a^4*b^2*c^7)) + (atan(((32*a^2*c^6*(4*a*c - b^2)^5 + 2*b^4*c^4*(4*a*c -
b^2)^5 - 16*a*b^2*c^5*(4*a*c - b^2)^5)*(x^2*(((((24*A*a^2*c^6 - 6*B*b^5*c^3
 + 52*B*a*b^3*c^4 - 124*B*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) -
 ((8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(2*B*b^10 - 2048*B*a^5*c^5 - 4
0*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))
/(2*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*
b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(B*b^5 -
 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2))/(8*c^3*(4*a*c - b^2)^(5/2))
 - ((8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(B*b^5 - 12*A*a^2*c^3 - 10*B
*a*b^3*c + 30*B*a^2*b*c^2)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*
B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(16*c^3*(4*a*c -
b^2)^(5/2)*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3
+ 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))/(
a*(4*a*c - b^2)^2) - (b*((((24*A*a^2*c^6 - 6*B*b^5*c^3 + 52*B*a*b^3*c^4 - 1
24*B*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) - ((8*b^5*c^6 - 64*a*b
^3*c^7 + 128*a^2*b*c^8)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a
^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(2*(16*a^2*c^6 + b^4
*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6
*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(2*B*b^10 - 2048*B*a^5*c^5 -
40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4)
)/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3
*b^4*c^6 - 5120*a^4*b^2*c^7)) - (B^2*b^5 - 6*A*B*a^2*c^3 - 9*B^2*a*b^3*c +
23*B^2*a^2*b*c^2)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) + (((b^5*c^6)/2 - 4*
a*b^3*c^7 + 8*a^2*b*c^8)*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*
c^2)^2)/(c^6*(4*a*c - b^2)^5*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5))))/(2*a*(
4*a*c - b^2)^(5/2))) - ((((8*B*a)/c + (8*a*c^2*(2*B*b^10 - 2048*B*a^5*c^5 -
 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4
))/(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b
^4*c^6 - 5120*a^4*b^2*c^7))*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2
*b*c^2))/(8*c^3*(4*a*c - b^2)^(5/2)) + (a*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^
```

```
3*c + 30*B*a^2*b*c^2)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2
*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(c*(4*a*c - b^2)^(5/2)
*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4
*c^6 - 5120*a^4*b^2*c^7)))/(a*(4*a*c - b^2)^2) + (b*((B^2*a)/c^4 + (((8*B*a
)/c + (8*a*c^2*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^
2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(4096*a^5*c^8 - 4*b^10*c^3 +
80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7))*(2*B
*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*
c^3 + 2560*B*a^4*b^2*c^4))/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 6
40*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) - (a*(B*b^5 - 12*A*a
^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2)/(c^4*(4*a*c - b^2)^5)))/(2*a*(4*
a*c - b^2)^(5/2))))/(B^2*b^10 + 144*A^2*a^4*c^6 + 160*B^2*a^2*b^6*c^2 - 600
*B^2*a^3*b^4*c^3 + 900*B^2*a^4*b^2*c^4 - 20*B^2*a*b^8*c - 24*A*B*a^2*b^5*c^
3 + 240*A*B*a^3*b^3*c^4 - 720*A*B*a^4*b*c^5))*(B*b^5 - 12*A*a^2*c^3 - 10*B*
a*b^3*c + 30*B*a^2*b*c^2))/(2*c^3*(4*a*c - b^2)^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

3.126 $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=146

$$\frac{3a(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3x^2(2a+bx^2)(Ab-2aB)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x^6(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[Out] -1/4*x^6*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/4*(A
*b-2*B*a)*x^2*(b*x^2+2*a)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3*a*(A*b-2*B*a)*ar
ctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] time = 0.14, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 804, 722, 618, 206}

$$-\frac{x^6(-2aB+x^2(-(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3x^2(2a+bx^2)(Ab-2aB)}{4(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3a(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^6*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (3*(A*b - 2*a*B)*x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 +
 c*x^4)) + (3*a*(A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^
2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 804

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(b*f - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(m*(b*(
e*f + d*g) - 2*(c*d*f + a*e*g)))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m
- 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
```

```
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0] && LtQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx,x,x^2\right)$$

$$= -\frac{x^6\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{(3(Ab-2aB))\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^6\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3(Ab-2aB)x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{(3a(Ab-2aB))\,\text{S}}{2}$$

$$= -\frac{x^6\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3(Ab-2aB)x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{(3a(Ab-2aB))\,\text{S}}{}$$

$$= -\frac{x^6\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3(Ab-2aB)x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3a(Ab-2aB)\,\text{ta}}{\left(b^2-4\right.}$$

Mathematica [A] time = 0.27, size = 261, normalized size = 1.79

$$\frac{1}{4}\left(\frac{a^2c\left(2c\left(A+Bx^2\right)-3bB\right)+ab\left(-bc\left(A+4Bx^2\right)+3Ac^2x^2+b^2B\right)+b^3x^2(bB-Ac)}{c^3\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{-4a^2c^3\left(4A+5Bx^2\right)}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b^5*B - 8*a*b^3*B*c - b^4*c*(A + 2*B*x^2) - 4*a^2*c^3*(4*A + 5*B*x^2) + a
*b^2*c^2*(5*A + 16*B*x^2) + 2*a*b*c^2*(11*a*B - 3*A*c*x^2))/(c^3*(b^2 - 4*a
*c)^2*(a + b*x^2 + c*x^4)) + (b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A
+ B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2)))/(c^3*(-b^2 + 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (12*a*(A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[
-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

fricas [B] time = 0.56, size = 1378, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^4*c^2 - 4*(10*B*a^3
 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6 + (B*b^7 - 64*A*a^3
```

```
*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3 - A*a*b^4)*c^2 - (
12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)*c^2 + 2*(B*a*b^6 -
4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3)*c^2 - (14*B*a^2*b^
4 - A*a*b^5)*c)*x^2 + 6*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*(2*B*a^2*b - A*a*b^2
)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B*a^3 - A*a^2*b)*c^3
+ (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b)*c^2)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^3*b^3 - A*a^2*b^4)*c)/(a^2*b^6*c^2 - 1
2*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*
a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5
- 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2
*c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 -
64*a^4*b*c^5)*x^2), -1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^
4*c^2 - 4*(10*B*a^3 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6
+ (B*b^7 - 64*A*a^3*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3
 - A*a*b^4)*c^2 - (12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)*
c^2 + 2*(B*a*b^6 - 4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3)
*c^2 - (14*B*a^2*b^4 - A*a*b^5)*c)*x^2 + 12*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*
(2*B*a^2*b - A*a*b^2)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B
*a^3 - A*a^2*b)*c^3 + (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b
)*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - (14*B*a^3*b^3 - A*a^2*b^4)*c)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a
^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3
*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^6
+ (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*
x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x^2)]
```

giac [B] time = 6.46, size = 318, normalized size = 2.18

$$\frac{3\left(2\,Ba^2 - Aab\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{-b^2 + 4\,ac}} - \frac{2\,Bb^4cx^6 - 16\,Bab^2c^2x^6 + 20\,Ba^2c^3x^6 + 6\,Aabc^3x^6 + Bb^5x^4 - 8\,Bab^3cx^4 + A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3*(2*B*a^2 - A*a*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^
2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*B*b^4*c*x^6 - 16*B*a*b^2*c^2
*x^6 + 20*B*a^2*c^3*x^6 + 6*A*a*b*c^3*x^6 + B*b^5*x^4 - 8*B*a*b^3*c*x^4 + A
*b^4*c*x^4 - 2*B*a^2*b*c^2*x^4 + A*a*b^2*c^2*x^4 + 16*A*a^2*c^3*x^4 + 2*B*a
*b^4*x^2 - 20*B*a^2*b^2*c*x^2 + 2*A*a*b^3*c*x^2 + 12*B*a^3*c^2*x^2 + 10*A*a
^2*b*c^2*x^2 + B*a^2*b^3 - 10*B*a^3*b*c + A*a^2*b^2*c + 8*A*a^3*c^2)/((b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.02, size = 398, normalized size = 2.73

$$-\frac{3Aab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2 - 8a\,b^2c + b^4\right)\sqrt{4ac-b^2}} + \frac{6B\,a^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2 - 8a\,b^2c + b^4\right)\sqrt{4ac-b^2}} + \frac{-\frac{\left(3aAb\,c^2+10a^2B\,c^2-8a\,b^2Bc+b^4B\right)x^6}{\left(16a^2c^2-8a\,b^2c+b^4\right)c} - \frac{\left(16A\,a^2c^3+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-(3*A*a*b*c^2+10*B*a^2*c^2-8*B*a*b^2*c+B*b^4)/c/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^6-1/2*(16*A*a^2*c^3+A*a*b^2*c^2+A*b^4*c-2*B*a^2*b*c^2-8*B*a*b^3*c+B*
b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^4-a*(5*A*a*b*c^2+A*b^3*c+6*B*a^2*c^2-
10*B*a*b^2*c+B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2-1/2*a^2/c^2*(8*A*a*c
^2+A*b^2*c-10*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-
```

```
3*a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-
b^2)^(1/2))*A*b+6*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.69, size = 593, normalized size = 4.06

$$\frac{3\,a\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{3\,(A\,b-2\,B\,a)\,(6\,B\,a^2\,c^2-3\,A\,a\,b\,c^2)}{(4\,a\,c-b^2)^{9/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{9\,a\,b\,(A\,b-2\,B\,a)^2\,(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2)}{2\,(4\,a\,c-b^2)^{15/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}\right)-\frac{18\,a^2\,b\,c^2\,(A\,b-2\,B\,a)^2}{(4\,a\,c-b^2)^{15/2}}\right)\left(b^4\,(4\,a\,c-b^2)^5+16\,a^2\,c^2\,(4\,a\,c-b^2\cdots\right.}{18\,A^2\,a^2\,b^2\,c^2-72\,A\,B\,a^3\,b\,c^2+72\,B^2\,a^4\,c^2}\right.}{(4\,a\,c-b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*a*atan(((x^2*((3*(A*b - 2*B*a)*(6*B*a^2*c^2 - 3*A*a*b*c^2))/((4*a*c - b^
2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (9*a*b*(A*b - 2*B*a)^2*(2*b^5*c^
2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c))) - (18*a^2*b*c^2*(A*b - 2*B*a)^2)/(4*a*c - b^2)^(15/2))*(b^4
*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))
/(72*B^2*a^4*c^2 + 18*A^2*a^2*b^2*c^2 - 72*A*B*a^3*b*c^2))*(A*b - 2*B*a))/(
4*a*c - b^2)^(5/2) - ((x^4*(B*b^5 + 16*A*a^2*c^3 + A*b^4*c - 8*B*a*b^3*c +
A*a*b^2*c^2 - 2*B*a^2*b*c^2))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2
*(B*b^3 + 8*A*a*c^2 + A*b^2*c - 10*B*a*b*c))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (x^6*(B*b^4 + 10*B*a^2*c^2 + 3*A*a*b*c^2 - 8*B*a*b^2*c))/(2*c*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x^2*(B*b^4 + 6*B*a^2*c^2 + A*b^3*c + 5*A
*a*b*c^2 - 10*B*a*b^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a
*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

sympy [B] time = 102.04, size = 775, normalized size = 5.31

$$-\frac{3a\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(-Ab+2Ba)\log\left(x^2+\frac{-3Aab^2+6Ba^2b-192a^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(-Ab+2Ba)+144a^3b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(-Ab+2Ba)-36a\cdots}{-6Aabc+12Ba^2c}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*a*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a)*log(x**2 + (-3*A*a*b**2 + 6*
B*a**2*b - 192*a**4*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) + 144*a*
*3*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) - 36*a**2*b**4*c*sqr
t(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) + 3*a*b**6*sqrt(-1/(4*a*c - b**2)**5
)*(-A*b + 2*B*a))/(-6*A*a*b*c + 12*B*a**2*c))/2 + 3*a*sqrt(-1/(4*a*c - b**2
)**5)*(-A*b + 2*B*a)*log(x**2 + (-3*A*a*b**2 + 6*B*a**2*b + 192*a**4*c**3*s
```

```
qrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) - 144*a**3*b**2*c**2*sqrt(-1/(4*a*
c - b**2)**5)*(-A*b + 2*B*a) + 36*a**2*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-
A*b + 2*B*a) - 3*a*b**6*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a))/(-6*A*a*
b*c + 12*B*a**2*c))/2 + (-8*A*a**3*c**2 - A*a**2*b**2*c + 10*B*a**3*b*c - B
*a**2*b**3 + x**6*(-6*A*a*b*c**3 - 20*B*a**2*c**3 + 16*B*a*b**2*c**2 - 2*B*
b**4*c) + x**4*(-16*A*a**2*c**3 - A*a*b**2*c**2 - A*b**4*c + 2*B*a**2*b*c**
2 + 8*B*a*b**3*c - B*b**5) + x**2*(-10*A*a**2*b*c**2 - 2*A*a*b**3*c - 12*B*
a**3*c**2 + 20*B*a**2*b**2*c - 2*B*a*b**4))/(64*a**4*c**4 - 32*a**3*b**2*c*
*3 + 4*a**2*b**4*c**2 + x**8*(64*a**2*c**6 - 32*a*b**2*c**5 + 4*b**4*c**4)
+ x**6*(128*a**2*b*c**5 - 64*a*b**3*c**4 + 8*b**5*c**3) + x**4*(128*a**3*c*
*5 - 24*a*b**4*c**3 + 4*b**6*c**2) + x**2*(128*a**3*b*c**4 - 64*a**2*b**3*c
**3 + 8*a*b**5*c**2))
```

3.127 $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=185

$$\frac{\left(3abB-A\left(2ac+b^2\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{x^4\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{a\left(8aBc-6Abc+b^2B\right)+x^2\left(4a\right.}{4c\left(b^2-4ac\right)^2\left(a\right.}$$

```
[Out] -1/4*x^4*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*(-
a*(-6*A*b*c+8*B*a*c+B*b^2)-(4*A*a*c^2-4*A*b^2*c+2*B*a*b*c+B*b^3)*x^2)/c/(-4
*a*c+b^2)^2/(c*x^4+b*x^2+a)+(3*a*b*B-A*(2*a*c+b^2))*arctanh((2*c*x^2+b)/(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] time = 0.26, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{1251, 820, 777, 618, 206\}$

$$-\frac{x^2\left(4aAc^2+2abBc-4Ab^2c+b^3B\right)+a\left(8aBc-6Abc+b^2B\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^4\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\left(3abB\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^4*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) - (a*(b^2*B - 6*A*b*c + 8*a*B*c) + (b^3*B - 4*A*b^2*c + 2*a*b*B*c + 4
*a*A*c^2)*x^2)/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*a*b*B - A*(b
^2 + 2*a*c))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
```

```
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^4\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{x(-2(Ab-2aB)-(bB-2Ac)x)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^4\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{a\left(b^2B-6Abc+8aBc\right)+\left(b^3B-4Ab^2c+2abBc+4\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x^4\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{a\left(b^2B-6Abc+8aBc\right)+\left(b^3B-4Ab^2c+2abBc+4\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x^4\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{a\left(b^2B-6Abc+8aBc\right)+\left(b^3B-4Ab^2c+2abBc+4\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 0.24, size = 233, normalized size = 1.26

$$\frac{1}{4}\left(\frac{2a^2Bc+a\left(bc\left(A+3Bx^2\right)-2Ac^2x^2+b^2(-B)\right)+b^2x^2(Ac-bB)}{c^2\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{4\left(A\left(2ac+b^2\right)-3abB\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-(b^4*B) + A*b^3*c + 2*a*b*c^2*(A - 3*B*x^2) + 4*a*c^2*(-4*a*B + A*c*x^2)
 + b^2*c*(5*a*B + 2*A*c*x^2))/(c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*(A + 3
*B*x^2)))/(c^2*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(-3*a*b*B + A*(b^
2 + 2*a*c))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))
/4
```

fricas [B] time = 0.62, size = 1369, normalized size = 7.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2*b - A*a*b^2)*c^3 + (3*B*a*b^
3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3*A*a^2*b)*c^3 + 6*(2*B*a^2*b^2
 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4 - 8*(4*B*a^4 - 3*A*a^3*b)*c^2
+ 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b - 11*A*a^2*b^2)*c^2 + (B*a^2*b^3
 - 5*A*a*b^4)*c)*x^2 - 2*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)*x^8 + 2*(2*A*
a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a^2*c^3 - 2*(3*
B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*A*a^2*b*c^2 -
(3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*b^2)*c)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*
c))/(c*x^4 + b*x^2 + a)) + 2*(2*B*a^3*b^2 - 3*A*a^2*b^3)*c)/(a^2*b^6*c - 12
*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a
^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 -
 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^
4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^
4*b*c^4)*x^2), -1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2*b - A*a*b^2)*c
^3 + (3*B*a*b^3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3*A*a^2*b)*c^3 +
6*(2*B*a^2*b^2 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4 - 8*(4*B*a^4 - 3
*A*a^3*b)*c^2 + 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b - 11*A*a^2*b^2)*c^
2 + (B*a^2*b^3 - 5*A*a*b^4)*c)*x^2 + 4*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)
*x^8 + 2*(2*A*a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a
^2*c^3 - 2*(3*B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*
A*a^2*b*c^2 - (3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*b^2)*c)*s
qrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 2*(2*B*a^3*b^2 - 3*A*a^2*b^3)*c)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*
c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x
^8 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*
c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(
a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2)]
```

giac [**A**] time = 6.59, size = 268, normalized size = 1.45

$$-\frac{\left(3\,Bab - Ab^2 - 2\,Aac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{-b^2+4\,ac}} - \frac{6\,Babc^2x^6 - 2\,Ab^2c^2x^6 - 4\,Aac^3x^6 + Bb^4x^4 + Bab^2cx^4 - 3\,Ab^3cx^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -(3*B*a*b - A*b^2 - 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4
 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*a*b*c^2*x^6 - 2*A
*b^2*c^2*x^6 - 4*A*a*c^3*x^6 + B*b^4*x^4 + B*a*b^2*c*x^4 - 3*A*b^3*c*x^4 +
16*B*a^2*c^2*x^4 - 6*A*a*b*c^2*x^4 + 2*B*a*b^3*x^2 + 10*B*a^2*b*c*x^2 - 10*
A*a*b^2*c*x^2 + 4*A*a^2*c^2*x^2 + B*a^2*b^2 + 8*B*a^3*c - 6*A*a^2*b*c)/((b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

maple [**B**] time = 0.02, size = 411, normalized size = 2.22

$$\frac{2Aac\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{A\,b^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}-\frac{3Bab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{(2aA}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(c*(2*A*a*c+A*b^2-3*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(6*A*a*b*
c^2+3*A*b^3*c-16*B*a^2*c^2-B*a*b^2*c-B*b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
4-a/c*(2*A*a*c^2-5*A*b^2*c+5*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+
1/2*a^2*(6*A*b*c-8*B*a*c-B*b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+
```

```
a)^2+2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a
*c-b^2)^(1/2))*a*A*c+1/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2-3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.68, size = 625, normalized size = 3.38

$$\frac{\operatorname{atan}\left(\frac{\left(x^2\left(\frac{\left(Ab^2c^2-3Babc^2+2Aac^3\right)\left(Ab^2-3Bab+2Aac\right)}{a\left(4ac-b^2\right)^{9/2}\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{b\left(Ab^2-3Bab+2Aac\right)^2\left(32a^2bc^4-16ab^3c^3+2b^5c^2\right)}{2a\left(4ac-b^2\right)^{15/2}\left(16a^2c^2-8ab^2c+b^4\right)}\right)+\frac{2bc^2\left(Ab^2-3Bab+2Aac\right)^2}{\left(4ac-b^2\right)^{15/2}}\right)\left(b^4\left(4ac-b^2\right)^5+\cdots}{8A^2a^2c^4+8A^2ab^2c^3+2A^2b^4c^2-24ABa^2bc^3-12ABab^3c^2+18B^2a^2b^2c^2}\right)\cdots}{\left(4ac-b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan(((x^2*(((A*b^2*c^2 + 2*A*a*c^3 - 3*B*a*b*c^2)*(A*b^2 + 2*A*a*c - 3*B*
a*b))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*(A*b^2 +
2*A*a*c - 3*B*a*b)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c
 - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b*c^2*(A*b^2 + 2*A*a*c
 - 3*B*a*b)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a
*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(8*A^2*a^2*c^4 + 2*A^2*b^4*c^2 +
18*B^2*a^2*b^2*c^2 + 8*A^2*a*b^2*c^3 - 12*A*B*a*b^3*c^2 - 24*A*B*a^2*b*c^3)
)*(A*b^2 + 2*A*a*c - 3*B*a*b))/(4*a*c - b^2)^(5/2) - ((x^4*(B*b^4 + 16*B*a^
2*c^2 - 3*A*b^3*c - 6*A*a*b*c^2 + B*a*b^2*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) - (c*x^6*(A*b^2 + 2*A*a*c - 3*B*a*b))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^
2*c)) + (a*(B*a*b^2 + 8*B*a^2*c - 6*A*a*b*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) + (x^2*(2*A*a^2*c^2 + B*a*b^3 - 5*A*a*b^2*c + 5*B*a^2*b*c))/(2*c*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x
^2 + 2*b*c*x^6)
```

sympy [B] time = 44.84, size = 833, normalized size = 4.50

$$\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-2Aac-Ab^2+3Bab\right)\log\left(x^2+\frac{-2Aabc-Ab^3+3Bab^2-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-2Aac-Ab^2+3Bab\right)+48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\cdots}{-4A\cdots}\right)\cdots}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b)*log(x**2 + (-2*A*a
*b*c - A*b**3 + 3*B*a*b**2 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*
a*c - A*b**2 + 3*B*a*b) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*
```

```
A*a*c - A*b**2 + 3*B*a*b) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*
c - A*b**2 + 3*B*a*b) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2
+ 3*B*a*b))/(-4*A*a*c**2 - 2*A*b**2*c + 6*B*a*b*c))/2 - sqrt(-1/(4*a*c - b*
*2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b)*log(x**2 + (-2*A*a*b*c - A*b**3 + 3*B
*a*b**2 + 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*
a*b) - 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*
B*a*b) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*
b) - b**6*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b))/(-4*A*a
*c**2 - 2*A*b**2*c + 6*B*a*b*c))/2 + (6*A*a**2*b*c - 8*B*a**3*c - B*a**2*b*
*2 + x**6*(4*A*a*c**3 + 2*A*b**2*c**2 - 6*B*a*b*c**2) + x**4*(6*A*a*b*c**2
+ 3*A*b**3*c - 16*B*a**2*c**2 - B*a*b**2*c - B*b**4) + x**2*(-4*A*a**2*c**2
 + 10*A*a*b**2*c - 10*B*a**2*b*c - 2*B*a*b**3))/(64*a**4*c**3 - 32*a**3*b**
2*c**2 + 4*a**2*b**4*c + x**8*(64*a**2*c**5 - 32*a*b**2*c**4 + 4*b**4*c**3)
 + x**6*(128*a**2*b*c**4 - 64*a*b**3*c**3 + 8*b**5*c**2) + x**4*(128*a**3*c
**4 - 24*a*b**4*c**2 + 4*b**6*c) + x**2*(128*a**3*b*c**3 - 64*a**2*b**3*c**
2 + 8*a*b**5*c))
```

3.128 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=170

$$-\frac{\left(2aBc-3Abc+b^2B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{\left(b+2cx^2\right)\left(2aBc-3Abc+b^2B\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] 1/4*(-a*(-2*A*c+B*b)-(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^
2+a)^2+1/4*(-3*A*b*c+2*B*a*c+B*b^2)*(2*c*x^2+b)/c/(-4*a*c+b^2)^2/(c*x^4+b*x
^2+a)-(-3*A*b*c+2*B*a*c+B*b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(5/2)
```

Rubi [A] time = 0.16, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 777, 614, 618, 206}

$$\frac{\left(b+2cx^2\right)\left(2aBc-3Abc+b^2B\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\left(2aBc-3Abc+b^2B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(4*c*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)^2) + ((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(4*c*(b^2 -
 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2*B - 3*A*b*c + 2*a*B*c)*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && LtQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3 (A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{(b^2B-3Abc+2aBc)\,\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\right.}{4c(b^2-4ac)}$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)(b+2cx^2)}{4c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(b^2B-}{}$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)(b+2cx^2)}{4c(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(b^2B-}{}$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)(b+2cx^2)}{4c(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(b^2B-}{}$$

Mathematica [A] time = 0.20, size = 172, normalized size = 1.01

$$\frac{1}{4}\left(\frac{4(2aBc-3Abc+b^2B)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}} + \frac{(b+2cx^2)(2aBc-3Abc+b^2B)}{c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{-2ac(A+Bx^2)+abB+bx^2}{c(4ac-b^2)(a+bx^2+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2
+ c*x^4)) + (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(c*(-b^2 + 4*a*
c)*(a + b*x^2 + c*x^4)^2) + (4*(b^2*B - 3*A*b*c + 2*a*B*c)*ArcTan[(b + 2*c*
x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

fricas [B] time = 0.86, size = 1226, normalized size = 7.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^2)*x
^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3*A*a
*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2*B*a^
3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 2*((B*b^2*c^2 + (2*B
```

```
*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^2*b^
2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 + 2*(
B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^
2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(6*B*a^3*b + A*a^2*b^2)*c)/((b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x
^2), 1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^
2)*x^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3
*A*a*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2*
B*a^3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 4*((B*b^2*c^2 +
(2*B*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^
2*b^2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 +
 2*(B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*sqr
t(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 4
*(6*B*a^3*b + A*a^2*b^2)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

giac [A] time = 6.06, size = 228, normalized size = 1.34

$$\frac{\left(Bb^2 + 2\,Bac - 3\,Abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{-b^2 + 4\,ac}} + \frac{2\,Bb^2cx^6 + 4\,Bac^2x^6 - 6\,Abc^2x^6 + 3\,Bb^3x^4 + 6\,Babcx^4 - 9\,Ab^2cx^4 + 1\cdots}{4\left(cx^4 + bx^2 + a\right)^2\left(b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (B*b^2 + 2*B*a*c - 3*A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4
- 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*B*b^2*c*x^6 + 4*B*a*
c^2*x^6 - 6*A*b*c^2*x^6 + 3*B*b^3*x^4 + 6*B*a*b*c*x^4 - 9*A*b^2*c*x^4 + 10*
B*a*b^2*x^2 - 2*A*b^3*x^2 - 4*B*a^2*c*x^2 - 10*A*a*b*c*x^2 + 6*B*a^2*b - A*
a*b^2 - 8*A*a^2*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

maple [B] time = 0.02, size = 379, normalized size = 2.23

$$-\frac{3Abc\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2 - 8a\,b^2c + b^4\right)\sqrt{4ac-b^2}} + \frac{2Bac\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2 - 8a\,b^2c + b^4\right)\sqrt{4ac-b^2}} + \frac{B\,b^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2 - 8a\,b^2c + b^4\right)\sqrt{4ac-b^2}} + \frac{-\frac{(3A\cdots}{16\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-c*(3*A*b*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-3/2*b*(3*A*b
*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*A*a*b*c+A*b^3+2*B*a^2*c
-5*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/2*a*(8*A*a*c+A*b^2-6*B*a*b)/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b*c+2/(16*a^2*c^2-8
*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*B*c
+1/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2))*b^2*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.66, size = 587, normalized size = 3.45

$$\frac{\mathrm{atan}\left(\frac{\left(x^2\left(\frac{\left(B\,b^2\,c^2-3\,A\,b\,c^3+2\,B\,a\,c^3\right)\left(B\,b^2-3\,A\,c\,b+2\,B\,a\,c\right)}{a\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b\left(B\,b^2-3\,A\,c\,b+2\,B\,a\,c\right)^2\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\,a\left(4\,a\,c-b^2\right)^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{2\,b\,c^2\left(B\,b^2-3\,A\,c\,b+2\,B\,a\,c\right)^2}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\left(b^4\left(4\,a\,c-b^2\right)^5\cdots}{18\,A^2\,b^2\,c^4-24\,A\,B\,a\,b\,c^4-12\,A\,B\,b^3\,c^3+8\,B^2\,a^2\,c^4+8\,B^2\,a\,b^2\,c^3+2\,B^2\,b^4\,c^2\cdots}\right)\cdots}{\left(4\,a\,c-b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan(((x^2*(((B*b^2*c^2 - 3*A*b*c^3 + 2*B*a*c^3)*(B*b^2 - 3*A*b*c + 2*B*a*
c))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*(B*b^2 - 3*
A*b*c + 2*B*a*c)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c -
 b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b*c^2*(B*b^2 - 3*A*b*c +
 2*B*a*c)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c
 - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*A^2*b^2*c^4 + 8*B^2*a^2*c^4 + 2
*B^2*b^4*c^2 - 12*A*B*b^3*c^3 + 8*B^2*a*b^2*c^3 - 24*A*B*a*b*c^4))*(B*b^2 -
 3*A*b*c + 2*B*a*c))/(4*a*c - b^2)^(5/2) - ((A*a*b^2 + 8*A*a^2*c - 6*B*a^2*
b)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(A*b^3 - 5*B*a*b^2 + 2*B*a^2*c
 + 5*A*a*b*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*b*x^4*(B*b^2 - 3*A*b
*c + 2*B*a*c))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (c*x^6*(B*b^2 - 3*A*b*c
 + 2*B*a*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 +
 c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

sympy [B] time = 21.37, size = 789, normalized size = 4.64

$$-\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-3Abc+2Bac+Bb^2\right)\log\left(x^2+\frac{-3Ab^2c+2Babc+Bb^3-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-3Abc+2Bac+Bb^2\right)+48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-\cdots\right)}}\cdots}{\cdots}\right)\cdots}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2)*log(x**2 + (-3*A*
b**2*c + 2*B*a*b*c + B*b**3 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-3*A
*b*c + 2*B*a*c + B*b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-3
*A*b*c + 2*B*a*c + B*b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b
*c + 2*B*a*c + B*b**2) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*
c + B*b**2))/(-6*A*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c))/2 + sqrt(-1/(4*a*c -
b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2)*log(x**2 + (-3*A*b**2*c + 2*B*a*b*c
 + B*b**3 + 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B
*b**2) - 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c +
 B*b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b
**2) - b**6*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2))/(-6*A
*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c))/2 + (-8*A*a**2*c - A*a*b**2 + 6*B*a**2*
b + x**6*(-6*A*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c) + x**4*(-9*A*b**2*c + 6*B*
```

```
a*b*c + 3*B*b**3) + x**2*(-10*A*a*b*c - 2*A*b**3 - 4*B*a**2*c + 10*B*a*b**2
))/(64*a**4*c**2 - 32*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a
*b**2*c**3 + 4*b**4*c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5
*c) + x**4*(128*a**3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 -
 64*a**2*b**3*c + 8*a*b**5))
```

3.129 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=139

$$\frac{3c(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{3(b+2cx^2)(bB-2Ac)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{-2aB-(x^2(bB-2Ac))+Ab}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[Out] 1/4*(-A*b+2*a*B+(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/4*(-2*A*
c+B*b)*(2*c*x^2+b)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3*c*(-2*A*c+B*b)*arctanh(
(2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1247, 638, 614, 618, 206}

$$-\frac{3(b+2cx^2)(bB-2Ac)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{-2aB+x^2(-(bB-2Ac))+Ab}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3c(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
- (3*(b*B - 2*A*c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
 (3*c*(b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)
^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{(3(bB-2Ac))\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3(bB-2Ac)\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{(3c(bB-2Ac))\,\text{Subst}}{2\left(b^2-\right.}$$

$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3(bB-2Ac)\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{(3c(bB-2Ac))\,\text{Subst}}{\left(b\right.}$$

$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3(bB-2Ac)\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3c(bB-2Ac)\tanh^{-1}}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.13, size = 142, normalized size = 1.02

$$\frac{-\frac{12c(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{\left(b^2-4ac\right)\left(B\left(2a+bx^2\right)-A\left(b+2cx^2\right)\right)}{\left(a+bx^2+cx^4\right)^2} - \frac{3\left(b+2cx^2\right)(bB-2Ac)}{a+bx^2+cx^4}}{4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-3*(b*B - 2*A*c)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + ((b^2 - 4*a*c)*(B*(
2*a + b*x^2) - A*(b + 2*c*x^2)))/(a + b*x^2 + c*x^4)^2 - (12*c*(b*B - 2*A*c
)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(4*(b^2 - 4
*a*c)^2)
```

fricas [B] time = 0.59, size = 1109, normalized size = 7.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 +
A*b^5 + 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*B*
a^3 - 5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2)*c
^2 + (B*a*b^3 - 2*A*b^4)*c)*x^2 + 6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2
 - 2*A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a
*b - A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/
(c*x^4 + b*x^2 + a)) + 2*(2*B*a^2*b^2 - 7*A*a*b^3)*c)/((b^6*c^2 - 12*a*b^4*
```

```
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)
*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), -1/4*
(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 + A*b^5
+ 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*B*a^3 -
5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2)*c^2 + (
B*a*b^3 - 2*A*b^4)*c)*x^2 - 12*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*
A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a*b -
A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(2*B*a^2*b^2 - 7*A*a
*b^3)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2
+ 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 +
 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c
^2 - 64*a^4*b*c^3)*x^2)]
```

giac [A] time = 5.57, size = 208, normalized size = 1.50

$$-\frac{3\left(Bbc-2\,Ac^{2}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}}-\frac{6\,Bbc^{2}x^{6}-12\,Ac^{3}x^{6}+9\,Bb^{2}cx^{4}-18\,Abc^{2}x^{4}+2\,Bb^{3}x^{2}+10\,Babcx^{2}-4}{4\left(cx^{4}+bx^{2}+a\right)^{2}\left(b^{4}-8\,ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*(B*b*c - 2*A*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b
^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*b*c^2*x^6 - 12*A*c^3*x^6
+ 9*B*b^2*c*x^4 - 18*A*b*c^2*x^4 + 2*B*b^3*x^2 + 10*B*a*b*c*x^2 - 4*A*b^2*c
*x^2 - 20*A*a*c^2*x^2 + B*a*b^2 + A*b^3 + 8*B*a^2*c - 10*A*a*b*c)/((c*x^4 +
 b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

maple [A] time = 0.01, size = 262, normalized size = 1.88

$$\frac{3A\,c^{2}x^{2}}{\left(4ac-b^{2}\right)^{2}\left(c\,x^{4}+b\,x^{2}+a\right)}-\frac{3Bbc\,x^{2}}{2\left(4ac-b^{2}\right)^{2}\left(c\,x^{4}+b\,x^{2}+a\right)}+\frac{6A\,c^{2}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{5}{2}}}-\frac{3Bbc\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/4*(A*b-2*B*a+(2*A*c-B*b)*x^2)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3/(4*a*c-b^2)
^2/(c*x^4+b*x^2+a)*c^2*x^2*A-3/2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*c*x^2*b*B+3/
2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b*A*c-3/4/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b^2
*B+6/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A-3/(4*a*c
-b^2)^(5/2)*c*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.59, size = 517, normalized size = 3.72

$$\frac{3\,c\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{3\,c\,(2\,A\,c-B\,b)\left(6\,A\,c^4-3\,B\,b\,c^3\right)}{a\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b\,c^2\,(2\,A\,c-B\,b)^2\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\,a\left(4\,a\,c-b^2\right)^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{18\,b\,c^4\,(2\,A\,c-B\,b)^2}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\left(b^4\left(4\,a\,c-b^2\right)^5+16\,a^2\,c^2\left(4\,a\,c-b^2\right)^5-8\cdots}{72\,A^2\,c^6-72\,A\,B\,b\,c^5+18\,B^2\,b^2\,c^4}\cdots\right.}{\left(4\,a\,c-b^2\right)^{5/2}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*c*atan(((x^2*((3*c*(2*A*c - B*b)*(6*A*c^4 - 3*B*b*c^3))/(a*(4*a*c - b^2)
^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*(2*A*c - B*b)^2*(2*b^5*c^
2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c))) + (18*b*c^4*(2*A*c - B*b)^2)/(4*a*c - b^2)^(15/2))*(b^4*(
4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(
72*A^2*c^6 + 18*B^2*b^2*c^4 - 72*A*B*b*c^5))*(2*A*c - B*b))/(4*a*c - b^2)^(
5/2) - ((A*b^3 + B*a*b^2 + 8*B*a^2*c - 10*A*a*b*c)/(4*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) - (9*x^4*(2*A*b*c^2 - B*b^2*c))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (x^2*(B*b^3 - 10*A*a*c^2 - 2*A*b^2*c + 5*B*a*b*c))/(2*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) - (3*c^2*x^6*(2*A*c - B*b))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

sympy [B] time = 12.40, size = 661, normalized size = 4.76

$$\frac{3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\,(-2Ac+Bb)\log\left(x^2+\frac{-6Abc^2+3Bb^2c-192a^3c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\,(-2Ac+Bb)+144a^2b^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\,(-2Ac+Bb)-36ab^4c^2\sqrt{\cdots}}{-12Ac^3+6Bbc^2}\cdots\right)}{2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B
*b**2*c - 192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 144*a**
2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 36*a*b**4*c**2*sqrt
(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)
*(-2*A*c + B*b))/(-12*A*c**3 + 6*B*b*c**2))/2 - 3*c*sqrt(-1/(4*a*c - b**2)*
*5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B*b**2*c + 192*a**3*c**4*sqr
t(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c
- b**2)**5)*(-2*A*c + B*b) + 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*
A*c + B*b) - 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b))/(-12*A*c**
3 + 6*B*b*c**2))/2 + (10*A*a*b*c - A*b**3 - 8*B*a**2*c - B*a*b**2 + x**6*(1
2*A*c**3 - 6*B*b*c**2) + x**4*(18*A*b*c**2 - 9*B*b**2*c) + x**2*(20*A*a*c**
2 + 4*A*b**2*c - 10*B*a*b*c - 2*B*b**3))/(64*a**4*c**2 - 32*a**3*b**2*c + 4
*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**6*(128
*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 24*a*b**4
*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

3.130 $\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=252

$$-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{A\log(x)}{a^3}+\frac{2cx^2\left(6a^2Bc+A\left(b^3-7abc\right)\right)+A\left(16a^2c^2-15ab^2c+2b^4\right)+6a^2bBc}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{(12}{}$$

```
[Out] 1/4*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a
)^2+1/4*(6*a^2*b*B*c+A*(16*a^2*c^2-15*a*b^2*c+2*b^4)+2*c*(6*a^2*B*c+A*(-7*a
*b*c+b^3))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-1/2*(12*a^3*B*c^2-A*(30*
a^2*b*c^2-10*a*b^3*c+b^5))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*
a*c+b^2)^(5/2)+A*ln(x)/a^3-1/4*A*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A] time = 0.54, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{2cx^2\left(6a^2Bc+A\left(b^3-7abc\right)\right)+A\left(16a^2c^2-15ab^2c+2b^4\right)+6a^2bBc}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(12a^3Bc^2-A\left(30a^2bc^2-10ab^3c+b^5\right)\right)}{2a^3\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] -(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)^2) + (6*a^2*b*B*c + A*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2) + 2*c*
(6*a^2*B*c + A*(b^3 - 7*a*b*c))*x^2)/(4*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) - ((12*a^3*B*c^2 - A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2))*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + (A*Log[x])/a^3 -
 (A*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-2A\left(b^2-4ac\right)-3(Ab-2aB)cx}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c}{4a^2\left(b^2-4ac\right)^2\left(a+bx\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c}{4a^2\left(b^2-4ac\right)^2\left(a+bx\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c}{4a^2\left(b^2-4ac\right)^2\left(a+bx\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c}{4a^2\left(b^2-4ac\right)^2\left(a+bx\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c}{4a^2\left(b^2-4ac\right)^2\left(a+bx\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c}{4a^2\left(b^2-4ac\right)^2\left(a+bx\right.}$$

Mathematica [A] time = 0.69, size = 396, normalized size = 1.57

$$\frac{\frac{a^2\left(A\left(-2ac+b^2+bcx^2\right)-aB\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{a\left(2a^2c\left(8Ac+3bB+6Bcx^2\right)-aAbc\left(15b+14cx^2\right)+2Ab^3\left(b+cx^2\right)\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(A\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c-8\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((a^2*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)^2) + (a*(2*A*b^3*(b + c*x^2) - a*A*b*c*(15*b + 14*c*x^2)
+ 2*a^2*c*(3*b*B + 8*A*c + 6*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4
)) + 4*A*Log[x] - ((-12*a^3*B*c^2 + A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^
4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4
*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - ((12*a^
3*B*c^2 + A*(-b^5 + 10*a*b^3*c - 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a
*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2
- 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

fricas [B] time = 4.99, size = 2494, normalized size = 9.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5*c^3 - 2*(A*a*b^5*c^2 - 4*(6*B*a^4
 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c -
64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*a^2
*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4*b^2)*c^2 - 2*(A*a*b^7 - 4*(10*B*a
^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a^3*b^3)*c^2 + 2*(B*a^3*b^4 - 5*A*a
^2*b^5)*c)*x^2 - ((A*b^5*c^2 - 10*A*a*b^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4
)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*(A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a
^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 - 8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3
*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)*c^2)*x^4 - 6*(2*B*a^5 - 5*A*a^4*b)*
c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^4*b^3 - 33*A*a^3*b^4)*c +
(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2
- 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A
*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*b^6*c
 + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b^7 -
12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2
+ a) - 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A
*b^6*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7
*c - 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*
A*a*b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A
*a*b^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(x))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^
4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^
2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b
^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48
*a^6*b^3*c^2 - 64*a^7*b*c^3)*x^2), -1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5
*c^3 - 2*(A*a*b^5*c^2 - 4*(6*B*a^4 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a
^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c - 64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b
^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*a^2*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4
*b^2)*c^2 - 2*(A*a*b^7 - 4*(10*B*a^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a
^3*b^3)*c^2 + 2*(B*a^3*b^4 - 5*A*a^2*b^5)*c)*x^2 - 2*((A*b^5*c^2 - 10*A*a*b
^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*
(A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 -
 8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)
*c^2)*x^4 - 6*(2*B*a^5 - 5*A*a^4*b)*c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(
2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (14*B*a^4*b^3 - 33*A*a^3*b^4)*c + (A*a
^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2 - 12
*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A*a*b
^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*b^6*c + 2
4*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b^7 - 12*A
*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a)
 - 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6
*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c -
 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*
b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b
^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(x))/(a^5*
b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^
4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 +
48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c
^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6
*b^3*c^2 - 64*a^7*b*c^3)*x^2)]
```

giac [A] time = 6.44, size = 421, normalized size = 1.67

$$-\frac{\left(Ab^{5}-10\,Aab^{3}c-12\,Ba^{3}c^{2}+30\,Aa^{2}bc^{2}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(a^{3}b^{4}-8\,a^{4}b^{2}c+16\,a^{5}c^{2}\right)\sqrt{-b^{2}+4\,ac}}-\frac{A\log\left(cx^{4}+bx^{2}+a\right)}{4\,a^{3}}+\frac{A\log\left(x^{2}\right)}{2\,a^{3}}+\frac{3\,Ab^{4}c^{2}x^{8}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(A*b^5 - 10*A*a*b^3*c - 12*B*a^3*c^2 + 30*A*a^2*b*c^2)*arctan((2*c*x^2
 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 +
 4*a*c)) - 1/4*A*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*A*log(x^2)/a^3 + 1/8*(3*A
*b^4*c^2*x^8 - 24*A*a*b^2*c^3*x^8 + 48*A*a^2*c^4*x^8 + 6*A*b^5*c*x^6 - 44*A
*a*b^3*c^2*x^6 + 24*B*a^3*c^3*x^6 + 68*A*a^2*b*c^3*x^6 + 3*A*b^6*x^4 - 10*A
*a*b^4*c*x^4 + 36*B*a^3*b*c^2*x^4 - 58*A*a^2*b^2*c^2*x^4 + 128*A*a^3*c^3*x^
4 + 10*A*a*b^5*x^2 + 8*B*a^3*b^2*c*x^2 - 72*A*a^2*b^3*c*x^2 + 40*B*a^4*c^2*
x^2 + 92*A*a^3*b*c^2*x^2 - 2*B*a^3*b^3 + 9*A*a^2*b^4 + 20*B*a^4*b*c - 66*A*
a^3*b^2*c + 96*A*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*x^4 + b*
x^2 + a)^2)
```

maple [**B**] time = 0.03, size = 1161, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 9/2/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b*B-1/2/(c*x^4+b*x
^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b*c^2+1/(c*x^4+b*x^2+a)^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^2*B*b^2*c+1/2/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^2*A*b^5+5*a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*c
^2+5/2*a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b*B*c+2/a^2/(16*a^2*c
^2-8*a*b^2*c+b^4)*c*ln(c*x^4+b*x^2+a)*A*b^2-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b
^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^5+A*ln(x)/a
^3-1/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^3-7/2/a/(c*x^4+b*x^
2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b+1/2/a^2/(c*x^4+b*x^2+a)^2*c^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^3-29/4/a/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^4*A*b^2+1/a^2/(c*x^4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^4*A*b^4-3/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^3
*c-15/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*
a*c-b^2)^(1/2))*A*b*c^2+5/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^3*c-21/4/(c*x^4+b*x^2+a)^2/(16*a^
2*c^2-8*a*b^2*c+b^4)*A*b^2*c+6*a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*A*c^2+3/4/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^4-4/a/(16*a
^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^4+b*x^2+a)*A-1/4/a^3/(16*a^2*c^2-8*a*b^2*c
+b^4)*ln(c*x^4+b*x^2+a)*A*b^4+6/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*c^2+3/(c*x^4+b*x^2+a)^2*c^3/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^6*B+4/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^4*A
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 11.57, size = 11674, normalized size = 46.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] ((3*A*b^4 + 24*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 10*B*a^2*b*c)/(4*a*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(A*b^5 + 10*B*a^3*c^2 - 6*A*a*b^3*c - A*
a^2*b*c^2 + 2*B*a^2*b^2*c))/(2*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(
16*A*a^2*c^3 + 4*A*b^4*c - 29*A*a*b^2*c^2 + 18*B*a^2*b*c^2))/(4*a^2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^6*(A*b^3 + 6*B*a^2*c - 7*A*a*b*c))/(2*a^2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*
b*x^2 + 2*b*c*x^6) + (A*log(x))/a^3 - (log(((c^5*x^2*(A*b^3 + 6*B*a^2*c - 7
*A*a*b*c)^3)/(a^6*(4*a*c - b^2)^6) - ((A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10
*A*a*b^3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((c^3*(4*A^2*b
^8 - 36*B^2*a^5*c^3 + 302*A^2*a^2*b^4*c^2 - 497*A^2*a^3*b^2*c^3 - 61*A^2*a*
b^6*c - 204*A*B*a^3*b^3*c^2 + 24*A*B*a^2*b^5*c + 468*A*B*a^4*b*c^3))/(a^4*(
4*a*c - b^2)^4) - ((A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a
^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((2*c^3*x^2*(A*b^5 + 60*B*a^3*c^2
 - 2*A*a*b^3*c + 10*A*a^2*b*c^2 - 24*B*a^2*b^2*c))/(a^2*(4*a*c - b^2)^2) +
(4*b*c^2*(A*b^5 - 6*B*a^3*c^2 - 9*A*a*b^3*c + 23*A*a^2*b*c^2))/(a^2*(4*a*c
- b^2)^2) + (b*c^2*(A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a
^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a
^3))/(4*a^3) + (c^4*x^2*(6*A^2*b^7 + 409*A^2*a^2*b^3*c^2 + 480*A*B*a^4*c^3
- 89*A^2*a*b^5*c - 560*A^2*a^3*b*c^3 + 36*B^2*a^4*b*c^2 - 324*A*B*a^3*b^2*c
^2 + 42*A*B*a^2*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) + (A*c^4*(A*b^3 + 6
*B*a^2*c - 7*A*a*b*c)^2)/(a^6*(4*a*c - b^2)^4))*((c^5*x^2*(A*b^3 + 6*B*a^2*
c - 7*A*a*b*c)^3)/(a^6*(4*a*c - b^2)^6) - ((A - a^3*(-(A*b^5 - 12*B*a^3*c^2
 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((c^3*(4*
A^2*b^8 - 36*B^2*a^5*c^3 + 302*A^2*a^2*b^4*c^2 - 497*A^2*a^3*b^2*c^3 - 61*A
^2*a*b^6*c - 204*A*B*a^3*b^3*c^2 + 24*A*B*a^2*b^5*c + 468*A*B*a^4*b*c^3))/(
a^4*(4*a*c - b^2)^4) - ((A - a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 3
0*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((2*c^3*x^2*(A*b^5 + 60*B*a^
3*c^2 - 2*A*a*b^3*c + 10*A*a^2*b*c^2 - 24*B*a^2*b^2*c))/(a^2*(4*a*c - b^2)^
2) + (4*b*c^2*(A*b^5 - 6*B*a^3*c^2 - 9*A*a*b^3*c + 23*A*a^2*b*c^2))/(a^2*(4
*a*c - b^2)^2) + (b*c^2*(A - a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 3
0*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^
2))/a^3))/(4*a^3) + (c^4*x^2*(6*A^2*b^7 + 409*A^2*a^2*b^3*c^2 + 480*A*B*a^4
*c^3 - 89*A^2*a*b^5*c - 560*A^2*a^3*b*c^3 + 36*B^2*a^4*b*c^2 - 324*A*B*a^3*
b^2*c^2 + 42*A*B*a^2*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) + (A*c^4*(A*b^
3 + 6*B*a^2*c - 7*A*a*b*c)^2)/(a^6*(4*a*c - b^2)^4)))*(2*A*b^10 - 2048*A*a^
5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*
b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 -
2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) - (atan((x^2*((((((((30720*B*a^11*c^9
 + 5120*A*a^10*b*c^9 + 2*A*a^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*a^6*b^9
*c^5 - 1216*A*a^7*b^7*c^6 + 3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8 - 48*B*
a^6*b^10*c^4 + 888*B*a^7*b^8*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*b^4*c^7
 - 43008*B*a^10*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^
8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2
*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^
4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*
b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227
328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80
*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^
12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3
840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*
c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c^2 -
 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c +
 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*
b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*
b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/
(8*a^3*(4*a*c - b^2)^(5/2)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*
```

a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^
6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4
- 6144*a^11*b^2*c^5)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^
2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
*c^4)) - (((6*A^2*a^2*b^11*c^4 - 137*A^2*a^3*b^9*c^5 + 1217*A^2*a^4*b^7*c^6
 - 5256*A^2*a^5*b^5*c^7 + 11024*A^2*a^6*b^3*c^8 + 36*B^2*a^6*b^5*c^6 - 288*
B^2*a^7*b^3*c^7 + 7680*A*B*a^8*c^9 - 8960*A^2*a^7*b*c^9 + 576*B^2*a^8*b*c^8
 + 42*A*B*a^4*b^8*c^5 - 660*A*B*a^5*b^6*c^6 + 3744*A*B*a^6*b^4*c^7 - 9024*A
*B*a^7*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2
 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - (((30720*B*a
^11*c^9 + 5120*A*a^10*b*c^9 + 2*A*a^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*
a^6*b^9*c^5 - 1216*A*a^7*b^7*c^6 + 3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8
- 48*B*a^6*b^10*c^4 + 888*B*a^7*b^8*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*
b^4*c^7 - 43008*B*a^10*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c +
 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5
) - ((2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A
*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 3
28*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^
6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c
^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*
(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*
c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*A*b^10 - 2048*A*a^5*c^5 -
 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4
))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^
6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A
*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) + ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*
b^3*c + 30*A*a^2*b*c^2)^3*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^
13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 22732
8*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(64*a^9*(4*a*c - b^2)^(15/2)*(a^6*b^
12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3
840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(3*A*b^8 + 160*A*a^4*c^4 - 39*A*a*b
^6*c + 18*B*a^4*b*c^3 + 180*A*a^2*b^4*c^2 - 325*A*a^3*b^2*c^3 - 6*B*a^3*b^3
*c^2))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*A^2*b^10 - 6400*A^2*a^5*c^5 - 36*
B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c^3 + 7775*A^2*a^4*b^2
*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^4*b^3*c^3 + 180*A*B*a
^5*b*c^4)) + (((A^3*b^9*c^5 + 216*B^3*a^6*c^8 + 147*A^3*a^2*b^5*c^7 - 343*A
^3*a^3*b^3*c^8 - 21*A^3*a*b^7*c^6 - 756*A*B^2*a^5*b*c^8 + 108*A*B^2*a^4*b^3
*c^7 + 18*A^2*B*a^2*b^6*c^6 - 252*A^2*B*a^3*b^4*c^7 + 882*A^2*B*a^4*b^2*c^8
)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^
6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - (((6*A^2*a^2*b^11*c^4 - 13
7*A^2*a^3*b^9*c^5 + 1217*A^2*a^4*b^7*c^6 - 5256*A^2*a^5*b^5*c^7 + 11024*A^2
*a^6*b^3*c^8 + 36*B^2*a^6*b^5*c^6 - 288*B^2*a^7*b^3*c^7 + 7680*A*B*a^8*c^9
- 8960*A^2*a^7*b*c^9 + 576*B^2*a^8*b*c^8 + 42*A*B*a^4*b^8*c^5 - 660*A*B*a^5
*b^6*c^6 + 3744*A*B*a^6*b^4*c^7 - 9024*A*B*a^7*b^2*c^8)/(a^6*b^12 + 4096*a^
12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4
*c^4 - 6144*a^11*b^2*c^5) - (((30720*B*a^11*c^9 + 5120*A*a^10*b*c^9 + 2*A*a
^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*a^6*b^9*c^5 - 1216*A*a^7*b^7*c^6 +
3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8 - 48*B*a^6*b^10*c^4 + 888*B*a^7*b^8
*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*b^4*c^7 - 43008*B*a^10*b^2*c^8)/(a^
6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3
 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*A*b^10 - 2048*A*a^5*c^5 - 4
0*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*
(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4
 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^
12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2
 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*
b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11
*b^2*c^5)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 -

```
 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 -
80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*
A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4
*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c +
640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) - (((((30720*B*a^11
*c^9 + 5120*A*a^10*b*c^9 + 2*A*a^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*a^6
*b^9*c^5 - 1216*A*a^7*b^7*c^6 + 3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8 - 4
8*B*a^6*b^10*c^4 + 888*B*a^7*b^8*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*b^4
*c^7 - 43008*B*a^10*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 24
0*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) -
 ((2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^
3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*
a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 +
 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5
- 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^
6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3
 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*
b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c
^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8
*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a
^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*
a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^
8))/(8*a^3*(4*a*c - b^2)^(5/2)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c +
640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^1
2*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*
c^4 - 6144*a^11*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*
b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) + ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c
 + 30*A*a^2*b*c^2)^2*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*
b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*
a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 9
7280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(32*a^6*(4*
a*c - b^2)^5*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 -
2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^1
0*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^
2*c^5)))*(3*A*b^7 + 6*B*a^4*c^3 - 33*A*a*b^5*c - 135*A*a^3*b*c^3 + 120*A*a^
2*b^3*c^2 - 6*B*a^3*b^2*c^2))/(8*a^3*c^2*(4*a*c - b^2)^6*(6*A^2*b^10 - 6400
*A^2*a^5*c^5 - 36*B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c^3
+ 7775*A^2*a^4*b^2*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^4*b
^3*c^3 + 180*A*B*a^5*b*c^4)))*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^1
5*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^
11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) +
 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)
^(15/2)))/(A^2*b^10*c^2 + 144*B^2*a^6*c^6 + 160*A^2*a^2*b^6*c^4 - 600*A^2*a
^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 - 20*A^2*a*b^8*c^3 - 24*A*B*a^3*b^5*c^4 +
240*A*B*a^4*b^3*c^5 - 720*A*B*a^5*b*c^6) - (((((((384*B*a^9*b*c^6 - 4*A*a^4
*b^10*c^2 + 68*A*a^5*b^8*c^3 - 444*A*a^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 147
2*A*a^8*b^2*c^6 + 24*B*a^7*b^5*c^4 - 192*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^10
*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - ((4*a^7*b^10*c^2
- 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)
*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3
*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c +
 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*
c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*
a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A
*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(4*a^7*b^10*c^2 - 64*a
^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*
b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c
^3 + 2560*A*a^4*b^2*c^4))/(8*a^3*(4*a*c - b^2)^(5/2)*(a^6*b^8 + 256*a^10*c^
4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8
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*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4
)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*
a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b
^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) - (((36*B^2*
a^7*c^6 - 4*A^2*a^2*b^8*c^3 + 61*A^2*a^3*b^6*c^4 - 302*A^2*a^4*b^4*c^5 + 49
7*A^2*a^5*b^2*c^6 - 24*A*B*a^4*b^5*c^4 + 204*A*B*a^5*b^3*c^5 - 468*A*B*a^6*
b*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^
2*c^3) - (((384*B*a^9*b*c^6 - 4*A*a^4*b^10*c^2 + 68*A*a^5*b^8*c^3 - 444*A*a
^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 1472*A*a^8*b^2*c^6 + 24*B*a^7*b^5*c^4 - 1
92*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 -
 256*a^9*b^2*c^3) - ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1
024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b
^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(a^
6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*
a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3
 + 5120*a^7*b^2*c^4)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^
2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a
*c - b^2)^(5/2)) + ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^
3*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 +
1024*a^11*b^2*c^6))/(64*a^9*(4*a*c - b^2)^(15/2)*(a^6*b^8 + 256*a^10*c^4 -
16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)))*(16*a^9*b^12*(4*a*c - b^
2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c -
b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(
4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*
b^2*c^5*(4*a*c - b^2)^(15/2))*(3*A*b^8 + 160*A*a^4*c^4 - 39*A*a*b^6*c + 18*
B*a^4*b*c^3 + 180*A*a^2*b^4*c^2 - 325*A*a^3*b^2*c^3 - 6*B*a^3*b^3*c^2))/(8*
a^3*c^2*(4*a*c - b^2)^(13/2)*(A^2*b^10*c^2 + 144*B^2*a^6*c^6 + 160*A^2*a^2*
b^6*c^4 - 600*A^2*a^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 - 20*A^2*a*b^8*c^3 - 24
*A*B*a^3*b^5*c^4 + 240*A*B*a^4*b^3*c^5 - 720*A*B*a^5*b*c^6)*(6*A^2*b^10 - 6
400*A^2*a^5*c^5 - 36*B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c
^3 + 7775*A^2*a^4*b^2*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^
4*b^3*c^3 + 180*A*B*a^5*b*c^4)) + (((A^3*b^6*c^4 + 49*A^3*a^2*b^2*c^6 + 36*
A*B^2*a^4*c^6 - 14*A^3*a*b^4*c^5 - 84*A^2*B*a^3*b*c^6 + 12*A^2*B*a^2*b^3*c^
5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^
3) + (((36*B^2*a^7*c^6 - 4*A^2*a^2*b^8*c^3 + 61*A^2*a^3*b^6*c^4 - 302*A^2*a
^4*b^4*c^5 + 497*A^2*a^5*b^2*c^6 - 24*A*B*a^4*b^5*c^4 + 204*A*B*a^5*b^3*c^5
 - 468*A*B*a^6*b*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c
^2 - 256*a^9*b^2*c^3) - (((384*B*a^9*b*c^6 - 4*A*a^4*b^10*c^2 + 68*A*a^5*b^
8*c^3 - 444*A*a^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 1472*A*a^8*b^2*c^6 + 24*B*
a^7*b^5*c^4 - 192*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 9
6*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*
a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5
*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b
^2*c^4))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a
^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2
560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b
^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*
a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3
 + 5120*a^7*b^2*c^4)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^
2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
*c^4)) + (((((384*B*a^9*b*c^6 - 4*A*a^4*b^10*c^2 + 68*A*a^5*b^8*c^3 - 444*A
*a^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 1472*A*a^8*b^2*c^6 + 24*B*a^7*b^5*c^4 -
 192*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2
 - 256*a^9*b^2*c^3) - ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 -
 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a
*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(
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a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(
4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c
^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*
c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c +
 30*A*a^2*b*c^2)*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*
a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c
 + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(8*a^3*(4*
a*c - b^2)^(5/2)*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 -
256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^
2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b
^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c^
2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^2*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384
*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^
5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*
b^2*c^4))/(32*a^6*(4*a*c - b^2)^5*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c +
96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c
 + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(3*A*b^7 + 6*B*
a^4*c^3 - 33*A*a*b^5*c - 135*A*a^3*b*c^3 + 120*A*a^2*b^3*c^2 - 6*B*a^3*b^2*
c^2)*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(15/2
) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b^2)^
(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(4*a*
c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a
*c - b^2)^6*(A^2*b^10*c^2 + 144*B^2*a^6*c^6 + 160*A^2*a^2*b^6*c^4 - 600*A^2
*a^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 - 20*A^2*a*b^8*c^3 - 24*A*B*a^3*b^5*c^4
+ 240*A*B*a^4*b^3*c^5 - 720*A*B*a^5*b*c^6)*(6*A^2*b^10 - 6400*A^2*a^5*c^5 -
 36*B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c^3 + 7775*A^2*a^4
*b^2*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^4*b^3*c^3 + 180*A
*B*a^5*b*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(2*
a^3*(4*a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.131 $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=363

$$\frac{(3Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{-A\left(20a^2c^2-20ab^2c+3b^4\right)+cx^2\left(aB\left(b^2-16ac\right)-3A\left(\right.\right.}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+c\right.}$$

```
[Out] 1/2*(a*b*B*(-7*a*c+b^2)-3*A*(10*a^2*c^2-7*a*b^2*c+b^4))/a^3/(-4*a*c+b^2)^2/
x^2+1/4*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x^2/(c*x^4
+b*x^2+a)^2+1/4*(-a*b*B*(-10*a*c+b^2)+A*(20*a^2*c^2-20*a*b^2*c+3*b^4)-c*(a*
B*(-16*a*c+b^2)-3*A*(-6*a*b*c+b^3))*x^2)/a^2/(-4*a*c+b^2)^2/x^2/(c*x^4+b*x^
2+a)+1/2*(a*b*B*(30*a^2*c^2-10*a*b^2*c+b^4)-3*A*(-20*a^3*c^3+30*a^2*b^2*c^2
-10*a*b^4*c+b^6))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^
(5/2)-(3*A*b-B*a)*ln(x)/a^4+1/4*(3*A*b-B*a)*ln(c*x^4+b*x^2+a)/a^4
```

Rubi [A] time = 0.77, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{abB\left(b^2-7ac\right)-3A\left(10a^2c^2-7ab^2c+b^4\right)}{2a^3x^2\left(b^2-4ac\right)^2}-\frac{-A\left(20a^2c^2-20ab^2c+3b^4\right)+cx^2\left(aB\left(b^2-16ac\right)-3A\left(b^3-6ab\right.\right.}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (a*b*B*(b^2 - 7*a*c) - 3*A*(b^4 - 7*a*b^2*c + 10*a^2*c^2))/(2*a^3*(b^2 - 4*
a*c)^2*x^2) - (a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4
*a*c)*x^2*(a + b*x^2 + c*x^4)^2) - (a*b*B*(b^2 - 10*a*c) - A*(3*b^4 - 20*a*
b^2*c + 20*a^2*c^2) + c*(a*B*(b^2 - 16*a*c) - 3*A*(b^3 - 6*a*b*c))*x^2)/(4*
a^2*(b^2 - 4*a*c)^2*x^2*(a + b*x^2 + c*x^4)) + ((a*b*B*(b^4 - 10*a*b^2*c +
30*a^2*c^2) - 3*A*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3))*ArcTanh
[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - ((3*A*b -
a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-3Ab^2+abB+10aAc-4(Ab-2aB)cx}{x^2\left(a+bx+cx^2\right)^2}\,dx,\right.}{4a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB\left(b^2-10ac\right)-A\left(3b^4-20ab^2c+20a\right.}{4a^2\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB\left(b^2-10ac\right)-A\left(3b^4-20ab^2c+20a\right.}{4a^2\left(b^2-4ac\right)}$$

$$= \frac{abB\left(b^2-7ac\right)-3A\left(b^4-7ab^2c+10a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}$$

$$= \frac{abB\left(b^2-7ac\right)-3A\left(b^4-7ab^2c+10a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}$$

$$= \frac{abB\left(b^2-7ac\right)-3A\left(b^4-7ab^2c+10a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}$$

$$= \frac{abB\left(b^2-7ac\right)-3A\left(b^4-7ab^2c+10a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}$$

Mathematica [A] time = 1.50, size = 642, normalized size = 1.77

$$\frac{-\frac{a^2\left(A\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)+aB\left(2ac-b^2-bcx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{a\left(aB\left(16a^2c^2-15ab^2c-14abc^2x^2+2b^4+2b^3cx^2\right)-A\left(46a^2bc^2+28a^2c^3x^2-29ab^3c-26ab^2c^2x\right.\right.}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*a*A)/x^2 - (a^2*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^2
*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(a*B*(2*
b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 - 14*a*b*c^2*x^2) - A*(4*b^5 -
29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3*x^2
)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + 4*(-3*A*b + a*B)*Log[x] + ((-(a
*B*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqr
t[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c])) + 3*A*(b^6 - 10*a*b^4*c + 3
0*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4
*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/(b^2 - 4*a*c)^(5/2) + ((a*B*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt
[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])
 + 3*A*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*
a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^4)
```

fricas [B] time = 10.17, size = 3956, normalized size = 10.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128*A*a^6*c^3 - 2*
(120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^3*b^3 - 3*A*a^
2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 - 69*A*a^4*b)*c
^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2
 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 + 200*A*a^5*
c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A*a^3*b^4)*c^2
- 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^2*b^7 - 8*(12*
B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^2 - (33*B*a^4*
b^4 - 104*A*a^3*b^5)*c)*x^2 - ((60*A*a^3*c^5 + 30*(B*a^3*b - 3*A*a^2*b^2)*c
^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c^2)*x^10 + 2*(60
*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2*b^4 - 3*A*a*b^5
)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 120*A*a^4*c^4 + 6
0*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4)*c^2 - 8*(B*a^2
*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*a^4*b*c^3 + 30*(
B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)*c)*x^4 + (B*a^3
*b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b^2)*c^2 - 10*(B*
a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((
64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b
^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*
a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6
)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*
a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5
)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(
B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b
^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b
)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*
x^2)*log(c*x^4 + b*x^2 + a) + 4*((64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^
2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)
*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^
4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a
*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)
*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6
 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^
3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3
*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2
- 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*x^2)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c
^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48
*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2
 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b
^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*x^2), -1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128*
A*a^6*c^3 - 2*(120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^
3*b^3 - 3*A*a^2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 -
 69*A*a^4*b)*c^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3*
A*a^2*b^5)*c^2 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^
8 + 200*A*a^5*c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A
*a^3*b^4)*c^2 - 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^
2*b^7 - 8*(12*B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^
2 - (33*B*a^4*b^4 - 104*A*a^3*b^5)*c)*x^2 - 2*((60*A*a^3*c^5 + 30*(B*a^3*b
- 3*A*a^2*b^2)*c^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c
^2)*x^10 + 2*(60*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2
```

```
*b^4 - 3*A*a*b^5)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 1
20*A*a^4*c^4 + 60*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4
)*c^2 - 8*(B*a^2*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*
a^4*b*c^3 + 30*(B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)
*c)*x^4 + (B*a^3*b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b
^2)*c^2 - 10*(B*a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((64*(B*a^4 - 3*A*a^3*b)*c
^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B
*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3
*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b
^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b
^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 -
3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c
^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4
 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 -
3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*x^2)*log(c*x^4 + b*x^2 +
 a) + 4*((64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 1
2*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^
4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 -
 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*
a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3
*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*
b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 1
2*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 -
 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^
3*b^5)*c)*x^2)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64
*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*
c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*
a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x
^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x^2)]
```

giac [A] time = 6.37, size = 648, normalized size = 1.79

$$-\frac{\left(Bab^5 - 3\,Ab^6 - 10\,Ba^2b^3c + 30\,Aab^4c + 30\,Ba^3bc^2 - 90\,Aa^2b^2c^2 + 60\,Aa^3c^3\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^4b^4 - 8\,a^5b^2c + 16\,a^6c^2\right)\sqrt{-b^2+4\,ac}} + \frac{3\,Bab^4c^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^5 - 3*A*b^6 - 10*B*a^2*b^3*c + 30*A*a*b^4*c + 30*B*a^3*b*c^2 -
90*A*a^2*b^2*c^2 + 60*A*a^3*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(
(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) + 1/8*(3*B*a*b^4*c
^2*x^8 - 9*A*b^5*c^2*x^8 - 24*B*a^2*b^2*c^3*x^8 + 72*A*a*b^3*c^3*x^8 + 48*B
*a^3*c^4*x^8 - 144*A*a^2*b*c^4*x^8 + 6*B*a*b^5*c*x^6 - 18*A*b^6*c*x^6 - 44*
B*a^2*b^3*c^2*x^6 + 136*A*a*b^4*c^2*x^6 + 68*B*a^3*b*c^3*x^6 - 236*A*a^2*b^
2*c^3*x^6 - 56*A*a^3*c^4*x^6 + 3*B*a*b^6*x^4 - 9*A*b^7*x^4 - 10*B*a^2*b^4*c
*x^4 + 38*A*a*b^5*c*x^4 - 58*B*a^3*b^2*c^2*x^4 + 110*A*a^2*b^3*c^2*x^4 + 12
8*B*a^4*c^3*x^4 - 436*A*a^3*b*c^3*x^4 + 10*B*a^2*b^5*x^2 - 26*A*a*b^6*x^2 -
 72*B*a^3*b^3*c*x^2 + 192*A*a^2*b^4*c*x^2 + 92*B*a^4*b*c^2*x^2 - 316*A*a^3*
b^2*c^2*x^2 - 72*A*a^4*c^3*x^2 + 9*B*a^3*b^4 - 19*A*a^2*b^5 - 66*B*a^4*b^2*
c + 144*A*a^3*b^3*c + 96*B*a^5*c^2 - 260*A*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2
*c + 16*a^6*c^2)*(c*x^4 + b*x^2 + a)^2) - 1/4*(B*a - 3*A*b)*log(c*x^4 + b*x
^2 + a)/a^4 + 1/2*(B*a - 3*A*b)*log(x^2)/a^4 - 1/2*(B*a*x^2 - 3*A*b*x^2 + A
*a)/(a^4*x^2)
```

maple [B] time = 0.04, size = 1862, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 6/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^4*c-3/a/(c*x^4+b
*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^3*c+13/2/a^2/(c*x^4+b*x^2+a)^2
*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^2-1/a^3/(c*x^4+b*x^2+a)^2*c^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^4-7/2/a/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^6*b*B+1/2/a^2/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^6*B*b^3-37/2/a/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b
+55/4/a^2/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^3+45/a^2
/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2
)^(1/2))*A*b^2*c^2-15/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^4*c-15/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*b*c^2+5/a^2/(16*a^
2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)
)*B*b^3*c-2/a^3/(c*x^4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^5-29
/4/a/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^2+1/a^2/(c*x^
4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^4-7/2/a/(c*x^4+b*x^2+a)^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^2*c^2-3/a^4*ln(x)*A*b+3/4/a/(c*x^4+b*x^
2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^4+3/4/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*
ln(c*x^4+b*x^2+a)*A*b^5-4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^4+b*x^2+a
)*B-1/4/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*b^4+4/(c*x^4+b*x
^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B-9/(c*x^4+b*x^2+a)^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^2*A*c^3-29/2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4
)*A*b*c^2-21/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^2*c-1/2/(c*
x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b*c^2-7/a/(c*x^4+b*x^2+a)^2
*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A-1/a^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^2*A*b^6+1/2/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^2*B*b^5+9/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^3*c+12/a^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^4+b*x^2+a)*A*b-6/a^3/(16*a^2*c^2-8*a
*b^2*c+b^4)*c*ln(c*x^4+b*x^2+a)*A*b^3+2/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln
(c*x^4+b*x^2+a)*B*b^2-30/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*c^3-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*b^5+3/2/a^4/(16*
a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2))*A*b^6-5/4/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^5+6*a/(c
*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*c^2-1/2*A/a^3/x^2+1/a^3*ln(x)*
B
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 15.91, size = 16265, normalized size = 44.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] (log(((c^5*x^2*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b
*c)^3)/(a^9*(4*a*c - b^2)^6) - ((B*a - 3*A*b + a^4*(-(60*A*a^3*c^3 - 3*A*b^
```

```
6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2
*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(((B*a - 3*A*b + a^4*(-(60*A*a^3*c^3
- 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A
*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((4*b*c^2*(30*A*a^3*c^3 - 3*A
*b^6 + B*a*b^5 + 27*A*a*b^4*c - 9*B*a^2*b^3*c + 23*B*a^3*b*c^2 - 69*A*a^2*b
^2*c^2))/(a^3*(4*a*c - b^2)^2) + (2*c^3*x^2*(B*a*b^5 - 300*A*a^3*c^3 - 3*A*
b^6 + 6*A*a*b^4*c - 2*B*a^2*b^3*c + 10*B*a^3*b*c^2 + 90*A*a^2*b^2*c^2))/(a^
3*(4*a*c - b^2)^2) + (b*c^2*(B*a - 3*A*b + a^4*(-(60*A*a^3*c^3 - 3*A*b^6 +
B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2
)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^4))/(4*
a^4) + (c^3*(900*A^2*a^5*c^5 - 36*A^2*b^10 - 4*B^2*a^2*b^8 + 24*A*B*a*b^9 -
 3078*A^2*a^2*b^6*c^2 + 7533*A^2*a^3*b^4*c^3 - 7020*A^2*a^4*b^2*c^4 - 302*B
^2*a^4*b^4*c^2 + 497*B^2*a^5*b^2*c^3 + 549*A^2*a*b^8*c + 61*B^2*a^3*b^6*c +
 1932*A*B*a^3*b^5*c^2 - 4002*A*B*a^4*b^3*c^3 - 366*A*B*a^2*b^7*c + 2340*A*B
*a^5*b*c^4))/(a^6*(4*a*c - b^2)^4) - (c^4*x^2*(54*A^2*b^9 + 6*B^2*a^2*b^7 -
 36*A*B*a*b^8 + 4311*A^2*a^2*b^5*c^2 - 9900*A^2*a^3*b^3*c^3 + 409*B^2*a^4*b
^3*c^2 - 2400*A*B*a^5*c^4 - 801*A^2*a*b^7*c + 8100*A^2*a^4*b*c^4 - 89*B^2*a
^3*b^5*c - 560*B^2*a^5*b*c^3 - 2664*A*B*a^3*b^4*c^2 + 4980*A*B*a^4*b^2*c^3
+ 534*A*B*a^2*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (c^4*(3*A*b - B*a)*
(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c)^2)/(a^9*(4*
a*c - b^2)^4))*((c^5*x^2*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c +
 7*B*a^2*b*c)^3)/(a^9*(4*a*c - b^2)^6) - ((3*A*b - B*a + a^4*(-(60*A*a^3*c^
3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90
*A*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(((3*A*b - B*a + a^4*(-(60*
A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*
c^2 - 90*A*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((4*b*c^2*(30*A*a^3
*c^3 - 3*A*b^6 + B*a*b^5 + 27*A*a*b^4*c - 9*B*a^2*b^3*c + 23*B*a^3*b*c^2 -
69*A*a^2*b^2*c^2))/(a^3*(4*a*c - b^2)^2) + (2*c^3*x^2*(B*a*b^5 - 300*A*a^3*
c^3 - 3*A*b^6 + 6*A*a*b^4*c - 2*B*a^2*b^3*c + 10*B*a^3*b*c^2 + 90*A*a^2*b^2
*c^2))/(a^3*(4*a*c - b^2)^2) - (b*c^2*(3*A*b - B*a + a^4*(-(60*A*a^3*c^3 -
3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a
^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))
/a^4))/(4*a^4) - (c^3*(900*A^2*a^5*c^5 - 36*A^2*b^10 - 4*B^2*a^2*b^8 + 24*A
*B*a*b^9 - 3078*A^2*a^2*b^6*c^2 + 7533*A^2*a^3*b^4*c^3 - 7020*A^2*a^4*b^2*c
^4 - 302*B^2*a^4*b^4*c^2 + 497*B^2*a^5*b^2*c^3 + 549*A^2*a*b^8*c + 61*B^2*a
^3*b^6*c + 1932*A*B*a^3*b^5*c^2 - 4002*A*B*a^4*b^3*c^3 - 366*A*B*a^2*b^7*c
+ 2340*A*B*a^5*b*c^4))/(a^6*(4*a*c - b^2)^4) + (c^4*x^2*(54*A^2*b^9 + 6*B^2
*a^2*b^7 - 36*A*B*a*b^8 + 4311*A^2*a^2*b^5*c^2 - 9900*A^2*a^3*b^3*c^3 + 409
*B^2*a^4*b^3*c^2 - 2400*A*B*a^5*c^4 - 801*A^2*a*b^7*c + 8100*A^2*a^4*b*c^4
- 89*B^2*a^3*b^5*c - 560*B^2*a^5*b*c^3 - 2664*A*B*a^3*b^4*c^2 + 4980*A*B*a^
4*b^2*c^3 + 534*A*B*a^2*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (c^4*(3*A
*b - B*a)*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c)^2
)/(a^9*(4*a*c - b^2)^4)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a
*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3
*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 25
60*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b
^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (log(x)*(3*A*b - B*a))/a^4
 - (A/(2*a) + (x^2*(9*A*b^5 - 24*B*a^3*c^2 - 3*B*a*b^4 - 68*A*a*b^3*c + 122
*A*a^2*b*c^2 + 21*B*a^2*b^2*c))/(4*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x
^6*(16*B*a^3*c^3 - 12*A*b^5*c + 4*B*a*b^4*c + 87*A*a*b^3*c^2 - 138*A*a^2*b*
c^3 - 29*B*a^2*b^2*c^2))/(4*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(3*A
*b^6 + 50*A*a^3*c^3 - B*a*b^5 - 18*A*a*b^4*c + 6*B*a^2*b^3*c + B*a^3*b*c^2
+ 7*A*a^2*b^2*c^2))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^8*(3*A*
b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c))/(2*a^3*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)))/(x^6*(2*a*c + b^2) + a^2*x^2 + c^2*x^10 + 2*a*b*x^
4 + 2*b*c*x^8) - (atan((x^2*((((((((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9
+ 6*A*a^6*b^14*c^3 - 108*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^
8*c^6 - 22272*A*a^10*b^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^
9 - 2*B*a^7*b^13*c^3 + 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*
```

b^7*c^6 - 3456*B*a^11*b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*
c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4
*c^4 - 6144*a^14*b^2*c^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^1
0*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 +
227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*
B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^
7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*
B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5
*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 +
4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 38
40*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 +
30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*
(4*a*c - b^2)^(5/2)) - ((60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c -
10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2)*(163840*a^16*b*c^9 - 12
*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5
- 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^1
1 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a
^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 32
0*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(8*a^4*(4*a*c -
b^2)^(5/2)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2
560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^1
0*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*
b^2*c^5)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A
*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680
*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^
4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a
^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (((54*A^2*a^3*b^13*c^4 - 1233*A^2*a^4*b^1
1*c^5 + 11583*A^2*a^5*b^9*c^6 - 57204*A^2*a^6*b^7*c^7 + 156276*A^2*a^7*b^5*
c^8 - 223200*A^2*a^8*b^3*c^9 + 6*B^2*a^5*b^11*c^4 - 137*B^2*a^6*b^9*c^5 + 1
217*B^2*a^7*b^7*c^6 - 5256*B^2*a^8*b^5*c^7 + 11024*B^2*a^9*b^3*c^8 - 38400*
A*B*a^10*c^10 + 129600*A^2*a^9*b*c^10 - 8960*B^2*a^10*b*c^9 - 36*A*B*a^4*b^
12*c^4 + 822*A*B*a^5*b^10*c^5 - 7512*A*B*a^6*b^8*c^6 + 34836*A*B*a^7*b^6*c^
7 - 84864*A*B*a^8*b^4*c^8 + 98880*A*B*a^9*b^2*c^9)/(a^9*b^12 + 4096*a^15*c^
6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c
^4 - 6144*a^14*b^2*c^5) - (((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9 + 6*A*a
^6*b^14*c^3 - 108*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^8*c^6 -
22272*A*a^10*b^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^9 - 2*B
*a^7*b^13*c^3 + 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*b^7*c^6
- 3456*B*a^11*b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*c^6 - 2
4*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 -
6144*a^14*b^2*c^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*
c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328
*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^1
0 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 -
3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b
^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c
+ 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a
^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13
*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 1
20*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840
*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^
3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640
*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(60*A*a^3*c^3 - 3*A*b
^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^
2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 +
30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2)^3*(16384
0*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 2
4960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*
b^3*c^8))/(64*a^12*(4*a*c - b^2)^(15/2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10

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*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a
^14*b^2*c^5)))*(9*A*b^9 - 160*B*a^5*c^4 - 3*B*a*b^8 - 117*A*a*b^7*c + 570*A
*a^4*b*c^4 + 39*B*a^2*b^6*c + 540*A*a^2*b^5*c^2 - 1005*A*a^3*b^3*c^3 - 180*
B*a^3*b^4*c^2 + 325*B*a^4*b^2*c^3))/(8*(4*a*c - b^2)^(13/2)*(900*A^2*a^9*c^
8 + 6400*B^2*a^10*c^7 - 54*A^2*a^3*b^12*c^2 - 960*a^6*b^6*c^4*(B^2*a - 36*A
^2*c) + 120*a^5*b^8*c^3*(B^2*a - 72*A^2*c) - 6*a^4*b^10*c^2*(B^2*a - 180*A^
2*c) - 25*a^8*b^2*c^6*(311*B^2*a - 2196*A^2*c) + 25*a^7*b^4*c^5*(154*B^2*a
- 2763*A^2*c) + 36*A*B*a^4*b^11*c^2 - 720*A*B*a^5*b^9*c^3 + 5760*A*B*a^6*b^
7*c^4 - 23070*A*B*a^7*b^5*c^5 + 46350*A*B*a^8*b^3*c^6 - 37500*A*B*a^9*b*c^7
)) - (((27000*A^3*a^6*c^11 + 27*A^3*b^12*c^5 + 4779*A^3*a^2*b^8*c^7 - 20601
*A^3*a^3*b^6*c^8 + 47790*A^3*a^4*b^4*c^9 - 56700*A^3*a^5*b^2*c^10 - B^3*a^3
*b^9*c^5 + 21*B^3*a^4*b^7*c^6 - 147*B^3*a^5*b^5*c^7 + 343*B^3*a^6*b^3*c^8 -
 567*A^3*a*b^10*c^6 - 27*A^2*B*a*b^11*c^5 + 18900*A^2*B*a^6*b*c^10 + 9*A*B^
2*a^2*b^10*c^5 - 189*A*B^2*a^3*b^8*c^6 + 1413*A*B^2*a^4*b^6*c^7 - 4347*A*B^
2*a^5*b^4*c^8 + 4410*A*B^2*a^6*b^2*c^9 + 567*A^2*B*a^2*b^9*c^6 - 4509*A^2*B
*a^3*b^7*c^7 + 16821*A^2*B*a^4*b^5*c^8 - 29160*A^2*B*a^5*b^3*c^9)/(a^9*b^12
 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 +
3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((54*A^2*a^3*b^13*c^4 - 1233*A^2*
a^4*b^11*c^5 + 11583*A^2*a^5*b^9*c^6 - 57204*A^2*a^6*b^7*c^7 + 156276*A^2*a
^7*b^5*c^8 - 223200*A^2*a^8*b^3*c^9 + 6*B^2*a^5*b^11*c^4 - 137*B^2*a^6*b^9*
c^5 + 1217*B^2*a^7*b^7*c^6 - 5256*B^2*a^8*b^5*c^7 + 11024*B^2*a^9*b^3*c^8 -
 38400*A*B*a^10*c^10 + 129600*A^2*a^9*b*c^10 - 8960*B^2*a^10*b*c^9 - 36*A*B
*a^4*b^12*c^4 + 822*A*B*a^5*b^10*c^5 - 7512*A*B*a^6*b^8*c^6 + 34836*A*B*a^7
*b^6*c^7 - 84864*A*B*a^8*b^4*c^8 + 98880*A*B*a^9*b^2*c^9)/(a^9*b^12 + 4096*
a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^1
3*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9
+ 6*A*a^6*b^14*c^3 - 108*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^
8*c^6 - 22272*A*a^10*b^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^
9 - 2*B*a^7*b^13*c^3 + 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*
b^7*c^6 - 3456*B*a^11*b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*
c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4
*c^4 - 6144*a^14*b^2*c^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^1
0*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 +
 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*
B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^
7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*
B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5
*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 +
 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 38
40*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b
^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2
 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4
*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*
c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b^11 + 20
48*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8
*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^
3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096
*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2
*c^4)) - (((((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9 + 6*A*a^6*b^14*c^3 - 1
08*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^8*c^6 - 22272*A*a^10*b
^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^9 - 2*B*a^7*b^13*c^3 +
 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*b^7*c^6 - 3456*B*a^11*
b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c +
 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c
^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11
*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 -
 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9
*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5
*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B
```

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*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c
^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^
10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144
*a^14*b^2*c^5)))*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^
2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) -
 ((60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*
a^3*b*c^2 - 90*A*a^2*b^2*c^2)*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^
10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6
+ 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2
*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b
^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280
*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(8*a^4*(4*a*c - b^2)^(5/2)*(4*a^4*b^1
0 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120
*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2
 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(60*A*a^3*c
^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 9
0*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((60*A*a^3*c^3 - 3*A*b^6 +
B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2
)^2*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^
11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 29
4912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c
- 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^
3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^
5*b^2*c^4))/(32*a^8*(4*a*c - b^2)^5*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8
*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 409
6*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a
^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(9*A*b^8 + 30*A*a^4*c^4 - 3*B*a*b^7 - 99
*A*a*b^6*c + 33*B*a^2*b^5*c + 135*B*a^4*b*c^3 + 360*A*a^2*b^4*c^2 - 435*A*a
^3*b^2*c^3 - 120*B*a^3*b^3*c^2))/(8*a^3*c^2*(4*a*c - b^2)^6*(900*A^2*a^6*c^
6 - 54*A^2*b^12 - 6*B^2*a^2*b^10 + 6400*B^2*a^7*c^5 + 36*A*B*a*b^11 - 8640*
A^2*a^2*b^8*c^2 + 34560*A^2*a^3*b^6*c^3 - 69075*A^2*a^4*b^4*c^4 + 54900*A^2
*a^5*b^2*c^5 - 960*B^2*a^4*b^6*c^2 + 3850*B^2*a^5*b^4*c^3 - 7775*B^2*a^6*b^
2*c^4 + 1080*A^2*a*b^10*c + 120*B^2*a^3*b^8*c + 5760*A*B*a^3*b^7*c^2 - 2307
0*A*B*a^4*b^5*c^3 + 46350*A*B*a^5*b^3*c^4 - 720*A*B*a^2*b^9*c - 37500*A*B*a
^6*b*c^5)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^
2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c
 - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c
^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(3600*A
^2*a^6*c^8 + 9*A^2*b^12*c^2 + 1440*A^2*a^2*b^8*c^4 - 5760*A^2*a^3*b^6*c^5 +
 11700*A^2*a^4*b^4*c^6 - 10800*A^2*a^5*b^2*c^7 + B^2*a^2*b^10*c^2 - 20*B^2*
a^3*b^8*c^3 + 160*B^2*a^4*b^6*c^4 - 600*B^2*a^5*b^4*c^5 + 900*B^2*a^6*b^2*c
^6 - 180*A^2*a*b^10*c^3 + 120*A*B*a^2*b^9*c^3 - 960*A*B*a^3*b^7*c^4 + 3720*
A*B*a^4*b^5*c^5 - 6600*A*B*a^5*b^3*c^6 - 6*A*B*a*b^11*c^2 + 3600*A*B*a^6*b*
c^7) - (((((((1920*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9*c^3 - 1
332*A*a^8*b^7*c^4 + 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*a^7*b^10
*c^2 - 68*B*a^8*b^8*c^3 + 444*B*a^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 + 1472*B*
a^11*b^2*c^6)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 2
56*a^12*b^2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 -
 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*
b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^
2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^
4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c
 + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*
b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(60*A*a^3*
c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 -
90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) - ((4*a^10*b^10*c^2 - 64*a^1
1*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(60*A
*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c
^2 - 90*A*a^2*b^2*c^2)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^
```

9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^
5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*
B*a^5*b^2*c^4))/(8*a^4*(4*a*c - b^2)^(5/2)*(a^9*b^8 + 256*a^13*c^4 - 16*a^1
0*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 -
80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*
A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 4
0*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4
 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*
b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5
120*a^8*b^2*c^4)) - (((900*A^2*a^8*c^8 - 36*A^2*a^3*b^10*c^3 + 549*A^2*a^4*
b^8*c^4 - 3078*A^2*a^5*b^6*c^5 + 7533*A^2*a^6*b^4*c^6 - 7020*A^2*a^7*b^2*c^
7 - 4*B^2*a^5*b^8*c^3 + 61*B^2*a^6*b^6*c^4 - 302*B^2*a^7*b^4*c^5 + 497*B^2*
a^8*b^2*c^6 + 24*A*B*a^4*b^9*c^3 - 366*A*B*a^5*b^7*c^4 + 1932*A*B*a^6*b^5*c
^5 - 4002*A*B*a^7*b^3*c^6 + 2340*A*B*a^8*b*c^7)/(a^9*b^8 + 256*a^13*c^4 - 1
6*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((1920*A*a^11*b*c^7 -
 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9*c^3 - 1332*A*a^8*b^7*c^4 + 4056*A*a^9*b^
5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*a^7*b^10*c^2 - 68*B*a^8*b^8*c^3 + 444*B*a
^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 + 1472*B*a^11*b^2*c^6)/(a^9*b^8 + 256*a^13
*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - ((4*a^10*b^10*
c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^
2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5
*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a
^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/
(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2
c^3)(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^
7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 1
20*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840
*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^
3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640
*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(60*A*a^3*c^3 - 3*A*b
^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^
2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 +
 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(60*A*a^3*c^3 -
3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a
^2*b^2*c^2)^3)/(64*a^12*(4*a*c - b^2)^(15/2)*(a^9*b^8 + 256*a^13*c^4 - 16*a
^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))*(16*a^12*b^12*(4*a*c - b^
2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c -
b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(
4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*
b^2*c^5*(4*a*c - b^2)^(15/2))*(9*A*b^9 - 160*B*a^5*c^4 - 3*B*a*b^8 - 117*A*
a*b^7*c + 570*A*a^4*b*c^4 + 39*B*a^2*b^6*c + 540*A*a^2*b^5*c^2 - 1005*A*a^3
*b^3*c^3 - 180*B*a^3*b^4*c^2 + 325*B*a^4*b^2*c^3))/(8*(4*a*c - b^2)^(13/2)*
(900*A^2*a^9*c^8 + 6400*B^2*a^10*c^7 - 54*A^2*a^3*b^12*c^2 - 960*a^6*b^6*c^
4*(B^2*a - 36*A^2*c) + 120*a^5*b^8*c^3*(B^2*a - 72*A^2*c) - 6*a^4*b^10*c^2*
(B^2*a - 180*A^2*c) - 25*a^8*b^2*c^6*(311*B^2*a - 2196*A^2*c) + 25*a^7*b^4*
c^5*(154*B^2*a - 2763*A^2*c) + 36*A*B*a^4*b^11*c^2 - 720*A*B*a^5*b^9*c^3 +
5760*A*B*a^6*b^7*c^4 - 23070*A*B*a^7*b^5*c^5 + 46350*A*B*a^8*b^3*c^6 - 3750
0*A*B*a^9*b*c^7)*(3600*A^2*a^6*c^8 + 9*A^2*b^12*c^2 + 1440*A^2*a^2*b^8*c^4
 - 5760*A^2*a^3*b^6*c^5 + 11700*A^2*a^4*b^4*c^6 - 10800*A^2*a^5*b^2*c^7 + B^
2*a^2*b^10*c^2 - 20*B^2*a^3*b^8*c^3 + 160*B^2*a^4*b^6*c^4 - 600*B^2*a^5*b^4
*c^5 + 900*B^2*a^6*b^2*c^6 - 180*A^2*a*b^10*c^3 + 120*A*B*a^2*b^9*c^3 - 960
*A*B*a^3*b^7*c^4 + 3720*A*B*a^4*b^5*c^5 - 6600*A*B*a^5*b^3*c^6 - 6*A*B*a*b^
11*c^2 + 3600*A*B*a^6*b*c^7)) + (((3780*A^3*a^3*b^3*c^7 - 1863*A^3*a^2*b^5*
c^6 - 27*A^3*b^9*c^4 + B^3*a^3*b^6*c^4 - 14*B^3*a^4*b^4*c^5 + 49*B^3*a^5*b^
2*c^6 + 900*A^2*B*a^5*c^8 + 378*A^3*a*b^7*c^5 - 2700*A^3*a^4*b*c^8 + 420*A*
B^2*a^5*b*c^7 + 27*A^2*B*a*b^8*c^4 - 9*A*B^2*a^2*b^7*c^4 + 126*A*B^2*a^3*b^
5*c^5 - 501*A*B^2*a^4*b^3*c^6 - 378*A^2*B*a^2*b^6*c^5 + 1683*A^2*B*a^3*b^4*
c^6 - 2520*A^2*B*a^4*b^2*c^7)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*
a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((900*A^2*a^8*c^8 - 36*A^2*a^3*b^10*c^3

+ 549*A^2*a^4*b^8*c^4 - 3078*A^2*a^5*b^6*c^5 + 7533*A^2*a^6*b^4*c^6 - 7020
*A^2*a^7*b^2*c^7 - 4*B^2*a^5*b^8*c^3 + 61*B^2*a^6*b^6*c^4 - 302*B^2*a^7*b^4
*c^5 + 497*B^2*a^8*b^2*c^6 + 24*A*B*a^4*b^9*c^3 - 366*A*B*a^5*b^7*c^4 + 193
2*A*B*a^6*b^5*c^5 - 4002*A*B*a^7*b^3*c^6 + 2340*A*B*a^8*b*c^7)/(a^9*b^8 + 2
56*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((1920
*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9*c^3 - 1332*A*a^8*b^7*c^4
+ 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*a^7*b^10*c^2 - 68*B*a^8*b^
8*c^3 + 444*B*a^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 + 1472*B*a^11*b^2*c^6)/(a^9
*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) -
((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5
+ 1024*a^14*b^2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9
*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5
*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B
*a^5*b^2*c^4))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2
- 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^
6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b^11 + 2048*B*a^6*c^5 -
2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2
*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 12
80*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*
a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b
^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B
*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 -
320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^1
0 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120
*a^8*b^2*c^4)) - (((((1920*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9
*c^3 - 1332*A*a^8*b^7*c^4 + 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*
a^7*b^10*c^2 - 68*B*a^8*b^8*c^3 + 444*B*a^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 +
1472*B*a^11*b^2*c^6)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4
*c^2 - 256*a^12*b^2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b
^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5
- 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^
2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1
280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^
10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 -
80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6
0*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*
b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) - ((4*a^10*b^10*c^2
- 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^
6)*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B
*a^3*b*c^2 - 90*A*a^2*b^2*c^2)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 12
0*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*
A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3
- 2560*B*a^5*b^2*c^4))/(8*a^4*(4*a*c - b^2)^(5/2)*(a^9*b^8 + 256*a^13*c^4
- 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^
9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^
4)))*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30
*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((4*a^10*b^
10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14
*b^2*c^6)*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c
+ 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2)^2*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*
b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^
2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^
4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(32*a^8*(4*a*c - b^2)^5*(a^9*b^8 + 256*a^1
3*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4
096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*
b^2*c^4)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2
)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c
- b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^
4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2))*(9*A*b^8

```
+ 30*A*a^4*c^4 - 3*B*a*b^7 - 99*A*a*b^6*c + 33*B*a^2*b^5*c + 135*B*a^4*b*c^
3 + 360*A*a^2*b^4*c^2 - 435*A*a^3*b^2*c^3 - 120*B*a^3*b^3*c^2))/(8*a^3*c^2*
(4*a*c - b^2)^6*(3600*A^2*a^6*c^8 + 9*A^2*b^12*c^2 + 1440*A^2*a^2*b^8*c^4 -
 5760*A^2*a^3*b^6*c^5 + 11700*A^2*a^4*b^4*c^6 - 10800*A^2*a^5*b^2*c^7 + B^2
*a^2*b^10*c^2 - 20*B^2*a^3*b^8*c^3 + 160*B^2*a^4*b^6*c^4 - 600*B^2*a^5*b^4*
c^5 + 900*B^2*a^6*b^2*c^6 - 180*A^2*a*b^10*c^3 + 120*A*B*a^2*b^9*c^3 - 960*
A*B*a^3*b^7*c^4 + 3720*A*B*a^4*b^5*c^5 - 6600*A*B*a^5*b^3*c^6 - 6*A*B*a*b^1
1*c^2 + 3600*A*B*a^6*b*c^7)*(900*A^2*a^6*c^6 - 54*A^2*b^12 - 6*B^2*a^2*b^10
 + 6400*B^2*a^7*c^5 + 36*A*B*a*b^11 - 8640*A^2*a^2*b^8*c^2 + 34560*A^2*a^3*
b^6*c^3 - 69075*A^2*a^4*b^4*c^4 + 54900*A^2*a^5*b^2*c^5 - 960*B^2*a^4*b^6*c
^2 + 3850*B^2*a^5*b^4*c^3 - 7775*B^2*a^6*b^2*c^4 + 1080*A^2*a*b^10*c + 120*
B^2*a^3*b^8*c + 5760*A*B*a^3*b^7*c^2 - 23070*A*B*a^4*b^5*c^3 + 46350*A*B*a^
5*b^3*c^4 - 720*A*B*a^2*b^9*c - 37500*A*B*a^6*b*c^5)))*(60*A*a^3*c^3 - 3*A*
b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b
^2*c^2))/(2*a^4*(4*a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out
```

3.132 $\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=554

$$\frac{\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4a}}}\right.}{8\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -1/8*(20*A*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)*x/c^2/(-4*a*c+b^2)^2+1/8*(12*A
*b*c-28*B*a*c+B*b^2)*x^3/c/(-4*a*c+b^2)^2-1/4*x^7*(A*b-2*a*B-(-2*A*c+B*b)*x
^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x^5*(7*A*b^2-12*a*b*B-4*a*A*c+(12*A*
b*c-28*B*a*c+B*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arctan(x*2^(1/
2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^4*B+A*b^3*c-27*a*b^2*B*c-16*a
*A*b*c^2+84*a^2*B*c^2+(40*A*a^2*c^3+18*A*a*b^2*c^2-A*b^4*c-132*B*a^2*b*c^2+
33*B*a*b^3*c-3*B*b^5)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)^2*2^(1/2)/(b
-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*(3*b^4*B+A*b^3*c-27*a*b^2*B*c-16*a*A*b*c^2+84*a^2*B*c^2+(-40*A*
a^2*c^3-18*A*a*b^2*c^2+A*b^4*c+132*B*a^2*b*c^2-33*B*a*b^3*c+3*B*b^5)/(-4*a*
c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 11.19, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4a}}}\right.}{8\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((3*b^3*B + A*b^2*c - 24*a*b*B*c + 20*a*A*c^2)*x)/(8*c^2*(b^2 - 4*a*c)^2)
+ ((b^2*B + 12*A*b*c - 28*a*B*c)*x^3)/(8*c*(b^2 - 4*a*c)^2) - (x^7*(A*b - 2
*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^5*(
7*A*b^2 - 12*a*b*B - 4*a*A*c + (b^2*B + 12*A*b*c - 28*a*B*c)*x^2))/(8*(b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a
*A*b*c^2 + 84*a^2*B*c^2 - (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^
2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a*A*b*c
^2 + 84*a^2*B*c^2 + (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^2 + 13
2*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b +
Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^8\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x^7\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\int \frac{x^6\left(7(Ab-2aB)+(-bB+2Ac)x^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^7\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^5\left(7Ab^2-12abB-4aAc+\left(b^2B+12Abc-28a\right.\right.}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= \frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^5\left(7Ab^2-12abB-\right.}{8\left(b^2-\right.}$$

$$= -\frac{\left(3b^3B+Ab^2c-24abBc+20aAc^2\right)x}{8c^2\left(b^2-4ac\right)^2}+\frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(Ab-2aB\right.}{4\left(b^2-4ac\right)}$$

$$= -\frac{\left(3b^3B+Ab^2c-24abBc+20aAc^2\right)x}{8c^2\left(b^2-4ac\right)^2}+\frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(Ab-2aB\right.}{4\left(b^2-4ac\right)}$$

$$= -\frac{\left(3b^3B+Ab^2c-24abBc+20aAc^2\right)x}{8c^2\left(b^2-4ac\right)^2}+\frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(Ab-2aB\right.}{4\left(b^2-4ac\right)}$$

Mathematica [A] time = 2.40, size = 644, normalized size = 1.16

$$-\frac{4x\left(a^2c\left(2c\left(A+Bx^2\right)-3bB\right)+ab\left(-bc\left(A+4Bx^2\right)+3Ac^2x^2+b^2B\right)+b^3x^2(bB-Ac)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{2}\sqrt{c}\left(4a^2c^2\left(21B\sqrt{b^2-4ac}+10Ac\right)-4abc^2\left(4A\sqrt{b^2-4ac}+33aB\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(2*b^5*B - b^4*c*(2*A + 5*B*x^2) - 4*a^2*c^3*(9*A + 11*B*x^2) + a*b^2
*c^2*(11*A + 37*B*x^2) + 16*a*b*c^2*(3*a*B - A*c*x^2) + b^3*c*(-17*a*B + A*
c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*x*(b^3*(b*B - A*c)*x^2
+ a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*
B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-3*b^5*
B + b^3*c*(33*a*B + A*Sqrt[b^2 - 4*a*c]) - 4*a*b*c^2*(33*a*B + 4*A*Sqrt[b^2
 - 4*a*c]) + 9*a*b^2*c*(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^4*(-(A*c) + 3*B*
Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(10*A*c + 21*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^5*B + 4*a*b*c^2*(33*a*B - 4*
A*Sqrt[b^2 - 4*a*c]) + b^4*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - 9*a*b^2*c*(2*A*c
 + 3*B*Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(-10*A*c + 21*B*Sqrt[b^2 - 4*a*c]) +
b^3*(-33*a*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(
16*c^3)
```

fricas [B] time = 14.45, size = 9636, normalized size = 17.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b^3)*c^2
)*x^7 + 2*(3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*c^2 - (20*B*a*b
^3 - A*b^4)*c)*x^5 + 2*(6*B*a*b^4 + 28*(B*a^3 + A*a^2*b)*c^2 - (49*B*a^2*b^
2 - 2*A*a*b^3)*c)*x^3 - sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8
 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16
*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 -
 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^
2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(
216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168
*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 20
*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^
5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*
B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*
B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*
A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4
- 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)
/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^
4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 -
640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-(1701*B^4*a^2*b^8
 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^4*a^3*b^2
)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*a^4*b^2 -
 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^5*b^2 - 129837
6*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 35*A^4*a*b
^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^2*a^2*b^6
 - 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7 + 35*A^2*
B^2*a*b^8)*c)*x + 1/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 - 640*(882
*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3*a^6*b +
5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*(93096*B^
3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b^6)*c^5 +
 (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17*A^3*a*b^
8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9 - A^3*b^
10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2 - 27*(31*
B^3*a*b^11 - A*B^2*b^12)*c - (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A*a^6*b)*c
```

```
^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4 - 45*A*a
^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a^3*b^8 -
5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b^12 - A*b
^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^
3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^
2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 444
6*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^
4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*
c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*
a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^
3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*
B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B
*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 20*a*b
^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^
10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^
2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*
a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^
3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 11
6*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^
10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^
2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*
a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + sqrt(1/2)*((b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2
*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a
^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^
2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b
^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c
^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2
*A*B*b^8)*c + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8
+ 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50
*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95
256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 -
6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5
)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33
*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12
- 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*
b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c
^10))*log(-(1701*B^4*a^2*b^8 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*
(6*A^3*B*a^4*b - A^4*a^3*b^2)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*
b + 287712*A^2*B^2*a^4*b^2 - 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (
1555848*B^4*a^5*b^2 - 1298376*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 1127
7*A^3*B*a^2*b^5 + 35*A^4*a*b^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^
3*b^5 + 3066*A^2*B^2*a^2*b^6 - 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 -
 819*A*B^3*a^2*b^7 + 35*A^2*B^2*a*b^8)*c)*x - 1/2*sqrt(1/2)*(27*B^3*b^13 +
32000*A^3*a^5*c^8 - 640*(882*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*
c^7 + 64*(10584*B^3*a^6*b + 5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^
3*a^3*b^4)*c^6 - 8*(93096*B^3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3
*b^5 + 49*A^3*a^2*b^6)*c^5 + (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84
*A^2*B*a^2*b^7 - 17*A^3*a*b^8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^
8 + 195*A^2*B*a*b^9 - A^3*b^10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^10
 + A^2*B*b^11)*c^2 - 27*(31*B^3*a*b^11 - A*B^2*b^12)*c - (3*B*b^14*c^5 - 40
96*(42*B*a^7 - 13*A*a^6*b)*c^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 -
 768*(194*B*a^5*b^4 - 45*A*a^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7
)*c^9 - 240*(42*B*a^3*b^8 - 5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^
11)*c^7 - (90*B*a*b^12 - A*b^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 -
50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 -
95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4
- 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b
^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(
```

```
33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^1
2 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(9*B^2*b^
9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 +
 A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 +
 (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B
*b^8)*c + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 12
80*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(44
1*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*
A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(1
4553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^
3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4
*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 64
0*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*
c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)
)) - sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (
b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16
*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*
(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 3
6*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^
7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^
2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^
4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2
)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A
^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324
*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 +
 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*
b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a
^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1
280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-(1701*B^4*a^2*b^8 - 945*A*B^3*a*b^9
- 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^4*a^3*b^2)*c^5 + 3*(1037232*
B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*a^4*b^2 - 32952*A^3*B*a^3*b^
3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^5*b^2 - 1298376*A*B^3*a^4*b^3 + 2
38464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 35*A^4*a*b^6)*c^3 + 9*(37701*
B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^2*a^2*b^6 - 35*A^3*B*a*b^7)*
c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7 + 35*A^2*B^2*a*b^8)*c)*x + 1
/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 - 640*(882*A*B^2*a^6 - 156*A^
2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3*a^6*b + 5562*A*B^2*a^5*b^2
- 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*(93096*B^3*a^5*b^3 + 3816*A*
B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b^6)*c^5 + (337392*B^3*a^4*b^
5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17*A^3*a*b^8)*c^4 - (81324*B^3
*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9 - A^3*b^10)*c^3 + 9*(1239*B
^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2 - 27*(31*B^3*a*b^11 - A*B^2*
b^12)*c + (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A*a^6*b)*c^12 + 6144*(40*B*a^
6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4 - 45*A*a^4*b^5)*c^10 + 1280
*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a^3*b^8 - 5*A*a^2*b^9)*c^8 +
24*(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b^12 - A*b^13)*c^6)*sqrt((81*
B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b
^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516
*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 3
24*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5
 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*
a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024
*a^5*c^15)))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*
B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*
B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c
^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^
6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^
8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^
```

```
5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B
*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2
*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A
^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*
c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c
^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*
a^4*b^2*c^9 - 1024*a^5*c^10))) + sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2
*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3
*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*
b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B
*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c
^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*
b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10
*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9
- 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 -
 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 1
7496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^
2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4
*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^
3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13
+ 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b
^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-(1701*B^
4*a^2*b^8 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^
4*a^3*b^2)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*
a^4*b^2 - 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^5*b^2
 - 1298376*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 3
5*A^4*a*b^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^
2*a^2*b^6 - 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7
+ 35*A^2*B^2*a*b^8)*c)*x - 1/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 -
 640*(882*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3
*a^6*b + 5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*
(93096*B^3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b
^6)*c^5 + (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17
*A^3*a*b^8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9
 - A^3*b^10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2
- 27*(31*B^3*a*b^11 - A*B^2*b^12)*c + (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A
*a^6*b)*c^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4
 - 45*A*a^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a
^3*b^8 - 5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b
^12 - A*b^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3
 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b +
 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*
b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B
^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*
B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^1
3 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4
 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 -
 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5
- 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10*c^5
 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 10
24*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108
*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496
*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 -
4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2
*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^
7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 12
80*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c
^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + 2*(3*B*a^2*b^3
 + 20*A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 +
```

```
16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8
*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 +
 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)
```

giac [B] time = 8.28, size = 3987, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 12*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^5*c^2 - 2*b^6*c^2 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^3 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 24*a*b^4*c^3 - 2*b^5*c^3 + 320*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 160*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^
4 + 288*a^2*b^2*c^4 + 112*a*b^3*c^4 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^5 - 640*a^3*c^5 - 416*a^2*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 208*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^3 + 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^3*c^3 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^4 + 2*(b^2 - 4*a*c)*b^4*c^2 + 32*(b^2 - 4*a*c)*a*b^2*c^3 + 2*(
b^2 - 4*a*c)*b^3*c^3 - 160*(b^2 - 4*a*c)*a^2*c^4 - 104*(b^2 - 4*a*c)*a*b*c^
4)*A + 3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^6*c - 2*b^7*c + 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 +
 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*a*b^5*c^2 - 2*b^6*c^2 - 128*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 -
 160*a^2*b^3*c^3 + 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^4 + 256*a^3*b*c^4 - 192*a^2*b^2*c^4 + 448*a^3*c^5 + sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 14*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*c^3 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^3 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*c^4 + 2*(b^2 - 4*a*c)*b^5*c - 24*(b^2 - 4*a*c)*a*b^3*c^2 + 2
*(b^2 - 4*a*c)*b^4*c^2 + 64*(b^2 - 4*a*c)*a^2*b*c^3 - 20*(b^2 - 4*a*c)*a*b^
2*c^3 + 112*(b^2 - 4*a*c)*a^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4 + sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)^2
- 4*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^8*c^2 - 16*a*b^6*c^3 -
2*b^7*c^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4 - 256*a^3*b^2*c^5 - 96*
a^2*b^3*c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b*c^6 + 48*a^2*b^2*c^6 -
 64*a^3*c^7)*abs(c)) + 1/32*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c
 + 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 2*b^6*c^2 - 144*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 24*a*b^4*c^3 +
2*b^5*c^3 + 320*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 160*sqrt(
```

```
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^4 - 288*a^2*b^2*c^4 - 112*a*b^3*c^4 - 80*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 640*a^3*c^5 + 416*a^2*b*c^5 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 56*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 208*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 104*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 - 32*(b^2 -
 4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 + 160*(b^2 - 4*a*c)*a^2*c^4 + 1
04*(b^2 - 4*a*c)*a*b*c^4)*A + 3*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
7 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^6*c + 2*b^7*c + 80*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^2 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 32*a*b^5*c^2 + 2*b^6*
c^2 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^3*c^3 + 160*a^2*b^3*c^3 - 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 256*a^3*b*c^4 + 192*a^2*b^2*c^4 - 448*
a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 1
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 96*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 224*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 112*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 56*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^5*c + 24*(b^2
 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*b^4*c^2 - 64*(b^2 - 4*a*c)*a^2*b*c^3
+ 20*(b^2 - 4*a*c)*a*b^2*c^3 - 112*(b^2 - 4*a*c)*a^2*c^4)*B)*arctan(2*sqrt(
1/2)*x/sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 - sqrt((b^5*c^2 - 8*a*b^3
*c^3 + 16*a^2*b*c^4)^2 - 4*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^
3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^
8*c^2 - 16*a*b^6*c^3 - 2*b^7*c^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4
- 256*a^3*b^2*c^5 - 96*a^2*b^3*c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b
*c^6 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*abs(c)) - 1/8*(5*B*b^4*c*x^7 - 37*B*a*b
^2*c^2*x^7 - A*b^3*c^2*x^7 + 44*B*a^2*c^3*x^7 + 16*A*a*b*c^3*x^7 + 3*B*b^5*
x^5 - 20*B*a*b^3*c*x^5 + A*b^4*c*x^5 - 4*B*a^2*b*c^2*x^5 + 5*A*a*b^2*c^2*x^
5 + 36*A*a^2*c^3*x^5 + 6*B*a*b^4*x^3 - 49*B*a^2*b^2*c*x^3 + 2*A*a*b^3*c*x^3
 + 28*B*a^3*c^2*x^3 + 28*A*a^2*b*c^2*x^3 + 3*B*a^2*b^3*x - 24*B*a^3*b*c*x +
 A*a^2*b^2*c*x + 20*A*a^3*c^2*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x
^4 + b*x^2 + a)^2)
```

maple [B] time = 0.07, size = 2015, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-1/8*(16*A*a*b*c^2-A*b^3*c+44*B*a^2*c^2-37*B*a*b^2*c+5*B*b^4)/(16*a^2*c^2-
8*a*b^2*c+b^4)/c*x^7-1/8*(36*A*a^2*c^3+5*A*a*b^2*c^2+A*b^4*c-4*B*a^2*b*c^2-
20*B*a*b^3*c+3*B*b^5)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*a/c^2*(28*A*a*
b*c^2+2*A*b^3*c+28*B*a^2*c^2-49*B*a*b^2*c+6*B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3-1/8*a^2*(20*A*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)/c^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*a*A*b-1/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(
```

```
1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3
-5/2*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
A*a^2-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*A*a*b^2+1/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*A*b^4-21/4/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*
B+27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*B-3/16/c^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*B+33/4/(16*a^2*c^2-8*
a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*a^2*b-33/16/c/(16*a
^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*a*b^3+3/16
/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*
b^5-1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A*b+1/16/c/(16*a^2*c^
2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-5/2*c/(16*a^2*c^2-8*a*b^2*c+b^4)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*a^2-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*a*b^2+1/16/c/(16*a^2*c^2-8*a*b^2*c+
b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^4+21/4/(16*a^2*c^2-8*a*b^2*c
+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)*a^2*B-27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*a*b^2*B+3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^
4*B+33/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B
*a^2*b-33/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
c*x)*B*a*b^3+3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*B*b^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(5\,Bb^4c+4\left(11\,Ba^2+4\,Aab\right)c^3-\left(37\,Bab^2+Ab^3\right)c^2\right)x^7+\left(3\,Bb^5+36\,Aa^2c^3-\left(4\,Ba^2b-5\,Aab^2\right)c^2-(20\,Bab^3\right)}{8\left(\left(b^4c^4-8\,ab^2c^5+16\,a^2c^6\right)x^8+a^2b^4c^2-8\,a^3b^2c^3+16\,a^4c^4+2\,(b^5c^3-8\,ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b^3)*c^2)*x
^7 + (3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*c^2 - (20*B*a*b^3 -
A*b^4)*c)*x^5 + (6*B*a*b^4 + 28*(B*a^3 + A*a^2*b)*c^2 - (49*B*a^2*b^2 - 2*A
*a*b^3)*c)*x^3 + (3*B*a^2*b^3 + 20*A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)*c)*
x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3
+ 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*
a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*
x^2) - 1/8*integrate(-(3*B*a*b^3 + 20*A*a^2*c^2 + (3*B*b^4 + 4*(21*B*a^2 -
```

```
4*A*a*b)*c^2 - (27*B*a*b^2 - A*b^3)*c)*x^2 - (24*B*a^2*b - A*a*b^2)*c)/(c*x
^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
```

mupad [B] time = 5.05, size = 22911, normalized size = 41.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((x^5*(3*B*b^5 + 36*A*a^2*c^3 + A*b^4*c - 20*B*a*b^3*c + 5*A*a*b^2*c^2 -
4*B*a^2*b*c^2))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^7*(5*B*b^4 + 44
*B*a^2*c^2 - A*b^3*c + 16*A*a*b*c^2 - 37*B*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)) + (x^3*(28*B*a^3*c^2 + 6*B*a*b^4 + 2*A*a*b^3*c + 28*A*a^2*b*
c^2 - 49*B*a^2*b^2*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*x*(3*B
*b^3 + 20*A*a*c^2 + A*b^2*c - 24*B*a*b*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - atan(
((((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^
7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^
8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1
474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*
(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c
^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^
2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*
c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^
7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^
2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*
c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8
*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15
*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15
482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377
280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 -
1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c
 - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1
/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 +
b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4
*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^
12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5
120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8
+ 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b
^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a
*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*
b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b
^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^
13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^
6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c
^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a
^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9
 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5
 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*
c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) -
288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^
2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18
*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a
^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4
*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 +
A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A
^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^
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4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 446
4*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7
 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^
19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 11
40*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 4377
6*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921
*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851
776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 +
27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*
a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c
 - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2
)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^
3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*
B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*
a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a
*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048
576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14
*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 19
66080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1
i - (((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B
*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4
*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5
 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(5
12*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^
6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17
*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^
13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7
*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15
*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b
^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*
a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b
^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) -
 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 +
377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7
 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*
a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)
^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15
 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*
a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6
*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5
 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c
^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^
2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(
4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a
^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^
6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3
*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2
*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^
2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
 + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^
2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*
c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*
c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b
^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
 - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b
*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b
^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 25804
8*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*
```

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b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6
 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 20
8*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2
*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 -
4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*
c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2
*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c +
 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 4
3776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6
921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2
851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7
 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B
^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^1
7*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B
*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032
*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B
^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(
4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1
048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b
^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 -
 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2
)*1i)/((((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 629145
6*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*
b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c
^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)
/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3
*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b
^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2
*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*
b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b
^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^
5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B
^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*
a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2
) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4
 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*
c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-
(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^
15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c
^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 537
60*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*
b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c
^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^
5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*
(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^
2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2
*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*
a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*
B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2
*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1
/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320
*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9
*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^
10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^
7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1
/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*
```

a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*
a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 25
8048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a
^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*
c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 +
 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*
B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3
 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a
^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*
B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*
c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6
+ 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9
+ 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4
- 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*
c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 44
1*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*
b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c
 - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*
A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860
032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 9
9*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*
(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512
*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^
3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^1
1 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(
1/2) + (((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 629145
6*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*
b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c
^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)
/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3
*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b
^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2
*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*
b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b
^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^
5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B
^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*
a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2
) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4
 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*
c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-
(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^
15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c
^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 537
60*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*
b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c
^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^
5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*
(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^
2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2
*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*
a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*
B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2
*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1
/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320
*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9
*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^

10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^
7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1
/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*
a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*
a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 25
8048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a
^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*
c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 +
208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*
B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3
- 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a
^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*
B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*
c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6
+ 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9
+ 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4
- 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*
c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 44
1*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*
b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*
A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860
032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 9
9*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*
(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512
*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^
3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^1
1 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(
1/2) + (35*A^3*a^2*b^7*c^2 - 592704*B^3*a^7*c^4 - 567*B^3*a^3*b^8 - 1176*A^
3*a^3*b^5*c^3 + 9456*A^3*a^4*b^3*c^4 - 89532*B^3*a^5*b^4*c^2 + 353808*B^3*a
^6*b^2*c^3 + 315*A*B^2*a^2*b^9 - 33600*A^2*B*a^6*c^5 + 6400*A^3*a^5*b*c^5 +
10935*B^3*a^4*b^6*c - 6552*A*B^2*a^3*b^7*c + 560448*A*B^2*a^6*b*c^4 + 210*
A^2*B*a^2*b^8*c + 61524*A*B^2*a^4*b^5*c^2 - 280800*A*B^2*a^5*b^3*c^3 - 5649
*A^2*B*a^3*b^6*c^2 + 42516*A^2*B*a^4*b^4*c^3 - 126192*A^2*B*a^5*b^2*c^4)/(2
56*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^
6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))))*(-(9*B^2*b^19 + A^2*b^17*c^
2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*
c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^
7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^
2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*
c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8
*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c
- b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15
*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15
482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377
280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 -
1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c
- b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1
/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 +
b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4
*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^
12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*2i - atan(((((256
*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*c^9
- 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 629
1456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*
B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(4096*a
^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 38
40*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B
^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 1

0160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680
960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 775
80*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9
628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^
8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 -
1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*
B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B
*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240
*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 1
08*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^
5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c
^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 29
49120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b
^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 32768
0*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5
- 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^
2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^
5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8
+ 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3
+ 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c
^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4
*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280
*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^
10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580
*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430
784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5
160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B
*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*
a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 +
720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10
*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 -
2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2*b^8
*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3*
b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c
^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a
^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b^8*
c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^
2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*
a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a
^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^
2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2
*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 2709504
0*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2
*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A
^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(
1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*
c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b
^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c
*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^
2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^1
0*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 +
53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a
^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i - (((
256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*
c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 +
6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 14745
60*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(409

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6*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17*c^2 -
9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4
- 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 -
680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 -
77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5
+ 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3
*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b
^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3
 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 154828
80*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*
A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290
240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2)
+ 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20
*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^1
2*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 +
 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*
a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 32
7680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c -
 b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11
*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c
^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c
^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^
7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881
280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b
*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5
580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1
430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8
- 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*
A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-
(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6
 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b
^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^1
3 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2*
b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a
^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^
2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*
B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b
^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 +
 A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A
^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^
2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2
*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*
B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 2709
5040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*
c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 5
5*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15
)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^
12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^
6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^
2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c -
 b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*
a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8
 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 196608
0*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i)/(
(((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7
```

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*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8
 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 14
74560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(
4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^
6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^2
 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c
^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7
 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2
 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c
^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*
b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*
c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 154
82880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 3772
80*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1
290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c
 - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/
2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b
^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*
b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^1
2 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 51
20*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 +
 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^
4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*
c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b
^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^
5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^1
3*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6
*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6
881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^
8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9
 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5
 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c
^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 2
88*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2
*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*
c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^
5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*
c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A
^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^
2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4
*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464
*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7
 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^1
9 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 114
0*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776
*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*
B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 28517
76*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 2
7095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c
 - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)
^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3
*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B
*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a
*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*
c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(10485
76*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*
```

```
c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 196
6080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) +
(((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7
*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8
 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 14
74560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(
4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^
6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17*c^2
 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c
^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7
 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2
 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c
^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*
b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c
- b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*
c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 154
82880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 3772
80*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1
290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c
- b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/
2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b
^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*
b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^1
2 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 51
20*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 +
 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^
4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*
c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b
^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^
5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^1
3*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6
*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6
881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^
8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9
+ 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5
- 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c
^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 2
88*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2
*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*
c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^
5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*
c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A
^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^
2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4
*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464
*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7
+ b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^1
9 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 114
0*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776
*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*
B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 28517
76*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 2
7095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c
- 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)
^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3
*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B
*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a
```

```
*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*
c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(10485
76*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*
c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 196
6080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) +
(35*A^3*a^2*b^7*c^2 - 592704*B^3*a^7*c^4 - 567*B^3*a^3*b^8 - 1176*A^3*a^3*b
^5*c^3 + 9456*A^3*a^4*b^3*c^4 - 89532*B^3*a^5*b^4*c^2 + 353808*B^3*a^6*b^2*
c^3 + 315*A*B^2*a^2*b^9 - 33600*A^2*B*a^6*c^5 + 6400*A^3*a^5*b*c^5 + 10935*
B^3*a^4*b^6*c - 6552*A*B^2*a^3*b^7*c + 560448*A*B^2*a^6*b*c^4 + 210*A^2*B*a
^2*b^8*c + 61524*A*B^2*a^4*b^5*c^2 - 280800*A*B^2*a^5*b^3*c^3 - 5649*A^2*B*
a^3*b^6*c^2 + 42516*A^2*B*a^4*b^4*c^3 - 126192*A^2*B*a^5*b^2*c^4)/(256*(409
6*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B
^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 1
0160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680
960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 775
80*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9
628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^
8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 -
1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*
B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B
*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240
*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 1
08*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^
5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c
^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 29
49120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.133 $\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=461

$$\frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Bc^2+36aAbc\cdots}{\sqrt{\cdots}}\cdots\right.}{\cdots}$$

```
[Out] -1/8*(-12*A*b*c+20*B*a*c+B*b^2)*x/c/(-4*a*c+b^2)^2-1/4*x^5*(A*b-2*a*B-(-2*A
*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x^3*(5*A*b^2-12*a*b*B+4*a*A
*c-(-12*A*b*c+20*B*a*c+B*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^3*B+3*A*b^2*c-16*a*b*
B*c+12*a*A*c^2+(-36*A*a*b*c^2-3*A*b^3*c+40*B*a^2*c^2+18*B*a*b^2*c-B*b^4)/(-
4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^3*B+3*A*b
^2*c-16*a*b*B*c+12*a*A*c^2+(36*A*a*b*c^2+3*A*b^3*c-40*B*a^2*c^2-18*B*a*b^2*
c+B*b^4)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] time = 4.62, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Bc^2+36aAbc\cdots}{\sqrt{\cdots}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b^2*B - 12*A*b*c + 20*a*B*c)*x)/(8*c*(b^2 - 4*a*c)^2) - (x^5*(A*b - 2*a*
B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^3*(5*A
*b^2 - 12*a*b*B + 4*a*A*c - (b^2*B - 12*A*b*c + 20*a*B*c)*x^2))/(8*(b^2 - 4
*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^
2 - (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B + 3*A*
b^2*c - 16*a*b*B*c + 12*a*A*c^2 + (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*
A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b
^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\int \frac{x^4\left(5(Ab-2aB)+(bB-2Ac)x^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^3\left(5Ab^2-12abB+4aAc-\left(b^2B-12Abc+20aBc\right)\right.}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{\left(b^2B-12Abc+20aBc\right)x}{8c\left(b^2-4ac\right)^2} - \frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^3\left(5Ab^2-12abB+4a\right.}{8\left(b^2-4a\right.}$$

$$= -\frac{\left(b^2B-12Abc+20aBc\right)x}{8c\left(b^2-4ac\right)^2} - \frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^3\left(5Ab^2-12abB+4a\right.}{8\left(b^2-4a\right.}$$

$$= -\frac{\left(b^2B-12Abc+20aBc\right)x}{8c\left(b^2-4ac\right)^2} - \frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^3\left(5Ab^2-12abB+4a\right.}{8\left(b^2-4a\right.}$$

Mathematica [A] time = 2.02, size = 543, normalized size = 1.18

$$-\frac{4x\left(2a^2Bc+a\left(bc\left(A+3Bx^2\right)-2Ac^2x^2+b^2(-B)\right)+b^2x^2(Ac-bB)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2x\left(b^2c\left(11aB+3Acx^2\right)+4abc^2\left(A-4Bx^2\right)+12ac^2\left(Acx^2-3aB\right)+b^3c\left(2A+Bx^2\right)-2b^4B\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(-2*b^4*B + 4*a*b*c^2*(A - 4*B*x^2) + b^3*c*(2*A + B*x^2) + 12*a*c^2*
(-3*a*B + A*c*x^2) + b^2*c*(11*a*B + 3*A*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x
^2 + c*x^4)) - (4*x*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A
```

```
*c^2*x^2 + b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sq
rt[2]*Sqrt[c]*(-(b^4*B) + 3*b^2*c*(6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(
10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(-3*A*c + B*Sqrt[b^2 - 4*a*c]) - 4*a*
b*c*(9*A*c + 4*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*Sqrt[c]*(b^4*B + 3*b^2*c*(-6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(-10
*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + 4*a*b*c*(9*A*c - 4*B*Sqrt[b^2 - 4*a*c]) + b
^3*(3*A*c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^2)
```

fricas [B] time = 5.17, size = 7060, normalized size = 15.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - 2*(B*b^4 +
4*(9*B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5 - 2*(2*B*a*b^3 + 4*A
*a^2*c^2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 - sqrt(1/2)*((b^4*c^3 - 8*a*b^2
*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^
2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^
4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 - 240*(4
*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*
b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5
 - 6*A*B*b^6)*c + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*
c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*
A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)
*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2
*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3
 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 10
24*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14
*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b + 9936
*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^
2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B
^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)*c)*x + 1/2*sqrt(1/2)*(B^3*b^
10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^2*a^4
*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284*
A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3*b^4
 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^
6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c -
(B*b^13*c^3 - 24576*A*a^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536
*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 -
160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10)*c^5
- 18*(4*B*a*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2
*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 -
2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^
8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^2*b^7 -
 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3
*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^
2*a*b^5 - 6*A*B*b^6)*c + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a
^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 -
18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B
^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 +
160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b
^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c
^7 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8
+ a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^
2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*
b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)
```

```
*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2
*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c + (b^10
*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7
 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*
c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2
 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*
c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a
^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(-(35*B
^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*
b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*b^2 + 1080
*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3
 + 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 315*A*B^3*
a*b^5 + 45*A^2*B^2*b^6)*c)*x - 1/2*sqrt(1/2)*(B^3*b^10 - 2304*(5*A^2*B*a^4
 - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^2*a^4*b + 198*A^2*B*a^3*b^2
 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2
*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*b^5 +
 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 6
3*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c - (B*b^13*c^3 - 24576*A*a
^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^5*b^3 - 5*A*a^
4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 - 160*(56*B*a^3*b^7 - 15*
A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10)*c^5 - 18*(4*B*a*b^11 - A*b^
12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (
625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B
^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1
280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2
*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*
B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c
 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b
^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^
3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4
*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a
^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4
 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) -
sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2
*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c -
 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)
*x^2)*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*
b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9
*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 1
60*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B
^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 30
0*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10
*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10
 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^
4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7
 - 1296*A^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a
^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*
b^4)*c^3 + 3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 -
135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)
*c)*x + 1/2*sqrt(1/2)*(B^3*b^10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64
*(500*B^3*a^5 - 420*A*B^2*a^4*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 -
 16*(1480*B^3*a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b
^5)*c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A
^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*
B^3*a*b^8 + 6*A*B^2*b^9)*c + (B*b^13*c^3 - 24576*A*a^6*c^10 + 4096*(13*B*a^
6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*
a^4*b^5 - 2*A*a^3*b^6)*c^7 - 160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25
*B*a^2*b^9 - 7*A*a*b^10)*c^5 - 18*(4*B*a*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4
+ 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3
```

```
*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 -
20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024
*a^5*c^11)))*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B
^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*
b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*
c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*
sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a
^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c
)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b
^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640
*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8
*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(
b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c
^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 -
240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*
A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2
*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^
3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 1
8*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^
2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 1
60*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^
10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^
7 - 1024*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 6
48*(14*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b
+ 9936*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*
a^3*b^2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*
(497*B^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)*c)*x - 1/2*sqrt(1/2)*(
B^3*b^10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B
^2*a^4*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 -
 1284*A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a
^3*b^4 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*
a^2*b^6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9
)*c + (B*b^13*c^3 - 24576*A*a^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9
- 1536*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*
c^7 - 160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10
)*c^5 - 18*(4*B*a*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A
^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*
c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*
b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^2
*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b
^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 -
(35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 -
 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*
c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54
*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*
c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11
)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4
*b^2*c^7 - 1024*a^5*c^8))) - 2*(B*a^2*b^2 + 4*(5*B*a^3 - 3*A*a^2*b)*c)*x)/(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a
^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^
2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)
```

giac [B] time = 11.66, size = 7578, normalized size = 16.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*(3*(2*b^4*c^3 - 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
```

(b^2 - 4*a*c)*c)*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^4)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2*A + (2*b^5*c^2 - 40*a*b^3*c^3 + 128*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a*b*c^3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2*B - 24*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 2*a*b^7*c^4 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 + 24*a^2*b^5*c^5 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 - 96*a^3*b^3*c^6 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^7 + 128*a^4*b*c^7 + 2*(b^2 - 4*a*c)*a*b^5*c^4 - 16*(b^2 - 4*a*c)*a^2*b^3*c^5 + 32*(b^2 - 4*a*c)*a^3*b*c^6)*A*abs(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 2*a*b^8*c^3 - 192*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 16*a^2*b^6*c^4 + 896*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + 288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + 384*a^3*b^4*c^5 - 1280*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^6 - 640*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 1792*a^4*b^2*c^6 + 320*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^7 + 2560*a^5*c^7 + 2*(b^2 - 4*a*c)*a*b^6*c^3 + 24*(b^2 - 4*a*c)*a^2*b^4*c^4 - 288*(b^2 - 4*a*c)*a^3*b^2*c^5 + 640*(b^2 - 4*a*c)*a^4*c^6)*B*abs(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3) - 3*(2*b^12*c^5 - 8*a*b^10*c^6 - 192*a^2*b^8*c^7 + 1792*a^3*b^6*c^8 - 5632*a^4*b^4*c^9 + 6144*a^5*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^12*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^10*c^5 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 2816*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 512*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 + 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 - 2*(b^2 - 4*a*c)*b^10*c^5 + 192*(b^2 - 4*a*c)*a^2*b^6*c^7 - 1024*(b^2 - 4*a*c)*a^3*b^4*c^8 + 1536*(b^2 - 4*a*c)*a^4*b^2*c^9)*A - (2*b^13*c^4 - 68*a*b^11*c^5 + 688*a^2*b^9*c^6 - 2688*a^3*b^7*c^7 + 2048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9 - 20480*a^6*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^13*c^2 + 34*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^11*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^12*c^3 -

344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 -
60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 1344*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 448*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 30*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9*c^5 - 1024*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 896
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 22
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 - 5
632*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 -
1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7
+ 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^
7 + 10240*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c
^8 + 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2
*c^8 + 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^
3*c^8 - 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*
b*c^9 - 2*(b^2 - 4*a*c)*b^11*c^4 + 60*(b^2 - 4*a*c)*a*b^9*c^5 - 448*(b^2 -
4*a*c)*a^2*b^7*c^6 + 896*(b^2 - 4*a*c)*a^3*b^5*c^7 + 1536*(b^2 - 4*a*c)*a^4
*b^3*c^8 - 5120*(b^2 - 4*a*c)*a^5*b*c^9)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + sqrt((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)^
2 - 4*(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)))/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/((a*b^10*c^3 - 20*a^2*b^8*c
^4 - 2*a*b^9*c^4 + 160*a^3*b^6*c^5 + 32*a^2*b^7*c^5 + a*b^8*c^5 - 640*a^4*b
^4*c^6 - 192*a^3*b^5*c^6 - 16*a^2*b^6*c^6 + 1280*a^5*b^2*c^7 + 512*a^4*b^3*
c^7 + 96*a^3*b^4*c^7 - 1024*a^6*c^8 - 512*a^5*b*c^8 - 256*a^4*b^2*c^8 + 256
*a^5*c^9)*abs(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*abs(c)) - 1/64*(3*(2*b^4*c^
3 - 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2
+ 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*
c^3 - 8*(b^2 - 4*a*c)*a*c^4)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2*A + (2*b^
5*c^2 - 40*a*b^3*c^3 + 128*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*
c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3
- 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a*b*c^3)*(b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)^2*B + 24*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 -
12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 + 2*a*b^7*c^4 + 48*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 - 24*a^2*b
^5*c^5 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 32*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^6 + 96*a^3*b^3*c^6 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b*c^7 - 128*a^4*b*c^7 - 2*(b^2 - 4*a*c)*a*b^5*c^4 + 16*(b^2
- 4*a*c)*a^2*b^3*c^5 - 32*(b^2 - 4*a*c)*a^3*b*c^6)*A*abs(b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 + 8
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 + 2*a*b^8*c^3 - 192*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^5*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 + 16*a^2*b^6
*c^4 + 896*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + 288*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 12*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^4*c^5 - 384*a^3*b^4*c^5 - 1280*sqrt(2)*sqrt(b*c - sqr

```
t(b^2 - 4*a*c)*c)*a^5*c^6 - 640*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4
*b*c^6 - 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 1792*a^4
*b^2*c^6 + 320*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 2560*a^5*c
^7 - 2*(b^2 - 4*a*c)*a*b^6*c^3 - 24*(b^2 - 4*a*c)*a^2*b^4*c^4 + 288*(b^2 -
4*a*c)*a^3*b^2*c^5 - 640*(b^2 - 4*a*c)*a^4*c^6)*B*abs(b^4*c - 8*a*b^2*c^2 +
 16*a^2*c^3) - 3*(2*b^12*c^5 - 8*a*b^10*c^6 - 192*a^2*b^8*c^7 + 1792*a^3*b^
6*c^8 - 5632*a^4*b^4*c^9 + 6144*a^5*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^12*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^10*c^5 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 2816*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 512*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 + 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 - 2*(b^2 - 4*a*c)*b^10*c^5 + 192*(b^
2 - 4*a*c)*a^2*b^6*c^7 - 1024*(b^2 - 4*a*c)*a^3*b^4*c^8 + 1536*(b^2 - 4*a*c
)*a^4*b^2*c^9)*A - (2*b^13*c^4 - 68*a*b^11*c^5 + 688*a^2*b^9*c^6 - 2688*a^3
*b^7*c^7 + 2048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9 - 20480*a^6*b*c^10 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^13*c^2 + 34*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^11*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^12*c^3 - 344*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 - 60*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 1344*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 448*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 30*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9*c^5 - 1024*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 896*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 224*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 - 5632*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 - 1536*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 448*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 10240*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 + 5120*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 + 768*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 2560*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 - 2*(b^2 -
4*a*c)*b^11*c^4 + 60*(b^2 - 4*a*c)*a*b^9*c^5 - 448*(b^2 - 4*a*c)*a^2*b^7*c^
6 + 896*(b^2 - 4*a*c)*a^3*b^5*c^7 + 1536*(b^2 - 4*a*c)*a^4*b^3*c^8 - 5120*(
b^2 - 4*a*c)*a^5*b*c^9)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5*c - 8*a*b^3*c^2 +
 16*a^2*b*c^3 - sqrt((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)^2 - 4*(a*b^4*c -
8*a^2*b^2*c^2 + 16*a^3*c^3)*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/(b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)))/((a*b^10*c^3 - 20*a^2*b^8*c^4 - 2*a*b^9*c^4
+ 160*a^3*b^6*c^5 + 32*a^2*b^7*c^5 + a*b^8*c^5 - 640*a^4*b^4*c^6 - 192*a^3*
b^5*c^6 - 16*a^2*b^6*c^6 + 1280*a^5*b^2*c^7 + 512*a^4*b^3*c^7 + 96*a^3*b^4*
c^7 - 1024*a^6*c^8 - 512*a^5*b*c^8 - 256*a^4*b^2*c^8 + 256*a^5*c^9)*abs(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*abs(c)) + 1/8*(B*b^3*c*x^7 - 16*B*a*b*c^2*x^
7 + 3*A*b^2*c^2*x^7 + 12*A*a*c^3*x^7 - B*b^4*x^5 - 5*B*a*b^2*c*x^5 + 5*A*b^
3*c*x^5 - 36*B*a^2*c^2*x^5 + 16*A*a*b*c^2*x^5 - 2*B*a*b^3*x^3 - 28*B*a^2*b*
c*x^3 + 19*A*a*b^2*c*x^3 - 4*A*a^2*c^2*x^3 - B*a^2*b^2*x - 20*B*a^3*c*x + 1
2*A*a^2*b*c*x)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.05, size = 1631, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (1/8*(12*A*a*c^2+3*A*b^2*c-16*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7
+1/8*(16*A*a*b*c^2+5*A*b^3*c-36*B*a^2*c^2-5*B*a*b^2*c-B*b^4)/c/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^5-1/8*a/c*(4*A*a*c^2-19*A*b^2*c+28*B*a*b*c+2*B*b^3)/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^3+1/8*a^2*(12*A*b*c-20*B*a*c-B*b^2)/c/(16*a^2*c^2-8*
a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2-3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*a*A-3/16/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b
^2+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*a*A*b+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*A*b^3+1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*B-1/16/(
16*a^2*c^2-8*a*b^2*c+b^4)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B-5/2/(16*a^2*c^2-8*a
*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*B-9/8/(16*a^2*c^
2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*B+1/16/(16*
a^2*c^2-8*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*B+3/4
/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A+3/16/(16*a^2*c^2-8*a*b
^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A*b+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*a*b*B+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/c*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B
-5/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2
*B-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b
^2*B+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*b^4*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - (B*b^4 + 4*(9*
B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5 - (2*B*a*b^3 + 4*A*a^2*c^
2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 - (B*a^2*b^2 + 4*(5*B*a^3 - 3*A*a^2*b)
*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^
```

```
2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*
a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^
2) + 1/8*integrate((B*a*b^2 + (B*b^3 + 12*A*a*c^2 - (16*B*a*b - 3*A*b^2)*c)
*x^2 + 4*(5*B*a^2 - 3*A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)
```

mupad [B] time = 3.95, size = 19041, normalized size = 41.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*
B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b
^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5
 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c
^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5
- 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040
*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A
^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2
*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*
a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*
c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*
a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^
10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*
b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 18
0*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*
a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8
 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 26214
40*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8
+ 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c
+ 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b
^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a
*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*
b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^
3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*
c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*
c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)
^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8
+ 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 +
992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 +
 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^
10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 +
 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^
7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(B^2
*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 57
6*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^
6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*
a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*
a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^
11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*
c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c
^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6
 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*
a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 -
 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 24
1920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 7
```

```
37280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^1
4*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^
5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a
^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*
c^12)))^(1/2)*1i - (((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6
*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 19
66080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*
B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c
 + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840
*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2
*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*
b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7
*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^
2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5
- 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 5
5*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3
- 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 240
00*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 178
1760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^1
5)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^
18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048
*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4
*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 2621
44*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7)
)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^
4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B
^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 3
7440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 5529
60*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880
*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680
*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a
*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B
^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a
^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*
a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(51
2*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a
^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9
 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1
/2) + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A
*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3
- 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c
^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 50
40*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216
*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B
^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^
2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^1
5*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^
2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*
b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^
6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) -
180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 72
0*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c
^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 262
1440*a^9*b^2*c^12)))^(1/2)*1i)/((((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 -
3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3
*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*
c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)
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/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b
^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^17 + 9*A^2*b^1
5*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(
1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 10368
0*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^
2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2
*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*
B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^
2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b
^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*
b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4
*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*
c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12
*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 29
49120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b
^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680
*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 2
56*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2
*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b
^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^1
1*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*
c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B
^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^
9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 -
 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7
 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*
b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16
*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 86016
0*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b
^2*c^12)))^(1/2) - (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*
a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2
*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 6
6*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c
^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*
B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b
^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*
c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^
5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 -
 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^
3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 2
4000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1
781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)
^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*
b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 2580
48*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b
^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (((5242880*B*a^7*c^8 + 3072*A*a*b
^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 49
1520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*
B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^
6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 -
1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 +
 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c -
b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c
^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8
 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 +
 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 +
 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2
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) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240
*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 99225
6*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*
B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^
13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 5376
0*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6
*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3
- 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^
6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 50
40*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216
*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B
^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^
2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^1
5*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^
2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*
b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^
6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) -
180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 72
0*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c
^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 262
1440*a^9*b^2*c^12)))^(1/2) + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2
+ 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2
 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b
^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 -
16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15
*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1
/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680
*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2
*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*
a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B
*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2
*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^
12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b
^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*
a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c
^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*
c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 294
9120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (1728*A^3*a^4*c^5 - 35*
B^3*a^2*b^7 + 1620*A^3*a^2*b^4*c^3 + 4752*A^3*a^3*b^2*c^4 - 9456*B^3*a^4*b^
3*c^2 + 15*A*B^2*a*b^8 + 4800*A*B^2*a^5*c^4 + 135*A^3*a*b^6*c^2 + 1176*B^3*
a^3*b^5*c - 6400*B^3*a^5*b*c^3 - 705*A*B^2*a^2*b^6*c - 15552*A^2*B*a^4*b*c^
4 + 6084*A*B^2*a^3*b^4*c^2 + 26256*A*B^2*a^4*b^2*c^3 - 1260*A^2*B*a^2*b^5*c
^2 - 13248*A^2*B*a^3*b^3*c^3 + 90*A^2*B*a*b^7*c)/(256*(b^12*c + 4096*a^6*c^
7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 -
 6144*a^5*b^2*c^6))))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b
^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2
*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a
^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3
*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*
b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c -
25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*
b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c
^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*
c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*
B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*
b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 8
60160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a
```

^9*b^2*c^12)))^(1/2)*2i - ((x^5*(B*b^4 + 36*B*a^2*c^2 - 5*A*b^3*c - 16*A*a*
b*c^2 + 5*B*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^7*(B*b^3 +
12*A*a*c^2 + 3*A*b^2*c - 16*B*a*b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) +
(x^3*(4*A*a^2*c^2 + 2*B*a*b^3 - 19*A*a*b^2*c + 28*B*a^2*b*c))/(8*c*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)) + (a^2*x*(B*b^2 - 12*A*b*c + 20*B*a*c))/(8*c*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 +
2*b*c*x^6) + atan(((((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^
6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1
966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360
*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*
c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 384
0*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^
2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B
*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^
7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c
^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5
- 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 -
55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3
- 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24
000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 17
81760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^
15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b
^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 25804
8*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^
4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262
144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7
))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c
^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) -
B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 +
37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552
960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 3488
0*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 186368
0*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*
a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*
B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*
a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B
*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(5
12*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*
a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^
9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(
1/2) - (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*
A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3
- 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*
c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*
b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5
040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 921
6*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*
B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B
^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^
15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A
^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3
*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a
^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) -
180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 7
20*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*
c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 26
21440*a^9*b^2*c^12)))^(1/2)*1i - (((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 -
3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^

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3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8
*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7
)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*
b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 + 9*A^2*b^
15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^
(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 1036
80*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B
^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^
2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A
*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A
^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*
b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5
*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(
4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20
*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^1
2*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2
949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*
b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 32768
0*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 -
256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^
2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*
b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^
11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5
*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*
B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c
^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4
- 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7
 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a
*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^1
6*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 8601
60*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*
b^2*c^12)))^(1/2) + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2
*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^
2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 -
66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*
c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*
A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A
*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*
b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13
*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c
^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9
- 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c
^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 +
24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 -
1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2
)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a
*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258
048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*
b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*1i)/((((5242880*B*a^7*c^8 + 3072*A
*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4
+ 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61
440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*
B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^
3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^
17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*
c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b
^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3
*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c
```

```
^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c
^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^
(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 +
 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 9
92256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 -
6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^1
0*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 +
53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7
*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*
c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^
5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a
^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c
- 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 -
9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 101
60*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 68096
0*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a
*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 73728
0*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*
a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*
B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2
) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4
+ 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^
10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 -
 2621440*a^9*b^2*c^12)))^(1/2) - (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*
c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4
*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a
^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^
5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*
b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15
)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 10
3680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140
*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*
B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040
*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180
*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^
2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a
^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(
-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^
20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b
^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 +
 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (((5242880*B*a^7*c^
8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^
2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^
3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6
- 6291456*B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*
a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x
*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b
^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440
*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A
^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2
*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2
*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c -
 b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a
^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b
^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*
b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1
048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b
^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1
```

```
966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*
(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163
840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*
c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*
A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A
*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*
b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13
*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c
^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9
- 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c
^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 +
24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 -
1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2
)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a
*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258
048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*
b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (x*(B^2*b^8 - 288*A^2*a^3*c^5 +
9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B
^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 -
 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c +
256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^1
7 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c
 - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^
9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*
c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^
4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^
7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(
1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 +
240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 99
2256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6
*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10
*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 5
3760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*
b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (1728*A^3
*a^4*c^5 - 35*B^3*a^2*b^7 + 1620*A^3*a^2*b^4*c^3 + 4752*A^3*a^3*b^2*c^4 - 9
456*B^3*a^4*b^3*c^2 + 15*A*B^2*a*b^8 + 4800*A*B^2*a^5*c^4 + 135*A^3*a*b^6*c
^2 + 1176*B^3*a^3*b^5*c - 6400*B^3*a^5*b*c^3 - 705*A*B^2*a^2*b^6*c - 15552*
A^2*B*a^4*b*c^4 + 6084*A*B^2*a^3*b^4*c^2 + 26256*A*B^2*a^4*b^2*c^3 - 1260*A
^2*B*a^2*b^5*c^2 - 13248*A^2*B*a^3*b^3*c^3 + 90*A^2*B*a*b^7*c)/(256*(b^12*c
 + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840
*a^4*b^4*c^5 - 6144*a^5*b^2*c^6))))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^1
6*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^
6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 -
 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 6
80960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B
^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 7
37280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*
A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 178176
0*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^
(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*
c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^
5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^
11 - 2621440*a^9*b^2*c^12)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.134 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=380

$$\frac{3x\left(x^2\left(4aBc-4Abc+b^2B\right)-A\left(4ac+b^2\right)+4abB\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\left(-\frac{-8aAc^2+12abBc-6A\cdots}{\sqrt{b^2-4ac}}\cdots\right.}{8\sqrt{\cdots}}$$

```
[Out] -1/4*x^3*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/8*x*
(4*a*b*B-A*(4*a*c+b^2)+(-4*A*b*c+4*B*a*c+B*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+
b*x^2+a)+3/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*B
-4*A*b*c+4*a*B*c+(8*A*a*c^2+6*A*b^2*c-12*B*a*b*c-B*b^3)/(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^2*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16*arctan(x*
2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*B-4*A*b*c+4*a*B*c+(-8*A*
a*c^2-6*A*b^2*c+12*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^2*2^(1/2
)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.41, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1275, 1166, 205}

$$\frac{3\left(-\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4\cdots\right.}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^3*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (3*x*(4*a*b*B - A*(b^2 + 4*a*c) + (b^2*B - 4*A*b*c + 4*a*B*c)*x^2))
/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^2*B - 4*A*b*c + 4*a*B*c -
(b^3*B - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a
*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b^2*B - 4*A*b*c + 4*a*B*c + (b^3*B
 - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^2*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
```

```
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\int \frac{x^2\left(3(Ab-2aB)+3(bB-2Ac)x^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x\left(4abB-A\left(b^2+4ac\right)+\left(b^2B-4Abc+4aBc\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x\left(4abB-A\left(b^2+4ac\right)+\left(b^2B-4Abc+4aBc\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x\left(4abB-A\left(b^2+4ac\right)+\left(b^2B-4Abc+4aBc\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 1.70, size = 447, normalized size = 1.18

$$\frac{\frac{8acx\left(A+Bx^2\right)-4abBx+4bx^3(Ac-bB)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2x\left(4bc\left(aB-3Acx^2\right)+4ac^2\left(A+3Bx^2\right)+b^2\left(3Bcx^2-7Ac\right)+2b^3B\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(b^2\left(B\sqrt{b^2-4ac}+6Ac\right)-4bc\left(A\sqrt{b^2-4\right.\right.}{\left(b^2\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*a*b*B*x + 4*b*(-(b*B) + A*c)*x^3 + 8*a*c*x*(A + B*x^2))/((b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)^2) + (2*x*(2*b^3*B + 4*a*c^2*(A + 3*B*x^2) + 4*b*c*(a*
B - 3*A*c*x^2) + b^2*(-7*A*c + 3*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*Sqrt[2]*Sqrt[c]*(-(b^3*B) - 4*b*c*(3*a*B + A*Sqrt[b^2 - 4*a*c])
 + 4*a*c*(2*A*c + B*Sqrt[b^2 - 4*a*c]) + b^2*(6*A*c + B*Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^3*B + 4*b*c*(3*a*B
 - A*Sqrt[b^2 - 4*a*c]) + b^2*(-6*A*c + B*Sqrt[b^2 - 4*a*c]) + 4*a*c*(-2*A*
c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c)
```

fricas [B] time = 4.43, size = 5650, normalized size = 14.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + 2*(5*B*b^3 + 4*A*a*c^2 + (16*B*
a*b - 19*A*b^2)*c)*x^5 + 2*(19*B*a*b^2 - 5*A*b^3 - 4*(B*a^2 + 4*A*a*b)*c)*x
```

^3 - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 -
6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*s
qrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*
B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c +
(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b
^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c
^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6
- 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^
4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-27*(5*B^4*a^2*b^4 - A*B^3*a*
b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*a*b^2)*c^3 + (16*B^4*a^4 - 8
0*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2 - 40*A*B^
3*a^2*b^3 + A^3*B*b^5)*c)*x + 27/2*sqrt(1/2)*(4*B^3*a^2*b^7 - A*B^2*a*b^8 -
 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a^4*b + A^3*a^3*b^2)*c^4 - 64
*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3)*c^3 + 8*(24*B^3*a^4*b^3
+ 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^3*b^5 - 8*A*B^2*a^2*b^6 + 4*
A^2*B*a*b^7 - A^3*b^8)*c - (4096*(2*B*a^8 - 3*A*a^7*b)*c^7 - 2048*(2*B*a^7*
b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(2*B*a
^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3 + 8*(14*B*a^3*b
^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^13)*c)*sqrt((B^4*a^2 - 2*A^2*
B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a
^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))*sqrt(-(B^2*a*b^5 - 16*(4*A*
B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2
 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^10*c - 20*a^2*b^8*c^2
 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqr
t((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*
a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*
c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 -
 1024*a^6*c^6))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 +
2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4
*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3
*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^
2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 +
 A^2*b^5)*c + (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^
4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 128
0*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^
3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-27*(5*B^4*a^2*
b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*a*b^2)*c^3 + (
16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3
*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x - 27/2*sqrt(1/2)*(4*B^3*a^2*b^7 -
 A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a^4*b + A^3*a^3
*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3)*c^3 + 8*(2
4*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^3*b^5 - 8*A*B^
2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c - (4096*(2*B*a^8 - 3*A*a^7*b)*c^7 -
2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*A*a^5*b^5)*c^5
 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3 +
 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^13)*c)*sqrt((B^
4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b
^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))*sqrt(-(B^2*a*
b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 +
A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^10*c -
20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 102
4*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*
b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c
^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280
*a^5*b^2*c^5 - 1024*a^6*c^6))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*
b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*

```
b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*
c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 -
20*A*B*a*b^4 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 -
640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^
2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5
*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 +
160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-
27*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*
a*b^2)*c^3 + (16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2
 + (40*B^4*a^3*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x + 27/2*sqrt(1/2)*(4
*B^3*a^2*b^7 - A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a
^4*b + A^3*a^3*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b
^3)*c^3 + 8*(24*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^
3*b^5 - 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c + (4096*(2*B*a^8 - 3*A
*a^7*b)*c^7 - 2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*
A*a^5*b^5)*c^5 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*
a^3*b^9)*c^3 + 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^1
3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c
^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))
*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*
A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c
 - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5
*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10
*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^
6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*
b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^
2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 -
 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*
B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*
a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*
a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6
*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*
a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a
^6*c^6))*log(-27*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*
B*a^2*b - A^4*a*b^2)*c^3 + (16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 -
5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x - 27/
2*sqrt(1/2)*(4*B^3*a^2*b^7 - A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a
^5 + 2*A^2*B*a^4*b + A^3*a^3*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 +
 3*A^2*B*a^3*b^3)*c^3 + 8*(24*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^
2 - (48*B^3*a^3*b^5 - 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c + (4096*
(2*B*a^8 - 3*A*a^7*b)*c^7 - 2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*
B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*
B*a^4*b^8 + A*a^3*b^9)*c^3 + 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*
b^12 - A*a*b^13)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2
 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1
024*a^7*c^7)))*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*
B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4
 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*
c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*
c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1
280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*
c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))) + 6*(4*B*a^2*b -
 A*a*b^2 - 4*A*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
 (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2
)*x^2)
```

giac [B] time = 8.26, size = 3162, normalized size = 8.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
*c - 2*b^6*c - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*a*b^4*c^2 + 2*b^5*c^2 + 64
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^2*b^2*c^3 + 16*a*b^3*c^3 - 16*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 128*a^3*c^4 - 96*a^2*b*c^4 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 48*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^4*c - 2*(b^2 - 4*a*c)*b^3
*c^2 - 32*(b^2 - 4*a*c)*a^2*c^3 - 24*(b^2 - 4*a*c)*a*b*c^3)*A - 2*(2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 4
*a*b^5*c + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 16*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^3*c^2 + 32*a^2*b^3*c^2 + 6*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 64*a^3*b*c^3 - 16*a^2*b^2*c^3 - 32*a^3
*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 4*(b^2 - 4*a*c)*a*b^3*c
 - 16*(b^2 - 4*a*c)*a^2*b*c^2 - 6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)
*a^2*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + sq
rt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))
/((a*b^8 - 16*a^2*b^6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b
^6*c^2 - 256*a^4*b^2*c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 +
128*a^4*b*c^4 + 48*a^3*b^2*c^4 - 64*a^4*c^5)*abs(c)) + 3/32*((sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c - 16*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 - 8*a*b^4*c^2 - 2*b^5*c^2 + 64*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^3 - 32*a^2*b^2*c^3 - 16*a*b^3*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*c^4 + 128*a^3*c^4 + 96*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^4*c - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^3*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c + 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a
*c)*a^2*c^3 + 24*(b^2 - 4*a*c)*a*b*c^3)*A - 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^5 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
- 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*a*b^5*c + 32*sqrt(2
```

```
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^2 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^2 - 32*a^2*b^3*c^2 - 6*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + 64*a^3*b*c^3 + 16*a^2*b^2*c^3 + 32*a^3*c^4 + 3*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 6*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 4*(b^2 - 4*a*c)*a*b^3*c + 16*(b^2 - 4*a*c)*
a^2*b*c^2 + 6*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(
2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - sqrt((b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((a*b^8 - 16*a^2*b^
6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b^6*c^2 - 256*a^4*b^2
*c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 + 128*a^4*b*c^4 + 48*a
^3*b^2*c^4 - 64*a^4*c^5)*abs(c)) + 1/8*(3*B*b^2*c*x^7 + 12*B*a*c^2*x^7 - 12
*A*b*c^2*x^7 + 5*B*b^3*x^5 + 16*B*a*b*c*x^5 - 19*A*b^2*c*x^5 + 4*A*a*c^2*x^
5 + 19*B*a*b^2*x^3 - 5*A*b^3*x^3 - 4*B*a^2*c*x^3 - 16*A*a*b*c*x^3 + 12*B*a^
2*b*x - 3*A*a*b^2*x - 12*A*a^2*c*x)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c
 + 16*a^2*c^2))
```

maple [B] time = 0.05, size = 1283, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-3/8*c*(4*A*b*c-4*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*(4*A*a*c
^2-19*A*b^2*c+16*B*a*b*c+5*B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(16*A*
a*b*c+5*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-3/8*a*(4
*A*a*c+A*b^2-4*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/4/(
16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-3/2/(16*a^2*c^2-8*a*b
^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A-9/8/(16*a^2*c^2-
8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2-3/4/(16*a^2
*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B-3/16/(16*a^2*c^2-8*a*b^2*c+
b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*B+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B-3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*A*b-3/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*a*A-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*A*b^2+3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a*B+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B+9/4/(16*a^
2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*B+3/16/(16
```

```
*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + (5*B*b^3 + 4*A*a*c^2 + (16*B*a*b
 - 19*A*b^2)*c)*x^5 + (19*B*a*b^2 - 5*A*b^3 - 4*(B*a^2 + 4*A*a*b)*c)*x^3 +
3*(4*B*a^2*b - A*a*b^2 - 4*A*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2)*x^2) - 3/8*integrate((4*B*a*b - A*b^2 - 4*A*a*c - (B*b^2 +
4*(B*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4 - 8*a*b^2*c + 16*a^2*c^2
)
```

mupad [B] time = 3.49, size = 16688, normalized size = 43.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B
*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*
c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 -
655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 +
240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 -
24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2
*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2
*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6
*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*
c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20
*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b
*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 +
 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 2
0*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c
^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*
a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*
c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7
+ 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*
b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^
7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2
 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61
440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7
*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 6
4*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360
*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*
a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*
a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c
^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) - (x*(
9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2
*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720
*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^
2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2
```

```
)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^
11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^
6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 1
1520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536
*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c
 - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*
A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*
B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2
 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b
^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 -
2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*1i - (((3*(1048576*A*a^6*c^8 - 25
6*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7
 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 2048
0*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a
^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3
 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(B^2*a*b^1
5 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15
)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c
^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 +
4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440
*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*
c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A
*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*
B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^1
1*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5
*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8
 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11
*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b
^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256
*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(
1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3
 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 614
40*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^
2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^
8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920
*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4
*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b
^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*
a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6
 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440
*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 23
4*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 14
4*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4
))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))
)*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(
-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11
520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^
2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^
6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 -
 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^
2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^
6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/
(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^1
4*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 196
6080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c))
)^(1/2)*1i)/((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 +
 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a
^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7
*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6
```

```
*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2
*c^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2
) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4
160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*
A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a
^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c
^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^
2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^
8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*
c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3
*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 +
860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^
10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5
*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*
(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*
(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c
 - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2
*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b
^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c
^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*
A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*
c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 -
 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1
048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 +
 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^
8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)
 - (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 +
576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^
2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*
c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2
*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5
*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*
c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6
+ 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*
b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 -
 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 8
1920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b
^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 25804
8*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4
*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (((3*(1048576*A*a^6*c^8
 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*
c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 -
20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720
*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6
*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(B^2*a
*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2
)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b
^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^
2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 6
1440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^
7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 -
64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 14336
0*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576
*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760
*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*
c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*
b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a
^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 -
```

256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^1
5)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*
c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 +
61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 1152
0*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*
B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 8
1920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B
*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a
^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 +
720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10
*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 262
1440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5
+ 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2
+ 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b
c^4))/(32(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6
c)))(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2
c(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4
- 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 56
0*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^
2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c
^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*
B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^
5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*
c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4
*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 -
1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20
c)))^(1/2) + (3(576*B^3*a^4*c^4 - 180*A^3*b^5*c^3 + 540*B^3*a^2*b^4*c^2 +
1584*B^3*a^3*b^2*c^3 - 9*A*B^2*b^7*c + 45*B^3*a*b^6*c + 576*A^2*B*a^3*c^5
+ 81*A^2*B*b^6*c^2 - 1440*A^3*a*b^3*c^4 - 576*A^3*a^2*b*c^5 - 576*A*B^2*a*b
^5*c^2 - 3456*A*B^2*a^3*b*c^4 + 1980*A^2*B*a*b^4*c^3 - 3600*A*B^2*a^2*b^3*c
^3 + 4464*A^2*B*a^2*b^2*c^4))/(256*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))))*(
-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4
*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520
*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a
^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b
^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81
920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b
^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c
^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(51
2*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c
^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 196608
0*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(
1/2)*2i - ((x^3*(5*A*b^3 - 19*B*a*b^2 + 4*B*a^2*c + 16*A*a*b*c))/(8*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) - (x^5*(5*B*b^3 + 4*A*a*c^2 - 19*A*b^2*c + 16*B*a*
b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*x*(A*b^2 + 4*A*a*c - 4*B*a*
b))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*c*x^7*(B*b^2 - 4*A*b*c + 4*B*a*
c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8
+ 2*a*b*x^2 + 2*b*c*x^6) + atan(((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 +
4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*
b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^
3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(
512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^
4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 - B^2*a*(-(4
*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*
A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*
a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b
^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c
^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a

^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^
3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6
+ 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^
2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 25
8048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*
b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*
b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 32768
0*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^1
5*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3
*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3
*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4
- 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2
*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7
+ 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 665
60*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*
B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 -
7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*
b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10
+ a*b^20*c)))^(1/2) - (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3
- 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^
4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*
b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^
7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2
+ 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61
440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7
*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 6
4*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360
*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*
a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*
a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c
^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*1i - (
((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11
*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1
048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*
B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2
*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^
10*c)) + (x*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c
+ A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^
9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^
7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1
024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*
b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 +
240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*
A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a
*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 76
80*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8
*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 +
a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960
*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4
*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 -
B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1
/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 -
1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160
*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2
*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8
+ 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a

^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*1i)/((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) - (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*

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c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 -
 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1
048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 +
 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^
8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)
 + (((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*
b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6
 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655
360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240
*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*
a*b^10*c)) + (x*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^
15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^
3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^
3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4
 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^
2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^
7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66
560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A
*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3
 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7
*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^1
0 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 4
0960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256
*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^1
5 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15
)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c
^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 +
4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440
*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*
c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A
*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*
B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^1
1*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5
*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8
 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (x*(9*B
^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^
2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*
B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 -
 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^1
5)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*
c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 +
 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 1152
0*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*
B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 8
1920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B
*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a
^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 +
720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10
*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 262
1440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (3*(576*B^3*a^4*c^4 - 180*A^3*b^5*
c^3 + 540*B^3*a^2*b^4*c^2 + 1584*B^3*a^3*b^2*c^3 - 9*A*B^2*b^7*c + 45*B^3*a
*b^6*c + 576*A^2*B*a^3*c^5 + 81*A^2*B*b^6*c^2 - 1440*A^3*a*b^3*c^4 - 576*A^
3*a^2*b*c^5 - 576*A*B^2*a*b^5*c^2 - 3456*A*B^2*a^3*b*c^4 + 1980*A^2*B*a*b^4
*c^3 - 3600*A*B^2*a^2*b^3*c^3 + 4464*A^2*B*a^2*b^2*c^4))/(256*(b^12 + 4096*
a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*
b^2*c^5 - 24*a*b^10*c))))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2
) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4
160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*
A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a
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^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c
^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^
2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^
8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*
c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3
*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 +
860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^
10*b^2*c^10 + a*b^20*c)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.135 $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=438

$$-\frac{x\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-A\left(8abc+b^3\right)+cx^2\left(12abB-A\left(20ac+b^2\right)\right)+aB\left(7b^2-4ac\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}}{}$$

```
[Out] -1/4*x*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x*(a
*B*(-4*a*c+7*b^2)-A*(8*a*b*c+b^3)+c*(12*a*b*B-A*(20*a*c+b^2))*x^2)/a/(-4*a*
c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*c^(1/2)*(6*a*B*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))+A*(b^3-52*a*b
*c+b^2*(-4*a*c+b^2)^(1/2)+20*a*c*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(5/2)*
2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2))*c^(1/2)*(6*a*B*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^(1/2))+A
*(b^3-52*a*b*c-b^2*(-4*a*c+b^2)^(1/2)-20*a*c*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c
+b^2)^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.09, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1275, 1178, 1166, 205}

$$-\frac{x\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(cx^2\left(12abB-A\left(20ac+b^2\right)\right)-A\left(8abc+b^3\right)+aB\left(7b^2-4ac\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
^2) - (x*(a*B*(7*b^2 - 4*a*c) - A*(b^3 + 8*a*b*c) + c*(12*a*b*B - A*(b^2 +
20*a*c))*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(6*a*B*
(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c + b^2*Sqrt[b^2
- 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 -
 52*a*b*c - b^2*Sqrt[b^2 - 4*a*c] - 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\int \frac{Ab-2aB+5(bB-2Ac)x^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(aB\left(7b^2-4ac\right)-A\left(b^3+8abc\right)+c\left(12abB-A\left(b^2\right.\right.\right.}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(aB\left(7b^2-4ac\right)-A\left(b^3+8abc\right)+c\left(12abB-A\left(b^2\right.\right.\right.}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(aB\left(7b^2-4ac\right)-A\left(b^3+8abc\right)+c\left(12abB-A\left(b^2\right.\right.\right.}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 1.65, size = 436, normalized size = 1.00

$$\frac{1}{16}\left(\frac{4x\left(B\left(2a+bx^2\right)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2x\left(A\left(8abc+20ac^2x^2+b^3+b^2cx^2\right)+aB\left(4ac-7b^2-12bcx^2\right)\right)}{a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\ldots}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (2*x*(a*B*(-7*b^2 + 4*a*c - 12*b*c*x^2) + A*(b^3 + 8*a*b*c + b^2*c*
x^2 + 20*a*c^2*x^2)))/(a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sq
rt[c]*(6*a*B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c +
b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]
) + A*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))
```

```
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^
(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/16
```

fricas [B] time = 6.68, size = 7270, normalized size = 16.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/16*(2*(20*A*a*c^3 - (12*B*a*b - A*b^2)*c^2)*x^7 + 2*(4*(B*a^2 + 7*A*a*b)*
c^2 - (19*B*a*b^2 - 2*A*b^3)*c)*x^5 - 2*(5*B*a*b^3 - A*b^4 - 36*A*a^2*c^2 +
 (16*B*a^2*b - 5*A*a*b^2)*c)*x^3 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b
^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6
 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*
a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a
*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 12
80*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*
B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^
3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^
2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a
^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*
c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*
(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3
 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^
2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^
3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*b
^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A
^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A
^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b
^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*
A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8
 + 53*A^3*a*b^9)*c - (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096*(
9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*
(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240
*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*sqrt
((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*
b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(
a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2
*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*
(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a
^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^
10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 -
1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*
A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b
 + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c
^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^
5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2
)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2
*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sq
rt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*
c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^
3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^
5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4
*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 62
5*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10
 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1
```

```
024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^
3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3
*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*
B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2
- 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^
2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)
*x - 1/2*sqrt(1/2)*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^
3*b^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*
a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324
*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a
^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A
*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c - (3*B*a^4*b^13 + A*a^3*
b^14 + 40960*A*a^10*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*
a^9*b^3 + A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a
^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a
^5*b^11 + 19*A*a^4*b^12)*c)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2
*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 +
 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 -
 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^
5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*
a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^
2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a
^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*
a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 5
0*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c +
 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(
a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*
c^4 - 1024*a^8*c^5))) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)
*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16
*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a
^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7
- 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7
*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (
a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*
c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2
 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B
*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9
*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c +
160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((
10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^
5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4*
a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 -
 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 +
 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*b^8 + 27*A*B
^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c
^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)
*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^
3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7
)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^3*a
*b^9)*c + (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096*(9*B*a^10*b
+ 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^
5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9
 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*sqrt((81*B^4*a^
4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A
^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 -
20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024
*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4
- 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2
 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a^4
```

```
*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^
5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3
 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^
2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a
^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 -
 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2)*((a*b^4*c
^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 +
 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32
*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*
a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(1
8*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*
A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 -
 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*
A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c
^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b
^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^
5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^
7*b^2*c^4 - 1024*a^8*c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b -
A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2
 + 3864*A^3*B*a^2*b^3 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3
*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*
B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x - 1/2*sq
rt(1/2)*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 64
00*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*
A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*
b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 10
2*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^
7 + 267*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c + (3*B*a^4*b^13 + A*a^3*b^14 + 4096
0*A*a^10*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A
*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17
*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 1
9*A*a^4*b^12)*c)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 +
12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^
2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^
4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a
*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*
A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A
^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3
+ 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*
A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^
2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^
6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 -
20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*
a^8*c^5))) - 2*(3*B*a^2*b^2 + A*a*b^3 + 4*(3*B*a^3 - 4*A*a^2*b)*c)*x)/((a*b
^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c
+ 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
```

giac [B] time = 11.77, size = 7267, normalized size = 16.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*((2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
```

a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c
^3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*b^2*c
 + 16*a^3*c^2)^2*A - 12*(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^4 - 8*a^2*b^2*c +
16*a^3*c^2)^2*B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9 - 28*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^8*c - 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^5*c^2 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2
 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 56*a^2*b^7*c^2 - 832
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 288*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^5*c^3 - 480*a^3*b^5*c^3 + 1024*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^5*b*c^4 + 512*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4
 + 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 1664*a^4*b^3*c
^4 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2048*a^5*b*c^5
 + 2*(b^2 - 4*a*c)*a*b^7*c - 48*(b^2 - 4*a*c)*a^2*b^5*c^2 + 288*(b^2 - 4*a*
c)*a^3*b^3*c^3 - 512*(b^2 - 4*a*c)*a^4*b*c^4)*A*abs(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2) + 6*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8 - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^7*c - 2*a^2*b^8*c + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^5*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 16
*a^3*b^6*c^2 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 32
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 4*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^6*c^4 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 256*a^5*b^2*c^4 + 64*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 512*a^6*c^5 + 2*(b^2 - 4*a
*c)*a^2*b^6*c - 8*(b^2 - 4*a*c)*a^3*b^4*c^2 - 32*(b^2 - 4*a*c)*a^4*b^2*c^3
 + 128*(b^2 - 4*a*c)*a^5*c^4)*B*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + (2*a
^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6*c^5 + 271
36*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2 - 4*a*c)*a^2*b^10*c^2
 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*a^4*b^6*c^4 + 5120*(b
^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c^6)*A + 6*(6*a^3*b^11
*c^2 - 88*a^4*b^9*c^3 + 448*a^5*b^7*c^4 - 768*a^6*b^5*c^5 - 512*a^7*b^3*c^6
 + 2048*a^8*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^11 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^4*b^9*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^10*c - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^5*b^7*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a

```
^4*b^8*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^9*c^2 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^6*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^5*b^6*c^3 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^4*b^7*c^3 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^7*b^3*c^4 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^5*b^5*c^4 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^8*b*c^5 - 512*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^7*b^2*c^5 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^7*b*c^6 - 6*(b^2 - 4*a*c)*a^3*b^9*c^2 + 64*(b^2 - 4*a*c)*a^4*b^7*c^3 -
 192*(b^2 - 4*a*c)*a^5*b^5*c^4 + 512*(b^2 - 4*a*c)*a^7*b*c^6)*B)*arctan(2*s
qrt(1/2)*x/sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + sqrt((a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(a*b^4*c -
8*a^2*b^2*c^2 + 16*a^3*c^3)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/((a^
3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 + 32*a^4*b^7*c^2 + a^
3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*b^6*c^3 + 1280*a^7*b
^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^5 - 512*a^7*b*c^5 -
256*a^6*b^2*c^5 + 256*a^7*c^6)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*abs(c)
) - 1/64*((2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*b
^2*c + 16*a^3*c^2)^2*A - 12*(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2)^2*B - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9 - 28
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^8*c + 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c^2 + 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6
*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 - 56*a^2*b^7*c^2 -
 832*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 288*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^5*c^3 + 480*a^3*b^5*c^3 + 1024*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^5*b*c^4 + 512*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2
*c^4 + 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 1664*a^4*b
^3*c^4 - 256*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 2048*a^5*b
*c^5 - 2*(b^2 - 4*a*c)*a*b^7*c + 48*(b^2 - 4*a*c)*a^2*b^5*c^2 - 288*(b^2 -
4*a*c)*a^3*b^3*c^3 + 512*(b^2 - 4*a*c)*a^4*b*c^4)*A*abs(a*b^4 - 8*a^2*b^2*c
 + 16*a^3*c^2) - 6*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8 - 8*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^7*c + 2*a^2*b^8*c + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2
- 16*a^3*b^6*c^2 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3
+ 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 4*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 256*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^6*c^4 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 -
16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 256*a^5*b^2*c^4 +
64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^5 - 512*a^6*c^5 - 2*(b^2 -
 4*a*c)*a^2*b^6*c + 8*(b^2 - 4*a*c)*a^3*b^4*c^2 + 32*(b^2 - 4*a*c)*a^4*b^2*
c^3 - 128*(b^2 - 4*a*c)*a^5*c^4)*B*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) +
(2*a^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6*c^5 +
 27136*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
```

```
sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2 - 4*a*c)*a^2*b^10
*c^2 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*a^4*b^6*c^4 + 512
0*(b^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c^6)*A + 6*(6*a^3*
b^11*c^2 - 88*a^4*b^9*c^3 + 448*a^5*b^7*c^4 - 768*a^6*b^5*c^5 - 512*a^7*b^3
*c^6 + 2048*a^8*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b^11 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^4*b^9*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^3*b^10*c - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^5*b^7*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*b^8*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^9*c^2 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^6*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b^6*c^3 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*b^7*c^3 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^7*b^3*c^4 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^5*b^5*c^4 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^8*b*c^5 - 512*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^7*b^2*c^5 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^7*b*c^6 - 6*(b^2 - 4*a*c)*a^3*b^9*c^2 + 64*(b^2 - 4*a*c)*a^4*b^7*c
^3 - 192*(b^2 - 4*a*c)*a^5*b^5*c^4 + 512*(b^2 - 4*a*c)*a^7*b*c^6)*B)*arctan
(2*sqrt(1/2)*x/sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 - sqrt((a*b^5 - 8*a
^2*b^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(a*b^4*
c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/
((a^3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 + 32*a^4*b^7*c^2
+ a^3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*b^6*c^3 + 1280*a
^7*b^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^5 - 512*a^7*b*c^
5 - 256*a^6*b^2*c^5 + 256*a^7*c^6)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*ab
s(c)) - 1/8*(12*B*a*b*c^2*x^7 - A*b^2*c^2*x^7 - 20*A*a*c^3*x^7 + 19*B*a*b^2
*c*x^5 - 2*A*b^3*c*x^5 - 4*B*a^2*c^2*x^5 - 28*A*a*b*c^2*x^5 + 5*B*a*b^3*x^3
 - A*b^4*x^3 + 16*B*a^2*b*c*x^3 - 5*A*a*b^2*c*x^3 - 36*A*a^2*c^2*x^3 + 3*B*
a^2*b^2*x + A*a*b^3*x + 12*B*a^3*c*x - 16*A*a^2*b*c*x)/((a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.05, size = 1335, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (1/8*c^2*(20*A*a*c+A*b^2-12*B*a*b)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8/a*c
*(28*A*a*b*c+2*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+1
/8*(36*A*a^2*c^2+5*A*a*b^2*c+A*b^4-16*B*a^2*b*c-5*B*a*b^3)/a/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^3+1/8*(16*A*a*b*c-A*b^3-12*B*a^2*c-3*B*a*b^2)/(16*a^2*c^2-8*
a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2-5/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
```

```
)*c)^(1/2)*c*x)*A-1/16/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*A*b^2+13/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*A*b-1/16/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*c*x)*A*b^3+3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*b*B-3/2*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*B-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*B*b^2+5/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A+1/16
/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+13/4/(16*a^2*c^2-8
*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-1/16/a/(16*a^2*
c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-3/4/(16*a^
2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B-3/2*a/(16*a^2*c^2-8*a*b^2*c+b
^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B-9/8/(16*a^2*c^2-8*a*b^2*c+b
^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((20*A*a*c^3 - (12*B*a*b - A*b^2)*c^2)*x^7 + (4*(B*a^2 + 7*A*a*b)*c^2 -
 (19*B*a*b^2 - 2*A*b^3)*c)*x^5 - (5*B*a*b^3 - A*b^4 - 36*A*a^2*c^2 + (16*B*
a^2*b - 5*A*a*b^2)*c)*x^3 - (3*B*a^2*b^2 + A*a*b^3 + 4*(3*B*a^3 - 4*A*a^2*b
)*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 -
 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x
^2) + 1/8*integrate((3*B*a*b^2 + A*b^3 + (20*A*a*c^2 - (12*B*a*b - A*b^2)*c
)*x^2 + 4*(3*B*a^2 - 4*A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)
```

mupad [B] time = 3.92, size = 18992, normalized size = 43.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^3*(A*b^4 + 36*A*a^2*c^2 - 5*B*a*b^3 + 5*A*a*b^2*c - 16*B*a^2*b*c))/(8*a
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x*(A*b^3 + 3*B*a*b^2 + 12*B*a^2*c - 16*
A*a*b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(4*B*a^2*c^2 + 2*A*b^3*
c + 28*A*a*b*c^2 - 19*B*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c
*x^7*(20*A*a*c^2 + A*b^2*c - 12*B*a*b*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((
(256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^
11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^
6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 614
```

```
40*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*
b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 +
3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A
^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*
A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4
*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7
*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b
^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8
 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8
*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 +
 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 -
 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^
2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*
a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 25
8048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^1
1*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*
c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^
6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 25
6*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)
^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*
b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b
^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^
11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*
c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^
2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*
c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 -
241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 +
 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2
*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*
c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160
*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b
^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 147
2*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^
4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a
^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(
-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2
*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A
^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^
2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2
*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a
^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b
^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9
*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8
*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^
2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3
*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^1
4*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 196
6080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*1i
 - (((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a
^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^5*b
^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3
+ 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*
(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c
^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(A^2*b^17 + 9*B^2*a^2*b^1
5 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2)
 + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^
2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^
2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*
a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^
```

```
9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^
2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*
c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*
c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c
 - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10
- 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4
 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 294912
0*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*
b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 3276
80*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2
 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2
)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2
*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*
a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a
^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7
*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c -
25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*
b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c
^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*
c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*
B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*
b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 8
60160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a
^12*b^2*c^9)))^(1/2) - (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5
+ 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2
*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(
32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3
)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^
2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10
160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 6809
60*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 3744
0*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*
B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*
c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^
2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^
5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a
^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512
*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^
6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6
- 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/
2)*1i)/((((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 921
6*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*
a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10
*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/
(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*
b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(A^2*b^17 + 9*B^2*a^
2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^
(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 348
80*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 18636
80*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680
*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A
*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 17203
20*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*
b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6
*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(
4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*
c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^1
2*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2
949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256
```

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*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 -
 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^
4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c
 - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 114
0*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776
*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*
B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^
2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15
*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2
*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b
^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7
*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 1
80*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720
*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^
5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621
440*a^12*b^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4
*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 3
4*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^
5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^
2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) +
 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2
 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 -
 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 +
 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 55
2960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-
(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 7372
80*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A
*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*
A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)
/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 76
80*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8
*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9))
)^(1/2) + (((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9
216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*
A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^
10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7
)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^
5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(A^2*b^17 + 9*B^2*
a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15
)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 3
4880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 186
3680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 1036
80*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040
*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 172
0320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^
3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a
^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(
-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^1
3*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b
^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 +
 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 2
56*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5
 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*
b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*
c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1
140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 437
76*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 504
0*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*
B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^
```

15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B
^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4
*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a
^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) -
180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 7
20*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*
c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 26
21440*a^12*b^2*c^9)))^(1/2) - (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a
^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 -
34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*
c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*
b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2)
+ 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c
^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5
- 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2
+ 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 +
552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*
(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 73
7280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920
*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 73728
0*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*
c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 -
7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b
^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9
)))^(1/2) + (35*A^3*b^6*c^4 - 8000*A^3*a^3*c^7 - 12720*A^3*a^2*b^2*c^6 + 54
0*B^3*a^2*b^5*c^3 + 4320*B^3*a^3*b^3*c^4 - 2880*A*B^2*a^4*c^6 - 15*A^2*B*b^
7*c^3 + 84*A^3*a*b^4*c^5 + 1728*B^3*a^4*b*c^5 + 135*A*B^2*a*b^6*c^3 - 360*A
^2*B*a*b^5*c^4 + 26880*A^2*B*a^3*b*c^6 - 5580*A*B^2*a^2*b^4*c^4 - 20592*A*B
^2*a^3*b^2*c^5 + 15696*A^2*B*a^2*b^3*c^5)/(256*(a^2*b^12 + 4096*a^8*c^6 - 2
4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144
*a^7*b^2*c^5))))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)
^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*
b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b
^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^
11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*
c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^
2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*
c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 -
241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 +
737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2
*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*
c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160
*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b
^2*c^9)))^(1/2)*2i + atan(((((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 419430
4*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b
^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2
- 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145
728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*
b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(
A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(
-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2
*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*
a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a
^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8
*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2
)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b
*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c
^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*
c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b

^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*
c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 19660
80*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262
144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 1
63840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^
3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15
- A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) +
6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*
a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*
a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^
6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*
c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*
a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^
2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^
5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c -
b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 -
40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 -
258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*
a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 - 800*A^2*a^
3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*
B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3
- 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b
^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c
- b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 11
40*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 4377
6*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040
*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B
^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^1
5*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^
2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*
b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^
7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) -
180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 72
0*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c
^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 262
1440*a^12*b^2*c^9)))^(1/2)*1i - (((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4
194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*
a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^1
2*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 +
3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240
*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (
x*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*
a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 1016
0*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960
*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*
B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^
2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c
- b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*
a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*
b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8
*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(
a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*
b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 -
1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)
*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^
4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 -
16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*
b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1
/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880

*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680 *A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B ^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B *a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320 *A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^ 12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b ^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4* a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^ 10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12* c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 294 9120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(A^2*b^6*c^3 - 800*A ^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^ 5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96* a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9 216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2* a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 1 80*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B *a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A *B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/ 2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b ^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*1i)/((((256*A*a*b^13*c^2 - 3145728*B*a^8*c^ 8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 8192 00*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^ 2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4* c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9 *B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 6 80960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 3 7440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 5529 60*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4 *a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280 *B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B *a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A* B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680 *a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c ^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^ (1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b ^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c ^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2 *a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^1 5)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 18 63680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103 680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 98304 0*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 17 20320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a ^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B* a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b* (-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^

13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*
b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7
+ 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 -
800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*
c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B
*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c
+ 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^
2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a
*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*
c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*
c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^
4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55
*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^
8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 2400
0*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781
760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15
)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b
^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*
a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4
*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (((256*A*a*b^13*c^2 - 3145728*B*a^8*
c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 81
9200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*
a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^
4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*
c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^
5)) + (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) -
9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2
- 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 -
680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 +
37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 55
2960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-
(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 7372
80*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A
*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*
A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)
/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 76
80*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8
*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9))
)^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4
*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6
*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B
^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)
^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3
+ 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 +
1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 1
03680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983
040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) -
1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B
*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*
B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*
b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*
a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^
7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^
7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(A^2*b^6*c^3
- 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^
4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A
*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*
c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*
b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B

```
*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^
9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^
3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*
c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 -
55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*
c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24
000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 17
81760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^
15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4
*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 25804
8*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b
^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (35*A^3*b^6*c^4 - 8000*A^3*a^3*c^7
 - 12720*A^3*a^2*b^2*c^6 + 540*B^3*a^2*b^5*c^3 + 4320*B^3*a^3*b^3*c^4 - 288
0*A*B^2*a^4*c^6 - 15*A^2*B*b^7*c^3 + 84*A^3*a*b^4*c^5 + 1728*B^3*a^4*b*c^5
+ 135*A*B^2*a*b^6*c^3 - 360*A^2*B*a*b^5*c^4 + 26880*A^2*B*a^3*b*c^6 - 5580*
A*B^2*a^2*b^4*c^4 - 20592*A*B^2*a^3*b^2*c^5 + 15696*A^2*B*a^2*b^3*c^5)/(256
*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*
c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))))*(-(A^2*b^17 + 9*B^2*a^2*b^15
- A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) +
 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*
a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*
a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^
6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*
c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*
a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^
2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^
5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c -
 b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 -
40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 -
 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*
a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.136 $\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=460

$$\frac{x\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx^2\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)+abB\left(8ac+b^2\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{3A\left(56a^2c^2-10ab^2c+b^4\right) \cdots}{\sqrt{b^2-4 \cdots}}\right. \cdots}{\cdots}$$

```
[Out] 1/4*x*(A*b^2-a*b*B-2*a*A*c+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a
)^2+1/8*x*(a*b*B*(8*a*c+b^2)+A*(28*a^2*c^2-25*a*b^2*c+3*b^4)+c*(a*B*(20*a*c
+b^2)+3*A*(-8*a*b*c+b^3))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(20*a*c+b^2
)+3*A*(-8*a*b*c+b^3)+(a*b*B*(-52*a*c+b^2)+3*A*(56*a^2*c^2-10*a*b^2*c+b^4))/
(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(
20*a*c+b^2)+3*A*(-8*a*b*c+b^3)+(-a*b*B*(-52*a*c+b^2)-3*A*(56*a^2*c^2-10*a*b
^2*c+b^4))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2)
```

Rubi [A] time = 1.35, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1178, 1166, 205}

$$\frac{x\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx^2\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)+abB\left(8ac+b^2\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{3A\left(56a^2c^2-10ab^2c+b^4\right) \cdots}{\sqrt{b^2-4 \cdots}}\right. \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(4*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)^2) + (x*(a*b*B*(b^2 + 8*a*c) + A*(3*b^4 - 25*a*b^2*c + 28*a
^2*c^2) + c*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c))*x^2))/(8*a^2*(b^2 -
4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8
*a*b*c) + (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*
Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(b
^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c) - (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*
a*b^2*c + 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{-3Ab^2-abB+14aAc-5(Ab-2aB)cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(abB\left(b^2+8ac\right)+A\left(3b^4-25ab^2c+28\right.\right.}{8a^2\left(b^2-4a\right.}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(abB\left(b^2+8ac\right)+A\left(3b^4-25ab^2c+28\right.\right.}{8a^2\left(b^2-4a\right.}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(abB\left(b^2+8ac\right)+A\left(3b^4-25ab^2c+28\right.\right.}{8a^2\left(b^2-4a\right.}$$

Mathematica [A] time = 2.19, size = 516, normalized size = 1.12

$$\frac{\frac{2x\left(A\left(28a^2c^2-25ab^2c-24abc^2x^2+3b^4+3b^3cx^2\right)+aB\left(8abc+20ac^2x^2+b^3+b^2cx^2\right)\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(3A\left(56a^2c^2-10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)+a\right.}{\left(b^2-4ac\right)^{5/2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*a*x*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)^2) + (2*x*(a*B*(b^3 + 8*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2)
 + A*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2)))/((b
^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(a*B*(b^3 - 52*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(b^4 - 10*a*b^2*c
 + 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(a*B*(-b^3 + 52*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(-b^4 + 10*a*b^2*c - 56*a^
2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqr
t[b^2 - 4*a*c]]))/(16*a^2)
```

fricas [B] time = 16.22, size = 9909, normalized size = 21.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(4*(5*B*a^2 - 6*A*a*b)*c^3 + (B*a*b^2 + 3*A*b^3)*c^2)*x^7 + 2*(28*A
*a^2*c^3 + 7*(4*B*a^2*b - 7*A*a*b^2)*c^2 + 2*(B*a*b^3 + 3*A*b^4)*c)*x^5 + 2
*(B*a*b^4 + 3*A*b^5 + 4*(9*B*a^3 - A*a^2*b)*c^2 + 5*(B*a^2*b^2 - 4*A*a*b^3)
*c)*x^3 - sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c
 + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(
4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3
*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*
(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*
c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*s
qrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7
+ 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b +
 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*
b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 25
8*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)
*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280
*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2
 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((3111696*A^4*a^
4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 - 90000*
A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309*A^4*a
^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B^2*a^3
*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^4 + 53
7*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^8)*c^3
 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^9)*c^2
)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^13 + 27
*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*b + 11
43*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*A^2*B*
a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*a^6*b^
4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5 + 674
4*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940*B^3*a
^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^10)*c^2
 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A^3*a*b
^12)*c - (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*c^7 -
2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*a^10*b
^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 + 123*A
*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^12 + 9
3*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6
+ 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 +
 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b +
17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(2
5*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^
5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a
^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7 + 6*A
*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*
b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^
4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7
)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a
^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B
^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^
2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B
^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)
*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A
^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*
c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 2
0*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a
^10*c^5))) + sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^
4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
```

c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((3111696*A^4*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 - 90000*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309*A^4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B^2*a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^4 + 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^8)*c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^9)*c^2)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^13 + 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*b + 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*A^2*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*a^6*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5 + 6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940*B^3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^10)*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A^3*a*b^12)*c - (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*c^7 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*a^10*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 + 123*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^12 + 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) - sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*

```
c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*
a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a
^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^
2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^
2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*
a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3
 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b
^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((3111696*
A^4*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 -
90000*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309
*A^4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B
^2*a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^
4 + 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^
8)*c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^
9)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^1
3 + 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*
b + 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*
A^2*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*
a^6*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5
 + 6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940
*B^3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^1
0)*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A
^3*a*b^12)*c + (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*
c^7 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*
a^10*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 +
 123*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^
12 + 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^
2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2
*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^
5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2
 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*
a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 -
 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7
 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^
2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*
a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2
*a*b^7)*c - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 +
1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54
*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*
(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 54
00*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^
2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 +
1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^1
2*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^
10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 -
1024*a^10*c^5))) + sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^
8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*
a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*
a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9
 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 -
9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)
*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c - (a^5*b^10 - 20
*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^
10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3
*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*
B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2
*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5
*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A
^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c
```

```
^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^
7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((31116
96*A^4*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6
 - 90000*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339
309*A^4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^
2*B^2*a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4
*b^4 + 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4
*b^8)*c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B
*b^9)*c^2)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*
b^13 + 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a
^7*b + 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 175
41*A^2*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B
^2*a^6*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*
b^5 + 6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (
940*B^3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*
b^10)*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 86
4*A^3*a*b^12)*c + (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*
b)*c^7 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169
*A*a^10*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^
8 + 123*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7
*b^12 + 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2
*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*
B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3
*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c
^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3
*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^
2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*
b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2
*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A
*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*
A^2*a*b^7)*c - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3
 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 +
 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 8
82*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 +
 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4
*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4
 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*
a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5
*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4
 - 1024*a^10*c^5))) - 2*(B*a^2*b^3 - 5*A*a*b^4 - 44*A*a^3*c^2 - (16*B*a^3*b
 - 37*A*a^2*b^2)*c)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^
4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^
3*c + 16*a^5*b*c^2)*x^2)
```

giac [B] time = 8.54, size = 4609, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^7*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 +
 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
3 - 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
```

c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 + 176
*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^
4*c^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c
 - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2
 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 176*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 88*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 11*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 44*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*(b^2 - 4*a
*c)*b^6*c - 26*(b^2 - 4*a*c)*a*b^4*c^2 - 2*(b^2 - 4*a*c)*b^5*c^2 + 128*(b^2
 - 4*a*c)*a^2*b^2*c^3 + 22*(b^2 - 4*a*c)*a*b^3*c^3 - 224*(b^2 - 4*a*c)*a^3*
c^4 - 88*(b^2 - 4*a*c)*a^2*b*c^4)*A + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^7 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*b^7*c + 144*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 48
*a^2*b^5*c^2 + 2*a*b^6*c^2 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 20*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 288*a^3*b^3*c^3 - 44*a^2*
b^4*c^3 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 512*a^4*b*
c^4 + 64*a^3*b^2*c^4 + 320*a^4*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^6 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^2*c^2 - 36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^2 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^3 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^3 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
c^4 + 2(b^2 - 4*a*c)*a*b^5*c - 40*(b^2 - 4*a*c)*a^2*b^3*c^2 - 2*(b^2 - 4*
a*c)*a*b^4*c^2 + 128*(b^2 - 4*a*c)*a^3*b*c^3 + 36*(b^2 - 4*a*c)*a^2*b^2*c^3
 + 80*(b^2 - 4*a*c)*a^3*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3*
b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a
^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)
))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2*
a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3
 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b
^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/32*(3*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^8 - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c + 2*b^8*c + 116*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 34*a*b^6*c^2
 - 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 232*a^2*b^4*c^3 + 30*a*b^5*c^3 + 448*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^4 - 736*a^3*b^2*c^4 - 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*c^5 + 896*a^4*c^5 + 352*a^3*b*c^5 + sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^

```
2 - 4*a*c)*c)*b^5*c^2 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b*c^3 - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^3 - 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^3*c^3 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^6*c + 26*(b^2 - 4*a*c)*a*b^4*c^2 + 2*(
b^2 - 4*a*c)*b^5*c^2 - 128*(b^2 - 4*a*c)*a^2*b^2*c^3 - 22*(b^2 - 4*a*c)*a*b
^3*c^3 + 224*(b^2 - 4*a*c)*a^3*c^4 + 88*(b^2 - 4*a*c)*a^2*b*c^4)*A + (sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c +
2*a*b^7*c + 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 40*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^5*c^2 - 48*a^2*b^5*c^2 - 2*a*b^6*c^2 - 256*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b^2*c^3 - 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c^3 + 288*a^3*b^3*c^3 + 44*a^2*b^4*c^3 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^4 - 512*a^4*b*c^4 - 64*a^3*b^2*c^4 - 320*a^4*c^5 + sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 22*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 32*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 36*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 160*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^5*c + 40*(b^2 - 4
*a*c)*a^2*b^3*c^2 + 2*(b^2 - 4*a*c)*a*b^4*c^2 - 128*(b^2 - 4*a*c)*a^3*b*c^3
 - 36*(b^2 - 4*a*c)*a^2*b^2*c^3 - 80*(b^2 - 4*a*c)*a^3*c^4)*B)*arctan(2*sqr
t(1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt((a^2*b^5 - 8*a^3
*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*
c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))
)/((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2
+ a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7
*c^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/8*(B*a*b^2*
c^2*x^7 + 3*A*b^3*c^2*x^7 + 20*B*a^2*c^3*x^7 - 24*A*a*b*c^3*x^7 + 2*B*a*b^3
*c*x^5 + 6*A*b^4*c*x^5 + 28*B*a^2*b*c^2*x^5 - 49*A*a*b^2*c^2*x^5 + 28*A*a^2
*c^3*x^5 + B*a*b^4*x^3 + 3*A*b^5*x^3 + 5*B*a^2*b^2*c*x^3 - 20*A*a*b^3*c*x^3
 + 36*B*a^3*c^2*x^3 - 4*A*a^2*b*c^2*x^3 - B*a^2*b^3*x + 5*A*a*b^4*x + 16*B*
a^3*b*c*x - 37*A*a^2*b^2*c*x + 44*A*a^3*c^2*x)/((a^2*b^4 - 8*a^3*b^2*c + 16
*a^4*c^2)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.28, size = 11936, normalized size = 25.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(4\left(5\,Ba^2-6\,Aab\right)c^3+\left(Bab^2+3\,Ab^3\right)c^2\right)x^7+\left(28\,Aa^2c^3+7\left(4\,Ba^2b-7\,Aab^2\right)c^2+2\left(Bab^3+3\,Ab^4\right)c\right)x^5+\left(}{8\left(\left(a^2b^4c^2-8\,a^3b^2c^3+16\,a^4c^4\right)x^8+a^4b^4-8\,a^5b^2c+16\,a^6c^2+2\left(a^2b^5c-8\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((4*(5*B*a^2 - 6*A*a*b)*c^3 + (B*a*b^2 + 3*A*b^3)*c^2)*x^7 + (28*A*a^2*
c^3 + 7*(4*B*a^2*b - 7*A*a*b^2)*c^2 + 2*(B*a*b^3 + 3*A*b^4)*c)*x^5 + (B*a*b
^4 + 3*A*b^5 + 4*(9*B*a^3 - A*a^2*b)*c^2 + 5*(B*a^2*b^2 - 4*A*a*b^3)*c)*x^3
 - (B*a^2*b^3 - 5*A*a*b^4 - 44*A*a^3*c^2 - (16*B*a^3*b - 37*A*a^2*b^2)*c)*x
)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c +
 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 -
 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x
^2) - 1/8*integrate(-(B*a*b^3 + 3*A*b^4 + 84*A*a^2*c^2 + (4*(5*B*a^2 - 6*A*
a*b)*c^2 + (B*a*b^2 + 3*A*b^3)*c)*x^2 - (16*B*a^2*b + 27*A*a*b^2)*c)/(c*x^4
 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

mupad [B] time = 4.61, size = 22914, normalized size = 49.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^3*(3*A*b^5 + 36*B*a^3*c^2 + B*a*b^4 - 20*A*a*b^3*c - 4*A*a^2*b*c^2 + 5*
B*a^2*b^2*c))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(28*A*a^2*c^3 +
 6*A*b^4*c + 2*B*a*b^3*c - 49*A*a*b^2*c^2 + 28*B*a^2*b*c^2))/(8*a^2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*A*b^4 + 44*A*a^2*c^2 - B*a*b^3 - 37*A*a*b^
2*c + 16*B*a^2*b*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20*B*a^
2*c^2 + 3*A*b^3*c - 24*A*a*b*c^2 + B*a*b^2*c))/(8*a^2*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + a
tan(((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 222
72*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360
*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^
3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*
c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*
a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^
4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-
(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*
a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*
A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441
*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1
/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^
4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^
7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B
^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(
1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10
*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*
b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^
2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^
6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 2580
48*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^1
3*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*
c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^
8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 25
6*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^1
5)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 +
570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6
 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*
a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a
^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B
*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c -
 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B
*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*
```

```
A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 51609
60*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3
*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c
- b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*
a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^
5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 262
1440*a^14*b^2*c^9)))^(1/2) + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^
2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 -
 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^
2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(
32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3
)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*
A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^
4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A
^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2
- 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 -
680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 3
69*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*
a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2
 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c
^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^
2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b
^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1
/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2
- 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^
11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2
*c^9)))^(1/2)*1i - (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*
b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8
*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*
c^8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819
200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^
4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3
 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 + B^2*a^2*b^17
+ 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^
2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*
c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8
*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c
 - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*
B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*
B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^
9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*
c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 37728
0*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 12
90240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c -
 b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*
c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^1
5*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b
^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7
 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 -
 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c
^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^
6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(
4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a
^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A
^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*
A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/
2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4
 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7
```

+ 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^
2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1
/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*
c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b
^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
+ 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2
*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6
*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 25804
8*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13
*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^
8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2
*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^
4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B
*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 2
56*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^
15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 +
570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^
6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2
*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*
a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*
B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c
- 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*
B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784
*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160
960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^
3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c
- b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720
*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c
^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 26
21440*a^14*b^2*c^9)))^(1/2)*1i)/((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^
9 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 202
7520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603
008*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5
*b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3
c^7)/(512(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 12
80*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^19 +
B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A
^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 285177
6*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27
095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^
2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^1
1*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*
c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 1
5482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2
*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^
12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^
7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^
2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288
*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20
+ 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3
+ 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 196608
0*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(2621
44*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 16
3840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5
*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 +
9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2
- 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c
^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*

b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (567*A^3*b^7*c^5 + 8000*B^3*a^5*c^7 + 67824*A^3*a^2*b^3*c^7 - 35*B^3*a^2*b^6*c^4 - 84*B^3*a^3*b^4*c^5 + 12720*B^3*a^4*b^2*c^6 + 141120*A^2*B*a^4*c^8 - 315*A^2*B*b^8*c^4 - 10368*A^3*a*b^5*c^6 - 169344*A^3*a^3*b*c^8 - 210*A*B^2*a*b^7*c^4 - 116160*A*B^2*a^4*b*c^7 + 6237*A^2*B*a*b^6*c^5 + 1764*A*B^2*a^2*b^5*c^5 + 4608*A*B^2*a^3*b^3*c^6 - 42372*A^2*B*a^2*b^4*c^6 + 96048*A^2*B*a^3*b^2*c^7)/(256*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*

```
A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20
+ 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3
+ 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080
*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(26214
4*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163
840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*
b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 +
9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2
- 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^
5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b
^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c -
 b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^
2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^
2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*
b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c
- b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*
A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290
240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c
- 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*
c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^1
2*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 +
 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c
^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*
b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180
*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c
^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^
6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(
4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a
^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A
^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*
A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/
2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4
 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7
 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^
2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1
/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*
c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b
^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
 + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2
*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6
*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 25804
8*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13
*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9
*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 -
 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5
 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^
3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c -
b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2
*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2
*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b
*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c -
 b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A
*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 12902
40*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^
2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c -
 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c
^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12
*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 +
```

2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*2i + atan(((((4194304*
B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^
3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 -
23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 921
6*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*
a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^
5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9
*b^2*c^5)) - (x*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15
)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 5
70960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6
- 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a
^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^
7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*
a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c -
1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*
a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A
*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 516096
0*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*
(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c -
b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a
^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5
+ 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621
440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b
^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*
(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))
*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B
*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b
^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*
a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15
)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 1
0160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680
960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*
A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^1
0*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 -
59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5
+ 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c
^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)
^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2)
)/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7
680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*
b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^
9)))^(1/2) + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530
*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*
c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104
*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 25
6*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^1
9 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 692
1*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 285
1776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 +
27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2
*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*
b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b
^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c
- 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*
B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4
*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B
*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a
*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) -

288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i - (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 -

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59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5
 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*
c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2
)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2
))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 -
7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11
*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c
^9)))^(1/2)*1i)/((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^
14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c
^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^
8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 81920
0*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*
b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 +
 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^19 + B^2*a^2*b^17 -
9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2
- 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^
5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b
^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c -
 b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^
2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^
2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*
b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c
- b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*
A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290
240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c
+ 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*
c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^1
2*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 +
 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 2
56*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5
 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*
b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*
a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3
*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2
*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^
2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2)
 + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 +
 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 +
 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*
a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2
) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^
4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4
*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) -
 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b
*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b
^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*
a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b
^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*
c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a
^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4
- 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a
^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256
*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15
)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 5
70960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6
- 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a
^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^
```

7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*
a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c -
1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*
a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A
*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 516096
0*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*
(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c -
b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a
^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5
+ 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621
440*a^14*b^2*c^9)))^(1/2) - (567*A^3*b^7*c^5 + 8000*B^3*a^5*c^7 + 67824*A^3
*a^2*b^3*c^7 - 35*B^3*a^2*b^6*c^4 - 84*B^3*a^3*b^4*c^5 + 12720*B^3*a^4*b^2*
c^6 + 141120*A^2*B*a^4*c^8 - 315*A^2*B*b^8*c^4 - 10368*A^3*a*b^5*c^6 - 1693
44*A^3*a^3*b*c^8 - 210*A*B^2*a*b^7*c^4 - 116160*A*B^2*a^4*b*c^7 + 6237*A^2*
B*a*b^6*c^5 + 1764*A*B^2*a^2*b^5*c^5 + 4608*A*B^2*a^3*b^3*c^6 - 42372*A^2*B
*a^2*b^4*c^6 + 96048*A^2*B*a^3*b^2*c^7)/(256*(a^4*b^12 + 4096*a^10*c^6 - 24
*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*
a^9*b^2*c^5)) + (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^1
4*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^
5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8
+ 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200
*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b
^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 +
3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9
*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 -
77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5
+ 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^
3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c -
b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2
*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2
*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b
*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c -
b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A
*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 12902
40*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^
2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c +
108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c
^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12
*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 +
2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 25
6*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5
- 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b
^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a
*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*
b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*
a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2
*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2)
+ 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 +
43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 +
6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a
^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2)
+ 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4
- 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*
c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) -
6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*
c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^
18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a
^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^
4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c

```
^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^
2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 -
 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^
4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*
a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)
^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 57
0960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 -
 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4
*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^
4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7
*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a
^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1
720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a
^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*
B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960
*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(
-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c -
b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^
7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5
+ 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 26214
40*a^14*b^2*c^9)))^(1/2)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*
c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b
^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a
^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*
a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) +
 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 4
3776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6
881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^
3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2)
+ 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4
- 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c
^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6
*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c
*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^1
8*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^
10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4
*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.137 $\int \frac{x(-7+4x^2)}{4-5x^2+x^4} dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[Out] 1/2*ln(-x^2+1)+3/2*ln(-x^2+4)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1247, 632, 31}

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(-7+4x^2\right)}{4-5x^2+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{-7+4x}{4-5x+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,x^2\right)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{-4+x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

fricas [A] time = 0.67, size = 17, normalized size = 0.68

$$\frac{1}{2}\log\left(x^2-1\right)+\frac{3}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

giac [A] time = 0.31, size = 19, normalized size = 0.76

$$\frac{1}{2}\log\left(\left|x^2-1\right|\right)+\frac{3}{2}\log\left(\left|x^2-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] 1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))
```

maple [A] time = 0.01, size = 18, normalized size = 0.72

$$\frac{3\ln\left(x^2-4\right)}{2}+\frac{\ln\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2-7)/(x^4-5*x^2+4),x)
[Out] 3/2*ln(x^2-4)+1/2*ln(x^2-1)
```

maxima [A] time = 0.72, size = 17, normalized size = 0.68

$$\frac{1}{2}\log\left(x^2-1\right)+\frac{3}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

mupad [B] time = 0.06, size = 17, normalized size = 0.68

$$\frac{\ln\left(x^2-1\right)}{2}+\frac{3\ln\left(x^2-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(4*x^2 - 7))/(x^4 - 5*x^2 + 4),x)
[Out] log(x^2 - 1)/2 + (3*log(x^2 - 4))/2
```

sympy [A] time = 0.12, size = 17, normalized size = 0.68

$$\frac{3\log\left(x^2-4\right)}{2}+\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2-7)/(x**4-5*x**2+4),x)
[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

3.138 $\int \frac{-7x+4x^3}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[Out] 1/2*ln(-x^2+1)+3/2*ln(-x^2+4)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 1247, 632, 31}

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-7x+4x^3}{4-5x^2+x^4} \, dx &= \int \frac{x\left(-7+4x^2\right)}{4-5x^2+x^4} \, dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-7+4x}{4-5x+x^2} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x} \, dx, x, x^2\right)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{-4+x} \, dx, x, x^2\right)\\
&= \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

fricas [A] time = 0.66, size = 17, normalized size = 0.68

$$\frac{1}{2}\log\left(x^2-1\right)+\frac{3}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

giac [A] time = 0.28, size = 19, normalized size = 0.76

$$\frac{1}{2}\log\left(\left|x^2-1\right|\right)+\frac{3}{2}\log\left(\left|x^2-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] 1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))
```

maple [A] time = 0.00, size = 18, normalized size = 0.72

$$\frac{3\ln\left(x^2-4\right)}{2}+\frac{\ln\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-7*x)/(x^4-5*x^2+4),x)
[Out] 3/2*ln(x^2-4)+1/2*ln(x^2-1)
```

maxima [A] time = 0.74, size = 25, normalized size = 1.00

$$\frac{3}{2}\log\left(x+2\right)+\frac{1}{2}\log\left(x+1\right)+\frac{1}{2}\log\left(x-1\right)+\frac{3}{2}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] 3/2*log(x + 2) + 1/2*log(x + 1) + 1/2*log(x - 1) + 3/2*log(x - 2)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.68

$$\frac{\ln\left(x^2-1\right)}{2}+\frac{3\,\ln\left(x^2-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(7*x - 4*x^3)/(x^4 - 5*x^2 + 4),x)
[Out] log(x^2 - 1)/2 + (3*log(x^2 - 4))/2
```

sympy [A] time = 0.11, size = 17, normalized size = 0.68

$$\frac{3\log\left(x^2-4\right)}{2}+\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-7*x)/(x**4-5*x**2+4),x)
```

```
[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

3.139 $\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

```
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1247, 634, 618, 204, 628}

$$\frac{1}{4}\log\left(x^4+x^2+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + x^2))/(1 + x^2 + x^4),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right) + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{4}\log\left(1+x^2+x^4\right) - \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right)\\
&= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right) + \frac{1}{4}\log\left(1+x^2+x^4\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + x^2))/(1 + x^2 + x^4),x]
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

fricas [A] time = 0.55, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

giac [A] time = 0.28, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$\frac{\sqrt{3}\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)}{2} + \frac{\ln\left(x^4+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+2)/(x^4+x^2+1),x)
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

maxima [A] time = 1.40, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="maxima")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

mupad [B] time = 0.21, size = 32, normalized size = 0.86

$$\frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 + 2))/(x^2 + x^4 + 1),x)
[Out] log(x^2 + x^4 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/2
```

sympy [A] time = 0.12, size = 37, normalized size = 1.00

$$\frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+2)/(x**4+x**2+1),x)
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2
```

3.140 $\int \frac{2x+x^3}{1+x^2+x^4}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

```
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 1247, 634, 618, 204, 628}

$$\frac{1}{4}\log\left(x^4+x^2+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^3)/(1 + x^2 + x^4),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2x+x^3}{1+x^2+x^4}\,dx &= \int \frac{x\left(2+x^2\right)}{1+x^2+x^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right)+\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{4}\log\left(1+x^2+x^4\right)-\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right) \\
&= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{4}\log\left(1+x^2+x^4\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^3)/(1 + x^2 + x^4),x]
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

fricas [A] time = 0.65, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

giac [A] time = 0.37, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$\frac{\sqrt{3}\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)}{2}+\frac{\ln\left(x^4+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2*x)/(x^4+x^2+1),x)
[Out] 1/2*3^(1/2)*arctan(1/3*(2*x^2+1)*3^(1/2))+1/4*ln(x^4+x^2+1)
```

maxima [A] time = 1.60, size = 53, normalized size = 1.43

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x + 1)
```

mupad [B] time = 0.03, size = 32, normalized size = 0.86

$$\frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^3)/(x^2 + x^4 + 1),x)
```

```
[Out] log(x^2 + x^4 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/2
```

sympy [A] time = 0.12, size = 37, normalized size = 1.00

$$\frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2*x)/(x**4+x**2+1),x)
```

```
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2
```

3.141 $\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=45

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{9x^2+5}{8(x^4+2x^2+3)}$$

```
[Out] 1/8*(9*x^2+5)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1593, 1247, 638, 618, 204}

$$\frac{9x^2+5}{8(x^4+2x^2+3)}+\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{11x+2x^3}{\left(3+2x^2+x^4\right)^2}\,dx &= \int \frac{x\left(11+2x^2\right)}{\left(3+2x^2+x^4\right)^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{11+2x}{\left(3+2x+x^2\right)^2}\,dx, x, x^2\right)\\ &= \frac{5+9x^2}{8\left(3+2x^2+x^4\right)}+\frac{9}{8}\,\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right)\\ &= \frac{5+9x^2}{8\left(3+2x^2+x^4\right)}-\frac{9}{4}\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+x^2\right)\right)\\ &= \frac{5+9x^2}{8\left(3+2x^2+x^4\right)}+\frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.00

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{9x^2+5}{8\left(x^4+2x^2+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

fricas [A] time = 0.62, size = 47, normalized size = 1.04

$$\frac{9\sqrt{2}\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+18x^2+10}{16\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 18*x^2 +
10)/(x^4 + 2*x^2 + 3)
```

giac [A] time = 0.94, size = 38, normalized size = 0.84

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{9x^2+5}{8\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/8*(9*x^2 + 5)/(x^4 + 2*x^2 +
 3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.91

$$\frac{9\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}+\frac{18x^2+10}{16x^4+32x^2+48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+11*x)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/16*(18*x^2+10)/(x^4+2*x^2+3)+9/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9\,x^2+5}{8\left(x^4+2\,x^2+3\right)}+\frac{9}{4}\int\frac{x}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(9*x^2 + 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 0.05, size = 41, normalized size = 0.91

$$\frac{\frac{9\,x^2}{8}+\frac{5}{8}}{x^4+2\,x^2+3}+\frac{9\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((11*x + 2*x^3)/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] ((9*x^2)/8 + 5/8)/(2*x^2 + x^4 + 3) + (9*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*
x^2)/2))/16
```

sympy [A] time = 0.15, size = 44, normalized size = 0.98

$$\frac{9x^2+5}{8x^4+16x^2+24}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+11*x)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (9*x**2 + 5)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt
(2)/2)/16
```

3.142 $\int x^5 \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\, dx$

Optimal. Leaf size=102

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{3/2}x^4+\frac{1}{480}\left(1837-510x^2\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1633}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{21229}{512}\tanh^{-1}\left(\frac{}{2}\right.$$

```
[Out] 3/10*x^4*(x^4+5*x^2+3)^(3/2)+1/480*(-510*x^2+1837)*(x^4+5*x^2+3)^(3/2)+2122
9/512*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1633/256*(2*x^2+5)*(x^4+5*
x^2+3)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 832, 779, 612, 621, 206}

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{3/2}x^4+\frac{1}{480}\left(1837-510x^2\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1633}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{21229}{512}\tanh^{-1}\left(\frac{}{2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-1633*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 + (3*x^4*(3 + 5*x^2 + x^4)^(3
/2))/10 + ((1837 - 510*x^2)*(3 + 5*x^2 + x^4)^(3/2))/480 + (21229*ArcTanh[(
5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
```

```
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^5\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x^2(2+3x)\sqrt{3+5x+x^2}\,dx,x,x^2\right)\\ &= \frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{10}\,\text{Subst}\left(\int\left(-18-\frac{85x}{2}\right)x\sqrt{3+5x+x^2}\,dx,x,\right.\\ &= \frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-510x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{1633}{64}\,\text{Su}\\ &= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-\right.\\ &= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-\right.\\ &= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-\right.\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 0.70

$$\frac{318435\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+2\sqrt{x^4+5x^2+3}\left(1152x^8+1680x^6-2248x^4+12250x^2-78387\right)}{7680}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-78387 + 12250*x^2 - 2248*x^4 + 1680*x^6 + 1152*x
^8) + 318435*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/7680
```

fricas [A] time = 0.84, size = 61, normalized size = 0.60

$$\frac{1}{3840}\left(1152\,x^8+1680\,x^6-2248\,x^4+12250\,x^2-78387\right)\sqrt{x^4+5\,x^2+3}-\frac{21229}{512}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3840*(1152*x^8 + 1680*x^6 - 2248*x^4 + 12250*x^2 - 78387)*sqrt(x^4 + 5*x^
2 + 3) - 21229/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.46, size = 102, normalized size = 1.00

$$\frac{1}{1280}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\left(8\,x^2+5\right)x^2-127\right)x^2+2635\right)x^2-33429\right)+\frac{1}{192}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\,x^2+5\right)x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x^
2 - 33429) + 1/192*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 +
1095) - 21229/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.03, size = 91, normalized size = 0.89

$$\frac{3\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^4}{10}-\frac{17\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2}{16}+\frac{21229\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{512}+\frac{1837\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{480}-\frac{1633\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/10*x^4*(x^4+5*x^2+3)^(3/2)-17/16*x^2*(x^4+5*x^2+3)^(3/2)+1837/480*(x^4+5*
x^2+3)^(3/2)-1633/256*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+21229/512*ln(5/2+x^2+(x
^4+5*x^2+3)^(1/2))
```

maxima [A] time = 0.67, size = 104, normalized size = 1.02

$$\frac{3}{10}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^4-\frac{17}{16}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2-\frac{1633}{128}\sqrt{x^4+5\,x^2+3}\,x^2+\frac{1837}{480}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{8165}{256}\sqrt{x^4+5\,x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5*x^2 + 3)^(3/2)*x^4 - 17/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 1633
/128*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1837/480*(x^4 + 5*x^2 + 3)^(3/2) - 8165/25
6*sqrt(x^4 + 5*x^2 + 3) + 21229/512*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5
)
```

mupad [B] time = 0.49, size = 102, normalized size = 1.00

$$\frac{21229\,\ln\left(\sqrt{x^4+5\,x^2+3}\,+x^2+\frac{5}{2}\right)}{512}-\frac{17\,x^2\,\left(x^4+5\,x^2+3\right)^{3/2}}{16}+\frac{3\,x^4\,\left(x^4+5\,x^2+3\right)^{3/2}}{10}+\frac{51\,\left(\frac{x^2}{2}+\frac{5}{4}\right)\,\sqrt{x^4+5\,x^2+}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (21229*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/512 - (17*x^2*(5*x^2 + x^4
 + 3)^(3/2))/16 + (3*x^4*(5*x^2 + x^4 + 3)^(3/2))/10 + (51*(x^2/2 + 5/4)*(5
*x^2 + x^4 + 3)^(1/2))/16 + (1837*(5*x^2 + x^4 + 3)^(1/2)*(10*x^2 + 8*x^4 -
 51))/3840
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

3.143 $\int x^3 \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\, dx$

Optimal. Leaf size=81

$$-\frac{1}{48}\left(59-18x^2\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{259}{128}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{3367}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] -1/48*(-18*x^2+59)*(x^4+5*x^2+3)^(3/2)-3367/256*arctanh(1/2*(2*x^2+5)/(x^4+
5*x^2+3)^(1/2))+259/128*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 779, 612, 621, 206}

$$-\frac{1}{48}\left(59-18x^2\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{259}{128}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{3367}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (259*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 - ((59 - 18*x^2)*(3 + 5*x^2 + x
^4)^(3/2))/48 - (3367*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3 \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\, dx = \frac{1}{2} \text{Subst}\left(\int x(2+3x)\sqrt{3+5x+x^2}\, dx, x, x^2\right)$$
$$= -\frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{259}{32}\text{Subst}\left(\int \sqrt{3+5x+x^2}\, dx, x, x^2\right)$$
$$= \frac{259}{128}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} - \frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2} - \frac{3367}{256}\text{Sub}$$
$$= \frac{259}{128}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} - \frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2} - \frac{3367}{128}\text{Sub}$$
$$= \frac{259}{128}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} - \frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2} - \frac{3367}{256}\tan$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.81

$$\frac{1}{768}\left(2\sqrt{x^4+5x^2+3}\left(144x^6+248x^4-374x^2+2469\right)-10101\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(2469 - 374*x^2 + 248*x^4 + 144*x^6) - 10101*ArcTa
nh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/768
```

fricas [A] time = 0.71, size = 56, normalized size = 0.69

$$\frac{1}{384}\left(144\,x^6+248\,x^4-374\,x^2+2469\right)\sqrt{x^4+5\,x^2+3}+\frac{3367}{256}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(144*x^6 + 248*x^4 - 374*x^2 + 2469)*sqrt(x^4 + 5*x^2 + 3) + 3367/256
*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.36, size = 88, normalized size = 1.09

$$\frac{1}{128}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\,x^2+5\right)x^2-89\right)x^2+1095\right)+\frac{1}{24}\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2+5\right)x^2-51\right)+\frac{3367}{256}\log\left(2\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 1/24*
sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 3367/256*log(2*x^2 - 2*sqr
t(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 74, normalized size = 0.91

$$\frac{3\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2}{8}-\frac{3367\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{256}-\frac{59\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{48}+\frac{259\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/8*(x^4+5*x^2+3)^(3/2)*x^2-59/48*(x^4+5*x^2+3)^(3/2)+259/128*(2*x^2+5)*(x^
4+5*x^2+3)^(1/2)-3367/256*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

maxima [A] time = 0.59, size = 87, normalized size = 1.07

$$\frac{3}{8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2+\frac{259}{64}\sqrt{x^4+5x^2+3}\,x^2-\frac{59}{48}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{1295}{128}\sqrt{x^4+5x^2+3}-\frac{3367}{256}\log\left(2x^2+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 259/64*sqrt(x^4 + 5*x^2 + 3)*x^2 - 59/48*
(x^4 + 5*x^2 + 3)^(3/2) + 1295/128*sqrt(x^4 + 5*x^2 + 3) - 3367/256*log(2*x
^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.43, size = 85, normalized size = 1.05

$$\frac{3x^2\left(x^4+5x^2+3\right)^{3/2}}{8}-\frac{3367\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{256}-\frac{9\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3}}{8}-\frac{59\sqrt{x^4+5x^2+3}\,\left(\right.}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (3*x^2*(5*x^2 + x^4 + 3)^(3/2))/8 - (3367*log((5*x^2 + x^4 + 3)^(1/2) + x^2
 + 5/2))/256 - (9*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1/2))/8 - (59*(5*x^2 + x
^4 + 3)^(1/2)*(10*x^2 + 8*x^4 - 51))/384
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

3.144 $\int x\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=74

$$\frac{1}{2}\left(x^4+5x^2+3\right)^{3/2}-\frac{11}{16}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{143}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] 1/2*(x^4+5*x^2+3)^(3/2)+143/32*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1
1/16*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1247, 640, 612, 621, 206}

$$\frac{1}{2}\left(x^4+5x^2+3\right)^{3/2}-\frac{11}{16}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{143}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-11*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 + (3 + 5*x^2 + x^4)^(3/2)/2 + (1
43*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int x\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx=\frac{1}{2}\text{Subst}\left(\int(2+3x)\sqrt{3+5x+x^2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\left(3+5x^2+x^4\right)^{3/2}-\frac{11}{4}\text{Subst}\left(\int\sqrt{3+5x+x^2}\,dx,x,x^2\right)$$
$$=-\frac{11}{16}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{2}\left(3+5x^2+x^4\right)^{3/2}+\frac{143}{32}\text{Subst}\left(\int\frac{}{\sqrt{3+}}\right.$$
$$=-\frac{11}{16}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{2}\left(3+5x^2+x^4\right)^{3/2}+\frac{143}{16}\text{Subst}\left(\int\frac{1}{4-x}\right.$$
$$=-\frac{11}{16}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{2}\left(3+5x^2+x^4\right)^{3/2}+\frac{143}{32}\tanh^{-1}\left(\frac{5}{2\sqrt{3+}}\right.$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.82

$$\frac{1}{32}\left(2\sqrt{x^4+5x^2+3}\left(8x^4+18x^2-31\right)+143\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-31 + 18*x^2 + 8*x^4) + 143*ArcTanh[(5 + 2*x^2)/(
2*Sqrt[3 + 5*x^2 + x^4])])/32
```

fricas [A] time = 0.66, size = 51, normalized size = 0.69

$$\frac{1}{16}\left(8\,x^4+18\,x^2-31\right)\sqrt{x^4+5\,x^2+3}-\frac{143}{32}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*x^4 + 18*x^2 - 31)*sqrt(x^4 + 5*x^2 + 3) - 143/32*log(-2*x^2 + 2*sq
rt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.34, size = 74, normalized size = 1.00

$$\frac{1}{16}\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2+5\right)x^2-51\right)+\frac{1}{4}\sqrt{x^4+5\,x^2+3}\left(2\,x^2+5\right)-\frac{143}{32}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 1/4*sqrt(x^4 + 5*x^2
+ 3)*(2*x^2 + 5) - 143/32*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.01, size = 57, normalized size = 0.77

$$\frac{143\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}+\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{2}-\frac{11\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/2*(x^4+5*x^2+3)^(3/2)-11/16*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+143/32*ln(x^2+5
/2+(x^4+5*x^2+3)^(1/2))
```

maxima [A] time = 0.65, size = 70, normalized size = 0.95

$$-\frac{11}{8}\sqrt{x^4+5x^2+3}\,x^2+\frac{1}{2}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{55}{16}\sqrt{x^4+5x^2+3}+\frac{143}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -11/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 55/16*sqrt(
x^4 + 5*x^2 + 3) + 143/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.29, size = 67, normalized size = 0.91

$$\frac{143\,\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{32}+\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3}+\frac{\sqrt{x^4+5x^2+3}\,\left(8x^4+10x^2-51\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (143*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/32 + (x^2/2 + 5/4)*(5*x^2 +
x^4 + 3)^(1/2) + ((5*x^2 + x^4 + 3)^(1/2)*(10*x^2 + 8*x^4 - 51))/16
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

3.145 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\,(6x^2+23)+\frac{1}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\sqrt{3}\,\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\,\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] 1/16*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-arctanh(1/6*(5*x^2+6)*3^(1/
2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/8*(6*x^2+23)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\,(6x^2+23)+\frac{1}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\sqrt{3}\,\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\,\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

```
[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*
x^2 + x^4])]/16 - Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x
^4])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(2+3x^2\right)\sqrt{3+5x^2+x^4}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x}\,dx, x, x^2\right)\\ &= \frac{1}{8}\left(23+6x^2\right)\sqrt{3+5x^2+x^4} - \frac{1}{8}\text{Subst}\left(\int \frac{-24-\frac{x}{2}}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{8}\left(23+6x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{16}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right) + 3\,\text{Subs}\\ &= \frac{1}{8}\left(23+6x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{8}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right) - 6\,\text{S}\\ &= \frac{1}{8}\left(23+6x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \sqrt{3}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.\end{aligned}$$

Mathematica [A] time = 0.04, size = 92, normalized size = 0.98

$$\frac{1}{16}\left(2\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-16\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

```
[Out] (2*(23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4] + ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x
^2 + x^4])] - 16*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^
4])])/16
```

fricas [A] time = 0.68, size = 95, normalized size = 1.01

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-\frac{1}{16}\log\left(-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 23) + sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5
*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1/16*log(-
2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.48, size = 98, normalized size = 1.04

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)-\frac{1}{16}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 23) + sqrt(3)*log((x^2 + sqrt(3) - sqrt(
x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) - 1/16*log(2*x^2
 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 85, normalized size = 0.90

$$-\sqrt{3}\ \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)+\frac{\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16}+\frac{3\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{8}+\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x)
```

```
[Out] 3/8*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+1/16*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+(x^4
+5*x^2+3)^(1/2)-arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

maxima [A] time = 1.50, size = 89, normalized size = 0.95

$$\frac{3}{4}\sqrt{x^4+5x^2+3}\,x^2-\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{23}{8}\sqrt{x^4+5x^2+3}+\frac{1}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)
/x^2 + 6/x^2 + 5) + 23/8*sqrt(x^4 + 5*x^2 + 3) + 1/16*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.43, size = 86, normalized size = 0.91

$$\frac{\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{16}-\sqrt{3}\ \ln\left(\frac{3}{x^2}+\frac{\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{5}{2}\right)+\frac{3\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3}}{2}+\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x,x)
```

```
[Out] log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2)/16 - 3^(1/2)*log(3/x^2 + (3^(1/2)*
(5*x^2 + x^4 + 3)^(1/2))/x^2 + 5/2) + (3*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1
/2))/2 + (5*x^2 + x^4 + 3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x, x)
```

3.146 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-3x^2)}{2x^2}+\frac{19}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{7\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

```
[Out] 19/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-7/3*arctanh(1/6*(5*x^2+6)*3
^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/2*(-3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 812, 843, 621, 206, 724}

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-3x^2)}{2x^2}+\frac{19}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{7\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] -((2 - 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (19*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^
2 + x^4])])/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^2}\,dx,x,x^2\right)$$
$$= -\frac{\left(2-3x^2\right)\sqrt{3+5x^2+x^4}}{2x^2}-\frac{1}{4}\text{Subst}\left(\int \frac{-28-19x}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{\left(2-3x^2\right)\sqrt{3+5x^2+x^4}}{2x^2}+\frac{19}{4}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)+7\,\text{Sub}$$
$$= -\frac{\left(2-3x^2\right)\sqrt{3+5x^2+x^4}}{2x^2}+\frac{19}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)-1$$
$$= -\frac{\left(2-3x^2\right)\sqrt{3+5x^2+x^4}}{2x^2}+\frac{19}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\frac{7\tanh^{-1}\left(\frac{\cdots}{2\sqrt{3}\cdots}\right)}{\sqrt{3}\cdots}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 1.00

$$\frac{\sqrt{x^4+5x^2+3}\left(3x^2-2\right)}{2x^2}+\frac{19}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{7\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] ((-2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (19*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^
2 + x^4])])/Sqrt[3]
```

fricas [A] time = 0.68, size = 112, normalized size = 1.15

$$\frac{56\sqrt{3}\,x^2\log\left(\frac{25\,x^2-2\,\sqrt{3}\left(5\,x^2+6\right)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)-114\,x^2\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+21\,x^2+1\cdots}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(56*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x
^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 114*x^2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) - 5) + 21*x^2 + 12*sqrt(x^4 + 5*x^2 + 3)*(3*x^2 - 2))/x^2
```

giac [A] time = 0.51, size = 138, normalized size = 1.42

$$\frac{7}{3}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{3}{2}\sqrt{x^4+5x^2+3}+\frac{5x^2-5\sqrt{x^4+5x^2+3}+6}{\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3}-\frac{19}{4}\log\left(2x^2-2\sqrt{x^4+\dots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 7/3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) + 3/2*sqrt(x^4 + 5*x^2 + 3) + (5*x^2 - 5*sqrt(x^4 + 5
*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3) - 19/4*log(2*x^2 - 2*s
qrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 104, normalized size = 1.07

$$-\frac{7\sqrt{3}\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{3}+\frac{19\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{3x^2}+\frac{7\sqrt{x^4+5x^2+3}}{3}+\frac{\left(2x^2+5\right)\sqrt{\dots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x)
```

```
[Out] 7/3*(x^4+5*x^2+3)^(1/2)+19/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-7/3*arctanh(1/
6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3/x^2*(x^4+5*x^2+3)^(3/2
)+1/6*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 1.35, size = 89, normalized size = 0.92

$$-\frac{7}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{\sqrt{x^4+5x^2+3}}{x^2}+\frac{19}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+\dots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -7/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*sqr
t(x^4 + 5*x^2 + 3) - sqrt(x^4 + 5*x^2 + 3)/x^2 + 19/4*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.88, size = 84, normalized size = 0.87

$$\frac{19\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{4}-\frac{\sqrt{x^4+5x^2+3}}{x^2}-\frac{7\sqrt{3}\ln\left(\frac{3}{x^2}+\frac{\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{5}{2}\right)}{3}+\frac{3\sqrt{x^4+5x^2+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^3,x)
```

```
[Out] (19*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/4 - (5*x^2 + x^4 + 3)^(1/2)/x
^2 - (7*3^(1/2)*log(3/x^2 + (3^(1/2)*(5*x^2 + x^4 + 3)^(1/2))/x^2 + 5/2))/3
 + (3*(5*x^2 + x^4 + 3)^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**3,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**3, x)
```

3.147 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{x^4+5x^2+3}\,(23x^2+6)}{12x^4}+\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{77\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

```
[Out] 3/2*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-77/72*arctanh(1/6*(5*x^2+6)*
3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/12*(23*x^2+6)*(x^4+5*x^2+3)^(1/2)/x^
4
```

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 810, 843, 621, 206, 724}

$$-\frac{\sqrt{x^4+5x^2+3}\,(23x^2+6)}{12x^4}+\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{77\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

```
[Out] -((6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^4) + (3*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/2 - (77*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*
x^2 + x^4])])/(24*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
```

```
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} - \frac{1}{24}\text{Subst}\left(\int \frac{-77-36x}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right) + \frac{77}{24}\text{Subs}$$

$$= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + 3\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right) - \frac{77}{12}$$

$$= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{77\tanh^{-1}\left(\frac{6}{2\sqrt{3}}\right.}{24\sqrt{3}}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 0.98

$$\frac{1}{72}\left(-\frac{6\sqrt{x^4+5x^2+3}\,(23x^2+6)}{x^4} + 108\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 77\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

```
[Out] ((-6*(6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + 108*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])] - 77*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 +
 5*x^2 + x^4])])/72
```

fricas [A] time = 0.65, size = 112, normalized size = 1.13

$$\frac{77\sqrt{3}\,x^4\log\left(\frac{25x^2-2\sqrt{3}\,(5x^2+6)-2\sqrt{x^4+5x^2+3}\,(5\sqrt{3}-6)+30}{x^2}\right) - 108x^4\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right) - 138x^4 - 6\sqrt{}}{72x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/72*(77*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x
^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 108*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) - 5) - 138*x^4 - 6*sqrt(x^4 + 5*x^2 + 3)*(23*x^2 + 6))/x^4
```

giac [B] time = 0.57, size = 169, normalized size = 1.71

$$\frac{77}{72}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{127\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+228\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-159x^2 -}{12\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 77/72*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
sqrt(x^4 + 5*x^2 + 3))) + 1/12*(127*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 228*(
x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 159*x^2 + 159*sqrt(x^4 + 5*x^2 + 3) - 324)
/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^2 - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*x
^2 + 3) + 5)
```

maple [A] time = 0.02, size = 121, normalized size = 1.22

$$-\frac{77\sqrt{3}\ \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{72}+\frac{3\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{13\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{36x^2}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{6x^4}+\frac{77\sqrt{x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x)
```

```
[Out] -1/6/x^4*(x^4+5*x^2+3)^(3/2)-13/36*(x^4+5*x^2+3)^(3/2)/x^2+77/72*(x^4+5*x^2
+3)^(1/2)-77/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+
13/72*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+3/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

maxima [A] time = 1.43, size = 106, normalized size = 1.07

$$-\frac{77}{72}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{1}{6}\sqrt{x^4+5x^2+3}-\frac{13\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{6x^4}+\frac{3}{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -77/72*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/6*s
qrt(x^4 + 5*x^2 + 3) - 13/12*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/6*(x^4 + 5*x^2 +
 3)^(3/2)/x^4 + 3/2*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^5,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**5,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**5, x)
```

3.148 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx$

Optimal. Leaf size=90

$$-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4}+\frac{13\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{36\sqrt{3}}-\frac{(x^4+5x^2+3)^{3/2}}{9x^6}$$

```
[Out] -1/9*(x^4+5*x^2+3)^(3/2)/x^6+13/108*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^
2+3)^(1/2))*3^(1/2)-1/18*(5*x^2+6)*(x^4+5*x^2+3)^(1/2)/x^4
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 806, 720, 724, 206}

$$-\frac{(x^4+5x^2+3)^{3/2}}{9x^6}-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4}+\frac{13\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{36\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

```
[Out] -((6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(18*x^4) - (3 + 5*x^2 + x^4)^(3/2)/(9*
x^6) + (13*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(36*Sqrt
[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} - \frac{13}{36}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\right.$$
$$= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{13}{18}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, -\right.$$
$$= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{13\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{36\sqrt{3}}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 0.82

$$\frac{1}{108}\left(13\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) - \frac{6\sqrt{x^4+5x^2+3}\,(7x^4+16x^2+6)}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

```
[Out] ((-6*Sqrt[3 + 5*x^2 + x^4]*(6 + 16*x^2 + 7*x^4))/x^6 + 13*Sqrt[3]*ArcTanh[(
6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/108
```

fricas [A] time = 0.74, size = 90, normalized size = 1.00

$$\frac{13\sqrt{3}\,x^6\log\left(\frac{25\,x^2+2\,\sqrt{3}\,(5\,x^2+6)+2\,\sqrt{x^4+5\,x^2+3}\,(5\,\sqrt{3}+6)+30}{x^2}\right) - 42\,x^6 - 6\,(7\,x^4+16\,x^2+6)\sqrt{x^4+5\,x^2+3}}{108\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/108*(13*sqrt(3)*x^6*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 42*x^6 - 6*(7*x^4 + 16*x^2 + 6)*sqrt(
x^4 + 5*x^2 + 3))/x^6
```

giac [B] time = 0.53, size = 189, normalized size = 2.10

$$-\frac{13}{108}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) + \frac{67\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^5 + 306\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^4 + 430\left(x^2-\right.}{18\left(\left(x^2-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -13/108*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3)
- sqrt(x^4 + 5*x^2 + 3))) + 1/18*(67*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 306*
(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 + 430*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 90*
(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 63*x^2 + 63*sqrt(x^4 + 5*x^2 + 3) + 108)/
((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^3
```

maple [A] time = 0.02, size = 118, normalized size = 1.31

$$\frac{13\sqrt{3}\ \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{108}+\frac{5\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{54x^2}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{9x^4}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{9x^6}-\frac{13\sqrt{x^4+5x^2+3}}{108}-\frac{5\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x)
```

```
[Out] -1/9*(x^4+5*x^2+3)^(3/2)/x^6-1/9*(x^4+5*x^2+3)^(3/2)/x^4+5/54*(x^4+5*x^2+3)
^(3/2)/x^2-13/108*(x^4+5*x^2+3)^(1/2)+13/108*arctanh(1/6*(5*x^2+6)*3^(1/2)/
(x^4+5*x^2+3)^(1/2))*3^(1/2)-5/108*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 1.45, size = 99, normalized size = 1.10

$$\frac{13}{108}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{1}{9}\sqrt{x^4+5x^2+3}+\frac{5\sqrt{x^4+5x^2+3}}{18x^2}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{9x^4}-\frac{\left(x^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 13/108*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/9*s
qrt(x^4 + 5*x^2 + 3) + 5/18*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/9*(x^4 + 5*x^2 +
3)^(3/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(3/2)/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^7,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**7,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**7, x)
```

3.149 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx$

Optimal. Leaf size=111

$$\frac{67\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{1728x^4}-\frac{871\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3456\sqrt{3}}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{12x^8}-\frac{11\left(x^4+5x^2+3\right)^{3/2}}{216x^6}$$

```
[Out] -1/12*(x^4+5*x^2+3)^(3/2)/x^8-11/216*(x^4+5*x^2+3)^(3/2)/x^6-871/10368*arct
anh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+67/1728*(5*x^2+6)*(x
^4+5*x^2+3)^(1/2)/x^4
```

Rubi [A] time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 834, 806, 720, 724, 206}

$$-\frac{11\left(x^4+5x^2+3\right)^{3/2}}{216x^6}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{12x^8}+\frac{67\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{1728x^4}-\frac{871\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3456\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

```
[Out] (67*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(1728*x^4) - (3 + 5*x^2 + x^4)^(3/2)
/(12*x^8) - (11*(3 + 5*x^2 + x^4)^(3/2))/(216*x^6) - (871*ArcTanh[(6 + 5*x^
2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(3456*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^5}\,dx,x,x^2\right)\\
&= -\frac{(3+5x^2+x^4)^{3/2}}{12x^8}-\frac{1}{24}\operatorname{Subst}\left(\int \frac{(-11+2x)\sqrt{3+5x+x^2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{(3+5x^2+x^4)^{3/2}}{12x^8}-\frac{11(3+5x^2+x^4)^{3/2}}{216x^6}-\frac{67}{144}\operatorname{Subst}\left(\int \frac{\sqrt{3+5x+x^2}}{x^3}\,d\right.\\
&= \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4}-\frac{(3+5x^2+x^4)^{3/2}}{12x^8}-\frac{11(3+5x^2+x^4)^{3/2}}{216x^6}+-\\
&= \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4}-\frac{(3+5x^2+x^4)^{3/2}}{12x^8}-\frac{11(3+5x^2+x^4)^{3/2}}{216x^6}--\\
&= \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4}-\frac{(3+5x^2+x^4)^{3/2}}{12x^8}-\frac{11(3+5x^2+x^4)^{3/2}}{216x^6}--
\end{aligned}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 0.74

$$\frac{6\sqrt{x^4+5x^2+3}\left(247x^6-182x^4-984x^2-432\right)-871\sqrt{3}\,x^8\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

```
[Out] (6*Sqrt[3 + 5*x^2 + x^4]*(-432 - 984*x^2 - 182*x^4 + 247*x^6) - 871*Sqrt[3]
*x^8*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(10368*x^8)
```

fricas [A] time = 0.63, size = 95, normalized size = 0.86

$$\frac{871\sqrt{3}\,x^8\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)+1482x^8+6\left(247x^6-182x^4-984x^2-432\right)\sqrt{x^4+5}}{10368x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/10368*(871*sqrt(3)*x^8*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 +
 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 1482*x^8 + 6*(247*x^6 - 182*x^4 -
984*x^2 - 432)*sqrt(x^4 + 5*x^2 + 3))/x^8
```

giac [B] time = 0.46, size = 233, normalized size = 2.10

$$\frac{871}{10368}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)-\frac{871\left(x^2-\sqrt{x^4+5x^2+3}\right)^7-5184\left(x^2-\sqrt{x^4+5x^2+3}\right)^6-57389}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 871/10368*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3
) - sqrt(x^4 + 5*x^2 + 3))) - 1/1728*(871*(x^2 - sqrt(x^4 + 5*x^2 + 3))^7 -
 5184*(x^2 - sqrt(x^4 + 5*x^2 + 3))^6 - 57389*(x^2 - sqrt(x^4 + 5*x^2 + 3))
^5 - 165888*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 - 204807*(x^2 - sqrt(x^4 + 5*x^
2 + 3))^3 - 93312*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 2403*x^2 + 2403*sqrt(x^
4 + 5*x^2 + 3) - 5184)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^4
```

maple [A] time = 0.02, size = 135, normalized size = 1.22

$$-\frac{871\sqrt{3}\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{10368}-\frac{335\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{5184x^2}+\frac{67\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{864x^4}-\frac{11\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{216x^6}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{12x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x)
```

```
[Out] -11/216*(x^4+5*x^2+3)^(3/2)/x^6+67/864*(x^4+5*x^2+3)^(3/2)/x^4-335/5184*(x^
4+5*x^2+3)^(3/2)/x^2+871/10368*(x^4+5*x^2+3)^(1/2)-871/10368*arctanh(1/6*(5
*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+335/10368*(2*x^2+5)*(x^4+5*x^2
+3)^(1/2)-1/12*(x^4+5*x^2+3)^(3/2)/x^8
```

maxima [A] time = 1.74, size = 116, normalized size = 1.05

$$-\frac{871}{10368}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{67}{864}\sqrt{x^4+5x^2+3}-\frac{335\sqrt{x^4+5x^2+3}}{1728x^2}+\frac{67\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{864x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -871/10368*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 6
7/864*sqrt(x^4 + 5*x^2 + 3) - 335/1728*sqrt(x^4 + 5*x^2 + 3)/x^2 + 67/864*(
x^4 + 5*x^2 + 3)^(3/2)/x^4 - 11/216*(x^4 + 5*x^2 + 3)^(3/2)/x^6 - 1/12*(x^4
 + 5*x^2 + 3)^(3/2)/x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^9,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**9,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**9, x)
```

3.150 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}} dx$

Optimal. Leaf size=132

$$-\frac{161\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{5184x^4}+\frac{2093\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368\sqrt{3}}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{15x^{10}}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{36x^8}+\frac{173\left(x^4+5\right.}{3240}$$

```
[Out] -1/15*(x^4+5*x^2+3)^(3/2)/x^10-1/36*(x^4+5*x^2+3)^(3/2)/x^8+173/3240*(x^4+5
*x^2+3)^(3/2)/x^6+2093/31104*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1
/2))*3^(1/2)-161/5184*(5*x^2+6)*(x^4+5*x^2+3)^(1/2)/x^4
```

Rubi [A] time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 834, 806, 720, 724, 206}

$$\frac{173\left(x^4+5x^2+3\right)^{3/2}}{3240x^6}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{36x^8}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{15x^{10}}-\frac{161\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{5184x^4}+\frac{2093\tanh^{-1}\left(\frac{5x^2}{2\sqrt{3}\sqrt{x^4}}\right.}{10368\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] (-161*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5184*x^4) - (3 + 5*x^2 + x^4)^(3/
2)/(15*x^10) - (3 + 5*x^2 + x^4)^(3/2)/(36*x^8) + (173*(3 + 5*x^2 + x^4)^(3
/2))/(3240*x^6) + (2093*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4
])])/(10368*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{1}{30}\text{Subst}\left(\int \frac{(-10+4x)\sqrt{3+5x+x^2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{1}{360}\text{Subst}\left(\int \frac{(-173-10x)\sqrt{3+}}{x^4}\right.$$

$$= -\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{173(3+5x^2+x^4)^{3/2}}{3240x^6} + \frac{161}{432}\text{Subs}$$

$$= -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{17}{}$$

$$= -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{17}{}$$

$$= -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{17}{}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 0.64

$$\frac{10465\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) - \frac{6\sqrt{x^4+5x^2+3}\left(2641x^8-1370x^6+1176x^4+10800x^2+5184\right)}{x^{10}}}{155520}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] ((-6*Sqrt[3 + 5*x^2 + x^4]*(5184 + 10800*x^2 + 1176*x^4 - 1370*x^6 + 2641*x
^8))/x^10 + 10465*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x
^4])])/155520
```

fricas [A] time = 0.72, size = 100, normalized size = 0.76

$$\frac{10465\,\sqrt{3}\,x^{10}\log\left(\frac{25\,x^2+2\,\sqrt{3}\,(5\,x^2+6)+2\,\sqrt{x^4+5\,x^2+3}\,(5\,\sqrt{3}+6)+30}{x^2}\right)-15846\,x^{10}-6\,\left(2641\,x^8-1370\,x^6+1176\,x^4+10800\right.}{155520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/155520*(10465*sqrt(3)*x^10*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x
^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 15846*x^10 - 6*(2641*x^8 - 137
0*x^6 + 1176*x^4 + 10800*x^2 + 5184)*sqrt(x^4 + 5*x^2 + 3))/x^10
```

giac [B] time = 0.56, size = 255, normalized size = 1.93

$$-\frac{2093}{31104}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right)+\frac{10465\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^9-42830\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^7+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] -2093/31104*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt
(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/25920*(10465*(x^2 - sqrt(x^4 + 5*x^2 + 3)
)^9 - 42830*(x^2 - sqrt(x^4 + 5*x^2 + 3))^7 + 1270080*(x^2 - sqrt(x^4 + 5*x
^2 + 3))^6 + 7060800*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 15310080*(x^2 - sqrt
(x^4 + 5*x^2 + 3))^4 + 16095870*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 7568640*(
x^2 - sqrt(x^4 + 5*x^2 + 3))^2 + 1096335*x^2 - 1096335*sqrt(x^4 + 5*x^2 + 3
) + 202176)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^5
```

maple [A] time = 0.02, size = 152, normalized size = 1.15

$$\frac{2093\sqrt{3}\ \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{31104}+\frac{805\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{15552x^2}-\frac{161\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{2592x^4}+\frac{173\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{3240x^6}-\frac{\left(x^4+5x^2+3\right.}{36x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x)
```

```
[Out] -1/15*(x^4+5*x^2+3)^(3/2)/x^10-1/36*(x^4+5*x^2+3)^(3/2)/x^8+173/3240*(x^4+5
*x^2+3)^(3/2)/x^6-161/2592*(x^4+5*x^2+3)^(3/2)/x^4+805/15552*(x^4+5*x^2+3)^
(3/2)/x^2-2093/31104*(x^4+5*x^2+3)^(1/2)+2093/31104*arctanh(1/6*(5*x^2+6)*3
^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-805/31104*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 1.73, size = 133, normalized size = 1.01

$$\frac{2093}{31104}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{161}{2592}\sqrt{x^4+5\,x^2+3}+\frac{805\sqrt{x^4+5\,x^2+3}}{5184\,x^2}-\frac{161\left(x^4+5\,x^2+3\right)}{2592\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] 2093/31104*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1
61/2592*sqrt(x^4 + 5*x^2 + 3) + 805/5184*sqrt(x^4 + 5*x^2 + 3)/x^2 - 161/25
92*(x^4 + 5*x^2 + 3)^(3/2)/x^4 + 173/3240*(x^4 + 5*x^2 + 3)^(3/2)/x^6 - 1/3
6*(x^4 + 5*x^2 + 3)^(3/2)/x^8 - 1/15*(x^4 + 5*x^2 + 3)^(3/2)/x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^11,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^11, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**11,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**11, x)
```

3.151 $\int x^4 \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\, dx$

Optimal. Leaf size=322

$$\frac{13}{3}\sqrt{x^4+5x^2+3}\,x-\frac{1924\left(2x^2+\sqrt{13}+5\right)x}{105\sqrt{x^4+5x^2+3}}-\frac{13\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\right.}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$

```
[Out] -1924/105*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+13/3*x*(x^4+5*x^2+3)^(1/
2)-26/35*x^3*(x^4+5*x^2+3)^(1/2)+1/21*x^5*(7*x^2+11)*(x^4+5*x^2+3)^(1/2)+96
2/315*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*Ell
ipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*1
3^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1
/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-13*(1/(36+x^2*(30+6*1
3^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))
^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(
5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3
)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1273, 1279, 1189, 1099, 1135}

$$\frac{1}{21}\left(7x^2+11\right)\sqrt{x^4+5x^2+3}\,x^5-\frac{26}{35}\sqrt{x^4+5x^2+3}\,x^3+\frac{13}{3}\sqrt{x^4+5x^2+3}\,x-\frac{1924\left(2x^2+\sqrt{13}+5\right)x}{105\sqrt{x^4+5x^2+3}}-\frac{13\sqrt{\frac{\left(5-\sqrt{13}\right.}{\left(5+\sqrt{13}\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-1924*x*(5 + Sqrt[13] + 2*x^2))/(105*Sqrt[3 + 5*x^2 + x^4]) + (13*x*Sqrt[3
 + 5*x^2 + x^4])/3 - (26*x^3*Sqrt[3 + 5*x^2 + x^4])/35 + (x^5*(11 + 7*x^2)*
Sqrt[3 + 5*x^2 + x^4])/21 + (962*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(105*Sqrt[3 + 5*x
^2 + x^4]) - (13*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1273

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*
p + 3)), x] + Dist[(2*p)/(c*(4*p + m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int x^4\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx=\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{63}\int\frac{x^4\left(-117-234x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx$$

$$=-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{315}\int\frac{x^2\left(-21\right.}{\sqrt{3\cdots}}$$

$$=\frac{13}{3}x\sqrt{3+5x^2+x^4}-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+\cdots}$$

$$=\frac{13}{3}x\sqrt{3+5x^2+x^4}-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+\cdots}$$

$$=-\frac{1924x\left(5+\sqrt{13}+2x^2\right)}{105\sqrt{3+5x^2+x^4}}+\frac{13}{3}x\sqrt{3+5x^2+x^4}-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\cdots$$

Mathematica [C] time = 0.38, size = 237, normalized size = 0.74

$$\frac{70x^{11}+460x^9+604x^7+460x^5+4082x^3+13i\sqrt{2}\left(148\sqrt{13}-635\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,F\left(i\sinh\cdots\right.}{2\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2730*x + 4082*x^3 + 460*x^5 + 604*x^7 + 460*x^9 + 70*x^11 - (1924*I)*Sqrt[
2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + S
qrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*S
qrt[13])/6] + (13*I)*Sqrt[2]*(-635 + 148*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*
x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2
/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(210*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(3\,x^6+2\,x^4\right)\sqrt{x^4+5\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)*sqrt(x^4 + 5*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4, x)
```

maple [A] time = 0.12, size = 260, normalized size = 0.81

$$\frac{\sqrt{x^4+5x^2+3}\;x^7}{3}+\frac{11\sqrt{x^4+5x^2+3}\;x^5}{21}-\frac{26\sqrt{x^4+5x^2+3}\;x^3}{35}+\frac{13\sqrt{x^4+5x^2+3}\;x}{3}-\frac{78\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\;\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/3*x^7*(x^4+5*x^2+3)^(1/2)+11/21*x^5*(x^4+5*x^2+3)^(1/2)-26/35*x^3*(x^4+5*
x^2+3)^(1/2)+13/3*x*(x^4+5*x^2+3)^(1/2)-78/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+
1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1
/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+46176/
35/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13
^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6
*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))
^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(3x^2+2\right) \sqrt{x^4+5x^2+3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

3.152 $\int x^2 \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\, dx$

Optimal. Leaf size=305

$$-\frac{4}{3}\sqrt{x^4+5x^2+3}\,x+\frac{1247\left(2x^2+\sqrt{13}+5\right)x}{210\sqrt{x^4+5x^2+3}}+\frac{2\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\right.\right.\right.}{\sqrt{x^4+5x^2+3}}$$

```
[Out] 1247/210*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-4/3*x*(x^4+5*x^2+3)^(1/2)
+1/35*x^3*(15*x^2+29)*(x^4+5*x^2+3)^(1/2)+2/3*(1/(36+x^2*(30+6*13^(1/2))))^
(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2))
)*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1
/2)/(x^4+5*x^2+3)^(1/2)-1247/1260*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^
2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13
^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(
1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)
^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1273, 1279, 1189, 1099, 1135}

$$\frac{1}{35}\left(15x^2+29\right)\sqrt{x^4+5x^2+3}\,x^3-\frac{4}{3}\sqrt{x^4+5x^2+3}\,x+\frac{1247\left(2x^2+\sqrt{13}+5\right)x}{210\sqrt{x^4+5x^2+3}}+\frac{2\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (1247*x*(5 + Sqrt[13] + 2*x^2))/(210*Sqrt[3 + 5*x^2 + x^4]) - (4*x*Sqrt[3 +
 5*x^2 + x^4])/3 + (x^3*(29 + 15*x^2)*Sqrt[3 + 5*x^2 + x^4])/35 - (1247*Sqr
t[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(210*Sqrt[3 + 5*x^2 + x^4]) + (2*Sqrt[2/(3*(5 + Sqrt[13])
)]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[1
3])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])
/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1273

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*
p + 3)), x] + Dist[(2*p)/(c*(4*p + m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int x^2\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx=\frac{1}{35}x^3\left(29+15x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{35}\int\frac{x^2\left(-51-140x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx$$

$$=-\frac{4}{3}x\sqrt{3+5x^2+x^4}+\frac{1}{35}x^3\left(29+15x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{105}\int\frac{-420-}{\sqrt{3+5x}}$$

$$=-\frac{4}{3}x\sqrt{3+5x^2+x^4}+\frac{1}{35}x^3\left(29+15x^2\right)\sqrt{3+5x^2+x^4}+4\int\frac{1}{\sqrt{3+5x^2+}}$$

$$=\frac{1247x\left(5+\sqrt{13}+2x^2\right)}{210\sqrt{3+5x^2+x^4}}-\frac{4}{3}x\sqrt{3+5x^2+x^4}+\frac{1}{35}x^3\left(29+15x^2\right)\sqrt{3+5x}$$

Mathematica [C] time = 0.26, size = 234, normalized size = 0.77

$$\frac{-i\sqrt{2}\left(1247\sqrt{13}-5395\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+1247i\sqrt{2}\left(\sqrt{1}\right.}{420}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (4*x*(-420 - 439*x^2 + 430*x^4 + 312*x^6 + 45*x^8) + (1247*I)*Sqrt[2]*(-5 +
 Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13]
+ 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])
```

```
/6] - I*Sqrt[2]*(-5395 + 1247*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 +
Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[
13])]*x], 19/6 + (5*Sqrt[13])/6])/(420*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(3\,x^4+2\,x^2\right)\sqrt{x^4+5\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)*sqrt(x^4 + 5*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2, x)
```

maple [A] time = 0.01, size = 243, normalized size = 0.80

$$\frac{3\sqrt{x^4+5x^2+3}\;x^5}{7}+\frac{29\sqrt{x^4+5x^2+3}\;x^3}{35}-\frac{4\sqrt{x^4+5x^2+3}\;x}{3}+\frac{24\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\;\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}}{\sqrt{-30+6\sqrt{13}}\;\sqrt{x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/7*(x^4+5*x^2+3)^(1/2)*x^5+29/35*(x^4+5*x^2+3)^(1/2)*x^3-4/3*(x^4+5*x^2+3)
^(1/2)*x+24/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5
/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*(-30+6*13^(
1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-14964/35/(-30+6*13^(1/2))^(1/2)*(-(
-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x
^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2
)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(
1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(3x^2+2\right) \sqrt{x^4+5x^2+3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
[Out] Integral(x**2*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

3.153 $\int \left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=279

$$-\frac{23x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{1}{15}x\left(9x^2+25\right)\sqrt{x^4+5x^2+3}+\frac{\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{\right.}\right.}\right.\right.}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$

```
[Out] -23/15*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+1/15*x*(9*x^2+25)*(x^4+5*x^
2+3)^(1/2)+23/90*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2))
)^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/
6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x
^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+(1/(36+x^2
*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*1
3^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*
(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4
+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1176, 1189, 1099, 1135}

$$-\frac{23x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{1}{15}x\left(9x^2+25\right)\sqrt{x^4+5x^2+3}+\frac{\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{\right.}\right.}\right.\right.}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-23*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) + (x*(25 + 9*x^2)
*Sqrt[3 + 5*x^2 + x^4])/15 + (23*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt
[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[Arc
Tan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(15*Sqrt[3 + 5*x^2 +
x^4]) + (Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13
])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{15}\int \frac{15-46x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4}-\frac{46}{15}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx+\int \frac{1}{\sqrt{3+5x^2+}}$$

$$= -\frac{23x\left(5+\sqrt{13}+2x^2\right)}{15\sqrt{3+5x^2+x^4}}+\frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4}+\frac{23\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}}{}$$

Mathematica [C] time = 0.24, size = 229, normalized size = 0.82

$$\frac{i\sqrt{2}\left(23\sqrt{13}-130\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-23i\sqrt{2}\left(\sqrt{13}-5\right)}{30\sqrt{x^4+5x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*x*(75 + 152*x^2 + 70*x^4 + 9*x^6) - (23*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[
(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellipti
cE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*
(-130 + 23*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
 Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5
*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+5\,x^2+3}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

maple [A] time = 0.01, size = 226, normalized size = 0.81

$$\frac{3\sqrt{x^4+5x^2+3}\;x^3}{5}+\frac{5\sqrt{x^4+5x^2+3}\;x}{3}+\frac{6\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\;\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\;x}{6}\right.}{\sqrt{-30+6\sqrt{13}}\;\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/5*(x^4+5*x^2+3)^(1/2)*x^3+5/3*(x^4+5*x^2+3)^(1/2)*x+6/(-30+6*13^(1/2))^(1
/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(
x^4+5*x^2+3)^(1/2)*EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*3
9^(1/2))+552/5/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-
(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(Elliptic
F(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-30
+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+5\,x^2+3}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

3.154 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$

Optimal. Leaf size=284

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-x^2)}{x}+\frac{9x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}+\frac{8\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\ldots}\right.\right.}{\sqrt{x^4+5x^2+3}}$$

```
[Out] 9/2*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-(-x^2+2)*(x^4+5*x^2+3)^(1/2)/x
+8/3*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*Elli
pticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13
^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-1
3^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-3/4*(1/(36+x^2*(3
0+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(
1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+
x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^
(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1271, 1189, 1099, 1135}

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-x^2)}{x}+\frac{9x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}+\frac{8\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\ldots}\right.\right.}{\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2,x]
```

```
[Out] (9*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - ((2 - x^2)*Sqrt[3
 + 5*x^2 + x^4])/x - (3*Sqrt[(3*(5 + Sqrt[13]))/2]*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sq
rt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) +
 (8*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt
[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6
]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx &= -\frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x} - \frac{1}{3}\int \frac{-48-27x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x} + 9\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx + 16\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx \\
&= \frac{9x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}} - \frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x} - \frac{3\sqrt{\frac{3}{2}}\left(5+\sqrt{13}\right)\sqrt{\frac{6+\left(5-\sqrt{13}\right)x}{6+\left(5+\sqrt{13}\right)x}}}{}
\end{aligned}$$

Mathematica [C] time = 0.26, size = 231, normalized size = 0.81

$$\frac{-i\sqrt{2}\left(9\sqrt{13}-13\right)x\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+9i\sqrt{2}\left(\sqrt{13}-5\right)x\sqrt{}}{4x\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2,x]
```

```
[Out] (4*(-6 - 7*x^2 + 3*x^4 + x^6) + (9*I)*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 +
Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*A
rcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-13 +
 9*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[
13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[
13])/6])/(4*x*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5\,x^2+3}\,\left(3\,x^2+2\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+5\,x^2+3}\,\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

maple [A] time = 0.02, size = 225, normalized size = 0.79

$$\sqrt{x^4+5x^2+3}\;x+\frac{96\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\;\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\;x}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\;\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x)
[Out] (x^4+5*x^2+3)^(1/2)*x+96/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1
)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/
6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-324/(-30+6*13^(1/2))^(
1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/
(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/
6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+
1/6*39^(1/2)))-2*(x^4+5*x^2+3)^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+5\,x^2+3}\,\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^2,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**2,x)
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**2, x)
```

3.155 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx$

Optimal. Leaf size=305

$$-\frac{64\sqrt{x^4+5x^2+3}}{9x}+\frac{32x\left(2x^2+\sqrt{13}+5\right)}{9\sqrt{x^4+5x^2+3}}+\frac{49\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(\right.\right.}{3\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$

```
[Out] 32/9*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-64/9*(x^4+5*x^2+3)^(1/2)/x-1/
3*(-9*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3-16/27*(1/(36+x^2*(30+6*13^(1/2))))^(1/
2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2
*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(
30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4
+5*x^2+3)^(1/2)+49/3*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1
/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2
),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(
6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1271, 1281, 1189, 1099, 1135}

$$-\frac{\sqrt{x^4+5x^2+3}\left(2-9x^2\right)}{3x^3}-\frac{64\sqrt{x^4+5x^2+3}}{9x}+\frac{32x\left(2x^2+\sqrt{13}+5\right)}{9\sqrt{x^4+5x^2+3}}+\frac{49\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ta}\right.}{3\sqrt{6\left(5+\sqrt{13}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4,x]
```

```
[Out] (32*x*(5 + Sqrt[13] + 2*x^2))/(9*Sqrt[3 + 5*x^2 + x^4]) - (64*Sqrt[3 + 5*x^
2 + x^4])/(9*x) - ((2 - 9*x^2)*Sqrt[3 + 5*x^2 + x^4])/(3*x^3) - (16*Sqrt[(2
*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(9*Sqrt[3 + 5*x^2 + x^4]) + (49*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sq
rt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3*Sqrt[6*(5 + Sqrt[13])]*S
qrt[3 + 5*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x^2\right)\sqrt{3+5x^2+x^4}}{x^4}\,dx &= -\frac{\left(2-9x^2\right)\sqrt{3+5x^2+x^4}}{3x^3}-\frac{1}{3}\int \frac{-64-49x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{64\sqrt{3+5x^2+x^4}}{9x}-\frac{\left(2-9x^2\right)\sqrt{3+5x^2+x^4}}{3x^3}+\frac{1}{9}\int \frac{147+64x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{64\sqrt{3+5x^2+x^4}}{9x}-\frac{\left(2-9x^2\right)\sqrt{3+5x^2+x^4}}{3x^3}+\frac{64}{9}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx \ldots \\
&= \frac{32x\left(5+\sqrt{13}+2x^2\right)}{9\sqrt{3+5x^2+x^4}}-\frac{64\sqrt{3+5x^2+x^4}}{9x}-\frac{\left(2-9x^2\right)\sqrt{3+5x^2+x^4}}{3x^3}-\frac{16\ldots}{\ldots}
\end{aligned}$$

Mathematica [C] time = 0.26, size = 237, normalized size = 0.78

$$\frac{-i\sqrt{2}\left(32\sqrt{13}-13\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,x^3F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+32i\sqrt{2}\left(\sqrt{13}-5\ldots\right.}{18x^3\sqrt{x^4+5\ldots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4,x]
```

```
[Out] (-2*(18 + 141*x^2 + 191*x^4 + 37*x^6) + (32*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3*
Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*El
```

```
lipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqr
t[2]*(-13 + 32*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*
Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 1
9/6 + (5*Sqrt[13])/6])/(18*x^3*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

maple [A] time = 0.02, size = 228, normalized size = 0.75

$$\frac{98\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\,x}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{37\sqrt{x^4+5x^2+3}}{9x}-\frac{2\sqrt{x^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x)
[Out] -37/9*(x^4+5*x^2+3)^(1/2)/x+98/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))
*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)*Ellipt
icF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-256/(-30+6*13^(1
/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^
(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30+6*13^(1/2))^(1/2
)*x,5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^
(1/2)+1/6*39^(1/2)))-2/3*(x^4+5*x^2+3)^(1/2)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^4,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**4,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**4, x)
```

3.156 $\int x^5 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} dx$

Optimal. Leaf size=127

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{5/2}x^4+\frac{\left(3313-1070x^2\right)\left(x^4+5x^2+3\right)^{5/2}}{1680}-\frac{2183}{768}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{28379\left(2x^2+5\right)\sqrt{\ }}{2048}$$

```
[Out] -2183/768*(2*x^2+5)*(x^4+5*x^2+3)^(3/2)+3/14*x^4*(x^4+5*x^2+3)^(5/2)+1/1680
*(-1070*x^2+3313)*(x^4+5*x^2+3)^(5/2)-368927/4096*arctanh(1/2*(2*x^2+5)/(x^
4+5*x^2+3)^(1/2))+28379/2048*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 832, 779, 612, 621, 206}

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{5/2}x^4+\frac{\left(3313-1070x^2\right)\left(x^4+5x^2+3\right)^{5/2}}{1680}-\frac{2183}{768}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{28379\left(2x^2+5\right)\sqrt{\ }}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (28379*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/2048 - (2183*(5 + 2*x^2)*(3 + 5*x
^2 + x^4)^(3/2))/768 + (3*x^4*(3 + 5*x^2 + x^4)^(5/2))/14 + ((3313 - 1070*x
^2)*(3 + 5*x^2 + x^4)^(5/2))/1680 - (368927*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 +
 5*x^2 + x^4])])/4096
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
```

```
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int x^5\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(2+3x)\left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2}+\frac{1}{14}\operatorname{Subst}\left(\int\left(-18-\frac{107x}{2}\right)x\left(3+5x+x^2\right)^{3/2}\right.\\
&= \frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2}+\frac{\left(3313-1070x^2\right)\left(3+5x^2+x^4\right)^{5/2}}{1680}-\frac{2183}{96}\operatorname{Su}\\
&= -\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2}+\frac{(3313-}{}\\
&= \frac{28379\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{2048}-\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\\
&= \frac{28379\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{2048}-\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\\
&= \frac{28379\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{2048}-\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+
\end{aligned}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 0.64

$$\frac{2\sqrt{x^4+5x^2+3}\left(46080x^{12}+323840x^{10}+482944x^8+154800x^6+283304x^4-1499570x^2+9546951\right)-387}{430080}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(9546951 - 1499570*x^2 + 283304*x^4 + 154800*x^6 +
 482944*x^8 + 323840*x^10 + 46080*x^12) - 38737335*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/430080
```

fricas [A] time = 0.84, size = 71, normalized size = 0.56

$$\frac{1}{215040}\left(46080\,x^{12}+323840\,x^{10}+482944\,x^8+154800\,x^6+283304\,x^4-1499570\,x^2+9546951\right)\sqrt{x^4+5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/215040*(46080*x^12 + 323840*x^10 + 482944*x^8 + 154800*x^6 + 283304*x^4 -
 1499570*x^2 + 9546951)*sqrt(x^4 + 5*x^2 + 3) + 368927/4096*log(-2*x^2 + 2*
sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.54, size = 207, normalized size = 1.63

$$\frac{1}{71680}\sqrt{x^4+5x^2+3}\left(2\left(4\left(2\left(8\left(10\left(12x^2+5\right)x^2-203\right)x^2+7635\right)x^2-76083\right)x^2+1627215\right)x^2-20756241\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/71680*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(10*(12*x^2 + 5)*x^2 - 203)*x^2 +
 7635)*x^2 - 76083)*x^2 + 1627215)*x^2 - 20756241) + 17/3072*sqrt(x^4 + 5*x
^2 + 3)*(2*(4*(2*(8*(2*x^2 + 1)*x^2 - 33)*x^2 + 321)*x^2 - 6837)*x^2 + 8714
7) + 19/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 26
35)*x^2 - 33429) + 1/64*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x
^2 + 1095) + 368927/4096*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.03, size = 138, normalized size = 1.09

$$\frac{3\sqrt{x^4+5x^2+3}\,x^{12}}{14}+\frac{253\sqrt{x^4+5x^2+3}\,x^{10}}{168}+\frac{539\sqrt{x^4+5x^2+3}\,x^{8}}{240}+\frac{645\sqrt{x^4+5x^2+3}\,x^{6}}{896}+\frac{5059\sqrt{x^4+5x^2+3}}{3840}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 645/896*x^6*(x^4+5*x^2+3)^(1/2)-149957/21504*x^2*(x^4+5*x^2+3)^(1/2)-368927
/4096*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+3/14*x^12*(x^4+5*x^2+3)^(1/2)+253/168
*x^10*(x^4+5*x^2+3)^(1/2)+539/240*x^8*(x^4+5*x^2+3)^(1/2)+3182317/71680*(x^
4+5*x^2+3)^(1/2)+5059/3840*x^4*(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 0.59, size = 135, normalized size = 1.06

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{\frac{5}{2}}x^4-\frac{107}{168}\left(x^4+5x^2+3\right)^{\frac{5}{2}}x^2-\frac{2183}{384}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2+\frac{3313}{1680}\left(x^4+5x^2+3\right)^{\frac{5}{2}}+\frac{28379}{1024}\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/14*(x^4 + 5*x^2 + 3)^(5/2)*x^4 - 107/168*(x^4 + 5*x^2 + 3)^(5/2)*x^2 - 21
83/384*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3313/1680*(x^4 + 5*x^2 + 3)^(5/2) + 28
379/1024*sqrt(x^4 + 5*x^2 + 3)*x^2 - 10915/768*(x^4 + 5*x^2 + 3)^(3/2) + 14
1895/2048*sqrt(x^4 + 5*x^2 + 3) - 368927/4096*log(2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.157 $\int x^3 (2 + 3x^2) (3 + 5x^2 + x^4)^{3/2} dx$

Optimal. Leaf size=106

$$-\frac{1}{40}(27 - 10x^2)(x^4 + 5x^2 + 3)^{5/2} + \frac{123}{128}(2x^2 + 5)(x^4 + 5x^2 + 3)^{3/2} - \frac{4797(2x^2 + 5)\sqrt{x^4 + 5x^2 + 3}}{1024} + \frac{62361 \tanh}{2}$$

```
[Out] 123/128*(2*x^2+5)*(x^4+5*x^2+3)^(3/2)-1/40*(-10*x^2+27)*(x^4+5*x^2+3)^(5/2)
+62361/2048*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-4797/1024*(2*x^2+5)*
(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 779, 612, 621, 206}

$$-\frac{1}{40}(27 - 10x^2)(x^4 + 5x^2 + 3)^{5/2} + \frac{123}{128}(2x^2 + 5)(x^4 + 5x^2 + 3)^{3/2} - \frac{4797(2x^2 + 5)\sqrt{x^4 + 5x^2 + 3}}{1024} + \frac{62361 \tanh}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-4797*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/1024 + (123*(5 + 2*x^2)*(3 + 5*x^
2 + x^4)^(3/2))/128 - ((27 - 10*x^2)*(3 + 5*x^2 + x^4)^(5/2))/40 + (62361*A
rcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2048
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3 \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x(2+3x)\left(3+5x+x^2\right)^{3/2} dx, x, x^2\right)$$
$$= -\frac{1}{40}\left(27-10x^2\right)\left(3+5x^2+x^4\right)^{5/2} + \frac{123}{16}\text{Subst}\left(\int \left(3+5x+x^2\right)^{3/2} dx$$
$$= \frac{123}{128}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} - \frac{1}{40}\left(27-10x^2\right)\left(3+5x^2+x^4\right)^{5/2} - \frac{4}{2}$$
$$= -\frac{4797\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{1024} + \frac{123}{128}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} - \frac{1}{40}$$
$$= -\frac{4797\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{1024} + \frac{123}{128}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} - \frac{1}{40}$$
$$= -\frac{4797\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{1024} + \frac{123}{128}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} - \frac{1}{40}$$

Mathematica [A] time = 0.03, size = 76, normalized size = 0.72

$$\frac{311805\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+2\sqrt{x^4+5x^2+3}\left(1280x^{10}+9344x^8+14960x^6+5064x^4+12390x^2-77229\right)}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-77229 + 12390*x^2 + 5064*x^4 + 14960*x^6 + 9344*
x^8 + 1280*x^10) + 311805*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/1
0240
```

fricas [A] time = 0.57, size = 66, normalized size = 0.62

$$\frac{1}{5120}\left(1280\,x^{10}+9344\,x^8+14960\,x^6+5064\,x^4+12390\,x^2-77229\right)\sqrt{x^4+5\,x^2+3}-\frac{62361}{2048}\log\left(-2\,x^2+2\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5120*(1280*x^10 + 9344*x^8 + 14960*x^6 + 5064*x^4 + 12390*x^2 - 77229)*sq
rt(x^4 + 5*x^2 + 3) - 62361/2048*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [B] time = 0.59, size = 179, normalized size = 1.69

$$\frac{1}{1024}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(2\left(8\left(2\,x^2+1\right)x^2-33\right)x^2+321\right)x^2-6837\right)x^2+87147\right)+\frac{17}{3840}\sqrt{x^4+5\,x^2+3}\left(2\left(4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1024*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(2*x^2 + 1)*x^2 - 33)*x^2 + 321)*x
^2 - 6837)*x^2 + 87147) + 17/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5
)*x^2 - 127)*x^2 + 2635)*x^2 - 33429) + 19/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*
(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 1/8*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5
)*x^2 - 51) - 62361/2048*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 121, normalized size = 1.14

$$\frac{\sqrt{x^4+5x^2+3}\;x^{10}}{4}+\frac{73\sqrt{x^4+5x^2+3}\;x^8}{40}+\frac{187\sqrt{x^4+5x^2+3}\;x^6}{64}+\frac{633\sqrt{x^4+5x^2+3}\;x^4}{640}+\frac{1239\sqrt{x^4+5x^2+3}\;x^2}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 187/64*(x^4+5*x^2+3)^(1/2)*x^6+1239/512*(x^4+5*x^2+3)^(1/2)*x^2+62361/2048*
ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+1/4*(x^4+5*x^2+3)^(1/2)*x^10+73/40*(x^4+5*x
^2+3)^(1/2)*x^8-77229/5120*(x^4+5*x^2+3)^(1/2)+633/640*(x^4+5*x^2+3)^(1/2)*
x^4
```

maxima [A] time = 0.88, size = 118, normalized size = 1.11

$$\frac{1}{4}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}x^2+\frac{123}{64}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2-\frac{27}{40}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}-\frac{4797}{512}\sqrt{x^4+5\,x^2+3}\,x^2+\frac{615}{128}\left(x^4+5\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^4 + 5*x^2 + 3)^(5/2)*x^2 + 123/64*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 27/4
0*(x^4 + 5*x^2 + 3)^(5/2) - 4797/512*sqrt(x^4 + 5*x^2 + 3)*x^2 + 615/128*(x
^4 + 5*x^2 + 3)^(3/2) - 23985/1024*sqrt(x^4 + 5*x^2 + 3) + 62361/2048*log(2
*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(3\,x^2+2\right)\,\left(x^4+5\,x^2+3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(3x^{2}+2\right)\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.158 $\int x\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=99

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{32}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{429}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{5577}{512}\tanh^{-1}\left(\frac{2x^2+}{2\sqrt{x^4+5x}}\right.$$

```
[Out] -11/32*(2*x^2+5)*(x^4+5*x^2+3)^(3/2)+3/10*(x^4+5*x^2+3)^(5/2)-5577/512*arct
anh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+429/256*(2*x^2+5)*(x^4+5*x^2+3)^(1/2
)
```

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1247, 640, 612, 621, 206}

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{32}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{429}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{5577}{512}\tanh^{-1}\left(\frac{2x^2+}{2\sqrt{x^4+5x}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (429*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 - (11*(5 + 2*x^2)*(3 + 5*x^2 +
x^4)^(3/2))/32 + (3*(3 + 5*x^2 + x^4)^(5/2))/10 - (5577*ArcTanh[(5 + 2*x^2)
/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int x\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{1}{2}\text{Subst}\left(\int (2+3x)\left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)$$
$$= \frac{3}{10}\left(3+5x^2+x^4\right)^{5/2} - \frac{11}{4}\text{Subst}\left(\int \left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)$$
$$= -\frac{11}{32}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{10}\left(3+5x^2+x^4\right)^{5/2} + \frac{429}{64}\text{Subst}\left(\int \sqrt{3}\right.$$
$$= \frac{429}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} - \frac{11}{32}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{10}\left(3+5\right.$$
$$= \frac{429}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} - \frac{11}{32}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{10}\left(3+5\right.$$
$$= \frac{429}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} - \frac{11}{32}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{10}\left(3+5\right.$$

Mathematica [A] time = 0.02, size = 71, normalized size = 0.72

$$\frac{2\sqrt{x^4+5x^2+3}\left(384x^8+2960x^6+5304x^4+2170x^2+7581\right)-27885\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{2560}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(7581 + 2170*x^2 + 5304*x^4 + 2960*x^6 + 384*x^8)
- 27885*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2560
```

fricas [A] time = 0.64, size = 61, normalized size = 0.62

$$\frac{1}{1280}\left(384\,x^8+2960\,x^6+5304\,x^4+2170\,x^2+7581\right)\sqrt{x^4+5\,x^2+3}+\frac{5577}{512}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(384*x^8 + 2960*x^6 + 5304*x^4 + 2170*x^2 + 7581)*sqrt(x^4 + 5*x^2 +
 3) + 5577/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.45, size = 151, normalized size = 1.53

$$\frac{1}{1280}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\left(8\,x^2+5\right)x^2-127\right)x^2+2635\right)x^2-33429\right)+\frac{17}{384}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\,x^2+5\right)x^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x^
2 - 33429) + 17/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 +
 1095) + 19/48*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 3/4*sqrt(x^
4 + 5*x^2 + 3)*(2*x^2 + 5) + 5577/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) +
 5)
```

maple [A] time = 0.02, size = 104, normalized size = 1.05

$$\frac{3\sqrt{x^4+5x^2+3}\;x^8}{10}+\frac{37\sqrt{x^4+5x^2+3}\;x^6}{16}+\frac{663\sqrt{x^4+5x^2+3}\;x^4}{160}+\frac{217\sqrt{x^4+5x^2+3}\;x^2}{128}-\frac{5577\ln\left(x^2+\frac{5}{2}+\sqrt{x^4\right.}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 3/10*(x^4+5*x^2+3)^(1/2)*x^8+37/16*(x^4+5*x^2+3)^(1/2)*x^6+663/160*(x^4+5*x
^2+3)^(1/2)*x^4+217/128*(x^4+5*x^2+3)^(1/2)*x^2+7581/1280*(x^4+5*x^2+3)^(1/
2)-5577/512*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

maxima [A] time = 0.67, size = 101, normalized size = 1.02

$$-\frac{11}{16}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2+\frac{3}{10}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}+\frac{429}{128}\sqrt{x^4+5\,x^2+3}\,x^2-\frac{55}{32}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}+\frac{2145}{256}\sqrt{x^4+5\,x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -11/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3/10*(x^4 + 5*x^2 + 3)^(5/2) + 429/128
*sqrt(x^4 + 5*x^2 + 3)*x^2 - 55/32*(x^4 + 5*x^2 + 3)^(3/2) + 2145/256*sqrt(
x^4 + 5*x^2 + 3) - 5577/512*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.53, size = 127, normalized size = 1.28

$$\frac{\left(x^2+\frac{5}{2}\right)\left(x^4+5\,x^2+3\right)^{3/2}}{4}-\frac{15\,x^2\left(x^4+5\,x^2+3\right)^{3/2}}{16}-\frac{5577\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{512}+\frac{585\left(2\,x^2+5\right)\sqrt{x}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] ((x^2 + 5/2)*(5*x^2 + x^4 + 3)^(3/2))/4 - (15*x^2*(5*x^2 + x^4 + 3)^(3/2))/
16 - (5577*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/512 + (585*(2*x^2 + 5)
*(5*x^2 + x^4 + 3)^(1/2))/256 - (39*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1/2))/
16 - (75*(5*x^2 + x^4 + 3)^(3/2))/32 + (3*(5*x^2 + x^4 + 3)^(5/2))/10
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.159 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x} \, dx$

Optimal. Leaf size=119

$$\frac{1}{48}\left(18x^2+61\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{1}{128}\left(199-74x^2\right)\sqrt{x^4+5x^2+3}+\frac{2401}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-3\sqrt{3}\tanh$$

```
[Out] 1/48*(18*x^2+61)*(x^4+5*x^2+3)^(3/2)+2401/256*arctanh(1/2*(2*x^2+5)/(x^4+5*
x^2+3)^(1/2))-3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+
1/128*(-74*x^2+199)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{1}{48}\left(18x^2+61\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{1}{128}\left(199-74x^2\right)\sqrt{x^4+5x^2+3}+\frac{2401}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-3\sqrt{3}\tanh$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

```
[Out] ((199 - 74*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 + ((61 + 18*x^2)*(3 + 5*x^2 + x^
4)^(3/2))/48 + (2401*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256 -
3*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\left(3+5x+x^2\right)^{3/2}}{x}\,dx,x,x^2\right)$$

$$= \frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2} - \frac{1}{16}\text{Subst}\left(\int \frac{\left(-48+\frac{37x}{2}\right)\sqrt{3+5x+x}}{x}\right.$$

$$= \frac{1}{128}\left(199-74x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{1}{64}$$

$$= \frac{1}{128}\left(199-74x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2} + 9\,\text{S}$$

$$= \frac{1}{128}\left(199-74x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2} - 18$$

$$= \frac{1}{128}\left(199-74x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{24}{25}$$

Mathematica [A] time = 0.06, size = 104, normalized size = 0.87

$$\frac{2401}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-3\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)+\frac{1}{384}\sqrt{x^4+5x^2+3}\left(144x^6+1208x^4+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(2061 + 2650*x^2 + 1208*x^4 + 144*x^6))/384 + (2401*
ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256 - 3*Sqrt[3]*ArcTanh[(6
+ 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

fricas [A] time = 0.69, size = 106, normalized size = 0.89

$$\frac{1}{384}\left(144x^6+1208x^4+2650x^2+2061\right)\sqrt{x^4+5x^2+3}+3\sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+}}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/384*(144*x^6 + 1208*x^4 + 2650*x^2 + 2061)*sqrt(x^4 + 5*x^2 + 3) + 3*sqrt
(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3
) - 6) + 30)/x^2) - 2401/256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.50, size = 113, normalized size = 0.95

$$\frac{1}{384}\sqrt{x^4+5x^2+3}\left(2\left(4\left(18x^2+151\right)x^2+1325\right)x^2+2061\right)+3\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)-\frac{2401}{256}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 + 151)*x^2 + 1325)*x^2 + 2061) +
3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt
(x^4 + 5*x^2 + 3))) - 2401/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 117, normalized size = 0.98

$$\frac{3\sqrt{x^4+5x^2+3}\,x^6}{8}+\frac{151\sqrt{x^4+5x^2+3}\,x^4}{48}+\frac{1325\sqrt{x^4+5x^2+3}\,x^2}{192}-3\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)+\frac{2401\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x)
```

```
[Out] 3/8*(x^4+5*x^2+3)^(1/2)*x^6+151/48*(x^4+5*x^2+3)^(1/2)*x^4+1325/192*(x^4+5*
x^2+3)^(1/2)*x^2+687/128*(x^4+5*x^2+3)^(1/2)+2401/256*ln(x^2+5/2+(x^4+5*x^2
+3)^(1/2))-3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

maxima [A] time = 1.40, size = 120, normalized size = 1.01

$$\frac{3}{8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2-\frac{37}{64}\sqrt{x^4+5x^2+3}\,x^2+\frac{61}{48}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-3\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{19}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 37/64*sqrt(x^4 + 5*x^2 + 3)*x^2 + 61/48*(
x^4 + 5*x^2 + 3)^(3/2) - 3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2
+ 6/x^2 + 5) + 199/128*sqrt(x^4 + 5*x^2 + 3) + 2401/256*log(2*x^2 + 2*sqrt(
x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x, x)
```

3.160 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=122

$$-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{2x^2}+\frac{3}{16}(18x^2+109)\sqrt{x^4+5x^2+3}+\frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-12\sqrt{3}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.$$

```
[Out] -1/2*(-x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2+609/32*arctanh(1/2*(2*x^2+5)/(x^4+5*x
^2+3)^(1/2))-12*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+
3/16*(18*x^2+109)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1251, 812, 814, 843, 621, 206, 724}

$$-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{2x^2}+\frac{3}{16}(18x^2+109)\sqrt{x^4+5x^2+3}+\frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-12\sqrt{3}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] (3*(109 + 18*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 - ((2 - x^2)*(3 + 5*x^2 + x^4)^
(3/2))/(2*x^2) + (609*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32 -
12*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\left(3+5x+x^2\right)^{3/2}}{x^2}\,dx,x,x^2\right)$$

$$= -\frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{(-48-27x)\sqrt{3+5x+x^2}}{x}\,dx,\right.$$

$$= \frac{3}{16}\left(109+18x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2} + \frac{1}{16}\text{Subst}$$

$$= \frac{3}{16}\left(109+18x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2} + \frac{609}{32}\text{Subst}$$

$$= \frac{3}{16}\left(109+18x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2} + \frac{609}{16}\text{Subst}$$

$$= \frac{3}{16}\left(109+18x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2} + \frac{609}{32}\tanh^{-}$$

Mathematica [A] time = 0.05, size = 107, normalized size = 0.88

$$\frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-12\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)+\frac{\sqrt{x^4+5x^2+3}\left(8x^6+78x^4+271x^2-\right.}{16x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-48 + 271*x^2 + 78*x^4 + 8*x^6))/(16*x^2) + (609*Ar
cTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32 - 12*Sqrt[3]*ArcTanh[(6 +
5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

fricas [A] time = 0.59, size = 122, normalized size = 1.00

$$\frac{1536\sqrt{3}\,x^2\log\left(\frac{25\,x^2-2\sqrt{3}\left(5\,x^2+6\right)-2\sqrt{x^4+5\,x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-2436\,x^2\log\left(-2\,x^2+2\sqrt{x^4+5\,x^2+3}-5\right)+1541\,x^2}{128\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/128*(1536*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 +
5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 2436*x^2*log(-2*x^2 + 2*sqrt(x^4 +
5*x^2 + 3) - 5) + 1541*x^2 + 8*(8*x^6 + 78*x^4 + 271*x^2 - 48)*sqrt(x^4 + 5
*x^2 + 3))/x^2
```

giac [A] time = 0.56, size = 153, normalized size = 1.25

$$\frac{1}{16}\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2+39\right)x^2+271\right)+12\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right)+\frac{3\left(5\,x^2-5\sqrt{x^4+5\,x^2+3}\right.}{\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 39)*x^2 + 271) + 12*sqrt(3)*log((x^2
 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))
) + 3*(5*x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^
2 - 3) - 609/32*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 117, normalized size = 0.96

$$\frac{\sqrt{x^4+5x^2+3}\;x^4}{2}+\frac{39\sqrt{x^4+5x^2+3}\;x^2}{8}-12\sqrt{3}\;\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)+\frac{609\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x)
```

```
[Out] 1/2*(x^4+5*x^2+3)^(1/2)*x^4+39/8*(x^4+5*x^2+3)^(1/2)*x^2+271/16*(x^4+5*x^2+
3)^(1/2)+609/32*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-12*arctanh(1/6*(5*x^2+6)*3^
(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-3*(x^4+5*x^2+3)^(1/2)/x^2
```

maxima [A] time = 1.75, size = 120, normalized size = 0.98

$$\frac{27}{8}\sqrt{x^4+5\,x^2+3}\,x^2+\frac{1}{2}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{327}{16}\sqrt{x^4+5\,x^2+3}-\frac{\left(x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 27/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 12*sqrt(3)*l
og(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 327/16*sqrt(x^4 + 5*x
^2 + 3) - (x^4 + 5*x^2 + 3)^(3/2)/x^2 + 609/32*log(2*x^2 + 2*sqrt(x^4 + 5*x
^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2+2\right)\,\left(x^4+5\,x^2+3\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^3,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**3,x)
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**3, x)
```

3.161 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=127

$$-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{4x^4}-\frac{3(28-19x^2)\sqrt{x^4+5x^2+3}}{8x^2}+\frac{453}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{127}{8}\sqrt{3}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.$$

```
[Out] -1/4*(-3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4+453/16*arctanh(1/2*(2*x^2+5)/(x^4+5
*x^2+3)^(1/2))-127/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(
1/2)-3/8*(-19*x^2+28)*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1251, 812, 843, 621, 206, 724}

$$-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{4x^4}-\frac{3(28-19x^2)\sqrt{x^4+5x^2+3}}{8x^2}+\frac{453}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{127}{8}\sqrt{3}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-3*(28 - 19*x^2)*Sqrt[3 + 5*x^2 + x^4])/(8*x^2) - ((2 - 3*x^2)*(3 + 5*x^2
+ x^4)^(3/2))/(4*x^4) + (453*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])]
)/16 - (127*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])
/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
```

```
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)(3+5x+x^2)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4}-\frac{3}{16}\operatorname{Subst}\left(\int \frac{(-56-38x)\sqrt{3+5x+x^2}}{x^2}\,dx\right.\\
&= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2}-\frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4}+\frac{3}{32}\operatorname{Subst}\\
&= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2}-\frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4}+\frac{453}{16}\operatorname{Subs}\\
&= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2}-\frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4}+\frac{453}{8}\operatorname{Subs}\\
&= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2}-\frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4}+\frac{453}{16}\tanh
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 107, normalized size = 0.84

$$
\frac{1}{16}\left(453\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-254\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)+\frac{2\sqrt{x^4+5x^2+3}\,(6x^6+83x^4-8}{x^4}\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] ((2*Sqrt[3 + 5*x^2 + x^4]*(-12 - 86*x^2 + 83*x^4 + 6*x^6))/x^4 + 453*ArcTan
h[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])] - 254*Sqrt[3]*ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/16
```

fricas [A] time = 0.80, size = 122, normalized size = 0.96

$$
\frac{1016\sqrt{3}\,x^4\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)-1812x^4\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)+67x^4}{64x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/64*(1016*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5
*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1812*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) - 5) + 67*x^4 + 8*(6*x^6 + 83*x^4 - 86*x^2 - 12)*sqrt(x^4 + 5*x^2
 + 3))/x^4
```

giac [A] time = 0.65, size = 190, normalized size = 1.50

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2+83\right)+\frac{127}{8}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{227\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+348\left(x^2-\right.}{4\left(\left(x^2-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 83) + 127/8*sqrt(3)*log((x^2 + sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/4*(227
*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 348*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 45
9*x^2 + 459*sqrt(x^4 + 5*x^2 + 3) - 684)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 -
 3)^2 - 453/16*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 117, normalized size = 0.92

$$\frac{3\sqrt{x^4+5x^2+3}\,x^2}{4}-\frac{127\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{8}+\frac{453\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16}-\frac{43\sqrt{x^4+5x^2+3}}{4x^2}-\frac{3\sqrt{x^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x)
```

```
[Out] 83/8*(x^4+5*x^2+3)^(1/2)+453/16*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-127/8*arcta
nh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-43/4*(x^4+5*x^2+3)^(1
/2)/x^2-3/2*(x^4+5*x^2+3)^(1/2)/x^4+3/4*(x^4+5*x^2+3)^(1/2)*x^2
```

maxima [A] time = 1.46, size = 137, normalized size = 1.08

$$\frac{7}{2}\sqrt{x^4+5x^2+3}\,x^2+\frac{1}{6}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{127}{8}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{197}{8}\sqrt{x^4+5x^2+3}-\frac{23}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 7/2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/6*(x^4 + 5*x^2 + 3)^(3/2) - 127/8*sqrt(3)
*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 197/8*sqrt(x^4 + 5*
x^2 + 3) - 23/12*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 1/6*(x^4 + 5*x^2 + 3)^(5/2)/
x^4 + 453/16*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^5,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**5,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**5, x)
```

3.162 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=127

$$-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{12x^2}+\frac{49}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{527\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{6x^6}$$

```
[Out] -1/6*(7*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6+49/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^
2+3)^(1/2))-527/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/
2)-1/12*(-32*x^2+67)*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 810, 812, 843, 621, 206, 724}

$$-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{6x^6}-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{12x^2}+\frac{49}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{527\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

```
[Out] -((67 - 32*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^2) - ((2 + 7*x^2)*(3 + 5*x^2 +
 x^4)^(3/2))/(6*x^6) + (49*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/
4 - (527*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(24*Sqrt[3
])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
```

```
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)(3+5x+x^2)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} - \frac{1}{24}\,\text{Subst}\left(\int \frac{(-134-64x)\sqrt{3+5x+x^2}}{x^2}\right.$$

$$= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{1}{48}\,\text{Subst}\Bigg($$

$$= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{49}{4}\,\text{Subst}\Bigg($$

$$= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{49}{2}\,\text{Subst}\Bigg($$

$$= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{49}{4}\tanh^{-1}\Bigg($$

Mathematica [A] time = 0.05, size = 107, normalized size = 0.84

$$\frac{1}{72}\left(882\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 527\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) + \frac{6\sqrt{x^4+5x^2+3}\,(18x^6-141x^4}{x^6}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

```
[Out] ((6*Sqrt[3 + 5*x^2 + x^4]*(-12 - 62*x^2 - 141*x^4 + 18*x^6))/x^6 + 882*ArcT
anh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])] - 527*Sqrt[3]*ArcTanh[(6 + 5*x^2
)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/72
```

fricas [A] time = 0.62, size = 122, normalized size = 0.96

$$\frac{527\sqrt{3}\,x^6\log\left(\frac{25\,x^2-2\,\sqrt{3}\left(5\,x^2+6\right)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)-882\,x^6\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}\,-5\right)-711\,x^6+6}{72\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/72*(527*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 882*x^6*log(-2*x^2 + 2*sqrt(x^4 + 5*x
^2 + 3) - 5) - 711*x^6 + 6*(18*x^6 - 141*x^4 - 62*x^2 - 12)*sqrt(x^4 + 5*x^
2 + 3))/x^6
```

giac [B] time = 0.68, size = 227, normalized size = 1.79

$$\frac{527}{72}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right)+\frac{3}{2}\sqrt{x^4+5\,x^2+3}+\frac{829\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^5+1824\left(x^2-\sqrt{x^4+5\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 527/72*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))) + 3/2*sqrt(x^4 + 5*x^2 + 3) + 1/12*(829*(x^2 - sqr
t(x^4 + 5*x^2 + 3))^5 + 1824*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 - 2200*(x^2 -
sqrt(x^4 + 5*x^2 + 3))^3 - 5292*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 + 2799*x^2
- 2799*sqrt(x^4 + 5*x^2 + 3) + 5724)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^
3 - 49/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 117, normalized size = 0.92

$$-\frac{527\sqrt{3}\;\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{72}+\frac{49\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{47\sqrt{x^4+5x^2+3}}{4x^2}-\frac{31\sqrt{x^4+5x^2+3}}{6x^4}-\frac{\sqrt{x^4+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x)
```

```
[Out] 49/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-527/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(
x^4+5*x^2+3)^(1/2))*3^(1/2)-47/4*(x^4+5*x^2+3)^(1/2)/x^2-(x^4+5*x^2+3)^(1/2
)/x^6-31/6*(x^4+5*x^2+3)^(1/2)/x^4+3/2*(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 1.58, size = 154, normalized size = 1.21

$$\frac{67}{36}\sqrt{x^4+5\,x^2+3}\,x^2+\frac{11}{54}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{527}{72}\sqrt{3}\log\left(\frac{2\,\sqrt{3}\,\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{431}{36}\sqrt{x^4+5\,x^2+3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 67/36*sqrt(x^4 + 5*x^2 + 3)*x^2 + 11/54*(x^4 + 5*x^2 + 3)^(3/2) - 527/72*sq
rt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 431/36*sqrt(x^
4 + 5*x^2 + 3) - 79/108*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 11/54*(x^4 + 5*x^2 +
3)^(5/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(5/2)/x^6 + 49/4*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2+2\right)\,\left(x^4+5\,x^2+3\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^7,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{2}+2\right)\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}}{x^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**7,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**7, x)
```

3.163 $\int x^4 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} dx$

Optimal. Leaf size=356

$$-\frac{4210}{429}\sqrt{x^4+5x^2+3}\,x+\frac{176723\left(2x^2+\sqrt{13}+5\right)x}{4290\sqrt{x^4+5x^2+3}}+\frac{2105\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\right.}{143\sqrt{x^4+5x^2+3}}$$

```
[Out] 1/143*x^5*(33*x^2+71)*(x^4+5*x^2+3)^(3/2)+176723/4290*x*(5+2*x^2+13^(1/2))/
(x^4+5*x^2+3)^(1/2)-4210/429*x*(x^4+5*x^2+3)^(1/2)+1251/715*x^3*(x^4+5*x^2+
3)^(1/2)-1/429*x^5*(272*x^2+283)*(x^4+5*x^2+3)^(1/2)+2105/429*(1/(36+x^2*(3
0+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(
1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+
x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(
5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-176723/25740*(1/(36+x^2*(30+6*13^(1
/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/
2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13
^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^
(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1273, 1279, 1189, 1099, 1135}

$$\frac{1}{143}\left(33x^2+71\right)\left(x^4+5x^2+3\right)^{3/2}x^5-\frac{1}{429}\left(272x^2+283\right)\sqrt{x^4+5x^2+3}\,x^5+\frac{1251}{715}\sqrt{x^4+5x^2+3}\,x^3-\frac{4210}{429}\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (176723*x*(5 + Sqrt[13] + 2*x^2))/(4290*Sqrt[3 + 5*x^2 + x^4]) - (4210*x*Sq
rt[3 + 5*x^2 + x^4])/429 + (1251*x^3*Sqrt[3 + 5*x^2 + x^4])/715 - (x^5*(283
 + 272*x^2)*Sqrt[3 + 5*x^2 + x^4])/429 + (x^5*(71 + 33*x^2)*(3 + 5*x^2 + x^
4)^(3/2))/143 - (176723*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2
)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[
(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(4290*Sqrt[3 + 5*x^2 + x^4]) +
 (2105*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + S
qrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13]
)/6]*x], (-13 + 5*Sqrt[13])/6])/(143*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1273

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*
p + 3)), x] + Dist[(2*p)/(c*(4*p + m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int x^4\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{1}{143}x^5\left(71+33x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{143}\int x^4\left(-69-272x^2\right)\sqrt{3+5$$

$$=-\frac{1}{429}x^5\left(283+272x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{143}x^5\left(71+33x^2\right)\left(3+5x^2+\right.$$

$$=\frac{1251}{715}x^3\sqrt{3+5x^2+x^4}-\frac{1}{429}x^5\left(283+272x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{143}x^5$$

$$=-\frac{4210}{429}x\sqrt{3+5x^2+x^4}+\frac{1251}{715}x^3\sqrt{3+5x^2+x^4}-\frac{1}{429}x^5\left(283+272x^2\right.$$

$$=-\frac{4210}{429}x\sqrt{3+5x^2+x^4}+\frac{1251}{715}x^3\sqrt{3+5x^2+x^4}-\frac{1}{429}x^5\left(283+272x^2\right.$$

$$=\frac{176723x\left(5+\sqrt{13}+2x^2\right)}{4290\sqrt{3+5x^2+x^4}}-\frac{4210}{429}x\sqrt{3+5x^2+x^4}+\frac{1251}{715}x^3\sqrt{3+5x^2}$$

Mathematica [C] time = 0.27, size = 249, normalized size = 0.70

$$-i\sqrt{2}\left(176723\sqrt{13}-757315\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+176723i\sqrt{2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(-63150 - 93991*x^2 + 3055*x^4 + 29003*x^6 + 39650*x^8 + 24635*x^10 +
6015*x^12 + 495*x^14) + (176723*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[
13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSin
h[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-757315 +
176723*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqr
t[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqr
t[13])/6])/(8580*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(3\,x^{10}+17\,x^{8}+19\,x^{6}+6\,x^{4}\right)\sqrt{x^{4}+5\,x^{2}+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^10 + 17*x^8 + 19*x^6 + 6*x^4)*sqrt(x^4 + 5*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^{4}+5\,x^{2}+3\right)^{\frac{3}{2}}\left(3\,x^{2}+2\right)x^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4, x)
```

maple [A] time = 0.02, size = 294, normalized size = 0.83

$$\frac{3\sqrt{x^4+5x^2+3}\;x^{11}}{13}+\frac{236\sqrt{x^4+5x^2+3}\;x^{9}}{143}+\frac{1090\sqrt{x^4+5x^2+3}\;x^{7}}{429}+\frac{356\sqrt{x^4+5x^2+3}\;x^{5}}{429}+\frac{1251\sqrt{x^4+5x^2+3}}{715}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 3/13*x^11*(x^4+5*x^2+3)^(1/2)+236/143*x^9*(x^4+5*x^2+3)^(1/2)+1090/429*(x^4
+5*x^2+3)^(1/2)*x^7+356/429*(x^4+5*x^2+3)^(1/2)*x^5+1251/715*(x^4+5*x^2+3)^
(1/2)*x^3-4210/429*(x^4+5*x^2+3)^(1/2)*x+25260/143/(-30+6*13^(1/2))^(1/2)*(
-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5
*x^2+3)^(1/2)*EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/
2))-2120676/715/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(
-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(Ellipti
cF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-3
0+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4 + 5\,x^2 + 3\right)^{\frac{3}{2}} \left(3\,x^2 + 2\right) x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(3\,x^2 + 2\right) \left(x^4 + 5\,x^2 + 3\right)^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(3x^2 + 2\right) \left(x^4 + 5x^2 + 3\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
[Out] Integral(x**4*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.164 $\int x^2 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=331

$$\frac{353}{99}\sqrt{x^4+5x^2+3}\,x-\frac{49949\left(2x^2+\sqrt{13}+5\right)x}{3465\sqrt{x^4+5x^2+3}}-\frac{353\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\right.\right.}{33\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$

```
[Out] 1/99*x^3*(27*x^2+67)*(x^4+5*x^2+3)^(3/2)-49949/3465*x*(5+2*x^2+13^(1/2))/(x
^4+5*x^2+3)^(1/2)+353/99*x*(x^4+5*x^2+3)^(1/2)-1/1155*x^3*(890*x^2+911)*(x^
4+5*x^2+3)^(1/2)+49949/20790*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30
+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2
)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))
^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2
)-353/33*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*
EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+3
0*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^
(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1273, 1279, 1189, 1099, 1135}

$$\frac{1}{99}\left(27x^2+67\right)\left(x^4+5x^2+3\right)^{3/2}x^3-\frac{\left(890x^2+911\right)\sqrt{x^4+5x^2+3}\,x^3}{1155}+\frac{353}{99}\sqrt{x^4+5x^2+3}\,x-\frac{49949\left(2x^2+\sqrt{13}\right.}{3465\sqrt{x^4+5x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-49949*x*(5 + Sqrt[13] + 2*x^2))/(3465*Sqrt[3 + 5*x^2 + x^4]) + (353*x*Sqr
t[3 + 5*x^2 + x^4])/99 - (x^3*(911 + 890*x^2)*Sqrt[3 + 5*x^2 + x^4])/1155 +
 (x^3*(67 + 27*x^2)*(3 + 5*x^2 + x^4)^(3/2))/99 + (49949*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(3465*Sqrt[3 + 5*x^2 + x^4]) - (353*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (
5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqr
t[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(33*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*
x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] && !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1273

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*
p + 3)), x] + Dist[(2*p)/(c*(4*p + m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int x^2\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{1}{99}x^3\left(67+27x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{33}\int x^2\left(-3-178x^2\right)\sqrt{3+5x^2+}$$

$$= -\frac{x^3\left(911+890x^2\right)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3\left(67+27x^2\right)\left(3+5x^2+x^4\right)^{3/}$$

$$= \frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3\left(911+890x^2\right)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3\left(67+\right.$$

$$= \frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3\left(911+890x^2\right)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3\left(67+\right.$$

$$= -\frac{49949x\left(5+\sqrt{13}+2x^2\right)}{3465\sqrt{3+5x^2+x^4}}+\frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3\left(911+890x^2\right)}{1155}$$

Mathematica [C] time = 0.25, size = 244, normalized size = 0.74

$$i\sqrt{2}\left(49949\sqrt{13}-212680\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-49949i\sqrt{2}\left(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*x*(37065 + 74681*x^2 + 69535*x^4 + 84962*x^6 + 50075*x^8 + 11795*x^10 +
945*x^12) - (49949*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/
(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 +
 Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-212680 + 49949*Sqrt[13
])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]
*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(69
30*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(3\,x^{8}+17\,x^{6}+19\,x^{4}+6\,x^{2}\right)\sqrt{x^{4}+5\,x^{2}+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^8 + 17*x^6 + 19*x^4 + 6*x^2)*sqrt(x^4 + 5*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^{4}+5\,x^{2}+3\right)^{\frac{3}{2}}\left(3\,x^{2}+2\right)x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2, x)
```

maple [A] time = 0.02, size = 277, normalized size = 0.84

$$\frac{3\sqrt{x^{4}+5x^{2}+3}\;x^{9}}{11}+\frac{202\sqrt{x^{4}+5x^{2}+3}\;x^{7}}{99}+\frac{2378\sqrt{x^{4}+5x^{2}+3}\;x^{5}}{693}+\frac{478\sqrt{x^{4}+5x^{2}+3}\;x^{3}}{385}+\frac{353\sqrt{x^{4}+5x^{2}+3}\;x}{99}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 3/11*(x^4+5*x^2+3)^(1/2)*x^9+202/99*(x^4+5*x^2+3)^(1/2)*x^7+2378/693*(x^4+5
*x^2+3)^(1/2)*x^5+478/385*(x^4+5*x^2+3)^(1/2)*x^3+353/99*(x^4+5*x^2+3)^(1/2
)*x-706/11/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/
6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*(-30+6*13^(1
/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))+399592/385/(-30+6*13^(1/2))^(1/2)*(-
(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*
x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/
2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^
(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^{4}+5\,x^{2}+3\right)^{\frac{3}{2}}\left(3\,x^{2}+2\right)x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(3\,x^2+2\right)\,\left(x^4+5\,x^2+3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(3x^{2}+2\right)\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
[Out] Integral(x**2*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.165 $\int \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=308

$$\frac{1}{3}x\left(x^2+3\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1}{15}x\left(12x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{203x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}+\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+}{\left(5+\sqrt{13}\right)x^2+}}}{}$$

```
[Out] 1/3*x*(x^2+3)*(x^4+5*x^2+3)^(3/2)+203/30*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)
^(1/2)-1/15*x*(12*x^2+5)*(x^4+5*x^2+3)^(1/2)+5/3*(1/(36+x^2*(30+6*13^(1/2))
))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(
36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/
2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))
^(1/2)/(x^4+5*x^2+3)^(1/2)-203/180*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x
^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*1
3^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^
(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3
)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1176, 1189, 1099, 1135}

$$\frac{1}{3}x\left(x^2+3\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1}{15}x\left(12x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{203x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}+\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+}{\left(5+\sqrt{13}\right)x^2+}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (203*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (x*(5 + 12*x^2)
*Sqrt[3 + 5*x^2 + x^4])/15 + (x*(3 + x^2)*(3 + 5*x^2 + x^4)^(3/2))/3 - (203
*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x
^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (
-13 + 5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4]) + (5*Sqrt[2/(3*(5 + Sqrt[1
3]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqr
t[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/
6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{21}\int \left(63-84x^2\right)\sqrt{3+5x^2+x^4}\,dx$$

$$= -\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{315}\int$$

$$= -\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}+10\int$$

$$= \frac{203x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}}-\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right)(3$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(3\,x^6+17\,x^4+19\,x^2+6\right)\sqrt{x^4+5\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\left(3\,x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2), x)
```

maple [A] time = 0.01, size = 260, normalized size = 0.84

$$\frac{\sqrt{x^4+5x^2+3}\;x^7}{3}+\frac{8\sqrt{x^4+5x^2+3}\;x^5}{3}+\frac{26\sqrt{x^4+5x^2+3}\;x^3}{5}+\frac{8\sqrt{x^4+5x^2+3}\;x}{3}+\frac{60\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\;\sqrt{-}}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 1/3*(x^4+5*x^2+3)^(1/2)*x^7+8/3*(x^4+5*x^2+3)^(1/2)*x^5+26/5*(x^4+5*x^2+3)^
(1/2)*x^3+8/3*(x^4+5*x^2+3)^(1/2)*x+60/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*1
3^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2
)*EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-2436/5/(
-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/
2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30+6*13^
(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(1/2))^(1/
2)*x,5/6*3^(1/2)+1/6*39^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\left(3\,x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(3\,x^2+2\right)\,\left(x^4+5\,x^2+3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.166 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=312

$$-\frac{(14-3x^2)(x^4+5x^2+3)^{3/2}}{7x}+\frac{1}{35}x(129x^2+655)\sqrt{x^4+5x^2+3}+\frac{412x(2x^2+\sqrt{13}+5)}{35\sqrt{x^4+5x^2+3}}+\frac{19\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{(\cdots}}{\cdots}$$

```
[Out] -1/7*(-3*x^2+14)*(x^4+5*x^2+3)^(3/2)/x+412/35*x*(5+2*x^2+13^(1/2))/(x^4+5*x
^2+3)^(1/2)+1/35*x*(129*x^2+655)*(x^4+5*x^2+3)^(1/2)+19/2*(1/(36+x^2*(30+6*
13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2)
)^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*
(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13
^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-206/105*(1/(36+x^2*(30+6*13^(1/2))))^(1
/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^
2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*
(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^
4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1271, 1176, 1189, 1099, 1135}

$$-\frac{(14-3x^2)(x^4+5x^2+3)^{3/2}}{7x}+\frac{1}{35}x(129x^2+655)\sqrt{x^4+5x^2+3}+\frac{412x(2x^2+\sqrt{13}+5)}{35\sqrt{x^4+5x^2+3}}+\frac{19\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{(\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2,x]
```

```
[Out] (412*x*(5 + Sqrt[13] + 2*x^2))/(35*Sqrt[3 + 5*x^2 + x^4]) + (x*(655 + 129*x
^2)*Sqrt[3 + 5*x^2 + x^4])/35 - ((14 - 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(7*x
) - (206*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
 Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[1
3])/6]*x], (-13 + 5*Sqrt[13])/6])/(35*Sqrt[3 + 5*x^2 + x^4]) + (19*Sqrt[3/(
2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*
(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 +
 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x^2}\,dx = -\frac{\left(14-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{7x} - \frac{3}{7}\int \left(-88-43x^2\right)\sqrt{3+5x^2+x^4}\,dx$$

$$= \frac{1}{35}x\left(655+129x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(14-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{7x} - \frac{1}{35}\int \frac{-1}{\sqrt{}}$$

$$= \frac{1}{35}x\left(655+129x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(14-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{7x} + \frac{824}{35}\int$$

$$= \frac{412x\left(5+\sqrt{13}+2x^2\right)}{35\sqrt{3+5x^2+x^4}} + \frac{1}{35}x\left(655+129x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(14-3x^2\right)\left(3\right.}{7}$$

Mathematica [C] time = 0.26, size = 235, normalized size = 0.75

$$\frac{30x^{10}+418x^8+2130x^6+3884x^4-i\sqrt{2}\left(412\sqrt{13}-65\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,xF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right.\right.}{70x\sqrt{x^4+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2,x]
```

```
[Out] (-1260 + 3884*x^4 + 2130*x^6 + 418*x^8 + 30*x^10 + (412*I)*Sqrt[2]*(-5 + Sq
rt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] +
```

```
2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/
6] - I*Sqrt[2]*(-65 + 412*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sq
rt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13
])]*x], 19/6 + (5*Sqrt[13])/6])/(70*x*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,x^6+17\,x^4+19\,x^2+6\right)\sqrt{x^4+5\,x^2+3}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2, x)
```

maple [A] time = 0.02, size = 260, normalized size = 0.83

$$\frac{3\sqrt{x^4+5x^2+3}\;x^5}{7}+\frac{134\sqrt{x^4+5x^2+3}\;x^3}{35}+10\sqrt{x^4+5x^2+3}\;x+\frac{342\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\;\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x}}{\sqrt{-30+6\sqrt{13}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x)
```

```
[Out] 3/7*(x^4+5*x^2+3)^(1/2)*x^5+134/35*(x^4+5*x^2+3)^(1/2)*x^3+10*(x^4+5*x^2+3)
^(1/2)*x+342/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-
5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*(-30+6*13^
(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-29664/35/(-30+6*13^(1/2))^(1/2)*(-
(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*
x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/
2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^
(1/2)))-6*(x^4+5*x^2+3)^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^2,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**2,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**2, x)
```

3.167 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=314

$$-\frac{13(24-5x^2)\sqrt{x^4+5x^2+3}}{15x}+\frac{949x(2x^2+\sqrt{13}+5)}{30\sqrt{x^4+5x^2+3}}+\frac{65\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)F\left(\text{ta}\right.}{\sqrt{x^4+5x^2+}}$$

```
[Out] -1/15*(-9*x^2+10)*(x^4+5*x^2+3)^(3/2)/x^3+949/30*x*(5+2*x^2+13^(1/2))/(x^4+
5*x^2+3)^(1/2)-13/15*(-5*x^2+24)*(x^4+5*x^2+3)^(1/2)/x+65/3*(1/(36+x^2*(30+
6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/
2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^
2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+
13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-949/180*(1/(36+x^2*(30+6*13^(1/2))))^
(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2))
)*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(
x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1271, 1189, 1099, 1135}

$$-\frac{(10-9x^2)(x^4+5x^2+3)^{3/2}}{15x^3}-\frac{13(24-5x^2)\sqrt{x^4+5x^2+3}}{15x}+\frac{949x(2x^2+\sqrt{13}+5)}{30\sqrt{x^4+5x^2+3}}+\frac{65\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13}}{(5+\sqrt{13}}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4,x]
```

```
[Out] (949*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (13*(24 - 5*x^2
)*Sqrt[3 + 5*x^2 + x^4])/(15*x) - ((10 - 9*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(1
5*x^3) - (949*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5
+ Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[
13])/6]*x], (-13 + 5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4]) + (65*Sqrt[2/
(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rubi steps

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = -\frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3}-\frac{1}{5}\int \frac{(-104-65x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$$

$$= -\frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x}-\frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3}+\frac{1}{15}\int \frac{1950}{\sqrt{3+}}$$

$$= -\frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x}-\frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3}+\frac{949}{15}\int \frac{}{\sqrt{3}}$$

$$= \frac{949x(5+\sqrt{13}+2x^2)}{30\sqrt{3+5x^2+x^4}}-\frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x}-\frac{(10-9x^2)(3+5x^2}{15x^3}$$

Mathematica [C] time = 0.29, size = 247, normalized size = 0.79

$$\frac{-13i\sqrt{2}\left(73\sqrt{13}-65\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,x^3F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+949i\sqrt{2}\left(\sqrt{13}-\right.}{60x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4,x]
```

```
[Out] (4*(-90 - 1155*x^2 - 1405*x^4 + 192*x^6 + 145*x^8 + 9*x^10) + (949*I)*Sqrt[
2]*(-5 + Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5
 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 +
(5*Sqrt[13])/6] - (13*I)*Sqrt[2]*(-65 + 73*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13
] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[
Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(60*x^3*Sqrt[3 + 5*x^2 +
 x^4])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(3x^6+17x^4+19x^2+6)\sqrt{x^4+5x^2+3}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="fricas")
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4, x)
```

maple [A] time = 0.02, size = 260, normalized size = 0.83

$$\frac{3\sqrt{x^4+5x^2+3}\;x^3}{5}+\frac{20\sqrt{x^4+5x^2+3}\;x}{3}+\frac{780\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\;\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-30}}{}\right.}{\sqrt{-30+6\sqrt{13}}\;\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x)
[Out] -67/3*(x^4+5*x^2+3)^(1/2)/x+3/5*(x^4+5*x^2+3)^(1/2)*x^3+20/3*(x^4+5*x^2+3)^
(1/2)*x+780/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5
/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*(-30+6*13^(
1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-11388/5/(-30+6*13^(1/2))^(1/2)*(-(-
5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^
2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)
+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1
/2)))-2*(x^4+5*x^2+3)^(1/2)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^4,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**4,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**4, x)
```

3.168 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=331

$$-\frac{722\sqrt{x^4+5x^2+3}}{15x}+\frac{361x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{103\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\right.\right.}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$

```
[Out] -1/5*(-5*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5+361/15*x*(5+2*x^2+13^(1/2))/(x^4+5*
x^2+3)^(1/2)-722/15*(x^4+5*x^2+3)^(1/2)/x-1/5*(-87*x^2+40)*(x^4+5*x^2+3)^(1
/2)/x^3-361/90*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^
(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*
(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2
*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+103*(1/(36+x
^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6
*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2)
)*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x
^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1271, 1281, 1189, 1099, 1135}

$$-\frac{\left(2-5x^2\right)\left(x^4+5x^2+3\right)^{3/2}}{5x^5}-\frac{\left(40-87x^2\right)\sqrt{x^4+5x^2+3}}{5x^3}-\frac{722\sqrt{x^4+5x^2+3}}{15x}+\frac{361x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{103\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6,x]
```

```
[Out] (361*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) - (722*Sqrt[3 + 5
*x^2 + x^4])/(15*x) - ((40 - 87*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5*x^3) - ((2 -
 5*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(5*x^5) - (361*Sqrt[(5 + Sqrt[13])/6]*Sqrt
[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2
)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(15*Sq
rt[3 + 5*x^2 + x^4]) + (103*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13
])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x
], (-13 + 5*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x^6}\,dx = -\frac{\left(2-5x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{5x^5} - \frac{1}{5}\int \frac{\left(-120-87x^2\right)\sqrt{3+5x^2+x^4}}{x^4}\,dx$$

$$= -\frac{\left(40-87x^2\right)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{\left(2-5x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{5x^5} + \frac{1}{15}\int \frac{2166+}{x^2\sqrt{3+}}$$

$$= -\frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{\left(40-87x^2\right)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{\left(2-5x^2\right)\left(3+5x^2+x^4\right.}{5x^5}$$

$$= -\frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{\left(40-87x^2\right)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{\left(2-5x^2\right)\left(3+5x^2+x^4\right.}{5x^5}$$

$$= \frac{361x\left(5+\sqrt{13}+2x^2\right)}{15\sqrt{3+5x^2+x^4}} - \frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{\left(40-87x^2\right)\sqrt{3+5x^2+x^4}}{5x^3} -$$

Mathematica [C] time = 0.28, size = 244, normalized size = 0.74

$$\frac{30x^{10}-634x^8-4040x^6-3438x^4-810x^2-i\sqrt{2}\left(361\sqrt{13}-260\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,x^5F\left(i\sinh^{-1}\right.}{30x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6,x]
```

```
[Out] (-108 - 810*x^2 - 3438*x^4 - 4040*x^6 - 634*x^8 + 30*x^10 + (361*I)*Sqrt[2]
*(-5 + Sqrt[13])*x^5*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
 Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5
*Sqrt[13])/6] - I*Sqrt[2]*(-260 + 361*Sqrt[13])*x^5*Sqrt[(-5 + Sqrt[13] - 2
*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[
2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(30*x^5*Sqrt[3 + 5*x^2 + x^4]
)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(3\,x^{6}+17\,x^{4}+19\,x^{2}+6\right)\sqrt{x^{4}+5\,x^{2}+3}}{x^{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+5\,x^{2}+3\right)^{\frac{3}{2}}\left(3\,x^{2}+2\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6, x)
```

maple [A] time = 0.02, size = 259, normalized size = 0.78

$$\sqrt{x^4+5x^2+3}\;x+\frac{618\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\;\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\;x}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\;\sqrt{x^4+5x^2+3}}-39$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x)
```

```
[Out] -7*(x^4+5*x^2+3)^(1/2)/x^3-392/15*(x^4+5*x^2+3)^(1/2)/x+(x^4+5*x^2+3)^(1/2)
*x+618/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/
6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*(-30+6*13^(1/2))
^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-8664/5/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/
6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(
1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*3
9^(1/2))-EllipticE(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2)))-
6/5/x^5*(x^4+5*x^2+3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+5\,x^{2}+3\right)^{\frac{3}{2}}\left(3\,x^{2}+2\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="maxima")
```

[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6, x)

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2}}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^6,x)

[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^6, x)

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{2}+2\right)\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**6,x)

[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**6, x)

3.169 $\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{a+bx^2+cx^4}\left(-16aBc-2cx^2(5bB-6Ac)-18Abc+15b^2B\right)}{48c^3}-\frac{\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\tanh^{-1}\Big(}{32c^{7/2}}$$

```
[Out] -1/32*(8*A*a*c^2-6*A*b^2*c-12*B*a*b*c+5*B*b^3)*arctanh(1/2*(2*c*x^2+b)/c^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/c^(7/2)+1/6*B*x^4*(c*x^4+b*x^2+a)^(1/2)/c+1/48*(
15*b^2*B-18*A*b*c-16*a*B*c-2*c*(-6*A*c+5*B*b)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^
3
```

Rubi [A] time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1251, 832, 779, 621, 206}

$$\frac{\sqrt{a+bx^2+cx^4}\left(-16aBc-2cx^2(5bB-6Ac)-18Abc+15b^2B\right)}{48c^3}-\frac{\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\tanh^{-1}\Big(}{32c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2*B - 18*A*b*c - 16*a*B*c -
2*c*(5*b*B - 6*A*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(48*c^3) - ((5*b^3*B - 6*
A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$

$$= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\text{Subst}\left(\int \frac{x\left(-2aB-\frac{1}{2}(5bB-6Ac)x\right)}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{6c}$$

$$= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} -$$

$$= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} -$$

$$= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} -$$

Mathematica [A] time = 0.11, size = 139, normalized size = 0.91

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(4c\left(-4aB+3Acx^2+2Bcx^4\right)-2bc\left(9A+5Bx^2\right)+15b^2B\right)-3\left(8aAc^2-12abBc-6Ab^2c+\right.}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^2*B - 2*b*c*(9*A + 5*B*x^2) + 4*c*
(-4*a*B + 3*A*c*x^2 + 2*B*c*x^4)) - 3*(5*b^3*B - 6*A*b^2*c - 12*a*b*B*c + 8
*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(96*c
^(7/2))
```

fricas [A] time = 0.84, size = 315, normalized size = 2.06

$$\left[\frac{3\left(5\,Bb^3+8\,Aac^2-6\left(2\,Bab+Ab^2\right)c\right)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)+}{192\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(5*B*b^3 + 8*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(c)*log(-8*c^2*
x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4
*a*c) + 4*(8*B*c^3*x^4 + 15*B*b^2*c - 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2
- 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/96*(3*(5*B*b^3 + 8*A*a*c^2
```

```
- 6*(2*B*a*b + A*b^2)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x
^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*B*c^3*x^4 + 15*B*b^2*c -
 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 +
a))/c^4]
```

giac [A] time = 0.54, size = 138, normalized size = 0.90

$$\frac{1}{48}\sqrt{cx^4+bx^2+a}\left(2\left(\frac{4\,Bx^2}{c}-\frac{5\,Bbc-6\,Ac^2}{c^3}\right)x^2+\frac{15\,Bb^2-16\,Bac-18\,Abc}{c^3}\right)+\frac{\left(5\,Bb^3-12\,Babc-6\,Ab^2c+8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*B*x^2/c - (5*B*b*c - 6*A*c^2)/c^3)*x^2 +
 (15*B*b^2 - 16*B*a*c - 18*A*b*c)/c^3) + 1/32*(5*B*b^3 - 12*B*a*b*c - 6*A*b
^2*c + 8*A*a*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c
) - b))/c^(7/2)
```

maple [B] time = 0.03, size = 286, normalized size = 1.87

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;B\,x^4}{6c}+\frac{\sqrt{c\,x^4+b\,x^2+a}\;A\,x^2}{4c}-\frac{5\sqrt{c\,x^4+b\,x^2+a}\;Bb\,x^2}{24c^2}-\frac{Aa\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4c^{\frac{3}{2}}}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*B*x^4*(c*x^4+b*x^2+a)^(1/2)/c-5/24*B*b/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)+5/
16*B*b^2/c^3*(c*x^4+b*x^2+a)^(1/2)-5/32*B*b^3/c^(7/2)*ln((1/2*b+c*x^2)/c^(1
/2)+(c*x^4+b*x^2+a)^(1/2))+3/8*B*b/c^(5/2)*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))-1/3*B*a/c^2*(c*x^4+b*x^2+a)^(1/2)+1/4*A*x^2/c*(c*x^4+b*x^
2+a)^(1/2)-3/8*A*b/c^2*(c*x^4+b*x^2+a)^(1/2)+3/16*A*b^2/c^(5/2)*ln((1/2*b+c
*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*A*a/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/
2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(A + Bx^2\right)}{\sqrt{a + bx^2 + cx^4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x**5*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.170 $\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=100

$$\frac{(-4aBc-4Abc+3b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{8c^2}$$

```
[Out] 1/16*(-4*A*b*c-4*B*a*c+3*B*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^
2+a)^(1/2))/c^(5/2)-1/8*(-2*B*c*x^2-4*A*c+3*B*b)*(c*x^4+b*x^2+a)^(1/2)/c^2
```

Rubi [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1251, 779, 621, 206}

$$\frac{(-4aBc-4Abc+3b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + ((3*b^2*B
- 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3 \left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{\left(3bB-4Ac-2Bcx^2\right)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{\left(3b^2B-4Abc-4aBc\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx\right.}{16c^2}$$

$$= -\frac{\left(3bB-4Ac-2Bcx^2\right)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{\left(3b^2B-4Abc-4aBc\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, -\right.}{8c^2}$$

$$= -\frac{\left(3bB-4Ac-2Bcx^2\right)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{\left(3b^2B-4Abc-4aBc\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right.}{16c^{5/2}}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 1.01

$$\frac{\left(-4aBc-4Abc+3b^2B\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(4Ac-3bB+2Bcx^2\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*(-3*b*B + 4*A*c + 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (3*b^2*B
- 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

fricas [A] time = 0.84, size = 233, normalized size = 2.33

$$\left[-\frac{\left(3\,Bb^2-4\,(Ba+Ab)c\right)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}\,-4\,ac\right)-4\left(2\,Bc^2x^2-\right.}{32\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*B*b^2 - 4*(B*a + A*b)*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^
2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(2*B*c^2*x
^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^3, -1/16*((3*B*b^2 - 4*(
B*a + A*b)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt
(c*x^4 + b*x^2 + a))/c^3]
```

giac [A] time = 0.45, size = 98, normalized size = 0.98

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(\frac{2\,Bx^2}{c}-\frac{3\,Bb-4\,Ac}{c^2}\right)-\frac{\left(3\,Bb^2-4\,Bac-4\,Abc\right)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}\,-b\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*B*x^2/c - (3*B*b - 4*A*c)/c^2) - 1/16*(3*B*b
^2 - 4*B*a*c - 4*A*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*
sqrt(c) - b))/c^(5/2)
```

maple [B] time = 0.02, size = 176, normalized size = 1.76

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;B\,x^{2}}{4c}-\frac{A b \ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}-\frac{B a \ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}+\frac{3B\,b^{2}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x^2/c*(c*x^4+b*x^2+a)^(1/2)-3/8*B*b/c^2*(c*x^4+b*x^2+a)^(1/2)+3/16*B*
b^2/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*B*a/c^(3/2)
*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/2*A/c*(c*x^4+b*x^2+a)^(1
/2)-1/4*A*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(A + Bx^{2}\right)}{\sqrt{a + bx^{2} + cx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.171 $\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=76

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

```
[Out] -1/4*(-2*A*c+B*b)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^
(3/2)+1/2*B*(c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1247, 640, 621, 206}

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$
$$= \frac{B\sqrt{a+bx^2+cx^4}}{2c} + \frac{(-bB+2Ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4c}$$
$$= \frac{B\sqrt{a+bx^2+cx^4}}{2c} + \frac{(-bB+2Ac)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2c}$$
$$= \frac{B\sqrt{a+bx^2+cx^4}}{2c} - \frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 1.03

$$\frac{1}{2}\left(\frac{(2Ac-bB)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}} + \frac{B\sqrt{a+bx^2+cx^4}}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((B*Sqrt[a + b*x^2 + c*x^4])/c + ((-(b*B) + 2*A*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*c^(3/2)))/2
```

fricas [A] time = 1.08, size = 178, normalized size = 2.34

$$\left[\frac{4\sqrt{cx^4+bx^2+a}\,Bc-(Bb-2\,Ac)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)}{8\,c^2}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*B*c - (B*b - 2*A*c)*sqrt(c)*log(-8*c^2*x^4
- 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c
))/c^2, 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*B*c + (B*b - 2*A*c)*sqrt(-c)*arctan(
1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c
)))/c^2]
```

giac [A] time = 0.46, size = 69, normalized size = 0.91

$$\frac{\sqrt{cx^4+bx^2+a}\,B}{2\,c} + \frac{(Bb-2\,Ac)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + b*x^2 + a)*B/c + 1/4*(B*b - 2*A*c)*log(abs(-2*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2)
```

maple [A] time = 0.01, size = 93, normalized size = 1.22

$$\frac{A\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}-\frac{Bb\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}+\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,B}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*B*(c*x^4+b*x^2+a)^(1/2)/c-1/4*B*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))+1/2*A*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^
(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 1.05, size = 92, normalized size = 1.21

$$\frac{A\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,\sqrt{c}}+\frac{B\,\sqrt{c\,x^4+b\,x^2+a}}{2\,c}-\frac{B\,b\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(2*c^(1/2)) + (B
*(a + b*x^2 + c*x^4)^(1/2))/(2*c) - (B*b*log((a + b*x^2 + c*x^4)^(1/2) + (b
/2 + c*x^2)/c^(1/2)))/(4*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(A+Bx^{2}\right)}{\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.172 $\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=90

$$\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*A*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)+1/2*B
*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 843, 621, 206, 724}

$$\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a])
 + (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c
])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}A\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)+\frac{1}{2}B\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\
&= -\left(A\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)\right)+B\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2
\end{aligned}$$

$$= -\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}+\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.03, size = 89, normalized size = 0.99

$$\frac{1}{2}\left(\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}-\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-((A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a])
+ (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c])/2
```

fricas [A] time = 1.07, size = 517, normalized size = 5.74

$$\left[\frac{Ba\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)+A\sqrt{a}c\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4}}{x^4}\right.}{4ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 +
 a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 +
8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/
(a*c), -1/4*(2*B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4
 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4
))/(a*c), 1/4*(2*A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2
*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*
c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(a*
c), 1/2*(A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt
(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^
2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(a*c)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Value
```

maple [A] time = 0.01, size = 76, normalized size = 0.84

$$-\frac{A\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{2\sqrt{a}}+\frac{B\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*B*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*A/a^(1/2)
*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 0.76, size = 81, normalized size = 0.90

$$\frac{B\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,\sqrt{c}}-\frac{A\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a}}-\frac{A\,\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (B*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(2*c^(1/2)) - (A
*log(1/x^2))/(2*a^(1/2)) - (A*log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2)
 + b*x^2))/(2*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

3.173 $\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

```
[Out] 1/4*(A*b-2*B*a)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3
/2)-1/2*A*(c*x^4+b*x^2+a)^(1/2)/a/x^2
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1251, 806, 724, 206}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(2*a*x^2) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*x^
2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} - \frac{(Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4a}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{(Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2a}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

Mathematica [A] time = 0.04, size = 82, normalized size = 1.02

$$\frac{1}{2}\left(\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}} - \frac{A\sqrt{a+bx^2+cx^4}}{ax^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-((A*Sqrt[a + b*x^2 + c*x^4])/(a*x^2)) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*x
^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)))/2
```

fricas [A] time = 1.15, size = 197, normalized size = 2.46

$$\left[-\frac{(2\,Ba-Ab)\sqrt{a}\,x^2\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\,\sqrt{cx^4+bx^2+a}\,Aa}{8\,a^2x^2}, \frac{(2\,Ba-Ab)\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*B*a - A*b)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sq
rt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 +
b*x^2 + a)*A*a)/(a^2*x^2), 1/4*((2*B*a - A*b)*sqrt(-a)*x^2*arctan(1/2*sqrt(
c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*sq
rt(c*x^4 + b*x^2 + a)*A*a)/(a^2*x^2)]
```

giac [A] time = 0.54, size = 124, normalized size = 1.55

$$\frac{(2\,Ba-Ab)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a} + \frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)Ab+2\,Aa\sqrt{c}}{2\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*a - A*b)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))
/(sqrt(-a)*a) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*b + 2*A*a*sq
rt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a)
```

maple [A] time = 0.02, size = 104, normalized size = 1.30

$$\frac{A b \ln\left(\frac{b x^2+2a+2\sqrt{c x^4+b x^2+a}\ \sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}}-\frac{B \ln\left(\frac{b x^2+2a+2\sqrt{c x^4+b x^2+a}\ \sqrt{a}}{x^2}\right)}{2\sqrt{a}}-\frac{\sqrt{c x^4+b x^2+a}\ A}{2a x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*B/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-1/2*A*(c
*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*A*b/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(
1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 0.78, size = 103, normalized size = 1.29

$$\frac{A\,b\,\mathrm{atanh}\left(\frac{\frac{b\,x^2}{2}+a}{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}\right)}{4\,a^{3/2}}-\frac{B\,\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\,\sqrt{a}}-\frac{A\,\sqrt{c\,x^4+b\,x^2+a}}{2\,a\,x^2}-\frac{B\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (A*b*atanh((a + (b*x^2)/2)/(a^(1/2)*(a + b*x^2 + c*x^4)^(1/2))))/(4*a^(3/2)
) - (B*log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) + b*x^2))/(2*a^(1/2))
- (A*(a + b*x^2 + c*x^4)^(1/2))/(2*a*x^2) - (B*log(1/x^2))/(2*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^3\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

3.174 $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=124

$$-\frac{\left(-4aAc-4abB+3Ab^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}$$

```
[Out] -1/16*(-4*A*a*c+3*A*b^2-4*B*a*b)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x
^2+a)^(1/2))/a^(5/2)-1/4*A*(c*x^4+b*x^2+a)^(1/2)/a/x^4+1/8*(3*A*b-4*B*a)*(c
*x^4+b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.15, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 834, 806, 724, 206}

$$-\frac{\left(-4aAc-4abB+3Ab^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(4*a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2 +
c*x^4])/(8*a^2*x^2) - ((3*A*b^2 - 4*a*b*B - 4*a*A*c)*ArcTanh[(2*a + b*x^2)/
(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
```

```
IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3Ab-4aB)+Acx}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4a}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} + \frac{\left(3Ab^2-4abB-4aAc\right)\text{Subst}\left(\int\right.}{16a^2}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{\left(3Ab^2-4abB-4aAc\right)\text{Subst}\left(\int\right.}{8a^2}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{\left(3Ab^2-4abB-4aAc\right)\tanh^{-1}\left(\right.}{16a^{5/2}}$$

Mathematica [A] time = 0.08, size = 107, normalized size = 0.86

$$\frac{\left(4aAc+4abB-3Ab^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}} + \frac{\sqrt{a+bx^2+cx^4}\left(3Abx^2-2a\left(A+2Bx^2\right)\right)}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(3*A*b*x^2 - 2*a*(A + 2*B*x^2)))/(8*a^2*x^4) + ((-
3*A*b^2 + 4*a*b*B + 4*a*A*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^
2 + c*x^4])])/(16*a^(5/2))
```

fricas [A] time = 0.87, size = 255, normalized size = 2.06

$$\left[\frac{\left(4\,Bab-3\,Ab^2+4\,Aac\right)\sqrt{a}\,x^4\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}\left(2\,Aa^2+\left(4\right.\right.}{32\,a^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 +
8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) -
 4*sqrt(c*x^4 + b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4),
-1/16*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b
*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*sqrt(c*x^4
+ b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4)]
```

giac [B] time = 0.52, size = 339, normalized size = 2.73

$$-\frac{\left(4\,Bab - 3\,Ab^2 + 4\,Aac\right)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^2}+\frac{4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3Bab-3\left(\sqrt{c}\,x^2-\sqrt{cx^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*B*a*b - 3*A*b^2 + 4*A*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^
2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/8*(4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
 + a))^3*B*a*b - 3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*b^2 + 4*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))^2*B*a^2*sqrt(c) - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*B*
a^2*b + 5*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*a*b^2 + 4*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))*A*a^2*c - 8*B*a^3*sqrt(c) + 8*A*a^2*b*sqrt(c))/(
((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^2)
```

maple [A] time = 0.02, size = 194, normalized size = 1.56

$$\frac{Ac\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}}-\frac{3A\,b^2\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{16a^{\frac{5}{2}}}+\frac{Bb\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}}+\frac{3\sqrt{c\,x^4+}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*A*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*A*b/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)-3/
16*A*b^2/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/4*A*
c/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-1/2*B/a/x^2*(
c*x^4+b*x^2+a)^(1/2)+1/4*B*b/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*
a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{x^5\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^5*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^5\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

3.175 $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=177

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-16aAc-18abB+15Ab^2\right)}{48a^3x^2}+\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4}+\frac{\left(8a^2Bc-12aAbc-6ab^2B+5A\right.}{32a^{7\!/}}$$

```
[Out] 1/32*(-12*A*a*b*c+5*A*b^3+8*B*a^2*c-6*B*a*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/a^(7/2)-1/6*A*(c*x^4+b*x^2+a)^(1/2)/a/x^6+1/24*(
5*A*b-6*B*a)*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4-1/48*(-16*A*a*c+15*A*b^2-18*B*a*
b)*(c*x^4+b*x^2+a)^(1/2)/a^3/x^2
```

Rubi [A] time = 0.24, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 834, 806, 724, 206}

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-16aAc-18abB+15Ab^2\right)}{48a^3x^2}+\frac{\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(6*a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a + b*x^2 +
c*x^4])/(24*a^2*x^4) - ((15*A*b^2 - 18*a*b*B - 16*a*A*c)*Sqrt[a + b*x^2 + c
*x^4])/(48*a^3*x^2) + ((5*A*b^3 - 6*a*b^2*B - 12*a*A*b*c + 8*a^2*B*c)*ArcTa
nh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
```

```
2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5Ab-6aB)+2Acx}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{6a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}\left(15Ab^2-18abB-16aAc\right)+\frac{1}{2}}{x^2\sqrt{a+bx+cx^2}}\right.}{12a^2}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15Ab^2-18abB-16aAc\right)\sqrt{a+}}{48a^3x^2}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15Ab^2-18abB-16aAc\right)\sqrt{a+}}{48a^3x^2}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15Ab^2-18abB-16aAc\right)\sqrt{a+}}{48a^3x^2}$$

Mathematica [A] time = 0.11, size = 148, normalized size = 0.84

$$\frac{\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(-4a^2\left(2A+3Bx^2\right)+2a\left(5Abx^2+\right.\right.}{48a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-15*A*b^2*x^4 - 4*a^2*(2*A + 3*B*x^2) + 2*a*(5*A*
b*x^2 + 9*b*B*x^4 + 8*A*c*x^4)))/(48*a^3*x^6) + ((5*A*b^3 - 6*a*b^2*B - 12*
a*A*b*c + 8*a^2*B*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^
4]])/(32*a^(7/2))
```

fricas [A] time = 0.96, size = 339, normalized size = 1.92

$$\left[\frac{3\left(6\,Bab^2-5\,Ab^3-4\left(2\,Ba^2-3\,Aab\right)c\right)\sqrt{a}\,x^6\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\left(\left(18\,Ba^2b-\right.\right.}{192\,a^4x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(a)*x^6*log(-
((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sq
rt(a) + 8*a^2)/x^4) + 4*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a
^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6), 1/96*
(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(-a)*x^6*arctan(1/2*
sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) +
 2*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 5*A
*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6)]
```

giac [B] time = 0.64, size = 571, normalized size = 3.23

$$\frac{\left(6\,Bab^2-5\,Ab^3-8\,Ba^2c+12\,Aabc\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}\,a^3}-\frac{18\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5Bab^2-15\left(\sqrt{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(6*B*a*b^2 - 5*A*b^3 - 8*B*a^2*c + 12*A*a*b*c)*arctan(-(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) - 1/48*(18*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))^5*B*a*b^2 - 15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^5*A*b^3 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*B*a^2*c + 36*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*A*a*b*c - 48*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))^3*B*a^2*b^2 + 40*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))
^3*A*a*b^3 - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*a^2*b*c - 48*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*B*a^3*b*sqrt(c) - 96*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))^2*A*a^3*c^(3/2) + 30*(sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 + a))*B*a^3*b^2 - 33*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*a^2*b^
3 + 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*B*a^4*c - 36*(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))*A*a^3*b*c + 48*B*a^4*b*sqrt(c) - 48*A*a^3*b^2*sqrt
(c) + 32*A*a^4*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^3*
a^3)
```

maple [A] time = 0.02, size = 311, normalized size = 1.76

$$-\frac{3Abc\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{8a^{\frac{5}{2}}}+\frac{5A\,b^3\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{32a^{\frac{7}{2}}}+\frac{Bc\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}}-\frac{3B\,b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*A*(c*x^4+b*x^2+a)^(1/2)/a/x^6+5/24*A*b/a^2/x^4*(c*x^4+b*x^2+a)^(1/2)-5
/16*A*b^2/a^3/x^2*(c*x^4+b*x^2+a)^(1/2)+5/32*A*b^3/a^(7/2)*ln((b*x^2+2*a+2*
(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-3/8*A*b/a^(5/2)*c*ln((b*x^2+2*a+2*(c*x^
4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/3*A/a^2*c/x^2*(c*x^4+b*x^2+a)^(1/2)-1/4*B/
a/x^4*(c*x^4+b*x^2+a)^(1/2)+3/8*B*b/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)-3/16*B*b^
2/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/4*B*c/a^(3/
2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{x^7\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^7*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**7*sqrt(a + b*x**2 + c*x**4)), x)
```

3.176 $\int \frac{x^4\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=403

$$\frac{x\sqrt{a+bx^2+cx^4}\left(-9aBc-10Abc+8b^2B\right)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{a}\sqrt{c}\,(4bB-5Ac)-9aBc-10\ldots\right.}{30c^{11/4}\sqrt{a+bx^2+cx\ldots}}$$

```
[Out] -1/15*(-5*A*c+4*B*b)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/5*B*x^3*(c*x^4+b*x^2+a)^
(1/2)/c+1/15*(-10*A*b*c-9*B*a*c+8*B*b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a
^(1/2)+x^2*c^(1/2))-1/15*a^(1/4)*(-10*A*b*c-9*B*a*c+8*B*b^2)*(cos(2*arctan(
c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin
(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2
*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b
*x^2+a)^(1/2)+1/30*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*
(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(8*b^2*B-10*A*b*c-9*a*B*
c+(-5*A*c+4*B*b)*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)
^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1279, 1197, 1103, 1195}

$$\frac{x\sqrt{a+bx^2+cx^4}\left(-9aBc-10Abc+8b^2B\right)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{a}\sqrt{c}\,(4bB-5Ac)-9aBc-10\ldots\right.}{30c^{11/4}\sqrt{a+bx^2+cx\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -((4*b*B - 5*A*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (B*x^3*Sqrt[a + b*x
^2 + c*x^4])/(5*c) + ((8*b^2*B - 10*A*b*c - 9*a*B*c)*x*Sqrt[a + b*x^2 + c*x
^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(8*b^2*B - 10*A*b*c -
9*a*B*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(15*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^2*B - 10*A*b*c
- 9*a*B*c + Sqrt[a]*Sqrt[c]*(4*b*B - 5*A*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4)*Sqrt[a + b*x^2 + c*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} - \frac{\int \frac{x^2\left(3aB+(4bB-5Ac)x^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx}{5c}$$

$$= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\int \frac{a(4bB-5Ac)+\left(8b^2B-10Abc-9aBc\right)x^2}{\sqrt{a+bx^2+cx^4}}}{15c^2}$$

$$= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} - \frac{\left(\sqrt{a}\left(8b^2B-10Abc-9aBc\right)\right)\int}{15c^{5/2}}$$

$$= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\left(8b^2B-10Abc-9aBc\right)x\sqrt{a+}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Mathematica [C] time = 2.21, size = 532, normalized size = 1.32

$$\frac{i\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(-9aBc-10Abc+8b^2B\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\right|}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b*B + 5*A*c + 3*B*c*x^2)*(a + b*
x^2 + c*x^4) + I*(8*b^2*B - 10*A*b*c - 9*a*B*c)*(-b + Sqrt[b^2 - 4*a*c])*Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2
*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])] - I*(-8*b^3*B + b*c*(17*a*B - 10*A*Sqrt[b^2 - 4*a*c]) + 2
*b^2*(5*A*c + 4*B*Sqrt[b^2 - 4*a*c]) - a*c*(10*A*c + 9*B*Sqrt[b^2 - 4*a*c])
)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
 - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcS
```

```
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x
^2 + c*x^4])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{Bx^6 + Ax^4}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^6 + A*x^4)/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^2 + A)x^4}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [B] time = 0.02, size = 815, normalized size = 2.02

$$\left(\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2}, \frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Ellip}\right.}{3\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
[Out] B*(1/5/c*x^3*(c*x^4+b*x^2+a)^(1/2)-4/15*b/c^2*x*(c*x^4+b*x^2+a)^(1/2)+1/15*
b/c^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5/c*a+8/15*b^2/c^2)*a*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(
4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+A*
(1/3/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c*
a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^2 + A)x^4}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(B\,x^2 + A)}{\sqrt{c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,(A + Bx^2)}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x**4*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.177 $\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=336

$$-\frac{x\sqrt{a+bx^2+cx^4}\,(2bB-3Ac)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{a}\,B\sqrt{c}-3Ac+2bB\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\right.\right.}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/3*B*x*(c*x^4+b*x^2+a)^(1/2)/c-1/3*(-3*A*c+2*B*b)*x*(c*x^4+b*x^2+a)^(1/2)/
c^(3/2)/(a^(1/2)+x^2*c^(1/2))+1/3*a^(1/4)*(-3*A*c+2*B*b)*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(
1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+
a)^(1/2)-1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a
^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b*B-3*A*c+B*a^(1/2)*c^(1/2)
)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(
1/2)
```

Rubi [A] time = 0.15, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1279, 1197, 1103, 1195}

$$-\frac{x\sqrt{a+bx^2+cx^4}\,(2bB-3Ac)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{a}\,B\sqrt{c}-3Ac+2bB\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\right.\right.}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - ((2*b*B - 3*A*c)*x*Sqrt[a + b*x^2 + c
*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(2*b*B - 3*A*c)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/
4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(2*b*B + Sqrt[a]*B*Sqrt[c] - 3*A*c)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*
c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} - \frac{\int \frac{aB+(2bB-3Ac)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c}$$

$$= \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} + \frac{\left(\sqrt{a}\,(2bB-3Ac)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}} - \frac{\left(\sqrt{a}\,\left(2bB+\sqrt{a}\,B\sqrt{c}\,-3Ac\right)\right.}{3c^{3/2}}$$

$$= \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} - \frac{(2bB-3Ac)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{\sqrt[4]{a}\,(2bB-3Ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{}}{3c^{7/4}}$$

Mathematica [C] time = 1.37, size = 479, normalized size = 1.43

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(-3Ac\sqrt{b^2-4ac}+2bB\sqrt{b^2-4ac}+2aBc+3Abc-2b^2B\right)F\left(i\sinh^{-1}\left(\sqrt{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*B*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) - I*(2*b*B - 3
*A*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x]
, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-2*b^2*B + 3*A*b*c
+ 2*a*B*c + 2*b*B*Sqrt[b^2 - 4*a*c] - 3*A*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2
- 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])])/(12*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bx^4+Ax^2}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^4 + A*x^2)/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.01, size = 607, normalized size = 1.81

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Elli}\cdots\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
[Out] B*(1/3*(c*x^4+b*x^2+a)^(1/2)/c*x-1/12/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2
))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+1/3*b/
c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a
*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4
)^(1/2))))-1/2*A*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*
c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-Ellip
ticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)
^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^2+A\right)x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx^2)}{\sqrt{a + bx^2 + cx^4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.178 $\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=283

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\,B\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)}}}{c^{3/4}\sqrt{a+}}$$

```
[Out] B*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*B*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x
^4+b*x^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(B+A*c^(1/2)/a^(1/2))
*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\,B\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)}}}{c^{3/4}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(B + (A*Sqrt[c])/Sqrt[a])*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \left(A+\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx - \frac{\left(\sqrt{a}\,B\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}}$$

$$= \frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,B\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.25, size = 302, normalized size = 1.07

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+B\left(\cdots\right.\right.}{2\sqrt{2}\,c\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*(-b + Sqrt[b^2 - 4*a*c])*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*Ellipt
icF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.01, size = 362, normalized size = 1.28

$$\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\;A\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}-\frac{\sqrt{2}\,\sqrt{-\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*
b/c-4)^(1/2)))+1/4*A*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*
x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2 + A}{\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((A + B*x^2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.179 $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=312

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B+A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E}{a^{3/4}\sqrt{a+bx}}$$

```
[Out] -A*(c*x^4+b*x^2+a)^(1/2)/a/x+A*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/(a^(1/2)+x
^2*c^(1/2))-A*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2
*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arcta
n(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(B*a^(1/2)+A*c^(1/2)
)*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(
3/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1281, 1197, 1103, 1195}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B+A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E}{a^{3/4}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2 + c*x^4])/(a*x)) + (A*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4]
)/(a*(Sqrt[a] + Sqrt[c]*x^2)) - (A*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) +
 ((Sqrt[a]*B + A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{ax} - \frac{\int \frac{-aB-Acx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{ax} + \left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx - \frac{\left(A\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,d\text{:}}{\sqrt{a}}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{ax} + \frac{A\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E}{a^{3/4}\sqrt{a+b:}}$$

Mathematica [C] time = 1.07, size = 448, normalized size = 1.44

$$\frac{-ix\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(A\left(\sqrt{b^2-4ac}-b\right)+2aB\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-4*A*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4) + I*A*(-b + Sqrt[
b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*El
lipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(2*a*B + A*(-b + Sqrt[b^2 - 4*a*c])
)*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2
*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*
x^2 + c*x^4])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(Bx^2+A\right)}{cx^6+bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(c*x^6 + b*x^4 + a*x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)

maple [A] time = 0.02, size = 386, normalized size = 1.24

$$\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\;B\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+\left(-\frac{\sqrt{2}\,\sqrt{-\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x)

[Out] 1/4*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+A*(-1/a*(c*x^4+b*x^2+a)^(1/2)/x-1/2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2 + A}{x^2\,\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^(1/2)),x)

[Out] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^(1/2)), x)

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^2\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*sqrt(a + b*x**2 + c*x**4)), x)
```

3.180 $\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=376

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}A\sqrt{c}-3aB+2Ab\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)(2 \cdots}{\cdots}$$

```
[Out] -1/3*A*(c*x^4+b*x^2+a)^(1/2)/a/x^3+1/3*(2*A*b-3*B*a)*(c*x^4+b*x^2+a)^(1/2)/
a^2/x-1/3*(2*A*b-3*B*a)*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(a^(1/2)+x^2*c^
(1/2))+1/3*(2*A*b-3*B*a)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/6*c^(1/4)*(co
s(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(
a^(1/2)+x^2*c^(1/2))*(2*A*b-3*a*B+A*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1
/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1281, 1197, 1103, 1195}

$$\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{\sqrt{c}x(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}A\sqrt{c}-3a \cdots\right.}{6a^{7/4}\sqrt{a+bx \cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(3*a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a + b*x^2 +
c*x^4])/(3*a^2*x) - ((2*A*b - 3*a*B)*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*
a^2*(Sqrt[a] + Sqrt[c]*x^2)) + ((2*A*b - 3*a*B)*c^(1/4)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(7/4)*Sqrt[a + b*
x^2 + c*x^4]) - ((2*A*b - 3*a*B + Sqrt[a]*A*Sqrt[c])*c^(1/4)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*a^(7/4)*Sqrt[a
 + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} - \frac{\int \frac{2Ab-3aB+Acx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{3a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} + \frac{\int \frac{-aAc-(2Ab-3aB)cx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^2}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} + \frac{\left((2Ab-3aB)\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+c}}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} - \frac{(2Ab-3aB)\sqrt{c}\,x\sqrt{a+bx^2+}}{3a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Mathematica [C] time = 0.70, size = 373, normalized size = 0.99

$$\frac{-\frac{4(a+bx^2+cx^4)(a(A+3Bx^2)-2Abx^2)}{x^3} + \frac{i\sqrt{2}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(\left(2A\left(b\sqrt{b^2-4ac}+ac-b^2\right)+3aB\left(b-\sqrt{b^2-4ac}\right)\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\right.\right.}{\sqrt{\frac{c}{\sqrt{b^2-4a}}}}}{12a^2\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-4*(a + b*x^2 + c*x^4)*(-2*A*b*x^2 + a*(A + 3*B*x^2)))/x^3 + (I*Sqrt[2]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(-((2*A*b - 3*a*B)*(-b + Sqrt[b^2 - 4*a*c])
*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (3*a*B*(b - Sqrt[b^2 - 4*a*c]) +
2*A*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*c]))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(12*a^2*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(Bx^2+A\right)}{cx^8+bx^6+ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(c*x^8 + b*x^6 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

maple [A] time = 0.02, size = 656, normalized size = 1.74

$$\left(\frac{\sqrt{2}\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\operatorname{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\text{Elliptic}\right.}{3\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] B*(-(c*x^4+b*x^2+a)^(1/2)/a/x-1/2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x
^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(
1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/
c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(
2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))))+A*(-1/3/a*(c*x^4+b*x^2+a)^(1/2)/
x^3+2/3/a^2*b*(c*x^4+b*x^2+a)^(1/2)/x-1/12/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)
^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+1
/3*b*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*
b/c-4)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2 + A}{x^4\,\sqrt{c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^4\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

3.181 $\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=98

$$-\frac{89}{48}\sqrt{x^4+5x^2+3}\,x^4-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{x^4+5x^2+3}+\frac{32801}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+\frac{3}{8}\sqrt{x^4+5x^2+$$

```
[Out] 32801/256*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-89/48*x^4*(x^4+5*x^2+3
)^(1/2)+3/8*x^6*(x^4+5*x^2+3)^(1/2)-1/384*(-3802*x^2+24243)*(x^4+5*x^2+3)^(
1/2)
```

Rubi [A] time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 832, 779, 621, 206}

$$\frac{3}{8}\sqrt{x^4+5x^2+3}\,x^6-\frac{89}{48}\sqrt{x^4+5x^2+3}\,x^4-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{x^4+5x^2+3}+\frac{32801}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-89*x^4*Sqrt[3 + 5*x^2 + x^4])/48 + (3*x^6*Sqrt[3 + 5*x^2 + x^4])/8 - ((24
243 - 3802*x^2)*Sqrt[3 + 5*x^2 + x^4])/384 + (32801*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/256
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7 \left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(2+3x)}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= \frac{3}{8}x^6\sqrt{3+5x^2+x^4} + \frac{1}{8}\text{Subst}\left(\int \frac{\left(-27-\frac{89x}{2}\right)x^2}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4} + \frac{3}{8}x^6\sqrt{3+5x^2+x^4} + \frac{1}{24}\text{Subst}\left(\int \frac{x\left(267+\frac{1901x}{4}\right)}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4} + \frac{3}{8}x^6\sqrt{3+5x^2+x^4} - \frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+x^4} +$$
$$= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4} + \frac{3}{8}x^6\sqrt{3+5x^2+x^4} - \frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+x^4} +$$
$$= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4} + \frac{3}{8}x^6\sqrt{3+5x^2+x^4} - \frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+x^4} +$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.67

$$\frac{1}{768}\left(98403\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+2\sqrt{x^4+5x^2+3}\left(144x^6-712x^4+3802x^2-24243\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-24243 + 3802*x^2 - 712*x^4 + 144*x^6) + 98403*Ar
cTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/768
```

fricas [A] time = 0.55, size = 56, normalized size = 0.57

$$\frac{1}{384}\left(144x^6-712x^4+3802x^2-24243\right)\sqrt{x^4+5x^2+3}-\frac{32801}{256}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(144*x^6 - 712*x^4 + 3802*x^2 - 24243)*sqrt(x^4 + 5*x^2 + 3) - 32801/
256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.37, size = 60, normalized size = 0.61

$$\frac{1}{384}\sqrt{x^4+5x^2+3}\left(2\left(4\left(18x^2-89\right)x^2+1901\right)x^2-24243\right)-\frac{32801}{256}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 - 89)*x^2 + 1901)*x^2 - 24243) -
32801/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.02, size = 87, normalized size = 0.89

$$\frac{3\sqrt{x^4+5x^2+3}\,x^6}{8}-\frac{89\sqrt{x^4+5x^2+3}\,x^4}{48}+\frac{1901\sqrt{x^4+5x^2+3}\,x^2}{192}+\frac{32801\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{256}-\frac{8081\sqrt{x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/8*(x^4+5*x^2+3)^(1/2)*x^6-89/48*(x^4+5*x^2+3)^(1/2)*x^4+1901/192*(x^4+5*x
^2+3)^(1/2)*x^2-8081/128*(x^4+5*x^2+3)^(1/2)+32801/256*ln(x^2+5/2+(x^4+5*x^
2+3)^(1/2))
```

maxima [A] time = 0.93, size = 90, normalized size = 0.92

$$\frac{3}{8}\sqrt{x^4+5x^2+3}\,x^6-\frac{89}{48}\sqrt{x^4+5x^2+3}\,x^4+\frac{1901}{192}\sqrt{x^4+5x^2+3}\,x^2-\frac{8081}{128}\sqrt{x^4+5x^2+3}+\frac{32801}{256}\log\left(2x^2+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(x^4 + 5*x^2 + 3)*x^6 - 89/48*sqrt(x^4 + 5*x^2 + 3)*x^4 + 1901/192*
sqrt(x^4 + 5*x^2 + 3)*x^2 - 8081/128*sqrt(x^4 + 5*x^2 + 3) + 32801/256*log(
2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((x^7*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**7*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.182 $\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=77

$$\frac{1}{2}\sqrt{x^4+5x^2+3}\,x^4+\frac{3}{16}\left(89-14x^2\right)\sqrt{x^4+5x^2+3}-\frac{1083}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] -1083/32*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+1/2*x^4*(x^4+5*x^2+3)^(
1/2)+3/16*(-14*x^2+89)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 832, 779, 621, 206}

$$\frac{1}{2}\sqrt{x^4+5x^2+3}\,x^4+\frac{3}{16}\left(89-14x^2\right)\sqrt{x^4+5x^2+3}-\frac{1083}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (x^4*Sqrt[3 + 5*x^2 + x^4])/2 + (3*(89 - 14*x^2)*Sqrt[3 + 5*x^2 + x^4])/16
- (1083*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(2+3x)}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}x^4\sqrt{3+5x^2+x^4} + \frac{1}{6}\text{Subst}\left(\int \frac{\left(-18-\frac{63x}{2}\right)x}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}x^4\sqrt{3+5x^2+x^4} + \frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4} - \frac{1083}{32}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,\right.\\
&= \frac{1}{2}x^4\sqrt{3+5x^2+x^4} + \frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4} - \frac{1083}{16}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5}{\sqrt{3+}}\right.\\
&= \frac{1}{2}x^4\sqrt{3+5x^2+x^4} + \frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4} - \frac{1083}{32}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.79

$$\frac{1}{32}\left(2\sqrt{x^4+5x^2+3}\left(8x^4-42x^2+267\right)-1083\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(267 - 42*x^2 + 8*x^4) - 1083*ArcTanh[(5 + 2*x^2)/
(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

fricas [A] time = 0.59, size = 51, normalized size = 0.66

$$\frac{1}{16}\left(8x^4-42x^2+267\right)\sqrt{x^4+5x^2+3}+\frac{1083}{32}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*x^4 - 42*x^2 + 267)*sqrt(x^4 + 5*x^2 + 3) + 1083/32*log(-2*x^2 + 2*
sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.34, size = 53, normalized size = 0.69

$$\frac{1}{16}\sqrt{x^4+5x^2+3}\left(2\left(4x^2-21\right)x^2+267\right)+\frac{1083}{32}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 - 21)*x^2 + 267) + 1083/32*log(2*x^2 -
 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.01, size = 70, normalized size = 0.91

$$\frac{\sqrt{x^4+5x^2+3}\,x^4}{2}-\frac{21\sqrt{x^4+5x^2+3}\,x^2}{8}-\frac{1083\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}+\frac{267\sqrt{x^4+5x^2+3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/2*(x^4+5*x^2+3)^(1/2)*x^4-21/8*(x^4+5*x^2+3)^(1/2)*x^2+267/16*(x^4+5*x^2+
3)^(1/2)-1083/32*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

maxima [A] time = 0.99, size = 73, normalized size = 0.95

$$\frac{1}{2}\sqrt{x^4+5x^2+3}\,x^4-\frac{21}{8}\sqrt{x^4+5x^2+3}\,x^2+\frac{267}{16}\sqrt{x^4+5x^2+3}-\frac{1083}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^4 + 5*x^2 + 3)*x^4 - 21/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 267/16*sqr
t(x^4 + 5*x^2 + 3) - 1083/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.183 $\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=56

$$\frac{149}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{8}\left(37-6x^2\right)\sqrt{x^4+5x^2+3}$$

```
[Out] 149/16*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1/8*(-6*x^2+37)*(x^4+5*x^^
2+3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = $\{1251, 779, 621, 206\}$

$$\frac{149}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{8}\left(37-6x^2\right)\sqrt{x^4+5x^2+3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] -((37 - 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + (149*ArcTanh[(5 + 2*x^2)/(2*Sqrt[
3 + 5*x^2 + x^4])])/16
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x(2+3x)}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4} + \frac{149}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4} + \frac{149}{8}\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)$$
$$= -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4} + \frac{149}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [**A**] time = 0.02, size = 56, normalized size = 1.00

$$\frac{1}{16}\left(2\sqrt{x^4+5x^2+3}\left(6x^2-37\right)+149\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*(-37 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4] + 149*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3
+ 5*x^2 + x^4])])/16
```

fricas [**A**] time = 0.54, size = 46, normalized size = 0.82

$$\frac{1}{8}\sqrt{x^4+5\,x^2+3}\left(6\,x^2-37\right)-\frac{149}{16}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(-2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) - 5)
```

giac [**A**] time = 0.35, size = 46, normalized size = 0.82

$$\frac{1}{8}\sqrt{x^4+5\,x^2+3}\left(6\,x^2-37\right)-\frac{149}{16}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(2*x^2 - 2*sqrt(x^4 + 5*
x^2 + 3) + 5)
```

maple [**A**] time = 0.01, size = 53, normalized size = 0.95

$$\frac{3\sqrt{x^4+5x^2+3}\;x^2}{4}+\frac{149\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16}-\frac{37\sqrt{x^4+5x^2+3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/4*(x^4+5*x^2+3)^(1/2)*x^2-37/8*(x^4+5*x^2+3)^(1/2)+149/16*ln(x^2+5/2+(x^4
+5*x^2+3)^(1/2))
```

maxima [A] time = 1.04, size = 56, normalized size = 1.00

$$\frac{3}{4}\sqrt{x^4+5\,x^2+3}\,x^2-\frac{37}{8}\sqrt{x^4+5\,x^2+3}+\frac{149}{16}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - 37/8*sqrt(x^4 + 5*x^2 + 3) + 149/16*log(2*x
^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^3\left(3\,x^2+2\right)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.184 $\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=49

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] -11/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+3/2*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1247, 640, 621, 206}

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x
^4])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{2+3x}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{2}\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)$$
$$= \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{3}{2}\sqrt{x^4+5x^2+3} - \frac{11}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x
^4])])/4
```

fricas [A] time = 0.90, size = 39, normalized size = 0.80

$$\frac{3}{2}\sqrt{x^4+5\,x^2+3} + \frac{11}{4}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.47, size = 39, normalized size = 0.80

$$\frac{3}{2}\sqrt{x^4+5\,x^2+3} + \frac{11}{4}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.01, size = 36, normalized size = 0.73

$$-\frac{11\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4} + \frac{3\sqrt{x^4+5x^2+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
[Out] 3/2*(x^4+5*x^2+3)^(1/2)-11/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

maxima [A] time = 0.92, size = 39, normalized size = 0.80

$$\frac{3}{2}\sqrt{x^4+5\,x^2+3} - \frac{11}{4}\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) - 11/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.52, size = 35, normalized size = 0.71

$$\frac{3\sqrt{x^4+5x^2+3}}{2}-\frac{11\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (3*(5*x^2 + x^4 + 3)^(1/2))/2 - (11*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2
))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.185 $\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=69

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

```
[Out] 3/2*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1/3*arctanh(1/6*(5*x^2+6)*3^
(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 843, 621, 206, 724}

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)+\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\right)+3\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x}{\sqrt{3+5x^2}}\right.\\
&= \frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 69, normalized size = 1.00

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/Sqrt[3]
```

fricas [A] time = 0.83, size = 75, normalized size = 1.09

$$\frac{1}{3}\sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-\frac{3}{2}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(
5*sqrt(3) - 6) + 30)/x^2) - 3/2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

giac [A] time = 0.43, size = 78, normalized size = 1.13

$$\frac{1}{3}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)-\frac{3}{2}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

maple [A] time = 0.01, size = 52, normalized size = 0.75

$$-\frac{\sqrt{3}\ \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{3}+\frac{3\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-1/3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+
5*x^2+3)^(1/2))*3^(1/2)
```

maxima [A] time = 1.93, size = 58, normalized size = 0.84

$$-\frac{1}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*log
(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 1.01, size = 56, normalized size = 0.81

$$\frac{3\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{2}-\frac{\sqrt{3}\left(\ln\left(\frac{1}{x^2}\right)+\ln\left(2\sqrt{3}\sqrt{x^4+5x^2+3}+5x^2+6\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] (3*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/2 - (3^(1/2)*(log(1/x^2) + log
(2*3^(1/2)*(5*x^2 + x^4 + 3)^(1/2) + 5*x^2 + 6)))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.186 $\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3*(x^4+5*
x^2+3)^(1/2)/x^2
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1251, 806, 724, 206}

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(3*x^2) - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 +
 5*x^2 + x^4])])/(3*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x^2\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{3+5x^2+x^4}}{3x^2} + \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{3+5x^2+x^4}}{3x^2} - \frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)$$
$$= -\frac{\sqrt{3+5x^2+x^4}}{3x^2} - \frac{2\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.01, size = 62, normalized size = 1.00

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2} - \frac{2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -1/3*Sqrt[3 + 5*x^2 + x^4]/x^2 - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 +
 5*x^2 + x^4])])/(3*Sqrt[3])
```

fricas [A] time = 0.52, size = 78, normalized size = 1.26

$$\frac{2\sqrt{3}\,x^2\log\left(\frac{25\,x^2-2\,\sqrt{3}\left(5\,x^2+6\right)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)-3\,x^2-3\,\sqrt{x^4+5\,x^2+3}}{9\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2
 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 3*x^2 - 3*sqrt(x^4 + 5*x^2 + 3))/x^2
```

giac [B] time = 0.39, size = 101, normalized size = 1.63

$$\frac{2}{9}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right)+\frac{5\,x^2-5\,\sqrt{x^4+5\,x^2+3}+6}{3\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) + 1/3*(5*x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - s
qrt(x^4 + 5*x^2 + 3))^2 - 3)
```

maple [A] time = 0.02, size = 49, normalized size = 0.79

$$-\frac{2\sqrt{3}\ \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{9}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -2/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3*(x^4+5*
x^2+3)^(1/2)/x^2
```

maxima [A] time = 2.02, size = 51, normalized size = 0.82

$$-\frac{2}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/3*sqr
t(x^4 + 5*x^2 + 3)/x^2
```

mupad [B] time = 0.66, size = 83, normalized size = 1.34

$$\frac{5\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{18}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{\sqrt{3}\,\left(\ln\left(\frac{1}{x^2}\right)+\ln\left(2\sqrt{3}\,\sqrt{x^4+5x^2+3}+5x^2+6\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] (5*3^(1/2)*atanh((3^(1/2)*(5*x^2 + 6))/(6*(5*x^2 + x^4 + 3)^(1/2))))/18 - (
5*x^2 + x^4 + 3)^(1/2)/(3*x^2) - (3^(1/2)*(log(1/x^2) + log(2*3^(1/2)*(5*x^
2 + x^4 + 3)^(1/2) + 5*x^2 + 6)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{3x^2+2}{x^3\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**3*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.187 $\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] 1/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/6*(x^4+5*x
^2+3)^(1/2)/x^4-1/12*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 834, 806, 724, 206}

$$-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(6*x^4) - Sqrt[3 + 5*x^2 + x^4]/(12*x^2) + (Sqrt[3]*
ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x^3\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{1}{12}\operatorname{Subst}\left(\int \frac{-3+2x}{x^2\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{\sqrt{3+5x^2+x^4}}{12x^2}-\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{\sqrt{3+5x^2+x^4}}{12x^2}+\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{\sqrt{3+5x^2+x^4}}{12x^2}+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 0.81

$$\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)-\frac{\left(x^2+2\right)\sqrt{x^4+5x^2+3}}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -1/12*((2 + x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + (Sqrt[3]*ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/8
```

fricas [A] time = 0.70, size = 83, normalized size = 1.00

$$\frac{3\sqrt{3}\,x^4\log\left(\frac{25\,x^2+2\sqrt{3}\left(5\,x^2+6\right)+2\sqrt{x^4+5\,x^2+3}\left(5\sqrt{3}+6\right)+30}{x^2}\right)-2\,x^4-2\sqrt{x^4+5\,x^2+3}\left(x^2+2\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(3)*x^4*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^
2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*(x^2 +
2))/x^4
```

giac [B] time = 0.52, size = 145, normalized size = 1.75

$$-\frac{1}{8}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right)+\frac{9\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^3+36\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2+47\,x^2-47}{12\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - s
qrt(x^4 + 5*x^2 + 3))) + 1/12*(9*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 36*(x^2
- sqrt(x^4 + 5*x^2 + 3))^2 + 47*x^2 - 47*sqrt(x^4 + 5*x^2 + 3) + 12)/((x^2
- sqrt(x^4 + 5*x^2 + 3))^2 - 3)^2
```

maple [A] time = 0.01, size = 66, normalized size = 0.80

$$\frac{\sqrt{3}\ \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{8}-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/6*(x^4+5*x
^2+3)^(1/2)/x^4-1/12*(x^4+5*x^2+3)^(1/2)/x^2
```

maxima [A] time = 2.00, size = 68, normalized size = 0.82

$$\frac{1}{8}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/12*sqr
t(x^4 + 5*x^2 + 3)/x^2 - 1/6*sqrt(x^4 + 5*x^2 + 3)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^5*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^5*(5*x^2 + x^4 + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**5/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**5*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.188 $\int \frac{2+3x^2}{x^7 \sqrt{3+5x^2+x^4}} \, dx$

Optimal. Leaf size=104

$$\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{61\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{216\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

```
[Out] -61/648*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/9*(x^4
+5*x^2+3)^(1/2)/x^6-1/54*(x^4+5*x^2+3)^(1/2)/x^4+13/108*(x^4+5*x^2+3)^(1/2)
/x^2
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 834, 806, 724, 206}

$$\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}-\frac{61\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{216\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(9*x^6) - Sqrt[3 + 5*x^2 + x^4]/(54*x^4) + (13*Sqrt[
3 + 5*x^2 + x^4])/(108*x^2) - (61*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5
*x^2 + x^4])])/(216*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
```

```
IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^4\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{1}{18}\text{Subst}\left(\int \frac{-2+4x}{x^3\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{\sqrt{3+5x^2+x^4}}{54x^4}+\frac{1}{108}\text{Subst}\left(\int \frac{-39-2x}{x^2\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{\sqrt{3+5x^2+x^4}}{54x^4}+\frac{13\sqrt{3+5x^2+x^4}}{108x^2}+\frac{61}{216}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\right.\\
&= -\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{\sqrt{3+5x^2+x^4}}{54x^4}+\frac{13\sqrt{3+5x^2+x^4}}{108x^2}-\frac{61}{108}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\right.\\
&= -\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{\sqrt{3+5x^2+x^4}}{54x^4}+\frac{13\sqrt{3+5x^2+x^4}}{108x^2}-\frac{61\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{216\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.74

$$\frac{6\sqrt{x^4+5x^2+3}\left(13x^4-2x^2-12\right)-61\sqrt{3}x^6\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{648x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (6*Sqrt[3 + 5*x^2 + x^4]*(-12 - 2*x^2 + 13*x^4) - 61*Sqrt[3]*x^6*ArcTanh[(6
 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(648*x^6)
```

fricas [A] time = 0.54, size = 90, normalized size = 0.87

$$\frac{61\sqrt{3}x^6\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)+78x^6+6\left(13x^4-2x^2-12\right)\sqrt{x^4+5x^2+3}}{648x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/648*(61*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 78*x^6 + 6*(13*x^4 - 2*x^2 - 12)*sqrt
(x^4 + 5*x^2 + 3))/x^6
```

giac [B] time = 0.51, size = 167, normalized size = 1.61

$$\frac{61}{648}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)-\frac{61\left(x^2-\sqrt{x^4+5x^2+3}\right)^5-920\left(x^2-\sqrt{x^4+5x^2+3}\right)^3-2052\left(x^2-\right.}{108\left(\left(x^2-\sqrt{x^4+5x^2-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 61/648*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))) - 1/108*(61*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 - 920*
(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 - 2052*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 14
49*x^2 + 1449*sqrt(x^4 + 5*x^2 + 3) - 108)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2
 - 3)^3
```

maple [A] time = 0.02, size = 83, normalized size = 0.80

$$-\frac{61\sqrt{3}\ \mathrm{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{648}+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -61/648*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/9*(x^4
+5*x^2+3)^(1/2)/x^6-1/54*(x^4+5*x^2+3)^(1/2)/x^4+13/108*(x^4+5*x^2+3)^(1/2)
/x^2
```

maxima [A] time = 1.98, size = 85, normalized size = 0.82

$$-\frac{61}{648}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -61/648*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 13/1
08*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/54*sqrt(x^4 + 5*x^2 + 3)/x^4 - 1/9*sqrt(x^
4 + 5*x^2 + 3)/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3x^2+2}{x^7\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^7*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^7*(5*x^2 + x^4 + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x^7\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**7/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**7*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.189 $\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}} \, dx$

Optimal. Leaf size=298

$$-\frac{10}{3}\sqrt{x^4+5x^2+3}\,x+\frac{419\left(2x^2+\sqrt{13}+5\right)x}{30\sqrt{x^4+5x^2+3}}+\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\right.\right.\right.}{\sqrt{x^4+5x^2+3}}$$

```
[Out] 419/30*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-10/3*x*(x^4+5*x^2+3)^(1/2)+
3/5*x^3*(x^4+5*x^2+3)^(1/2)+5/3*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*
(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(
1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+1
3^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2
+3)^(1/2)-419/180*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)
))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1
/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+
x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1279, 1189, 1099, 1135}

$$\frac{3}{5}\sqrt{x^4+5x^2+3}\,x^3-\frac{10}{3}\sqrt{x^4+5x^2+3}\,x+\frac{419\left(2x^2+\sqrt{13}+5\right)x}{30\sqrt{x^4+5x^2+3}}+\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (419*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (10*x*Sqrt[3 +
5*x^2 + x^4])/3 + (3*x^3*Sqrt[3 + 5*x^2 + x^4])/5 - (419*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(30*Sqrt[3 + 5*x^2 + x^4]) + (5*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5
- Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellipti
cF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2
+ x^4]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{5}x^3\sqrt{3+5x^2+x^4} - \frac{1}{5}\int \frac{x^2\left(27+50x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{10}{3}x\sqrt{3+5x^2+x^4} + \frac{3}{5}x^3\sqrt{3+5x^2+x^4} + \frac{1}{15}\int \frac{150+419x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{10}{3}x\sqrt{3+5x^2+x^4} + \frac{3}{5}x^3\sqrt{3+5x^2+x^4} + 10\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx + \frac{419}{15}\int \frac{}{\sqrt{3+}}$$

$$= \frac{419x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}} - \frac{10}{3}x\sqrt{3+5x^2+x^4} + \frac{3}{5}x^3\sqrt{3+5x^2+x^4} - \frac{419\sqrt{\frac{1}{6}}\left(5+\sqrt{13}\right.}{}$$

Mathematica [C] time = 0.27, size = 229, normalized size = 0.77

$$\frac{-i\sqrt{2}\left(419\sqrt{13}-1795\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+419i\sqrt{2}\left(\sqrt{13}\right.}{60\sqrt{x^4+5x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (4*x*(-150 - 223*x^2 - 5*x^4 + 9*x^6) + (419*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqr
t[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellip
ticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2
]*(-1795 + 419*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt
[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6
+ (5*Sqrt[13])/6])/(60*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{3\,x^6+2\,x^4}{\sqrt{x^4+5\,x^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

[Out] integral((3*x^6 + 2*x^4)/sqrt(x^4 + 5*x^2 + 3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)x^4}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3), x)

maple [A] time = 0.02, size = 226, normalized size = 0.76

$$\frac{3\sqrt{x^4+5x^2+3}\,x^3}{5}-\frac{10\sqrt{x^4+5x^2+3}\,x}{3}+\frac{60\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)

[Out] 3/5*(x^4+5*x^2+3)^(1/2)*x^3-10/3*(x^4+5*x^2+3)^(1/2)*x+60/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-5028/5/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)x^4}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")

[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)

[Out] int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)

[Out] Integral(x**4*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)

3.190 $\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=270

$$\sqrt{x^4+5x^2+3}\,x-\frac{4\left(2x^2+\sqrt{13}+5\right)x}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right.}\right.\right.}{\sqrt{x^4+5x^2+3}}$$

```
[Out] -4*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+x*(x^4+5*x^2+3)^(1/2)-1/2*(1/(3
6+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(3
0+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1
/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/
(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+2/3*(1/(36+x^2*(30+6*13^(1/
2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2
)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^
(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(
1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1279, 1189, 1099, 1135}

$$\sqrt{x^4+5x^2+3}\,x-\frac{4\left(2x^2+\sqrt{13}+5\right)x}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right.}\right.\right.}{\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-4*x*(5 + Sqrt[13] + 2*x^2))/Sqrt[3 + 5*x^2 + x^4] + x*Sqrt[3 + 5*x^2 + x^
4] + (2*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13
])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4] - (Sqrt[3/(2*(5 + Sq
rt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
 Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx &= x\sqrt{3+5x^2+x^4}-\frac{1}{3}\int \frac{9+24x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= x\sqrt{3+5x^2+x^4}-3\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx-8\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{4x\left(5+\sqrt{13}+2x^2\right)}{\sqrt{3+5x^2+x^4}}+x\sqrt{3+5x^2+x^4}+\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right.\right.}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.24, size = 222, normalized size = 0.82

$$\frac{i\sqrt{2}\left(4\sqrt{13}-17\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-4i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{\frac{-2x^2+}{\sqrt{1}}}}{2\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*x*(3 + 5*x^2 + x^4) - (4*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13]
- 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sq
rt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-17 + 4*Sqrt[1
3])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2
]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(2
*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{3\,x^4+2\,x^2}{\sqrt{x^4+5\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)/sqrt(x^4 + 5*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^2+2\right)x^2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3), x)
```

maple [A] time = 0.01, size = 208, normalized size = 0.77

$$\sqrt{x^4+5x^2+3}\,x-\frac{18\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\,x}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}+\frac{288}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] (x^4+5*x^2+3)^(1/2)*x-18/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1
)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/
6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))+288/(-30+6*13^(1/2))^(
1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/
(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/
6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+
1/6*39^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^2+2\right)x^2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(3\,x^2+2\right)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.191 $\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}} \, dx$

Optimal. Leaf size=257

$$\frac{3x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}+\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right.}{\sqrt{x^4+5x^2+3}}$$

```
[Out] 3/2*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+1/3*(1/(36+x^2*(30+6*13^(1/2))
))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(
36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/
2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))
^(1/2)/(x^4+5*x^2+3)^(1/2)-1/4*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(
30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1
/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2
))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1189, 1099, 1135}

$$\frac{3x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}+\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right.}{\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(3*(5 + Sqrt
[13]))/2]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
 Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (
5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellip
ticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^
2 + x^4]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx = 2\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx + 3\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{3x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}} - \frac{\sqrt{\frac{3}{2}}\left(5+\sqrt{13}\right)\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\right.\right.}{2\sqrt{3+5x^2+x^4}}$$

Mathematica [C] time = 0.12, size = 159, normalized size = 0.62

$$\frac{i\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\left(\left(11-3\sqrt{13}\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+3\left(\sqrt{13}-5\right)E\left(i\sinh^{-1}\right.\right.}{2\sqrt{2}\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] ((I/2)*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*
x^2]*(3*(-5 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6
 + (5*Sqrt[13])/6] + (11 - 3*Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt
[13])]*x], 19/6 + (5*Sqrt[13])/6]))/(Sqrt[2]*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

maple [A] time = 0.01, size = 194, normalized size = 0.75

$$\frac{12\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\,x}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}} - \frac{108\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -108/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*
13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30
+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(1/2
))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2)))+12/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6
*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1
/2)*EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.192 $\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=278

$$\frac{x\left(2x^2+\sqrt{13}+5\right)}{3\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}+\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\right.\right.}{\sqrt{x^4+5x^2+3}}$$

```
[Out] 1/3*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-2/3*(x^4+5*x^2+3)^(1/2)/x+1/2*
(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF
(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2
))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/
2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-1/18*(1/(36+x^2*(30+6*
13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2)
)^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*
(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2
))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1281, 1189, 1099, 1135}

$$\frac{x\left(2x^2+\sqrt{13}+5\right)}{3\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}+\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\right.\right.}{\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (x*(5 + Sqrt[13] + 2*x^2))/(3*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 +
x^4])/(3*x) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5
 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt
[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[3/(2*
(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx &= -\frac{2\sqrt{3+5x^2+x^4}}{3x}-\frac{1}{3}\int \frac{-9-2x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{2\sqrt{3+5x^2+x^4}}{3x}+\frac{2}{3}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx+3\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx\\ &= \frac{x\left(5+\sqrt{13}+2x^2\right)}{3\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{3x}-\frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right.\right.}{3\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.25, size = 224, normalized size = 0.81

$$\frac{-i\sqrt{2}\left(4+\sqrt{13}\right)x\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+i\sqrt{2}\left(\sqrt{13}-5\right)x\sqrt{\frac{-2x^2}{\sqrt{}}}}{6x\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (-4*(3 + 5*x^2 + x^4) + I*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2
*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[
2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(4 + Sqrt[13])*x*S
qrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ell
ipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(6*x*Sq
rt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x^6+5x^4+3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/(x^6 + 5*x^4 + 3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{\sqrt{x^4+5x^2+3}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

maple [A] time = 0.02, size = 211, normalized size = 0.76

$$\frac{18\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\,x}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}-\frac{24}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 18/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13
^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*(-30+6*13^(1/2))^(1/
2)*x,5/6*3^(1/2)+1/6*39^(1/2))-2/3*(x^4+5*x^2+3)^(1/2)/x-24/(-30+6*13^(1/2)
)^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/
2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x
,5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/
2)+1/6*39^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{x^2\,\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x^2\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**2*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.193 $\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=302

$$\frac{7x\left(2x^2+\sqrt{13}+5\right)}{54\sqrt{x^4+5x^2+3}}-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right.\right.}{9\sqrt{x^4+5x^2+3}}$$

```
[Out] 7/54*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-2/9*(x^4+5*x^2+3)^(1/2)/x^3-7
/27*(x^4+5*x^2+3)^(1/2)/x-1/27*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(
30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1
/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13
^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+
3)^(1/2)-7/324*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^
(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*
(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2
*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1281, 1189, 1099, 1135}

$$\frac{7x\left(2x^2+\sqrt{13}+5\right)}{54\sqrt{x^4+5x^2+3}}-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{2\sqrt{x^4+5x^2+3}}{9x^3}-\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\right.}{9\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^4*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (7*x*(5 + Sqrt[13] + 2*x^2))/(54*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2
 + x^4])/(9*x^3) - (7*Sqrt[3 + 5*x^2 + x^4])/(27*x) - (7*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(54*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(9*Sqrt[3 + 5*x^2
 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx &= -\frac{2\sqrt{3+5x^2+x^4}}{9x^3}-\frac{1}{9}\int \frac{-7+2x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{2\sqrt{3+5x^2+x^4}}{9x^3}-\frac{7\sqrt{3+5x^2+x^4}}{27x}+\frac{1}{27}\int \frac{-6+7x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{2\sqrt{3+5x^2+x^4}}{9x^3}-\frac{7\sqrt{3+5x^2+x^4}}{27x}-\frac{2}{9}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx+\frac{7}{27}\int \frac{x^2}{\sqrt{3+5x^2-}}\\ &= \frac{7x\left(5+\sqrt{13}+2x^2\right)}{54\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{9x^3}-\frac{7\sqrt{3+5x^2+x^4}}{27x}-\frac{7\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+(}{6+(}}}{}\end{aligned}$$

Mathematica [C] time = 0.26, size = 237, normalized size = 0.78

$$\frac{-i\sqrt{2}\left(7\sqrt{13}-47\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,x^3F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+7i\sqrt{2}\left(\sqrt{13}-5\right)}{108x^3\sqrt{x^4+5x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(x^4*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (-4*(18 + 51*x^2 + 41*x^4 + 7*x^6) + (7*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3*Sqrt
[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellipt
icE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]
*(-47 + 7*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[
5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 +
 (5*Sqrt[13])/6])/(108*x^3*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^8+5\,x^6+3\,x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/(x^8 + 5*x^6 + 3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

maple [A] time = 0.02, size = 228, normalized size = 0.75

$$-\frac{4\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\,x}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{3\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{2\sqrt{x^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -7/27*(x^4+5*x^2+3)^(1/2)/x-28/3/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2
))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^
(1/2)+5)*(EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-
EllipticE(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2)))-2/9*(x^4+
5*x^2+3)^(1/2)/x^3-4/3/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^
(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*
(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{x^4\,\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x^4\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**4*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.194 $\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{(47x^2+33)\,x^2}{13\sqrt{x^4+5x^2+3}}+\frac{133}{26}\sqrt{x^4+5x^2+3}-\frac{41}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] -41/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1/13*x^2*(47*x^2+33)/(x^4+
5*x^2+3)^(1/2)+133/26*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 818, 640, 621, 206}

$$-\frac{(47x^2+33)\,x^2}{13\sqrt{x^4+5x^2+3}}+\frac{133}{26}\sqrt{x^4+5x^2+3}-\frac{41}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -(x^2*(33 + 47*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (133*Sqrt[3 + 5*x^2 + x^4
])/26 - (41*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(2+3x^2\right)}{\left(3+5x^2+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(2+3x)}{\left(3+5x+x^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{x^2\left(33+47x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{1}{13}\operatorname{Subst}\left(\int \frac{33+\frac{133x}{2}}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{x^2\left(33+47x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{133}{26}\sqrt{3+5x^2+x^4}-\frac{41}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{x^2\left(33+47x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{133}{26}\sqrt{3+5x^2+x^4}-\frac{41}{2}\operatorname{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{x^2\left(33+47x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{133}{26}\sqrt{3+5x^2+x^4}-\frac{41}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 72, normalized size = 0.94

$$\frac{78x^4+1198x^2-533\sqrt{x^4+5x^2+3}\,\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+798}{52\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (798 + 1198*x^2 + 78*x^4 - 533*Sqrt[3 + 5*x^2 + x^4]*ArcTanh[(5 + 2*x^2)/(2
*Sqrt[3 + 5*x^2 + x^4])])/(52*Sqrt[3 + 5*x^2 + x^4])
```

fricas [A] time = 0.56, size = 86, normalized size = 1.12

$$\frac{1811\,x^4+9055\,x^2+1066\left(x^4+5\,x^2+3\right)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+4\left(39\,x^4+599\,x^2+399\right)\sqrt{x^4+5\,x}}{104\left(x^4+5\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/104*(1811*x^4 + 9055*x^2 + 1066*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4
 + 5*x^2 + 3) - 5) + 4*(39*x^4 + 599*x^2 + 399)*sqrt(x^4 + 5*x^2 + 3) + 543
3)/(x^4 + 5*x^2 + 3)
```

giac [A] time = 0.38, size = 52, normalized size = 0.68

$$\frac{\left(39\,x^2+599\right)x^2+399}{26\,\sqrt{x^4+5\,x^2+3}}+\frac{41}{4}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

[Out] 1/26*((39*x^2 + 599)*x^2 + 399)/sqrt(x^4 + 5*x^2 + 3) + 41/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.02, size = 91, normalized size = 1.18

$$\frac{3x^4}{2\sqrt{x^4+5x^2+3}}+\frac{41x^2}{4\sqrt{x^4+5x^2+3}}-\frac{41\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{133}{8\sqrt{x^4+5x^2+3}}+\frac{\frac{665x^2}{52}+\frac{3325}{104}}{\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)

[Out] 3/2*x^4/(x^4+5*x^2+3)^(1/2)+41/4*x^2/(x^4+5*x^2+3)^(1/2)-133/8/(x^4+5*x^2+3)^(1/2)+665/104*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)-41/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))

maxima [A] time = 0.92, size = 73, normalized size = 0.95

$$\frac{3\,x^4}{2\,\sqrt{x^4+5\,x^2+3}}+\frac{599\,x^2}{26\,\sqrt{x^4+5\,x^2+3}}+\frac{399}{26\,\sqrt{x^4+5\,x^2+3}}-\frac{41}{4}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")

[Out] 3/2*x^4/sqrt(x^4 + 5*x^2 + 3) + 599/26*x^2/sqrt(x^4 + 5*x^2 + 3) + 399/26/sqrt(x^4 + 5*x^2 + 3) - 41/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(3\,x^2+2\right)}{{\left(x^4+5\,x^2+3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)

[Out] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(3x^2+2\right)}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)

[Out] Integral(x**5*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)

3.195 $\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{-47x^2-33}{13\sqrt{x^4+5x^2+3}}+\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] 3/2*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+1/13*(-47*x^2-33)/(x^4+5*x^2
+3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1251, 777, 621, 206}

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -(33 + 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) + (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[
3 + 5*x^2 + x^4])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{(3+5x+x^2)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}} + 3\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)$$
$$= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}} + \frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] time = 0.12, size = 54, normalized size = 0.96

$$\frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right) - \frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/13*(33 + 47*x^2)/Sqrt[3 + 5*x^2 + x^4] + (3*Log[5 + 2*x^2 + 2*Sqrt[3 + 5
*x^2 + x^4]])/2
```

fricas [A] time = 0.61, size = 81, normalized size = 1.45

$$-\frac{94\,x^4+470\,x^2+39\left(x^4+5\,x^2+3\right)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+2\,\sqrt{x^4+5\,x^2+3}\left(47\,x^2+33\right)+282}{26\left(x^4+5\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/26*(94*x^4 + 470*x^2 + 39*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4 + 5*
x^2 + 3) - 5) + 2*sqrt(x^4 + 5*x^2 + 3)*(47*x^2 + 33) + 282)/(x^4 + 5*x^2 +
 3)
```

giac [A] time = 0.51, size = 46, normalized size = 0.82

$$-\frac{47\,x^2+33}{13\,\sqrt{x^4+5\,x^2+3}} - \frac{3}{2}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/13*(47*x^2 + 33)/sqrt(x^4 + 5*x^2 + 3) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*
x^2 + 3) + 5)
```

maple [B] time = 0.01, size = 95, normalized size = 1.70

$$-\frac{3x^2}{2\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2}+\frac{15}{4\sqrt{x^4+5x^2+3}}-\frac{75\left(2x^2+5\right)}{52\sqrt{x^4+5x^2+3}}+\frac{\frac{10x^2}{13}+\frac{12}{13}}{\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -3/2/(x^4+5*x^2+3)^(1/2)*x^2+15/4/(x^4+5*x^2+3)^(1/2)-75/52*(2*x^2+5)/(x^4+
5*x^2+3)^(1/2)+3/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+2/13/(x^4+5*x^2+3)^(1/2)
*(5*x^2+6)
```

maxima [A] time = 0.93, size = 56, normalized size = 1.00

$$-\frac{47\,x^2}{13\,\sqrt{x^4+5\,x^2+3}}-\frac{33}{13\,\sqrt{x^4+5\,x^2+3}}+\frac{3}{2}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}\,+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -47/13*x^2/sqrt(x^4 + 5*x^2 + 3) - 33/13/sqrt(x^4 + 5*x^2 + 3) + 3/2*log(2*
x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

mupad [B] time = 0.31, size = 52, normalized size = 0.93

$$\frac{3\,\ln\left(\sqrt{x^4+5\,x^2+3}\,+x^2+\frac{5}{2}\right)}{2}-\frac{47\,x^2}{13\,\sqrt{x^4+5\,x^2+3}}-\frac{33}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] (3*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/2 - (47*x^2)/(13*(5*x^2 + x^4
+ 3)^(1/2)) - 33/(13*(5*x^2 + x^4 + 3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\left(3x^2+2\right)}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.196 $\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

```
[Out] 1/13*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1247, 636}

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{(3+5x+x^2)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{8+11x^2}{13\sqrt{3+5x^2+x^4}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 25, normalized size = 1.00

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

fricas [B] time = 0.74, size = 46, normalized size = 1.84

$$\frac{11\,x^4+55\,x^2+\sqrt{x^4+5\,x^2+3}\,(11\,x^2+8)+33}{13\,(x^4+5\,x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*(11*x^4 + 55*x^2 + sqrt(x^4 + 5*x^2 + 3)*(11*x^2 + 8) + 33)/(x^4 + 5*x
^2 + 3)
```

giac [A] time = 0.36, size = 21, normalized size = 0.84

$$\frac{11\,x^2+8}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*(11*x^2 + 8)/sqrt(x^4 + 5*x^2 + 3)
```

maple [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 1/13*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 0.94, size = 32, normalized size = 1.28

$$\frac{11\,x^2}{13\,\sqrt{x^4+5\,x^2+3}}+\frac{8}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 11/13*x^2/sqrt(x^4 + 5*x^2 + 3) + 8/13/sqrt(x^4 + 5*x^2 + 3)
```

mupad [B] time = 0.24, size = 21, normalized size = 0.84

$$\frac{11\,x^2+8}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] (11*x^2 + 8)/(13*(5*x^2 + x^4 + 3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(3x^2+2\right)}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.197 $\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{-8x^2-7}{39\sqrt{x^4+5x^2+3}}-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/39*(-8*x^
2-7)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 822, 12, 724, 206}

$$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(7 + 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) - ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sq
rt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x\left(3+5x+x^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}} - \frac{1}{39}\operatorname{Subst}\left(\int -\frac{13}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}} + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}} - \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}} - \frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 1.00

$$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}} - \frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/39*(7 + 8*x^2)/Sqrt[3 + 5*x^2 + x^4] - ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sq
rt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

fricas [B] time = 0.72, size = 107, normalized size = 1.62

$$-\frac{24\,x^4 - 13\,\sqrt{3}\left(x^4+5\,x^2+3\right)\log\left(\frac{25\,x^2-2\,\sqrt{3}\left(5\,x^2+6\right)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right) + 120\,x^2 + 3\,\sqrt{x^4+5\,x^2+3}\left(8\,x^2+7\right)}{117\left(x^4+5\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/117*(24*x^4 - 13*sqrt(3)*(x^4 + 5*x^2 + 3)*log((25*x^2 - 2*sqrt(3)*(5*x^
2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 120*x^2 + 3*s
qrt(x^4 + 5*x^2 + 3)*(8*x^2 + 7) + 72)/(x^4 + 5*x^2 + 3)
```

giac [A] time = 0.56, size = 78, normalized size = 1.18

$$-\frac{1}{9}\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4+5\,x^2+3}\right)+\frac{1}{9}\sqrt{3}\log\left(-x^2-\sqrt{3}+\sqrt{x^4+5\,x^2+3}\right)-\frac{8\,x^2+7}{39\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*log(-x^2 + sqrt(3) + sqrt(x^4 + 5*x^2 + 3)) + 1/9*sqrt(3)*log(
-x^2 - sqrt(3) + sqrt(x^4 + 5*x^2 + 3)) - 1/39*(8*x^2 + 7)/sqrt(x^4 + 5*x^2
 + 3)
```

maple [A] time = 0.02, size = 67, normalized size = 1.02

$$-\frac{\sqrt{3}\ \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{9}-\frac{4\left(2x^2+5\right)}{39\sqrt{x^4+5x^2+3}}+\frac{1}{3\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -4/39*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)+1/3/(x^4+5*x^2+3)^(1/2)-1/9*arctanh(1/6
*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

maxima [A] time = 2.01, size = 65, normalized size = 0.98

$$-\frac{8\,x^2}{39\,\sqrt{x^4+5\,x^2+3}}-\frac{1}{9}\,\sqrt{3}\,\log\left(\frac{2\,\sqrt{3}\,\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{7}{39\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/39*x^2/sqrt(x^4 + 5*x^2 + 3) - 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^
2 + 3)/x^2 + 6/x^2 + 5) - 7/39/sqrt(x^4 + 5*x^2 + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{3\,x^2+2}{x\,\left(x^4+5\,x^2+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(3/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

3.198 $\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{-8x^2-7}{39x^2\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{39x^2}+\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/39*(-8*x^2
-7)/x^2/(x^4+5*x^2+3)^(1/2)-2/39*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1251, 822, 806, 724, 206}

$$-\frac{8x^2+7}{39x^2\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{39x^2}+\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(7 + 8*x^2)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 + x^4])/(39
*x^2) + ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
```

```
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{2+3x}{x^2\left(3+5x+x^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}}-\frac{1}{39}\mathrm{Subst}\left(\int \frac{-6+8x}{x^2\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{39x^2}-\frac{1}{3}\mathrm{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{39x^2}+\frac{2}{3}\mathrm{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\ &= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{39x^2}+\frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 88, normalized size = 0.98

$$\frac{-6x^4-54x^2+13\sqrt{3}\sqrt{x^4+5x^2+3}\,x^2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)-39}{117x^2\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-39 - 54*x^2 - 6*x^4 + 13*Sqrt[3]*x^2*Sqrt[3 + 5*x^2 + x^4]*ArcTanh[(6 + 5
*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(117*x^2*Sqrt[3 + 5*x^2 + x^4])
```

fricas [A] time = 0.87, size = 124, normalized size = 1.38

$$-\frac{6\,x^6+30\,x^4-13\,\sqrt{3}\left(x^6+5\,x^4+3\,x^2\right)\log\left(\frac{25\,x^2+2\,\sqrt{3}\left(5\,x^2+6\right)+2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}+6\right)+30}{x^2}\right)+18\,x^2+3\left(2\,x^4+18\,x^2+13\right)\sqrt{x^4+5\,x^2+3}}{117\left(x^6+5\,x^4+3\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/117*(6*x^6 + 30*x^4 - 13*sqrt(3)*(x^6 + 5*x^4 + 3*x^2)*log((25*x^2 + 2*s
qrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) + 1
8*x^2 + 3*(2*x^4 + 18*x^2 + 13)*sqrt(x^4 + 5*x^2 + 3))/(x^6 + 5*x^4 + 3*x^2
)
```

giac [A] time = 0.49, size = 122, normalized size = 1.36

$$-\frac{1}{9}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{7x^2+11}{117\sqrt{x^4+5x^2+3}}+\frac{5x^2-5\sqrt{x^4+5x^2+3}+6}{9\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - s
qrt(x^4 + 5*x^2 + 3))) + 1/117*(7*x^2 + 11)/sqrt(x^4 + 5*x^2 + 3) + 1/9*(5*
x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)
```

maple [A] time = 0.02, size = 84, normalized size = 0.93

$$\frac{\sqrt{3}\ \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{9}-\frac{1}{3\sqrt{x^4+5x^2+3}\ x^2}-\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{2x^2+5}{39\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -1/3/(x^4+5*x^2+3)^(1/2)-1/39*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)+1/9*arctanh(1/6
*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3/x^2/(x^4+5*x^2+3)^(1/2)
```

maxima [A] time = 2.09, size = 82, normalized size = 0.91

$$-\frac{2x^2}{39\sqrt{x^4+5x^2+3}}+\frac{1}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{6}{13\sqrt{x^4+5x^2+3}}-\frac{1}{3\sqrt{x^4+5x^2+3}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/39*x^2/sqrt(x^4 + 5*x^2 + 3) + 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^
2 + 3)/x^2 + 6/x^2 + 5) - 6/13/sqrt(x^4 + 5*x^2 + 3) - 1/3/(sqrt(x^4 + 5*x^
2 + 3)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{3x^2+2}{x^3\left(x^4+5x^2+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(3/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{3x^2+2}{x^3\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**3*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

3.199 $\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{11}{13}\sqrt{x^4+5x^2+3}\,x+\frac{43\left(2x^2+\sqrt{13}+5\right)x}{13\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}(5\right.\right.}{13\sqrt{x^4+5x^2+3}}$$

```
[Out] 1/13*x^3*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)+43/13*x*(5+2*x^2+13^(1/2))/(x^4+5*x
^2+3)^(1/2)-11/13*x*(x^4+5*x^2+3)^(1/2)+11/26*(1/(36+x^2*(30+6*13^(1/2))))^
(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2))
)*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1
/2)/(x^4+5*x^2+3)^(1/2)-43/78*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(3
0+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/
2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2)
)^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/
2)
```

Rubi [A] time = 0.16, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1275, 1279, 1189, 1099, 1135}

$$\frac{\left(11x^2+8\right)x^3}{13\sqrt{x^4+5x^2+3}}-\frac{11}{13}\sqrt{x^4+5x^2+3}\,x+\frac{43\left(2x^2+\sqrt{13}+5\right)x}{13\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+\right.}{13\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (43*x*(5 + Sqrt[13] + 2*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (x^3*(8 + 11*x^2
))/(13*Sqrt[3 + 5*x^2 + x^4]) - (11*x*Sqrt[3 + 5*x^2 + x^4])/13 - (43*Sqrt[
(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(
6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 +
5*Sqrt[13])/6])/(13*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[3/(2*(5 + Sqrt[13]))]
*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13]
)*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(
13*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4\left(2+3x^2\right)}{\left(3+5x^2+x^4\right)^{3/2}}\,dx = \frac{x^3\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{1}{13}\int \frac{x^2\left(-24-33x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{x^3\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}-\frac{11}{13}x\sqrt{3+5x^2+x^4}-\frac{1}{39}\int \frac{-99-258x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{x^3\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}-\frac{11}{13}x\sqrt{3+5x^2+x^4}+\frac{33}{13}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx+\frac{86}{13}\int \frac{x^2}{\sqrt{3+5x^2\ldots}}$$

$$= \frac{43x\left(5+\sqrt{13}+2x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{x^3\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}-\frac{11}{13}x\sqrt{3+5x^2+x^4}-\frac{43\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\ldots}}{\ldots}$$

Mathematica [C] time = 0.26, size = 219, normalized size = 0.71

$$\frac{-2x\left(47x^2+33\right)-i\sqrt{2}\left(43\sqrt{13}-182\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+43\ldots}{26\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-2*x*(33 + 47*x^2) + (43*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] -
2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt
```

```
[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-182 + 43*Sqrt[1
3])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2
]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(2
6*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(3\,x^{6}+2\,x^{4}\right)\sqrt{x^{4}+5\,x^{2}+3}}{x^{8}+10\,x^{6}+31\,x^{4}+30\,x^{2}+9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)*sqrt(x^4 + 5*x^2 + 3)/(x^8 + 10*x^6 + 31*x^4 + 30*
x^2 + 9), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^{2}+2\right)x^{4}}{\left(x^{4}+5\,x^{2}+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2), x)
```

maple [A] time = 0.02, size = 240, normalized size = 0.78

$$\frac{198\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^{2}+1}\,\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^{2}+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\,x}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^{4}+5x^{2}+3}}-\frac{6\left(\frac{19}{26}x^{3}+\frac{15}{26}x\right)}{\sqrt{x^{4}+5x^{2}+3}}-\frac{309\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -6*(19/26*x^3+15/26*x)/(x^4+5*x^2+3)^(1/2)+198/13/(-30+6*13^(1/2))^(1/2)*(-
(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*
x^2+3)^(1/2)*EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2
))-3096/13/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/
6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/
6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*1
3^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2)))-4*(-5/26*x^3-3/13*x)/(x^4+5*x^2
+3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^{2}+2\right)x^{4}}{\left(x^{4}+5\,x^{2}+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(3\,x^2+2\right)}{{\left(x^4+5\,x^2+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(3x^{2} + 2\right)}{\left(x^{4} + 5x^{2} + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
[Out] Integral(x**4*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.200 $\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=286

$$-\frac{11x\left(2x^2+\sqrt{13}+5\right)}{26\sqrt{x^4+5x^2+3}}+\frac{x\left(11x^2+8\right)}{13\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\right.}\right.\right.}{13\sqrt{x^4+5x^2+3}}$$

```
[Out] 1/13*x*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)-11/26*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2
+3)^(1/2)-4/39*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^
(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*
(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((
6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+11/156*
(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE
(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2
))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/
(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1275, 1189, 1099, 1135}

$$-\frac{11x\left(2x^2+\sqrt{13}+5\right)}{26\sqrt{x^4+5x^2+3}}+\frac{x\left(11x^2+8\right)}{13\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\right.}\right.\right.}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-11*x*(5 + Sqrt[13] + 2*x^2))/(26*Sqrt[3 + 5*x^2 + x^4]) + (x*(8 + 11*x^2)
)/(13*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[A
rcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(26*Sqrt[3 + 5*x^2
+ x^4]) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 +
(5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sq
rt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(2+3x^2\right)}{\left(3+5x^2+x^4\right)^{3/2}}\,dx &= \frac{x\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{1}{13}\int \frac{-8-11x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= \frac{x\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}-\frac{8}{13}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx-\frac{11}{13}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{11x\left(5+\sqrt{13}+2x^2\right)}{26\sqrt{3+5x^2+x^4}}+\frac{x\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{11\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5\right.\right.}{}\end{aligned}$$

Mathematica [C] time = 0.25, size = 219, normalized size = 0.77

$$\frac{4x\left(11x^2+8\right)+i\sqrt{2}\left(11\sqrt{13}-39\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-11i\sqrt{2}\ (}{52\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(8 + 11*x^2) - (11*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*
x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2
/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-39 + 11*Sqrt[13])
*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*E
llipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(52*S
qrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(3\,x^4+2\,x^2\right)\sqrt{x^4+5\,x^2+3}}{x^8+10\,x^6+31\,x^4+30\,x^2+9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)*sqrt(x^4 + 5*x^2 + 3)/(x^8 + 10*x^6 + 31*x^4 + 30*
x^2 + 9), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^2+2\right)x^2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2), x)
```

maple [A] time = 0.02, size = 240, normalized size = 0.84

$$-\frac{48\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\,x}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{6\left(-\frac{5}{26}x^3-\frac{3}{13}x\right)}{\sqrt{x^4+5x^2+3}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -6*(-5/26*x^3-3/13*x)/(x^4+5*x^2+3)^(1/2)-48/13/(-30+6*13^(1/2))^(1/2)*(-(-
5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^
2+3)^(1/2)*EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))
+396/13/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1
/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(
-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(
1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2)))-4*(1/13*x^3+5/26*x)/(x^4+5*x^2+3)^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^2+2\right)x^2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(3\,x^2+2\right)}{{\left(x^4+5\,x^2+3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(3x^{2}+2\right)}{\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.201 $\int \frac{2+3x^2}{\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=282

$$\frac{4x\left(2x^2+\sqrt{13}+5\right)}{39\sqrt{x^4+5x^2+3}}-\frac{x\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(\right.\right.}{13\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$

```
[Out] -1/39*x*(8*x^2+7)/(x^4+5*x^2+3)^(1/2)+4/39*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+
3)^(1/2)-2/117*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^
(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*
(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2
*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+11/13*(1/(36
+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30
+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/
2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/
(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1178, 1189, 1099, 1135}

$$\frac{4x\left(2x^2+\sqrt{13}+5\right)}{39\sqrt{x^4+5x^2+3}}-\frac{x\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(\right.\right.}{13\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(5 + Sqrt[13] + 2*x^2))/(39*Sqrt[3 + 5*x^2 + x^4]) - (x*(7 + 8*x^2))/(
39*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[A
rcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(39*Sqrt[3 + 5*x^2
+ x^4]) + (11*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 +
(5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sq
rt[13])/6])/(13*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{2+3x^2}{\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{x\left(7+8x^2\right)}{39\sqrt{3+5x^2+x^4}} - \frac{1}{39}\int \frac{-33-8x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{x\left(7+8x^2\right)}{39\sqrt{3+5x^2+x^4}} + \frac{8}{39}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx + \frac{11}{13}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{4x\left(5+\sqrt{13}+2x^2\right)}{39\sqrt{3+5x^2+x^4}} - \frac{x\left(7+8x^2\right)}{39\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{\frac{2}{3}}\left(5+\sqrt{13}\right)\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{\cdots}\right.\right.}{39\cdots}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x^8+10x^6+31x^4+30x^2+9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/(x^8 + 10*x^6 + 31*x^4 + 30*x^2
+ 9), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

maple [A] time = 0.02, size = 240, normalized size = 0.85

$$\frac{66\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\,x}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{6\left(\frac{1}{13}x^3+\frac{5}{26}x\right)}{\sqrt{x^4+5x^2+3}}-\frac{96\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -6*(1/13*x^3+5/26*x)/(x^4+5*x^2+3)^(1/2)+66/13/(-30+6*13^(1/2))^(1/2)*(-(-5
/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2
+3)^(1/2)*EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-
96/13/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6
*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(-3
0+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(1/
2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2)))-4*(-19/78*x-5/78*x^3)/(x^4+5*x^2+3)^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{{\left(x^4+5\,x^2+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.202 $\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=309

$$\frac{19x\left(2x^2+\sqrt{13}+5\right)}{234\sqrt{x^4+5x^2+3}}-\frac{19\sqrt{x^4+5x^2+3}}{117x}-\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\right.}{39\sqrt{x^4+5x}}$$

```
[Out] 1/39*(-8*x^2-7)/x/(x^4+5*x^2+3)^(1/2)+19/234*x*(5+2*x^2+13^(1/2))/(x^4+5*x^
2+3)^(1/2)-19/117*(x^4+5*x^2+3)^(1/2)/x-4/117*(1/(36+x^2*(30+6*13^(1/2))))^
(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2))
)*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1
/2)/(x^4+5*x^2+3)^(1/2)-19/1404*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*
(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(
1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/
2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(
1/2)
```

Rubi [A] time = 0.16, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1277, 1281, 1189, 1099, 1135}

$$\frac{19x\left(2x^2+\sqrt{13}+5\right)}{234\sqrt{x^4+5x^2+3}}-\frac{19\sqrt{x^4+5x^2+3}}{117x}-\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\right.}{39\sqrt{x^4+5x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^2*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (19*x*(5 + Sqrt[13] + 2*x^2))/(234*Sqrt[3 + 5*x^2 + x^4]) - (7 + 8*x^2)/(39
*x*Sqrt[3 + 5*x^2 + x^4]) - (19*Sqrt[3 + 5*x^2 + x^4])/(117*x) - (19*Sqrt[(
5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/(234*Sqrt[3 + 5*x^2 + x^4]) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*
Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])
*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3
9*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx &= -\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}}-\frac{1}{39}\int \frac{-19+8x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}}-\frac{19\sqrt{3+5x^2+x^4}}{117x}+\frac{1}{117}\int \frac{-24+19x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}}-\frac{19\sqrt{3+5x^2+x^4}}{117x}+\frac{19}{117}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx-\frac{8}{39}\int \frac{}{\sqrt{}}\\ &= \frac{19x\left(5+\sqrt{13}+2x^2\right)}{234\sqrt{3+5x^2+x^4}}-\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}}-\frac{19\sqrt{3+5x^2+x^4}}{117x}-\frac{19\sqrt{\frac{1}{6}\left(5+\sqrt{1}\right.}}{}\end{aligned}$$

Mathematica [C] time = 0.27, size = 228, normalized size = 0.74

$$\frac{-i\sqrt{2}\left(19\sqrt{13}-143\right)x\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+19i\sqrt{2}\left(\sqrt{13}-5\right.}{468x\sqrt{x^4+5x^2+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(x^2*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-4*(78 + 119*x^2 + 19*x^4) + (19*I)*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 + S
qrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*Ar
```

```
cSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-143 +
 19*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt
[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt
[13])/6])/(468*x*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x^{10}+10x^8+31x^6+30x^4+9x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/(x^10 + 10*x^8 + 31*x^6 + 30*x^4
 + 9*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2), x)
```

maple [A] time = 0.03, size = 257, normalized size = 0.83

$$-\frac{16\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\,x}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{9x}-\frac{6\left(-\right.}{\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -6*(-5/78*x^3-19/78*x)/(x^4+5*x^2+3)^(1/2)-16/13/(-30+6*13^(1/2))^(1/2)*(-(
-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x
^2+3)^(1/2)*EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2)
)-76/13/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1
/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*(
-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*(-30+6*13^(
1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2)))-2/9*(x^4+5*x^2+3)^(1/2)/x-4*(19/23
4*x^3+40/117*x)/(x^4+5*x^2+3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{x^2\,{\left(x^4+5\,x^2+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(3/2)),x)
[Out] int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^{2}+2}{x^{2}\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(3/2),x)
[Out] Integral((3*x**2 + 2)/(x**2*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

3.203 $\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{133x\left(2x^2+\sqrt{13}+5\right)}{1053\sqrt{x^4+5x^2+3}}+\frac{266\sqrt{x^4+5x^2+3}}{1053x}-\frac{5\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\right.}{351\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$

```
[Out] 1/39*(-8*x^2-7)/x^3/(x^4+5*x^2+3)^(1/2)-133/1053*x*(5+2*x^2+13^(1/2))/(x^4+
5*x^2+3)^(1/2)-5/351*(x^4+5*x^2+3)^(1/2)/x^3+266/1053*(x^4+5*x^2+3)^(1/2)/x
+133/6318*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)
*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+
30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-1
3^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-5/351*(1/(36+x^2*
(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13
^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(
6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+
5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1277, 1281, 1189, 1099, 1135}

$$-\frac{133x\left(2x^2+\sqrt{13}+5\right)}{1053\sqrt{x^4+5x^2+3}}+\frac{266\sqrt{x^4+5x^2+3}}{1053x}-\frac{5\sqrt{x^4+5x^2+3}}{351x^3}-\frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}-\frac{5\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right.\right.}{35}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^4*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-133*x*(5 + Sqrt[13] + 2*x^2))/(1053*Sqrt[3 + 5*x^2 + x^4]) - (7 + 8*x^2)/
(39*x^3*Sqrt[3 + 5*x^2 + x^4]) - (5*Sqrt[3 + 5*x^2 + x^4])/(351*x^3) + (266
*Sqrt[3 + 5*x^2 + x^4])/(1053*x) + (133*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5
 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellipt
icE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(1053*Sqrt[3 +
 5*x^2 + x^4]) - (5*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(351*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{1}{39}\int \frac{-5+24x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{1}{351}\int \frac{-266-5x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{266\sqrt{3+5x^2+x^4}}{1053x}-\frac{\int \frac{15+266x^2}{\sqrt{3+5x^2+x^4}}\,dx}{1053}$$

$$= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{266\sqrt{3+5x^2+x^4}}{1053x}-\frac{5}{351}\int \frac{1}{\sqrt{3+5x^2}}$$

$$= -\frac{133x\left(5+\sqrt{13}+2x^2\right)}{1053\sqrt{3+5x^2+x^4}}-\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{266\sqrt{3+5x^2}}{1053}$$

Mathematica [C] time = 0.27, size = 234, normalized size = 0.72

$$\frac{532x^6+2630x^4+1014x^2+i\sqrt{2}\left(133\sqrt{13}-650\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,x^3F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)\middle|\right.}{2106x^3\sqrt{x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(x^4*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-468 + 1014*x^2 + 2630*x^4 + 532*x^6 - (133*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3
*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*E
llipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sq
rt[2]*(-650 + 133*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13]
)]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x]
, 19/6 + (5*Sqrt[13])/6])/(2106*x^3*Sqrt[3 + 5*x^2 + x^4])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^{12}+10\,x^{10}+31\,x^8+30\,x^6+9\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/(x^12 + 10*x^10 + 31*x^8 + 30*x^
6 + 9*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4), x)
```

maple [A] time = 0.02, size = 274, normalized size = 0.84

$$-\frac{10\sqrt{-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-30+6\sqrt{13}}\,x}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{117\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}+\frac{23\sqrt{x^4+5x^2+3}}{81x}-2\sqrt{\;}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 23/81*(x^4+5*x^2+3)^(1/2)/x-6*(19/234*x^3+40/117*x)/(x^4+5*x^2+3)^(1/2)-10/
117/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*1
3^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*(-30+6*13^(1/2))^(1
/2)*x,5/6*3^(1/2)+1/6*39^(1/2))+1064/117/(-30+6*13^(1/2))^(1/2)*(-(-5/6+1/6
*13^(1/2))*x^2+1)^(1/2)*(-(-5/6-1/6*13^(1/2))*x^2+1)^(1/2)/(x^4+5*x^2+3)^(1
/2)/(13^(1/2)+5)*(EllipticF(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39
^(1/2))-EllipticE(1/6*(-30+6*13^(1/2))^(1/2)*x,5/6*3^(1/2)+1/6*39^(1/2)))-2
/27*(x^4+5*x^2+3)^(1/2)/x^3-4*(-40/351*x^3-343/702*x)/(x^4+5*x^2+3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{x^4\,{\left(x^4+5\,x^2+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(3/2)),x)
[Out] int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^{2} + 2}{x^{4} \left(x^{4} + 5x^{2} + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(3/2),x)
[Out] Integral((3*x**2 + 2)/(x**4*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

3.204 $\int (fx)^{3/2}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{9}{4};-\frac{1}{2},-\frac{1}{2};\frac{13}{4};-\right.}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}}}}$$

```
[Out] 2/5*d*(f*x)^(5/2)*AppellF1(5/4,-1/2,-1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/9*e*(f*
x)^(9/2)*AppellF1(9/4,-1/2,-1/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{9}{4};-\frac{1}{2},-\frac{1}{2};\frac{13}{4};-\right.}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(3/2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*e*(f*x)^(9/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4, -1/2
, -1/2, 13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int (fx)^{3/2}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx=\int\left(d(fx)^{3/2}\sqrt{a+bx^2+cx^4}+\frac{e(fx)^{7/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$
$$=d\int(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx+\frac{e\int(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$
$$=\frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\ldots$$
$$=\frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\ldots$$

Mathematica [A] time = 0.98, size = 430, normalized size = 1.45

$$\frac{2f\sqrt{fx}\left(10a\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(-18ace+7b^2e-13bcd\right)\ldots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^(3/2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*f*Sqrt[f*x]*(5*(a + b*x^2 + c*x^4)*(-14*b^2*e + 2*b*c*(13*d + 5*e*x^2) +
 c*(36*a*e + 65*c*d*x^2 + 45*c*e*x^4)) + 10*a*(-13*b*c*d + 7*b^2*e - 18*a*c
*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2,
 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4
*a*c])] + 2*(-39*b^2*c*d + 130*a*c^2*d + 21*b^3*e - 79*a*b*c*e)*x^2*Sqrt[(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4,
(-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(
2925*c^2*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(efx^3+dfx\right)\sqrt{cx^4+bx^2+a}\,\sqrt{fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*f*x^3 + d*f*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\left(fx\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \left(f x\right)^{\frac{3}{2}} \left(e\,x^{2}+d\right) \sqrt{c\,x^{4}+b\,x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^{4}+bx^{2}+a}\left(ex^{2}+d\right)\left(fx\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f\,x\right)}^{3/2}\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f x\right)^{\frac{3}{2}} \left(d + e x^{2}\right) \sqrt{a + b x^{2} + c x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f*x)**(3/2)*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

3.205 $\int \sqrt{fx}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{7}{4};-\frac{1}{2},-\frac{1}{2};\frac{11}{4}\right.}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2}{\sqrt{b^2}}}}$$

```
[Out] 2/3*d*(f*x)^(3/2)*AppellF1(3/4,-1/2,-1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/7*e*(f*
x)^(7/2)*AppellF1(7/4,-1/2,-1/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{7}{4};-\frac{1}{2},-\frac{1}{2};\frac{11}{4}\right.}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2}{\sqrt{b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f*x]*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*e*(f*x)^(7/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4, -1/2
, -1/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \sqrt{fx}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx=\int\left(d\sqrt{fx}\sqrt{a+bx^2+cx^4}+\frac{e(fx)^{5/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$

$$=d\int\sqrt{fx}\sqrt{a+bx^2+cx^4}\,dx+\frac{e\int(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$

$$=\frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{\left(e\sqrt{a+\right.}}{}$$

$$=\frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2e(}{}$$

Mathematica [A] time = 5.71, size = 386, normalized size = 1.30

$$\frac{2x\sqrt{fx}\left(6x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(14ace-5b^2e+11bcd\right)+1\right.}{16}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[f*x]*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*Sqrt[f*x]*(21*(11*c*d + 2*b*e + 7*c*e*x^2)*(a + b*x^2 + c*x^4) + 14*a*
(22*c*d - 3*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])] + 6*(11*b*c*d - 5*b^2*e + 14*a*c*e)*x^2*Sqrt[(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(1617*c
*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\sqrt{fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\sqrt{fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{f x}\, \left(e\,x^{2}+d\right) \sqrt{c\,x^{4}+b\,x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^{4}+bx^{2}+a}\left(ex^{2}+d\right)\sqrt{fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f\,x}\, \left(e\,x^{2}+d\right)\, \sqrt{c\,x^{4}+b\,x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{f x}\, \left(d+ex^{2}\right) \sqrt{a+bx^{2}+cx^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f*x)*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

3.206 $\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx$

Optimal. Leaf size=295

$$\frac{2d\sqrt{fx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{}{b-\sqrt{}}\right.}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}}}$$

```
[Out] 2/5*e*(f*x)^(5/2)*AppellF1(5/4,-1/2,-1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2*d*App
ellF1(1/4,-1/2,-1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c
+b^2)^(1/2)))*(f*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-4*a*c+b^2
)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d\sqrt{fx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{}{b-\sqrt{}}\right.}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[f*x],x]
```

```
[Out] (2*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])]) + (2*e*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1
/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(5*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \left(\frac{d\sqrt{a+bx^2+cx^4}}{\sqrt{fx}} + \frac{e(fx)^{3/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$
$$= d\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx + \frac{e\int (fx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$
$$= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{\sqrt{fx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(e\sqrt{a+bx^2+cx^4}\right)\int (}{f^2\sqrt{1 -}}$$
$$= \frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2e(fx)^5}{}$$

Mathematica [A] time = 0.71, size = 386, normalized size = 1.31

$$\frac{2x\left(2x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(10ace-3b^2e+9bcd\right)+10a\sqrt{}\right.}{225c\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[f*x],x]
```

```
[Out] (2*x*(5*(9*c*d + 2*b*e + 5*c*e*x^2)*(a + b*x^2 + c*x^4) + 10*a*(18*c*d - b*
e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2,
1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*
a*c])] + 2*(9*b*c*d - 3*b^2*e + 10*a*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(225*c*Sqrt[f*x]*Sqrt[a
 + b*x^2 + c*x^4])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\sqrt{fx}}{fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(f*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{\sqrt{f x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{4}+bx^{2}+a}\left(ex^{2}+d\right)}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^{2}+d\right)\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{\sqrt{f\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(1/2),x)
```

```
[Out] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^{2}\right)\sqrt{a+bx^{2}+cx^{4}}}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/sqrt(f*x), x)
```

3.207 $\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx$

Optimal. Leaf size=295

$$\frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2d\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 2/3*e*(f*x)^(3/2)*AppellF1(3/4,-1/2,-1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)-2*d*App
ellF1(-1/4,-1/2,-1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*
c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(f*x)^(1/2)/(1+2*c*x^2/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1335, 1141, 510}

$$\frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2d\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(f*x)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]) + (2*e*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2,
-1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])])/(3*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \int \left(\frac{d\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}} + \frac{e\sqrt{fx}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$
$$= d\int \frac{\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx + \frac{e\int \sqrt{fx}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$
$$= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{(fx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(e\sqrt{a+bx^2+cx^4}\right)\int \sqrt{f}}{f^2\sqrt{1+\frac{}{b-}}}$$
$$= -\frac{2d\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2e(fx)^{3/2}\sqrt{a}}{}$$

Mathematica [A] time = 0.86, size = 370, normalized size = 1.25

$$\frac{x\left(28x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}(2ae+7bd)F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+12x^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+}{b-\sqrt{b^2-4ac}}}\right.}{147(fx)^{3/2}\sqrt{a+bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(f*x)^(3/2),x]
```

```
[Out] (x*(-42*(7*d - e*x^2)*(a + b*x^2 + c*x^4) + 28*(7*b*d + 2*a*e)*x^2*Sqrt[(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 12
*(14*c*d + b*e)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])]))/(147*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\sqrt{fx}}{f^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(f^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)}{\left(fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(f*x)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{\left(f x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{4}+bx^{2}+a}\left(ex^{2}+d\right)}{\left(f x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(f*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}{{\left(f\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(3/2),x)
```

```
[Out] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex^{2}\right)\sqrt{a + bx^{2} + cx^{4}}}{\left(f x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/(f*x)**(3/2), x)
```

3.208 $\int (fx)^{3/2}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=299

$$\frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{9}{4};-\frac{3}{2},-\frac{3}{2};\frac{13}{4};\right.}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c}{\sqrt{b^2-4}}}}$$

```
[Out] 2/5*a*d*(f*x)^(5/2)*AppellF1(5/4,-3/2,-3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/9*a*e
*(f*x)^(9/2)*AppellF1(9/4,-3/2,-3/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1335, 1141, 510}

$$\frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{9}{4};-\frac{3}{2},-\frac{3}{2};\frac{13}{4};\right.}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c}{\sqrt{b^2-4}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*
f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]) + (2*a*e*(f*x)^(9/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4,
-3/2, -3/2, 13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int (fx)^{3/2}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int\left(d(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{7/2}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right)dx$$

$$=d\int (fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}\,dx+\frac{e\int (fx)^{7/2}\left(a+bx^2+cx^4\right)^{3/2}\,d}{f^2}$$

$$=\frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int (fx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$=\frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right.}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cefx^7+(cd+be)fx^5+(bd+ae)fx^3+adfx\right)\sqrt{cx^4+bx^2+a}\,\sqrt{fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
[Out] integral((c*e*f*x^7 + (c*d + b*e)*f*x^5 + (b*d + a*e)*f*x^3 + a*d*f*x)*sqrt
(c*x^4 + b*x^2 + a)*sqrt(f*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)(fx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (fx)^{\frac{3}{2}}\left(e\,x^2+d\right)\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \left(ex^2 + d\right) (fx)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f\,x\right)}^{3/2}\,\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^{\frac{3}{2}} \left(d + ex^2\right) \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((f*x)**(3/2)*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.209 $\int \sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=299

$$\frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{7}{4};-\frac{3}{2},-\frac{3}{2};\right.}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\vphantom{x}}}$$

```
[Out] 2/3*a*d*(f*x)^(3/2)*AppellF1(3/4,-3/2,-3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/7*a*e
*(f*x)^(7/2)*AppellF1(7/4,-3/2,-3/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{7}{4};-\frac{3}{2},-\frac{3}{2};\right.}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\vphantom{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*
f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]) + (2*a*e*(f*x)^(7/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4,
-3/2, -3/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}dx=\int\left(d\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{5/2}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right)dx$$

$$=d\int\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}dx+\frac{e\int(fx)^{5/2}\left(a+bx^2+cx^4\right)^{3/2}dx}{f^2}$$

$$=\frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int\sqrt{fx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+$$

$$=\frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+$$

Mathematica [A] time = 6.14, size = 490, normalized size = 1.64

$$\frac{2x\sqrt{fx}\left(12x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(420a^2c^2e-309ab^2ce+6\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[f*x]*(7*(a + b*x^2 + c*x^4)*(-108*b^3*e + 12*b^2*c*(19*d + 7*e*x^
2) + b*c*(624*a*e + 7*c*x^2*(323*d + 231*e*x^2)) + c^2*(77*c*x^4*(19*d + 15
*e*x^2) + a*(3971*d + 2415*e*x^2))) + 28*a*(-57*b^2*c*d + 836*a*c^2*d + 27*
b^3*e - 156*a*b*c*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*A
ppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + 12*(-95*b^3*c*d + 684*a*b*c^2*d + 45*b^4*e - 309
*a*b^2*c*e + 420*a^2*c^2*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(153615*c^2*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cex^6+(cd+be)x^4+(bd+ae)x^2+ad\right)\sqrt{cx^4+bx^2+a}\sqrt{fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 + b
*x^2 + a)*sqrt(f*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)\sqrt{fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*sqrt(f*x), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{fx}\, \left(e\,x^{2}+d\right) \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}\left(ex^{2}+d\right)\sqrt{fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*sqrt(f*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f\,x}\, \left(e\,x^2+d\right)\, \left(c\,x^4+b\,x^2+a\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{fx}\, \left(d+ex^{2}\right) \left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(f*x)*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.210 $\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx$

Optimal. Leaf size=297

$$\frac{2ad\sqrt{fx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\right.}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}}}}$$

```
[Out] 2/5*a*e*(f*x)^(5/2)*AppellF1(5/4,-3/2,-3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2*a*d
*AppellF1(1/4,-3/2,-3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4
*a*c+b^2)^(1/2)))*(f*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-4*a*c
+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2ad\sqrt{fx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\right.}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
```

```
[Out] (2*a*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2, 5/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]) + (2*a*e*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2
, -3/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(5*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx = \int \left(\frac{d(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}} + \frac{e(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}{f^2}\right)dx$$

$$= d\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx + \frac{e\int (fx)^{3/2}(a+bx^2+cx^4)^{3/2}\,dx}{f^2}$$

$$= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{\sqrt{fx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(ae\sqrt{a+bx^2+\right.}{}$$

$$= \frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(}{}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
[Out] $Aborted
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cex^6+(cd+be)x^4+(bd+ae)x^2+ad)\sqrt{cx^4+bx^2+a}\,\sqrt{fx}}{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="fricas"
)
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 + b
*x^2 + a)*sqrt(f*x)/(f*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/sqrt(f*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/sqrt(f*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2 + d\right)\,\left(c\,x^4 + b\,x^2 + a\right)^{3/2}}{\sqrt{f\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(1/2),x)
```

```
[Out] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex^2\right)\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)/sqrt(f*x), x)
```

3.211 $\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx$

Optimal. Leaf size=297

$$\frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2ad\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 2/3*a*e*(f*x)^(3/2)*AppellF1(3/4,-3/2,-3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)-2*a*d
*AppellF1(-1/4,-3/2,-3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-
4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(f*x)^(1/2)/(1+2*c*x^2/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2ad\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x]
```

```
[Out] (-2*a*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*a*e*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3
/2, -3/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(3*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{(fx)^{3/2}}\,dx = \int \left(\frac{d\left(a+bx^2+cx^4\right)^{3/2}}{(fx)^{3/2}} + \frac{e\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right) dx$$

$$= d\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{(fx)^{3/2}}\,dx + \frac{e\int \sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}\,dx}{f^2}$$

$$= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{(fx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(ae\sqrt{a+bx^2+cx^4}\right.}{f^2\ldots}$$

$$= -\frac{2ad\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(fx)^{3/\ldots}}{\ldots}$$

Mathematica [A] time = 0.99, size = 447, normalized size = 1.51

$$\frac{x\left(-56ax^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(-44ace+3b^2e-240bcd\right)+2\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x]
```

```
[Out] (x*(14*(a + b*x^2 + c*x^4)*(a*c*(-1155*d + 209*e*x^2) + x^2*(12*b^2*e + 7*c
^2*x^2*(15*d + 11*e*x^2) + b*c*(195*d + 119*e*x^2))) - 56*a*(-240*b*c*d + 3
*b^2*e - 44*a*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])] + 24*(15*b^2*c*d + 420*a*c^2*d - 5*b^3*e + 36*a*
b*c*e)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/
4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])]))/(8085*c*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cex^6+(cd+be)x^4+(bd+ae)x^2+ad\right)\sqrt{cx^4+bx^2+a}\sqrt{fx}}{f^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 + b
*x^2 + a)*sqrt(f*x)/(f^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)}{(fx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/(f*x)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{\left(fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x)
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}\left(ex^{2}+d\right)}{\left(fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="maxima"
)
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/(f*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{{\left(f\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x)
[Out] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^{2}\right)\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}{\left(fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(3/2),x)
[Out] Integral((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(f*x)**(3/2), x)
```

3.212 $\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{}{\sqrt{b}}}}{}$$

```
[Out] 2/5*d*(f*x)^(5/2)*AppellF1(5/4,1/2,1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(c*x^4+b*x^2+a)^(1/2)+2/9*e*(f*x)
^(9/2)*AppellF1(9/4,1/2,1/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{}{\sqrt{b}}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^(3/2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 1/2, 1/2, 13/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(9*
f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \left(\frac{d(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}} + \frac{e(fx)^{7/2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$= d\int \frac{(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx + \frac{e\int \frac{(fx)^{7/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{3/2}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}} + \frac{\left(e\sqrt{1+\frac{2c}{b-\sqrt{b}}}\right.}{}$$

$$= \frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}} + \cdots$$

Mathematica [A] time = 0.59, size = 354, normalized size = 1.19

$$\frac{2f\sqrt{fx}\left(x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}(5cd-3be)F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-5ae\sqrt{\frac{-\sqrt{b^2-4}}{b-\sqrt{}}}\cdots\right.}{25c\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^(3/2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*f*Sqrt[f*x]*(5*e*(a + b*x^2 + c*x^4) - 5*a*e*Sqrt[(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (5*c*d - 3*b*e)*x^2*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2,
 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)]))/(25*c*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(efx^3+dfx\right)\sqrt{fx}}{\sqrt{cx^4+bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*f*x^3 + d*f*x)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)\left(fx\right)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{\frac{3}{2}} (e\,x^2 + d)}{\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)

[Out] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)(fx)^{\frac{3}{2}}}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)

[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x)^{3/2}\,(e\,x^2 + d)}{\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)

[Out] int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{\frac{3}{2}} (d + ex^2)}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((f*x)**(3/2)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)

3.213 $\int \frac{\sqrt{fx}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{}}{}$$

```
[Out] 2/3*d*(f*x)^(3/2)*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(c*x^4+b*x^2+a)^(1/2)+2/7*e*(f*x)
^(7/2)*AppellF1(7/4,1/2,1/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f*x]*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*
f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{\sqrt{fx}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx=\int\left(\frac{d\sqrt{fx}}{\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{5/2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$
$$=d\int \frac{\sqrt{fx}}{\sqrt{a+bx^2+cx^4}}\,dx+\frac{e\int \frac{(fx)^{5/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$
$$=\frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{fx}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\right.}{}$$
$$=\frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}}+\frac{2e(f}{}$$

Mathematica [A] time = 5.14, size = 242, normalized size = 0.81

$$\frac{2\sqrt{fx}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\left(7dxF_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+3ex^3F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-}}\right.\right.}{21\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f*x]*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[f*x]*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(7*d*x*Ap
pellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])] + 3*e*x^3*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*Sqrt[a +
 b*x^2 + c*x^4])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex^2+d\right)\sqrt{fx}}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)\sqrt{fx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f x}\, \left(e\,x^{2}+d\right)}{\sqrt{c\,x^{4}+b\,x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^{2}+d\right)\sqrt{fx}}{\sqrt{cx^{4}+bx^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f\,x}\, \left(e\,x^{2}+d\right)}{\sqrt{c\,x^{4}+b\,x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{fx}\, \left(d+ex^{2}\right)}{\sqrt{a+bx^{2}+cx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f*x)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.214 $\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=295

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c}{\sqrt{b^2-}}}}{5f}$$

```
[Out] 2/5*e*(f*x)^(5/2)*AppellF1(5/4,1/2,1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)+2*d*Appel
lF1(1/4,1/2,1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2
)^(1/2)))*(f*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c}{\sqrt{b^2-}}}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]])/(f*Sqrt[a + b*x^2 +
c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]])/(5*f^3*S
qrt[a + b*x^2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \int \left(\frac{d}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{3/2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$
$$= d\int \frac{1}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx + \frac{e\int \frac{(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$
$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}} + \frac{\left(e\sqrt{1\right.}{}$$
$$= \frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.20, size = 241, normalized size = 0.82

$$\frac{2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\left(5dxF_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+ex^3F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\right.\right.}{5\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/(Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(5*d*x*AppellF1[1/4
, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])] + e*x^3*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*Sqrt[f*x]*Sqrt[a + b*x
^2 + c*x^4])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\sqrt{fx}}{cfx^5+bfx^3+afx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c*f*x^5 + b*f*x^3 +
 a*f*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e\,x^{2}+d}{\sqrt{f x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^{2}+d}{\sqrt{cx^{4}+bx^{2}+a}\,\sqrt{f x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^{2}+d}{\sqrt{f\,x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^{2}}{\sqrt{f x}\,\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/(sqrt(f*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.215 $\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=295

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4a}}}}{f}$$

```
[Out] 2/3*e*(f*x)^(3/2)*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)-2*d*Appel
lF1(-1/4,1/2,1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2)/f/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4a}}}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*Sqrt[a + b*x^2
+ c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f^3
*Sqrt[a + b*x^2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \left(\frac{d}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}} + \frac{e\sqrt{fx}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$= d\int \frac{1}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx + \frac{e\int \frac{\sqrt{fx}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}} + \frac{\left(e\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{a+bx^2+cx^4}} + \frac{2\cdots}{\cdots}$$

Mathematica [A] time = 0.62, size = 356, normalized size = 1.21

$$\frac{2x\left(7x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}(ae+bd)F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+9cdx^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+2c\cdots}{b-\sqrt{b^2-4ac}}}\cdots\right.}{21a(fx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*x*(-21*d*(a + b*x^2 + c*x^4) + 7*(b*d + a*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*c*d*x^4*Sqrt[(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4,
 (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/
(21*a*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\sqrt{fx}}{cf^2x^6+bf^2x^4+af^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c*f^2*x^6 + b*f^2*x
^4 + a*f^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\,(fx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e\,x^{2}+d}{\left(f x\right)^{\frac{3}{2}} \sqrt{c\,x^{4}+b\,x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^{2}+d}{\sqrt{cx^{4}+bx^{2}+a} \left(f x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^{2}+d}{\left(f\,x\right)^{3/2}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^{2}}{\left(f x\right)^{\frac{3}{2}} \sqrt{a + bx^{2} + cx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/((f*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.216 $\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{}{\sqrt{b}}}}{9}$$

```
[Out] 2/5*d*(f*x)^(5/2)*AppellF1(5/4,3/2,3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(c*x^4+b*x^2+a)^(1/2)+2/9*e*(f*
x)^(9/2)*AppellF1(9/4,3/2,3/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2
/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{}{\sqrt{b}}}}{9}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f*Sqrt[a + b*
x^2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 3/2, 3/2, 13/4,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
9*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \left(\frac{d(fx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}+\frac{e(fx)^{7/2}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$= d\int \frac{(fx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx+\frac{e\int \frac{(fx)^{7/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{3/2}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}+\frac{\left(e\sqrt{1+\right.}{}$$

$$= \frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}}+$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(efx^3+dfx\right)\sqrt{cx^4+bx^2+a}\,\sqrt{fx}}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
[Out] integral((e*f*x^3 + d*f*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)/(c^2*x^8 + 2*b
*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)\left(fx\right)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{\frac{3}{2}}\left(e\,x^2+d\right)}{\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)\left(fx\right)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f\,x\right)}^{3/2}\,\left(e\,x^2+d\right)}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

3.217 $\int \frac{\sqrt{fx}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{}}{}$$

```
[Out] 2/3*d*(f*x)^(3/2)*AppellF1(3/4,3/2,3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(c*x^4+b*x^2+a)^(1/2)+2/7*e*(f*
x)^(7/2)*AppellF1(7/4,3/2,3/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2
/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1335, 1141, 510}

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f*x]*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*f*Sqrt[a + b*
x^2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 3/2, 3/2, 11/4,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
7*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{\sqrt{fx}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \left(\frac{d\sqrt{fx}}{\left(a+bx^2+cx^4\right)^{3/2}} + \frac{e(fx)^{5/2}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$= d\int \frac{\sqrt{fx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx + \frac{e\int \frac{(fx)^{5/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{fx}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}} + \frac{\left(e\sqrt{1+\frac{2}{b-\sqrt{}}}\right.}{}$$

$$= \frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}} + \frac{2e}{}$$

Mathematica [A] time = 5.70, size = 397, normalized size = 1.31

$$\frac{x\sqrt{fx}\left(7\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(-3abe+2acd+b^2d\right)+9cx^2\sqrt{}\right.}{21a\left(4ac\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f*x]*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[f*x]*(-21*b^2*d + 21*b*(a*e - c*d*x^2) + 42*a*c*(d + e*x^2) + 7*(b^
2*d + 2*a*c*d - 3*a*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*c*(b*d - 2*a*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*a*(-b^2 + 4
*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\sqrt{fx}}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c^2*x^8 + 2*b*c*x^6
 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)\sqrt{fx}}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f x}\, \left(e\,x^{2}+d\right)}{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^{2}+d\right)\sqrt{fx}}{\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f\,x}\,\left(e\,x^2+d\right)}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Timed out
```

3.218 $\int \frac{d+ex^2}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=301

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c}{\sqrt{b^2-}}}}{5a}$$

```
[Out] 2/5*e*(f*x)^(5/2)*AppellF1(5/4,3/2,3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)+2*d*App
ellF1(1/4,3/2,3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))*(f*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2
/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c}{\sqrt{b^2-}}}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*Sqrt[a + b*x^2
+ c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f
^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \left(\frac{d}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}} + \frac{e(fx)^{3/2}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$= d\int \frac{1}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx + \frac{e\int \frac{(fx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{fx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}} +$$

$$= \frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right.}{af\sqrt{a+bx^2+cx^4}}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
[Out] $Aborted
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\sqrt{fx}}{c^2fx^9+2\,bcfx^7+\left(b^2+2\,ac\right)fx^5+2\,abfx^3+a^2fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c^2*f*x^9 + 2*b*c*f
*x^7 + (b^2 + 2*a*c)*f*x^5 + 2*a*b*f*x^3 + a^2*f*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{e\,x^2+d}{\sqrt{fx}\,\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \sqrt{fx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2 + d}{\sqrt{f\,x}\,\left(c\,x^4 + b\,x^2 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{\sqrt{fx}\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(sqrt(f*x)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.219 $\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=301

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4a}}}}{af}$$

```
[Out] 2/3*e*(f*x)^(3/2)*AppellF1(3/4,3/2,3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)-2*d*App
ellF1(-1/4,3/2,3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+
b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c
+b^2)^(1/2)))^(1/2)/a/f/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4a}}}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*Sqrt[f*x]*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a
*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int\left(\frac{d}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}+\frac{e\sqrt{fx}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$= d\int \frac{1}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx + \frac{e\int \frac{\sqrt{fx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{(fx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}} +$$

$$= -\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{fx}\sqrt{a+bx^2+cx^4}} +$$

Mathematica [A] time = 0.97, size = 460, normalized size = 1.53

$$-\frac{x\left(7x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(2a^2ce+ab^2e+9abcd-3b^3d\right)-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/21*(x*(-21*(-3*b^2*d*x^2*(b + c*x^2) + a^2*c*(8*d - 2*e*x^2) + a*(10*c^2
*d*x^4 + b^2*(-2*d + e*x^2) + b*c*x^2*(11*d + e*x^2))) + 7*(-3*b^3*d + 9*a*
b*c*d + a*b^2*e + 2*a^2*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 9*c*(3*b^2*d - 10*a*c*d - a*b*e)*x^4
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/
2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])]))/(a^2*(b^2 - 4*a*c)*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\sqrt{fx}}{c^2f^2x^{10}+2\,bcf^2x^8+\left(b^2+2\,ac\right)f^2x^6+2\,abf^2x^4+a^2f^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c^2*f^2*x^10 + 2*b*
c*f^2*x^8 + (b^2 + 2*a*c)*f^2*x^6 + 2*a*b*f^2*x^4 + a^2*f^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2+d}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{e\,x^{2}+d}{\left(f x\right)^{\frac{3}{2}} \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^{2}+d}{\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}} \left(fx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{{\left(f\,x\right)}^{3/2}\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Timed out
```

3.220 $\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=243

$$\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{f^7(m+7)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{3c(fx)^{m+11}\left(ace+b^2e+bcd\right)}{f^{11}(m+11)}+\frac{3a(f}{}$$

```
[Out] a^3*d*(f*x)^(1+m)/f/(1+m)+a^2*(a*e+3*b*d)*(f*x)^(3+m)/f^3/(3+m)+3*a*(a*b*e+
a*c*d+b^2*d)*(f*x)^(5+m)/f^5/(5+m)+(3*a^2*c*e+3*a*b^2*e+6*a*b*c*d+b^3*d)*(f
*x)^(7+m)/f^7/(7+m)+(6*a*b*c*e+3*a*c^2*d+b^3*e+3*b^2*c*d)*(f*x)^(9+m)/f^9/(
9+m)+3*c*(a*c*e+b^2*e+b*c*d)*(f*x)^(11+m)/f^11/(11+m)+c^2*(3*b*e+c*d)*(f*x)
^(13+m)/f^13/(13+m)+c^3*e*(f*x)^(15+m)/f^15/(15+m)
```

Rubi [A] time = 0.18, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1261}

$$\frac{(fx)^{m+7}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{f^7(m+7)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+9}\left(6abce+3ac^2d+3b^2cd+b\right.}{f^9(m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*d*(f*x)^(1 + m))/(f*(1 + m)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m))/(f^3*
(3 + m)) + (3*a*(b^2*d + a*c*d + a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((b^
3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + ((3
*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (3
*c*(b*c*d + b^2*e + a*c*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (c^2*(c*d + 3*
b*e)*(f*x)^(13 + m))/(f^13*(13 + m)) + (c^3*e*(f*x)^(15 + m))/(f^15*(15 + m
))
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int (fx)^m\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^3\,dx=\int \left(a^3d(fx)^m+\frac{a^2(3bd+ae)(fx)^{2+m}}{f^2}+\frac{3a\left(b^2d+acd+abe\right)(fx)^{4+m}}{f^4}+\right.$$
$$=\frac{a^3d(fx)^{1+m}}{f(1+m)}+\frac{a^2(3bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{3a\left(b^2d+acd+abe\right)(fx)^{5+m}}{f^5(5+m)}+$$

Mathematica [A] time = 0.30, size = 191, normalized size = 0.79

$$x(fx)^m\left(\frac{a^3d}{m+1}+\frac{x^6\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{m+7}+\frac{a^2x^2(ae+3bd)}{m+3}+\frac{3cx^{10}\left(ace+b^2e+bcd\right)}{m+11}+\frac{3ax^4\left(abe+ac\right.}{m+5}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(f*x)^m*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2)/(3 + m) + (3*a*(b^2*d
+ a*c*d + a*b*e)*x^4)/(5 + m) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e
)*x^6)/(7 + m) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^8)/(9 + m)
+ (3*c*(b*c*d + b^2*e + a*c*e)*x^10)/(11 + m) + (c^2*(c*d + 3*b*e)*x^12)/(1
3 + m) + (c^3*e*x^14)/(15 + m))
```

fricas [**B**] time = 0.74, size = 1357, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*e*m^7 + 49*c^3*e*m^6 + 973*c^3*e*m^5 + 10045*c^3*e*m^4 + 57379*c^3*e*
m^3 + 177331*c^3*e*m^2 + 264207*c^3*e*m + 135135*c^3*e)*x^15 + ((c^3*d + 3*
b*c^2*e)*m^7 + 51*(c^3*d + 3*b*c^2*e)*m^6 + 1045*(c^3*d + 3*b*c^2*e)*m^5 +
11055*(c^3*d + 3*b*c^2*e)*m^4 + 155925*c^3*d + 467775*b*c^2*e + 64339*(c^3*
d + 3*b*c^2*e)*m^3 + 201609*(c^3*d + 3*b*c^2*e)*m^2 + 303255*(c^3*d + 3*b*c
^2*e)*m)*x^13 + 3*((b*c^2*d + (b^2*c + a*c^2)*e)*m^7 + 53*(b*c^2*d + (b^2*c
 + a*c^2)*e)*m^6 + 1125*(b*c^2*d + (b^2*c + a*c^2)*e)*m^5 + 12265*(b*c^2*d
+ (b^2*c + a*c^2)*e)*m^4 + 184275*b*c^2*d + 73139*(b*c^2*d + (b^2*c + a*c^2
)*e)*m^3 + 233487*(b*c^2*d + (b^2*c + a*c^2)*e)*m^2 + 184275*(b^2*c + a*c^2
)*e + 355815*(b*c^2*d + (b^2*c + a*c^2)*e)*m)*x^11 + ((3*(b^2*c + a*c^2)*d
+ (b^3 + 6*a*b*c)*e)*m^7 + 55*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^6
 + 1213*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^5 + 13723*(3*(b^2*c + a
*c^2)*d + (b^3 + 6*a*b*c)*e)*m^4 + 84547*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*
b*c)*e)*m^3 + 277093*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^2 + 675675
*(b^2*c + a*c^2)*d + 225225*(b^3 + 6*a*b*c)*e + 430335*(3*(b^2*c + a*c^2)*d
 + (b^3 + 6*a*b*c)*e)*m)*x^9 + (((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m
^7 + 57*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^6 + 1309*((b^3 + 6*a*b*
c)*d + 3*(a*b^2 + a^2*c)*e)*m^5 + 15477*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2
*c)*e)*m^4 + 99715*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^3 + 340011*(
(b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^2 + 289575*(b^3 + 6*a*b*c)*d + 8
68725*(a*b^2 + a^2*c)*e + 544095*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*
m)*x^7 + 3*((a^2*b*e + (a*b^2 + a^2*c)*d)*m^7 + 59*(a^2*b*e + (a*b^2 + a^2*
c)*d)*m^6 + 1413*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^5 + 17575*(a^2*b*e + (a*b^
2 + a^2*c)*d)*m^4 + 405405*a^2*b*e + 120179*(a^2*b*e + (a*b^2 + a^2*c)*d)*m
^3 + 437121*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^2 + 405405*(a*b^2 + a^2*c)*d +
738567*(a^2*b*e + (a*b^2 + a^2*c)*d)*m)*x^5 + ((3*a^2*b*d + a^3*e)*m^7 + 61
*(3*a^2*b*d + a^3*e)*m^6 + 1525*(3*a^2*b*d + a^3*e)*m^5 + 20065*(3*a^2*b*d
+ a^3*e)*m^4 + 2027025*a^2*b*d + 675675*a^3*e + 147859*(3*a^2*b*d + a^3*e)*
m^3 + 594439*(3*a^2*b*d + a^3*e)*m^2 + 1140855*(3*a^2*b*d + a^3*e)*m)*x^3 +
 (a^3*d*m^7 + 63*a^3*d*m^6 + 1645*a^3*d*m^5 + 22995*a^3*d*m^4 + 185059*a^3*
d*m^3 + 852957*a^3*d*m^2 + 2071215*a^3*d*m + 2027025*a^3*d)*x)*(f*x)^m/(m^8
 + 64*m^7 + 1708*m^6 + 24640*m^5 + 208054*m^4 + 1038016*m^3 + 2924172*m^2 +
 4098240*m + 2027025)
```

giac [**B**] time = 0.75, size = 2816, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((f*x)^m*c^3*m^7*x^15*e + 49*(f*x)^m*c^3*m^6*x^15*e + (f*x)^m*c^3*d*m^7*x^1
3 + 3*(f*x)^m*b*c^2*m^7*x^13*e + 973*(f*x)^m*c^3*m^5*x^15*e + 51*(f*x)^m*c^
3*d*m^6*x^13 + 153*(f*x)^m*b*c^2*m^6*x^13*e + 10045*(f*x)^m*c^3*m^4*x^15*e
+ 3*(f*x)^m*b*c^2*d*m^7*x^11 + 1045*(f*x)^m*c^3*d*m^5*x^13 + 3*(f*x)^m*b^2*
c*m^7*x^11*e + 3*(f*x)^m*a*c^2*m^7*x^11*e + 3135*(f*x)^m*b*c^2*m^5*x^13*e +
 57379*(f*x)^m*c^3*m^3*x^15*e + 159*(f*x)^m*b*c^2*d*m^6*x^11 + 11055*(f*x)^
```

m*c^3*d*m^4*x^13 + 159*(f*x)^m*b^2*c*m^6*x^11*e + 159*(f*x)^m*a*c^2*m^6*x^1
1*e + 33165*(f*x)^m*b*c^2*m^4*x^13*e + 177331*(f*x)^m*c^3*m^2*x^15*e + 3*(f
*x)^m*b^2*c*d*m^7*x^9 + 3*(f*x)^m*a*c^2*d*m^7*x^9 + 3375*(f*x)^m*b*c^2*d*m^
5*x^11 + 64339*(f*x)^m*c^3*d*m^3*x^13 + (f*x)^m*b^3*m^7*x^9*e + 6*(f*x)^m*a
*b*c*m^7*x^9*e + 3375*(f*x)^m*b^2*c*m^5*x^11*e + 3375*(f*x)^m*a*c^2*m^5*x^1
1*e + 193017*(f*x)^m*b*c^2*m^3*x^13*e + 264207*(f*x)^m*c^3*m*x^15*e + 165*(
f*x)^m*b^2*c*d*m^6*x^9 + 165*(f*x)^m*a*c^2*d*m^6*x^9 + 36795*(f*x)^m*b*c^2*
d*m^4*x^11 + 201609*(f*x)^m*c^3*d*m^2*x^13 + 55*(f*x)^m*b^3*m^6*x^9*e + 330
*(f*x)^m*a*b*c*m^6*x^9*e + 36795*(f*x)^m*b^2*c*m^4*x^11*e + 36795*(f*x)^m*a
*c^2*m^4*x^11*e + 604827*(f*x)^m*b*c^2*m^2*x^13*e + 135135*(f*x)^m*c^3*x^15
*e + (f*x)^m*b^3*d*m^7*x^7 + 6*(f*x)^m*a*b*c*d*m^7*x^7 + 3639*(f*x)^m*b^2*c
*d*m^5*x^9 + 3639*(f*x)^m*a*c^2*d*m^5*x^9 + 219417*(f*x)^m*b*c^2*d*m^3*x^11
 + 303255*(f*x)^m*c^3*d*m*x^13 + 3*(f*x)^m*a*b^2*m^7*x^7*e + 3*(f*x)^m*a^2*
c*m^7*x^7*e + 1213*(f*x)^m*b^3*m^5*x^9*e + 7278*(f*x)^m*a*b*c*m^5*x^9*e + 2
19417*(f*x)^m*b^2*c*m^3*x^11*e + 219417*(f*x)^m*a*c^2*m^3*x^11*e + 909765*(
f*x)^m*b*c^2*m*x^13*e + 57*(f*x)^m*b^3*d*m^6*x^7 + 342*(f*x)^m*a*b*c*d*m^6*
x^7 + 41169*(f*x)^m*b^2*c*d*m^4*x^9 + 41169*(f*x)^m*a*c^2*d*m^4*x^9 + 70046
1*(f*x)^m*b*c^2*d*m^2*x^11 + 155925*(f*x)^m*c^3*d*x^13 + 171*(f*x)^m*a*b^2*
m^6*x^7*e + 171*(f*x)^m*a^2*c*m^6*x^7*e + 13723*(f*x)^m*b^3*m^4*x^9*e + 823
38*(f*x)^m*a*b*c*m^4*x^9*e + 700461*(f*x)^m*b^2*c*m^2*x^11*e + 700461*(f*x)
^m*a*c^2*m^2*x^11*e + 467775*(f*x)^m*b*c^2*x^13*e + 3*(f*x)^m*a*b^2*d*m^7*x
^5 + 3*(f*x)^m*a^2*c*d*m^7*x^5 + 1309*(f*x)^m*b^3*d*m^5*x^7 + 7854*(f*x)^m*
a*b*c*d*m^5*x^7 + 253641*(f*x)^m*b^2*c*d*m^3*x^9 + 253641*(f*x)^m*a*c^2*d*m
^3*x^9 + 1067445*(f*x)^m*b*c^2*d*m*x^11 + 3*(f*x)^m*a^2*b*m^7*x^5*e + 3927*
(f*x)^m*a*b^2*m^5*x^7*e + 3927*(f*x)^m*a^2*c*m^5*x^7*e + 84547*(f*x)^m*b^3*
m^3*x^9*e + 507282*(f*x)^m*a*b*c*m^3*x^9*e + 1067445*(f*x)^m*b^2*c*m*x^11*e
 + 1067445*(f*x)^m*a*c^2*m*x^11*e + 177*(f*x)^m*a*b^2*d*m^6*x^5 + 177*(f*x)
^m*a^2*c*d*m^6*x^5 + 15477*(f*x)^m*b^3*d*m^4*x^7 + 92862*(f*x)^m*a*b*c*d*m^
4*x^7 + 831279*(f*x)^m*b^2*c*d*m^2*x^9 + 831279*(f*x)^m*a*c^2*d*m^2*x^9 + 5
52825*(f*x)^m*b*c^2*d*x^11 + 177*(f*x)^m*a^2*b*m^6*x^5*e + 46431*(f*x)^m*a*
b^2*m^4*x^7*e + 46431*(f*x)^m*a^2*c*m^4*x^7*e + 277093*(f*x)^m*b^3*m^2*x^9*
e + 1662558*(f*x)^m*a*b*c*m^2*x^9*e + 552825*(f*x)^m*b^2*c*x^11*e + 552825*
(f*x)^m*a*c^2*x^11*e + 3*(f*x)^m*a^2*b*d*m^7*x^3 + 4239*(f*x)^m*a*b^2*d*m^5
x^5 + 4239(f*x)^m*a^2*c*d*m^5*x^5 + 99715*(f*x)^m*b^3*d*m^3*x^7 + 598290*
(f*x)^m*a*b*c*d*m^3*x^7 + 1291005*(f*x)^m*b^2*c*d*m*x^9 + 1291005*(f*x)^m*a
*c^2*d*m*x^9 + (f*x)^m*a^3*m^7*x^3*e + 4239*(f*x)^m*a^2*b*m^5*x^5*e + 29914
5*(f*x)^m*a*b^2*m^3*x^7*e + 299145*(f*x)^m*a^2*c*m^3*x^7*e + 430335*(f*x)^m
*b^3*m*x^9*e + 2582010*(f*x)^m*a*b*c*m*x^9*e + 183*(f*x)^m*a^2*b*d*m^6*x^3
 + 52725*(f*x)^m*a*b^2*d*m^4*x^5 + 52725*(f*x)^m*a^2*c*d*m^4*x^5 + 340011*(f
*x)^m*b^3*d*m^2*x^7 + 2040066*(f*x)^m*a*b*c*d*m^2*x^7 + 675675*(f*x)^m*b^2*
c*d*x^9 + 675675*(f*x)^m*a*c^2*d*x^9 + 61*(f*x)^m*a^3*m^6*x^3*e + 52725*(f*
x)^m*a^2*b*m^4*x^5*e + 1020033*(f*x)^m*a*b^2*m^2*x^7*e + 1020033*(f*x)^m*a^
2*c*m^2*x^7*e + 225225*(f*x)^m*b^3*x^9*e + 1351350*(f*x)^m*a*b*c*x^9*e + (f
*x)^m*a^3*d*m^7*x + 4575*(f*x)^m*a^2*b*d*m^5*x^3 + 360537*(f*x)^m*a*b^2*d*m
^3*x^5 + 360537*(f*x)^m*a^2*c*d*m^3*x^5 + 544095*(f*x)^m*b^3*d*m*x^7 + 3264
570*(f*x)^m*a*b*c*d*m*x^7 + 1525*(f*x)^m*a^3*m^5*x^3*e + 360537*(f*x)^m*a^2
*b*m^3*x^5*e + 1632285*(f*x)^m*a*b^2*m*x^7*e + 1632285*(f*x)^m*a^2*c*m*x^7*
e + 63*(f*x)^m*a^3*d*m^6*x + 60195*(f*x)^m*a^2*b*d*m^4*x^3 + 1311363*(f*x)^
m*a*b^2*d*m^2*x^5 + 1311363*(f*x)^m*a^2*c*d*m^2*x^5 + 289575*(f*x)^m*b^3*d*
x^7 + 1737450*(f*x)^m*a*b*c*d*x^7 + 20065*(f*x)^m*a^3*m^4*x^3*e + 1311363*(
f*x)^m*a^2*b*m^2*x^5*e + 868725*(f*x)^m*a*b^2*x^7*e + 868725*(f*x)^m*a^2*c*
x^7*e + 1645*(f*x)^m*a^3*d*m^5*x + 443577*(f*x)^m*a^2*b*d*m^3*x^3 + 2215701
*(f*x)^m*a*b^2*d*m*x^5 + 2215701*(f*x)^m*a^2*c*d*m*x^5 + 147859*(f*x)^m*a^3
*m^3*x^3*e + 2215701*(f*x)^m*a^2*b*m*x^5*e + 22995*(f*x)^m*a^3*d*m^4*x + 17
83317*(f*x)^m*a^2*b*d*m^2*x^3 + 1216215*(f*x)^m*a*b^2*d*x^5 + 1216215*(f*x)
^m*a^2*c*d*x^5 + 594439*(f*x)^m*a^3*m^2*x^3*e + 1216215*(f*x)^m*a^2*b*x^5*e
 + 185059*(f*x)^m*a^3*d*m^3*x + 3422565*(f*x)^m*a^2*b*d*m*x^3 + 1140855*(f*
x)^m*a^3*m*x^3*e + 852957*(f*x)^m*a^3*d*m^2*x + 2027025*(f*x)^m*a^2*b*d*x^3
 + 675675*(f*x)^m*a^3*x^3*e + 2071215*(f*x)^m*a^3*d*m*x + 2027025*(f*x)^m*a

```
^3*d*x)/(m^8 + 64*m^7 + 1708*m^6 + 24640*m^5 + 208054*m^4 + 1038016*m^3 + 2
924172*m^2 + 4098240*m + 2027025)
```

maple [B] time = 0.01, size = 1935, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] x*(c^3*e*m^7*x^14+49*c^3*e*m^6*x^14+3*b*c^2*e*m^7*x^12+c^3*d*m^7*x^12+973*c
^3*e*m^5*x^14+153*b*c^2*e*m^6*x^12+51*c^3*d*m^6*x^12+10045*c^3*e*m^4*x^14+3
*a*c^2*e*m^7*x^10+3*b^2*c*e*m^7*x^10+3*b*c^2*d*m^7*x^10+3135*b*c^2*e*m^5*x^
12+1045*c^3*d*m^5*x^12+57379*c^3*e*m^3*x^14+159*a*c^2*e*m^6*x^10+159*b^2*c*
e*m^6*x^10+159*b*c^2*d*m^6*x^10+33165*b*c^2*e*m^4*x^12+11055*c^3*d*m^4*x^12
+177331*c^3*e*m^2*x^14+6*a*b*c*e*m^7*x^8+3*a*c^2*d*m^7*x^8+3375*a*c^2*e*m^5
*x^10+b^3*e*m^7*x^8+3*b^2*c*d*m^7*x^8+3375*b^2*c*e*m^5*x^10+3375*b*c^2*d*m^
5*x^10+193017*b*c^2*e*m^3*x^12+64339*c^3*d*m^3*x^12+264207*c^3*e*m*x^14+330
*a*b*c*e*m^6*x^8+165*a*c^2*d*m^6*x^8+36795*a*c^2*e*m^4*x^10+55*b^3*e*m^6*x^
8+165*b^2*c*d*m^6*x^8+36795*b^2*c*e*m^4*x^10+36795*b*c^2*d*m^4*x^10+604827*
b*c^2*e*m^2*x^12+201609*c^3*d*m^2*x^12+135135*c^3*e*x^14+3*a^2*c*e*m^7*x^6+
3*a*b^2*e*m^7*x^6+6*a*b*c*d*m^7*x^6+7278*a*b*c*e*m^5*x^8+3639*a*c^2*d*m^5*x
^8+219417*a*c^2*e*m^3*x^10+b^3*d*m^7*x^6+1213*b^3*e*m^5*x^8+3639*b^2*c*d*m^
5*x^8+219417*b^2*c*e*m^3*x^10+219417*b*c^2*d*m^3*x^10+909765*b*c^2*e*m*x^12
+303255*c^3*d*m*x^12+171*a^2*c*e*m^6*x^6+171*a*b^2*e*m^6*x^6+342*a*b*c*d*m^
6*x^6+82338*a*b*c*e*m^4*x^8+41169*a*c^2*d*m^4*x^8+700461*a*c^2*e*m^2*x^10+5
7*b^3*d*m^6*x^6+13723*b^3*e*m^4*x^8+41169*b^2*c*d*m^4*x^8+700461*b^2*c*e*m^
2*x^10+700461*b*c^2*d*m^2*x^10+467775*b*c^2*e*x^12+155925*c^3*d*x^12+3*a^2*
b*e*m^7*x^4+3*a^2*c*d*m^7*x^4+3927*a^2*c*e*m^5*x^6+3*a*b^2*d*m^7*x^4+3927*a
*b^2*e*m^5*x^6+7854*a*b*c*d*m^5*x^6+507282*a*b*c*e*m^3*x^8+253641*a*c^2*d*m
^3*x^8+1067445*a*c^2*e*m*x^10+1309*b^3*d*m^5*x^6+84547*b^3*e*m^3*x^8+253641
*b^2*c*d*m^3*x^8+1067445*b^2*c*e*m*x^10+1067445*b*c^2*d*m*x^10+177*a^2*b*e*
m^6*x^4+177*a^2*c*d*m^6*x^4+46431*a^2*c*e*m^4*x^6+177*a*b^2*d*m^6*x^4+46431
*a*b^2*e*m^4*x^6+92862*a*b*c*d*m^4*x^6+1662558*a*b*c*e*m^2*x^8+831279*a*c^2
*d*m^2*x^8+552825*a*c^2*e*x^10+15477*b^3*d*m^4*x^6+277093*b^3*e*m^2*x^8+831
279*b^2*c*d*m^2*x^8+552825*b^2*c*e*x^10+552825*b*c^2*d*x^10+a^3*e*m^7*x^2+3
*a^2*b*d*m^7*x^2+4239*a^2*b*e*m^5*x^4+4239*a^2*c*d*m^5*x^4+299145*a^2*c*e*m
^3*x^6+4239*a*b^2*d*m^5*x^4+299145*a*b^2*e*m^3*x^6+598290*a*b*c*d*m^3*x^6+2
582010*a*b*c*e*m*x^8+1291005*a*c^2*d*m*x^8+99715*b^3*d*m^3*x^6+430335*b^3*e
*m*x^8+1291005*b^2*c*d*m*x^8+61*a^3*e*m^6*x^2+183*a^2*b*d*m^6*x^2+52725*a^2
*b*e*m^4*x^4+52725*a^2*c*d*m^4*x^4+1020033*a^2*c*e*m^2*x^6+52725*a*b^2*d*m^
4*x^4+1020033*a*b^2*e*m^2*x^6+2040066*a*b*c*d*m^2*x^6+1351350*a*b*c*e*x^8+6
75675*a*c^2*d*x^8+340011*b^3*d*m^2*x^6+225225*b^3*e*x^8+675675*b^2*c*d*x^8+
a^3*d*m^7+1525*a^3*e*m^5*x^2+4575*a^2*b*d*m^5*x^2+360537*a^2*b*e*m^3*x^4+36
0537*a^2*c*d*m^3*x^4+1632285*a^2*c*e*m*x^6+360537*a*b^2*d*m^3*x^4+1632285*a
*b^2*e*m*x^6+3264570*a*b*c*d*m*x^6+544095*b^3*d*m*x^6+63*a^3*d*m^6+20065*a^
3*e*m^4*x^2+60195*a^2*b*d*m^4*x^2+1311363*a^2*b*e*m^2*x^4+1311363*a^2*c*d*m
^2*x^4+868725*a^2*c*e*x^6+1311363*a*b^2*d*m^2*x^4+868725*a*b^2*e*x^6+173745
0*a*b*c*d*x^6+289575*b^3*d*x^6+1645*a^3*d*m^5+147859*a^3*e*m^3*x^2+443577*a
^2*b*d*m^3*x^2+2215701*a^2*b*e*m*x^4+2215701*a^2*c*d*m*x^4+2215701*a*b^2*d*
m*x^4+22995*a^3*d*m^4+594439*a^3*e*m^2*x^2+1783317*a^2*b*d*m^2*x^2+1216215*
a^2*b*e*x^4+1216215*a^2*c*d*x^4+1216215*a*b^2*d*x^4+185059*a^3*d*m^3+114085
5*a^3*e*m*x^2+3422565*a^2*b*d*m*x^2+852957*a^3*d*m^2+675675*a^3*e*x^2+20270
25*a^2*b*d*x^2+2071215*a^3*d*m+2027025*a^3*d)*(f*x)^m/(m+1)/(m+3)/(m+5)/(m+
7)/(m+9)/(m+11)/(m+13)/(m+15)
```

maxima [A] time = 1.39, size = 408, normalized size = 1.68

$$\frac{c^3ef^mx^{15}x^m}{m+15}+\frac{c^3df^mx^{13}x^m}{m+13}+\frac{3\,bc^2ef^mx^{13}x^m}{m+13}+\frac{3\,bc^2df^mx^{11}x^m}{m+11}+\frac{3\,b^2cef^mx^{11}x^m}{m+11}+\frac{3\,ac^2ef^mx^{11}x^m}{m+11}+\frac{3\,b^2cdf^mx^9x^n}{m+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] c^3*e*f^m*x^15*x^m/(m + 15) + c^3*d*f^m*x^13*x^m/(m + 13) + 3*b*c^2*e*f^m*x
^13*x^m/(m + 13) + 3*b*c^2*d*f^m*x^11*x^m/(m + 11) + 3*b^2*c*e*f^m*x^11*x^m
/(m + 11) + 3*a*c^2*e*f^m*x^11*x^m/(m + 11) + 3*b^2*c*d*f^m*x^9*x^m/(m + 9)
 + 3*a*c^2*d*f^m*x^9*x^m/(m + 9) + b^3*e*f^m*x^9*x^m/(m + 9) + 6*a*b*c*e*f^
m*x^9*x^m/(m + 9) + b^3*d*f^m*x^7*x^m/(m + 7) + 6*a*b*c*d*f^m*x^7*x^m/(m +
7) + 3*a*b^2*e*f^m*x^7*x^m/(m + 7) + 3*a^2*c*e*f^m*x^7*x^m/(m + 7) + 3*a*b^
2*d*f^m*x^5*x^m/(m + 5) + 3*a^2*c*d*f^m*x^5*x^m/(m + 5) + 3*a^2*b*e*f^m*x^5
*x^m/(m + 5) + 3*a^2*b*d*f^m*x^3*x^m/(m + 3) + a^3*e*f^m*x^3*x^m/(m + 3) +
(f*x)^(m + 1)*a^3*d/(f*(m + 1))
```

mupad [B] time = 1.06, size = 769, normalized size = 3.16

$$\frac{x^7\left(f\,x\right)^m\,\left(3\,c\,e\,a^2+3\,e\,a\,b^2+6\,c\,d\,a\,b+d\,b^3\right)\,\left(m^7+57\,m^6+1309\,m^5+15477\,m^4+99715\,m^3+340011\,m^2+54\right.}{m^8+64\,m^7+1708\,m^6+24640\,m^5+208054\,m^4+1038016\,m^3+2924172\,m^2+4098240\,m+202}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^7*(f*x)^m*(b^3*d + 3*a*b^2*e + 3*a^2*c*e + 6*a*b*c*d)*(544095*m + 340011
*m^2 + 99715*m^3 + 15477*m^4 + 1309*m^5 + 57*m^6 + m^7 + 289575))/(4098240*
m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7
+ m^8 + 2027025) + (x^9*(f*x)^m*(b^3*e + 3*a*c^2*d + 3*b^2*c*d + 6*a*b*c*e)
*(430335*m + 277093*m^2 + 84547*m^3 + 13723*m^4 + 1213*m^5 + 55*m^6 + m^7 +
 225225))/(4098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 +
 1708*m^6 + 64*m^7 + m^8 + 2027025) + (a^3*d*x*(f*x)^m*(2071215*m + 852957*
m^2 + 185059*m^3 + 22995*m^4 + 1645*m^5 + 63*m^6 + m^7 + 2027025))/(4098240
*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7
 + m^8 + 2027025) + (c^3*e*x^15*(f*x)^m*(264207*m + 177331*m^2 + 57379*m^3
+ 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135))/(4098240*m + 2924172*m^2 +
1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) +
 (3*a*x^5*(f*x)^m*(b^2*d + a*b*e + a*c*d)*(738567*m + 437121*m^2 + 120179*m
^3 + 17575*m^4 + 1413*m^5 + 59*m^6 + m^7 + 405405))/(4098240*m + 2924172*m^
2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 202702
5) + (3*c*x^11*(f*x)^m*(b^2*e + a*c*e + b*c*d)*(355815*m + 233487*m^2 + 731
39*m^3 + 12265*m^4 + 1125*m^5 + 53*m^6 + m^7 + 184275))/(4098240*m + 292417
2*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 20
27025) + (a^2*x^3*(f*x)^m*(a*e + 3*b*d)*(1140855*m + 594439*m^2 + 147859*m^
3 + 20065*m^4 + 1525*m^5 + 61*m^6 + m^7 + 675675))/(4098240*m + 2924172*m^2
 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025
) + (c^2*x^13*(f*x)^m*(3*b*e + c*d)*(303255*m + 201609*m^2 + 64339*m^3 + 11
055*m^4 + 1045*m^5 + 51*m^6 + m^7 + 155925))/(4098240*m + 2924172*m^2 + 103
8016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025)
```

sympy [A] time = 12.38, size = 11538, normalized size = 47.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Piecewise(((-a**3*d/(14*x**14) - a**3*e/(12*x**12) - a**2*b*d/(4*x**12) - 3
*a**2*b*e/(10*x**10) - 3*a**2*c*d/(10*x**10) - 3*a**2*c*e/(8*x**8) - 3*a*b*
*2*d/(10*x**10) - 3*a*b**2*e/(8*x**8) - 3*a*b*c*d/(4*x**8) - a*b*c*e/x**6 -
 a*c**2*d/(2*x**6) - 3*a*c**2*e/(4*x**4) - b**3*d/(8*x**8) - b**3*e/(6*x**6
) - b**2*c*d/(2*x**6) - 3*b**2*c*e/(4*x**4) - 3*b*c**2*d/(4*x**4) - 3*b*c**
2*e/(2*x**2) - c**3*d/(2*x**2) + c**3*e*log(x))/f**15, Eq(m, -15)), ((-a**3
```

```
*d/(12*x**12) - a**3*e/(10*x**10) - 3*a**2*b*d/(10*x**10) - 3*a**2*b*e/(8*x
**8) - 3*a**2*c*d/(8*x**8) - a**2*c*e/(2*x**6) - 3*a*b**2*d/(8*x**8) - a*b*
*2*e/(2*x**6) - a*b*c*d/x**6 - 3*a*b*c*e/(2*x**4) - 3*a*c**2*d/(4*x**4) - 3
*a*c**2*e/(2*x**2) - b**3*d/(6*x**6) - b**3*e/(4*x**4) - 3*b**2*c*d/(4*x**4
) - 3*b**2*c*e/(2*x**2) - 3*b*c**2*d/(2*x**2) + 3*b*c**2*e*log(x) + c**3*d*
log(x) + c**3*e*x**2/2)/f**13, Eq(m, -13)), ((-a**3*d/(10*x**10) - a**3*e/(
8*x**8) - 3*a**2*b*d/(8*x**8) - a**2*b*e/(2*x**6) - a**2*c*d/(2*x**6) - 3*a
**2*c*e/(4*x**4) - a*b**2*d/(2*x**6) - 3*a*b**2*e/(4*x**4) - 3*a*b*c*d/(2*x
**4) - 3*a*b*c*e/x**2 - 3*a*c**2*d/(2*x**2) + 3*a*c**2*e*log(x) - b**3*d/(4
*x**4) - b**3*e/(2*x**2) - 3*b**2*c*d/(2*x**2) + 3*b**2*c*e*log(x) + 3*b*c*
*2*d*log(x) + 3*b*c**2*e*x**2/2 + c**3*d*x**2/2 + c**3*e*x**4/4)/f**11, Eq(
m, -11)), ((-a**3*d/(8*x**8) - a**3*e/(6*x**6) - a**2*b*d/(2*x**6) - 3*a**2
*b*e/(4*x**4) - 3*a**2*c*d/(4*x**4) - 3*a**2*c*e/(2*x**2) - 3*a*b**2*d/(4*x
**4) - 3*a*b**2*e/(2*x**2) - 3*a*b*c*d/x**2 + 6*a*b*c*e*log(x) + 3*a*c**2*d
*log(x) + 3*a*c**2*e*x**2/2 - b**3*d/(2*x**2) + b**3*e*log(x) + 3*b**2*c*d*
log(x) + 3*b**2*c*e*x**2/2 + 3*b*c**2*d*x**2/2 + 3*b*c**2*e*x**4/4 + c**3*d
*x**4/4 + c**3*e*x**6/6)/f**9, Eq(m, -9)), ((-a**3*d/(6*x**6) - a**3*e/(4*x
**4) - 3*a**2*b*d/(4*x**4) - 3*a**2*b*e/(2*x**2) - 3*a**2*c*d/(2*x**2) + 3*
a**2*c*e*log(x) - 3*a*b**2*d/(2*x**2) + 3*a*b**2*e*log(x) + 6*a*b*c*d*log(x
) + 3*a*b*c*e*x**2 + 3*a*c**2*d*x**2/2 + 3*a*c**2*e*x**4/4 + b**3*d*log(x)
+ b**3*e*x**2/2 + 3*b**2*c*d*x**2/2 + 3*b**2*c*e*x**4/4 + 3*b*c**2*d*x**4/4
 + b*c**2*e*x**6/2 + c**3*d*x**6/6 + c**3*e*x**8/8)/f**7, Eq(m, -7)), ((-a*
*3*d/(4*x**4) - a**3*e/(2*x**2) - 3*a**2*b*d/(2*x**2) + 3*a**2*b*e*log(x) +
 3*a**2*c*d*log(x) + 3*a**2*c*e*x**2/2 + 3*a*b**2*d*log(x) + 3*a*b**2*e*x**
2/2 + 3*a*b*c*d*x**2 + 3*a*b*c*e*x**4/2 + 3*a*c**2*d*x**4/4 + a*c**2*e*x**6
/2 + b**3*d*x**2/2 + b**3*e*x**4/4 + 3*b**2*c*d*x**4/4 + b**2*c*e*x**6/2 +
b*c**2*d*x**6/2 + 3*b*c**2*e*x**8/8 + c**3*d*x**8/8 + c**3*e*x**10/10)/f**5
, Eq(m, -5)), ((-a**3*d/(2*x**2) + a**3*e*log(x) + 3*a**2*b*d*log(x) + 3*a*
*2*b*e*x**2/2 + 3*a**2*c*d*x**2/2 + 3*a**2*c*e*x**4/4 + 3*a*b**2*d*x**2/2 +
 3*a*b**2*e*x**4/4 + 3*a*b*c*d*x**4/2 + a*b*c*e*x**6 + a*c**2*d*x**6/2 + 3*
a*c**2*e*x**8/8 + b**3*d*x**4/4 + b**3*e*x**6/6 + b**2*c*d*x**6/2 + 3*b**2*
c*e*x**8/8 + 3*b*c**2*d*x**8/8 + 3*b*c**2*e*x**10/10 + c**3*d*x**10/10 + c*
*3*e*x**12/12)/f**3, Eq(m, -3)), ((a**3*d*log(x) + a**3*e*x**2/2 + 3*a**2*b
*d*x**2/2 + 3*a**2*b*e*x**4/4 + 3*a**2*c*d*x**4/4 + a**2*c*e*x**6/2 + 3*a*b
**2*d*x**4/4 + a*b**2*e*x**6/2 + a*b*c*d*x**6 + 3*a*b*c*e*x**8/4 + 3*a*c**2
*d*x**8/8 + 3*a*c**2*e*x**10/10 + b**3*d*x**6/6 + b**3*e*x**8/8 + 3*b**2*c*
d*x**8/8 + 3*b**2*c*e*x**10/10 + 3*b*c**2*d*x**10/10 + b*c**2*e*x**12/4 + c
**3*d*x**12/12 + c**3*e*x**14/14)/f, Eq(m, -1)), (a**3*d*f**m*m**7*x*x**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924
172*m**2 + 4098240*m + 2027025) + 63*a**3*d*f**m*m**6*x*x**m/(m**8 + 64*m**
7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40
98240*m + 2027025) + 1645*a**3*d*f**m*m**5*x*x**m/(m**8 + 64*m**7 + 1708*m*
*6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 22995*a**3*d*f**m*m**4*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
185059*a**3*d*f**m*m**3*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 852957*a*
*3*d*f**m*m**2*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2071215*a**3*d*f**
m*m*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016
*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2027025*a**3*d*f**m*x*x**m/(m
**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29241
72*m**2 + 4098240*m + 2027025) + a**3*e*f**m*m**7*x**3*x**m/(m**8 + 64*m**7
 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409
8240*m + 2027025) + 61*a**3*e*f**m*m**6*x**3*x**m/(m**8 + 64*m**7 + 1708*m*
*6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 1525*a**3*e*f**m*m**5*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 20065*a**3*e*f**m*m**4*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
```

```
 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 14785
9*a**3*e*f**m*m**3*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208
054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 594439*a**3
*e*f**m*m**2*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1140855*a**3*e*f*
*m*m*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103
8016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 675675*a**3*e*f**m*x**3*x
**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*b*d*f**m*m**7*x**3*x**m/(m**8
 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*
m**2 + 4098240*m + 2027025) + 183*a**2*b*d*f**m*m**6*x**3*x**m/(m**8 + 64*m
**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 4575*a**2*b*d*f**m*m**5*x**3*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 60195*a**2*b*d*f**m*m**4*x**3*x**m/(m**8 + 64*m**7 + 1708*
m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
 2027025) + 443577*a**2*b*d*f**m*m**3*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6
 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202
7025) + 1783317*a**2*b*d*f**m*m**2*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 3422565*a**2*b*d*f**m*m*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2
027025*a**2*b*d*f**m*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*b*
e*f**m*m**7*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 177*a**2*b*e*f**m*
m**6*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103
8016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 4239*a**2*b*e*f**m*m**5*x
**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m
**3 + 2924172*m**2 + 4098240*m + 2027025) + 52725*a**2*b*e*f**m*m**4*x**5*x
**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 360537*a**2*b*e*f**m*m**3*x**5*x**m/
(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292
4172*m**2 + 4098240*m + 2027025) + 1311363*a**2*b*e*f**m*m**2*x**5*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 2215701*a**2*b*e*f**m*m*x**5*x**m/(m**8 + 6
4*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
 + 4098240*m + 2027025) + 1216215*a**2*b*e*f**m*x**5*x**m/(m**8 + 64*m**7 +
 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 3*a**2*c*d*f**m*m**7*x**5*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 177*a**2*c*d*f**m*m**6*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 4239*a**2*c*d*f**m*m**5*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 5272
5*a**2*c*d*f**m*m**4*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 360537*a*
*2*c*d*f**m*m**3*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805
4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1311363*a**2*
c*d*f**m*m**2*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m
**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2215701*a**2*c*d
*f**m*m*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 +
1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1216215*a**2*c*d*f**m*
x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*
m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*c*e*f**m*m**7*x**7*x**m
/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29
24172*m**2 + 4098240*m + 2027025) + 171*a**2*c*e*f**m*m**6*x**7*x**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
**2 + 4098240*m + 2027025) + 3927*a**2*c*e*f**m*m**5*x**7*x**m/(m**8 + 64*m
```

7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 46431*a**2*c*e*f**m*m**4*x**7*x**m/(m**8 + 64*m**7 +
 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 299145*a**2*c*e*f**m*m**3*x**7*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
 + 2027025) + 1020033*a**2*c*e*f**m*m**2*x**7*x**m/(m**8 + 64*m**7 + 1708*m
6 + 24640*m5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 1632285*a**2*c*e*f**m*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 868725*a**2*c*e*f**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a*
b**2*d*f**m*m**7*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805
4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 177*a*b**2*d*
f**m*m**6*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
+ 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 4239*a*b**2*d*f**m*m
5*x5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038
016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 52725*a*b**2*d*f**m*m**4*x
5*xm/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m
3 + 2924172*m2 + 4098240*m + 2027025) + 360537*a*b**2*d*f**m*m**3*x**5*
x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3
+ 2924172*m**2 + 4098240*m + 2027025) + 1311363*a*b**2*d*f**m*m**2*x**5*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2
924172*m**2 + 4098240*m + 2027025) + 2215701*a*b**2*d*f**m*m*x**5*x**m/(m**
8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172
*m**2 + 4098240*m + 2027025) + 1216215*a*b**2*d*f**m*x**5*x**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 3*a*b**2*e*f**m*m**7*x**7*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
 + 2027025) + 171*a*b**2*e*f**m*m**6*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6
+ 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027
025) + 3927*a*b**2*e*f**m*m**5*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 2464
0*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
 46431*a*b**2*e*f**m*m**4*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2991
45*a*b**2*e*f**m*m**3*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1020033*
a*b**2*e*f**m*m**2*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208
054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1632285*a*b
2*e*fm*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 868725*a*b**2*e*f
m*x7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038
016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 6*a*b*c*d*f**m*m**7*x**7*x
m/(m8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 342*a*b*c*d*f**m*m**6*x**7*x**m/(m**
8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172
*m**2 + 4098240*m + 2027025) + 7854*a*b*c*d*f**m*m**5*x**7*x**m/(m**8 + 64*
m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
 4098240*m + 2027025) + 92862*a*b*c*d*f**m*m**4*x**7*x**m/(m**8 + 64*m**7 +
 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 598290*a*b*c*d*f**m*m**3*x**7*x**m/(m**8 + 64*m**7 + 1708
*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
+ 2027025) + 2040066*a*b*c*d*f**m*m**2*x**7*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 3264570*a*b*c*d*f**m*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 1737450*a*b*c*d*f**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 6*a*b*c
*e*f**m*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 330*a*b*c*e*f**m*

m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103
8016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 7278*a*b*c*e*f**m*m**5*x*
*9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m*
*3 + 2924172*m**2 + 4098240*m + 2027025) + 82338*a*b*c*e*f**m*m**4*x**9*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2
924172*m**2 + 4098240*m + 2027025) + 507282*a*b*c*e*f**m*m**3*x**9*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 1662558*a*b*c*e*f**m*m**2*x**9*x**m/(m**8 +
64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + 2582010*a*b*c*e*f**m*m*x**9*x**m/(m**8 + 64*m**
7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40
98240*m + 2027025) + 1351350*a*b*c*e*f**m*x**9*x**m/(m**8 + 64*m**7 + 1708*
m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 3*a*c**2*d*f**m*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 165*a*c**2*d*f**m*m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3639
*a*c**2*d*f**m*m**5*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20
8054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 41169*a*c*
*2*d*f**m*m**4*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*
m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 253641*a*c**2*d
*f**m*m**3*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
+ 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 831279*a*c**2*d*f**
m*m**2*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1
038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1291005*a*c**2*d*f**m*m
*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016
*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 675675*a*c**2*d*f**m*x**9*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2
924172*m**2 + 4098240*m + 2027025) + 3*a*c**2*e*f**m*m**7*x**11*x**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
2 + 4098240*m + 2027025) + 159*a*c2*e*f**m*m**6*x**11*x**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 3375*a*c**2*e*f**m*m**5*x**11*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 36795*a*c**2*e*f**m*m**4*x**11*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
+ 2027025) + 219417*a*c**2*e*f**m*m**3*x**11*x**m/(m**8 + 64*m**7 + 1708*m
6 + 24640*m5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 700461*a*c**2*e*f**m*m**2*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6
+ 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202
7025) + 1067445*a*c**2*e*f**m*m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 552825*a*c**2*e*f**m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + b**3*
d*f**m*m**7*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 57*b**3*d*f**m*m**
6*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1309*b**3*d*f**m*m**5*x**7*x
m/(m8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
2924172*m**2 + 4098240*m + 2027025) + 15477*b**3*d*f**m*m**4*x**7*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 99715*b**3*d*f**m*m**3*x**7*x**m/(m**8 + 64
*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
+ 4098240*m + 2027025) + 340011*b**3*d*f**m*m**2*x**7*x**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 544095*b**3*d*f**m*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m*
*6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 289575*b**3*d*f**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + b

```
**3*e*f**m*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054
*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 55*b**3*e*f**m
*m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1213*b**3*e*f**m*m**5*x*
*9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m*
*3 + 2924172*m**2 + 4098240*m + 2027025) + 13723*b**3*e*f**m*m**4*x**9*x**m
/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29
24172*m**2 + 4098240*m + 2027025) + 84547*b**3*e*f**m*m**3*x**9*x**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
**2 + 4098240*m + 2027025) + 277093*b**3*e*f**m*m**2*x**9*x**m/(m**8 + 64*m
**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 430335*b**3*e*f**m*m*x**9*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
 + 2027025) + 225225*b**3*e*f**m*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
 + 3*b**2*c*d*f**m*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 165*b*
*2*c*d*f**m*m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805
4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3639*b**2*c*d
*f**m*m**5*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 41169*b**2*c*d*f**m
*m**4*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 253641*b**2*c*d*f**m*m**
3*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 831279*b**2*c*d*f**m*m**2*x*
*9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m*
*3 + 2924172*m**2 + 4098240*m + 2027025) + 1291005*b**2*c*d*f**m*m*x**9*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2
924172*m**2 + 4098240*m + 2027025) + 675675*b**2*c*d*f**m*x**9*x**m/(m**8 +
 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + 3*b**2*c*e*f**m*m**7*x**11*x**m/(m**8 + 64*m**7
 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409
8240*m + 2027025) + 159*b**2*c*e*f**m*m**6*x**11*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
 + 2027025) + 3375*b**2*c*e*f**m*m**5*x**11*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 36795*b**2*c*e*f**m*m**4*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 219417*b**2*c*e*f**m*m**3*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 700461*b**2*c*e*f**m*m**2*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1
067445*b**2*c*e*f**m*m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 552825
*b**2*c*e*f**m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054
*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*b*c**2*d*f**
m*m**7*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 +
1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 159*b*c**2*d*f**m*m**6
*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3375*b*c**2*d*f**m*m**5*x**1
1*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 36795*b*c**2*d*f**m*m**4*x**11*x*
*m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
2924172*m**2 + 4098240*m + 2027025) + 219417*b*c**2*d*f**m*m**3*x**11*x**m/
(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292
4172*m**2 + 4098240*m + 2027025) + 700461*b*c**2*d*f**m*m**2*x**11*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 1067445*b*c**2*d*f**m*m*x**11*x**m/(m**8 +
64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**
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2 + 4098240*m + 2027025) + 552825*b*c**2*d*f**m*x**11*x**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 3*b*c**2*e*f**m*m**7*x**13*x**m/(m**8 + 64*m**7 + 1708*m
**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 153*b*c**2*e*f**m*m**6*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 3135*b*c**2*e*f**m*m**5*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
33165*b*c**2*e*f**m*m**4*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1930
17*b*c**2*e*f**m*m**3*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 604827*
b*c**2*e*f**m*m**2*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20
8054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 909765*b*c
**2*e*f**m*m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m
**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 467775*b*c**2*e*
f**m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + c**3*d*f**m*m**7*x**13*x
**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 51*c**3*d*f**m*m**6*x**13*x**m/(m**8
 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*
m**2 + 4098240*m + 2027025) + 1045*c**3*d*f**m*m**5*x**13*x**m/(m**8 + 64*m
**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 11055*c**3*d*f**m*m**4*x**13*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 64339*c**3*d*f**m*m**3*x**13*x**m/(m**8 + 64*m**7 + 1708*m
**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 201609*c**3*d*f**m*m**2*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 +
 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20270
25) + 303255*c**3*d*f**m*m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m
**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 15
5925*c**3*d*f**m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2080
54*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + c**3*e*f**m*
m**7*x**15*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 49*c**3*e*f**m*m**6*x**1
5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 973*c**3*e*f**m*m**5*x**15*x**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924
172*m**2 + 4098240*m + 2027025) + 10045*c**3*e*f**m*m**4*x**15*x**m/(m**8 +
 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + 57379*c**3*e*f**m*m**3*x**15*x**m/(m**8 + 64*m*
*7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4
098240*m + 2027025) + 177331*c**3*e*f**m*m**2*x**15*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 264207*c**3*e*f**m*m*x**15*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 135135*c**3*e*f**m*x**15*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025), Tr
ue))
```

3.221 $\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=155

$$\frac{a^2d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\left(2ace+b^2e+2bcd\right)}{f^7(m+7)}+\frac{(fx)^{m+5}\left(2abe+2acd+b^2d\right)}{f^5(m+5)}+\frac{a(fx)^{m+3}(ae+2bd)}{f^3(m+3)}+\frac{c(fx)^{m+9}(2be}{f^9(m+9}$$

```
[Out] a^2*d*(f*x)^(1+m)/f/(1+m)+a*(a*e+2*b*d)*(f*x)^(3+m)/f^3/(3+m)+(2*a*b*e+2*a*
c*d+b^2*d)*(f*x)^(5+m)/f^5/(5+m)+(2*a*c*e+b^2*e+2*b*c*d)*(f*x)^(7+m)/f^7/(7
+m)+c*(2*b*e+c*d)*(f*x)^(9+m)/f^9/(9+m)+c^2*e*(f*x)^(11+m)/f^11/(11+m)
```

Rubi [A] time = 0.10, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1261}

$$\frac{a^2d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5}\left(2abe+2acd+b^2d\right)}{f^5(m+5)}+\frac{(fx)^{m+7}\left(2ace+b^2e+2bcd\right)}{f^7(m+7)}+\frac{a(fx)^{m+3}(ae+2bd)}{f^3(m+3)}+\frac{c(fx)^{m+9}(2be}{f^9(m+9}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*d*(f*x)^(1 + m))/(f*(1 + m)) + (a*(2*b*d + a*e)*(f*x)^(3 + m))/(f^3*(3
 + m)) + ((b^2*d + 2*a*c*d + 2*a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((2*b*
c*d + b^2*e + 2*a*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + (c*(c*d + 2*b*e)*(f*x
)^(9 + m))/(f^9*(9 + m)) + (c^2*e*(f*x)^(11 + m))/(f^11*(11 + m))
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2 \, dx = \int \left(a^2d(fx)^m+\frac{a(2bd+ae)(fx)^{2+m}}{f^2}+\frac{\left(b^2d+2acd+2abe\right)(fx)^{4+m}}{f^4}\right.$$

$$=\frac{a^2d(fx)^{1+m}}{f(1+m)}+\frac{a(2bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{\left(b^2d+2acd+2abe\right)(fx)^{5+m}}{f^5(5+m)}+$$

Mathematica [A] time = 0.12, size = 117, normalized size = 0.75

$$x(fx)^m\left(\frac{a^2d}{m+1}+\frac{x^6\left(2ace+b^2e+2bcd\right)}{m+7}+\frac{x^4\left(2abe+2acd+b^2d\right)}{m+5}+\frac{ax^2(ae+2bd)}{m+3}+\frac{cx^8(2be+cd)}{m+9}+\frac{c^2ex^{10}}{m+11}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] x*(f*x)^m*((a^2*d)/(1 + m) + (a*(2*b*d + a*e)*x^2)/(3 + m) + ((b^2*d + 2*a*
c*d + 2*a*b*e)*x^4)/(5 + m) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^6)/(7 + m) + (
c*(c*d + 2*b*e)*x^8)/(9 + m) + (c^2*e*x^10)/(11 + m))
```

fricas [B] time = 0.64, size = 573, normalized size = 3.70

$$\frac{\left(\left(c^2em^5+25\,c^2em^4+230\,c^2em^3+950\,c^2em^2+1689\,c^2em+945\,c^2e\right)x^{11}+\left(\left(c^2d+2\,bce\right)m^5+27\,\left(c^2d+2\,bce\right)m\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*e*m^5 + 25*c^2*e*m^4 + 230*c^2*e*m^3 + 950*c^2*e*m^2 + 1689*c^2*e*m +
 945*c^2*e)*x^11 + ((c^2*d + 2*b*c*e)*m^5 + 27*(c^2*d + 2*b*c*e)*m^4 + 262*
(c^2*d + 2*b*c*e)*m^3 + 1155*c^2*d + 2310*b*c*e + 1122*(c^2*d + 2*b*c*e)*m^
2 + 2041*(c^2*d + 2*b*c*e)*m)*x^9 + ((2*b*c*d + (b^2 + 2*a*c)*e)*m^5 + 29*(
2*b*c*d + (b^2 + 2*a*c)*e)*m^4 + 302*(2*b*c*d + (b^2 + 2*a*c)*e)*m^3 + 2970
*b*c*d + 1366*(2*b*c*d + (b^2 + 2*a*c)*e)*m^2 + 1485*(b^2 + 2*a*c)*e + 2577
*(2*b*c*d + (b^2 + 2*a*c)*e)*m)*x^7 + ((2*a*b*e + (b^2 + 2*a*c)*d)*m^5 + 31
*(2*a*b*e + (b^2 + 2*a*c)*d)*m^4 + 350*(2*a*b*e + (b^2 + 2*a*c)*d)*m^3 + 41
58*a*b*e + 1730*(2*a*b*e + (b^2 + 2*a*c)*d)*m^2 + 2079*(b^2 + 2*a*c)*d + 34
89*(2*a*b*e + (b^2 + 2*a*c)*d)*m)*x^5 + ((2*a*b*d + a^2*e)*m^5 + 33*(2*a*b*
d + a^2*e)*m^4 + 406*(2*a*b*d + a^2*e)*m^3 + 6930*a*b*d + 3465*a^2*e + 2262
*(2*a*b*d + a^2*e)*m^2 + 5353*(2*a*b*d + a^2*e)*m)*x^3 + (a^2*d*m^5 + 35*a^
2*d*m^4 + 470*a^2*d*m^3 + 3010*a^2*d*m^2 + 9129*a^2*d*m + 10395*a^2*d)*x)*(
f*x)^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

giac [B] time = 0.40, size = 1178, normalized size = 7.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((f*x)^m*c^2*m^5*x^11*e + 25*(f*x)^m*c^2*m^4*x^11*e + (f*x)^m*c^2*d*m^5*x^9
 + 2*(f*x)^m*b*c*m^5*x^9*e + 230*(f*x)^m*c^2*m^3*x^11*e + 27*(f*x)^m*c^2*d*
m^4*x^9 + 54*(f*x)^m*b*c*m^4*x^9*e + 950*(f*x)^m*c^2*m^2*x^11*e + 2*(f*x)^m
*b*c*d*m^5*x^7 + 262*(f*x)^m*c^2*d*m^3*x^9 + (f*x)^m*b^2*m^5*x^7*e + 2*(f*x
)^m*a*c*m^5*x^7*e + 524*(f*x)^m*b*c*m^3*x^9*e + 1689*(f*x)^m*c^2*m*x^11*e +
 58*(f*x)^m*b*c*d*m^4*x^7 + 1122*(f*x)^m*c^2*d*m^2*x^9 + 29*(f*x)^m*b^2*m^4
*x^7*e + 58*(f*x)^m*a*c*m^4*x^7*e + 2244*(f*x)^m*b*c*m^2*x^9*e + 945*(f*x)^
m*c^2*x^11*e + (f*x)^m*b^2*d*m^5*x^5 + 2*(f*x)^m*a*c*d*m^5*x^5 + 604*(f*x)^
m*b*c*d*m^3*x^7 + 2041*(f*x)^m*c^2*d*m*x^9 + 2*(f*x)^m*a*b*m^5*x^5*e + 302*
(f*x)^m*b^2*m^3*x^7*e + 604*(f*x)^m*a*c*m^3*x^7*e + 4082*(f*x)^m*b*c*m*x^9*
e + 31*(f*x)^m*b^2*d*m^4*x^5 + 62*(f*x)^m*a*c*d*m^4*x^5 + 2732*(f*x)^m*b*c*
d*m^2*x^7 + 1155*(f*x)^m*c^2*d*x^9 + 62*(f*x)^m*a*b*m^4*x^5*e + 1366*(f*x)^
m*b^2*m^2*x^7*e + 2732*(f*x)^m*a*c*m^2*x^7*e + 2310*(f*x)^m*b*c*x^9*e + 2*(
f*x)^m*a*b*d*m^5*x^3 + 350*(f*x)^m*b^2*d*m^3*x^5 + 700*(f*x)^m*a*c*d*m^3*x^
5 + 5154*(f*x)^m*b*c*d*m*x^7 + (f*x)^m*a^2*m^5*x^3*e + 700*(f*x)^m*a*b*m^3*
x^5*e + 2577*(f*x)^m*b^2*m*x^7*e + 5154*(f*x)^m*a*c*m*x^7*e + 66*(f*x)^m*a*
b*d*m^4*x^3 + 1730*(f*x)^m*b^2*d*m^2*x^5 + 3460*(f*x)^m*a*c*d*m^2*x^5 + 297
0*(f*x)^m*b*c*d*x^7 + 33*(f*x)^m*a^2*m^4*x^3*e + 3460*(f*x)^m*a*b*m^2*x^5*e
 + 1485*(f*x)^m*b^2*x^7*e + 2970*(f*x)^m*a*c*x^7*e + (f*x)^m*a^2*d*m^5*x +
812*(f*x)^m*a*b*d*m^3*x^3 + 3489*(f*x)^m*b^2*d*m*x^5 + 6978*(f*x)^m*a*c*d*m
*x^5 + 406*(f*x)^m*a^2*m^3*x^3*e + 6978*(f*x)^m*a*b*m*x^5*e + 35*(f*x)^m*a^
2*d*m^4*x + 4524*(f*x)^m*a*b*d*m^2*x^3 + 2079*(f*x)^m*b^2*d*x^5 + 4158*(f*x
)^m*a*c*d*x^5 + 2262*(f*x)^m*a^2*m^2*x^3*e + 4158*(f*x)^m*a*b*x^5*e + 470*(
f*x)^m*a^2*d*m^3*x + 10706*(f*x)^m*a*b*d*m*x^3 + 5353*(f*x)^m*a^2*m*x^3*e +
 3010*(f*x)^m*a^2*d*m^2*x + 6930*(f*x)^m*a*b*d*x^3 + 3465*(f*x)^m*a^2*x^3*e
 + 9129*(f*x)^m*a^2*d*m*x + 10395*(f*x)^m*a^2*d*x)/(m^6 + 36*m^5 + 505*m^4
+ 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

maple [B] time = 0.01, size = 783, normalized size = 5.05

$$\frac{\left(c^2 e\, m^5 x^{10} + 25 c^2 e\, m^4 x^{10} + 2bce\, m^5 x^8 + c^2 d\, m^5 x^8 + 230 c^2 e\, m^3 x^{10} + 54 bce\, m^4 x^8 + 27 c^2 d\, m^4 x^8 + 950 c^2 e\, m^2 x^{10}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x*(c^2*e*m^5*x^10+25*c^2*e*m^4*x^10+2*b*c*e*m^5*x^8+c^2*d*m^5*x^8+230*c^2*e
*m^3*x^10+54*b*c*e*m^4*x^8+27*c^2*d*m^4*x^8+950*c^2*e*m^2*x^10+2*a*c*e*m^5*
x^6+b^2*e*m^5*x^6+2*b*c*d*m^5*x^6+524*b*c*e*m^3*x^8+262*c^2*d*m^3*x^8+1689*
c^2*e*m*x^10+58*a*c*e*m^4*x^6+29*b^2*e*m^4*x^6+58*b*c*d*m^4*x^6+2244*b*c*e*
m^2*x^8+1122*c^2*d*m^2*x^8+945*c^2*e*x^10+2*a*b*e*m^5*x^4+2*a*c*d*m^5*x^4+6
04*a*c*e*m^3*x^6+b^2*d*m^5*x^4+302*b^2*e*m^3*x^6+604*b*c*d*m^3*x^6+4082*b*c
*e*m*x^8+2041*c^2*d*m*x^8+62*a*b*e*m^4*x^4+62*a*c*d*m^4*x^4+2732*a*c*e*m^2*
x^6+31*b^2*d*m^4*x^4+1366*b^2*e*m^2*x^6+2732*b*c*d*m^2*x^6+2310*b*c*e*x^8+1
155*c^2*d*x^8+a^2*e*m^5*x^2+2*a*b*d*m^5*x^2+700*a*b*e*m^3*x^4+700*a*c*d*m^3
*x^4+5154*a*c*e*m*x^6+350*b^2*d*m^3*x^4+2577*b^2*e*m*x^6+5154*b*c*d*m*x^6+3
3*a^2*e*m^4*x^2+66*a*b*d*m^4*x^2+3460*a*b*e*m^2*x^4+3460*a*c*d*m^2*x^4+2970
*a*c*e*x^6+1730*b^2*d*m^2*x^4+1485*b^2*e*x^6+2970*b*c*d*x^6+a^2*d*m^5+406*a
^2*e*m^3*x^2+812*a*b*d*m^3*x^2+6978*a*b*e*m*x^4+6978*a*c*d*m*x^4+3489*b^2*d
*m*x^4+35*a^2*d*m^4+2262*a^2*e*m^2*x^2+4524*a*b*d*m^2*x^2+4158*a*b*e*x^4+41
58*a*c*d*x^4+2079*b^2*d*x^4+470*a^2*d*m^3+5353*a^2*e*m*x^2+10706*a*b*d*m*x^
2+3010*a^2*d*m^2+3465*a^2*e*x^2+6930*a*b*d*x^2+9129*a^2*d*m+10395*a^2*d)*(f
*x)^m/(m+11)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.22, size = 230, normalized size = 1.48

$$\frac{c^2 e f^m x^{11} x^m}{m+11} + \frac{c^2 d f^m x^9 x^m}{m+9} + \frac{2\,bce f^m x^9 x^m}{m+9} + \frac{2\,bcd f^m x^7 x^m}{m+7} + \frac{b^2 e f^m x^7 x^m}{m+7} + \frac{2\,ace f^m x^7 x^m}{m+7} + \frac{b^2 d f^m x^5 x^m}{m+5} + \frac{2\,acd f^m x^5}{m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] c^2*e*f^m*x^11*x^m/(m + 11) + c^2*d*f^m*x^9*x^m/(m + 9) + 2*b*c*e*f^m*x^9*x
^m/(m + 9) + 2*b*c*d*f^m*x^7*x^m/(m + 7) + b^2*e*f^m*x^7*x^m/(m + 7) + 2*a*
c*e*f^m*x^7*x^m/(m + 7) + b^2*d*f^m*x^5*x^m/(m + 5) + 2*a*c*d*f^m*x^5*x^m/(
m + 5) + 2*a*b*e*f^m*x^5*x^m/(m + 5) + 2*a*b*d*f^m*x^3*x^m/(m + 3) + a^2*e*
f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a^2*d/(f*(m + 1))
```

mupad [B] time = 0.60, size = 429, normalized size = 2.77

$$\frac{x^5 \left(f\,x\right)^m \left(d\,b^2 + 2\,a\,e\,b + 2\,a\,c\,d\right) \left(m^5 + 31\,m^4 + 350\,m^3 + 1730\,m^2 + 3489\,m + 2079\right)}{m^6 + 36\,m^5 + 505\,m^4 + 3480\,m^3 + 12139\,m^2 + 19524\,m + 10395} + \frac{x^7 \left(f\,x\right)^m \left(e\,b^2 + 2\,c\right.}{m^6 + 36\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (x^5*(f*x)^m*(b^2*d + 2*a*b*e + 2*a*c*d)*(3489*m + 1730*m^2 + 350*m^3 + 31*
m^4 + m^5 + 2079))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6
 + 10395) + (x^7*(f*x)^m*(b^2*e + 2*a*c*e + 2*b*c*d)*(2577*m + 1366*m^2 + 3
02*m^3 + 29*m^4 + m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 +
36*m^5 + m^6 + 10395) + (a^2*d*x*(f*x)^m*(9129*m + 3010*m^2 + 470*m^3 + 35*
m^4 + m^5 + 10395))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^
6 + 10395) + (a*x^3*(f*x)^m*(a*e + 2*b*d)*(5353*m + 2262*m^2 + 406*m^3 + 33
*m^4 + m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^
6 + 10395) + (c*x^9*(f*x)^m*(2*b*e + c*d)*(2041*m + 1122*m^2 + 262*m^3 + 27
```

```
*m^4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^
6 + 10395) + (c^2*e*x^11*(f*x)^m*(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5
 + 945))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395)
```

sympy [A] time = 5.44, size = 4190, normalized size = 27.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-a**2*d/(10*x**10) - a**2*e/(8*x**8) - a*b*d/(4*x**8) - a*b*e/(
3*x**6) - a*c*d/(3*x**6) - a*c*e/(2*x**4) - b**2*d/(6*x**6) - b**2*e/(4*x**
4) - b*c*d/(2*x**4) - b*c*e/x**2 - c**2*d/(2*x**2) + c**2*e*log(x))/f**11,
Eq(m, -11)), ((-a**2*d/(8*x**8) - a**2*e/(6*x**6) - a*b*d/(3*x**6) - a*b*e/
(2*x**4) - a*c*d/(2*x**4) - a*c*e/x**2 - b**2*d/(4*x**4) - b**2*e/(2*x**2)
- b*c*d/x**2 + 2*b*c*e*log(x) + c**2*d*log(x) + c**2*e*x**2/2)/f**9, Eq(m,
-9)), ((-a**2*d/(6*x**6) - a**2*e/(4*x**4) - a*b*d/(2*x**4) - a*b*e/x**2 -
a*c*d/x**2 + 2*a*c*e*log(x) - b**2*d/(2*x**2) + b**2*e*log(x) + 2*b*c*d*log
(x) + b*c*e*x**2 + c**2*d*x**2/2 + c**2*e*x**4/4)/f**7, Eq(m, -7)), ((-a**2
*d/(4*x**4) - a**2*e/(2*x**2) - a*b*d/x**2 + 2*a*b*e*log(x) + 2*a*c*d*log(x
) + a*c*e*x**2 + b**2*d*log(x) + b**2*e*x**2/2 + b*c*d*x**2 + b*c*e*x**4/2
+ c**2*d*x**4/4 + c**2*e*x**6/6)/f**5, Eq(m, -5)), ((-a**2*d/(2*x**2) + a**
2*e*log(x) + 2*a*b*d*log(x) + a*b*e*x**2 + a*c*d*x**2 + a*c*e*x**4/2 + b**2
*d*x**2/2 + b**2*e*x**4/4 + b*c*d*x**4/2 + b*c*e*x**6/3 + c**2*d*x**6/6 + c
**2*e*x**8/8)/f**3, Eq(m, -3)), ((a**2*d*log(x) + a**2*e*x**2/2 + a*b*d*x**
2 + a*b*e*x**4/2 + a*c*d*x**4/2 + a*c*e*x**6/3 + b**2*d*x**4/4 + b**2*e*x**
6/6 + b*c*d*x**6/3 + b*c*e*x**8/4 + c**2*d*x**8/8 + c**2*e*x**10/10)/f, Eq(
m, -1)), (a**2*d*f**m*m**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 35*a**2*d*f**m*m**4*x*x**m/(m**6 + 36*m**5
+ 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 470*a**2*d*f**m*m*
*3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 3010*a**2*d*f**m*m**2*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
 + 12139*m**2 + 19524*m + 10395) + 9129*a**2*d*f**m*m*x*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10395*a**2*d*f**
m*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10
395) + a**2*e*f**m*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 33*a**2*e*f**m*m**4*x**3*x**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 406*a**2*e*f**m
*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 2262*a**2*e*f**m*m**2*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 +
3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5353*a**2*e*f**m*m*x**3*x**m/(m
**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3465
*a**2*e*f**m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
+ 19524*m + 10395) + 2*a*b*d*f**m*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4
 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 66*a*b*d*f**m*m**4*x**3*x**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 8
12*a*b*d*f**m*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139
*m**2 + 19524*m + 10395) + 4524*a*b*d*f**m*m**2*x**3*x**m/(m**6 + 36*m**5 +
 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10706*a*b*d*f**m*m*
x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 6930*a*b*d*f**m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 2*a*b*e*f**m*m**5*x**5*x**m/(m**6 + 36*m**5
 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*b*e*f**m*m**
4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
 10395) + 700*a*b*e*f**m*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m
**3 + 12139*m**2 + 19524*m + 10395) + 3460*a*b*e*f**m*m**2*x**5*x**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6978*a*b
*e*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 4158*a*b*e*f**m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
```

80*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*c*d*f**m*m**5*x**5*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*c*
d*f**m*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 700*a*c*d*f**m*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3460*a*c*d*f**m*m**2*x**5*x
m/(m6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 6978*a*c*d*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1213
9*m**2 + 19524*m + 10395) + 4158*a*c*d*f**m*x**5*x**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*c*e*f**m*m**5*x**7*
x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 58*a*c*e*f**m*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 604*a*c*e*f**m*m**3*x**7*x**m/(m**6 + 36*m**5
 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2732*a*c*e*f**m*m
2*x7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 5154*a*c*e*f**m*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m
3 + 12139*m2 + 19524*m + 10395) + 2970*a*c*e*f**m*x**7*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + b**2*d*f**m*m
5*x5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 31*b**2*d*f**m*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 350*b**2*d*f**m*m**3*x**5*x**m/(m**
6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1730*b
2*d*fm*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m*
*2 + 19524*m + 10395) + 3489*b**2*d*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2079*b**2*d*f**m*x**5*x*
*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 b**2*e*f**m*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 29*b**2*e*f**m*m**4*x**7*x**m/(m**6 + 36*m**5 + 5
05*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 302*b**2*e*f**m*m**3*
x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 1366*b**2*e*f**m*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m
3 + 12139*m2 + 19524*m + 10395) + 2577*b**2*e*f**m*m*x**7*x**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1485*b**2*
e*f**m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1952
4*m + 10395) + 2*b*c*d*f**m*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 348
0*m**3 + 12139*m**2 + 19524*m + 10395) + 58*b*c*d*f**m*m**4*x**7*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 604*b*c
*d*f**m*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
 + 19524*m + 10395) + 2732*b*c*d*f**m*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m
4 + 3480*m3 + 12139*m**2 + 19524*m + 10395) + 5154*b*c*d*f**m*m*x**7*x*
*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 2970*b*c*d*f**m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
2 + 19524*m + 10395) + 2*b*c*e*fm*m**5*x**9*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 54*b*c*e*f**m*m**4*x**9*
x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 524*b*c*e*f**m*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 2244*b*c*e*f**m*m**2*x**9*x**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4082*b*c*e*f**m
*m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 2310*b*c*e*f**m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
 + 12139*m**2 + 19524*m + 10395) + c**2*d*f**m*m**5*x**9*x**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 27*c**2*d*f**m*
m**4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 262*c**2*d*f**m*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 1122*c**2*d*f**m*m**2*x**9*x**m/(
m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 204
1*c**2*d*f**m*m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m*
*2 + 19524*m + 10395) + 1155*c**2*d*f**m*x**9*x**m/(m**6 + 36*m**5 + 505*m*
*4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + c**2*e*f**m*m**5*x**11*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +

```
25*c**2*e*f**m*m**4*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 230*c**2*e*f**m*m**3*x**11*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 950*c**2*e*f**m*
m**2*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 1689*c**2*e*f**m*m*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 945*c**2*e*f**m*x**11*x**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395), True))
```

3.222 $\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=83

$$\frac{(fx)^{m+3}(ae+bd)}{f^3(m+3)}+\frac{ad(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5}(be+cd)}{f^5(m+5)}+\frac{ce(fx)^{m+7}}{f^7(m+7)}$$

```
[Out] a*d*(f*x)^(1+m)/f/(1+m)+(a*e+b*d)*(f*x)^(3+m)/f^3/(3+m)+(b*e+c*d)*(f*x)^(5+
m)/f^5/(5+m)+c*e*(f*x)^(7+m)/f^7/(7+m)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1261}

$$\frac{(fx)^{m+3}(ae+bd)}{f^3(m+3)}+\frac{ad(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5}(be+cd)}{f^5(m+5)}+\frac{ce(fx)^{m+7}}{f^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d*(f*x)^(1 + m))/(f*(1 + m)) + ((b*d + a*e)*(f*x)^(3 + m))/(f^3*(3 + m))
 + ((c*d + b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (c*e*(f*x)^(7 + m))/(f^7*(7
+ m))
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right) \, dx &= \int \left(ad(fx)^m+\frac{(bd+ae)(fx)^{2+m}}{f^2}+\frac{(cd+be)(fx)^{4+m}}{f^4}+\frac{ce(fx)^{6+m}}{f^6}\right) \, dx\\ &= \frac{ad(fx)^{1+m}}{f(1+m)}+\frac{(bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{(cd+be)(fx)^{5+m}}{f^5(5+m)}+\frac{ce(fx)^{7+m}}{f^7(7+m)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.71

$$x(fx)^m \left(\frac{x^2(ae+bd)}{m+3}+\frac{ad}{m+1}+\frac{x^4(be+cd)}{m+5}+\frac{cex^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] x*(f*x)^m*((a*d)/(1 + m) + ((b*d + a*e)*x^2)/(3 + m) + ((c*d + b*e)*x^4)/(5
 + m) + (c*e*x^6)/(7 + m))
```

fricas [B] time = 0.91, size = 171, normalized size = 2.06

$$\frac{\left(\left(cem^3+9\,cem^2+23\,cem+15\,ce\right)x^7+\left((cd+be)m^3+11\,(cd+be)m^2+21\,cd+21\,be+31\,(cd+be)m\right)x^5+\left((b\right.\right.}{m^4+16\,m^3+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((c*e*m^3 + 9*c*e*m^2 + 23*c*e*m + 15*c*e)*x^7 + ((c*d + b*e)*m^3 + 11*(c*d
 + b*e)*m^2 + 21*c*d + 21*b*e + 31*(c*d + b*e)*m)*x^5 + ((b*d + a*e)*m^3 +
13*(b*d + a*e)*m^2 + 35*b*d + 35*a*e + 47*(b*d + a*e)*m)*x^3 + (a*d*m^3 + 1
5*a*d*m^2 + 71*a*d*m + 105*a*d)*x)*(f*x)^m/(m^4 + 16*m^3 + 86*m^2 + 176*m +
 105)
```

giac [B] time = 0.42, size = 350, normalized size = 4.22

$$\left(fx\right)^{m}cm^{3}x^{7}e + 9\left(fx\right)^{m}cm^{2}x^{7}e + \left(fx\right)^{m}cdm^{3}x^{5} + \left(fx\right)^{m}bm^{3}x^{5}e + 23\left(fx\right)^{m}cmx^{7}e + 11\left(fx\right)^{m}cdm^{2}x^{5} + 11\left(f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] ((f*x)^m*c*m^3*x^7*e + 9*(f*x)^m*c*m^2*x^7*e + (f*x)^m*c*d*m^3*x^5 + (f*x)^
m*b*m^3*x^5*e + 23*(f*x)^m*c*m*x^7*e + 11*(f*x)^m*c*d*m^2*x^5 + 11*(f*x)^m*
b*m^2*x^5*e + 15*(f*x)^m*c*x^7*e + (f*x)^m*b*d*m^3*x^3 + 31*(f*x)^m*c*d*m*x
^5 + (f*x)^m*a*m^3*x^3*e + 31*(f*x)^m*b*m*x^5*e + 13*(f*x)^m*b*d*m^2*x^3 +
21*(f*x)^m*c*d*x^5 + 13*(f*x)^m*a*m^2*x^3*e + 21*(f*x)^m*b*x^5*e + (f*x)^m*
a*d*m^3*x + 47*(f*x)^m*b*d*m*x^3 + 47*(f*x)^m*a*m*x^3*e + 15*(f*x)^m*a*d*m^
2*x + 35*(f*x)^m*b*d*x^3 + 35*(f*x)^m*a*x^3*e + 71*(f*x)^m*a*d*m*x + 105*(f
*x)^m*a*d*x)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

maple [B] time = 0.00, size = 221, normalized size = 2.66

$$\left(ce\,m^{3}x^{6} + 9ce\,m^{2}x^{6} + be\,m^{3}x^{4} + cd\,m^{3}x^{4} + 23cem\,x^{6} + 11be\,m^{2}x^{4} + 11cd\,m^{2}x^{4} + 15ce\,x^{6} + ae\,m^{3}x^{2} + bd\,m^{3}x^{2} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x)
```

```
[Out] x*(c*e*m^3*x^6+9*c*e*m^2*x^6+b*e*m^3*x^4+c*d*m^3*x^4+23*c*e*m*x^6+11*b*e*m^
2*x^4+11*c*d*m^2*x^4+15*c*e*x^6+a*e*m^3*x^2+b*d*m^3*x^2+31*b*e*m*x^4+31*c*d
*m*x^4+13*a*e*m^2*x^2+13*b*d*m^2*x^2+21*b*e*x^4+21*c*d*x^4+a*d*m^3+47*a*e*m
*x^2+47*b*d*m*x^2+15*a*d*m^2+35*a*e*x^2+35*b*d*x^2+71*a*d*m+105*a*d)*(f*x)^
m/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.06, size = 104, normalized size = 1.25

$$\frac{cef^{m}x^{7}x^{m}}{m+7} + \frac{cdf^{m}x^{5}x^{m}}{m+5} + \frac{bef^{m}x^{5}x^{m}}{m+5} + \frac{bdf^{m}x^{3}x^{m}}{m+3} + \frac{aef^{m}x^{3}x^{m}}{m+3} + \frac{\left(fx\right)^{m+1}ad}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] c*e*f^m*x^7*x^m/(m + 7) + c*d*f^m*x^5*x^m/(m + 5) + b*e*f^m*x^5*x^m/(m + 5)
 + b*d*f^m*x^3*x^m/(m + 3) + a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f
*(m + 1))
```

mupad [B] time = 0.34, size = 171, normalized size = 2.06

$$\left(f\,x\right)^{m}\left(\frac{x^{3}\,\left(a\,e+b\,d\right)\,\left(m^{3}+13\,m^{2}+47\,m+35\right)}{m^{4}+16\,m^{3}+86\,m^{2}+176\,m+105} + \frac{x^{5}\,\left(b\,e+c\,d\right)\,\left(m^{3}+11\,m^{2}+31\,m+21\right)}{m^{4}+16\,m^{3}+86\,m^{2}+176\,m+105} + \frac{a\,d\,x\,\left(m^{3}+15\,m\right.}{m^{4}+16\,m^{3}+86\,n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] (f*x)^m*((x^3*(a*e + b*d)*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 + 16*
m^3 + m^4 + 105) + (x^5*(b*e + c*d)*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86
*m^2 + 16*m^3 + m^4 + 105) + (a*d*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 8
6*m^2 + 16*m^3 + m^4 + 105) + (c*e*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m +
86*m^2 + 16*m^3 + m^4 + 105))
```

sympy [A] time = 1.86, size = 1056, normalized size = 12.72

$$\begin{cases} \frac{-\frac{ad}{6x^6}-\frac{ae}{4x^4}-\frac{bd}{4x^4}-\frac{be}{2x^2}-\frac{cd}{2x^2}+ce\log(x)}{f^7} \\ \frac{-\frac{ad}{4x^4}-\frac{ae}{2x^2}-\frac{bd}{2x^2}+be\log(x)+cd\log(x)+\frac{cex^2}{2}}{f^5} \\ \frac{-\frac{ad}{2x^2}+ae\log(x)+bd\log(x)+\frac{bex^2}{2}+\frac{cdx^2}{2}+\frac{cex^4}{4}}{f^3} \\ \frac{ad\log(x)+\frac{aex^2}{2}+\frac{bdx^2}{2}+\frac{bex^4}{4}+\frac{cdx^4}{4}+\frac{cex^6}{6}}{f} \\ \frac{adf^m m^3 xx^m}{m^4+16m^3+86m^2+176m+105}+\frac{15adf^m m^2 xx^m}{m^4+16m^3+86m^2+176m+105}+\frac{71adf^m mxx^m}{m^4+16m^3+86m^2+176m+105}+\frac{105adf^m xx^m}{m^4+16m^3+86m^2+176m+105}+\frac{}{m^4+16m} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-a*d/(6*x**6) - a*e/(4*x**4) - b*d/(4*x**4) - b*e/(2*x**2) - c*
d/(2*x**2) + c*e*log(x))/f**7, Eq(m, -7)), ((-a*d/(4*x**4) - a*e/(2*x**2) -
 b*d/(2*x**2) + b*e*log(x) + c*d*log(x) + c*e*x**2/2)/f**5, Eq(m, -5)), ((-
a*d/(2*x**2) + a*e*log(x) + b*d*log(x) + b*e*x**2/2 + c*d*x**2/2 + c*e*x**4
/4)/f**3, Eq(m, -3)), ((a*d*log(x) + a*e*x**2/2 + b*d*x**2/2 + b*e*x**4/4 +
 c*d*x**4/4 + c*e*x**6/6)/f, Eq(m, -1)), (a*d*f**m*m**3*x*x**m/(m**4 + 16*m
**3 + 86*m**2 + 176*m + 105) + 15*a*d*f**m*m**2*x*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 71*a*d*f**m*m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176
*m + 105) + 105*a*d*f**m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
a*e*f**m*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 13*a*e*f
**m*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 47*a*e*f**m*m
*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*a*e*f**m*x**3*x**m
/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b*d*f**m*m**3*x**3*x**m/(m**4 +
 16*m**3 + 86*m**2 + 176*m + 105) + 13*b*d*f**m*m**2*x**3*x**m/(m**4 + 16*m
**3 + 86*m**2 + 176*m + 105) + 47*b*d*f**m*m*x**3*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 35*b*d*f**m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 17
6*m + 105) + b*e*f**m*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 10
5) + 11*b*e*f**m*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
31*b*e*f**m*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*b*e*f
**m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + c*d*f**m*m**3*x**5
*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*c*d*f**m*m**2*x**5*x**m
/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*c*d*f**m*m*x**5*x**m/(m**4 +
 16*m**3 + 86*m**2 + 176*m + 105) + 21*c*d*f**m*x**5*x**m/(m**4 + 16*m**3 +
 86*m**2 + 176*m + 105) + c*e*f**m*m**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2
 + 176*m + 105) + 9*c*e*f**m*m**2*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176
*m + 105) + 23*c*e*f**m*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105
) + 15*c*e*f**m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

3.223 $\int \frac{(fx)^m(d+ex^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=194

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))+(
f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.30, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1285, 364}

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 -
4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1285

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(fx)^m\left(d+ex^2\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4a}\\ &= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}{}_2F_1\left(1,\frac{1+m}{2}\right.}{\left(b+\sqrt{b^2-4ac}\right)f(1}\end{aligned}$$

Mathematica [A] time = 0.24, size = 156, normalized size = 0.80

$$\frac{x(fx)^m\left(\left(d\sqrt{b^2-4ac}-2ae+bd\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(d\sqrt{b^2-4ac}+2ae-bd\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\right.\right.}{2a(m+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*(f*x)^m*((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, (1 + m
)/2, (3 + m)/2, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (-(b*d) + Sqrt[b^2 -
4*a*c]*d + 2*a*e)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]))/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex^2+d\right)\left(fx\right)^m}{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^2+d\right)\left(fx\right)^m}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{\left(e\,x^2+d\right)\left(fx\right)^m}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^2+d\right)\left(fx\right)^m}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(f\,x\right)^m\,\left(e\,x^2+d\right)}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (d + ex^2)}{a + bx^2 + cx^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.224 $\int \frac{(fx)^m\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=392

$$\frac{c(fx)^{m+1}\left(b\left(d(1-m)\sqrt{b^2-4ac}+4ae\right)-2a\left(e(1-m)\sqrt{b^2-4ac}+2cd(3-m)\right)+b^2(d-dm)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m-}{2}\right.}{2af(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$

```
[Out] 1/2*(f*x)^(1+m)*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^2)/a/(-4*a*c+b^2)/f/(
c*x^4+b*x^2+a)-1/2*c*(f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*
x^2/(b+(-4*a*c+b^2)^(1/2)))*(b^2*d*(1-m)+b*(4*a*e-d*(1-m)*(-4*a*c+b^2)^(1/2
))+2*a*(-2*c*d*(3-m)+e*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/f/(1
+m)/(b+(-4*a*c+b^2)^(1/2))+1/2*c*(f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+
1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(b^2*(-d*m+d)+b*(4*a*e+d*(1-m)*(-4*
a*c+b^2)^(1/2))-2*a*(2*c*d*(3-m)+e*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2
)^(3/2)/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 2.65, antiderivative size = 358, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1277, 1285, 364}

$$\frac{c(fx)^{m+1}\left((1-m)\sqrt{b^2-4ac}\,(bd-2ae)+4abe-4acd(3-m)+b^2(d-dm)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2af(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 -
 4*a*c)*f*(a + b*x^2 + c*x^4)) + (c*(4*a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - 2*a
*e)*(1 - m) - 4*a*c*d*(3 - m) + b^2*(d - d*m))*(f*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2
 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)) - (c*(4*a*b*e - Sqrt[b^2
 - 4*a*c]*(b*d - 2*a*e)*(1 - m) - 4*a*c*d*(3 - m) + b^2*(d - d*m))*(f*x)^(1
 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1285

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(fx)^m \left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)f\left(a+bx^2+cx^4\right)} - \frac{\int \frac{(fx)^m\left(-b^2d(1-m)+2acd(3-m)-abe(1+m)-c(bd-2a\right.}{a+bx^2+cx^4}}{2a\left(b^2-4ac\right)}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)f\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(4abe+b^2d(1-m)+\sqrt{b^2-4ac}\,(bd-\right.\right.}{4a\,(}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)f\left(a+bx^2+cx^4\right)} + \frac{c\left(4abe+b^2d(1-m)+\sqrt{b^2-4ac}\,(bd-2\right.}{2a\left(b^2-\right.}$$

Mathematica [C] time = 0.23, size = 160, normalized size = 0.41

$$\frac{x(fx)^m\left(d(m+3)F_1\left(\frac{m+1}{2};2,2;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+e(m+1)x^2F_1\left(\frac{m+3}{2};2,2;\frac{m+5}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right.\right.}{a^2(m+1)(m+3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(f*x)^m*(d*(3 + m)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^2*App
ellF1[(3 + m)/2, 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(a^2*(1 + m)*(3 + m))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex^2+d\right)\left(fx\right)^m}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a
*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)\left(fx\right)^m}{\left(cx^4+bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)\left(f x\right)^{m}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^{2}+d\right)\left(f x\right)^{m}}{\left(cx^{4}+bx^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x\right)^{m}\,\left(e\,x^{2}+d\right)}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x)
[Out] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

3.225 $\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=319

$$\frac{ad(fx)^{m+1}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{ae(fx)^{m+3}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+3}{2};-\frac{3}{2}\right.}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}$$

```
[Out] a*d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,-3/2,-3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+
b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+m)/
(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))
^(1/2)+a*e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,-3/2,-3/2,5/2+1/2*m,-2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f
^3/(3+m)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1335, 1141, 510}

$$\frac{ad(fx)^{m+1}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{ae(fx)^{m+3}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+3}{2};-\frac{3}{2}\right.}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2, -3/2,
(3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (a*e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x
^4]*AppellF1[(3 + m)/2, -3/2, -3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \left(d(fx)^m\left(a+bx^2+cx^4\right)^{3/2} + \frac{e(fx)^{2+m}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right)dx$$

$$= d\int (fx)^m\left(a+bx^2+cx^4\right)^{3/2}\,dx + \frac{e\int (fx)^{2+m}\left(a+bx^2+cx^4\right)^{3/2}\,dx}{f^2}$$

$$= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int (fx)^m\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2} a}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{ad(fx)^{1+m}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1+m}{2};-\frac{3}{2},-\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2c}{b+\sqrt{b^2}}\right.}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A] time = 0.53, size = 466, normalized size = 1.46

$$\frac{x(fx)^m\sqrt{a+bx^2+cx^4}\left((m+1)x^2\left(\left(m^2+12m+35\right)(ae+bd)F_1\left(\frac{m+3}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+5}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*Sqrt[a + b*x^2 + c*x^4]*(a*d*(105 + 71*m + 15*m^2 + m^3)*AppellF
1[(1 + m)/2, -1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*x^2*((b*d + a*e)*(35 + 12*m + m^
2)*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (3 + m)*x^2*((c*d + b*e)*(7 +
m)*AppellF1[(5 + m)/2, -1/2, -1/2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + c*e*(5 + m)*x^2*AppellF1[(7 +
m)/2, -1/2, -1/2, (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])]))))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cex^6+(cd+be)x^4+(bd+ae)x^2+ad\right)\sqrt{cx^4+bx^2+a}\,\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 + b
*x^2 + a)*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right)\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}\left(fx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}\left(ex^{2}+d\right)\left(fx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f\,x\right)}^m\,\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m}\left(d+ex^{2}\right)\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.226 $\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\, dx$

Optimal. Leaf size=317

$$\frac{d(fx)^{m+1}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{e(fx)^{m+3}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+3}{2};-\frac{1}{2}\right.}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}$$

```
[Out] d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,-1/2,-1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^
2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+m)/(1
+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)+e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,-1/2,-1/2,5/2+1/2*m,-2*c*x^2/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(
3+m)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1335, 1141, 510}

$$\frac{d(fx)^{m+1}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{e(fx)^{m+3}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+3}{2};-\frac{1}{2}\right.}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x^4]*
AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx = \int \left(d(fx)^m\sqrt{a+bx^2+cx^4} + \frac{e(fx)^{2+m}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$

$$= d\int (fx)^m\sqrt{a+bx^2+cx^4}\,dx + \frac{e\int (fx)^{2+m}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int (fx)^m\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(e\sqrt{a\right.}}{}$$

$$= \frac{d(fx)^{1+m}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A] time = 0.20, size = 267, normalized size = 0.84

$$\frac{x(fx)^m\sqrt{a+bx^2+cx^4}\left(d(m+3)F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+e(m+1)x^2F_1\left(\frac{m+3}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+5}{2}\right.\right.}{(m+1)(m+3)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[a + b*x^2 + c*x^4]*(d*(3 + m)*AppellF1[(1 + m)/2, -1/2, -1/
2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])] + e*(1 + m)*x^2*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/((1 +
m)*(3 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+d\right)\sqrt{c\,x^4+b\,x^2+a}\,\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)\left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f\,x\right)}^m\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(d + ex^{2}\right) \sqrt{a + bx^{2} + cx^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

3.227 $\int \frac{(fx)^m\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=317

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{}$$

```
[Out] d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,1/2,1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(1+m)/(c*x^4+b*x^2+a)^(1/
2)+e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,1/2,1/2,5/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b
^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2
)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(3+m)/(c*x^4+b*x^2+a
)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1335, 1141, 510}

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*(1
+ m)*Sqrt[a + b*x^2 + c*x^4]) + (e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 +
 m)/2, 1/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{(fx)^m\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx=\int\left(\frac{d(fx)^m}{\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{2+m}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$=d\int\frac{(fx)^m}{\sqrt{a+bx^2+cx^4}}\,dx+\frac{e\int\frac{(fx)^{2+m}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$

$$=\frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int\frac{(fx)^m}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2}}}\right.}{}$$

$$=\frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{2};\frac{1}{2},\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right.}{f(1+m)\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.26, size = 267, normalized size = 0.84

$$\frac{x(fx)^m\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\left(d(m+3)F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+e(m+1)x^2F\right.}{(m+1)(m+3)\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(d*(3 + m)*
AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^2*AppellF1[(3 + m)/2, 1
/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])]))/((1 + m)*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex^2+d\right)\left(fx\right)^m}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^2+d\right)\left(fx\right)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)\left(f x\right)^{m}}{\sqrt{c\,x^{4}+b\,x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^{2}+d\right)\left(f x\right)^{m}}{\sqrt{cx^{4}+bx^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x\right)^{m}\,\left(e\,x^{2}+d\right)}{\sqrt{c\,x^{4}+b\,x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m}\left(d+ex^{2}\right)}{\sqrt{a+bx^{2}+cx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.228 $\int \frac{(fx)^m\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=323

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{}$$

```
[Out] d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,3/2,3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(1+m)/(c*x^4+b*x^2+a)^(
1/2)+e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,3/2,3/2,5/2+1/2*m,-2*c*x^2/(b-(-4*a*c
+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1
/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(3+m)/(c*x^4+b*x
^2+a)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1335, 1141, 510}

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*(
1 + m)*Sqrt[a + b*x^2 + c*x^4]) + (e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3
 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])/(a*f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{(fx)^m\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int\left(\frac{d(fx)^m}{\left(a+bx^2+cx^4\right)^{3/2}}+\frac{e(fx)^{2+m}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$=d\int\frac{(fx)^m}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx+\frac{e\int\frac{(fx)^{2+m}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx}{f^2}$$

$$=\frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int\frac{(fx)^m}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}+\frac{\left(e\sqrt{1+\frac{2}{b-\sqrt{}}}\right.}{}$$

$$=\frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{2};\frac{3}{2},\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af(1+m)\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.41, size = 307, normalized size = 0.95

$$\frac{x(fx)^m\left(\sqrt{b^2-4ac}-b-2cx^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{3/2}\left(d(m+3)F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\right.\right.}{(m+1)(m+3)\left(\sqrt{b^2-4ac}-b\right)\left(a+bx^2+cx^4\right)^{3/}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]))^(3/2)*(d*(3 + m)*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/
2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
+ e*(1 + m)*x^2*AppellF1[(3 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/((-b + Sqrt[b^2 - 4
*a*c])*(1 + m)*(3 + m)*(a + b*x^2 + c*x^4)^(3/2))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\left(fx\right)^m}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m/(c^2*x^8 + 2*b*c*x^6 +
 (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^2+d\right)\left(fx\right)^m}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right) \left(f x\right)^{m}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^{2}+d\right) \left(fx\right)^{m}}{\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f\,x\right)}^m\,\left(e\,x^2+d\right)}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(d + ex^{2}\right)}{\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.229 $\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=134

$$\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

```
[Out] -1/2*d*x^2/c/e^2+1/4*x^4/c/e+1/2*a^(3/2)*d*arctan(x^2*c^(1/2)/a^(1/2))/c^(3
/2)/(a*e^2+c*d^2)+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2+c*d^2)-1/4*a^2*e*ln(c*x^4+
a)/c^2/(a*e^2+c*d^2)
```

Rubi [A] time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 1629, 635, 205, 260}

$$-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(d*x^2)/(2*c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^2)
) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^9}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(d+ex)\left(a+cx^2\right)}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{d}{ce^2}+\frac{x}{ce}+\frac{d^4}{e^2\left(cd^2+ae^2\right)(d+ex)}+\frac{a^2(d-ex)}{c\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx,x,$$
$$= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}+\frac{a^2\,\text{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx,x,x^2\right)}{2c\left(cd^2+ae^2\right)}$$
$$= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}+\frac{\left(a^2d\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2c\left(cd^2+ae^2\right)}-\frac{\left(a^2e\right)\text{Subst}\left(\right.}{2c\left(c\right.}$$
$$= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(cd^2+ae^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.06, size = 134, normalized size = 1.00

$$\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/2*(d*x^2)/(c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^
2)) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))
```

fricas [A] time = 8.36, size = 277, normalized size = 2.07

$$\left[\frac{acde^3\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)-a^2e^4\log\left(cx^4+a\right)+2c^2d^4\log\left(ex^2+d\right)+\left(c^2d^2e^2+ace^4\right)x^4-2\left(c^2d^3e+a\right.}{4\left(c^3d^2e^3+ac^2e^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*c*d*e^3*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)
) - a^2*e^4*log(c*x^4 + a) + 2*c^2*d^4*log(e*x^2 + d) + (c^2*d^2*e^2 + a*c*
e^4)*x^4 - 2*(c^2*d^3*e + a*c*d*e^3)*x^2)/(c^3*d^2*e^3 + a*c^2*e^5), 1/4*(2
*a*c*d*e^3*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - a^2*e^4*log(c*x^4 + a) + 2
*c^2*d^4*log(e*x^2 + d) + (c^2*d^2*e^2 + a*c*e^4)*x^4 - 2*(c^2*d^3*e + a*c*
d*e^3)*x^2)/(c^3*d^2*e^3 + a*c^2*e^5)]
```

giac [A] time = 0.31, size = 121, normalized size = 0.90

$$\frac{d^4\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e^3+ae^5\right)}-\frac{a^2e\log\left(cx^4+a\right)}{4\left(c^3d^2+ac^2e^2\right)}+\frac{a^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(c^2d^2+ace^2\right)\sqrt{ac}}+\frac{\left(cx^4e-2cdx^2\right)e^{(-2)}}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(x^2*e + d))/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*log(c*x^4 + a)/
(c^3*d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))/((c^2*d^2 + a*c*e
^2)*sqrt(a*c)) + 1/4*(c*x^4*e - 2*c*d*x^2)*e^(-2)/c^2
```

maple [A] time = 0.01, size = 122, normalized size = 0.91

$$\frac{a^2 d \arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\left(a\,e^2+c\,d^2\right)\sqrt{ac}\;c}-\frac{a^2 e\ln\left(c\,x^4+a\right)}{4\left(a\,e^2+c\,d^2\right)c^2}+\frac{x^4}{4ce}+\frac{d^4\ln\left(e\,x^2+d\right)}{2\left(a\,e^2+c\,d^2\right)e^3}-\frac{d\,x^2}{2c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*x^4/c/e-1/2*d*x^2/c/e^2-1/4*a^2*e*ln(c*x^4+a)/c^2/(a*e^2+c*d^2)+1/2*a^2
/(a*e^2+c*d^2)/c*d/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))+1/2*d^4*ln(e*x^2+d
)/e^3/(a*e^2+c*d^2)
```

maxima [A] time = 2.00, size = 120, normalized size = 0.90

$$\frac{d^4\log\left(ex^2+d\right)}{2\left(cd^2e^3+ae^5\right)}-\frac{a^2e\log\left(cx^4+a\right)}{4\left(c^3d^2+ac^2e^2\right)}+\frac{a^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(c^2d^2+ace^2\right)\sqrt{ac}}+\frac{ex^4-2\,dx^2}{4\,ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*d^4*log(e*x^2 + d)/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*log(c*x^4 + a)/(c^3*
d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))/((c^2*d^2 + a*c*e^2)*s
qrt(a*c)) + 1/4*(e*x^4 - 2*d*x^2)/(c*e^2)
```

mupad [B] time = 0.87, size = 181, normalized size = 1.35

$$\frac{\ln\left(\sqrt{-a^3\,c^5}+a\,c^3\,x^2\right)\left(d\,\sqrt{-a^3\,c^5}-a^2\,c^2\,e\right)}{4\,c^5\,d^2+4\,a\,c^4\,e^2}-\frac{\ln\left(\sqrt{-a^3\,c^5}-a\,c^3\,x^2\right)\left(d\,\sqrt{-a^3\,c^5}+a^2\,c^2\,e\right)}{4\,\left(c^5\,d^2+a\,c^4\,e^2\right)}+\frac{d^4\,\ln\left(e\,x^2+d\right)}{2\,c\,d^2\,e^3+2\,a\,e^5}+\frac{x^{\cdots}}{4\,c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log((-a^3*c^5)^(1/2) + a*c^3*x^2)*(d*(-a^3*c^5)^(1/2) - a^2*c^2*e))/(4*c^5
*d^2 + 4*a*c^4*e^2) - (log((-a^3*c^5)^(1/2) - a*c^3*x^2)*(d*(-a^3*c^5)^(1/2
) + a^2*c^2*e))/(4*(c^5*d^2 + a*c^4*e^2)) + (d^4*log(d + e*x^2))/(2*a*e^5 +
 2*c*d^2*e^3) + x^4/(4*c*e) - (d*x^2)/(2*c*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.230 $\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=118

$$-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{ad\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2+cd^2\right)}+\frac{x^2}{2ce}$$

```
[Out] 1/2*x^2/c/e-1/2*a^(3/2)*e*arctan(x^2*c^(1/2)/a^(1/2))/c^(3/2)/(a*e^2+c*d^2)
-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2+c*d^2)-1/4*a*d*ln(c*x^4+a)/c/(a*e^2+c*d^2)
```

Rubi [A] time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 1629, 635, 205, 260}

$$-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2+cd^2\right)}-\frac{ad\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}+\frac{x^2}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x^2/(2*c*e) - (a^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2)*(c*d^2 +
 a*e^2)) - (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 + a*e^2)) - (a*d*Log[a + c*x^
4])/(4*c*(c*d^2 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)(a+cx^2)}dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ce} - \frac{d^3}{e(cd^2+ae^2)(d+ex)} - \frac{a(ae+cdx)}{c(cd^2+ae^2)(a+cx^2)}\right)dx, x, x^2\right)$$
$$= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2+ae^2)} - \frac{a\,\text{Subst}\left(\int \frac{ae+cdx}{a+cx^2}dx, x, x^2\right)}{2c(cd^2+ae^2)}$$
$$= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2+ae^2)} - \frac{(ad)\,\text{Subst}\left(\int \frac{x}{a+cx^2}dx, x, x^2\right)}{2(cd^2+ae^2)} - \frac{(a^2e)\,\text{Subst}\left(\int \frac{1}{a+cx^2}dx, x, x\right.}{2c(cd^2+ae^2)}$$
$$= \frac{x^2}{2ce} - \frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(cd^2+ae^2)} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2+ae^2)} - \frac{ad\log(a+cx^4)}{4c(cd^2+ae^2)}$$

Mathematica [A] time = 0.09, size = 99, normalized size = 0.84

$$\frac{-\frac{2a^{3/2}e^3\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{3/2}} + \frac{e\left(2x^2(ae^2+cd^2)-ade\log(a+cx^4)\right)}{c} - 2d^3\log(d+ex^2)}{4e^2(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((-2*a^(3/2)*e^3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) - 2*d^3*Log[d + e*x
^2] + (e*(2*(c*d^2 + a*e^2)*x^2 - a*d*e*Log[a + c*x^4]))/c)/(4*e^2*(c*d^2 +
 a*e^2))
```

fricas [A] time = 3.47, size = 212, normalized size = 1.80

$$\left[\frac{ae^3\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right) - ade^2\log(cx^4+a) - 2cd^3\log(ex^2+d) + 2(cd^2e+ae^3)x^2}{4(c^2d^2e^2+ace^4)}, -\frac{2ae^3\sqrt{\frac{a}{c}}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*e^3*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) -
a*d*e^2*log(c*x^4 + a) - 2*c*d^3*log(e*x^2 + d) + 2*(c*d^2*e + a*e^3)*x^2)/
(c^2*d^2*e^2 + a*c*e^4), -1/4*(2*a*e^3*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a)
+ a*d*e^2*log(c*x^4 + a) + 2*c*d^3*log(e*x^2 + d) - 2*(c*d^2*e + a*e^3)*x^2
)/(c^2*d^2*e^2 + a*c*e^4)]
```

giac [A] time = 0.31, size = 105, normalized size = 0.89

$$-\frac{d^3\log\left(|x^2e+d|\right)}{2(cd^2e^2+ae^4)} - \frac{a^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2(c^2d^2+ace^2)\sqrt{ac}} + \frac{x^2e^{(-1)}}{2c} - \frac{ad\log(cx^4+a)}{4(c^2d^2+ace^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*d^3*log(abs(x^2*e + d))/(c*d^2*e^2 + a*e^4) - 1/2*a^2*arctan(c*x^2/sqr
t(a*c))*e/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) + 1/2*x^2*e^(-1)/c - 1/4*a*d*log(
c*x^4 + a)/(c^2*d^2 + a*c*e^2)
```

maple [A] time = 0.01, size = 108, normalized size = 0.92

$$-\frac{a^{2} e \arctan\left(\frac{c\,x^{2}}{\sqrt{a c}}\right)}{2\left(a\,e^{2}+c\,d^{2}\right)\sqrt{a c}\;c}-\frac{a d \ln\left(c\,x^{4}+a\right)}{4\left(a\,e^{2}+c\,d^{2}\right)c}-\frac{d^{3} \ln\left(e\,x^{2}+d\right)}{2\left(a\,e^{2}+c\,d^{2}\right)e^{2}}+\frac{x^{2}}{2 c e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/2*x^2/c/e-1/4*a*d*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/2*a^2/(a*e^2+c*d^2)/c*e/(
a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2+c*d^2
)
```

maxima [A] time = 2.02, size = 107, normalized size = 0.91

$$-\frac{d^{3} \log\left(e x^{2}+d\right)}{2\left(c d^{2} e^{2}+a e^{4}\right)}-\frac{a^{2} e \arctan\left(\frac{c x^{2}}{\sqrt{a c}}\right)}{2\left(c^{2} d^{2}+a c e^{2}\right)\sqrt{a c}}-\frac{a d \log\left(c x^{4}+a\right)}{4\left(c^{2} d^{2}+a c e^{2}\right)}+\frac{x^{2}}{2\,c e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*d^3*log(e*x^2 + d)/(c*d^2*e^2 + a*e^4) - 1/2*a^2*e*arctan(c*x^2/sqrt(a
*c))/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) - 1/4*a*d*log(c*x^4 + a)/(c^2*d^2 + a*
c*e^2) + 1/2*x^2/(c*e)
```

mupad [B] time = 0.72, size = 166, normalized size = 1.41

$$\frac{x^{2}}{2\,c\,e}-\frac{d^{3} \ln\left(e\,x^{2}+d\right)}{2\,c\,d^{2}\,e^{2}+2\,a\,e^{4}}-\frac{\ln\left(\sqrt{-a^{3}\,c^{3}}+a\,c^{2}\,x^{2}\right)\left(e\,\sqrt{-a^{3}\,c^{3}}+a\,c^{2}\,d\right)}{4\left(c^{4}\,d^{2}+a\,c^{3}\,e^{2}\right)}+\frac{\ln\left(\sqrt{-a^{3}\,c^{3}}-a\,c^{2}\,x^{2}\right)\left(e\,\sqrt{-a^{3}\,c^{3}}-a\,c^{2}\right.}{4\,c^{4}\,d^{2}+4\,a\,c^{3}\,e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] x^2/(2*c*e) - (d^3*log(d + e*x^2))/(2*a*e^4 + 2*c*d^2*e^2) - (log((-a^3*c^3
)^(1/2) + a*c^2*x^2)*(e*(-a^3*c^3)^(1/2) + a*c^2*d))/(4*(c^4*d^2 + a*c^3*e^
2)) + (log((-a^3*c^3)^(1/2) - a*c^2*x^2)*(e*(-a^3*c^3)^(1/2) - a*c^2*d))/(4
*c^4*d^2 + 4*a*c^3*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.231 $\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=105

$$\frac{ae\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}+\frac{d^2\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)}-\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}$$

```
[Out] 1/2*d^2*ln(e*x^2+d)/e/(a*e^2+c*d^2)+1/4*a*e*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/2
*d*arctan(x^2*c^(1/2)/a^(1/2))*a^(1/2)/(a*e^2+c*d^2)/c^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 1629, 635, 205, 260}

$$\frac{d^2\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)}+\frac{ae\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}-\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(Sqrt[a]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*e^2)) + (d
^2*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)) + (a*e*Log[a + c*x^4])/(4*c*(c*d^2
 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^5}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d^2}{\left(cd^2+ae^2\right)(d+ex)} - \frac{a(d-ex)}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right) dx, x, x^2\right)$$
$$= \frac{d^2\log\left(d+ex^2\right)}{2e\left(cd^2+ae^2\right)} - \frac{a\operatorname{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}$$
$$= \frac{d^2\log\left(d+ex^2\right)}{2e\left(cd^2+ae^2\right)} - \frac{(ad)\operatorname{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)} + \frac{(ae)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}$$
$$= -\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,\left(cd^2+ae^2\right)} + \frac{d^2\log\left(d+ex^2\right)}{2e\left(cd^2+ae^2\right)} + \frac{ae\log\left(a+cx^4\right)}{4c\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.73

$$\frac{-2\sqrt{a}\,\sqrt{c}\,de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right) + ae^2\log\left(a+cx^4\right) + 2cd^2\log\left(d+ex^2\right)}{4ace^3+4c^2d^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + 2*c*d^2*Log[d + e*x
^2] + a*e^2*Log[a + c*x^4])/(4*c^2*d^2*e + 4*a*c*e^3)
```

fricas [A] time = 1.96, size = 170, normalized size = 1.62

$$\left[\frac{cde\sqrt{-\frac{a}{c}}\,\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right) + ae^2\log\left(cx^4+a\right) + 2\,cd^2\log\left(ex^2+d\right)}{4\left(c^2d^2e+ace^3\right)}, -\frac{2\,cde\sqrt{\frac{a}{c}}\,\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right) - ae^2\log\left(\right.}{4\left(c^2d^2e+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(c*d*e*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) +
a*e^2*log(c*x^4 + a) + 2*c*d^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3), -1/4*
(2*c*d*e*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - a*e^2*log(c*x^4 + a) - 2*c*d
^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3)]
```

giac [A] time = 0.36, size = 90, normalized size = 0.86

$$\frac{ae\log\left(cx^4+a\right)}{4\left(c^2d^2+ace^2\right)} + \frac{d^2\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e+ae^3\right)} - \frac{ad\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(abs(x^2*e + d))/(c
*d^2*e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c
))
```

maple [A] time = 0.01, size = 92, normalized size = 0.88

$$-\frac{ad\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{2\left(a\,e^{2}+c\,d^{2}\right)\sqrt{ac}}+\frac{ae\ln\left(c\,x^{4}+a\right)}{4\left(a\,e^{2}+c\,d^{2}\right)c}+\frac{d^{2}\ln\left(e\,x^{2}+d\right)}{2\left(a\,e^{2}+c\,d^{2}\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*a*e*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/2*a/(a*e^2+c*d^2)*d/(a*c)^(1/2)*arcta
n(1/(a*c)^(1/2)*c*x^2)+1/2*d^2*ln(e*x^2+d)/e/(a*e^2+c*d^2)
```

maxima [A] time = 2.00, size = 89, normalized size = 0.85

$$\frac{ae\log\left(cx^{4}+a\right)}{4\left(c^{2}d^{2}+ace^{2}\right)}+\frac{d^{2}\log\left(ex^{2}+d\right)}{2\left(cd^{2}e+ae^{3}\right)}-\frac{ad\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{2\left(cd^{2}+ae^{2}\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(e*x^2 + d)/(c*d^2*
e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

mupad [B] time = 0.99, size = 138, normalized size = 1.31

$$\frac{d^{2}\,\ln\left(e\,x^{2}+d\right)}{2\,c\,d^{2}\,e+2\,a\,e^{3}}-\frac{\ln\left(\sqrt{-a\,c^{3}}\,+c^{2}\,x^{2}\right)\left(d\,\sqrt{-a\,c^{3}}\,-a\,c\,e\right)}{4\,\left(c^{3}\,d^{2}+a\,c^{2}\,e^{2}\right)}+\frac{\ln\left(\sqrt{-a\,c^{3}}\,-c^{2}\,x^{2}\right)\left(d\,\sqrt{-a\,c^{3}}\,+a\,c\,e\right)}{4\,c^{3}\,d^{2}+4\,a\,c^{2}\,e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (d^2*log(d + e*x^2))/(2*a*e^3 + 2*c*d^2*e) - (log((-a*c^3)^(1/2) + c^2*x^2)
*(d*(-a*c^3)^(1/2) - a*c*e))/(4*(c^3*d^2 + a*c^2*e^2)) + (log((-a*c^3)^(1/2
) - c^2*x^2)*(d*(-a*c^3)^(1/2) + a*c*e))/(4*c^3*d^2 + 4*a*c^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.232 $\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=96

$$\frac{d\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}+\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}$$

```
[Out] -1/2*d*ln(e*x^2+d)/(a*e^2+c*d^2)+1/4*d*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*e*arct
an(x^2*c^(1/2)/a^(1/2))*a^(1/2)/(a*e^2+c*d^2)/c^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 801, 635, 205, 260}

$$-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}+\frac{d\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}+\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*e^2)) - (d*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) + (d*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{de}{(cd^2+ae^2)(d+ex)} + \frac{ae+cdx}{(cd^2+ae^2)(a+cx^2)}\right)dx, x, x^2\right)$$
$$= -\frac{d\log(d+ex^2)}{2(cd^2+ae^2)} + \frac{\operatorname{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}$$
$$= -\frac{d\log(d+ex^2)}{2(cd^2+ae^2)} + \frac{(cd)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)} + \frac{(ae)\operatorname{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}$$
$$= \frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(cd^2+ae^2)} - \frac{d\log(d+ex^2)}{2(cd^2+ae^2)} + \frac{d\log(a+cx^4)}{4(cd^2+ae^2)}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.69

$$\frac{d\log(a+cx^4) + \frac{2\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{c}} - 2d\log(d+ex^2)}{4ae^2+4cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((2*Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[c] - 2*d*Log[d + e*x^2] +
 d*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

fricas [A] time = 0.84, size = 145, normalized size = 1.51

$$\left[\frac{e\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right) + d\log(cx^4+a) - 2\,d\log(ex^2+d)}{4(cd^2+ae^2)}, \frac{2\,e\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right) + d\log(cx^4+a) - 2\,d\log\ldots}{4(cd^2+ae^2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(e*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + d*lo
g(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 + a*e^2), 1/4*(2*e*sqrt(a/c)*arct
an(c*x^2*sqrt(a/c)/a) + d*log(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 + a*e
^2)]
```

giac [A] time = 0.39, size = 86, normalized size = 0.90

$$-\frac{de\log(|x^2e+d|)}{2(cd^2e+ae^3)} + \frac{a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2(cd^2+ae^2)\sqrt{ac}} + \frac{d\log(cx^4+a)}{4(cd^2+ae^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*d*e*log(abs(x^2*e + d))/(c*d^2*e + a*e^3) + 1/2*a*arctan(c*x^2/sqrt(a*
c))*e/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*x^4 + a)/(c*d^2 + a*e^2)
```

maple [A] time = 0.01, size = 83, normalized size = 0.86

$$\frac{ae\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\left(a\,e^2+c\,d^2\right)\sqrt{ac}}+\frac{d\ln\left(c\,x^4+a\right)}{4a\,e^2+4c\,d^2}-\frac{d\ln\left(e\,x^2+d\right)}{2\left(a\,e^2+c\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*d*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2/(a*e^2+c*d^2)*a*e/(a*c)^(1/2)*arctan(1/
(a*c)^(1/2)*c*x^2)-1/2*d*ln(e*x^2+d)/(a*e^2+c*d^2)
```

maxima [A] time = 1.99, size = 82, normalized size = 0.85

$$\frac{ae\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}}+\frac{d\log\left(cx^4+a\right)}{4\left(cd^2+ae^2\right)}-\frac{d\log\left(ex^2+d\right)}{2\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*x
^4 + a)/(c*d^2 + a*e^2) - 1/2*d*log(e*x^2 + d)/(c*d^2 + a*e^2)
```

mupad [B] time = 1.94, size = 944, normalized size = 9.83

$$\frac{c\,d\,\ln\left(a^4\,e^6-9\,a\,c^3\,d^6-39\,a^3\,c\,d^2\,e^4+a^3\,e^6\,x^2\,\sqrt{-a\,c}-9\,c^3\,d^6\,x^2\,\sqrt{-a\,c}+79\,a^2\,c^2\,d^4\,e^2-42\,c\,d^5\,e\,{\left(-a\,c\right)}^{3/2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (c*d*log(a^4*e^6 - 9*a*c^3*d^6 - 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2
) - 9*c^3*d^6*x^2*(-a*c)^(1/2) + 79*a^2*c^2*d^4*e^2 - 42*c*d^5*e*(-a*c)^(3/
2) + 76*a*d^3*e^3*(-a*c)^(3/2) + 10*a^3*d*e^5*(-a*c)^(1/2) + 76*a^2*c^2*d^3
*e^3*x^2 - 42*a*c^3*d^5*e*x^2 - 10*a^3*c*d*e^5*x^2 + 39*a*d^2*e^4*x^2*(-a*c
)^(3/2) - 79*c*d^4*e^2*x^2*(-a*c)^(3/2)))/(4*c^2*d^2 + 4*a*c*e^2) - (d*log(
d + e*x^2))/(2*(a*e^2 + c*d^2)) + (c*d*log(9*a*c^3*d^6 - a^4*e^6 + 39*a^3*c
*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) - 79*a^2*c
^2*d^4*e^2 + 10*a^3*d*e^5*(-a*c)^(1/2) + 42*a*c^2*d^5*e*(-a*c)^(1/2) - 76*a
^2*c^2*d^3*e^3*x^2 + 42*a*c^3*d^5*e*x^2 + 10*a^3*c*d*e^5*x^2 - 76*a^2*c*d^3
*e^3*(-a*c)^(1/2) + 79*a*c^2*d^4*e^2*x^2*(-a*c)^(1/2) - 39*a^2*c*d^2*e^4*x^
2*(-a*c)^(1/2)))/(4*c^2*d^2 + 4*a*c*e^2) - (e*log(a^4*e^6 - 9*a*c^3*d^6 - 3
9*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) + 7
9*a^2*c^2*d^4*e^2 - 42*c*d^5*e*(-a*c)^(3/2) + 76*a*d^3*e^3*(-a*c)^(3/2) + 1
0*a^3*d*e^5*(-a*c)^(1/2) + 76*a^2*c^2*d^3*e^3*x^2 - 42*a*c^3*d^5*e*x^2 - 10
*a^3*c*d*e^5*x^2 + 39*a*d^2*e^4*x^2*(-a*c)^(3/2) - 79*c*d^4*e^2*x^2*(-a*c)^
(3/2))*(-a*c)^(1/2))/(4*c^2*d^2 + 4*a*c*e^2) + (e*log(9*a*c^3*d^6 - a^4*e^6
 + 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2)
 - 79*a^2*c^2*d^4*e^2 + 10*a^3*d*e^5*(-a*c)^(1/2) + 42*a*c^2*d^5*e*(-a*c)^(
1/2) - 76*a^2*c^2*d^3*e^3*x^2 + 42*a*c^3*d^5*e*x^2 + 10*a^3*c*d*e^5*x^2 - 7
6*a^2*c*d^3*e^3*(-a*c)^(1/2) + 79*a*c^2*d^4*e^2*x^2*(-a*c)^(1/2) - 39*a^2*c
*d^2*e^4*x^2*(-a*c)^(1/2))*(-a*c)^(1/2))/(4*c^2*d^2 + 4*a*c*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.233 $\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=96

$$-\frac{e\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}+\frac{e\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}+\frac{\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}$$

```
[Out] 1/2*e*ln(e*x^2+d)/(a*e^2+c*d^2)-1/4*e*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*d*arcta
n(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^2+c*d^2)/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1248, 706, 31, 635, 205, 260}

$$\frac{e\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}-\frac{e\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}+\frac{\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)) + (e*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) - (e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{cd-cex}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)} + \frac{e^2\operatorname{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{2(cd^2+ae^2)}\\ &= \frac{e\log(d+ex^2)}{2(cd^2+ae^2)} + \frac{(cd)\operatorname{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)} - \frac{(ce)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}\\ &= \frac{\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(cd^2+ae^2)} + \frac{e\log(d+ex^2)}{2(cd^2+ae^2)} - \frac{e\log(a+cx^4)}{4(cd^2+ae^2)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.70

$$\frac{\frac{2\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}} - e\log(a+cx^4) + 2e\log(d+ex^2)}{4ae^2+4cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((2*Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[a] + 2*e*Log[d + e*x^2] -
 e*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

fricas [A] time = 0.95, size = 146, normalized size = 1.52

$$\left[\frac{d\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right) - e\log(cx^4+a) + 2e\log(ex^2+d)}{4(cd^2+ae^2)}, -\frac{2d\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right) + e\log(cx^4+a) - 2e\,\mathrm{l}}{4(cd^2+ae^2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - e*lo
g(c*x^4 + a) + 2*e*log(e*x^2 + d))/(c*d^2 + a*e^2), -1/4*(2*d*sqrt(c/a)*arc
tan(a*sqrt(c/a)/(c*x^2)) + e*log(c*x^4 + a) - 2*e*log(e*x^2 + d))/(c*d^2 +
a*e^2)]
```

giac [A] time = 0.35, size = 85, normalized size = 0.89

$$\frac{cd\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2+ae^2)\sqrt{ac}} - \frac{e\log(cx^4+a)}{4(cd^2+ae^2)} + \frac{e^2\log(|x^2e+d|)}{2(cd^2e+ae^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*c*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4*e*log(c*x
^4 + a)/(c*d^2 + a*e^2) + 1/2*e^2*log(abs(x^2*e + d))/(c*d^2*e + a*e^3)
```

maple [A] time = 0.01, size = 83, normalized size = 0.86

$$\frac{cd \arctan\left(\frac{c x^2}{\sqrt{ac}}\right)}{2\left(a e^2+c d^2\right)\sqrt{ac}}-\frac{e \ln\left(c x^4+a\right)}{4\left(a e^2+c d^2\right)}+\frac{e \ln\left(e x^2+d\right)}{2a e^2+2c d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/4*e*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*c/(a*e^2+c*d^2)*d/(a*c)^(1/2)*arctan(1
/(a*c)^(1/2)*c*x^2)+1/2*e*ln(e*x^2+d)/(a*e^2+c*d^2)
```

maxima [A] time = 1.98, size = 82, normalized size = 0.85

$$\frac{cd \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}}-\frac{e \log\left(cx^4+a\right)}{4\left(cd^2+ae^2\right)}+\frac{e \log\left(ex^2+d\right)}{2\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4*e*log(c*x
^4 + a)/(c*d^2 + a*e^2) + 1/2*e*log(e*x^2 + d)/(c*d^2 + a*e^2)
```

mupad [B] time = 1.02, size = 328, normalized size = 3.42

$$\frac{e \ln\left(e x^2+d\right)}{2 c d^2+2 a e^2}-\frac{\ln\left(a c^5 d^6 x^2-c^3 d^6 (-a c)^{3/2}-9 a^3 e^6 (-a c)^{3/2}+9 a^4 c^2 e^6 x^2+19 a d^2 e^4 (-a c)^{5/2}+11 c d^4 e^2 (\right.}{4\left(a^2 e^2+c a d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (e*log(d + e*x^2))/(2*a*e^2 + 2*c*d^2) - (log(a*c^5*d^6*x^2 - c^3*d^6*(-a*c
)^(3/2) - 9*a^3*e^6*(-a*c)^(3/2) + 9*a^4*c^2*e^6*x^2 + 19*a*d^2*e^4*(-a*c)^
(5/2) + 11*c*d^4*e^2*(-a*c)^(5/2) + 11*a^2*c^4*d^4*e^2*x^2 + 19*a^3*c^3*d^2
*e^4*x^2)*(a*e - d*(-a*c)^(1/2)))/(4*(a^2*e^2 + a*c*d^2)) - (log(9*a^3*e^6*
(-a*c)^(3/2) + c^3*d^6*(-a*c)^(3/2) + a*c^5*d^6*x^2 + 9*a^4*c^2*e^6*x^2 - 1
9*a*d^2*e^4*(-a*c)^(5/2) - 11*c*d^4*e^2*(-a*c)^(5/2) + 11*a^2*c^4*d^4*e^2*x
^2 + 19*a^3*c^3*d^2*e^4*x^2)*(a*e + d*(-a*c)^(1/2)))/(4*(a^2*e^2 + a*c*d^2)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.234 $\int \frac{1}{x(d+ex^2)(a+cx^4)} dx$

Optimal. Leaf size=114

$$-\frac{cd\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)}-\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}+\frac{\log(x)}{ad}$$

```
[Out] ln(x)/a/d-1/2*e^2*ln(e*x^2+d)/d/(a*e^2+c*d^2)-1/4*c*d*ln(c*x^4+a)/a/(a*e^2+
c*d^2)-1/2*e*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^2+c*d^2)/a^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 894, 635, 205, 260}

$$-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)}-\frac{cd\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}+\frac{\log(x)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(Sqrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)) + Lo
g[x]/(a*d) - (e^2*Log[d + e*x^2])/(2*d*(c*d^2 + a*e^2)) - (c*d*Log[a + c*x^
4])/(4*a*(c*d^2 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+cx^2\right)}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx}-\frac{e^3}{d\left(cd^2+ae^2\right)(d+ex)}-\frac{c(ae+cdx)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx,x,x^2\right)$$
$$=\frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)}-\frac{c\,\text{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx,x,x^2\right)}{2a\left(cd^2+ae^2\right)}$$
$$=\frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)}-\frac{\left(c^2d\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{2a\left(cd^2+ae^2\right)}-\frac{(ce)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\right.}{2\left(cd^2+ae\right.}$$
$$=-\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^2+ae^2\right)}+\frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)}-\frac{cd\log\left(a+cx^4\right)}{4a\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.07, size = 134, normalized size = 1.18

$$\frac{-cd^2\log\left(a+cx^4\right)+2\sqrt{a}\sqrt{c}\,de\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\sqrt{a}\sqrt{c}\,de\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)-2ae^2\log\left(d+ex^2\right)+4ae}{4a^2de^2+4acd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*
Sqrt[c]*d*e*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c*d^2*Log[x] + 4*a*
e^2*Log[x] - 2*a*e^2*Log[d + e*x^2] - c*d^2*Log[a + c*x^4])/(4*a*c*d^3 + 4*
a^2*d*e^2)
```

fricas [A] time = 11.29, size = 201, normalized size = 1.76

$$\left[\frac{ade\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-cd^2\log\left(cx^4+a\right)-2\,ae^2\log\left(ex^2+d\right)+4\left(cd^2+ae^2\right)\log(x)}{4\left(acd^3+a^2de^2\right)},\frac{2\,ade\sqrt{\frac{c}{a}}\,\text{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*e*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) -
c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x))/(
a*c*d^3 + a^2*d*e^2), 1/4*(2*a*d*e*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) -
c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x))/(
a*c*d^3 + a^2*d*e^2)]
```

giac [A] time = 0.29, size = 102, normalized size = 0.89

$$-\frac{cd\log\left(cx^4+a\right)}{4\left(acd^2+a^2e^2\right)}-\frac{c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(cd^2+ae^2\right)\sqrt{ac}}-\frac{e^3\log\left(\left|x^2e+d\right|\right)}{2\left(cd^3e+ade^3\right)}+\frac{\log\left(x^2\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*c*d*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*c*arctan(c*x^2/sqrt(a*c))
*e/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/2*e^3*log(abs(x^2*e + d))/(c*d^3*e + a*d
*e^3) + 1/2*log(x^2)/(a*d)
```

maple [A] time = 0.01, size = 101, normalized size = 0.89

$$-\frac{ce\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\left(a\,e^2+c\,d^2\right)\sqrt{ac}}-\frac{cd\ln\left(c\,x^4+a\right)}{4\left(a\,e^2+c\,d^2\right)a}-\frac{e^2\ln\left(e\,x^2+d\right)}{2\left(a\,e^2+c\,d^2\right)d}+\frac{\ln(x)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] ln(x)/a/d-1/4*c*d*ln(c*x^4+a)/a/(a*e^2+c*d^2)-1/2*c/(a*e^2+c*d^2)*e/(a*c)^(
1/2)*arctan(1/(a*c)^(1/2)*c*x^2)-1/2*e^2*ln(e*x^2+d)/d/(a*e^2+c*d^2)
```

maxima [A] time = 1.97, size = 101, normalized size = 0.89

$$-\frac{cd\log\left(cx^4+a\right)}{4\left(acd^2+a^2e^2\right)}-\frac{e^2\log\left(ex^2+d\right)}{2\left(cd^3+ade^2\right)}-\frac{ce\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}}+\frac{\log\left(x^2\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/4*c*d*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*e^2*log(e*x^2 + d)/(c*d^3
 + a*d*e^2) - 1/2*c*e*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) +
 1/2*log(x^2)/(a*d)
```

mupad [B] time = 0.96, size = 527, normalized size = 4.62

$$\ln\Big(64\,a^7\,c\,e^{10}\,x^2-64\,a^6\,e^{10}\,\sqrt{-a^3\,c}-25\,a\,c^5\,d^{10}\,\sqrt{-a^3\,c}+25\,a^2\,c^6\,d^{10}\,x^2+180\,a^2\,d^2\,e^8\,\left(-a^3\,c\right)^{3/2}-41\,c^2\,d^6\,e^4\,\big(-a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(64*a^7*c*e^10*x^2 - 64*a^6*e^10*(-a^3*c)^(1/2) - 25*a*c^5*d^10*(-a^3*c
)^(1/2) + 25*a^2*c^6*d^10*x^2 + 180*a^2*d^2*e^8*(-a^3*c)^(3/2) - 41*c^2*d^6
*e^4*(-a^3*c)^(3/2) - 9*a^3*c^5*d^8*e^2*x^2 - 41*a^4*c^4*d^6*e^4*x^2 + 109*
a^5*c^3*d^4*e^6*x^2 + 180*a^6*c^2*d^2*e^8*x^2 + 9*a^2*c^4*d^8*e^2*(-a^3*c)^
(1/2) + 109*a*c*d^4*e^6*(-a^3*c)^(3/2))*(e*(-a^3*c)^(1/2) - a*c*d))/(4*a^3*
e^2 + 4*a^2*c*d^2) - (log(64*a^6*e^10*(-a^3*c)^(1/2) + 64*a^7*c*e^10*x^2 +
25*a*c^5*d^10*(-a^3*c)^(1/2) + 25*a^2*c^6*d^10*x^2 - 180*a^2*d^2*e^8*(-a^3*
c)^(3/2) + 41*c^2*d^6*e^4*(-a^3*c)^(3/2) - 9*a^3*c^5*d^8*e^2*x^2 - 41*a^4*c
^4*d^6*e^4*x^2 + 109*a^5*c^3*d^4*e^6*x^2 + 180*a^6*c^2*d^2*e^8*x^2 - 9*a^2*
c^4*d^8*e^2*(-a^3*c)^(1/2) - 109*a*c*d^4*e^6*(-a^3*c)^(3/2))*(e*(-a^3*c)^(1
/2) + a*c*d))/(4*(a^3*e^2 + a^2*c*d^2)) - (e^2*log(d + e*x^2))/(2*c*d^3 + 2
*a*d*e^2) + log(x)/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.235 $\int \frac{1}{x^3(d+ex^2)(a+cx^4)} dx$

Optimal. Leaf size=129

$$-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)}-\frac{e\log(x)}{ad^2}-\frac{1}{2adx^2}$$

```
[Out] -1/2/a/d/x^2-1/2*c^(3/2)*d*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2
)-e*ln(x)/a/d^2+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)+1/4*c*e*ln(c*x^4+a)/a
/(a*e^2+c*d^2)
```

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 894, 635, 205, 260}

$$-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{e\log(x)}{ad^2}-\frac{1}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/(2*a*d*x^2) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^
2 + a*e^2)) - (e*Log[x])/(a*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 + a*e
^2)) + (c*e*Log[a + c*x^4])/(4*a*(c*d^2 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx^2}-\frac{e}{ad^2x}+\frac{e^4}{d^2\left(cd^2+ae^2\right)(d+ex)}-\frac{c^2(d-ex)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx$$

$$= -\frac{1}{2adx^2}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)}-\frac{c^2\,\text{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx,x,x^2\right)}{2a\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{2adx^2}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)}-\frac{\left(c^2d\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2a\left(cd^2+ae^2\right)}+\frac{\left(c^2e\right)\text{Su}}{2}$$

$$= -\frac{1}{2adx^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^2+ae^2\right)}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.10, size = 169, normalized size = 1.31

$$\frac{2c^{3/2}d^3x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2c^{3/2}d^3x^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)+\sqrt{a}\left(-4ex^2\log(x)\left(ae^2+cd^2\right)+cd^2ex^2\log\left(a+cx\right.\right.}{4a^{3/2}d^2x^2\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (2*c^(3/2)*d^3*x^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(3/2)*d^3*
x^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*(-2*c*d^3 - 2*a*d*e^2
 - 4*e*(c*d^2 + a*e^2)*x^2*Log[x] + 2*a*e^3*x^2*Log[d + e*x^2] + c*d^2*e*x^
2*Log[a + c*x^4]))/(4*a^(3/2)*d^2*(c*d^2 + a*e^2)*x^2)
```

fricas [A] time = 68.84, size = 265, normalized size = 2.05

$$\left[\frac{cd^3x^2\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+cd^2ex^2\log\left(cx^4+a\right)+2\,ae^3x^2\log\left(ex^2+d\right)-2\,cd^3-2\,ade^2-4\left(cd^2e+ae^3\right)x}{4\left(acd^4+a^2d^2e^2\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(c*d^3*x^2*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)
) + c*d^2*e*x^2*log(c*x^4 + a) + 2*a*e^3*x^2*log(e*x^2 + d) - 2*c*d^3 - 2*a
*d*e^2 - 4*(c*d^2*e + a*e^3)*x^2*log(x))/((a*c*d^4 + a^2*d^2*e^2)*x^2), 1/4
*(2*c*d^3*x^2*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + c*d^2*e*x^2*log(c*x^4
 + a) + 2*a*e^3*x^2*log(e*x^2 + d) - 2*c*d^3 - 2*a*d*e^2 - 4*(c*d^2*e + a*e
^3)*x^2*log(x))/((a*c*d^4 + a^2*d^2*e^2)*x^2)]
```

giac [A] time = 0.30, size = 132, normalized size = 1.02

$$-\frac{c^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(acd^2+a^2e^2\right)\sqrt{ac}}+\frac{ce\log\left(cx^4+a\right)}{4\left(acd^2+a^2e^2\right)}+\frac{e^4\log\left(\left|x^2e+d\right|\right)}{2\left(cd^4e+ad^2e^3\right)}-\frac{e\log\left(x^2\right)}{2\,ad^2}+\frac{x^2e-d}{2\,ad^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*c^2*d*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) + 1/4*c*
e*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) + 1/2*e^4*log(abs(x^2*e + d))/(c*d^4*e
 + a*d^2*e^3) - 1/2*e*log(x^2)/(a*d^2) + 1/2*(x^2*e - d)/(a*d^2*x^2)
```

maple [A] time = 0.01, size = 119, normalized size = 0.92

$$-\frac{c^{2} d \arctan\left(\frac{c\,x^{2}}{\sqrt{a c}}\right)}{2\left(a\,e^{2}+c\,d^{2}\right)\sqrt{a c}\;a}+\frac{c e \ln\left(c\,x^{4}+a\right)}{4\left(a\,e^{2}+c\,d^{2}\right)a}+\frac{e^{3} \ln\left(e\,x^{2}+d\right)}{2\left(a\,e^{2}+c\,d^{2}\right)d^{2}}-\frac{e \ln(x)}{a\,d^{2}}-\frac{1}{2 a d\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/2/a/d/x^2-e*ln(x)/a/d^2+1/4*c*e*ln(c*x^4+a)/a/(a*e^2+c*d^2)-1/2*c^2/(a*e
^2+c*d^2)/a*d/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)+1/2*e^3*ln(e*x^2+d)/d
^2/(a*e^2+c*d^2)
```

maxima [A] time = 2.00, size = 120, normalized size = 0.93

$$\frac{e^{3} \log\left(ex^{2}+d\right)}{2\left(cd^{4}+ad^{2}e^{2}\right)}-\frac{c^{2} d \arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{2\left(acd^{2}+a^{2}e^{2}\right)\sqrt{ac}}+\frac{ce \log\left(cx^{4}+a\right)}{4\left(acd^{2}+a^{2}e^{2}\right)}-\frac{e \log\left(x^{2}\right)}{2\,ad^{2}}-\frac{1}{2\,adx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*e^3*log(e*x^2 + d)/(c*d^4 + a*d^2*e^2) - 1/2*c^2*d*arctan(c*x^2/sqrt(a*
c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) + 1/4*c*e*log(c*x^4 + a)/(a*c*d^2 + a^2
*e^2) - 1/2*e*log(x^2)/(a*d^2) - 1/2/(a*d*x^2)
```

mupad [B] time = 1.38, size = 820, normalized size = 6.36

$$\ln\left(a^{6} c^{12} d^{16} x^{2}+64\,a^{14} c^{4} e^{16} x^{2}+a^{2} c^{7} d^{16}\left(-a^{3} c^{3}\right)^{3/2}-64\,a^{13} c^{2} e^{16}\sqrt{-a^{3} c^{3}}+63\,a^{3} d^{8} e^{8}\left(-a^{3} c^{3}\right)^{5/2}+224\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(a^6*c^12*d^16*x^2 + 64*a^14*c^4*e^16*x^2 + a^2*c^7*d^16*(-a^3*c^3)^(3/
2) - 64*a^13*c^2*e^16*(-a^3*c^3)^(1/2) + 63*a^3*d^8*e^8*(-a^3*c^3)^(5/2) +
224*a^9*d^2*e^14*(-a^3*c^3)^(3/2) - 28*c^3*d^14*e^2*(-a^3*c^3)^(5/2) + 28*a
^7*c^11*d^14*e^2*x^2 + 114*a^8*c^10*d^12*e^4*x^2 + 108*a^9*c^9*d^10*e^6*x^2
 - 63*a^10*c^8*d^8*e^8*x^2 - 32*a^11*c^7*d^6*e^10*x^2 + 212*a^12*c^6*d^4*e^
12*x^2 + 224*a^13*c^5*d^2*e^14*x^2 - 114*a*c^2*d^12*e^4*(-a^3*c^3)^(5/2) -
108*a^2*c*d^10*e^6*(-a^3*c^3)^(5/2) + 212*a^8*c*d^4*e^12*(-a^3*c^3)^(3/2) -
 32*a^7*c^2*d^6*e^10*(-a^3*c^3)^(3/2))*(d*(-a^3*c^3)^(1/2) + a^2*c*e))/(4*a
^4*e^2 + 4*a^3*c*d^2) - (log(a^6*c^12*d^16*x^2 + 64*a^14*c^4*e^16*x^2 - a^2
*c^7*d^16*(-a^3*c^3)^(3/2) + 64*a^13*c^2*e^16*(-a^3*c^3)^(1/2) - 63*a^3*d^8
*e^8*(-a^3*c^3)^(5/2) - 224*a^9*d^2*e^14*(-a^3*c^3)^(3/2) + 28*c^3*d^14*e^2
*(-a^3*c^3)^(5/2) + 28*a^7*c^11*d^14*e^2*x^2 + 114*a^8*c^10*d^12*e^4*x^2 +
108*a^9*c^9*d^10*e^6*x^2 - 63*a^10*c^8*d^8*e^8*x^2 - 32*a^11*c^7*d^6*e^10*x
^2 + 212*a^12*c^6*d^4*e^12*x^2 + 224*a^13*c^5*d^2*e^14*x^2 + 114*a*c^2*d^12
*e^4*(-a^3*c^3)^(5/2) + 108*a^2*c*d^10*e^6*(-a^3*c^3)^(5/2) - 212*a^8*c*d^4
*e^12*(-a^3*c^3)^(3/2) + 32*a^7*c^2*d^6*e^10*(-a^3*c^3)^(3/2))*(d*(-a^3*c^3
)^(1/2) - a^2*c*e))/(4*(a^4*e^2 + a^3*c*d^2)) + (e^3*log(d + e*x^2))/(2*c*d
^4 + 2*a*d^2*e^2) - 1/(2*a*d*x^2) - (e*log(x))/(a*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.236 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=156

$$\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{c^2d\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)}-\frac{\log(x)\left(cd^2-ae^2\right)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)}+\frac{e}{2ad^2x^2}-\frac{1}{4adx^4}$$

```
[Out] -1/4/a/d/x^4+1/2*e/a/d^2/x^2+1/2*c^(3/2)*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(3
/2)/(a*e^2+c*d^2)-(-a*e^2+c*d^2)*ln(x)/a^2/d^3-1/2*e^4*ln(e*x^2+d)/d^3/(a*e
^2+c*d^2)+1/4*c^2*d*ln(c*x^4+a)/a^2/(a*e^2+c*d^2)
```

Rubi [A] time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 894, 635, 205, 260}

$$\frac{c^2d\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}-\frac{\log(x)\left(cd^2-ae^2\right)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)}+\frac{e}{2ad^2x^2}-\frac{1}{4adx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/(4*a*d*x^4) + e/(2*a*d^2*x^2) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*a^(3/2)*(c*d^2 + a*e^2)) - ((c*d^2 - a*e^2)*Log[x])/(a^2*d^3) - (e^4*L
og[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)) + (c^2*d*Log[a + c*x^4])/(4*a^2*(c*d
^2 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{1}{adx^3}-\frac{e}{ad^2x^2}+\frac{-cd^2+ae^2}{a^2d^3x}-\frac{e^5}{d^3\left(cd^2+ae^2\right)(d+ex)}+\frac{c^2(ae-}{a^2\left(cd^2+ae\right.}\right.\right.$$
$$= -\frac{1}{4adx^4}+\frac{e}{2ad^2x^2}-\frac{\left(cd^2-ae^2\right)\log(x)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)}+\frac{c^2\operatorname{Subst}\left(\int\frac{ae+cdx}{a+cx^2}\,dx\right.}{2a^2\left(cd^2+ae^2\right)}$$
$$= -\frac{1}{4adx^4}+\frac{e}{2ad^2x^2}-\frac{\left(cd^2-ae^2\right)\log(x)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)}+\frac{\left(c^3d\right)\operatorname{Subst}\left(\int\frac{x}{a+cx^2}\right.}{2a^2\left(cd^2+ae^2\right.}$$
$$= -\frac{1}{4adx^4}+\frac{e}{2ad^2x^2}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^2+ae^2\right)}-\frac{\left(cd^2-ae^2\right)\log(x)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)}+$$

Mathematica [A] time = 0.09, size = 209, normalized size = 1.34

$$-\frac{a^2d^2e^2+2a^2e^4x^4\log\left(d+ex^2\right)-2a^2de^3x^2-4a^2e^4x^4\log(x)+2\sqrt{a}\,c^{3/2}d^3ex^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt{a}\,c^{3/2}d^3ex^4}{4a^2d^3x^4\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/4*(a*c*d^4 + a^2*d^2*e^2 - 2*a*c*d^3*e*x^2 - 2*a^2*d*e^3*x^2 + 2*Sqrt[a]
*c^(3/2)*d^3*e*x^4*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*c^(3
/2)*d^3*e*x^4*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c^2*d^4*x^4*Log[x
] - 4*a^2*e^4*x^4*Log[x] + 2*a^2*e^4*x^4*Log[d + e*x^2] - c^2*d^4*x^4*Log[a
 + c*x^4])/(a^2*d^3*(c*d^2 + a*e^2)*x^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.34, size = 168, normalized size = 1.08

$$\frac{c^2d\log\left(cx^4+a\right)}{4\left(a^2cd^2+a^3e^2\right)}+\frac{c^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(acd^2+a^2e^2\right)\sqrt{ac}}-\frac{e^5\log\left(\left|x^2e+d\right|\right)}{2\left(cd^5e+ad^3e^3\right)}-\frac{\left(cd^2-ae^2\right)\log\left(x^2\right)}{2a^2d^3}+\frac{3cd^2x^4-3ax^4e^2+2adx^2e-ad}{4a^2d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*c^2*d*log(c*x^4 + a)/(a^2*c*d^2 + a^3*e^2) + 1/2*c^2*arctan(c*x^2/sqrt(
a*c))*e/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) - 1/2*e^5*log(abs(x^2*e + d))/(c*d^
5*e + a*d^3*e^3) - 1/2*(c*d^2 - a*e^2)*log(x^2)/(a^2*d^3) + 1/4*(3*c*d^2*x^
4 - 3*a*x^4*e^2 + 2*a*d*x^2*e - a*d^2)/(a^2*d^3*x^4)
```

maple [A] time = 0.02, size = 145, normalized size = 0.93

$$\frac{c^2 e \arctan\left(\frac{c x^2}{\sqrt{ac}}\right)}{2\left(a e^2+c d^2\right)\sqrt{ac}\, a}+\frac{c^2 d \ln\left(c x^4+a\right)}{4\left(a e^2+c d^2\right) a^2}-\frac{e^4 \ln\left(e x^2+d\right)}{2\left(a e^2+c d^2\right) d^3}+\frac{e^2 \ln(x)}{a d^3}-\frac{c \ln(x)}{a^2 d}+\frac{e}{2a d^2 x^2}-\frac{1}{4ad x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/4/a/d/x^4+1/d^3/a*ln(x)*e^2-1/d/a^2*ln(x)*c+1/2*e/a/d^2/x^2+1/4*c^2*d*ln
(c*x^4+a)/a^2/(a*e^2+c*d^2)+1/2*c^2/(a*e^2+c*d^2)/a*e/(a*c)^(1/2)*arctan(1/
(a*c)^(1/2)*c*x^2)-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)
```

maxima [A] time = 2.05, size = 145, normalized size = 0.93

$$-\frac{e^4 \log\left(ex^2+d\right)}{2\left(cd^5+ad^3e^2\right)}+\frac{c^2 d \log\left(cx^4+a\right)}{4\left(a^2cd^2+a^3e^2\right)}+\frac{c^2 e \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(acd^2+a^2e^2\right)\sqrt{ac}}-\frac{\left(cd^2-ae^2\right)\log\left(x^2\right)}{2\,a^2d^3}+\frac{2\,ex^2-d}{4\,ad^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*e^4*log(e*x^2 + d)/(c*d^5 + a*d^3*e^2) + 1/4*c^2*d*log(c*x^4 + a)/(a^2
*c*d^2 + a^3*e^2) + 1/2*c^2*e*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 + a^2*e^2)*
sqrt(a*c)) - 1/2*(c*d^2 - a*e^2)*log(x^2)/(a^2*d^3) + 1/4*(2*e*x^2 - d)/(a*
d^2*x^4)
```

mupad [B] time = 1.87, size = 1017, normalized size = 6.52

$$\frac{\ln\left(25\,a^2\,c^9\,d^{20}\,\left(-a^5\,c^3\right)^{3/2}-64\,a^{19}\,c^4\,e^{20}\,x^2-25\,a^9\,c^{14}\,d^{20}\,x^2-64\,a^{17}\,c^2\,e^{20}\,\sqrt{-a^5\,c^3}+100\,a^3\,d^8\,e^{12}\,\left(-a^5\,c^3\right)^{5/2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(25*a^2*c^9*d^20*(-a^5*c^3)^(3/2) - 64*a^19*c^4*e^20*x^2 - 25*a^9*c^14*
d^20*x^2 - 64*a^17*c^2*e^20*(-a^5*c^3)^(1/2) + 100*a^3*d^8*e^12*(-a^5*c^3)^
(5/2) + 128*a^11*d^2*e^18*(-a^5*c^3)^(3/2) - 112*c^3*d^14*e^6*(-a^5*c^3)^(5
/2) - 76*a^10*c^13*d^18*e^2*x^2 - 138*a^11*c^12*d^16*e^4*x^2 - 112*a^12*c^1
1*d^14*e^6*x^2 + 55*a^13*c^10*d^12*e^8*x^2 + 104*a^14*c^9*d^10*e^10*x^2 + 1
00*a^15*c^8*d^8*e^12*x^2 + 172*a^16*c^7*d^6*e^14*x^2 + 32*a^17*c^6*d^4*e^16
*x^2 - 128*a^18*c^5*d^2*e^18*x^2 + 55*a*c^2*d^12*e^8*(-a^5*c^3)^(5/2) + 104
*a^2*c*d^10*e^10*(-a^5*c^3)^(5/2) - 32*a^10*c*d^4*e^16*(-a^5*c^3)^(3/2) + 7
6*a^3*c^8*d^18*e^2*(-a^5*c^3)^(3/2) + 138*a^4*c^7*d^16*e^4*(-a^5*c^3)^(3/2)
 - 172*a^9*c^2*d^6*e^14*(-a^5*c^3)^(3/2))*(e*(-a^5*c^3)^(1/2) + a^2*c^2*d))
/(4*a^5*e^2 + 4*a^4*c*d^2) - (e^4*log(d + e*x^2))/(2*(c*d^5 + a*d^3*e^2)) -
 (log(25*a^9*c^14*d^20*x^2 + 64*a^19*c^4*e^20*x^2 + 25*a^2*c^9*d^20*(-a^5*c
^3)^(3/2) - 64*a^17*c^2*e^20*(-a^5*c^3)^(1/2) + 100*a^3*d^8*e^12*(-a^5*c^3)
^(5/2) + 128*a^11*d^2*e^18*(-a^5*c^3)^(3/2) - 112*c^3*d^14*e^6*(-a^5*c^3)^(
5/2) + 76*a^10*c^13*d^18*e^2*x^2 + 138*a^11*c^12*d^16*e^4*x^2 + 112*a^12*c^
11*d^14*e^6*x^2 - 55*a^13*c^10*d^12*e^8*x^2 - 104*a^14*c^9*d^10*e^10*x^2 -
100*a^15*c^8*d^8*e^12*x^2 - 172*a^16*c^7*d^6*e^14*x^2 - 32*a^17*c^6*d^4*e^1
6*x^2 + 128*a^18*c^5*d^2*e^18*x^2 + 55*a*c^2*d^12*e^8*(-a^5*c^3)^(5/2) + 10
4*a^2*c*d^10*e^10*(-a^5*c^3)^(5/2) - 32*a^10*c*d^4*e^16*(-a^5*c^3)^(3/2) +
76*a^3*c^8*d^18*e^2*(-a^5*c^3)^(3/2) + 138*a^4*c^7*d^16*e^4*(-a^5*c^3)^(3/2
) - 172*a^9*c^2*d^6*e^14*(-a^5*c^3)^(3/2))*(e*(-a^5*c^3)^(1/2) - a^2*c^2*d)
)/(4*(a^5*e^2 + a^4*c*d^2)) - (1/(4*a*d) - (e*x^2)/(2*a*d^2))/x^4 + (log(x)
*(a*e^2 - c*d^2))/(a^2*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.237 $\int \frac{x^8}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=359

$$-\frac{a^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\left(ae^2+cd^2\right)}-\frac{a}{}$$

```
[Out] -d*x/c/e^2+1/3*x^3/c/e+d^(7/2)*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)/(a*e^2+c*d
^2)+1/4*a^(5/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))
/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*a^(5/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/
4))*(-e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*a^(5/4)*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(7/4)/
(a*e^2+c*d^2)*2^(1/2)+1/8*a^(5/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{a^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\left(ae^2+cd^2\right)}-\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -((d*x)/(c*e^2)) + x^3/(3*c*e) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(
5/2)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(5/4)*(Sqrt
[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7
/4)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) +
(a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^8}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(-\frac{d}{ce^2}+\frac{x^2}{ce}+\frac{d^4}{e^2\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{a^2\left(d-ex^2\right)}{c\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{a^2\int \frac{d-ex^2}{a+cx^4}\,dx}{c\left(cd^2+ae^2\right)}+\frac{d^4\int \frac{1}{d+ex^2}\,dx}{e^2\left(cd^2+ae^2\right)}$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2+ae^2\right)}+\frac{\left(a^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c^2\left(cd^2+ae^2\right)}+\frac{\left(a^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\right)\int \cdots}{2c^2\left(cd^2+\cdots\right.}$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2+ae^2\right)}+\frac{\left(a^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c^2\left(cd^2+ae^2\right)}+\frac{\left(a^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right.\cdots}{4c^2\left(\cdots\right.}$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2+ae^2\right)}-\frac{a^{5/4}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\left(cd^2+ae^2\right)}\cdots$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2+ae^2\right)}-\frac{a^{7/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{7/4}\left(cd^2+ae^2\right)}+\frac{a^{7/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\cdots}{2\sqrt{2}\,c^{7/4}\cdots}$$

Mathematica [A] time = 0.36, size = 344, normalized size = 0.96

$$-3\sqrt{2}ae^{5/2}\left(a^{3/4}e+\sqrt[4]{a}\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+3\sqrt{2}ae^{5/2}\left(a^{3/4}e+\sqrt[4]{a}\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-24*c^(3/4)*d*Sqrt[e]*(c*d^2 + a*e^2)*x + 8*c^(3/4)*e^(3/2)*(c*d^2 + a*e^2
)*x^3 + 24*c^(7/4)*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^(5/4)*
e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
- 6*Sqrt[2]*a^(5/4)*e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*a*e^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*a*e^(5/2)
*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(24*c^(7/4)*e^(5/2)*(c*d^2 + a*e^2))
```

fricas [B] time = 21.11, size = 4414, normalized size = 12.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*c*d^3*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) +
 4*(c*d^2*e + a*e^3)*x^3 - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5
*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2
+ a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2
*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*
c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^
2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*
d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^
8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5
*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*
c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2
 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5*d^4 +
2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e
^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 +
a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 -
 a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^2*e^3 +
 a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4
*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sq
rt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^
4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4
*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*
c^3*e^4))) - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4
*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^
11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7
*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^
2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^
5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10
*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a
^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a
^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 +
4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4
))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^
2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8
+ 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))
```

```
/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x -
(a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*
sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^
2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e
- (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^
2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c
^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) - 1
2*(c*d^3 + a*d*e^2)*x)/(c^2*d^2*e^2 + a*c*e^4), 1/12*(12*c*d^3*sqrt(d/e)*ar
ctan(e*x*sqrt(d/e)/d) + 4*(c*d^2*e + a*e^3)*x^3 - 3*(c^2*d^2*e^2 + a*c*e^4)
*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2
*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*
d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a
^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 +
(c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^
2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c
^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 +
a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*
a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^
5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((
2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 -
2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4
 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*
e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^
4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 +
 a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*
e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3
*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*
d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 +
 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d
*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c
*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^
3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*l
og(-(a^3*c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2
*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^
4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a
^4*c^7*e^8)))*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*s
qrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2
 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^
4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c
^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^
2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d
^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^
3*c*d^2 - a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*
d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^1
1*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*
e^8)))*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a
^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^
2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e
^2 + a^2*c^3*e^4))) - 12*(c*d^3 + a*d*e^2)*x)/(c^2*d^2*e^2 + a*c*e^4)]
```

giac [A] time = 0.54, size = 363, normalized size = 1.01

$$\frac{d^{\frac{7}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2e^2+ae^4}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d-\left(ac^3\right)^{\frac{3}{4}}ae\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,c^5d^2+\sqrt{2}\,ac^4e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d-\left(ac^3\right)^{\frac{3}{4}}ae\right)\arctan\left(\frac{\sqrt{2}}{}\right.}{2\left(\sqrt{2}\,c^5d^2+\sqrt{2}\,ac^4e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e^2 + a*e^4) + 1/2*((a*c^
3)^(1/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/2*((a*c^3)
^(1/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/4*((a*c^3)^(
1/4)*a*c^2*d + (a*c^3)^(3/4)*a*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) - 1/4*((a*c^3)^(1/4)*a*c^2*d + (a
*c^3)^(3/4)*a*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^5*
d^2 + sqrt(2)*a*c^4*e^2) + 1/3*(c^2*x^3*e^2 - 3*c^2*d*x*e)*e^(-3)/c^3
```

maple [A] time = 0.02, size = 405, normalized size = 1.13

$$\frac{d^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a e^2+c d^2\right)\sqrt{de}\,e^2}-\frac{\sqrt{2}\,a^2 e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a e^2+c d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}c^2}-\frac{\sqrt{2}\,a^2 e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a e^2+c d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}c^2}-\frac{\sqrt{2}\,a^2 e\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(a e^2+c d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}c^2}+\frac{\left(\frac{a}{c}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/3*x^3/c/e-d*x/c/e^2+1/8*a/(a*e^2+c*d^2)/c*d*(a/c)^(1/4)*2^(1/2)*ln((x^2+(
a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+
1/4*a/(a*e^2+c*d^2)/c*d*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)
+1/4*a/(a*e^2+c*d^2)/c*d*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)-1/8*a^2/(a*e^2+c*d^2)/c^2*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(
1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))-1/4*a^2/(a*e^2+c
*d^2)/c^2*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/4*a^2/(a*
e^2+c*d^2)/c^2*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/e^2*
d^4/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

maxima [A] time = 2.05, size = 294, normalized size = 0.82

$$\frac{d^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2e^2+ae^4\right)\sqrt{de}}+\frac{a^2\left(\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d+\cdots\right.}{\cdots}\cdots\right)}{8\left(c^2d^2+ace^2\right)}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] d^4*arctan(e*x/sqrt(d*e))/((c*d^2*e^2 + a*e^4)*sqrt(d*e)) + 1/8*a^2*(2*sqrt
(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/
4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c))
+ 2*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*s
qrt(c)) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c
^2*d^2 + a*c*e^2) + 1/3*(e*x^3 - 3*d*x)/(c*e^2)
```

mupad [B] time = 2.06, size = 6097, normalized size = 16.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(a^7*d^4*e^26 + 16*c^7*d^18*e^12 - 16*c^7*x*(-d^7*e^5)^(5/2) + 2*a^6*c*
d^6*e^24 + 16*a^3*c^4*d^12*e^18 + a^5*c^2*d^8*e^22 - a^7*e^24*x*(-d^7*e^5)^
(1/2) - a^5*c^2*d^4*e^20*x*(-d^7*e^5)^(1/2) + 16*a^3*c^4*d*e^11*x*(-d^7*e^5
)^(3/2) - 2*a^6*c*d^2*e^22*x*(-d^7*e^5)^(1/2))*(-d^7*e^5)^(1/2))/(2*a*e^7 +
 2*c*d^2*e^5) - atan((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*
a^5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^
7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(
1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a
^4*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1
/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)
+ (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^
5*d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) +
2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*
a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4
*a^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1
/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)
- (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e
^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d
^2*e^2)))^(1/2)*1i - (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a
^5*c^6*d^2*e^9)/(c^3*e^3) + (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7
)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1
/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^
4*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/
2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) -
 (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5
*d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2
*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a
^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*
a^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/
2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) +
 (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^
2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^
2*e^2)))^(1/2)*1i)/((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^
5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)
^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/
2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4
*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2
) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) +
(2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*
d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*
a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^
3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a
^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2
) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) -
(2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2
*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2
*e^2)))^(1/2) + (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^
6*d^2*e^9)/(c^3*e^3) + (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/
2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(
256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8
*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) +
2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x
*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*
e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*a^3*
c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^
6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c
```

^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*(a^7*d^4*e^3 - a^6*c*d^6*e))/(c^3*e^3)))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*2i - (log(a^7*d^4*e^26 + 16*c^7*d^18*e^12 + 16*c^7*x*(-d^7*e^5)^(5/2) + 2*a^6*c*d^6*e^24 + 16*a^3*c^4*d^12*e^18 + a^5*c^2*d^8*e^22 + a^7*e^24*x*(-d^7*e^5)^(1/2) + a^5*c^2*d^4*e^20*x*(-d^7*e^5)^(1/2) - 16*a^3*c^4*d*e^11*x*(-d^7*e^5)^(3/2) + 2*a^6*c*d^2*e^22*x*(-d^7*e^5)^(1/2))*(-d^7*e^5)^(1/2))/(2*(a*e^7 + c*d^2*e^5)) - atan((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*1i - (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) + (2*x*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*1i)/((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) + (2*x*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a

```
^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) -
 c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c
^8*d^2*e^2)))^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^
2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(
1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 +
 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((a*
e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4
 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*(a^7*d^4*e^3 - a^6*c*d^6*e))
/(c^3*e^3)))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*
d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*2i + x^3/(3*c*e)
 - (d*x)/(c*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.238 $\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=345

$$-\frac{a^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}+\cdots$$

```
[Out] x/c/e-d^(5/2)*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)/(a*e^2+c*d^2)-1/8*a^(3/4)*l
n(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/c^
(5/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*a^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*a^(
3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/c^(5/4)/(a*
e^2+c*d^2)*2^(1/2)-1/4*a^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/
2)+d*c^(1/2))/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2)) +
 (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) -
(a^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) + (a^(3/4)*(Sqrt[c]*d - S
qrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2
]*c^(5/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^6}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{1}{ce} - \frac{d^3}{e\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{a\left(ae+cdx^2\right)}{c\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= \frac{x}{ce} - \frac{a\int \frac{ae+cdx^2}{a+cx^4}\,dx}{c\left(cd^2+ae^2\right)} - \frac{d^3\int \frac{1}{d+ex^2}\,dx}{e\left(cd^2+ae^2\right)}$$

$$= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)} + \frac{\left(a\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)} - \frac{\left(a\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)}$$

$$= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)} - \frac{\left(a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)} - \frac{\left(a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{}{-}}{4\sqrt{2}c^{3/4}\left(cd^2\right.}$$

$$= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)} - \frac{a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)} + \frac{a^{3/4}\left(d-\right.}{}$$

$$= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)} + \frac{a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}\left(cd^2+ae^2\right)} - \frac{a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\text{ta}}{2\sqrt{2}c^{5/4}\left(c\right.}$$

Mathematica [A] time = 0.24, size = 373, normalized size = 1.08

$$-\frac{\left(a^{3/4}cd-a^{5/4}\sqrt{c}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\left(ae^2+cd^2\right)}+\frac{\left(a^{3/4}cd-a^{5/4}\sqrt{c}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\left(ae^2+cd^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2)) -
 ((a^(3/4)*c*d + a^(5/4)*Sqrt[c]*e)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*c^(1/4)*
x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d
+ a^(5/4)*Sqrt[c]*e)*ArcTan[(Sqrt[2]*a^(1/4) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4
))])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*
e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/
4)*(c*d^2 + a*e^2)) + ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*e)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

fricas [B] time = 3.58, size = 4354, normalized size = 12.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*c*d^2*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) +
(c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c
^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*
d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 +
 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^
2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^
2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d
^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2
*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^
4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4
*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d^2*e + a*c*
e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^
3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c
^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2
+ a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 -
 (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d
^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^
6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 +
a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a
*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*
d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2
*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4
*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^
3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*l
og(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c
^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)
/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c
^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt
(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*
a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2
*e^2 + a^2*c^2*e^4))) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (c^4*d^4 +
 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*
e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a
^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 -
```

```
a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*
c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c
^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(
2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 -
2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 +
 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^
4))) + 4*(c*d^2 + a*e^2)*x)/(c^2*d^2*e + a*c*e^3), -1/4*(4*c*d^2*sqrt(d/e)*
arctan(e*x*sqrt(d/e)/d) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4
 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^
5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 +
 a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2
- a^3*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^
2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a
*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(
-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4
- 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4
 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*
e^4))) + (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^
2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 +
 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(
c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a
^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt
(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*
a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^
4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2
 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*
e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d^2
*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*
sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2
+ 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3
*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^2*e - a^
3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 -
2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 +
 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d
^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*
d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8
)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*d^2*e + a*c*e^3)*sqr
t(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^
4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e
^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^
2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^
5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 +
 a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^
6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*
e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6
*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*
a*c^3*d^2*e^2 + a^2*c^2*e^4))) - 4*(c*d^2 + a*e^2)*x)/(c^2*d^2*e + a*c*e^3)
]
```

giac [A] time = 0.44, size = 333, normalized size = 0.97

$$-\frac{d^{\frac{5}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2e+ae^3}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x\right.}{2}\right.}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e + a*e^3) - 1/2*((a*c^3
)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/2*((a*c^3)^(1/4
)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) + x*e^(-1)/c - 1/4*((a*c^
3)^(1/4)*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*a*c*e - (a*c
^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2
+ sqrt(2)*a*c^3*e^2)
```

maple [A] time = 0.01, size = 387, normalized size = 1.12

$$-\frac{d^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{de}\;e}-\frac{\sqrt{2}\;ad\arctan\left(\frac{\sqrt{2}\;x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^2+c\,d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}c}-\frac{\sqrt{2}\;ad\arctan\left(\frac{\sqrt{2}\;x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^2+c\,d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}c}-\frac{\sqrt{2}\;ad\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;x+\sqrt{\frac{a}{c}}}\right)}{8\left(a\,e^2+c\,d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] x/c/e-1/4*a/(a*e^2+c*d^2)/c*e*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x-1)-1/8*a/(a*e^2+c*d^2)/c*e*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4*a/(a*e^2+c*
d^2)/c*e*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/8*a/(a*e^2+c
*d^2)/c*d/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x
^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4*a/(a*e^2+c*d^2)/c*d/(a/c)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/4*a/(a*e^2+c*d^2)/c*d/(a/c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/e*d^3/(a*e^2+c*d^2)/(d*e)^(1/2)*
arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.09, size = 289, normalized size = 0.84

$$-\frac{d^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2e+ae^3\right)\sqrt{de}}-\frac{a\left(\frac{2\sqrt{2}\left(\sqrt{a}\,cd+a\sqrt{c}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{a}\,cd+a\sqrt{c}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\left(\sqrt{a}\cdots\right.}{\cdots}\cdots\right)}{8\left(c^2d^2+ace^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -d^3*arctan(e*x/sqrt(d*e))/((c*d^2*e + a*e^3)*sqrt(d*e)) - 1/8*a*(2*sqrt(2)
*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1
/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c))
 + 2*sqrt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x -
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
c))*sqrt(c)) - sqrt(2)*(sqrt(a)*c*d - a*sqrt(c)*e)*log(sqrt(c)*x^2 + sqrt(2
)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(sqrt(a)*c*d - a
*sqrt(c)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)
*c^(3/4)))/(c^2*d^2 + a*c*e^2) + x/(c*e)
```

mupad [B] time = 1.83, size = 5908, normalized size = 17.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/((a + c*x^4)*(d + e*x^2)),x)

[Out] atan(((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*1i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*1i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*(a^4*c*d^5 - a^5*d^3*e^2))/(c*e)))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*2i + atan(((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e

^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*
e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16
*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^6*d^7*e
 - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(c*
d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4
 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*
d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3
*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))
)^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2)
- a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*
c^6*d^2*e^2)))^(1/2)*1i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*
a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^
(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2
)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c
^6*d^2*e^8))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*
a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(
64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^
5))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d
e)/(16(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6
*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/
2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2
*a*c^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(c*d^2*(
-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^
2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*1i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^
6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-(c*d^2*(-a^3*c^5)^(1/2) - a
e^2(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6
*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4
*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3
*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))
)^(1/2) + (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 1
6*a^4*c^4*d^3*e^5))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/
2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) -
 (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(c*d^
2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 +
 a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(
c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(
16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (((((64*a^5*c^4*d*e^
8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(c*d^2*(-a^3*c
^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*
e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 25
6*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2)
- a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*
c^6*d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^
5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(
-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e
^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(
c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(
16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^
3*c^3*d^6))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a
^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*(a^4
*c*d^5 - a^5*d^3*e^2))/(c*e)))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)
^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/
2)*2i + x/(c*e) - (log(16*c^5*x*(-d^5*e^3)^(5/2) - 16*c^5*d^13*e^7 - a^5*d^
3*e^17 - 2*a^4*c*d^5*e^15 + 16*a^2*c^3*d^9*e^11 - a^3*c^2*d^7*e^13 + a^5*e^
16*x*(-d^5*e^3)^(1/2) + a^3*c^2*d^4*e^12*x*(-d^5*e^3)^(1/2) + 16*a^2*c^3*d*
e^7*x*(-d^5*e^3)^(3/2) + 2*a^4*c*d^2*e^14*x*(-d^5*e^3)^(1/2))*(-d^5*e^3)^(1
/2))/(2*(a*e^5 + c*d^2*e^3)) + (log(a^5*d^3*e^17 + 16*c^5*d^13*e^7 + 16*c^5
x(-d^5*e^3)^(5/2) + 2*a^4*c*d^5*e^15 - 16*a^2*c^3*d^9*e^11 + a^3*c^2*d^7*
e^13 + a^5*e^16*x*(-d^5*e^3)^(1/2) + a^3*c^2*d^4*e^12*x*(-d^5*e^3)^(1/2) +

```
16*a^2*c^3*d*e^7*x*(-d^5*e^3)^(3/2) + 2*a^4*c*d^2*e^14*x*(-d^5*e^3)^(1/2))*
(-d^5*e^3)^(1/2))/(2*a*e^5 + 2*c*d^2*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.239 $\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$\frac{\sqrt[4]{a}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{a}}{}$$

```
[Out] -1/4*a^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/c^
(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*a^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))
*(-e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*a^(1/4)*ln(-a^(1/
4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*
e^2+c*d^2)*2^(1/2)-1/8*a^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)+d^(3/2)*arctan(x*
e^(1/2)/d^(1/2))/(a*e^2+c*d^2)/e^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)) + (a^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) + (a^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*
(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{d^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{a\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= -\frac{a\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2} + \frac{d^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)} - \frac{\left(a\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)} - \frac{\left(a\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)} - \frac{\left(a\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c\left(cd^2+ae^2\right)} - \frac{\left(a\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}}{4c\left(cd^2+ae^2\right)}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)} + \frac{\sqrt[4]{a}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(cd^2+ae^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{c}\,d+\right.}{}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)} + \frac{a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{3/4}\left(cd^2+ae^2\right)} - \frac{a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\right.}{2\sqrt{2}\,c^{3/4}\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.15, size = 233, normalized size = 0.69

$$\frac{\sqrt{2}\sqrt[4]{a}\sqrt{e}\left(\left(\sqrt{a}e+\sqrt{c}d\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)+2\left(\sqrt{c}d-\right.\right.}{8c^{3/4}\sqrt{e}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*c^(3/4)*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*a^(1/4)*Sqrt[e]*(2
*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (-2*Sqrt
[c]*d + 2*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*c^(3/4)*Sqrt[e]*(c*
d^2 + a*e^2))
```

fricas [B] time = 1.68, size = 4040, normalized size = 12.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*((c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 + (c^
4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2
 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*
e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4
)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
+ 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2
*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d
^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)
))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d
^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d
^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*
e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2
*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^
8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))
)/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) + (c*d^2 + a*e^2)*sqrt((2*a*d*e
 - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e
^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^
2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2
 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2
*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*
e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e
- (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^
2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 +
a*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2
*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^
4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2
*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c
^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^
7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e
^8)))*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*
d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4
```

```
*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*
c*e^4))) + 2*d*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)))/
(c*d^2 + a*e^2), 1/4*(4*d*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + (c*d^2 + a*e^
2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4
 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^
4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4))*log(-(c*d^2 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d
^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^
8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))
)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4
- 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4
 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^
2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c
^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 + (c
^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^
2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4))) + (c*d^2 + a*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c
^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*
d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8
)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x + (c^2*
d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*
d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4
*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d
^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)
))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*
e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*
e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d
^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^
2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^
2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6
*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e
 - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e
^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^
2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))))/(c*d^2 +
a*e^2)]
```

giac [A] time = 0.43, size = 327, normalized size = 0.97

$$\frac{d^{\frac{3}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2+ae^2}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\right.}{2\left(\frac{a}{c}\right.}\right.}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2 + a*e^2) - 1/2*((a*c^3)^(
1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/2*((a*c^3)^(1/4)*c
^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d +
 (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4
```

```
*d^2 + sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*
e^2)
```

maple [A] time = 0.01, size = 363, normalized size = 1.08

$$\frac{d^{2}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a e^{2}+c d^{2}\right)\sqrt{de}}+\frac{\sqrt{2}\,ae\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a e^{2}+c d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,ae\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a e^{2}+c d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,ae\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(a e^{2}+c d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/8/(a*e^2+c*d^2)*d*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c
)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4/(a*e^2+c*d^2)*d*(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/4/(a*e^2+c*d^2)*d*(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/8*a/(a*e^2+c*d^2)*e/c/(a/c)^(
1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^
(1/2)*x+(a/c)^(1/2)))+1/4*a/(a*e^2+c*d^2)*e/c/(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x+1)+1/4*a/(a*e^2+c*d^2)*e/c/(a/c)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/c)^(1/4)*x-1)+d^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*
e*x)
```

maxima [A] time = 2.63, size = 268, normalized size = 0.80

$$\frac{d^{2}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^{2}+ae^{2}\right)\sqrt{de}}-\frac{a\left(\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\cdots\right.}{\cdots}\right.}{8\left(cd^{2}+ae^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] d^2*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) - 1/8*a*(2*sqrt(2)*(s
qrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(
1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sq
rt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(
1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)
) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqr
t(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c*d^2 +
 a*e^2)
```

mupad [B] time = 2.20, size = 5111, normalized size = 15.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(-((((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c
^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 11
```

2*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) + ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-
a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))
^(1/2)*(64*a^2*c^6*d^6*e^2 - x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2
) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512
*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*
e^7) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) -
c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^
2*e^2)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) -
x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)
^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)
*1i + (((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^
5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112
*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) - ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a
*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^
(1/2)*(x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(
c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2
*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*
e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) - c
d^2(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2
*e^2)))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) -
x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^
(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*
1i)/((((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5
*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*
a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) + ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*
c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(
1/2)*(64*a^2*c^6*d^6*e^2 - x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2)
+ 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a
^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^
7) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) - c*
d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*
e^2)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x
*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(
1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) -
(((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4
+ a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*
c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) - ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)
^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)
(x((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d
^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*
d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 +
128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*
(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)
))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*
a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2)
+ 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) + 2*a
^3*c*d^2*e^2))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)
/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*2i + atan(-((((c*d^2
*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^
3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e
^2 - 32*a^3*c^4*d^3*e^4) + ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) +
2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(64*a^2*
c^6*d^6*e^2 - x*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e
)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 -
512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 128*a^3*
c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^
(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)
+ 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5
+ 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^

```
2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*1i + (((c*d^2*
(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3
*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^
2 - 32*a^3*c^4*d^3*e^4) - ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2
*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(x*((c*d^
2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c
^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 -
512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c
^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(
1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) -
 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5
+ 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2
*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*1i)/((((c*d^2*(
-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*
e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^2
 - 32*a^3*c^4*d^3*e^4) + ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*
a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(64*a^2*c^
6*d^6*e^2 - x*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/
(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 51
2*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 128*a^3*c^
5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1
/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) +
16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5 +
 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*
d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) - (((c*d^2*(-a*c
^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4
+ 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^2 - 3
2*a^3*c^4*d^3*e^4) - ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^
2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(x*((c*d^2*(-a
*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^
4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a
^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^
4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2)
+ 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) - 16*a
^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5 + 4*a
^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)
/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) + 2*a^3*c*d^2*e^2))*
((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 +
 a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*2i - (log(16*c^3*x*(-d^3*e)^(5/2) +
 a^3*d^2*e^8 + 16*c^3*d^8*e^2 + 17*a*c^2*d^6*e^4 + 2*a^2*c*d^4*e^6 + a^3*e^
8*x*(-d^3*e)^(1/2) - 17*a*c^2*d*e^3*x*(-d^3*e)^(3/2) + 2*a^2*c*d^2*e^6*x*(-
d^3*e)^(1/2))*(-d^3*e)^(1/2))/(2*(a*e^3 + c*d^2*e)) + (log(a^3*d^2*e^8 - 16
*c^3*x*(-d^3*e)^(5/2) + 16*c^3*d^8*e^2 + 17*a*c^2*d^6*e^4 + 2*a^2*c*d^4*e^6
 - a^3*e^8*x*(-d^3*e)^(1/2) + 17*a*c^2*d*e^3*x*(-d^3*e)^(3/2) - 2*a^2*c*d^2
*e^6*x*(-d^3*e)^(1/2))*(-d^3*e)^(1/2))/(2*a*e^3 + 2*c*d^2*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.240 $\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=337

$$\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}-\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\right.}{ae^2+cd^2}$$

```
[Out] 1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2
))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)*2^
(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(1/4
)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*
a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-arctan(x*e^(1/2)/d
^(1/2))*d^(1/2)*e^(1/2)/(a*e^2+c*d^2)
```

Rubi [A] time = 0.27, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}-\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\right.}{ae^2+cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -((Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)) - ((Sqrt[c
]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4
)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]
*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)
*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx &= \int \left(-\frac{de}{\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{ae+cdx^2}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right) dx\\
&= \frac{\int \frac{ae+cdx^2}{a+cx^4}\,dx}{cd^2+ae^2}-\frac{(de)\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}\\
&= -\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{cd^2+ae^2}-\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)}+\frac{\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)}\\
&= -\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{cd^2+ae^2}+\frac{\left(\sqrt[4]{c}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,\sqrt[4]{a}\left(cd^2+ae^2\right)}+\frac{\left(\sqrt[4]{c}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int}{4\sqrt{2}\,\sqrt[4]{a}\,(}\\
&= -\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{cd^2+ae^2}+\frac{\sqrt[4]{c}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\left(cd^2+ae^2\right)}-\frac{\sqrt[4]{c}\,(d-}{}\\
&= -\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{cd^2+ae^2}-\frac{\sqrt[4]{c}\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\left(cd^2+ae^2\right)}+\frac{\sqrt[4]{c}\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}(}{2\sqrt{2}\,\sqrt[4]{a}\,(cd^2+}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 232, normalized size = 0.69

$$\frac{\sqrt{2}\left(\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)\right)-2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-}\right.}{8\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a^(1/4)*c^(1/4)*Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*(
-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqr
t[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*d -
Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(1/4)*c^(1/4)*(c*d
^2 + a*e^2))
```

fricas [B] time = 1.20, size = 3892, normalized size = 11.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*s
qrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6
*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 +
2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^
4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4
*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
)) - (c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqr
t(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a
^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2
+ a^2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*
a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*
c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4
- 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e
^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)))
 + (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(
-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3
*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^
2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c
^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*
e^8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) -
 (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^
2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*
c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5
*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^
8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*
a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) - 2
```

```
*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(c*d^2 + a*e^2),
 -1/4*((c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*s
qrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6
*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 +
2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^
4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4
*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
)) - (c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqr
t(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a
^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2
+ a^2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*
a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*
c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4
- 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e
^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)))
 + (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(
-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3
*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^
2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c
^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*
e^8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) -
 (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^
2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*
c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5
*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^
8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*
a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) + 4
*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(c*d^2 + a*e^2)]
```

giac [A] time = 0.38, size = 336, normalized size = 1.00

$$-\frac{\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{cd^2+ae^2}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^3d^2+\sqrt{2}\,a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\right.}{}\right.}{2\left(\sqrt{2}\,ac^3d^2+\sqrt{2}\,a^2c^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c*d^2 + a*e^2) + 1/2*((a*c^3)^(
1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/
4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)
*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqr
t(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*a*c*e - (a*c^3)^
(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2 +
sqrt(2)*a^2*c^2*e^2)
```

maple [A] time = 0.01, size = 351, normalized size = 1.04

$$-\frac{de\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{de}}+\frac{\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^2+c\,d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^2+c\,d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,d\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(a\,e^2+c\,d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,e\,\mathrm{ar}}{4a\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4/(a*e^2+c*d^2)*e*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/8
/(a*e^2+c*d^2)*e*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1
/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4/(a*e^2+c*d^2)*e*(a/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/8/(a*e^2+c*d^2)*d/(a/c)^(1/4)*
2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)
*x+(a/c)^(1/2)))+1/4/(a*e^2+c*d^2)*d/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)+1/4/(a*e^2+c*d^2)*d/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x-1)-d*e/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 1.43, size = 275, normalized size = 0.82

$$-\frac{de\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2+ae^2\right)\sqrt{de}}+\frac{\frac{2\sqrt{2}\left(\sqrt{a}\,cd+a\sqrt{c}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{a}\,cd+a\sqrt{c}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\left(\sqrt{a}\,cd-a\sqrt{c}\right.}{}}{8\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -d*e*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) + 1/8*(2*sqrt(2)*(sq
rt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*
c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2
*sqrt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*
sqrt(c)) - sqrt(2)*(sqrt(a)*c*d - a*sqrt(c)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^
(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(sqrt(a)*c*d - a*sqr
t(c)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(
3/4)))/(c*d^2 + a*e^2)
```

mupad [B] time = 1.59, size = 4720, normalized size = 14.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(a^2*d*e^7 + c^2*d^5*e^3 - c^2*d*x*(-d*e)^(7/2) + 2*a*c*d^3*e^5 + a^2*e
^7*x*(-d*e)^(1/2) + 2*a*c*e^3*x*(-d*e)^(5/2))*(-d*e)^(1/2))/(2*a*e^2 + 2*c*
d^2) - atan((((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(
a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2)
- a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2)*(x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 5
12*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^4*c
^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^5*e^2
```

- 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*
(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))
^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*a*c^4
*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c
^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*1i - ((-(c*d^2*(-a*c)^(1/2)
- a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d
^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7 -
x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 +
a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*
e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 + 38
4*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*
d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^
3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c
^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) -
a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*
e^2)))^(1/2)*1i)/(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/
(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(
1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c
^2*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d
e)/(16(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^
9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*
a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^
5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a
e^2(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e
^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*
a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16
*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + ((-(c*d^2*(-a*c)^(1/
2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2
*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)
/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7
- x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4
+ a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^
6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 +
384*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^
5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*
c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2
*c^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2)
- a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2) + 2*a*c^3*d*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2)
+ 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*2i -
(log(a^2*d*e^7 + c^2*d^5*e^3 + c^2*d*x*(-d*e)^(7/2) + 2*a*c*d^3*e^5 - a^2*e
^7*x*(-d*e)^(1/2) - 2*a*c*e^3*x*(-d*e)^(5/2))*(-d*e)^(1/2))/(2*(a*e^2 + c*d
^2)) - atan((((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(
a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(1/2)
- c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2)*(x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 5
12*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^4*c
^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^5*e^2
- 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*
(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))
^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*a*c^4
*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c
^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*1i - ((-(a*e^2*(-a*c)^(1/2)
- c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d
^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7 -
x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 +

```
 a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*
e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 + 38
4*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*
d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^
3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c
^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2) -
c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*
e^2)))^(1/2)*1i)/(((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/
(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(
1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c
^2*d^2*e^2)))^(1/2)*(x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d
*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^
9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*
a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^
5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c
*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e
^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*
a*c^4*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16
*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + ((-(a*e^2*(-a*c)^(1/
2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2
*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)
/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7
 - x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4
 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^
6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 +
384*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^
5*d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*
c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2
*c^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2)
- c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2) + 2*a*c^3*d*e^3))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2)
 + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.241 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}-$$

```
[Out] 1/4*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(
3/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*
(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*c^(1/4)*ln(-a^(1/4
)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e
^2+c*d^2)*2^(1/2)+1/8*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1
/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)+e^(3/2)*arctan(x*e
^(1/2)/d^(1/2))/(a*e^2+c*d^2)/d^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1171, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{e^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= \frac{c\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)} - \cdots$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)} - \frac{\sqrt[4]{c}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)} + \frac{\sqrt[4]{c}\left(\sqrt{c}\,d+\cdots\right.}{\cdots}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)} - \frac{\sqrt[4]{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)} + \frac{\sqrt[4]{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\cdots\right)}{2\sqrt{2}\,a^{3/4}\left(cd^2+a\cdots\right)}$$

Mathematica [A] time = 0.15, size = 234, normalized size = 0.70

$$\frac{8a^{3/4}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+\sqrt{2}\,\sqrt[4]{c}\,\sqrt{d}\,\left(-\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\cdots\right.\right.\right.}{8a^{3/4}\sqrt{d}\,\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*((
-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d]*(c*
d^2 + a*e^2))
```

fricas [B] time = 2.28, size = 4084, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e
^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*
d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a
^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*
e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*
d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2
+ a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^
4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d
^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^
4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
 a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^
2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^
5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (
a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a
^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^
2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e
^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4
- 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2
*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*
e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4
)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c
^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*
a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 -
 a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
```

```
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - 2*e*sqrt(-e/d)*log((e*x^2 + 2*d
*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(c*d^2 + a*e^2), 1/4*(4*e*sqrt(e/d)*arctan
(x*sqrt(e/d)) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 -
 a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e +
 (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2
 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3
 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*
a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*
c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^
2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4
*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*
d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-
(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 +
6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e
^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3
*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d
^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)
*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6
*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2
*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^
4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))
)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*
c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(
c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4
+ 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^
7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))))/(c*d^2 + a*e^2)]
```

giac [A] time = 0.42, size = 339, normalized size = 1.01

$$\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/
2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*
((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^
```

```
2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2
)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*
d^2 + a*e^2)*sqrt(d))
```

maple [A] time = 0.01, size = 363, normalized size = 1.08

$$\frac{e^{2}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{de}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^{2}+c\,d^{2}\right)a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^{2}+c\,d^{2}\right)a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;cd\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+}\right.}{8\left(a\,e^{2}+c\,d^{2}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/8*c/(a*e^2+c*d^2)*d*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(
a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*c/(a*e^2+c*d^2)*d*
(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*c/(a*e^2+c*d^2)*d
*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/8/(a*e^2+c*d^2)*e/
(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(
1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x+1)-1/4/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x-1)+e^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e
*x)
```

maxima [A] time = 1.09, size = 268, normalized size = 0.80

$$\frac{e^{2}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^{2}+ae^{2}\right)\sqrt{de}}+\frac{c\left(\frac{2\sqrt{2}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}d+\sqrt{a}e\right)}{}\right.}{8\left(cd^{2}+ae^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] e^2*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) + 1/8*c*(2*sqrt(2)*(s
qrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(
1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sq
rt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(
1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)
) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqr
t(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c*d^2 +
 a*e^2)
```

mupad [B] time = 1.67, size = 4802, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^
5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-
```

```
a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
 c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d
^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^
5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3
*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*
((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
 a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*
(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)
))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) +
2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4
*c^4*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(
16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512
*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^
6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32
*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c
)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2
) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^
(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*
1i)/((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5
*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a
^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5
 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*
c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*(
(a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((a
*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^
3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^
3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(
1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^
2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4
*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(
a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2
*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^
2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c
^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1
/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) +
20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2
) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)))*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*2i + atan(((((c*d^2*(-a^3*c)^(1/2) -
a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1
/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(2
56*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*
d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9
 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*
c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^
2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(
-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2))
)^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^
(1/2)*1i - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(1
6*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d
```

```
^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*
c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^(1
/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d
^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 44
8*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e
^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*
e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) + 6*c^5*e^5*
x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^
4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i)/((((c*d^2*(-a^3*c)^(1/2) - a
*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2
*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/
2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(25
6*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d
*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9
- 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c
^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2
 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-
a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))
^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^
3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(
1/2) + (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a
^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(
-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*
d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^(1/2)
- a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*
d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e
^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^
3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))
*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4
+ a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(
(c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^
(1/2)*2i - (log(16*a^2*e^2*(-d*e^3)^(3/2) + c^2*d^5*e^3*x - c^2*d^5*e*(-d*e
^3)^(1/2) + 16*a^2*d*e^7*x + a*c*d^2*(-d*e^3)^(3/2) + a*c*d^3*e^5*x)*(-d*e^
3)^(1/2))/(2*(c*d^3 + a*d*e^2)) + (log(c^2*d^5*e^3*x - 16*a^2*e^2*(-d*e^3)^
(3/2) + c^2*d^5*e*(-d*e^3)^(1/2) + 16*a^2*d*e^7*x + 4*a*c*d^2*(-d*e^3)^(3/2
) + a*c*d^3*e^5*x + 5*a*c*d^3*e^3*(-d*e^3)^(1/2))*(-d*e^3)^(1/2))/(2*c*d^3
+ 2*a*d*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.242 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=348

$$-\frac{c^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\right.}{}$$

```
[Out] -1/a/d/x-e^(5/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2+c*d^2)-1/8*c^(3/4
)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))
/a^(5/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(5/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*
c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(5/4)/
(a*e^2+c*d^2)*2^(1/2)-1/4*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^
(1/2)+d*c^(1/2))/a^(5/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{c^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^
2)) + (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2
)) - (c^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(Sqrt[c]*
d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(5/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)}\,dx=\int\left(\frac{1}{adx^2}-\frac{e^3}{d\left(cd^2+ae^2\right)\left(d+ex^2\right)}-\frac{c\left(ae+cdx^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$=-\frac{1}{adx}-\frac{c\int\frac{ae+cdx^2}{a+cx^4}\,dx}{a\left(cd^2+ae^2\right)}-\frac{e^3\int\frac{1}{d+ex^2}\,dx}{d\left(cd^2+ae^2\right)}$$

$$=-\frac{1}{adx}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)}+\frac{\left(c\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2a\left(cd^2+ae^2\right)}-$$

$$=-\frac{1}{adx}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)}-\frac{\left(c^{5/4}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int\frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)}-$$

$$=-\frac{1}{adx}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)}-\frac{c^{5/4}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)}+$$

$$=-\frac{1}{adx}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)}+\frac{c^{3/4}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)}-$$

Mathematica [A] time = 0.25, size = 389, normalized size = 1.12

$$-\sqrt{d}\left(8a^{5/4}e^2+\sqrt{2}\,c^{5/4}d^2x\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\sqrt{2}\,c^{5/4}d^2x\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\sqrt{2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a^(5/4)*e^(5/2)*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - Sqrt[d]*(8*a^(1/4)*c*d^
2 + 8*a^(5/4)*e^2 - 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sqrt[a]*e)*x*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sqrt[a]*e
)*x*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*c^(5/4)*d^2*x*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*Sqrt[a]*c^(3/4)*d
*e*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*c^(5/
4)*d^2*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*S
qrt[a]*c^(3/4)*d*e*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
))/(8*a^(5/4)*d^(3/2)*(c*d^2 + a*e^2)*x)
```

fricas [B] time = 6.21, size = 4362, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*e^2*x*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d))
+ (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 +
4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c
^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^
2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^
5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6
*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2
*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^
3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e
^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a*c*d^3 + a^
2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqr
t(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e
^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*
c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c
*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4
*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^
4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*
d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4
*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))
/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + (a*c*d^3 + a^2*d*e^2)*x*sqrt
(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2
*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*
d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4
*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2
*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*
c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2
*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e
^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^
3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 +
 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e -
(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2
+ a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8
```

```
*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^
3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*
d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c
^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)
))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*
d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a
^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^
2 + a^4*e^4))) - 4*c*d^2 - 4*a*e^2)/((a*c*d^3 + a^2*d*e^2)*x), -1/4*(4*a*e^
2*x*sqrt(e/d)*arctan(x*sqrt(e/d)) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*
e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*
e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4
*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(
-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^
5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a
^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*
e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(
c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 +
 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4))) + (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^
4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e
^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6
+ a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c
^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a
^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*
a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2
*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c
^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*
e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4
))) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2
*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^
8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a
^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^
2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(
-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2
 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2
*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^
2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d
^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + (a*c*d^3
+ a^2*d*e^2)*x*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)
*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d
^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*
a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a
^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a
*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^
4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^
3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5
*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^
8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + 4*c*d^2 + 4*a*e^2)/((a*c
*d^3 + a^2*d*e^2)*x)]
```

giac [A] time = 0.40, size = 348, normalized size = 1.00

$$-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}ace+\left(ac^{3}\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}ace+\left(ac^{3}\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1
/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4
*((a*c^3)^(1/4)*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*a
*c*e - (a*c^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(
2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - arctan(x*e^(1/2)/sqrt(d))*e^(5/2)/((c
*d^3 + a*d*e^2)*sqrt(d)) - 1/(a*d*x)
```

maple [A] time = 0.01, size = 390, normalized size = 1.12

$$-\frac{e^{3}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{de}\;d}-\frac{\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,cd\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/a/d/x-1/4*c/(a*e^2+c*d^2)/a*e*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x-1)-1/8*c/(a*e^2+c*d^2)/a*e*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2
^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4*c/(a*e^2
+c*d^2)/a*e*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/8*c/(a*e^
2+c*d^2)/a*d/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))
/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4*c/(a*e^2+c*d^2)/a*d/(a/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/4*c/(a*e^2+c*d^2)/a*d/(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/d*e^3/(a*e^2+c*d^2)/(d*e)^(1/
2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 1.27, size = 292, normalized size = 0.84

$$-\frac{e^{3}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^{3}+ade^{2}\right)\sqrt{de}}-\frac{c\left(\frac{2\sqrt{2}\left(\sqrt{a}cd+a\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{a}cd+a\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\left(\sqrt{a}cd-\right.}{}\right.}{8\left(acd^{2}+a^{2}e^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -e^3*arctan(e*x/sqrt(d*e))/((c*d^3 + a*d*e^2)*sqrt(d*e)) - 1/8*c*(2*sqrt(2)
*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1
/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c))
 + 2*sqrt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x -
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
c))*sqrt(c)) - sqrt(2)*(sqrt(a)*c*d - a*sqrt(c)*e)*log(sqrt(c)*x^2 + sqrt(2
)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(sqrt(a)*c*d - a
*sqrt(c)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)
*c^(3/4)))/(a*c*d^2 + a^2*e^2) - 1/(a*d*x)
```

mupad [B] time = 2.00, size = 5761, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2
) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*
a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2)
 + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*
(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^
7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512
*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) - 192*a^
10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 + 256*a^13*
c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*
d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c
^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^
(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^7*e^8))*(-(
a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e
^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*1i + (x*(2*a^7*c^7*d^9*e^5 - 4*
a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a
^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^
2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 +
 a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^
2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^
2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^
5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d
^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^1
4*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8)
)*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(
a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a
^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(
-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e
^2)))^(1/2)*1i)/((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^
5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c
^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5
*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))
)^(1/2)*(x*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d
*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^1
5*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^
9) - 192*a^10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7
+ 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 11
2*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*
d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*
d^2*e^2)))^(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^
7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)
/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) - (x*(2*a^7*c^7*d^9*
e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/
2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(
((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a
^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(a*e^2*(-a^5*c^3)^(1/2
) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*
a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512
*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^
11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8
*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*
d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*
e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e
^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) -
 c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6
*c*d^2*e^2)))^(1/2)))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) +
2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*2i + a
```

```
tan(((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5*c^3)^(1/2)
 - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a
^6*c*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2)
+ 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(
-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7
*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*
a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) - 192*a^1
0*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 + 256*a^13*c
^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d
^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^
3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(
1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^7*e^8))*(-(c
*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^
4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*1i + (x*(2*a^7*c^7*d^9*e^5 - 4*a
^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^
3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(c*d^2
*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 +
a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2
*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2
*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5
*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^
12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14
*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))
*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a
^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^
8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-
a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^
2)))^(1/2)*1i)/((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5
*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^
2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*
c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))
^(1/2)*(x*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*
e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15
*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9
) - 192*a^10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 +
 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112
*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e
^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d
^2*e^2)))^(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^7
*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/
(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) - (x*(2*a^7*c^7*d^9*e
^5 - 4*a^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2
) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*((
(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^
7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a^5*c^3)^(1/2)
 - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a
^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*
a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^1
1*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8*
c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d
^8*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e
)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^
2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) -
a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*
c*d^2*e^2)))^(1/2)))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2
*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*2i - 1/
(a*d*x) - (log(c^4*d^11*(-d^3*e^5)^(1/2) - 16*a^4*e^3*(-d^3*e^5)^(3/2) + 16
*a^4*d^4*e^11*x + c^4*d^12*e^3*x + a*c^3*d^9*e^2*(-d^3*e^5)^(1/2) + a*c^3*d
^10*e^5*x - 16*a^3*c*d^6*e^9*x + 16*a^3*c*d^2*e*(-d^3*e^5)^(3/2))*(-d^3*e^5
```

```
)^(1/2))/(2*(c*d^5 + a*d^3*e^2)) + (log(16*a^4*e^3*(-d^3*e^5)^(3/2) - c^4*d
^11*(-d^3*e^5)^(1/2) + 16*a^4*d^4*e^11*x + c^4*d^12*e^3*x - a*c^3*d^9*e^2*(
-d^3*e^5)^(1/2) + a*c^3*d^10*e^5*x - 16*a^3*c*d^6*e^9*x - 16*a^3*c*d^2*e*(-
d^3*e^5)^(3/2))*(-d^3*e^5)^(1/2))/(2*c*d^5 + 2*a*d^3*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+a),x)
[Out] Timed out
```

3.243 $\int \frac{1}{x^4(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=360

$$\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{\cdots}\right.}{}$$

```
[Out] -1/3/a/d/x^3+e/a/d^2/x+e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/(a*e^2+c*d
^2)-1/4*c^(5/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))
/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*c^(5/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/
4))*(-e*a^(1/2)+d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*c^(5/4)*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(7/4)/
(a*e^2+c*d^2)*2^(1/2)-1/8*c^(5/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{\cdots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/(3*a*d*x^3) + e/(a*d^2*x) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/
2)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c
]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4
)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c
^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{1}{adx^4}-\frac{e}{ad^2x^2}+\frac{e^4}{d^2\left(cd^2+ae^2\right)\left(d+ex^2\right)}-\frac{c^2\left(d-ex^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}-\frac{c^2\int \frac{d-ex^2}{a+cx^4}\,dx}{a\left(cd^2+ae^2\right)}+\frac{e^4\int \frac{1}{d+ex^2}\,dx}{d^2\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}-\frac{\left(c\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2a\left(cd^2+ae^2\right)}-\frac{\left(c\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+\right.\right.}{2a}$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}-\frac{\left(c\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4a\left(cd^2+ae^2\right)}-$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}+\frac{c^{5/4}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x\right.}{4\sqrt{2}\,a^{7/4}\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}+\frac{c^{5/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}\left(cd^2+ae^2\right)}-$$

Mathematica [A] time = 0.40, size = 367, normalized size = 1.02

$$3\sqrt{2}\,c^{5/4}d^{5/2}x^3\left(a^{3/4}e+\sqrt[4]{a}\,\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-3\sqrt{2}\,c^{5/4}d^{5/2}x^3\left(a^{3/4}e+\sqrt[4]{a}\,\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a*d^(3/2)*(c*d^2 + a*e^2) + 24*a*Sqrt[d]*e*(c*d^2 + a*e^2)*x^2 + 24*a^2
*e^(7/2)*x^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^(1/4)*c^(5/4)*d^(5/2
)*(Sqrt[c]*d - Sqrt[a]*e)*x^3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*S
qrt[2]*a^(1/4)*c^(5/4)*d^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*x^3*ArcTan[1 + (S
qrt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d +
 a^(3/4)*e)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - 3*
Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*x^3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^2*d^(5/2)*(c*d^2 + a*e^2)*x^
3)
```

fricas [B] time = 19.68, size = 4442, normalized size = 12.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a*e^3*x^3*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d
)) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c
*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^
4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8
)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x
 + (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^
5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3
*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d
*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2
*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 +
4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)))
- 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d
^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))
/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x -
(a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*
sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^
6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e
+ (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^
2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a
^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) + 3
*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8
+ 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^
2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqr
t(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e
^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 +
 a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10
*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a
*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
+ a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4
```

```
*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*
c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c
^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-
(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3
*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^
2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*
d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 4*c*d^3
 - 4*a*d*e^2 + 12*(c*d^2*e + a*e^3)*x^2)/((a*c*d^4 + a^2*d^2*e^2)*x^3), 1/1
2*(12*a*e^3*x^3*sqrt(e/d)*arctan(x*sqrt(e/d)) + 3*(a*c*d^4 + a^2*d^2*e^2)*x
^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d
^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^
9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2
+ (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*
e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4
*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*
e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8
 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*
sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4
- 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c
^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
+ a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (
a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2
 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^
10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 +
4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3
*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqr
t((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2
*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*
d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a
^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6
*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 +
a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*
c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a
^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^
2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((
2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*
c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4
*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*
e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^
2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2
*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d
^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5
*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*
c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 4*c*d^3 - 4*a*d*e^2 + 12*(c*d^2*e + a*e
^3)*x^2)/((a*c*d^4 + a^2*d^2*e^2)*x^3)]
```

giac [A] time = 0.40, size = 364, normalized size = 1.01

$$-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d-\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d-\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1
/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4
*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*c
^2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(
2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/((c
*d^4 + a*d^2*e^2)*sqrt(d)) + 1/3*(3*x^2*e - d)/(a*d^2*x^3)
```

maple [A] time = 0.01, size = 406, normalized size = 1.13

$$\frac{e^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a e^2+c d^2\right)\sqrt{de}\, d^2}+\frac{\sqrt{2}\, ce \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a e^2+c d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} a}+\frac{\sqrt{2}\, ce \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a e^2+c d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} a}+\frac{\sqrt{2}\, ce \ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{a}{c}}}\right)}{8\left(a e^2+c d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\, \cdots}{4\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/3/a/d/x^3+e/a/d^2/x-1/8*c^2/(a*e^2+c*d^2)/a^2*d*(a/c)^(1/4)*2^(1/2)*ln((
x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/
2)))-1/4*c^2/(a*e^2+c*d^2)/a^2*d*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)-1/4*c^2/(a*e^2+c*d^2)/a^2*d*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/c)^(1/4)*x-1)+1/8*c/(a*e^2+c*d^2)/a*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1
/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*c/(
a*e^2+c*d^2)/a*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*c/
(a*e^2+c*d^2)/a*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/d^2
*e^4/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.12, size = 297, normalized size = 0.82

$$\frac{e^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^4+ad^2e^2\right)\sqrt{de}}-\frac{c^2\left(\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\cdots}{\cdots}\right.}{8\left(acd^2+a^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c*d^4 + a*d^2*e^2)*sqrt(d*e)) - 1/8*c^2*(2*sqrt
(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/
4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c))
+ 2*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*s
qrt(c)) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a
*c*d^2 + a^2*e^2) + 1/3*(3*e*x^2 - d)/(a*d^2*x^3)
```

mupad [B] time = 2.26, size = 5972, normalized size = 16.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/
2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2
*a^8*c*d^2*e^2)))^(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2)
 + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*
((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9
*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*
a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9
*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20*e^6 - 256*a^12*c^
5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*
d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^
5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d
^4 + 2*a^8*c*d^2*e^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^7*c^8*d^19*e^5 + 4
8*a^9*c^6*d^15*e^9))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*
a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*1i + (x*
(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/2) - c*d^
2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^
2*e^2)))^(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*
c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((a*e^2*(
-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^
7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*
d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24
*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^
8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4
- 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^5)^(1/2)
- c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^
8*c*d^2*e^2)))^(1/2) + 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6
*d^15*e^9))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d
*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*1i)/((x*(2*a^5*c^
9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c
^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^
(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/
(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((a*e^2*(-a^7*c^5)
^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4
 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5
- 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9*c^8*d^24*e^2 + 12
8*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20*e^6 - 256*a^12*c^5*d^18*e^8 - 256*a
^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9
*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(
-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e
^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^7*c^8*d^19*e^5 + 48*a^9*c^6*d^15*e^9
))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(
a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) - (x*(2*a^5*c^9*d^18*e^5 +
 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) +
2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((a*
e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4
 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((a*e^2*(-a^7*c^5)^(1/2) - c*d
^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d
^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c
^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24*e^2 - 128*a^10*c^7*d
^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^8 + 256*a^13*c^4*d^16
*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^
6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1
```

```
/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)
+ 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6*d^15*e^9))*((a*e^2*(
-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^
7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) + 2*a^5*c^8*d^14*e^8))*((a*e^2*(-a^7*c
^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*
d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*2i + atan(((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^
7*d^14*e^9) - ((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3
*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7
*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^
2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*
c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))
^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e
^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 +
 192*a^11*c^6*d^20*e^6 - 256*a^12*c^5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) -
x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a
^11*c^5*d^15*e^10))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a
^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) - 4*a^6*c
^9*d^21*e^3 - 4*a^7*c^8*d^19*e^5 + 48*a^9*c^6*d^15*e^9))*((c*d^2*(-a^7*c^5)
^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4
 + 2*a^8*c*d^2*e^2)))^(1/2)*1i + (x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^
9) - ((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16
*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7*c^5)^(1/
2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2
*a^8*c*d^2*e^2)))^(1/2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2
) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(5
12*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*
a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11
*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7
*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d
^15*e^10))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*
e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) + 4*a^6*c^9*d^21*e
^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6*d^15*e^9))*((c*d^2*(-a^7*c^5)^(1/2) -
a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*
c*d^2*e^2)))^(1/2)*1i)/((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((c*
d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4
 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7*c^5)^(1/2) - a*e^
2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^
2*e^2)))^(1/2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4
*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c
^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*
d^18*e^9) - 64*a^9*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20
*e^6 - 256*a^12*c^5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23
*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10)
)*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a
^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^
7*c^8*d^19*e^5 + 48*a^9*c^6*d^15*e^9))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a
^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2
)))^(1/2) - (x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((c*d^2*(-a^7*c^
5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d
^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)
^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/
2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(1
6*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3
+ 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) +
64*a^9*c^8*d^24*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a
^12*c^5*d^18*e^8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^
8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((c*d^2*(-
a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7
*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) + 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e
```

```
^5 - 48*a^9*c^6*d^15*e^9))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2
) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) +
2*a^5*c^8*d^14*e^8))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*
a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*2i - (1/
(3*a*d) - (e*x^2)/(a*d^2))/x^3 - (log(16*a^7*d^13*e^20 + c^7*d^27*e^6 + 2*a
*c^6*d^25*e^8 + a^2*c^5*d^23*e^10 + 16*a^4*c^3*d^19*e^14 + 16*a^7*e^3*x*(-d
^5*e^7)^(5/2) - a^2*c^5*d^15*x*(-d^5*e^7)^(3/2) + c^7*d^24*e^3*x*(-d^5*e^7)
^(1/2) - 16*a^4*c^3*d^11*e^4*x*(-d^5*e^7)^(3/2) + 2*a*c^6*d^22*e^5*x*(-d^5*
e^7)^(1/2))*(-d^5*e^7)^(1/2))/(2*(c*d^7 + a*d^5*e^2)) + (log(16*a^7*d^13*e^
20 + c^7*d^27*e^6 + 2*a*c^6*d^25*e^8 + a^2*c^5*d^23*e^10 + 16*a^4*c^3*d^19*
e^14 - 16*a^7*e^3*x*(-d^5*e^7)^(5/2) + a^2*c^5*d^15*x*(-d^5*e^7)^(3/2) - c^
7*d^24*e^3*x*(-d^5*e^7)^(1/2) + 16*a^4*c^3*d^11*e^4*x*(-d^5*e^7)^(3/2) - 2*
a*c^6*d^22*e^5*x*(-d^5*e^7)^(1/2))*(-d^5*e^7)^(1/2))/(2*c*d^7 + 2*a*d^5*e^2
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.244 $\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=169

$$-\frac{\sqrt{a}\,d\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae\left(ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)^2}+\frac{a\left(ae+cdx^2\right)}{4c^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)^2}$$

```
[Out] 1/4*a*(c*d*x^2+a*e)/c^2/(a*e^2+c*d^2)/(c*x^4+a)+1/2*d^4*ln(e*x^2+d)/e/(a*e^
2+c*d^2)^2+1/4*a*e*(a*e^2+2*c*d^2)*ln(c*x^4+a)/c^2/(a*e^2+c*d^2)^2-1/4*d*(a
*e^2+3*c*d^2)*arctan(x^2*c^(1/2)/a^(1/2))*a^(1/2)/c^(3/2)/(a*e^2+c*d^2)^2
```

Rubi [A] time = 0.37, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 1647, 1629, 635, 205, 260}

$$\frac{a\left(ae+cdx^2\right)}{4c^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{ae\left(ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)^2}-\frac{\sqrt{a}\,d\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{d^4\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*(a*e + c*d*x^2))/(4*c^2*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[a]*d*(3*c*d
^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) +
(d^4*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)^2) + (a*e*(2*c*d^2 + a*e^2)*Log[a
 + c*x^4])/(4*c^2*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(d+ex)(a+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2d^2}{cd^2+ae^2}-\frac{a^2dex}{cd^2+ae^2}-2ax^2}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4ac}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(-\frac{2acd^4}{(cd^2+ae^2)^2(d+ex)} + \frac{a^2(d(3cd^2+ae^2)-2e(2cd^2+ae^2)x}{(cd^2+ae^2)^2(a+cx^2)}\right.\right.}{4ac}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} - \frac{a\,\text{Subst}\left(\int \frac{d(3cd^2+ae^2)-2e(2cd^2+ae^2)x}{a+cx^2}\right.}{4c(cd^2+ae^2)^2}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} + \frac{(ae(2cd^2+ae^2))\,\text{Subst}\left(\int \frac{x}{a+cx^2}\right.}{2c(cd^2+ae^2)^2}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{a}\,d(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} +$$

Mathematica [A] time = 0.21, size = 135, normalized size = 0.80

$$\frac{-\frac{\sqrt{a}\,d(ae^2+3cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{3/2}} + \frac{ae(ae^2+2cd^2)\log(a+cx^4)}{c^2} + \frac{a(ae^2+cd^2)(ae+cdx^2)}{c^2(a+cx^4)} + \frac{2d^4\log(d+ex^2)}{e}}{4(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((a*(c*d^2 + a*e^2)*(a*e + c*d*x^2))/(c^2*(a + c*x^4)) - (Sqrt[a]*d*(3*c*d^
2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) + (2*d^4*Log[d + e*x^2])/
e + (a*e*(2*c*d^2 + a*e^2)*Log[a + c*x^4])/c^2)/(4*(c*d^2 + a*e^2)^2)
```

fricas [A] time = 31.51, size = 555, normalized size = 3.28

$$\left[\frac{2a^2cd^2e^2 + 2a^3e^4 + 2(ac^2d^3e + a^2cde^3)x^2 + (3ac^2d^3e + a^2cde^3 + (3c^3d^3e + ac^2de^3)x^4)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2cx^2\sqrt{-}}{cx^4+a}\right.}{8(ac^4d^4e + 2a^2c^3d^2e^3 + a^3c^2e^5 + (c^5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a^2*c*d^2*e^2 + 2*a^3*e^4 + 2*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 + (3*
a*c^2*d^3*e + a^2*c*d*e^3 + (3*c^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(-a/c)*log
((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + 2*(2*a^2*c*d^2*e^2 + a^3*e
^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(c*x^4 + a) + 4*(c^3*d^4*x^4 + a
*c^2*d^4)*log(e*x^2 + d))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 +
(c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4), 1/4*(a^2*c*d^2*e^2 + a^3*
e^4 + (a*c^2*d^3*e + a^2*c*d*e^3)*x^2 - (3*a*c^2*d^3*e + a^2*c*d*e^3 + (3*c
^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + (2*a^2*c
*d^2*e^2 + a^3*e^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(c*x^4 + a) + 2*
(c^3*d^4*x^4 + a*c^2*d^4)*log(e*x^2 + d))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3
+ a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4)]
```

giac [A] time = 0.36, size = 251, normalized size = 1.49

$$\frac{d^4\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}+\frac{\left(2\,acd^2e+a^2e^3\right)\log\left(cx^4+a\right)}{4\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)}-\frac{\left(3\,acd^3+a^2de^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ac}}-\frac{2\,acd^2x^4e-acd^3x}{4\left(c^3d^4+2\,ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(2*
a*c*d^2*e + a^2*e^3)*log(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^
4) - 1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c)) - 1/4*(2*a*c*d^2*x^4*e - a*c*d^3*x^2 + a^
2*x^4*e^3 - a^2*d*x^2*e^2 + a^2*d^2*e)/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*
e^4)*(c*x^4 + a))
```

maple [A] time = 0.02, size = 305, normalized size = 1.80

$$\frac{a^2d\,e^2x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)c}+\frac{a\,d^3x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)}-\frac{a^2d\,e^2\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}\,c}-\frac{3a\,d^3\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}+\frac{a}{4\left(a\,e^2+c\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*d/c*x^2*e^2+1/4*a/(a*e^2+c*d^2)^2/(c*x^4+
a)*x^2*d^3+1/4*a^3/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3/c^2+1/4*a^2/(a*e^2+c*d^2)^
2/(c*x^4+a)*e/c*d^2+1/4*a^2/(a*e^2+c*d^2)^2/c^2*ln(c*x^4+a)*e^3+1/2*a/(a*e^
2+c*d^2)^2/c*ln(c*x^4+a)*d^2*e-1/4*a^2/(a*e^2+c*d^2)^2/c/(a*c)^(1/2)*arctan
(1/(a*c)^(1/2)*c*x^2)*d*e^2-3/4*a/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c
)^(1/2)*c*x^2)*d^3+1/2*d^4*ln(e*x^2+d)/e/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.08, size = 220, normalized size = 1.30

$$\frac{d^4\log\left(ex^2+d\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}+\frac{\left(2\,acd^2e+a^2e^3\right)\log\left(cx^4+a\right)}{4\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)}-\frac{\left(3\,acd^3+a^2de^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ac}}+\frac{acdx}{4\left(ac^3d^2+a^2c^2e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d^4*log(e*x^2 + d)/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(2*a*c*d
^2*e + a^2*e^3)*log(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4) -
```

```
1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2
*e^2 + a^2*c*e^4)*sqrt(a*c)) + 1/4*(a*c*d*x^2 + a^2*e)/(a*c^3*d^2 + a^2*c^2
*e^2 + (c^4*d^2 + a*c^3*e^2)*x^4)
```

mupad [B] time = 1.30, size = 305, normalized size = 1.80

$$\frac{\frac{a^2 e}{4 c^2 \left(c d^2+a e^2\right)}+\frac{a d x^2}{4 c \left(c d^2+a e^2\right)}}{c x^4+a}-\frac{\ln\left(\sqrt{-a c^5}+c^3 x^2\right) \left(3 c d^3 \sqrt{-a c^5}-2 a^2 c^2 e^3-4 a c^3 d^2 e+a d e^2 \sqrt{-a c^5}\right)}{8 \left(a^2 c^4 e^4+2 a c^5 d^2 e^2+c^6 d^4\right)}+\frac{\ln\left(\sqrt{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((a^2*e)/(4*c^2*(a*e^2 + c*d^2)) + (a*d*x^2)/(4*c*(a*e^2 + c*d^2)))/(a + c*
x^4) - (log((-a*c^5)^(1/2) + c^3*x^2)*(3*c*d^3*(-a*c^5)^(1/2) - 2*a^2*c^2*e
^3 - 4*a*c^3*d^2*e + a*d*e^2*(-a*c^5)^(1/2)))/(8*(c^6*d^4 + a^2*c^4*e^4 + 2
*a*c^5*d^2*e^2)) + (log((-a*c^5)^(1/2) - c^3*x^2)*(3*c*d^3*(-a*c^5)^(1/2) +
 2*a^2*c^2*e^3 + 4*a*c^3*d^2*e + a*d*e^2*(-a*c^5)^(1/2)))/(8*(c^6*d^4 + a^2
*c^4*e^4 + 2*a*c^5*d^2*e^2)) + (d^4*log(d + e*x^2))/(2*a^2*e^5 + 2*c^2*d^4*
e + 4*a*c*d^2*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.245 $\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{a}\,e\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{a\left(d-ex^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}-\frac{d^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}$$

```
[Out] 1/4*a*(-e*x^2+d)/c/(a*e^2+c*d^2)/(c*x^4+a)-1/2*d^3*ln(e*x^2+d)/(a*e^2+c*d^2
)^2+1/4*d^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4*e*(a*e^2+3*c*d^2)*arctan(x^2*c^
(1/2)/a^(1/2))*a^(1/2)/c^(3/2)/(a*e^2+c*d^2)^2
```

Rubi [A] time = 0.25, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 1647, 801, 635, 205, 260}

$$\frac{\sqrt{a}\,e\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{a\left(d-ex^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{d^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*(d - e*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[a]*e*(3*c*d^2 + a
*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) - (d^3*L
og[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d^3*Log[a + c*x^4])/(4*(c*d^2 + a*e
^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(d+ex)(a+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\operatorname{Subst}\left(\int \frac{-\frac{a^2de}{cd^2+ae^2}-\frac{a(2cd^2+ae^2)x}{cd^2+ae^2}}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4ac}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\operatorname{Subst}\left(\int \left(\frac{2acd^3e}{(cd^2+ae^2)^2(d+ex)} - \frac{a(3acd^2e+a^2e^3+2c^2d^3x)}{(cd^2+ae^2)^2(a+cx^2)}\right) dx, x,$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{\operatorname{Subst}\left(\int \frac{3acd^2e+a^2e^3+2c^2d^3x}{a+cx^2}\,dx, x, x^2\right.}{4c(cd^2+ae^2)^2}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{(cd^3)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)^2} +$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{a}\,e(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} +$$

Mathematica [A] time = 0.11, size = 142, normalized size = 0.95

$$\frac{\sqrt{a}\,e(a+cx^4)(ae^2+3cd^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)+\sqrt{c}\left(-2cd^3(a+cx^4)\log(d+ex^2)+cd^3(a+cx^4)\log(a+cx^4)+\right.}{4c^{3/2}(a+cx^4)(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (Sqrt[a]*e*(3*c*d^2 + a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + Sq
rt[c]*(a*(c*d^2 + a*e^2)*(d - e*x^2) - 2*c*d^3*(a + c*x^4)*Log[d + e*x^2] +
 c*d^3*(a + c*x^4)*Log[a + c*x^4]))/(4*c^(3/2)*(c*d^2 + a*e^2)^2*(a + c*x^4
))
```

fricas [A] time = 15.33, size = 457, normalized size = 3.05

$$\left[\frac{2acd^3+2a^2de^2-2(acd^2e+a^2e^3)x^2+\left(3acd^2e+a^2e^3+(3c^2d^2e+ace^3)x^4\right)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+2(\right.}{8\left(ac^3d^4+2a^2c^2d^2e^2+a^3ce^4+(c^4d^4+2ac^3d^2e^2+a^2c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c*d^3 + 2*a^2*d*e^2 - 2*(a*c*d^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e
+ a^2*e^3 + (3*c^2*d^2*e + a*c*e^3)*x^4)*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sq
rt(-a/c) - a)/(c*x^4 + a)) + 2*(c^2*d^3*x^4 + a*c*d^3)*log(c*x^4 + a) - 4*(
c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3
*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4), 1/4*(a*c*d^3 + a^2
*d*e^2 - (a*c*d^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e + a^2*e^3 + (3*c^2*d^2*e
+ a*c*e^3)*x^4)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + (c^2*d^3*x^4 + a*c*d^
3)*log(c*x^4 + a) - 2*(c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a*c^3*d^4 +
2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x
^4)]
```

giac [A] time = 0.35, size = 223, normalized size = 1.49

$$-\frac{d^3e\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}+\frac{d^3\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{\left(3\,acd^2e+a^2e^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ac}}-\frac{c^2d^3x^4+acd^2x^2e+}{4\left(c^3d^4+2\,ac^2d^2e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*e*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*
d^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e +
 a^2*e^3)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*
sqrt(a*c)) - 1/4*(c^2*d^3*x^4 + a*c*d^2*x^2*e + a^2*x^2*e^3 - a^2*d*e^2)/((
c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

maple [A] time = 0.02, size = 260, normalized size = 1.73

$$-\frac{a^2e^3x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)c}-\frac{a\,d^2e\,x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)}+\frac{a^2e^3\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}\;c}+\frac{3a\,d^2e\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}+\frac{}{4\left(a\,e^2+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c*x^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*
a*e*x^2*d^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*d/c*e^2+1/4/(a*e^2+c*d^2)^2/(
c*x^4+a)*a*d^3+1/4*d^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4/(a*e^2+c*d^2)^2/c/(a
*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)*a^2*e^3+3/4/(a*e^2+c*d^2)^2/(a*c)^(1/
2)*arctan(1/(a*c)^(1/2)*c*x^2)*a*d^2*e-1/2*d^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.05, size = 197, normalized size = 1.31

$$\frac{d^3\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{d^3\log\left(ex^2+d\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{\left(3\,acd^2e+a^2e^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ac}}-\frac{aex^2-ac}{4\left(ac^2d^2+a^2ce^2+\left(c^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*d^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d^3*log(e*
x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e + a^2*e^3)*
arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c))
 - 1/4*(a*e*x^2 - a*d)/(a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x^4)
```

mupad [B] time = 1.49, size = 647, normalized size = 4.31

$$\frac{\frac{a\,d}{4\,c\,\left(c\,d^2+a\,e^2\right)}-\frac{a\,e\,x^2}{4\,c\,\left(c\,d^2+a\,e^2\right)}}{c\,x^4+a}-\frac{d^3\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}+\frac{\ln\left(36\,c^8\,d^{10}\,x^2+36\,c^6\,d^{10}\,\sqrt{-a\,c^3}\,+a^5\,c\,e^{10}\,\sqrt{-a\,c^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((a*d)/(4*c*(a*e^2 + c*d^2)) - (a*e*x^2)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4)
 - (d^3*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (log(36*c
^8*d^10*x^2 + 36*c^6*d^10*(-a*c^3)^(1/2) + a^5*c*e^10*(-a*c^3)^(1/2) + a^5*
c^3*e^10*x^2 - 22*a^2*d^4*e^6*(-a*c^3)^(3/2) - 81*c^2*d^8*e^2*(-a*c^3)^(3/2
) + 60*a^2*c^6*d^6*e^4*x^2 + 22*a^3*c^5*d^4*e^6*x^2 + 8*a^4*c^4*d^2*e^8*x^2
 + 8*a^4*c^2*d^2*e^8*(-a*c^3)^(1/2) - 60*a*c*d^6*e^4*(-a*c^3)^(3/2) + 81*a*
c^7*d^8*e^2*x^2)*(2*c^3*d^3 + a*e^3*(-a*c^3)^(1/2) + 3*c*d^2*e*(-a*c^3)^(1/
2)))/(8*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)) - (log(36*c^8*d^10*x^2 -
 36*c^6*d^10*(-a*c^3)^(1/2) - a^5*c*e^10*(-a*c^3)^(1/2) + a^5*c^3*e^10*x^2
+ 22*a^2*d^4*e^6*(-a*c^3)^(3/2) + 81*c^2*d^8*e^2*(-a*c^3)^(3/2) + 60*a^2*c^
6*d^6*e^4*x^2 + 22*a^3*c^5*d^4*e^6*x^2 + 8*a^4*c^4*d^2*e^8*x^2 - 8*a^4*c^2*
d^2*e^8*(-a*c^3)^(1/2) + 60*a*c*d^6*e^4*(-a*c^3)^(3/2) + 81*a*c^7*d^8*e^2*x
^2)*(a*e^3*(-a*c^3)^(1/2) - 2*c^3*d^3 + 3*c*d^2*e*(-a*c^3)^(1/2)))/(8*(c^5*
d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.246 $\int \frac{x^5}{(d+ex^2)(a+cx^4)^2} \, dx$

Optimal. Leaf size=155

$$-\frac{d^2 e \log \left(a+cx^4\right)}{4 \left(ae^2+cd^2\right)^2}+\frac{d^2 e \log \left(d+ex^2\right)}{2 \left(ae^2+cd^2\right)^2}+\frac{d \left(cd^2-ae^2\right) \tan^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{4 \sqrt{a} \sqrt{c} \left(ae^2+cd^2\right)^2}+\frac{-ae-cdx^2}{4c \left(a+cx^4\right) \left(ae^2+cd^2\right)}$$

```
[Out] 1/4*(-c*d*x^2-a*e)/c/(a*e^2+c*d^2)/(c*x^4+a)+1/2*d^2*e*ln(e*x^2+d)/(a*e^2+c
*d^2)^2-1/4*d^2*e*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4*d*(-a*e^2+c*d^2)*arctan(x
^2*c^(1/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 153, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 1647, 801, 635, 205, 260}

$$-\frac{ae+cdx^2}{4c \left(a+cx^4\right) \left(ae^2+cd^2\right)}+\frac{d^2 e \log \left(d+ex^2\right)}{2 \left(ae^2+cd^2\right)^2}-\frac{d^2 e \log \left(a+cx^4\right)}{4 \left(ae^2+cd^2\right)^2}+\frac{d \left(cd^2-ae^2\right) \tan^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{4 \sqrt{a} \sqrt{c} \left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(a*e + c*d*x^2)/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (d*(c*d^2 - a*e^2)*Arc
Tan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) + (d^2*e*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (d^2*e*Log[a + c*x^4])/(4*(c*d^2 +
a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{ae+cdx^2}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{acd^2}{cd^2+ae^2}+\frac{acdex}{cd^2+ae^2}}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)}{4ac}$$

$$= -\frac{ae+cdx^2}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \left(-\frac{2acd^2e^2}{\left(cd^2+ae^2\right)^2(d+ex)} + \frac{acd\left(-cd^2+ae^2+2cdex\right)}{\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}\right) dx, x$$

$$= -\frac{ae+cdx^2}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^2e\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} - \frac{d\,\text{Subst}\left(\int \frac{-cd^2+ae^2+2cdex}{a+cx^2}\,dx, x, x\right.}{4\left(cd^2+ae^2\right)^2}$$

$$= -\frac{ae+cdx^2}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^2e\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} - \frac{\left(cd^2e\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)^2}$$

$$= -\frac{ae+cdx^2}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(cd^2+ae^2\right)^2} + \frac{d^2e\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} - \frac{d^2}{4}$$

Mathematica [A] time = 0.15, size = 120, normalized size = 0.77

$$\frac{\frac{d\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - \frac{\left(ae^2+cd^2\right)\left(ae+cdx^2\right)}{c\left(a+cx^4\right)} - d^2e\log\left(a+cx^4\right) + 2d^2e\log\left(d+ex^2\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-(((c*d^2 + a*e^2)*(a*e + c*d*x^2))/(c*(a + c*x^4))) + (d*(c*d^2 - a*e^2)*
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + 2*d^2*e*Log[d + e*x^2] -
 d^2*e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

fricas [A] time = 6.53, size = 487, normalized size = 3.14

$$\left[-\frac{2a^2cd^2e + 2a^3e^3 + 2\left(ac^2d^3 + a^2cde^2\right)x^2 - \left(acd^3 - a^2de^2 + \left(c^2d^3 - acde^2\right)x^4\right)\sqrt{-ac}\log\left(\frac{cx^4+2\sqrt{-ac}\,x^2-a}{cx^4+a}\right) + 2}{8\left(a^2c^3d^4 + 2a^3c^2d^2e^2 + a^4ce^4 + \left(ac^4d^4 + 2a^2c^3d^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a^2*c*d^2*e + 2*a^3*e^3 + 2*(a*c^2*d^3 + a^2*c*d*e^2)*x^2 - (a*c*d
^3 - a^2*d*e^2 + (c^2*d^3 - a*c*d*e^2)*x^4)*sqrt(-a*c)*log((c*x^4 + 2*sqrt(
-a*c)*x^2 - a)/(c*x^4 + a)) + 2*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(c*x^4 +
 a) - 4*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^
3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x
^4), -1/4*(a^2*c*d^2*e + a^3*e^3 + (a*c^2*d^3 + a^2*c*d*e^2)*x^2 + (a*c*d^3
 - a^2*d*e^2 + (c^2*d^3 - a*c*d*e^2)*x^4)*sqrt(a*c)*arctan(sqrt(a*c)/(c*x^2
)) + (a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(c*x^4 + a) - 2*(a*c^2*d^2*e*x^4 +
a^2*c*d^2*e)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4 +
 (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4)]
```

giac [A] time = 0.33, size = 220, normalized size = 1.42

$$-\frac{d^2 e \log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{d^2e^2\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}+\frac{\left(cd^3-ade^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{ac}}+\frac{c^2d^2x^4e-c^2d^3x^2-a}{4\left(c^3d^4+2\,ac^2d^2e^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*d^2*e*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*d^2*e^2
*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(c*d^3 - a
*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a
*c)) + 1/4*(c^2*d^2*x^4*e - c^2*d^3*x^2 - a*c*d*x^2*e^2 - a^2*e^3)/((c^3*d^
4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

maple [A] time = 0.02, size = 252, normalized size = 1.63

$$-\frac{ad\,e^2x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)}-\frac{c\,d^3x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)}-\frac{ad\,e^2\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}+\frac{c\,d^3\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}-\frac{a^2e}{4\left(a\,e^2+c\,d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*a*d*e^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^
2*c*d^3-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c-1/4/(a*e^2+c*d^2)^2/(c*x^4+
a)*a*e*d^2-1/4*d^2*e*ln(c*x^4+a)/(a*e^2+c*d^2)^2-1/4/(a*e^2+c*d^2)^2*d/(a*c
)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)*a*e^2+1/4/(a*e^2+c*d^2)^2/(a*c)^(1/2)*a
rctan(1/(a*c)^(1/2)*c*x^2)*c*d^3+1/2*d^2*e*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.08, size = 192, normalized size = 1.24

$$-\frac{d^2e\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{d^2e\log\left(ex^2+d\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{\left(cd^3-ade^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{ac}}-\frac{cdx^2+ae}{4\left(ac^2d^2+a^2ce^2+\left(c^3d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*d^2*e*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*d^2*e*l
og(e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c*d^3 - a*d*e^2)*a
rctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) - 1/
4*(c*d*x^2 + a*e)/(a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x^4)
```

mupad [B] time = 1.52, size = 528, normalized size = 3.41

$$\frac{\ln\left(a^4\,e^8\,\sqrt{-a\,c}+c^4\,d^8\,\sqrt{-a\,c}+70\,d^4\,e^4\,{\left(-a\,c\right)}^{5/2}+c^5\,d^8\,x^2+a^4\,c\,e^8\,x^2-36\,a^2\,d^2\,e^6\,{\left(-a\,c\right)}^{3/2}-36\,c^2\,d^6\,e^2\,{\left(-a\,c\right)}^{3/2}\right.}{a^3\,c\,e^4+2\,a^2\,c^2\,d^2\,e^2+a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(a^4*e^8*(-a*c)^(1/2) + c^4*d^8*(-a*c)^(1/2) + 70*d^4*e^4*(-a*c)^(5/2)
+ c^5*d^8*x^2 + a^4*c*e^8*x^2 - 36*a^2*d^2*e^6*(-a*c)^(3/2) - 36*c^2*d^6*e^
2*(-a*c)^(3/2) + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4
*d^6*e^2*x^2)*(c*((d^3*(-a*c)^(1/2))/8 - (a*d^2*e)/4) - (a*d*e^2*(-a*c)^(1/
2))/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - ((d*x^2)/(4*(a*e^2 +
c*d^2)) + (a*e)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4) - (log(c^5*d^8*x^2 - c^4
*d^8*(-a*c)^(1/2) - 70*d^4*e^4*(-a*c)^(5/2) - a^4*e^8*(-a*c)^(1/2) + a^4*c*
e^8*x^2 + 36*a^2*d^2*e^6*(-a*c)^(3/2) + 36*c^2*d^6*e^2*(-a*c)^(3/2) + 70*a^
2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(c*((d^3
*(-a*c)^(1/2))/8 + (a*d^2*e)/4) - (a*d*e^2*(-a*c)^(1/2))/8))/(a*c^3*d^4 + a
^3*c*e^4 + 2*a^2*c^2*d^2*e^2) + (d^2*e*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^
4 + 2*a*c*d^2*e^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.247 $\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=149

$$\frac{de^2\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}-\frac{de^2\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{e\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)^2}+\frac{ex^2-d}{4\left(a+cx^4\right)\left(ae^2+cd^2\right)}$$

```
[Out] 1/4*(e*x^2-d)/(a*e^2+c*d^2)/(c*x^4+a)-1/2*d*e^2*ln(e*x^2+d)/(a*e^2+c*d^2)^2
+1/4*d*e^2*ln(c*x^4+a)/(a*e^2+c*d^2)^2-1/4*e*(-a*e^2+c*d^2)*arctan(x^2*c^(1
/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 823, 801, 635, 205, 260}

$$-\frac{d-ex^2}{4\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{de^2\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{de^2\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}-\frac{e\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(d - e*x^2)/(4*(c*d^2 + a*e^2)*(a + c*x^4)) - (e*(c*d^2 - a*e^2)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - (d*e^2*Log[d
+ e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d*e^2*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{x^3}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{acde-ace^2x}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)}{4ac\left(cd^2+ae^2\right)}$$

$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{2acde^3}{\left(cd^2+ae^2\right)(d+ex)} - \frac{ace\left(-cd^2+ae^2+2cdex\right)}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right) dx, x, x^2\right.}{4ac\left(cd^2+ae^2\right)}$$

$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{de^2\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{e\,\text{Subst}\left(\int \frac{-cd^2+ae^2+2cdex}{a+cx^2}\,dx, x, x^2\right)}{4\left(cd^2+ae^2\right)^2}$$

$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{de^2\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{\left(cde^2\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)^2} -$$

$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,\sqrt{c}\,\left(cd^2+ae^2\right)^2} - \frac{de^2\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{de^2\,\mathrm{l}}{4(}$$

Mathematica [A] time = 0.14, size = 114, normalized size = 0.77

$$\frac{\frac{e\left(ae^2-cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} + \frac{\left(ex^2-d\right)\left(ae^2+cd^2\right)}{a+cx^4} + de^2\log\left(a+cx^4\right) - 2de^2\log\left(d+ex^2\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(-d + e*x^2))/(a + c*x^4) + (e*(-(c*d^2) + a*e^2)*ArcTan[
(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - 2*d*e^2*Log[d + e*x^2] + d*e^2*
Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

fricas [A] time = 6.69, size = 492, normalized size = 3.30

$$\left[-\frac{2\,ac^2d^3 + 2\,a^2cde^2 - 2\left(ac^2d^2e + a^2ce^3\right)x^2 - \left(acd^2e - a^2e^3 + \left(c^2d^2e - ace^3\right)x^4\right)\sqrt{-ac}\,\log\left(\frac{cx^4 - 2\,\sqrt{-ac}\,x^2 - a}{cx^4+a}\right) - 2}{8\left(a^2c^3d^4 + 2\,a^3c^2d^2e^2 + a^4ce^4 + \left(ac^4d^4 + 2\,a^2c^3d^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a*c^2*d^3 + 2*a^2*c*d*e^2 - 2*(a*c^2*d^2*e + a^2*c*e^3)*x^2 - (a*c
*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*sqrt(-a*c)*log((c*x^4 - 2*sqr
t(-a*c)*x^2 - a)/(c*x^4 + a)) - 2*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(c*x^4
 + a) + 4*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*
a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)
*x^4), -1/4*(a*c^2*d^3 + a^2*c*d*e^2 - (a*c^2*d^2*e + a^2*c*e^3)*x^2 - (a*c
*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*sqrt(a*c)*arctan(sqrt(a*c)/(c
*x^2)) - (a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(c*x^4 + a) + 2*(a*c^2*d*e^2*x^
4 + a^2*c*d*e^2)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e
^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4)]
```

giac [A] time = 0.28, size = 188, normalized size = 1.26

$$\frac{de^2\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{de^3\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}-\frac{\left(cd^2e-ae^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{ac}}-\frac{cd^3-\left(cd^2e+ae^3\right)x^2+a}{4\left(cx^4+a\right)\left(cd^2+ae^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*d*e^2*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d*e^3*lo
g(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) - 1/4*(c*d^2*e - a*
e^3)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)
) - 1/4*(c*d^3 - (c*d^2*e + a*e^3)*x^2 + a*d*e^2)/((c*x^4 + a)*(c*d^2 + a*e
^2)^2)
```

maple [A] time = 0.02, size = 247, normalized size = 1.66

$$\frac{a\,e^3x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)}+\frac{c\,d^2e\,x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)}+\frac{a\,e^3\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}-\frac{c\,d^2e\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}-\frac{ad\,e^2}{4\left(a\,e^2+c\,d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*e^3*a+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*e
*c*d^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a*d*e^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*
c*d^3+1/4*d*e^2*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4/(a*e^2+c*d^2)^2/(a*c)^(1/2)
*arctan(1/(a*c)^(1/2)*c*x^2)*a*e^3-1/4/(a*e^2+c*d^2)^2*e/(a*c)^(1/2)*arctan
(1/(a*c)^(1/2)*c*x^2)*c*d^2-1/2*d*e^2*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.03, size = 186, normalized size = 1.25

$$\frac{de^2\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{de^2\log\left(ex^2+d\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{\left(cd^2e-ae^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{ac}}+\frac{ex^2-d}{4\left(\left(c^2d^2+ace^2\right)x^4+acd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*d*e^2*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d*e^2*lo
g(e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/4*(c*d^2*e - a*e^3)*ar
ctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) + 1/4
*(e*x^2 - d)/((c^2*d^2 + a*c*e^2)*x^4 + a*c*d^2 + a^2*e^2)
```

mupad [B] time = 1.41, size = 527, normalized size = 3.54

$$\frac{\ln\left(a^4\,e^8\,\sqrt{-a\,c}\,+c^4\,d^8\,\sqrt{-a\,c}\,+70\,d^4\,e^4\,{\left(-a\,c\right)}^{5/2}+c^5\,d^8\,x^2+a^4\,c\,e^8\,x^2-36\,a^2\,d^2\,e^6\,{\left(-a\,c\right)}^{3/2}-36\,c^2\,d^6\,e^2\,(-a\,c}{a^3\,c\,e^4+2\,a^2\,c^2\,d^2\,e^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(a^4*e^8*(-a*c)^(1/2) + c^4*d^8*(-a*c)^(1/2) + 70*d^4*e^4*(-a*c)^(5/2)
+ c^5*d^8*x^2 + a^4*c*e^8*x^2 - 36*a^2*d^2*e^6*(-a*c)^(3/2) - 36*c^2*d^6*e^
2*(-a*c)^(3/2) + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4
*d^6*e^2*x^2)*(a*((e^3*(-a*c)^(1/2))/8 + (c*d*e^2)/4) - (c*d^2*e*(-a*c)^(1/
2))/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (d/(4*(a*e^2 + c*d^2)
) - (e*x^2)/(4*(a*e^2 + c*d^2)))/(a + c*x^4) - (log(c^5*d^8*x^2 - c^4*d^8*(
-a*c)^(1/2) - 70*d^4*e^4*(-a*c)^(5/2) - a^4*e^8*(-a*c)^(1/2) + a^4*c*e^8*x^
2 + 36*a^2*d^2*e^6*(-a*c)^(3/2) + 36*c^2*d^6*e^2*(-a*c)^(3/2) + 70*a^2*c^3*
d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(a*((e^3*(-a*c
)^(1/2))/8 - (c*d*e^2)/4) - (c*d^2*e*(-a*c)^(1/2))/8))/(a*c^3*d^4 + a^3*c*e
^4 + 2*a^2*c^2*d^2*e^2) - (d*e^2*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*
a*c*d^2*e^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.248 $\int \frac{x}{(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=151

$$\frac{\sqrt{c}\, d \left(3ae^2+cd^2\right) \tan^{-1}\left(\frac{\sqrt{c}\, x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx^2}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^3 \log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}+\frac{e^3 \log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}$$

```
[Out] 1/4*(c*d*x^2+a*e)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/2*e^3*ln(e*x^2+d)/(a*e^2+c*d^
2)^2-1/4*e^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4*d*(3*a*e^2+c*d^2)*arctan(x^2*c
^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^2+c*d^2)^2
```

Rubi [A] time = 0.18, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = $\{1248, 741, 801, 635, 205, 260\}$

$$\frac{\sqrt{c}\, d \left(3ae^2+cd^2\right) \tan^{-1}\left(\frac{\sqrt{c}\, x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx^2}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e^3 \log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{e^3 \log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*e + c*d*x^2)/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[c]*d*(c*d^2 + 3*a
*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)^2) + (e^3*L
og[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (e^3*Log[a + c*x^4])/(4*(c*d^2 + a*e
^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(d+ex)(a+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{-cd^2-2ae^2-cdex}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4a(cd^2+ae^2)}$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(-\frac{2ae^4}{(cd^2+ae^2)(d+ex)} - \frac{c(cd^3+3ade^2-2ae^3x)}{(cd^2+ae^2)(a+cx^2)}\right) dx, x, x^2\right)}{4a(cd^2+ae^2)}$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{c\,\text{Subst}\left(\int \frac{cd^3+3ade^2-2ae^3x}{a+cx^2}\,dx, x, x^2\right)}{4a(cd^2+ae^2)^2}$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{(ce^3)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)^2} + \frac{(c$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{c}\,d(cd^2+3ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^2+ae^2)^2} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{e$$

Mathematica [A] time = 0.13, size = 117, normalized size = 0.77

$$\frac{\frac{\sqrt{c}\,d(3ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{a^{3/2}} + \frac{(ae^2+cd^2)(ae+cdx^2)}{a(a+cx^4)} - e^3\log(a+cx^4) + 2e^3\log(d+ex^2)}{4(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(a*e + c*d*x^2))/(a*(a + c*x^4)) + (Sqrt[c]*d*(c*d^2 + 3*
a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/a^(3/2) + 2*e^3*Log[d + e*x^2] - e^3*
Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

fricas [A] time = 15.72, size = 458, normalized size = 3.03

$$\left[\frac{2acd^2e + 2a^2e^3 + 2(c^2d^3 + acde^2)x^2 + (acd^3 + 3a^2de^2 + (c^2d^3 + 3acde^2)x^4)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right) - 2(}{8(a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + 2a^2c^2d^2e^2 + a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c*d^2*e + 2*a^2*e^3 + 2*(c^2*d^3 + a*c*d*e^2)*x^2 + (a*c*d^3 + 3*
a^2*d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^4)*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sq
rt(-c/a) - a)/(c*x^4 + a)) - 2*(a*c*e^3*x^4 + a^2*e^3)*log(c*x^4 + a) + 4*(
a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 + d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4
*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), 1/4*(a*c*d^2*e + a
^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x^2 - (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*
a*c*d*e^2)*x^4)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) - (a*c*e^3*x^4 + a^2*
e^3)*log(c*x^4 + a) + 2*(a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 + d))/(a^2*c^2*d^
4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)
*x^4)]
```

giac [A] time = 0.38, size = 199, normalized size = 1.32

$$-\frac{e^3\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{e^4\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}+\frac{\left(c^2d^3+3\,acde^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}+\frac{acd^2e+\left(c^2d^3+acd\right.}{4\left(cx^4+a\right)\left(cd^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*e^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*e^4*log(a
bs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(c^2*d^3 + 3*a*c
*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sq
rt(a*c)) + 1/4*(a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x^2 + a^2*e^3)/((c*x^4 +
a)*(c*d^2 + a*e^2)^2*a)
```

maple [A] time = 0.02, size = 255, normalized size = 1.69

$$\frac{c^2d^3x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)a}+\frac{cd\,e^2x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)}+\frac{c^2d^3\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}\;a}+\frac{3cd\,e^2\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}+\frac{a}{4\left(a\,e^2+c\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*x^2*e^2+1/4*c^2/(a*e^2+c*d^2)^2/(c*x^4+a)
*d^3/a*x^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*a+1/4*c/(a*e^2+c*d^2)^2/(c*x^4
+a)*e*d^2-1/4*e^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+3/4*c/(a*e^2+c*d^2)^2/(a*c)^(
1/2)*arctan(1/(a*c)^(1/2)*c*x^2)*d*e^2+1/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2
)*arctan(1/(a*c)^(1/2)*c*x^2)*d^3+1/2*e^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.03, size = 196, normalized size = 1.30

$$-\frac{e^3\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{e^3\log\left(ex^2+d\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{\left(c^2d^3+3\,acde^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}+\frac{cdx^2+}{4\left(a^2cd^2+a^3e^2+\left(ac\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*e^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*e^3*log(e
*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c^2*d^3 + 3*a*c*d*e^2)
*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)
) + 1/4*(c*d*x^2 + a*e)/(a^2*c*d^2 + a^3*e^2 + (a*c^2*d^2 + a^2*c*e^2)*x^4)
```

mupad [B] time = 1.49, size = 649, normalized size = 4.30

$$\frac{\frac{e}{4\left(c\,d^2+a\,e^2\right)}+\frac{c\,d\,x^2}{4\,a\left(c\,d^2+a\,e^2\right)}}{c\,x^4+a}+\frac{e^3\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}+\frac{\ln\left(36\,a^6\,e^{10}\,\sqrt{-a^3\,c}+36\,a^7\,c\,e^{10}\,x^2+a\,c^5\,d^{10}\,\sqrt{-a^3\,c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (e/(4*(a*e^2 + c*d^2)) + (c*d*x^2)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) + (e^
3*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (log(36*a^6*e^1
0*(-a^3*c)^(1/2) + 36*a^7*c*e^10*x^2 + a*c^5*d^10*(-a^3*c)^(1/2) + a^2*c^6*
d^10*x^2 - 81*a^2*d^2*e^8*(-a^3*c)^(3/2) - 22*c^2*d^6*e^4*(-a^3*c)^(3/2) +
8*a^3*c^5*d^8*e^2*x^2 + 22*a^4*c^4*d^6*e^4*x^2 + 60*a^5*c^3*d^4*e^6*x^2 + 8
1*a^6*c^2*d^2*e^8*x^2 + 8*a^2*c^4*d^8*e^2*(-a^3*c)^(1/2) - 60*a*c*d^4*e^6*(
-a^3*c)^(3/2))*(c*d^3*(-a^3*c)^(1/2) - 2*a^3*e^3 + 3*a*d*e^2*(-a^3*c)^(1/2)
))/(8*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)) - (log(36*a^7*c*e^10*x^2 -
 36*a^6*e^10*(-a^3*c)^(1/2) - a*c^5*d^10*(-a^3*c)^(1/2) + a^2*c^6*d^10*x^2
+ 81*a^2*d^2*e^8*(-a^3*c)^(3/2) + 22*c^2*d^6*e^4*(-a^3*c)^(3/2) + 8*a^3*c^5
*d^8*e^2*x^2 + 22*a^4*c^4*d^6*e^4*x^2 + 60*a^5*c^3*d^4*e^6*x^2 + 81*a^6*c^2
*d^2*e^8*x^2 - 8*a^2*c^4*d^8*e^2*(-a^3*c)^(1/2) + 60*a*c*d^4*e^6*(-a^3*c)^(
3/2))*(2*a^3*e^3 + c*d^3*(-a^3*c)^(1/2) + 3*a*d*e^2*(-a^3*c)^(1/2)))/(8*(a^
5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.249 $\int \frac{1}{x(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=209

$$-\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)}-\frac{cd\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}+\frac{\log(x)}{a^2d}+\frac{c\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^4\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}\,e^3\tan}{2\sqrt{a}\,(ae}$$

```
[Out] 1/4*c*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+ln(x)/a^2/d-1/2*e^4*ln(e*x^2+d)/
d/(a*e^2+c*d^2)^2-1/4*c*d*(2*a*e^2+c*d^2)*ln(c*x^4+a)/a^2/(a*e^2+c*d^2)^2-1
/4*e*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^2+c*d^2)-1/2*e^3*arct
an(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^2+c*d^2)^2/a^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 894, 639, 205, 635, 260}

$$-\frac{cd\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)}+\frac{\log(x)}{a^2d}+\frac{c\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^4\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}\,e^3\tan}{2\sqrt{a}\,(ae}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[c]*e^3*ArcTan[(Sq
rt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)^2) - (Sqrt[c]*e*ArcTan[(Sqr
t[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)) + Log[x]/(a^2*d) - (e^4*Log
[d + e*x^2])/(2*d*(c*d^2 + a*e^2)^2) - (c*d*(c*d^2 + 2*a*e^2)*Log[a + c*x^4
])/(4*a^2*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^2dx} - \frac{e^5}{d\left(cd^2+ae^2\right)^2(d+ex)} - \frac{c(ae+cdx)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{c\left(-a\right.}{a^2\left(\right.}\right.\right.$$

$$= \frac{\log(x)}{a^2d} - \frac{e^4\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)^2} + \frac{c\,\text{Subst}\left(\int \frac{-a^2e^3-cd\left(cd^2+2ae^2\right)x}{a+cx^2}\,dx, x, x^2\right)}{2a^2\left(cd^2+ae^2\right)^2} - \frac{c\,\text{Subst}\left(\right.}{2a}$$

$$= \frac{c\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{\log(x)}{a^2d} - \frac{e^4\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)^2} - \frac{\left(ce^3\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,\right.}{2\left(cd^2+ae^2\right)^2}$$

$$= \frac{c\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{c}\,e^3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^2+ae^2\right)^2} - \frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^2+ae^2\right)} + \frac{\log(x)}{a^2d} -$$

Mathematica [A] time = 0.18, size = 241, normalized size = 1.15

$$\frac{-2a^2e^4\left(a+cx^4\right)\log\left(d+ex^2\right)+4\log(x)\left(a+cx^4\right)\left(ae^2+cd^2\right)^2-cd^2\left(a+cx^4\right)\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)+\sqrt{}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*c*d*(c*d^2 + a*e^2)*(d - e*x^2) + Sqrt[a]*Sqrt[c]*d*e*(c*d^2 + 3*a*e^2)*
(a + c*x^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*Sqrt[c]*d*e*(
c*d^2 + 3*a*e^2)*(a + c*x^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*(c
*d^2 + a*e^2)^2*(a + c*x^4)*Log[x] - 2*a^2*e^4*(a + c*x^4)*Log[d + e*x^2] -
 c*d^2*(c*d^2 + 2*a*e^2)*(a + c*x^4)*Log[a + c*x^4])/(4*a^2*d*(c*d^2 + a*e^
2)^2*(a + c*x^4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 0.37, size = 279, normalized size = 1.33

$$-\frac{\left(c^2d^3+2\,acde^2\right)\log\left(cx^4+a\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)}-\frac{e^5\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^5e+2\,acd^3e^3+a^2de^5\right)}-\frac{\left(c^2d^2e+3\,ace^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}+\frac{c^3d^3x^4+2\,ac^2}{4(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(c^2*d^3 + 2*a*c*d*e^2)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
+ a^4*e^4) - 1/2*e^5*log(abs(x^2*e + d))/(c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d
*e^5) - 1/4*(c^2*d^2*e + 3*a*c*e^3)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2
*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) + 1/4*(c^3*d^3*x^4 + 2*a*c^2*d*x^4*e^2
 - a*c^2*d^2*x^2*e + 2*a*c^2*d^3 - a^2*c*x^2*e^3 + 3*a^2*c*d*e^2)/((a^2*c^2
*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*(c*x^4 + a)) + 1/2*log(x^2)/(a^2*d)
```

maple [**A**] time = 0.02, size = 309, normalized size = 1.48

$$-\frac{c^2d^2e\,x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)a}-\frac{c\,e^3x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)}-\frac{c^2d^2e\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}\;a}-\frac{3c\,e^3\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}+\frac{}{4\left(a\,e^2+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] ln(x)/a^2/d-1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*x^2-1/4*c^2/(a*e^2+c*d^2)^2
/a/(c*x^4+a)*x^2*e*d^2+1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*e^2+1/4*c^2/(a*e^2
+c*d^2)^2/a/(c*x^4+a)*d^3-1/2*c/(a*e^2+c*d^2)^2/a*ln(c*x^4+a)*d*e^2-1/4*c^2
/(a*e^2+c*d^2)^2/a^2*ln(c*x^4+a)*d^3-3/4*c/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arct
an(1/(a*c)^(1/2)*c*x^2)*e^3-1/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*arctan(1/
(a*c)^(1/2)*c*x^2)*e*d^2-1/2*e^4*ln(e*x^2+d)/d/(a*e^2+c*d^2)^2
```

maxima [**A**] time = 2.05, size = 228, normalized size = 1.09

$$-\frac{e^4\log\left(ex^2+d\right)}{2\left(c^2d^5+2\,acd^3e^2+a^2de^4\right)}-\frac{\left(c^2d^3+2\,acde^2\right)\log\left(cx^4+a\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)}-\frac{\left(c^2d^2e+3\,ace^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}-\frac{ce}{4\left(a^2cd^2+a^3e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^4*log(e*x^2 + d)/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4) - 1/4*(c^2*d^
3 + 2*a*c*d*e^2)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) -
 1/4*(c^2*d^2*e + 3*a*c*e^3)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*
d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/4*(c*e*x^2 - c*d)/(a^2*c*d^2 + a^3*e^2 +
(a*c^2*d^2 + a^2*c*e^2)*x^4) + 1/2*log(x^2)/(a^2*d)
```

mupad [**B**] time = 2.58, size = 1082, normalized size = 5.18

$$\frac{\frac{c\,d}{4\,a\,\left(c\,d^2+a\,e^2\right)}-\frac{c\,e\,x^2}{4\,a\,\left(c\,d^2+a\,e^2\right)}}{c\,x^4+a}-\frac{\ln\left(400\,a^9\,c^{12}\,d^{20}\,x^2-10481\,d^4\,e^{16}\,{\left(-a^5\,c\right)}^{7/2}-1024\,a^{12}\,e^{20}\,{\left(-a^5\,c\right)}^{3/2}+1024\,a^{19}\,c^2\,e^{20}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((c*d)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^2)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4)
 - (log(400*a^9*c^12*d^20*x^2 - 10481*d^4*e^16*(-a^5*c)^(7/2) - 1024*a^12*e
```

```
^20*(-a^5*c)^(3/2) + 1024*a^19*c^2*e^20*x^2 - 400*a^2*c^10*d^20*(-a^5*c)^(3
/2) + 5840*a^6*d^2*e^18*(-a^5*c)^(5/2) + 33710*c^6*d^14*e^6*(-a^5*c)^(5/2)
+ 4104*a^10*c^11*d^18*e^2*x^2 + 16689*a^11*c^10*d^16*e^4*x^2 + 33710*a^12*c
^9*d^14*e^6*x^2 + 33391*a^13*c^8*d^12*e^8*x^2 + 10748*a^14*c^7*d^10*e^10*x^
2 - 3585*a^15*c^6*d^8*e^12*x^2 + 3998*a^16*c^5*d^6*e^14*x^2 + 10481*a^17*c^
4*d^4*e^16*x^2 + 5840*a^18*c^3*d^2*e^18*x^2 + 10748*a^2*c^4*d^10*e^10*(-a^5
*c)^(5/2) - 3585*a^3*c^3*d^8*e^12*(-a^5*c)^(5/2) + 3998*a^4*c^2*d^6*e^14*(-
a^5*c)^(5/2) - 4104*a^3*c^9*d^18*e^2*(-a^5*c)^(3/2) - 16689*a^4*c^8*d^16*e^
4*(-a^5*c)^(3/2) + 33391*a*c^5*d^12*e^8*(-a^5*c)^(5/2))*(3*a*e^3*(-a^5*c)^(
1/2) + 2*a^2*c^2*d^3 + 4*a^3*c*d*e^2 + c*d^2*e*(-a^5*c)^(1/2)))/(8*(a^6*e^4
 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)) + (log(1024*a^12*e^20*(-a^5*c)^(3/2) + 1
0481*d^4*e^16*(-a^5*c)^(7/2) + 400*a^9*c^12*d^20*x^2 + 1024*a^19*c^2*e^20*x
^2 + 400*a^2*c^10*d^20*(-a^5*c)^(3/2) - 5840*a^6*d^2*e^18*(-a^5*c)^(5/2) -
33710*c^6*d^14*e^6*(-a^5*c)^(5/2) + 4104*a^10*c^11*d^18*e^2*x^2 + 16689*a^1
1*c^10*d^16*e^4*x^2 + 33710*a^12*c^9*d^14*e^6*x^2 + 33391*a^13*c^8*d^12*e^8
*x^2 + 10748*a^14*c^7*d^10*e^10*x^2 - 3585*a^15*c^6*d^8*e^12*x^2 + 3998*a^1
6*c^5*d^6*e^14*x^2 + 10481*a^17*c^4*d^4*e^16*x^2 + 5840*a^18*c^3*d^2*e^18*x
^2 - 10748*a^2*c^4*d^10*e^10*(-a^5*c)^(5/2) + 3585*a^3*c^3*d^8*e^12*(-a^5*c
)^(5/2) - 3998*a^4*c^2*d^6*e^14*(-a^5*c)^(5/2) + 4104*a^3*c^9*d^18*e^2*(-a^
5*c)^(3/2) + 16689*a^4*c^8*d^16*e^4*(-a^5*c)^(3/2) - 33391*a*c^5*d^12*e^8*(
-a^5*c)^(5/2))*(3*a*e^3*(-a^5*c)^(1/2) - 2*a^2*c^2*d^3 - 4*a^3*c*d*e^2 + c*
d^2*e*(-a^5*c)^(1/2)))/(8*(a^6*e^4 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)) - (e^4
*log(d + e*x^2))/(2*c^2*d^5 + 2*a^2*d*e^4 + 4*a*c*d^3*e^2) + log(x)/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.250 $\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)^2} dx$

Optimal. Leaf size=236

$$-\frac{c^{3/2}d\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}+\frac{ce\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}-$$

```
[Out] -1/2/a^2/d/x^2-1/4*c*(c*d*x^2+a*e)/a^2/(a*e^2+c*d^2)/(c*x^4+a)-1/4*c^(3/2)*
d*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)-1/2*c^(3/2)*d*(2*a*e^2+
c*d^2)*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)^2-e*ln(x)/a^2/d^2+
1/2*e^5*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)^2+1/4*c*e*(2*a*e^2+c*d^2)*ln(c*x^4+a)
/a^2/(a*e^2+c*d^2)^2
```

Rubi [A] time = 0.26, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1252, 894, 639, 205, 635, 260}

$$-\frac{c^{3/2}d\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{ce\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/(2*a^2*d*x^2) - (c*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*(c*d^2 + a*e^2)) - (
c^(3/2)*d*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(5/2)*(c*d^
2 + a*e^2)^2) - (e*Log[x])/(a^2*d^2) + (e^5*Log[d + e*x^2])/(2*d^2*(c*d^2 +
 a*e^2)^2) + (c*e*(c*d^2 + 2*a*e^2)*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2)^
2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2dx^2} - \frac{e}{a^2d^2x} + \frac{e^6}{d^2\left(cd^2+ae^2\right)^2(d+ex)} - \frac{c^2(d-ex)}{a\left(cd^2+ae^2\right)\left(a+cx\right.}\right.\right.$$

$$= -\frac{1}{2a^2dx^2} - \frac{e\log(x)}{a^2d^2} + \frac{e^5\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)^2} - \frac{c^2\,\text{Subst}\left(\int \frac{d-ex}{\left(a+cx^2\right)^2}\,dx, x, x^2\right)}{2a\left(cd^2+ae^2\right)} - \frac{\left(c^2\right.}{}$$

$$= -\frac{1}{2a^2dx^2} - \frac{c\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e\log(x)}{a^2d^2} + \frac{e^5\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)^2} - \frac{\left(c^2d\right)\text{Su}}{}$$

$$= -\frac{1}{2a^2dx^2} - \frac{c\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(cd^2+ae^2\right)} - \frac{c^{3/2}d\left(cd^2+2ae^2\right.}{2a^{5/2}\left(cd^2\right.}$$

Mathematica [A] time = 0.43, size = 248, normalized size = 1.05

$$\frac{1}{4}\left(\frac{c^{3/2}d\left(5ae^2+3cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2}\left(ae^2+cd^2\right)^2} + \frac{c^{3/2}d\left(5ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{5/2}\left(ae^2+cd^2\right)^2} - \frac{c\left(ae+cdx^2\right)}{a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-2/(a^2*d*x^2) - (c*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4)) + (
c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(
5/2)*(c*d^2 + a*e^2)^2) + (c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) - (4*e*Log[x])/(a^2*d^2)
 + (2*e^5*Log[d + e*x^2])/(c*d^3 + a*d*e^2)^2 + (c*(c*d^2*e + 2*a*e^3)*Log[
a + c*x^4])/(a^2*(c*d^2 + a*e^2)^2))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.35, size = 344, normalized size = 1.46

$$\frac{\left(c^2d^2e+2\,ace^3\right)\log\left(cx^4+a\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)}+\frac{e^6\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^6e+2\,acd^4e^3+a^2d^2e^5\right)}-\frac{\left(3\,c^3d^3+5\,ac^2de^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)\sqrt{ac}}-\frac{9\,c^3d^5x^4+15\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(c^2*d^2*e + 2*a*c*e^3)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
 a^4*e^4) + 1/2*e^6*log(abs(x^2*e + d))/(c^2*d^6*e + 2*a*c*d^4*e^3 + a^2*d^
2*e^5) - 1/4*(3*c^3*d^3 + 5*a*c^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*
d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/12*(9*c^3*d^5*x^4 + 15*a*c^
2*d^3*x^4*e^2 - 2*a^2*c*x^6*e^5 + 3*a*c^2*d^4*x^2*e + 6*a^2*c*d*x^4*e^4 + 6
*a*c^2*d^5 + 3*a^2*c*d^2*x^2*e^3 + 12*a^2*c*d^3*e^2 - 2*a^3*x^2*e^5 + 6*a^3
*d*e^4)/((a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*e^4)*(c*x^6 + a*x^2)) - 1
/2*e*log(x^2)/(a^2*d^2)
```

maple [A] time = 0.02, size = 332, normalized size = 1.41

$$-\frac{c^2d\,e^2x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)a}-\frac{c^3d^3x^2}{4\left(a\,e^2+c\,d^2\right)^2\left(c\,x^4+a\right)a^2}-\frac{5c^2d\,e^2\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}\;a}-\frac{3c^3d^3\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}\;a^2}-\frac{}{4\left(a\,e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/2/a^2/d/x^2-e*ln(x)/a^2/d^2-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^2*d*e^
2-1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x^2*d^3-1/4*c/(a*e^2+c*d^2)^2/(c*x^
4+a)*e^3-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*e*d^2+1/2*c/(a*e^2+c*d^2)^2/a*
ln(c*x^4+a)*e^3+1/4*c^2/(a*e^2+c*d^2)^2/a^2*ln(c*x^4+a)*e*d^2-5/4*c^2/(a*e^
2+c*d^2)^2/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)*d*e^2-3/4*c^3/(a*e^2+c
*d^2)^2/a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)*d^3+1/2*e^5*ln(e*x^2+d)
/d^2/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.01, size = 278, normalized size = 1.18

$$\frac{e^5\log\left(ex^2+d\right)}{2\left(c^2d^6+2\,acd^4e^2+a^2d^2e^4\right)}+\frac{\left(c^2d^2e+2\,ace^3\right)\log\left(cx^4+a\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)}-\frac{\left(3\,c^3d^3+5\,ac^2de^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)\sqrt{ac}}-\frac{acdex^2+\left(3\,c^2d\right.}{4\left(\left(a^2c^2d^3+a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*e^5*log(e*x^2 + d)/(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4) + 1/4*(c^2*d
^2*e + 2*a*c*e^3)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)
- 1/4*(3*c^3*d^3 + 5*a*c^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*d^4 + 2
*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/4*(a*c*d*e*x^2 + (3*c^2*d^2 + 2*a*
c*e^2)*x^4 + 2*a*c*d^2 + 2*a^2*e^2)/((a^2*c^2*d^3 + a^3*c*d*e^2)*x^6 + (a^3
*c*d^3 + a^4*d*e^2)*x^2) - 1/2*e*log(x^2)/(a^2*d^2)
```

mupad [B] time = 2.94, size = 1337, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(81*a^10*c^16*d^24*x^2 + 1024*a^22*c^4*e^24*x^2 - 81*a^3*c^11*d^24*(-a^
5*c^3)^(3/2) + 1024*a^20*c^2*e^24*(-a^5*c^3)^(1/2) - 14496*a^6*d^8*e^16*(-a
^5*c^3)^(5/2) - 5120*a^14*d^2*e^22*(-a^5*c^3)^(3/2) + 11647*c^6*d^20*e^4*(-
a^5*c^3)^(5/2) + 1638*a^11*c^15*d^22*e^2*x^2 + 11647*a^12*c^14*d^20*e^4*x^2
 + 43524*a^13*c^13*d^18*e^6*x^2 + 97311*a^14*c^12*d^16*e^8*x^2 + 133334*a^1
5*c^11*d^14*e^10*x^2 + 103633*a^16*c^10*d^12*e^12*x^2 + 29456*a^17*c^9*d^10
*e^14*x^2 - 14496*a^18*c^8*d^8*e^16*x^2 - 7984*a^19*c^7*d^6*e^18*x^2 + 5888
*a^20*c^6*d^4*e^20*x^2 + 5120*a^21*c^5*d^2*e^22*x^2 + 43524*a*c^5*d^18*e^6*
(-a^5*c^3)^(5/2) + 29456*a^5*c*d^10*e^14*(-a^5*c^3)^(5/2) - 5888*a^13*c*d^4
*e^20*(-a^5*c^3)^(3/2) + 97311*a^2*c^4*d^16*e^8*(-a^5*c^3)^(5/2) + 133334*a
^3*c^3*d^14*e^10*(-a^5*c^3)^(5/2) + 103633*a^4*c^2*d^12*e^12*(-a^5*c^3)^(5/
2) - 1638*a^4*c^10*d^22*e^2*(-a^5*c^3)^(3/2) + 7984*a^12*c^2*d^6*e^18*(-a^5
*c^3)^(3/2))*(4*a^4*c*e^3 - 3*c*d^3*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d^2*e - 5*
a*d*e^2*(-a^5*c^3)^(1/2)))/(8*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)) -
(1/(2*a*d) + (c*e*x^2)/(4*a*(a*e^2 + c*d^2)) + (c*x^4*(2*a*e^2 + 3*c*d^2))/
(4*a^2*d*(a*e^2 + c*d^2)))/(a*x^2 + c*x^6) + (log(81*a^10*c^16*d^24*x^2 + 1
024*a^22*c^4*e^24*x^2 + 81*a^3*c^11*d^24*(-a^5*c^3)^(3/2) - 1024*a^20*c^2*e
^24*(-a^5*c^3)^(1/2) + 14496*a^6*d^8*e^16*(-a^5*c^3)^(5/2) + 5120*a^14*d^2*
e^22*(-a^5*c^3)^(3/2) - 11647*c^6*d^20*e^4*(-a^5*c^3)^(5/2) + 1638*a^11*c^1
5*d^22*e^2*x^2 + 11647*a^12*c^14*d^20*e^4*x^2 + 43524*a^13*c^13*d^18*e^6*x^
2 + 97311*a^14*c^12*d^16*e^8*x^2 + 133334*a^15*c^11*d^14*e^10*x^2 + 103633*
a^16*c^10*d^12*e^12*x^2 + 29456*a^17*c^9*d^10*e^14*x^2 - 14496*a^18*c^8*d^8
*e^16*x^2 - 7984*a^19*c^7*d^6*e^18*x^2 + 5888*a^20*c^6*d^4*e^20*x^2 + 5120*
a^21*c^5*d^2*e^22*x^2 - 43524*a*c^5*d^18*e^6*(-a^5*c^3)^(5/2) - 29456*a^5*c
*d^10*e^14*(-a^5*c^3)^(5/2) + 5888*a^13*c*d^4*e^20*(-a^5*c^3)^(3/2) - 97311
*a^2*c^4*d^16*e^8*(-a^5*c^3)^(5/2) - 133334*a^3*c^3*d^14*e^10*(-a^5*c^3)^(5
/2) - 103633*a^4*c^2*d^12*e^12*(-a^5*c^3)^(5/2) + 1638*a^4*c^10*d^22*e^2*(-
a^5*c^3)^(3/2) - 7984*a^12*c^2*d^6*e^18*(-a^5*c^3)^(3/2))*(4*a^4*c*e^3 + 3*
c*d^3*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d^2*e + 5*a*d*e^2*(-a^5*c^3)^(1/2)))/(8*
(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)) + (e^5*log(d + e*x^2))/(2*c^2*d^
6 + 2*a^2*d^2*e^4 + 4*a*c*d^4*e^2) - (e*log(x))/(a^2*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.251 $\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=265

$$\frac{c^{3/2}e\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}+\frac{c^2d\left(3ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4a^3\left(ae^2+cd^2\right)^2}-\frac{\log(x)\left(2cd^2-ae^2\right)}{a^3d^3}-\frac{}{4a^2\left(a\right.}$$

```
[Out] -1/4/a^2/d/x^4+1/2*e/a^2/d^2/x^2-1/4*c^2*(-e*x^2+d)/a^2/(a*e^2+c*d^2)/(c*x^
4+a)+1/4*c^(3/2)*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)+1/2*c^
(3/2)*e*(2*a*e^2+c*d^2)*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)^2
-(-a*e^2+2*c*d^2)*ln(x)/a^3/d^3-1/2*e^6*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)^2+1/4
*c^2*d*(3*a*e^2+2*c*d^2)*ln(c*x^4+a)/a^3/(a*e^2+c*d^2)^2
```

Rubi [A] time = 0.33, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 894, 639, 205, 635, 260}

$$-\frac{c^2\left(d-ex^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{c^2d\left(3ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4a^3\left(ae^2+cd^2\right)^2}+\frac{c^{3/2}e\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/(4*a^2*d*x^4) + e/(2*a^2*d^2*x^2) - (c^2*(d - e*x^2))/(4*a^2*(c*d^2 + a*
e^2)*(a + c*x^4)) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*(c
*d^2 + a*e^2)) + (c^(3/2)*e*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*a^(5/2)*(c*d^2 + a*e^2)^2) - ((2*c*d^2 - a*e^2)*Log[x])/(a^3*d^3) - (e
^6*Log[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)^2) + (c^2*d*(2*c*d^2 + 3*a*e^2)*L
og[a + c*x^4])/(4*a^3*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^2dx^3}-\frac{e}{a^2d^2x^2}+\frac{-2cd^2+ae^2}{a^3d^3x}-\frac{e^7}{d^3\left(cd^2+ae^2\right)^2(d+ex)}+\frac{}{a^2\left(\right.}\right.\right.$$
$$= -\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}-\frac{\left(2cd^2-ae^2\right)\log(x)}{a^3d^3}-\frac{e^6\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)^2}+\frac{c^2\,\text{Subst}\left(\int \frac{a}{}\right.}{}$$
$$= -\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}-\frac{c^2\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{\left(2cd^2-ae^2\right)\log(x)}{a^3d^3}-\frac{e^6\log}{2d^3\left(\right.}$$
$$= -\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}-\frac{c^2\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(cd^2+ae^2\right)}+\frac{c^{3/2}e\left(\right.}{}$$

Mathematica [A] time = 0.39, size = 278, normalized size = 1.05

$$\frac{1}{4}\left(-\frac{c^{3/2}e\left(5ae^2+3cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}e\left(5ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{a^{5/2}\left(ae^2+cd^2\right)^2}+\frac{c^2\left(3ade^2+2cd^3\right)\log\left(\right.}{a^3\left(ae^2+cd^2\right)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-(1/(a^2*d*x^4)) + (2*e)/(a^2*d^2*x^2) + (c^2*(-d + e*x^2))/(a^2*(c*d^2 +
a*e^2)*(a + c*x^4)) - (c^(3/2)*e*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) - (c^(3/2)*e*(3*c*d^2 + 5*a*
e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) +
 (4*(-2*c*d^2 + a*e^2)*Log[x])/(a^3*d^3) - (2*e^6*Log[d + e*x^2])/(d^3*(c*d
^2 + a*e^2)^2) + (c^2*(2*c*d^3 + 3*a*d*e^2)*Log[a + c*x^4])/(a^3*(c*d^2 + a
*e^2)^2))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.37, size = 350, normalized size = 1.32

$$\frac{\left(2\,c^{3}d^{3}+3\,ac^{2}de^{2}\right)\log\left(cx^{4}+a\right)}{4\left(a^{3}c^{2}d^{4}+2\,a^{4}cd^{2}e^{2}+a^{5}e^{4}\right)}-\frac{e^{7}\log\left(\left|x^{2}e+d\right|\right)}{2\left(c^{2}d^{7}e+2\,acd^{5}e^{3}+a^{2}d^{3}e^{5}\right)}+\frac{\left(3\,c^{3}d^{2}e+5\,ac^{2}e^{3}\right)\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{4\left(a^{2}c^{2}d^{4}+2\,a^{3}cd^{2}e^{2}+a^{4}e^{4}\right)\sqrt{ac}}-\frac{2\,c^{4}d^{3}x^{4}+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*c^3*d^3 + 3*a*c^2*d*e^2)*log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4) - 1/2*e^7*log(abs(x^2*e + d))/(c^2*d^7*e + 2*a*c*d^5*e^3 + a^
2*d^3*e^5) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*arctan(c*x^2/sqrt(a*c))/((a^2*
c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/4*(2*c^4*d^3*x^4 + 3*a*
c^3*d*x^4*e^2 - a*c^3*d^2*x^2*e + 3*a*c^3*d^3 - a^2*c^2*x^2*e^3 + 4*a^2*c^2
*d*e^2)/((a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*(c*x^4 + a)) - 1/2*(2*c*
d^2 - a*e^2)*log(x^2)/(a^3*d^3) + 1/4*(6*c*d^2*x^4 - 3*a*x^4*e^2 + 2*a*d*x^
2*e - a*d^2)/(a^3*d^3*x^4)
```

maple [A] time = 0.02, size = 363, normalized size = 1.37

$$\frac{c^{2}e^{3}x^{2}}{4\left(a\,e^{2}+c\,d^{2}\right)^{2}\left(c\,x^{4}+a\right)a}+\frac{c^{3}d^{2}e\,x^{2}}{4\left(a\,e^{2}+c\,d^{2}\right)^{2}\left(c\,x^{4}+a\right)a^{2}}+\frac{5c^{2}e^{3}\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{4\left(a\,e^{2}+c\,d^{2}\right)^{2}\sqrt{ac}\;a}+\frac{3c^{3}d^{2}e\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{4\left(a\,e^{2}+c\,d^{2}\right)^{2}\sqrt{ac}\;a^{2}}-\frac{}{4\left(a\,e^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/a^2/d/x^4+1/a^2/d^3*ln(x)*e^2-2/a^3/d*ln(x)*c+1/2*e/a^2/d^2/x^2+1/4*c^
2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*e^3*x^2+1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)
*x^2*e*d^2-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*d*e^2-1/4*c^3/(a*e^2+c*d^2)^
2/a^2/(c*x^4+a)*d^3+3/4*c^2/(a*e^2+c*d^2)^2/a^2*ln(c*x^4+a)*d*e^2+1/2*c^3/(
a*e^2+c*d^2)^2/a^3*ln(c*x^4+a)*d^3+5/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*ar
ctan(1/(a*c)^(1/2)*c*x^2)*e^3+3/4*c^3/(a*e^2+c*d^2)^2/a^2/(a*c)^(1/2)*arcta
n(1/(a*c)^(1/2)*c*x^2)*e*d^2-1/2*e^6*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)^2
```

maxima [A] time = 2.08, size = 332, normalized size = 1.25

$$-\frac{e^{6}\log\left(ex^{2}+d\right)}{2\left(c^{2}d^{7}+2\,acd^{5}e^{2}+a^{2}d^{3}e^{4}\right)}+\frac{\left(2\,c^{3}d^{3}+3\,ac^{2}de^{2}\right)\log\left(cx^{4}+a\right)}{4\left(a^{3}c^{2}d^{4}+2\,a^{4}cd^{2}e^{2}+a^{5}e^{4}\right)}+\frac{\left(3\,c^{3}d^{2}e+5\,ac^{2}e^{3}\right)\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{4\left(a^{2}c^{2}d^{4}+2\,a^{3}cd^{2}e^{2}+a^{4}e^{4}\right)\sqrt{ac}}+\frac{\left(3\,c^{2}d^{2}e+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^6*log(e*x^2 + d)/(c^2*d^7 + 2*a*c*d^5*e^2 + a^2*d^3*e^4) + 1/4*(2*c^
3*d^3 + 3*a*c^2*d*e^2)*log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*
e^4) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*d^
4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) + 1/4*((3*c^2*d^2*e + 2*a*c*e^3)*
x^6 - a*c*d^3 - a^2*d*e^2 - (2*c^2*d^3 + a*c*d*e^2)*x^4 + 2*(a*c*d^2*e + a^
2*e^3)*x^2)/((a^2*c^2*d^4 + a^3*c*d^2*e^2)*x^8 + (a^3*c*d^4 + a^4*d^2*e^2)*
x^4) - 1/2*(2*c*d^2 - a*e^2)*log(x^2)/(a^3*d^3)
```

mupad [B] time = 3.48, size = 1545, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(6400*a^13*c^18*d^28*x^2 + 1024*a^27*c^4*e^28*x^2 - 6400*a^3*c^13*d^28*
(-a^7*c^3)^(3/2) + 1024*a^24*c^2*e^28*(-a^7*c^3)^(1/2) - 10688*a^6*d^8*e^20
*(-a^7*c^3)^(5/2) - 2048*a^16*d^2*e^26*(-a^7*c^3)^(3/2) + 536959*c^6*d^20*e
^8*(-a^7*c^3)^(5/2) + 54944*a^14*c^17*d^26*e^2*x^2 + 200881*a^15*c^16*d^24*
e^4*x^2 + 413414*a^16*c^15*d^22*e^6*x^2 + 536959*a^17*c^14*d^20*e^8*x^2 + 4
65092*a^18*c^13*d^18*e^10*x^2 + 256991*a^19*c^12*d^16*e^12*x^2 + 52822*a^20
*c^11*d^14*e^14*x^2 - 37423*a^21*c^10*d^12*e^16*x^2 - 27472*a^22*c^9*d^10*e
^18*x^2 - 10688*a^23*c^8*d^8*e^20*x^2 - 10288*a^24*c^7*d^6*e^22*x^2 - 3584*
a^25*c^6*d^4*e^24*x^2 + 2048*a^26*c^5*d^2*e^26*x^2 + 465092*a*c^5*d^18*e^10
*(-a^7*c^3)^(5/2) - 27472*a^5*c*d^10*e^18*(-a^7*c^3)^(5/2) + 3584*a^15*c*d^
4*e^24*(-a^7*c^3)^(3/2) + 256991*a^2*c^4*d^16*e^12*(-a^7*c^3)^(5/2) + 52822
*a^3*c^3*d^14*e^14*(-a^7*c^3)^(5/2) - 37423*a^4*c^2*d^12*e^16*(-a^7*c^3)^(5
/2) - 54944*a^4*c^12*d^26*e^2*(-a^7*c^3)^(3/2) - 200881*a^5*c^11*d^24*e^4*(
-a^7*c^3)^(3/2) - 413414*a^6*c^10*d^22*e^6*(-a^7*c^3)^(3/2) + 10288*a^14*c^
2*d^6*e^22*(-a^7*c^3)^(3/2))*(4*a^3*c^3*d^3 + 5*a*e^3*(-a^7*c^3)^(1/2) + 6*
a^4*c^2*d*e^2 + 3*c*d^2*e*(-a^7*c^3)^(1/2)))/(8*(a^8*e^4 + a^6*c^2*d^4 + 2*
a^7*c*d^2*e^2)) - (e^6*log(d + e*x^2))/(2*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^
5*e^2)) - (1/(4*a*d) - (e*x^2)/(2*a*d^2) + (x^4*(2*c^2*d^2 + a*c*e^2))/(4*a
^2*d*(a*e^2 + c*d^2)) - (c*e*x^6*(2*a*e^2 + 3*c*d^2))/(4*a^2*d^2*(a*e^2 + c
*d^2)))/(a*x^4 + c*x^8) + (log(6400*a^13*c^18*d^28*x^2 + 1024*a^27*c^4*e^28
*x^2 + 6400*a^3*c^13*d^28*(-a^7*c^3)^(3/2) - 1024*a^24*c^2*e^28*(-a^7*c^3)^
(1/2) + 10688*a^6*d^8*e^20*(-a^7*c^3)^(5/2) + 2048*a^16*d^2*e^26*(-a^7*c^3)
^(3/2) - 536959*c^6*d^20*e^8*(-a^7*c^3)^(5/2) + 54944*a^14*c^17*d^26*e^2*x^
2 + 200881*a^15*c^16*d^24*e^4*x^2 + 413414*a^16*c^15*d^22*e^6*x^2 + 536959*
a^17*c^14*d^20*e^8*x^2 + 465092*a^18*c^13*d^18*e^10*x^2 + 256991*a^19*c^12*
d^16*e^12*x^2 + 52822*a^20*c^11*d^14*e^14*x^2 - 37423*a^21*c^10*d^12*e^16*x
^2 - 27472*a^22*c^9*d^10*e^18*x^2 - 10688*a^23*c^8*d^8*e^20*x^2 - 10288*a^2
4*c^7*d^6*e^22*x^2 - 3584*a^25*c^6*d^4*e^24*x^2 + 2048*a^26*c^5*d^2*e^26*x^
2 - 465092*a*c^5*d^18*e^10*(-a^7*c^3)^(5/2) + 27472*a^5*c*d^10*e^18*(-a^7*c
^3)^(5/2) - 3584*a^15*c*d^4*e^24*(-a^7*c^3)^(3/2) - 256991*a^2*c^4*d^16*e^1
2*(-a^7*c^3)^(5/2) - 52822*a^3*c^3*d^14*e^14*(-a^7*c^3)^(5/2) + 37423*a^4*c
^2*d^12*e^16*(-a^7*c^3)^(5/2) + 54944*a^4*c^12*d^26*e^2*(-a^7*c^3)^(3/2) +
200881*a^5*c^11*d^24*e^4*(-a^7*c^3)^(3/2) + 413414*a^6*c^10*d^22*e^6*(-a^7*
c^3)^(3/2) - 10288*a^14*c^2*d^6*e^22*(-a^7*c^3)^(3/2))*(4*a^3*c^3*d^3 - 5*a
*e^3*(-a^7*c^3)^(1/2) + 6*a^4*c^2*d*e^2 - 3*c*d^2*e*(-a^7*c^3)^(1/2)))/(8*(
a^8*e^4 + a^6*c^2*d^4 + 2*a^7*c*d^2*e^2)) + (log(x)*(a*e^2 - 2*c*d^2))/(a^3
*d^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.252 $\int \frac{x^8}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=712

$$\frac{\sqrt[4]{a}\,d^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{a}\,d^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)^2}+\cdots$$

```
[Out] 1/4*d*x/c/(a*e^2+c*d^2)-1/4*x^3*(c*d*x^2+a*e)/c/(a*e^2+c*d^2)/(c*x^4+a)-1/1
6*a^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-3*e*a^(1/2)+d*c^(1/2))/c^(
7/4)/(a*e^2+c*d^2)*2^(1/2)-1/16*a^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))
*(-3*e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*a^(1/4)*d^2*arc
tan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d
^2)^2*2^(1/2)-1/4*a^(1/4)*d^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/
2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/8*a^(1/4)*d^2*ln(-a^(1/4)*c
^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+
c*d^2)^2*2^(1/2)-1/8*a^(1/4)*d^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c
^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/32*a^(1/4)*
ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*e*a^(1/2)+d*c^(1/2))/
c^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*a^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(3*e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)+d^(
7/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2+c*d^2)^2/e^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1314, 1276, 1280, 1168, 1162, 617, 204, 1165, 628, 1288, 205}

$$\frac{\sqrt[4]{a}\,d^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{a}\,d^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\left(ae^2+cd^2\right)^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (d*x)/(4*c*(c*d^2 + a*e^2)) - (x^3*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a
 + c*x^4)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)
^2) + (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) + (a^(1/4)*(Sqrt[c]*d - 3*Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)*(c*d^2
 + a*e^2)) - (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d
 - 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)
*(c*d^2 + a*e^2)) + (a^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2)
 + (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a^(1/4)*d^2*(Sqr
t[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4
)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*(a*e - c*d*x^2))/(4*a
*c*(p + 1)), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 1314

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> -Dist[(a*f^4)/(c*d^2 + a*e^2), Int[(f*x)^(m - 4)*(d - e*x^2)*
(a + c*x^4)^p, x], x] + Dist[(d^2*f^4)/(c*d^2 + a*e^2), Int[((f*x)^(m - 4)*
(a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && Lt
Q[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\int \frac{x^8}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = -\frac{a\int \frac{x^4\left(d-ex^2\right)}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2}+\frac{d^2\int \frac{x^4}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx}{cd^2+ae^2}$$

$$= -\frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{\int \frac{x^2\left(-3ae-cdx^2\right)}{a+cx^4}\,dx}{4c\left(cd^2+ae^2\right)}+\frac{d^2\int \left(\frac{d^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)}-\frac{a\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx\right.}\right.}{cd^2+ae^2}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)}-\frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{\left(ad^2\right)\int \frac{d-ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{d^4\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{\int \frac{-a}{}}{4c^2\,(}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)}-\frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2}-\frac{\left(ad^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \sqrt{}}{2c\left(cd^2+ae^2\right.}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)}-\frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2}-\frac{\left(ad^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{}}{\sqrt{}}}{4c\left(cd^2+\right.}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)}-\frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2}+\frac{\sqrt[4]{a}\,d^2\left(\sqrt{c}\,d+\sqrt{a}\,e\right)}{4\sqrt{2}}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)}-\frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2}+\frac{a^{3/4}d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\tan}{2\sqrt{2}\,c^{3/4}\left(cd^2\right.}$$

Mathematica [A] time = 0.30, size = 431, normalized size = 0.61

$$\frac{\frac{\sqrt{2}\sqrt[4]{a}\left(3a^{3/2}e^3+7\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+5c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{7/4}}-\frac{\sqrt{2}\sqrt[4]{a}\left(3a^{3/2}e^3+7\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+5c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\right.}{c^{7/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*a*(c*d^2 + a*e^2)*x*(d - e*x^2))/(c*(a + c*x^4)) + (32*d^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[
a]*c*d^2*e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
```

```
)/a^(1/4)])/c^(7/4) + (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*
e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/c^(7/4) + (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + a*Sqrt[c
]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/c^(7/4) - (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + a*Sqr
t[c]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/c^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

fricas [B] time = 37.53, size = 9856, normalized size = 13.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(a*c*d^2*e + a^2*e^3)*x^3 - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*
c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e
 + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*
d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2
*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d
^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 +
 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5
*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))
/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c
^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594
*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2
*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a
*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9
 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*
c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e
^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 +
56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*
c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e +
44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4
*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^
5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*
e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28
*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^
10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c
^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*
e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^
2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d
*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 +
 a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d
^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 +
 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^
3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d
^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 +
 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 -
750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*
x - (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3
*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6
*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625
*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^
6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16
 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4
*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*
e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^
5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^
4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*
```

```
e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81
*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c
^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*
e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*
a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) - (a*c^3*d^4 + 2*a^2*c
^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqr
t((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6
*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^
6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6
 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*
a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11
*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14
+ a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^
4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*
c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*
d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c
^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 1
9*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^
10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8
+ 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2
*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d
^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((
70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^
2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d
^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 +
2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c
^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^
8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a
^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d
^2*e^6 + a^4*c^3*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4
*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*
d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*
a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^
2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738
*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13
*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^
10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 +
4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log
(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e
^6 - 81*a^4*e^8)*x - (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4
 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d^8*e^3
 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5
*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4
+ 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7
*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*
d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12
 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3
*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3
*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 -
 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^
6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^
12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10
+ 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a
*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) - 8*(
c^2*d^3*x^4 + a*c*d^3)*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2
 + d)) - 4*(a*c*d^3 + a^2*d*e^2)*x)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*
e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4), -1/16*(4*(a*c*d^2*e +
 a^2*e^3)*x^3 - 16*(c^2*d^3*x^4 + a*c*d^3)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d
) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2
```

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 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5
+ (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*
c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^
4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a
^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^1
2*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^
12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^
2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a
*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (
125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6
- 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5
+ 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^
6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6
 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*
a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11
*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14
+ a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (
c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3
*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 +
 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*
e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d
^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12
+ 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c
^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^
2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70
*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2
+ 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^1
2 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 23
83*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^1
4*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*
e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8
*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d
^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9 - 170*a*c^5*d^7*e
^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d
^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4
*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^
2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738
*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13
*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^
10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*
c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6
*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 -
 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*
a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d
^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8
 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^
7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^
6 + a^4*c^3*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4
+ 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e
^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c
^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 5
29*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*
c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12
*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 +
28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c
^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(62
5*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 -
81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86
```

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*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54
*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11
)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 274
8*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12
)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*
e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*
a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3
+ 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*
d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*
a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d
^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^
4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*
a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*
d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) + (a*c^3*d
^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e
^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 +
 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqr
t(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4
*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^
15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 +
 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c
^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^
4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2
 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9
- 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*
d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7
*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 195
0*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*
c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*
e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 5
6*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^
16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*
a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-
(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^
3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*
d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70
*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*
d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 +
 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) - 4*(a*c*d^3 + a^2*d*e^2)*x)/(a*c^3*d^4
 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4
)*x^4)]
```

giac [A] time = 0.57, size = 581, normalized size = 0.82

$$\frac{d^{\frac{7}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{c^2d^4+2\,acd^2e^2+a^2e^4}-\frac{\left(5\,\left(ac^3\right)^{\frac{1}{4}}c^3d^3+\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-7\,\left(ac^3\right)^{\frac{3}{4}}cd^2e-3\,\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,c^6d^4+2\,\sqrt{2}\,ac^5d^2e^2+\sqrt{2}\,a^2c^4e^4\right)}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) - 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - 7*(a*c^3)^
(3/4)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + sqrt(2
)*a^2*c^4*e^4) - 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 -
 7*(a*c^3)^(3/4)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
```

```
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e
^2 + sqrt(2)*a^2*c^4*e^4) - 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a
*c^2*d*e^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*log(x^2 + sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2
+ sqrt(2)*a^2*c^4*e^4) + 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^
2*d*e^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2
)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + s
qrt(2)*a^2*c^4*e^4) - 1/4*(a*x^3*e - a*d*x)/((c*x^4 + a)*(c^2*d^2 + a*c*e^2
))
```

maple [A] time = 0.02, size = 873, normalized size = 1.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3/c*x^3-1/4*a/(a*e^2+c*d^2)^2/(c*x^4+a
)*e*x^3*d^2+1/4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*d/c*x*e^2+1/4*a/(a*e^2+c*d^2)
^2/(c*x^4+a)*d^3*x-1/16*a/(a*e^2+c*d^2)^2/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x-1)*d*e^2-5/16/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x-1)*d^3-1/32*a/(a*e^2+c*d^2)^2/c*(a/c)^(1/4)*2^(1/2)*l
n((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^
(1/2)))*d*e^2-5/32/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*
2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3-1/16*a/
(a*e^2+c*d^2)^2/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2
-5/16/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d
^3+3/32*a^2/(a*e^2+c*d^2)^2/c^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(
1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^3+7/32*a/(a*
e^2+c*d^2)^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2
))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e+3/16*a^2/(a*e^2+c*d^2)^2/
c^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^3+7/16*a/(a*e^2+c
*d^2)^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e+3/16*a^
2/(a*e^2+c*d^2)^2/c^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e
^3+7/16*a/(a*e^2+c*d^2)^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^2*e+d^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.06, size = 504, normalized size = 0.71

$$\frac{d^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{de}} - \frac{a\left(\frac{2\,\sqrt{2}\left(5\,c^{\frac{3}{2}}d^3-7\,\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2-3\,a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}} + \frac{2\,\sqrt{2}\left(5\,c^{\frac{3}{2}}d^3-7\,\sqrt{a}\,cd^2e+\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] d^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)) -
 1/32*a*(2*sqrt(2)*(5*c^(3/2)*d^3 - 7*sqrt(a)*c*d^2*e + a*sqrt(c)*d*e^2 - 3
*a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sq
rt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(5
*c^(3/2)*d^3 - 7*sqrt(a)*c*d^2*e + a*sqrt(c)*d*e^2 - 3*a^(3/2)*e^3)*arctan(
1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(5*c^(3/2)*d^3 + 7*sqrt(a
)*c*d^2*e + a*sqrt(c)*d*e^2 + 3*a^(3/2)*e^3)*log(sqrt(c)*x^2 + sqrt(2)*a^(1
```

```
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(5*c^(3/2)*d^3 + 7*sqr
t(a)*c*d^2*e + a*sqrt(c)*d*e^2 + 3*a^(3/2)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a
^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 +
 a^2*c*e^4) - 1/4*((a*c*d^2*e + a^2*e^3)*x^3 - (a*c*d^3 + a^2*d*e^2)*x)/(a*
c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*
c^2*e^4)*x^4)
```

mupad [B] time = 2.86, size = 18343, normalized size = 25.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((a*d*x)/(4*c*(a*e^2 + c*d^2)) - (a*e*x^3)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^
4) + atan(((((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11
*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9
 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6
*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a
^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 1843
20*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(25
6*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*
d^2*e^6)) - (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a
^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c
^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 +
 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a
^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a
^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589
824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^
8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6
*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d
^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^
4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c
^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920*a^8*c^4*d*e^14 + 13184*a
^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a
^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^
7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e
^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*
e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2)
 - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10
*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((25*c^3*d^6*(-a
*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e
^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-
a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d
^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e
+ 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^
5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^
6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^
(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 7
0*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)
^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4
 + 4*a^3*c^8*d^2*e^6)))^(1/2)*1i - (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e
^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^
7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e
^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a
^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*
a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 286
72*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*
c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*
e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e
```

- 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(
c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d
^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*
c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680
*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(1
28*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4
*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*
c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)
^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*
a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(1920*
a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a
^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7
*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d
^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c
^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^
2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8
+ a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))
^(1/2))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*
d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/
2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^
10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(81*a^8*e^
13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a
^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^
8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))
)*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5
+ 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 4
1*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6
*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*1i)/((81*a^6*d^4*e^8
+ 450*a^5*c*d^6*e^6 + 300*a^3*c^3*d^10*e^2 + 733*a^4*c^2*d^8*e^4)/(128*(c^7
*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^
6)) + (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3
- 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 29
28*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*
c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^
11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^
6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^
7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e
^6)) - (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^
4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(
1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*
c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^
7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^
12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a
^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4
*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*
c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^
3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a
c^7)^(1/2))/(256(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^
4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^
10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^
7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8
+ a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))
*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 +
44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41
*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*
e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((25*c^3*d^6*(-a*c^7)
^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 +
70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7
)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^

4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612
*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3
*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6
*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2)
 - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c
^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2
))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*
a^3*c^8*d^2*e^6)))^(1/2) + (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17
232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320
*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a
*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d
^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12
*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c
^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*
e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*
c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c
^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8
 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6))
)^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^1
2*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^1
0*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*
d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6
)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^
5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) -
 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d
^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(1920*a^8*c^4*
d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d
^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3
e^12))/(128(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 +
 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2
) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^
2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^
7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*
((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 +
44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*
a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e
^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(81*a^8*e^13 + 800
*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d
^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*
c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c
^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2
*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*
d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6
*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)))*((25*c^3*d^6*(-a*c^7)^(1/2)
 - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c
^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2
))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*
a^3*c^8*d^2*e^6)))^(1/2)*2i + atan(((((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*
e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e
^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*
e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*
a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480
*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28
672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2
*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3
d^6(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e
 + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*
(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*
d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3

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*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 32768
0*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(
128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^
4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3
*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7
)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4
*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920
*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*
a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^
7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*
d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*
c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c
^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8
 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6))
)^(1/2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4
*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1
/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c
^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e
^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*
a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d
^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)
))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5
 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) +
41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^
6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*1i - (((5120*a^2*c^8
*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e
^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/
(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c
^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a
^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 1228
80*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 +
 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((9*a^3*e^6
*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*
d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e
^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*
c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^
14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*
a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327
680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^
2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^
6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e +
39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^
11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2
*e^6)))^(1/2) - (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a
^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6
*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*
c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)
^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 +
 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^
7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e
^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-
a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a
*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d
^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6
)))^(1/2) - (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832
*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c
^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*
e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)
^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d
```

^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*1i)/((81*a^6*d^4*e^8 + 450*a^5*c*d^6*e^6 + 300*a^3*c^3*d^10*e^2 + 733*a^4*c^2*d^8*e^4)/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^

(1/2) - (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d
^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5
*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*
e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) -
25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^
6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2)
)/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a
^3*c^8*d^2*e^6)))^(1/2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(
1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4
e^2(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4
*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2
) - (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5
*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e
^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*
a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) +
6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(
-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e
^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)))*((
9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44
*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^
2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2
+ 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*2i + (atan(((((x*(81*a^8*
e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913
*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(256*(c^7*
d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6
)) + (((20*a^2*c^8*d^13*e + (27*a^8*c^2*d*e^13)/16 - (1077*a^3*c^7*d^11*e^3
)/16 - (2361*a^4*c^6*d^9*e^5)/16 - (425*a^5*c^5*d^7*e^7)/8 + (135*a^6*c^4*d
^5*e^9)/8 + (183*a^7*c^3*d^3*e^11)/16)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*
d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - ((-d^7*e)^(1/2)*(((-d^7
e)^(1/2)((320*a^5*c^9*d^8*e^8 - 288*a^3*c^11*d^12*e^4 - 240*a^4*c^10*d^10
*e^6 - 80*a^2*c^12*d^14*e^2 + 720*a^6*c^8*d^6*e^10 + 480*a^7*c^7*d^4*e^12 +
112*a^8*c^6*d^2*e^14)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*
c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*(-d^7*e)^(1/2)*(65536*a^9*c^7*e^17 -
65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*
e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d
^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(512*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e
^3)*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^
4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)) + (x*(1920*a^8*c^4*
d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d
^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3
e^12))/(256(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 +
4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^
(1/2))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2)*1i)/(a^2*e
^5 + c^2*d^4*e + 2*a*c*d^2*e^3) + (((x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e +
612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*
c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*
d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((20*a^2*c^8*d^13*e +
(27*a^8*c^2*d*e^13)/16 - (1077*a^3*c^7*d^11*e^3)/16 - (2361*a^4*c^6*d^9*e^5
)/16 - (425*a^5*c^5*d^7*e^7)/8 + (135*a^6*c^4*d^5*e^9)/8 + (183*a^7*c^3*d^3
e^11)/16)/(2(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4
+ 4*a^3*c^4*d^2*e^6)) - ((-d^7*e)^(1/2)*(((-d^7*e)^(1/2)*((320*a^5*c^9*d^8*
e^8 - 288*a^3*c^11*d^12*e^4 - 240*a^4*c^10*d^10*e^6 - 80*a^2*c^12*d^14*e^2
+ 720*a^6*c^8*d^6*e^10 + 480*a^7*c^7*d^4*e^12 + 112*a^8*c^6*d^2*e^14)/(2*(c
^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*
e^6)) + (x*(-d^7*e)^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 3
27680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^
9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2
e^15))/(512(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)*(c^7*d^8 + a^4*c^3*e^8 +
4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 +

```
c^2*d^4*e + 2*a*c*d^2*e^3)) - (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13
*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e
^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(256*(c^7*d^8 + a^4*
c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a
^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(2*(a^2*e^5 + c^2*d^4
*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2)*1i)/(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e
^3))/(((81*a^6*d^4*e^8)/128 + (225*a^5*c*d^6*e^6)/64 + (75*a^3*c^3*d^10*e^2
)/32 + (733*a^4*c^2*d^8*e^4)/128)/(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2
+ 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6) + (((x*(81*a^8*e^13 + 800*a^2*c^6*
d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 +
1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(256*(c^7*d^8 + a^4*c^3*e^8 +
 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (((20*a^2*c^8*
d^13*e + (27*a^8*c^2*d*e^13)/16 - (1077*a^3*c^7*d^11*e^3)/16 - (2361*a^4*c^
6*d^9*e^5)/16 - (425*a^5*c^5*d^7*e^7)/8 + (135*a^6*c^4*d^5*e^9)/8 + (183*a^
7*c^3*d^3*e^11)/16)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5
*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - ((-d^7*e)^(1/2)*(((-d^7*e)^(1/2)*((320*a^5
*c^9*d^8*e^8 - 288*a^3*c^11*d^12*e^4 - 240*a^4*c^10*d^10*e^6 - 80*a^2*c^12*
d^14*e^2 + 720*a^6*c^8*d^6*e^10 + 480*a^7*c^7*d^4*e^12 + 112*a^8*c^6*d^2*e^
14)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3
*c^4*d^2*e^6)) - (x*(-d^7*e)^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^1
4*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^
11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^
8*c^8*d^2*e^15))/(512*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)*(c^7*d^8 + a^4*
c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a
^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)) + (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*
c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*
c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(256*(c^7*d
^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)
)))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(2*(a^2*e^5
+ c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(a^2*e^5 + c^2*d^4*e + 2*a*c
*d^2*e^3) - (((x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 8
32*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6
*c^2*d^4*e^9))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^
4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((20*a^2*c^8*d^13*e + (27*a^8*c^2*d*e^13)/16
 - (1077*a^3*c^7*d^11*e^3)/16 - (2361*a^4*c^6*d^9*e^5)/16 - (425*a^5*c^5*d^
7*e^7)/8 + (135*a^6*c^4*d^5*e^9)/8 + (183*a^7*c^3*d^3*e^11)/16)/(2*(c^7*d^8
 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))
- ((-d^7*e)^(1/2)*(((-d^7*e)^(1/2)*((320*a^5*c^9*d^8*e^8 - 288*a^3*c^11*d^1
2*e^4 - 240*a^4*c^10*d^10*e^6 - 80*a^2*c^12*d^14*e^2 + 720*a^6*c^8*d^6*e^10
 + 480*a^7*c^7*d^4*e^12 + 112*a^8*c^6*d^2*e^14)/(2*(c^7*d^8 + a^4*c^3*e^8 +
 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*(-d^7*e)^(1
/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^
5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^
6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(512*(a^2*e^5
+ c^2*d^4*e + 2*a*c*d^2*e^3)*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a
^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*
e^3)) - (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d
^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5
*e^10 + 20736*a^7*c^5*d^3*e^12))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*
e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2
*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*
(-d^7*e)^(1/2))/(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2)*1i)/
(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.253 $\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=687

$$-\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\sqrt[4]{a}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\sqrt[4]{a}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right.}{8\sqrt{2}\sqrt[4]{a}}$$

```
[Out] -1/4*x*(c*d*x^2+a*e)/c/(a*e^2+c*d^2)/(c*x^4+a)-1/16*arctan(-1+c^(1/4)*x*2^(
1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)-
1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(
5/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*d^2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2)-1
/8*d^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1
/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/4*d^2*arctan(-1+c^(1/4)*x*2^
(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2
)+1/4*d^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(1/4)
/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)+1
/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))
/a^(1/4)/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)-d^(5/2)*arctan(x*e^(1/2)/d^(1/2))*e^
(1/2)/(a*e^2+c*d^2)^2
```

Rubi [A] time = 0.60, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1314, 1276, 1168, 1162, 617, 204, 1165, 628, 1288, 205}

$$-\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\sqrt[4]{a}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\sqrt[4]{a}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right.}{8\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(x*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) - (d^(5/2)*Sqrt[e]*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (d^2*(Sqrt[c]*d + Sqrt[a]*e
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^
2 + a*e^2)^2) + ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d + Sqrt
[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*
(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d -
 Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt
[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*
(c*d^2 + a*e^2)) - (d^2*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2)
+ ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*(a*e - c*d*x^2))/(4*a
*c*(p + 1)), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 1314

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> -Dist[(a*f^4)/(c*d^2 + a*e^2), Int[(f*x)^(m - 4)*(d - e*x^2)*
(a + c*x^4)^p, x], x] + Dist[(d^2*f^4)/(c*d^2 + a*e^2), Int[((f*x)^(m - 4)*
(a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && Lt
```

```
Q[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{a\int \frac{x^2(d-ex^2)}{(a+cx^4)^2}\,dx}{cd^2+ae^2} + \frac{d^2\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx}{cd^2+ae^2}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\int \frac{-ae+cdx^2}{a+cx^4}\,dx}{4c(cd^2+ae^2)} + \frac{d^2\int \left(-\frac{de}{(cd^2+ae^2)(d+ex^2)} + \frac{ae+cdx^2}{(cd^2+ae^2)(a+cx^4)}\right.}{cd^2+ae^2}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^2\int \frac{ae+cdx^2}{a+cx^4}\,dx}{(cd^2+ae^2)^2} - \frac{(d^3e)\int \frac{1}{d+ex^2}\,dx}{(cd^2+ae^2)^2} - \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}+}{a+cx}}{8c(cd^2+ae^2}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\left(d^2\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2(cd^2+ae^2)^2} +$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{(\sqrt{c}\,d+\sqrt{a}\,e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{16\sqrt{2}\sqrt[4]{a}\,c^{5/4}(cd^2+a}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}\,c^{3/4}(cd^2+ae^2)} -$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}\,d^2\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{2\sqrt{2}\sqrt[4]{a}\,(cd^2+ae^2)^2}$$

Mathematica [A] time = 0.37, size = 428, normalized size = 0.62

$$-\frac{\frac{\sqrt{2}\left(a^{3/2}e^3+5\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2-3c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{a}\,c^{5/4}} - \frac{\sqrt{2}\left(a^{3/2}e^3+5\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2-3c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{a}\,c^{5/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/32*((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(c*(a + c*x^4)) + 32*d^(5/2)*S
qrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a]*
c*d^2*e - a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(a^(1/4)*c^(5/4)) - (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e -
a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^
(1/4)*c^(5/4)) + (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d
*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(a^(1/4)*c^(5/4)) - (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + a*Sqrt[
c]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(a^(1/4)*c^(5/4)))/(c*d^2 + a*e^2)^2
```

fricas [B] time = 22.08, size = 9822, normalized size = 14.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*
c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e
- 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*
e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*
e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*
a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11
*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10
 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*
a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-
(81*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 -
a^4*e^8)*x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 1
4*a^4*c^2*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3
*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-
(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e
^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a
^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9
*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 +
 a^9*c^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d
^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)
*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^
3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^1
6 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*
a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2
*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*
a^3*c^3*d^2*e^6 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e
^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4
*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4
 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2
 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5
*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^
12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 +
28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c
^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81
*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4
*e^8)*x - (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a
^4*c^2*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^
7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81
*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6
+ 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*
c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^
8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^
9*c^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8
+ 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sq
rt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d
^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 +
 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5
*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^
14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3
*c^3*d^2*e^6 + a^4*c^2*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4
+ (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*
c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 +
4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 +
```

```
799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*
d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*
e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*
a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*
d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^
4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^
8)*x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*
c^2*d^2*e^7 - a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d
^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^
6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 1
43*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^1
2*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e
^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c
^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4
*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(
-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*
e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*
a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^
9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14
 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^
3*d^2*e^6 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (
c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d
^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a
^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799
*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2
*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4
 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7
*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6
*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d
^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*
x - (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2
*d^2*e^7 - a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*
e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d
^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*
a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d
^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8
+ 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*
e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*
c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(8
1*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6
 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2
*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d
^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a
^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d
^2*e^6 + a^4*c^2*e^8))) - 8*(c^2*d^2*x^4 + a*c*d^2)*sqrt(-d*e)*log((e*x^2 -
 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 4*(a*c*d^2*e + a^2*e^3)*x)/(a*c^3*d^4 +
 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*
x^4), -1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + 16*(c^2*d^2*x^4 + a*c*d^2)*sqrt(
d*e)*arctan(sqrt(d*e)*x/d) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (
c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d
^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a
^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799
*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2
*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4
 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7
*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6
*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d
^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*
x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2
```

*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x - (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8

```
 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5
*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6
 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 +
 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 -
2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^
2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*
d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a
^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4
*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*
e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*
a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8 - 270*
a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x - (45*a
*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7
- a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a
^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 - 558
*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d
^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 +
 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*
c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*s
qrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e
^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^1
2 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^
4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^1
4*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 +
56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^
16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 +
a^4*c^2*e^8))) + 4*(a*c*d^2*e + a^2*e^3)*x)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2
+ a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)]
```

giac [A] time = 0.59, size = 595, normalized size = 0.87

$$-\frac{d^{\frac{5}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{c^2d^4+2\,acd^2e^2+a^2e^4}+\frac{\left(5\,\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+3\,\left(ac^3\right)^{\frac{3}{4}}cd^3+\left(ac^3\right)^{\frac{1}{4}}a^2ce^3-\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,ac^5d^4+2\,\sqrt{2}\,a^2c^4d^2e^2+\sqrt{2}\,a^3c^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + 3*(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1
/4)*a^2*c*e^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sq
rt(2)*a^3*c^3*e^4) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + 3*(a*c^3)^(3/4)*c*d
^3 + (a*c^3)^(1/4)*a^2*c*e^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c
^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*(5*(a*c^3)^(1/4)*a*c^2*d^2*e - 3*(
a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*a*d*e^2)*log(x
^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*
c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/16*(5*(a*c^3)^(1/4)*a*c^2*d^2*e - 3*
(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*a*d*e^2)*log(
x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2
*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/4*(c*d*x^3 + a*x*e)/((c*x^4 + a)*(c
^2*d^2 + a*c*e^2))
```

maple [A] time = 0.02, size = 852, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*a*d*e^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^
3*c*d^3-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c*x-1/4/(a*e^2+c*d^2)^2/(c*x^
4+a)*e*a*d^2*x+1/16/(a*e^2+c*d^2)^2/c*(a/c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x-1)*e^3+5/16/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/c)^(1/4)*x-1)*d^2*e+1/32/(a*e^2+c*d^2)^2/c*(a/c)^(1/4)*a*2^(1/2)*ln((x
^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2
)))*e^3+5/32/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2
)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e+1/16/(a*e^2
+c*d^2)^2/c*(a/c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^3+5/16/
(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e-1
/32/(a*e^2+c*d^2)^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/
c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*a*d*e^2+3/32/(a*e^2+c*d^
2)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a
/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3-1/16/(a*e^2+c*d^2)^2/c/(a/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*a*d*e^2+3/16/(a*e^2+c*d^2)^2/(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3-1/16/(a*e^2+c*d^2)^2/c/(a/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*a*d*e^2+3/16/(a*e^2+c*d^2)^
2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3-d^3*e/(a*e^2+c*d^
2)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.08, size = 476, normalized size = 0.69

$$-\frac{d^3 e \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\sqrt{de}} - \frac{cdx^3 + aex}{4\left(ac^2d^2 + a^2ce^2 + \left(c^3d^2 + ac^2e^2\right)x^4\right)} + \frac{\frac{2\,\sqrt{2}\left(3\,\sqrt{a}\,c^2d^3 + 5\,ac^{\frac{3}{2}}d^2e - a^{\frac{3}{2}}cde^2 + a^2\,\sqrt{c}\,e^3\right)\arctan\ldots}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -d^3*e*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)
) - 1/4*(c*d*x^3 + a*e*x)/(a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x^
4) + 1/32*(2*sqrt(2)*(3*sqrt(a)*c^2*d^3 + 5*a*c^(3/2)*d^2*e - a^(3/2)*c*d*e
^2 + a^2*sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(
1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sq
rt(2)*(3*sqrt(a)*c^2*d^3 + 5*a*c^(3/2)*d^2*e - a^(3/2)*c*d*e^2 + a^2*sqrt(c
)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*(3*sqrt(a)*
c^2*d^3 - 5*a*c^(3/2)*d^2*e - a^(3/2)*c*d*e^2 - a^2*sqrt(c)*e^3)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(3
*sqrt(a)*c^2*d^3 - 5*a*c^(3/2)*d^2*e - a^(3/2)*c*d*e^2 - a^2*sqrt(c)*e^3)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c
^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)
```

mupad [B] time = 2.82, size = 17909, normalized size = 26.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] atan(((((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^
6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256
*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2
```

*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*
d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6
*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6
*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(-(a^3*e^6*(-a*c^5)^(1/
2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*
c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2
))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 +
4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 -
327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^
9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*
e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*
a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2
*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*
c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8
+ 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(
1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*
a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c
^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e
^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1
/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e
^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c
^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)
)*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 -
4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^
2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e
^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(a^6*e^13 - 288*a*
c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7
 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^
2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*
(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*
a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*
d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e
^6)))^(1/2)*1i - (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3
*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2
e^12)/(256(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*
a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 2211
84*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184
320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 +
4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-(a^3*e^6*(
-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*
e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-
a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7
*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*
d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5
*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a
^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d
^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5
)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d
^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a
^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(
1/2) - (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e
^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 +
 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^
2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*
(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*
a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*
d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e
^6)))^(1/2))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c
^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(

1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*1i)/((((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5

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*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/
(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a
^4*c^6*d^2*e^6)))^(1/2) - (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 2
1248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296
*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a
*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)
^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 3
0*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^
(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^
4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c
^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2
*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 +
 a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))
^(1/2) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4
*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*
c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e
^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*
d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^
4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3
*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (a^4*d^3*e^9 + 108*a*c^3*d^9*e^
3 + 18*a^3*c*d^5*e^7 + 93*a^2*c^2*d^7*e^5)/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*
c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6))))*(-(a^3*e^6*(-a*c^5)
^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 3
0*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^
(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^
4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*2i - ((d*x^3)/(4*(a*e^2 + c*d^2)) + (a*e*x)/
(4*c*(a*e^2 + c*d^2)))/(a + c*x^4) + atan(((((432*a*c^7*d^12*e^2 + 13040*a^
2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3
*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^
2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 -
4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 40
9600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(2
56*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d
^2*e^6)) - (x*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c
^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(
1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^
2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c
^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c
^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^
7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*
c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(
1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*
a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1
/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4
+ 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^1
4 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 -
 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8
+ 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*
c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3
 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c
^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^
4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-
a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*
c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^
8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6
)))^(1/2) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*
c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a
^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3
*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^
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4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*1i - (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*1i)/((((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((

a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (a^4*d^3*e^9 + 108*a*c^3*d^9*e^3 + 18*a^3*c*d^5*e^7 + 93*a^2*c^2*d^7*e^5)/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6))))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*2i - (atan(-(((-d^5*e)^(1/2)*(((-d^5*e)^(1/2)*(((27*a*c^7*d^12*e^2)/16 + (815*a^2*c^6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (33*a^4*c^4*d^6*e^8)/8 - (25*a^5*c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c

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^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d
^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(256*(c^5*d^8 + a^
4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((17
6*a^2*c^10*d^13*e^3 - 16*a^8*c^4*d*e^15 + 864*a^3*c^9*d^11*e^5 + 1680*a^4*c
^8*d^9*e^7 + 1600*a^5*c^7*d^7*e^9 + 720*a^6*c^6*d^5*e^11 + 96*a^7*c^5*d^3*e
^13)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*
c^2*d^2*e^6)) - (x*(-d^5*e)^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14
*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9
*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c
^6*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(c^5*d^8 + a^4*c*e^8
 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-d^5*e)^(1/2
))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c
^2*d^4 + 2*a*c*d^2*e^2))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*(a^
6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*
a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4
*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*1i)/(a^2*e^4 + c^2*d^4
+ 2*a*c*d^2*e^2) - ((-d^5*e)^(1/2)*(((-d^5*e)^(1/2)*(((27*a*c^7*d^12*e^2)/1
6 + (815*a^2*c^6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (33*a^4*c^4*d^6*e
^8)/8 - (25*a^5*c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16)/(2*(c^5*d^8 + a
^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (((x
*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 2547
2*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6
*c^4*d^3*e^12))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4
*e^4 + 4*a^3*c^2*d^2*e^6)) - (((176*a^2*c^10*d^13*e^3 - 16*a^8*c^4*d*e^15 +
 864*a^3*c^9*d^11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7*d^7*e^9 + 720*a
^6*c^6*d^5*e^11 + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^
6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-d^5*e)^(1/2)*(65536*
a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*
a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 5898
24*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2
*a*c*d^2*e^2)*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 +
4*a^3*c^2*d^2*e^6)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)
))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))))/(2*(a^2*e^4 +
c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^
2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(
256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*
d^2*e^6)))*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(((a^4*d^3*e^9)/128 + (
27*a*c^3*d^9*e^3)/32 + (9*a^3*c*d^5*e^7)/64 + (93*a^2*c^2*d^7*e^5)/128)/(c^
5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6
) + ((-d^5*e)^(1/2)*(((-d^5*e)^(1/2)*(((27*a*c^7*d^12*e^2)/16 + (815*a^2*c^
6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (33*a^4*c^4*d^6*e^8)/8 - (25*a^5
*c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*
c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((x*(1152*a*c^9*d^
13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e
^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))
/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^
2*d^2*e^6)) + (((176*a^2*c^10*d^13*e^3 - 16*a^8*c^4*d*e^15 + 864*a^3*c^9*d^
11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7*d^7*e^9 + 720*a^6*c^6*d^5*e^11
 + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c
^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(-d^5*e)^(1/2)*(65536*a^9*c^5*e^17 -
65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e
^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*
e^13 + 327680*a^8*c^6*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(
c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e
^6)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^5*e)^(1/
2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c
*d^2*e^2)) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2
*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(256*(c^5*d^8 +
a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6))))/(a^
```

```
2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) + ((-d^5*e)^(1/2)*(((-d^5*e)^(1/2)*(((27*a
*c^7*d^12*e^2)/16 + (815*a^2*c^6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (
33*a^4*c^4*d^6*e^8)/8 - (25*a^5*c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16)
/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*
d^2*e^6)) - (((x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8
*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5
*e^10 + 4864*a^6*c^4*d^3*e^12))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2
 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (((176*a^2*c^10*d^13*e^3 - 16*
a^8*c^4*d*e^15 + 864*a^3*c^9*d^11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7
*d^7*e^9 + 720*a^6*c^6*d^5*e^11 + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c*
e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-d^5*
e)^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^
12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8
*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(512*(a^2*e
^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^
2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4
+ 2*a*c*d^2*e^2)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))
)/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^6*e^13 - 288*a*c^5*d^10*e
^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4
*c^2*d^4*e^9))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*
e^4 + 4*a^3*c^2*d^2*e^6))))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^5*e)^
(1/2)*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.254 $\int \frac{x^4}{(d+ex^2)(a+cx^4)^2} \, dx$

Optimal. Leaf size=685

$$\frac{\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}+\frac{\left(3\sqrt{c}\,d-\right.}{8\sqrt{}}$$

```
[Out] -1/4*x*(-e*x^2+d)/(a*e^2+c*d^2)/(c*x^4+a)+d^(3/2)*e^(3/2)*arctan(x*e^(1/2)/
d^(1/2))/(a*e^2+c*d^2)^2+1/4*c^(1/4)*d^2*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/4*c^(1/4)*d^2*a
rctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*
d^2)^2*2^(1/2)-1/8*c^(1/4)*d^2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^
(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/8*c^(1/4)*d^
2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a
^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-
e*a^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/16*arctan(1+
c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/4)/(a*e^2+
c*d^2)*2^(1/2)+1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a
^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*ln(a^(1/4)*c
^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/
4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1314, 1179, 1168, 1162, 617, 204, 1165, 628, 1171, 205}

$$\frac{\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}+\frac{\left(3\sqrt{c}\,d-\right.}{8\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(x*(d - e*x^2))/(4*(c*d^2 + a*e^2)*(a + c*x^4)) + (d^(3/2)*e^(3/2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]
*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*
e^2)^2) + ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d -
Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^2) - ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d^2*(Sqrt[
c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*
(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
- ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1314

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> -Dist[(a*f^4)/(c*d^2 + a*e^2), Int[(f*x)^(m - 4)*(d - e*x^2)*
(a + c*x^4)^p, x], x] + Dist[(d^2*f^4)/(c*d^2 + a*e^2), Int[((f*x)^(m - 4)*
(a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && Lt
```

```
Q[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\int \frac{x^4}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = -\frac{a\int \frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2} + \frac{d^2\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx}{cd^2+ae^2}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{\int \frac{-3d+ex^2}{a+cx^4}\,dx}{4\left(cd^2+ae^2\right)} + \frac{d^2\int \left(\frac{e^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right.}{cd^2+ae^2}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{\left(cd^2\right)\int \frac{d-ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{\left(d^2e^2\right)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2} - \frac{\left(\frac{3\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{1}{\ }}{8c\left(cd^2\right.}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\left(d^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\left(3\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\right.}{16\sqrt{2}\,a^{3/4}c^{3/4}\left(cd^2\right.}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{\ }}{\sqrt[4]{\ }}\right.}{8\sqrt{2}\,a^{3/4}c^{3/4}\left(cd^2+ae^2\right)}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\,d^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\right.}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.27, size = 423, normalized size = 0.62

$$\frac{\frac{\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}\,cd^2e+3a\sqrt{c}\,de^2-c^{3/2}d^3\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{3/4}c^{3/4}} + \frac{\sqrt{2}\left(-a^{3/2}e^3+3\sqrt{a}\,cd^2e-3a\sqrt{c}\,de^2+c^{3/2}d^3\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x\right.}{a^{3/4}c^{3/4}}}{\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x^2)*(a + c*x^4)^2),x]

[Out] ((8*(c*d^2 + a*e^2)*(-(d*x) + e*x^3))/(a + c*x^4) + 32*d^(3/2)*e^(3/2)*ArcT
an[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a
*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(
3/4)*c^(3/4)) + (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d
*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(3/4)*c^(3/
4)) + (Sqrt[2]*(-(c^(3/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^
(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(3/4)
*c^(3/4)) + (Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 -
 a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(3
/4)*c^(3/4)))/(32*(c*d^2 + a*e^2)^2)
```

fricas [B] time = 18.40, size = 9678, normalized size = 14.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(c*d^2*e + a*e^3)*x^3 - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (
c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*
e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*
c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10
 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 5
6*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*
d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^
6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 -
 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4
*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3
*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 -
 a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*
a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*
e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4
 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*
c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 2
0*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^
4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2
+ 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*
c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^
12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 2
8*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a
*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-
(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 +
 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4
*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8
*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^
11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c
^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^
6 - a^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*
a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 +
14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sq
rt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e
^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8
*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7
*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14
+ a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^
5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e
^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3
*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^
16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a
^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2
*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^
4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c
*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255
*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2
```

```
*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^
4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9
*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c
^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*
d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^
2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (
3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*
e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 +
 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c
*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^1
2*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28
*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*
e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10
*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30
*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c
^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^1
0 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8
 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))
 + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^
2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8
 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*s
qrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*
e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 +
8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^
7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14
 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d
^2*e^6 - a^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4
- 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e
^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^1
1)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*
d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^1
6 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^
7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*
e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 -
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^
3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^
11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 +
 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^
4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^
4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + 8*(c*d*e*x^4 + a*d*e)*sqrt(-d*e)
*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 4*(c*d^3 + a*d*e^2)*x)/(a*
c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4)*x^4), 1/16*(4*(c*d^2*e + a*e^3)*x^3 + 16*(c*d*e*x^4 + a*d*e)*sqrt(d*e)*a
rctan(sqrt(d*e)*x/d) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 +
2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^
2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^
2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^
4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^1
2)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*
d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 +
 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*
a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*
d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 +
 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*
e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d
^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^
```

```
8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6
*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*
c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^
12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*
e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*
c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10
 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 5
6*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*
d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^
6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a*c^2*d^4 +
 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*s
qrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^
6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12
- 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c
^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14
*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*
a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^1
6)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6
 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^
8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^
3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5
*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d
^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a
^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*
d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 +
 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3
*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-
(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 +
 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4
*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8
*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^
11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c
^2*d^2*e^6 + a^5*c*e^8))) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d
^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 +
 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c
^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d
^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^
6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6
*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e
^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2
 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a
*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4*d^7*
e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (3*a^3*c^7*
d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*
c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c
^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10
+ a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56
*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d
^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c
*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255
*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2
*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^
4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9
*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c
^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x
```

```
^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c
^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*
d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*
a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10
*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8
+ 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^
3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^
2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a
^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c
^2*d^3*e^6 + 3*a^5*c*d*e^8 - (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^
5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(
c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 +
255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*
c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*
e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^1
1*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8
 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*s
qrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*
e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 +
8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^
7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14
 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8))) - 4*(c*d^3 + a*d*e^2)*x)/(a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)]
```

giac [A] time = 0.47, size = 586, normalized size = 0.86

$$\frac{d^{\frac{3}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{3}{2}}}{c^2d^4+2\,acd^2e^2+a^2e^4}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3-3\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-3\left(ac^3\right)^{\frac{3}{4}}cd^2e+\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,ac^5d^4+2\,\sqrt{2}\,a^2c^4d^2e^2+\sqrt{2}\,a^3c^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^
4) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(
3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sqrt(
2)*a^3*c^3*e^4) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2
- 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^
2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4
)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 + sq
rt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2
*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)
*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*
e^2 + sqrt(2)*a^3*c^3*e^4) + 1/4*(x^3*e - d*x)/((c*x^4 + a)*(c*d^2 + a*e^2)
)
```

maple [A] time = 0.02, size = 848, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*e^3*a+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*e
*c*d^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x*a*d*e^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a
)*x*c*d^3-3/16/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/
4)*x-1)*d*e^2+1/16/(a*e^2+c*d^2)^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x-1)*c*d^3-3/32/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^
(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^2
+1/32/(a*e^2+c*d^2)^2*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(
a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*c*d^3-3/16/(a*e^2+c*d^
2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2+1/16/(a*e^2+
c*d^2)^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*c*d^3+1/32/(
a*e^2+c*d^2)^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1
/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*a*e^3-3/32/(a*e^2+c*d^2)^2/(a
/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/
4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e+1/16/(a*e^2+c*d^2)^2/c/(a/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*a*e^3-3/16/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e+1/16/(a*e^2+c*d^2)^2/c/(a/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*a*e^3-3/16/(a*e^2+c*d^2)^2/(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e+e^2/(a*e^2+c*d^2)^2*d^
2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.09, size = 490, normalized size = 0.72

$$\frac{d^2e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{de}}+\frac{\left(cd^2e+ae^3\right)x^3-\left(cd^3+ade^2\right)x}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4+\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)x^4\right)}+\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^3-3\sqrt{a}\,cd^2e-3\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] d^2*e^2*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e
)) + 1/4*((c*d^2*e + a*e^3)*x^3 - (c*d^3 + a*d*e^2)*x)/(a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4) + 1/32*(2
*sqrt(2)*(c^(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*s
qrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^3 -
 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2
*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2*e - 3*a*
sqrt(c)*d*e^2 - a^(3/2)*e^3)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x +
sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2*e - 3*a
*sqrt(c)*d*e^2 - a^(3/2)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x +
 sqrt(a))/(a^(3/4)*c^(3/4)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
```

mupad [B] time = 4.87, size = 17180, normalized size = 25.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] - atan((((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*
d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*
d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (
x*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e +
 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) -
 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4
```

```
*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^
4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^
9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*
c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*
d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2)
 - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^
3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)
^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4
*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e
^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 +
4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4
*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3
)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20
*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a
^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*
c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*
e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256
*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*
c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 -
 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*
(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a
^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 + 33*c^5*d^8*e
^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^
4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*
((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6
*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 1
5*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c
^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i - (((((28672
*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a
^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^
3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((a^3*e^6*(-a^3*
c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 -
 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*
(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a
^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2
*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 32768
0*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327
680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*
d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c
^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c
^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*
c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d
^2*e^6)))^(1/2) - (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*
d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e
^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4
*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*
(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 +
 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(25
6*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^
6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d
*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^
6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d
^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^
3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/
(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4
*a^6*c^4*d^2*e^6)))^(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6
*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 +
4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^
3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 2
```

0*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i)/((5*c^2*d^4*e^6 + a*c*d^2*e^8)/(128*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/

2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4
*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2
+ 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2)
+ 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2
*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 +
a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^
(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^
4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*
a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(
-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 +
15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256
*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6
*c^4*d^2*e^6)))^(1/2)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/
2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*
e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8
 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)
))^(1/2)*2i - atan((((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155
648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45
056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d
^2*e^4)) - (x*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2
*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*
c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3
*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(
65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 5
89824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 +
 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^
8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^
3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5
 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^
4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6
*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 12
8*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^
6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 +
 c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d
^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^
2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c
*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*
e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 96
0*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d
^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^
3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4
*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^
2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d
^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 +
33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^1
1))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2
*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c
^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^
3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e
^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i
- (((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^
6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^
12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((c^
3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4
*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^
2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d
^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17
 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10
*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^

4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i)/((5*c^2*d^4*e^6 + a*c*d^2*e^8)/(128*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^

6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (((((28672*a^2*c^8
*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d
^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 +
c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((c^3*d^6*(-a^3*c^3)^(1/
2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*
c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^
3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d
^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^
14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^
8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*
c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6
+ 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2
) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e
^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8
+ a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6))
)^(1/2) - (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 +
21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 38
40*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d
^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^
3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^
2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c
^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^
2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 +
8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*
c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3
*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a
*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^
3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4
*d^2*e^6)))^(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 3
8*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*
d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2)
- a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^
3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)
^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4
*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-
a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 1
5*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*
(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*
c^4*d^2*e^6)))^(1/2)*2i - ((d*x)/(4*(a*e^2 + c*d^2)) - (e*x^3)/(4*(a*e^2 +
c*d^2)))/(a + c*x^4) - (atan(((((((((c^6*d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4
+ (a^4*c^2*d*e^11)/16 + (259*a^2*c^4*d^5*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(
2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (((x*(256*a*c^
8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6
+ 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/
(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4
*e^4)) + (((112*a^2*c^8*d^10*e^4 - 16*a*c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6
+ 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^
3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*(-d^3*e^3)^(1/2)
*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 -
589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11
+ 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*
d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6
+ 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2
e^2)))(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^
3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*(a^4*c*e^13 + 33*c^5
*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(2
56*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e
^4)))*(-d^3*e^3)^(1/2)*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) - ((((((c^6*
d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d*e^11)/16 + (259*a^2*c^4*d^5

```
*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 +
 3*a^2*c*d^2*e^4)) - (((x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6
*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*
d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^
2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (((112*a^2*c^8*d^10*e^4 - 16*a*
c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*
e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^
2*c*d^2*e^4)) + (x*(-d^3*e^3)^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^
14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^
8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*
c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8
+ 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2)
)/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 +
c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*
d^2*e^2)) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^
3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2
+ 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2)*1i)/(a^2*e^4 + c^
2*d^4 + 2*a*c*d^2*e^2))/(((5*c^2*d^4*e^6)/128 + (a*c*d^2*e^8)/128)/(a^3*e^6
 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4) + ((((((c^6*d^9*e^3)/16 - (
15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d*e^11)/16 + (259*a^2*c^4*d^5*e^7)/8 - (47*a
^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^
4)) + (((x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 2
1632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840
*a^5*c^4*d^3*e^12))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2
*e^6 + 6*a^2*c^2*d^4*e^4)) + (((112*a^2*c^8*d^10*e^4 - 16*a*c^9*d^12*e^2 +
608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*e^10 + 176*a^6*
c^4*d^2*e^12)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) -
 (x*(-d^3*e^3)^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680
*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327
680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/
(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e
^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 +
 c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c
*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*
(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a
^3*c^2*d^2*e^11))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e
^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*
e^2) + ((((((c^6*d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d*e^11)/16 +
 (259*a^2*c^4*d^5*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 +
3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (((x*(256*a*c^8*d^11*e^4 - 128*c^9*d^
13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^
8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(256*(a^4*e^8 + c^4*d^8
 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (((112*a^2*c^8
*d^10*e^4 - 16*a*c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 +
 688*a^5*c^5*d^4*e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c
^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*(-d^3*e^3)^(1/2)*(65536*a^9*c^4*e^17 -
65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^
7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e
^13 + 327680*a^8*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a
^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))
*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2
))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 +
 c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^
6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(256*(a^4*e^8 + c^4*d^8 +
 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))
/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2)*1i)/(a^2*e^4 + c^2*
d^4 + 2*a*c*d^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.255 $\int \frac{x^2}{(d+ex^2)(a+cx^4)^2} \, dx$

Optimal. Leaf size=685

$$\frac{\sqrt[4]{c}\,de\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\,de\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{(\cdots}{}$$

```
[Out] 1/4*x*(c*d*x^2+a*e)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/32*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e*a^(1/2)+d*c^(1/2))/a^(5/4)/c^(1/4)/(a*e^2+
c*d^2)*2^(1/2)-1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e
*a^(1/2)+d*c^(1/2))/a^(5/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*c^(1/4)*d*e*a
rctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c
*d^2)^2*2^(1/2)-1/4*c^(1/4)*d*e*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(
1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/8*c^(1/4)*d*e*ln(-a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^
2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*d*e*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/16*arctan
(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c^(1/2))/a^(5/4)/c^(1/4)/(a*e
^2+c*d^2)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c
^(1/2))/a^(5/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-e^(5/2)*arctan(x*e^(1/2)/d^(1
/2))*d^(1/2)/(a*e^2+c*d^2)^2
```

Rubi [A] time = 0.56, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1316, 1179, 1168, 1162, 617, 204, 1165, 628, 1171, 205}

$$\frac{\sqrt[4]{c}\,de\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\,de\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{(\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (x*(a*e + c*d*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[d]*e^(5/2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (c^(1/4)*d*e*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d*e*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(
1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) - ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1316

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[f^2/(c*d^2 + a*e^2), Int[(f*x)^(m - 2)*(a*e + c*d*x^2)*(
a + c*x^4)^p, x], x] - Dist[(d*e*f^2)/(c*d^2 + a*e^2), Int[((f*x)^(m - 2)*(
a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && LtQ
```

```
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{\int \frac{ae+cdx^2}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2} - \frac{(de)\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx}{cd^2+ae^2}$$

$$= \frac{x\left(ae+cdx^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\int \frac{-3ae-cdx^2}{a+cx^4}\,dx}{4a\left(cd^2+ae^2\right)} - \frac{(de)\int \left(\frac{e^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)}{cd^2+ae^2}$$

$$= \frac{x\left(ae+cdx^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{(cde)\int \frac{d-ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2} - \frac{\left(de^3\right)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2} - \frac{\left(d-\frac{3\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\,\sqrt{c}}{a+c}}{8a\left(cd^2+ae^2\right.}$$

$$= \frac{x\left(ae+cdx^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\left(d\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)e\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2} - \left(\right.$$

$$= \frac{x\left(ae+cdx^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{c}\left(d-\frac{3\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\right.}{16\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)}$$

$$= \frac{x\left(ae+cdx^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\left(d+\frac{3\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)}$$

$$= \frac{x\left(ae+cdx^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{c}\,de\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.28, size = 428, normalized size = 0.62

$$\frac{\frac{\sqrt{2}\left(-3a^{3/2}e^3+\sqrt{a}\,cd^2e+5a\sqrt{c}\,de^2+c^{3/2}d^3\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{5/4}\sqrt[4]{c}} - \frac{\sqrt{2}\left(-3a^{3/2}e^3+\sqrt{a}\,cd^2e+5a\sqrt{c}\,de^2+c^{3/2}d^3\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{5/4}\sqrt[4]{c}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(a*(a + c*x^4)) - 32*Sqrt[d]*e^(5/2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e +
5*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(a^(5/4)*c^(1/4)) + (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + 5*a*Sqrt[c
]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/4)*
c^(1/4)) + (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 - 3*
a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(5/
4)*c^(1/4)) - (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 -
 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^
(5/4)*c^(1/4)))/(32*(c*d^2 + a*e^2)^2)
```

fricas [B] time = 20.92, size = 9774, normalized size = 14.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e
^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4
*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*
d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 +
143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c
*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^
12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 +
28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 +
4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c
^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^
4*e^8)*x + (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5
*c*d^2*e^7 + 27*a^6*e^9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^
7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^
6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 79
9*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6
*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*
e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a
^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d
^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqr
t(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^
6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 +
 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^
5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^
14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 +
4*a^5*c*d^2*e^6 + a^6*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (
a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d
^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^
4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^
2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e
^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4
 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^1
1*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*
c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8
 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)
*x - (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2
*e^7 + 27*a^6*e^9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4
+ 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12
 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*
c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d
^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 +
56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*
e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4
*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^
6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 79
9*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6
*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*
e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a
^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*
c*d^2*e^6 + a^6*e^8))) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*
d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3
 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*
a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*
```

d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 + (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 + (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) + 8*(a*c*e^2*x^4 + a^2*e^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 4*(a*c*d^2*e + a^2*e^3)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), 1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 - 16*(a*c*e^2*x^4 + a^2*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^

9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5
*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10
*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 55
8*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7
*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*
e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*
c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2
 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^
5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^
8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 +
28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^
4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a
^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*
e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*
d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5
+ (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 +
a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^
3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^
5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6
 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^1
2*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^
2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 +
112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (a^2*c^4*d^8*e +
6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 - (a
^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 +
 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 +
 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*
c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d
^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 +
 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^
5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a
^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10
*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 55
8*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7
*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*
e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4
*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)))
 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^
2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2
*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^
8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*
d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*
d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*
a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*
d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*
e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^
2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (a^2*c^4*d^8*e + 6*a^3*
c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 + (a^4*c^6
*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^
8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a
^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*
e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^
4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^
11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e +
4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2
*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 +
 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*
c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d
^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 +

```
28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 +
 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) - (a^
2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^
3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d
^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqr
t(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^
6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 +
 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^
5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^
14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 +
4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*
d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (a^2*c^4*d^8*e + 6*a^3*c^3*d^
6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 + (a^4*c^6*d^11
+ 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*
d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4
*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 +
 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56
*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3
*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*
d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e
^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a
^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*
e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^
4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^
11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3
*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) + 4*(a*c*d^
2*e + a^2*e^3)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2
*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)]
```

giac [A] time = 0.50, size = 603, normalized size = 0.88

$$-\frac{\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{5}{2}}}{c^2d^4+2\,acd^2e^2+a^2e^4}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e-\left(ac^3\right)^{\frac{3}{4}}cd^3-3\left(ac^3\right)^{\frac{1}{4}}a^2ce^3-5\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^2c^4d^4+2\sqrt{2}\,a^3c^3d^2e^2+\sqrt{2}\,a^4c^2e^4\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(5/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - (a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4
)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 +
sqrt(2)*a^4*c^2*e^4) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - (a*c^3)^(3/4)*c*d^3
 - 3*(a*c^3)^(1/4)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a
^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*((a*c^3)^(1/4)*a*c^2*d^2*e + (
a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4)*a^2*c*e^3 + 5*(a*c^3)^(3/4)*a*d*e^2)*l
og(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2
)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*((a*c^3)^(1/4)*a*c^2*d^2*e
+ (a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4)*a^2*c*e^3 + 5*(a*c^3)^(3/4)*a*d*e^2
)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqr
t(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/4*(c*d*x^3 + a*x*e)/((c*x^4
 + a)*(a*c*d^2 + a^2*e^2))
```

maple [A] time = 0.02, size = 852, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*c*d*x^3*e^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*c^2
*d^3/a*x^3+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x*e^3*a+1/4/(a*e^2+c*d^2)^2/(c*x^4
+a)*x*e*c*d^2+3/16/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)
^(1/4)*x-1)*e^3-1/16/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/c)^(1/4)*x-1)*c*d^2*e+3/32/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a
/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e
^3-1/32/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x
+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*c*d^2*e+3/16/(a*e^2+
c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^3-1/16/(a*e^
2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*c*d^2*e+5/
32/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^
(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^2+1/32/(a*e^2+c*d^2)^2/
a*c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/
c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3+5/16/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^2+1/16/(a*e^2+c*d^2)^2/a*c/(a/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3+5/16/(a*e^2+c*d^2)^2/(a/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2+1/16/(a*e^2+c*d^2)^2/a*c
/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3-d*e^3/(a*e^2+c*d^2
)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.15, size = 472, normalized size = 0.69

$$-\frac{de^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{de}}+\frac{cdx^3+aex}{4\left(a^2cd^2+a^3e^2+\left(ac^2d^2+a^2ce^2\right)x^4\right)}+\frac{\frac{2\sqrt{2}\left(\sqrt{a}c^2d^3-ac^{\frac{3}{2}}d^2e+5\,a^{\frac{3}{2}}cde^2+3\,a^2\sqrt{c}e^3\right)\text{arcta}}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -d*e^3*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)
) + 1/4*(c*d*x^3 + a*e*x)/(a^2*c*d^2 + a^3*e^2 + (a*c^2*d^2 + a^2*c*e^2)*x^
4) + 1/32*(2*sqrt(2)*(sqrt(a)*c^2*d^3 - a*c^(3/2)*d^2*e + 5*a^(3/2)*c*d*e^2
 + 3*a^2*sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(
1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sq
rt(2)*(sqrt(a)*c^2*d^3 - a*c^(3/2)*d^2*e + 5*a^(3/2)*c*d*e^2 + 3*a^2*sqrt(c
)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*(sqrt(a)*c^
2*d^3 + a*c^(3/2)*d^2*e + 5*a^(3/2)*c*d*e^2 - 3*a^2*sqrt(c)*e^3)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(s
qrt(a)*c^2*d^3 + a*c^(3/2)*d^2*e + 5*a^(3/2)*c*d*e^2 - 3*a^2*sqrt(c)*e^3)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a
*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)
```

mupad [B] time = 2.87, size = 17812, normalized size = 26.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((e*x)/(4*(a*e^2 + c*d^2)) + (c*d*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) +
 atan((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d
^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*
```

d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*1i - (((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e -

4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*1i)/(((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(128*a*c^10*d^

13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12)) /(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (5*c^5*d^5*e^7 + 54*a*c^4*d^3*e^9 + 81*a^2*c^3*d*e^11)/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*2i + atan(((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(2

56*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*1i - (((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*1i)/((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^

6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/
(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c
^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c
^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1
/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^
6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4
*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c
^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6
*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(
1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e
^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*
c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2
) + (((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11
*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5
*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*
e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*((c^3*d^6*(-a^5*c)^(1/2)
 - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*
c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))
/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*
a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 3
27680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9
+ 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e
^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6
*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2
*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5
*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8
+ 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(
128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^
3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c
^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6
*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(
1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e
^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*
c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2
) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c
^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8
 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))
*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*
a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*
c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2
 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(81*a^4*c^3*e^13 + c
^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))
/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*
c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*
c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(
1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a
^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (5*c^5*d^
5*e^7 + 54*a*c^4*d^3*e^9 + 81*a^2*c^3*d*e^11)/(128*(a^6*e^8 + a^2*c^4*d^8 +
 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))*((c^3*d^6*(-a^
5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3
 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5
c)^(1/2))/(256(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^
4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*2i - (atan((((-d*e^5)^(1/2)*((x*(81*a^4*
c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^
4*d^2*e^11))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*
e^2 + 6*a^4*c^2*d^4*e^4)) - ((((c^9*d^12*e^2)/16 + (13*a*c^8*d^10*e^4)/16 +
 (21*a^2*c^7*d^8*e^6)/8 + (29*a^3*c^6*d^6*e^8)/8 + (805*a^4*c^5*d^4*e^10)/1
6 + (777*a^5*c^4*d^2*e^12)/16)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6
+ 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((-d*e^5)^(1/2)*((208*a^9*c^4

```
*d*e^15 + 16*a^3*c^10*d^13*e^3 + 288*a^4*c^9*d^11*e^5 + 1200*a^5*c^8*d^9*e^
7 + 2240*a^6*c^7*d^7*e^9 + 2160*a^7*c^6*d^5*e^11 + 1056*a^8*c^5*d^3*e^13)/(
2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*
d^4*e^4)) - (x*(-d*e^5)^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^
3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8
*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*
d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^6*e^8 + a^2*c^4*d^8
+ 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))/(2*(a^2*e^4 +
 c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14
 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30
592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(256*(a^6*e^8 + a^2*c^4*d^8
+ 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-d*e^5)^(1/2)
)/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^
2*d^4 + 2*a*c*d^2*e^2)))*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) + ((-d*e^5
)^(1/2)*((x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*
d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*
e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((c^9*d^12*e^2)/16 + (13*
a*c^8*d^10*e^4)/16 + (21*a^2*c^7*d^8*e^6)/8 + (29*a^3*c^6*d^6*e^8)/8 + (805
*a^4*c^5*d^4*e^10)/16 + (777*a^5*c^4*d^2*e^12)/16)/(2*(a^6*e^8 + a^2*c^4*d^
8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((-d*e^5)
^(1/2)*((208*a^9*c^4*d*e^15 + 16*a^3*c^10*d^13*e^3 + 288*a^4*c^9*d^11*e^5 +
 1200*a^5*c^8*d^9*e^7 + 2240*a^6*c^7*d^7*e^9 + 2160*a^7*c^6*d^5*e^11 + 1056
*a^8*c^5*d^3*e^13)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*
d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*(-d*e^5)^(1/2)*(65536*a^11*c^4*e^17 - 65
536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7
- 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^1
3 + 327680*a^10*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^
6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e
^4))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*(128*a*c^10*d^13*e^2 -
14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*
a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(256*(a^
6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e
^4)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^(1/
2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c
*d^2*e^2))/(((5*c^5*d^5*e^7)/128 + (27*a*c^4*d^3*e^9)/64 + (81*a^2*c^3*d*e^
11)/128)/(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a
^4*c^2*d^4*e^4) - ((-d*e^5)^(1/2)*((x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a
*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(256*(a^6*e^8 +
a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (
(((c^9*d^12*e^2)/16 + (13*a*c^8*d^10*e^4)/16 + (21*a^2*c^7*d^8*e^6)/8 + (29
*a^3*c^6*d^6*e^8)/8 + (805*a^4*c^5*d^4*e^10)/16 + (777*a^5*c^4*d^2*e^12)/16
)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c
^2*d^4*e^4)) + ((((-d*e^5)^(1/2)*((208*a^9*c^4*d*e^15 + 16*a^3*c^10*d^13*e^
3 + 288*a^4*c^9*d^11*e^5 + 1200*a^5*c^8*d^9*e^7 + 2240*a^6*c^7*d^7*e^9 + 21
60*a^7*c^6*d^5*e^11 + 1056*a^8*c^5*d^3*e^13)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*
a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-d*e^5)^(1/2)
*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5
- 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^1
1 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(512*(a^2*e^4 + c^
2*d^4 + 2*a*c*d^2*e^2)*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3
*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) +
(x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 396
8*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a
^6*c^5*d^3*e^12))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3
*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a
*c*d^2*e^2)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))))/(a^
2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) + ((-d*e^5)^(1/2)*((x*(81*a^4*c^3*e^13 + c
^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))
/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*
```

```
c^2*d^4*e^4)) + ((((c^9*d^12*e^2)/16 + (13*a*c^8*d^10*e^4)/16 + (21*a^2*c^7
*d^8*e^6)/8 + (29*a^3*c^6*d^6*e^8)/8 + (805*a^4*c^5*d^4*e^10)/16 + (777*a^5
*c^4*d^2*e^12)/16)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*
d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((-d*e^5)^(1/2)*((208*a^9*c^4*d*e^15 + 16
*a^3*c^10*d^13*e^3 + 288*a^4*c^9*d^11*e^5 + 1200*a^5*c^8*d^9*e^7 + 2240*a^6
*c^7*d^7*e^9 + 2160*a^7*c^6*d^5*e^11 + 1056*a^8*c^5*d^3*e^13)/(2*(a^6*e^8 +
 a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) +
(x*(-d*e^5)^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a
^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 32768
0*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(
512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^
2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))/(2*(a^2*e^4 + c^2*d^4 + 2
*a*c*d^2*e^2)) - (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c
^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*
d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^
2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-d*e^5)^(1/2))/(2*(a^2*e^
4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*
c*d^2*e^2))))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^(1/2)*1i)/(a^2
*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.256 $\int \frac{1}{(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=689

$$-\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}$$

```
[Out] 1/4*c*x*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/4*c^(1/4)*e^2*arctan(-1+c^(1
/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/
2)+1/4*c^(1/4)*e^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2
))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*e^2*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^
(1/2)+1/8*c^(1/4)*e^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*
a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/16*c^(1/4)*arctan(-1+c
^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^
(1/2)+1/16*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1
/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/
2)+1/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2
)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)+e^(7/2)*arctan(x*e^(1/2)/d^(1/
2))/(a*e^2+c*d^2)^2/d^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1239, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]
*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1239

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \int\left(\frac{e^4}{\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)^2} - \frac{ce^2\left(-d+ex^2\right)}{\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}\right) dx$$

$$= -\frac{\left(ce^2\right)\int\frac{-d+ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{e^4\int\frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{c\int\frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} + \frac{\left(\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)e^2\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2} + \ldots$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} + \frac{\left(\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)e^2\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)^2} \ldots$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\ldots\right.}{4\sqrt{2}\,a^{3/4}\left(cd^2+ae^{2}\ldots\right.}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\ldots}{\sqrt[4]{\ldots}}\right.}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)^2}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\ldots}{\sqrt[4]{\ldots}}\right.}{2\sqrt{2}\,a^{3/4}\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.29, size = 429, normalized size = 0.62

$$-\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}\,cd^2e+7a\sqrt{c}\,de^2+3c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{7/4}} + \frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}\,cd^2e+7a\sqrt{c}\,de^2+3c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x\ldots\right.}{a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]
*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e
- 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c
]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqr
t[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

fricas [B] time = 40.14, size = 9892, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*
e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e +
44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^
5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d
^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8
 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e
^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*
a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))
/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a
^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*
a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2
+ 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^
10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^1
0*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a
^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*
d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*
d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^1
0 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5
*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 +
 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*
c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^
4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d
^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8
+ 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^
16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^
6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*
a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70
*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6
*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^
5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e
^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*
e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 2
8*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4
*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5
*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625
*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4
 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*
e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*
e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748
*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*
e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^
5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e
^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3
 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a
^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*
d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*
d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^1
0 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2
*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3
*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c
^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)
*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*
c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)
/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^1
0*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*
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c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - 8*(a*c*e^3*x^4 + a^2*e^3)*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), -1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 - 16*(a*c*e^3*x^4 + a^2*e^3)*sqrt(e/d)*arctan(x*sqrt(e/d)) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5

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*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e
^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 4
4*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5
*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^
10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8
- 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^
2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a
^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^
7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^
2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c
*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2
*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*
e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 +
625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 +
56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13
*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 +
 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c
*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198
*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 +
26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*
sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c
^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*
a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*
c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e
^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*
c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2*c^2*d
^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4
)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(
-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^
6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*
c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6
+ 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^1
4*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^
4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 +
1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5
*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 17
5*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 +
34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 +
738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*
c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8
*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^
8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a
^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^
4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*
sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c
^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
```

8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)]

giac [A] time = 0.47, size = 603, normalized size = 0.88

$$\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-\left(ac^3\right)^{\frac{3}{4}}cd^2e-5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^2c^4d^4+2\sqrt{2}\,a^3c^3d^2e^2+\sqrt{2}\,a^4c^2e^4\right)}+\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d)) - 1/4*(c*x^3*e - c*d*x)/((c*x^4 + a)*(a*c*d^2 + a^2*e^2))

maple [A] time = 0.02, size = 873, normalized size = 1.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*x^3-1/4*c^2/(a*e^2+c*d^2)^2/(c*x^4+a)*
e/a*x^3*d^2+1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*x*e^2+1/4*c^2/(a*e^2+c*d^2)^2
/(c*x^4+a)*d^3/a*x+7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x-1)*d*e^2+3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3+7/32*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2
^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2)))*d*e^2+3/32*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((
x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/
2)))*d^3+7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x+1)*d*e^2+3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x+1)*d^3-5/32/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2
-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))
)*e^3-1/32*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e-5/16/(a*e
^2+c*d^2)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^3-1/16*c/
(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e
-5/16/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e
^3-1/16*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^2*e+e^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.06, size = 506, normalized size = 0.73

$$\frac{e^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{de}}+\frac{c\left(\frac{2\sqrt{2}\left(3c^{\frac{3}{2}}d^3-\sqrt{a}\,cd^2e+7\,a\sqrt{c}\,de^2-5\,a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(3c^{\frac{3}{2}}d^3-\sqrt{a}\,cd^2e+7\,a\sqrt{c}\cdots\right.}{\sqrt{\cdots}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)) +
 1/32*c*(2*sqrt(2)*(3*c^(3/2)*d^3 - sqrt(a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 - 5
*a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sq
rt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3
*c^(3/2)*d^3 - sqrt(a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 - 5*a^(3/2)*e^3)*arctan(
1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*c^(3/2)*d^3 + sqrt(a)*
c*d^2*e + 7*a*sqrt(c)*d*e^2 + 5*a^(3/2)*e^3)*log(sqrt(c)*x^2 + sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*c^(3/2)*d^3 + sqrt(
a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 + 5*a^(3/2)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a
^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
 + a^3*e^4) - 1/4*((c^2*d^2*e + a*c*e^3)*x^3 - (c^2*d^3 + a*c*d*e^2)*x)/(a^
2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^
3*c*e^4)*x^4)
```

mupad [B] time = 2.73, size = 17945, normalized size = 26.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^
4) - atan((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c
^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8
*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a
^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*
e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c
^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^
(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a
^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*
c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*
c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a
^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 +
 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a
^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*
e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(
-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*
d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a
^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c
^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e
^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)
))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) +
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e
^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 +
21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8
*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(
128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^
2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4
*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c
)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4
*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i - (((((
65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 3
6864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 +
663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4
*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((9*c
^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^
5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c
*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 +
4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536
*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 3
27680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13
 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25
*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d
*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(
256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*
c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 +
7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 666
88*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
```

3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c
^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^
5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e
^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) -
25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c
*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))
/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^
9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8
*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9
*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*
a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6
+ 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i)/((125*a^2*c^5*e^12 + 8
1*c^7*d^4*e^8 + 270*a*c^6*d^2*e^10)/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d
^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^11*c^4*e^16
 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e
^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*
e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*
e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/
2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*
a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(
1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 +
 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3
- 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e
^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*
d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^
2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(
1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4
e^2(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7
*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2
) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*
e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10
 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
+ 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*
a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*
e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(2
56*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c
^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^
2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c
^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*
e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)
^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d
^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a
^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1
/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2
*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*
c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(
1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 7
0*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)
^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d
^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8
*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*
c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6
*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a
^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*
c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^
8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)

))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*2i - atan((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70

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*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^
(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612
*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e
^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)
))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) -
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i - (((((65536*a^11
*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c
^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9
*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a
^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(-
a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*
e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(
-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*
d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*
d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*
c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*
a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(
-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*
a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*
e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^
4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c
^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7
*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c
*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(
1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70
*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^
(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13
+ 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33
296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5
*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6
*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 4
1*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^1
1*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*
e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 +
 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*
(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i)/((125*a^2*c^5*e^12 + 81*c^7*d^4*
e^8 + 270*a*c^6*d^2*e^10)/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4
*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^11*c^4*e^16 - 12288*a
^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 24576
0*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331
776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^
5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^
3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^
5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256
*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2
*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a
^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 32768
0*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))
/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*
c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a
```

^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7
*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 +
4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(11
52*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 2035
2*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*
a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^
3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-
a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a
*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e
^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4
)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*
e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^1
1)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^
6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6
*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a
^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8
 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(
1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e
^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*
c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*
e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(2
56*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c
^2*d^4*e^4)))^(1/2) + (((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 -
57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 +
 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^
14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a
^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2
) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^
2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^
4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(
65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 -
589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11
 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^
4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25
*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a
^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*
c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 +
 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^
2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 +
 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/
(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c
^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^
4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*
c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 +
4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*
c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8
*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 +
a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((
25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44
*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^
2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6
 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 +
 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d
^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1
/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*
e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*
c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)

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))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) -
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*2i + (atan(-(((((((4
5*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*e^5)/16
 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6*d^3*e^
11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6
*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 - 224*a^
5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*a^8*c^7
*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(2*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x
*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7
*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*
a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(
512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*
d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*
d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*
c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*
d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a^8*e^8
+ a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*
(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/
(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1425*a^4*c^5*e^13 + 81*c^9*
d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11)
)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6
*c^2*d^4*e^4)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2) -
((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*
e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6
*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*
e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 -
 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*
a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(2*(a^8*
e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4
)) + (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327
680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 +
327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e
^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4
*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(
2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1152*a^2*c^11*d^13*e^2 - 490
24*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a
^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a
^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*
e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^
(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1425*a^4*c^5*e^13 +
81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^
2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
+ 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*
e^2))/(((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c
^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*
a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^1
4*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8
+ 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(
2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*
d^4*e^4)) - (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^
3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8
*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^
5*d^2*e^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*
d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(
1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1152*a^2*c^11*d^13*e^
```

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2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 +
 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/
(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c
^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-
d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1425*a^4*c^5*
e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3
*c^6*d^2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d
^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(c^2*d^5 + a^2*d*e^4 + 2*a*c*
d^3*e^2) - ((125*a^2*c^5*e^12)/128 + (81*c^7*d^4*e^8)/128 + (135*a*c^6*d^2*
e^10)/64)/(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*
a^6*c^2*d^4*e^4) + ((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16
+ (305*a^2*c^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9
)/8 + (2081*a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e
^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*
a^4*c^11*d^14*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*
c^8*d^8*e^8 + 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5
*d^2*e^14)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
+ 6*a^6*c^2*d^4*e^4)) + (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*
c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680
*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 32
7680*a^12*c^5*d^2*e^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^
8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))
)*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1152*a^2*
c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c
^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6
*d^3*e^12))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e
^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d
^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1
425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^
9 + 2532*a^3*c^6*d^2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 +
 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(c^2*d^5 + a^2*d*
e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*
e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.257 $\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=745

$$-\frac{c^{3/4}\left(\sqrt{c}\,d-3\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}\,d-3\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)}+\frac{c^{3/}}{}$$

```
[Out] -1/a^2/d/x-1/4*c*x*(c*d*x^2+a*e)/a^2/(a*e^2+c*d^2)/(c*x^4+a)-e^(9/2)*arctan
(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2+c*d^2)^2-1/32*c^(3/4)*ln(-a^(1/4)*c^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e*a^(1/2)+d*c^(1/2))/a^(9/4)/(a*e^2+c*
d^2)*2^(1/2)+1/32*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))
*(-3*e*a^(1/2)+d*c^(1/2))/a^(9/4)/(a*e^2+c*d^2)*2^(1/2)-1/16*c^(3/4)*arctan
(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c^(1/2))/a^(9/4)/(a*e^2+c*d^2
)*2^(1/2)-1/16*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c
^(1/2))/a^(9/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*c^(3/4)*ln(-a^(1/4)*c^(1/4)*x*2^(
1/2)+a^(1/2)+x^2*c^(1/2))*(a^(3/2)*e^3-d*(2*a*e^2+c*d^2)*c^(1/2))/a^(9/4)/(
a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*c^(1/2))*(a^(3/2)*e^3-d*(2*a*e^2+c*d^2)*c^(1/2))/a^(9/4)/(a*e^2+c*d^2)^2*2
^(1/2)-1/4*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(a^(3/2)*e^3+d*(2*a
*e^2+c*d^2)*c^(1/2))/a^(9/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/4*c^(3/4)*arctan(1+c
^(1/4)*x*2^(1/2)/a^(1/4))*(a^(3/2)*e^3+d*(2*a*e^2+c*d^2)*c^(1/2))/a^(9/4)/(
a*e^2+c*d^2)^2*2^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1336, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c^{3/4}\left(a^{3/2}e^3-\sqrt{c}\,d\left(2ae^2+cd^2\right)\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(\sqrt{c}\,d-3\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\right.}{16\sqrt{2}\,a^{9/4}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(1/(a^2*d*x)) - (c*x*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) + (c^(3
/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(9/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 + Sqrt[c]*d*(c*d^2
+ 2*a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(c*
d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(a^(3/2)*
e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4
)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(
c*d^2 + a*e^2)^2) + (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) -
(c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)^2}dx = \int\left(\frac{1}{a^2dx^2} - \frac{e^5}{d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} - \frac{c\left(ae+cdx^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)^2} + \frac{c\left(-a^2e^3-cd\left(\right.\right.}{a^2\left(cd^2+ae\right.}\right.$$

$$= -\frac{1}{a^2dx} + \frac{c\int \frac{-a^2e^3-cd\left(cd^2+2ae^2\right)x^2}{a+cx^4}dx}{a^2\left(cd^2+ae^2\right)^2} - \frac{e^5\int \frac{1}{d+ex^2}dx}{d\left(cd^2+ae^2\right)^2} - \frac{c\int \frac{ae+cdx^2}{\left(a+cx^4\right)^2}dx}{a\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{a^2dx} - \frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2} + \frac{c\int \frac{-3ae-cdx^2}{a+cx^4}dx}{4a^2\left(cd^2+ae^2\right)} + \frac{\left(c\left(c\right.\right.}{}$$

$$= -\frac{1}{a^2dx} - \frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2} + \frac{\left(c\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}-}{a+cx^4}}{8a^2\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{a^2dx} - \frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2} - \frac{c^{5/4}\left(cd^3+2ade^2-\frac{a^{3/2}e^3}{\sqrt{c}}\right.}{4\sqrt{2}a}$$

$$= -\frac{1}{a^2dx} - \frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2} + \frac{c^{5/4}\left(cd^3+2ade^2+\frac{a^{3/2}e^3}{\sqrt{c}}\right.}{2\sqrt{2}a^{9/4}\left(cd\right.}$$

$$= -\frac{1}{a^2dx} - \frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2} + \frac{c^{3/4}\left(\sqrt{c}d+3\sqrt{a}e\right)\tan^{-}}{8\sqrt{2}a^{9/4}\left(cd^2+\right.}$$

Mathematica [A] time = 0.37, size = 499, normalized size = 0.67

$$\frac{1}{32}\left(\frac{\sqrt{2}c^{3/4}\left(7a^{3/2}e^3+3\sqrt{a}cd^2e-9a\sqrt{c}de^2-5c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{9/4}\left(ae^2+cd^2\right)^2} + \frac{\sqrt{2}c^{3/4}\left(-7a^{3/2}e^3-3\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-32/(a^2*d*x) - (8*c*x*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (32*e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) + (2
*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 + 7
*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(9/4)*(c*d^2 + a*
e^2)^2) - (2*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[
c]*d*e^2 + 7*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(9/4)
*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(-5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e
- 9*a*Sqrt[c]*d*e^2 + 7*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(5*c^(3/2)
*d^3 - 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 - 7*a^(3/2)*e^3)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2))/32
```

fricas [B] time = 98.08, size = 10188, normalized size = 13.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a*c^2*d^4 + 32*a^2*c*d^2*e^2 + 16*a^3*e^4 + 4*(5*c^3*d^4 + 9*a*c
^2*d^2*e^2 + 4*a^2*c*e^4)*x^4 + 4*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 - ((a^2*c
^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*
e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d
*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e
^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*
e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 +
 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*
e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 2
8*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5
*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-(625*c^
6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2
401*a^4*c^2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*
d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 - (5*a^7*c^5*d^11 + 29*a^8*c^
4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 +
9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8
*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10
+ 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12
*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 +
28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e
 + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 405
0*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c
^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a
^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^
4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14
 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^
7*c*d^2*e^6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)
*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e +
 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 +
 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*
a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5
*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^1
0*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*
d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 +
 a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*
c*d^2*e^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4
*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (75*a^3*c^5*d^8*e +
418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c
*e^9 - (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*
c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050
*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^
5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^
10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4
*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14
+ a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5
+ (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 +
a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 +
 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401
*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 +
 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^1
5*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*
d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) - ((a^2*c^3*d^5
+ 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a
^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 -
(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^
8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3
868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a
```

^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 5
6*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*
c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^
6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-(625*c^6*d^8 +
 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4
*c^2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5
 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e
^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*
d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 +
3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*
a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 +
56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15
*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*
a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^
6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8
*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*
e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7
*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e
^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17
*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2
*e^6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 +
(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*
c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*
c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d
^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^
8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d
^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8
 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e
^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e
^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^
4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (75*a^3*c^5*d^8*e + 418*a^4
*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 +
(5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5
*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*
d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e
^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*
d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^
8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*
e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*
c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8
)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a
^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^
3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^1
2*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d
^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) - 8*(a^2*c*e^4*x^5 + a^
3*e^4*x)*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/((a^2*
c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3
*e^2 + a^5*d*e^4)*x), -1/16*(16*a*c^2*d^4 + 32*a^2*c*d^2*e^2 + 16*a^3*e^4 +
 4*(5*c^3*d^4 + 9*a*c^2*d^2*e^2 + 4*a^2*c*e^4)*x^4 + 4*(a*c^2*d^3*e + a^2*c
*d*e^3)*x^2 + 16*(a^2*c*e^4*x^5 + a^3*e^4*x)*sqrt(e/d)*arctan(x*sqrt(e/d))
- ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a
^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*
a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^
7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2
*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d
^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*
c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6
*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^

8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log
(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^
2*e^6 - 2401*a^4*c^2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684
*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 - (5*a^7*c^5*d^11 +
29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d
^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^
2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*
d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11
*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^
6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*
c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^
3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d
^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6
417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d
^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 7
0*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c
*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e
^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4
*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^
4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*
d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^1
2 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 641
7*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^1
6 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*
a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d
^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4
 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 494
4*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (75*a^3*c^5
*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 -
343*a^7*c*e^9 - (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 +
 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^
12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 64
17*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^
16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70
*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*
d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c
^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d
^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*
d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^
10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d
^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10
 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4
*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) - ((a^2
*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^
3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2
*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2
*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^
8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10
 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^1
2*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 +
 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a
^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-(625*
c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 -
 2401*a^4*c^2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^
3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*
c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8
+ 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d
^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^1
0 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^

```
12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10
+ 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*d^5
*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e
^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4
050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4
*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8
*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*
c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^
14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*
a^7*c*d^2*e^6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^
4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e
 + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 405
0*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c
^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a
^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^
4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14
 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^
7*c*d^2*e^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c
^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (75*a^3*c^5*d^8*e
+ 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7
*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^1
0*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 40
50*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*
c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*
a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c
^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^1
4 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^
5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6
+ a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4
 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 24
01*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4
 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a
^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^
3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))))/((a^2*c^3*d^5
 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 +
a^5*d*e^4)*x)]
```

giac [A] time = 0.45, size = 639, normalized size = 0.86

$$-\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+5\left(ac^3\right)^{\frac{3}{4}}cd^3+7\left(ac^3\right)^{\frac{1}{4}}a^2ce^3+9\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^3c^3d^4+2\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)}-\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)
*a^2*c*e^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + s
qrt(2)*a^5*c*e^4) - 1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 5*(a*c^3)^(3/4)*c*d^
3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*
a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(3*(a*c^3)^(1/4)*a*c^2*d^2*e -
5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 9*(a*c^3)^(3/4)*a*d*e^2
)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sqr
t(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(3*(a*c^3)^(1/4)*a*c^2*d^2
```

```
*e - 5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 9*(a*c^3)^(3/4)*a*
d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 +
2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - arctan(x*e^(1/2)/sqrt(d))*
e^(9/2)/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d)) - 1/4*(5*c^2*d^2*x^
4 + 4*a*c*x^4*e^2 + a*c*d*x^2*e + 4*a*c*d^2 + 4*a^2*e^2)/((a^2*c*d^3 + a^3*
d*e^2)*(c*x^5 + a*x))
```

maple [A] time = 0.02, size = 911, normalized size = 1.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/a^2/d/x-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^3*d*e^2-1/4*c^3/(a*e^2+c*d
^2)^2/a^2/(c*x^4+a)*x^3*d^3-1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*x*e^3-1/4*c^2/(
a*e^2+c*d^2)^2/a/(c*x^4+a)*x*e*d^2-7/32*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(
1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+
(a/c)^(1/2)))*e^3-3/32*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+
(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))
*d^2*e-7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1
/4)*x-1)*e^3-3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/c)^(1/4)*x-1)*d^2*e-7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/c)^(1/4)*x+1)*e^3-3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e-9/32*c/(a*e^2+c*d^2)^2/a/(a/c)^(
1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^
(1/2)*x+(a/c)^(1/2)))*d*e^2-5/32*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^(1/4)*2^(1/2
)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/
c)^(1/2)))*d^3-9/16*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x-1)*d*e^2-5/16*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3-9/16*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2-5/16*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3-1/d*e^5/(a*e^2+c*d^2)^2/
(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [A] time = 2.11, size = 521, normalized size = 0.70

$$-\frac{e^5\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^5+2\,acd^3e^2+a^2de^4\right)\sqrt{de}}-\frac{c\left(\frac{2\,\sqrt{2}\left(5\,\sqrt{a}\,c^2d^3+3\,ac^{\frac{3}{2}}d^2e+9\,a^{\frac{3}{2}}cde^2+7\,a^2\sqrt{c}\,e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\left(5\,\sqrt{a}\,c^2d^3\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -e^5*arctan(e*x/sqrt(d*e))/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d*e)
) - 1/32*c*(2*sqrt(2)*(5*sqrt(a)*c^2*d^3 + 3*a*c^(3/2)*d^2*e + 9*a^(3/2)*c*
d*e^2 + 7*a^2*sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4
)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) +
 2*sqrt(2)*(5*sqrt(a)*c^2*d^3 + 3*a*c^(3/2)*d^2*e + 9*a^(3/2)*c*d*e^2 + 7*a
^2*sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*(5
*sqrt(a)*c^2*d^3 - 3*a*c^(3/2)*d^2*e + 9*a^(3/2)*c*d*e^2 - 7*a^2*sqrt(c)*e^
3)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4))
 + sqrt(2)*(5*sqrt(a)*c^2*d^3 - 3*a*c^(3/2)*d^2*e + 9*a^(3/2)*c*d*e^2 - 7*a
```

```
^2*sqrt(c)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(
3/4)*c^(3/4)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) - 1/4*(a*c*d*e*x^2
 + (5*c^2*d^2 + 4*a*c*e^2)*x^4 + 4*a*c*d^2 + 4*a^2*e^2)/((a^2*c^2*d^3 + a^3
*c*d*e^2)*x^5 + (a^3*c*d^3 + a^4*d*e^2)*x)
```

mupad [B] time = 5.16, size = 24015, normalized size = 32.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] - (1/(a*d) + (c*e*x^2)/(4*a*(a*e^2 + c*d^2)) + (c*x^4*(4*a*e^2 + 5*c*d^2))/
(4*a^2*d*(a*e^2 + c*d^2)))/(a*x + c*x^5) - atan(((11875*a^5*c^10*d^15*e - a
^9*c^3*(72128*a^3*d*e^15 + 265655*c^3*d^7*e^9 - 76440*a*c^2*d^5*e^11 - 1785
85*a^2*c*d^3*e^13) + 68800*a^6*c^9*d^13*e^3 + 89403*a^7*c^8*d^11*e^5 - 1264
88*a^8*c^7*d^9*e^7)*(a^25*d^2*e^19*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^
3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3
*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^
(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*
a^11*c^2*d^4*e^4))^(5/2)*2i - a^15*c^2*e^17*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2
) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 12
4*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-
a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6
*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*3136i - a^11*c^10*d^19*x*(-(49*a^3*e^6*(-
a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*25i - a^16*c^9*d^20*e*x*(-(4
9*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e
 + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1
/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c
*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i + a^24*c*d^4*
e^17*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^
5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-
a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8
 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i +
 a^8*c^9*d^14*e^3*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^
(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c
^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8
+ a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4)
)^(1/2)*1250i + a^9*c^8*d^12*e^5*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*
d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d
^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1
/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^
11*c^2*d^4*e^4))^(1/2)*9900i + a^10*c^7*d^10*e^7*x*(-(49*a^3*e^6*(-a^9*c^3)
^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5
 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e
^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^
3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*31902i + a^11*c^6*d^8*e^9*x*(-(49*a^
3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 1
26*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2)
- 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2
*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*52008i + a^12*c^5*d^
6*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*
a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*
(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d
^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*422
38i + a^13*c^4*d^4*e^13*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9
*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 -
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81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^1
3*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^
4*e^4))^(1/2)*10924i - a^14*c^3*d^2*e^15*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) -
 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a
^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9
*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^
2 + 6*a^11*c^2*d^4*e^4))^(1/2)*5694i - a^12*c^9*d^17*e^2*x*(-(49*a^3*e^6*(-
a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*216i - a^13*c^8*d^15*e^4*x*(
-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^
5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)
^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^1
2*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*700i - a^14*c
^7*d^13*e^6*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4
*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*
c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2
)*808i + a^15*c^6*d^11*e^8*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-
a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3
 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(
a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2
*d^4*e^4))^(3/2)*778i + a^16*c^5*d^9*e^10*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2)
- 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*
a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^
9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e
^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3224i + a^17*c^4*d^7*e^12*x*(-(49*a^3*e^6*(
-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*
c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^
2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 +
4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3460i + a^18*c^3*d^5*e^14*x
*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*
d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^
3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a
^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*1384i - a^1
9*c^2*d^3*e^16*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/
2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*
d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a
^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(
3/2)*57i - a^17*c^8*d^18*e^3*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*
(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e
^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))
/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c
^2*d^4*e^4))^(5/2)*14i - a^18*c^7*d^16*e^5*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2)
 - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124
*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a
^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*
e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^19*c^6*d^14*e^7*x*(-(49*a^3*e^6*(-
a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i - a^20*c^5*d^12*e^9*x*(-
(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5
*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^
(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12
*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^21*c^4
*d^10*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) +
 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*
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e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c
^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)
*28i + a^22*c^3*d^8*e^13*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^
9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 -
81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^
13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d
^4*e^4))^(5/2)*56i + a^23*c^2*d^6*e^15*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 2
5*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6
*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c
^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2
+ 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^20*c*d*e^18*x*(-(49*a^3*e^6*(-a^9*c^3)
^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5
+ 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e
^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^
3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i) + (-a^9*c^3)^(1/2)*(3125*c^9*d
^16 + 21952*a^8*c*e^16 + 3000*a*c^8*d^14*e^2 - 77435*a^2*c^7*d^12*e^4 - 242
104*a^3*c^6*d^10*e^6 - 127665*a^4*c^5*d^8*e^8 + 240064*a^5*c^4*d^6*e^10 + 1
18199*a^6*c^3*d^4*e^12 - 130368*a^7*c^2*d^2*e^14)*(a^25*d^2*e^19*x*(-(49*a^
3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 1
26*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2)
- 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2
*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i - a^15*c^2*e^17*x
*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*
d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^
3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a
^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*3136i - a^1
1*c^10*d^19*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4
e^2(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*
c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2
)*25i - a^16*c^9*d^20*e*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9
*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 -
81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^1
3*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^
4*e^4))^(5/2)*2i + a^24*c*d^4*e^17*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^
3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3
*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^
(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*
a^11*c^2*d^4*e^4))^(5/2)*14i + a^8*c^9*d^14*e^3*x*(-(49*a^3*e^6*(-a^9*c^3)^
(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5
+ 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^
4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3
*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*1250i + a^9*c^8*d^12*e^5*x*(-(49*a^3*
e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126
*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) -
39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e
^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*9900i + a^10*c^7*d^10*
e^7*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5
*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a
^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8
+ 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*31902i
+ a^11*c^6*d^8*e^9*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3
)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a
*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^
8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^
4))^(1/2)*52008i + a^12*c^5*d^6*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*
c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c
^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3
)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 +

6*a^11*c^2*d^4*e^4))^(1/2)*42238i + a^13*c^4*d^4*e^13*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*10924i - a^14*c^3*d^2*e^15*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*5694i - a^12*c^9*d^17*e^2*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*216i - a^13*c^8*d^15*e^4*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*700i - a^14*c^7*d^13*e^6*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*808i + a^15*c^6*d^11*e^8*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*778i + a^16*c^5*d^9*e^10*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3224i + a^17*c^4*d^7*e^12*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3460i + a^18*c^3*d^5*e^14*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*1384i - a^19*c^2*d^3*e^16*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*57i - a^17*c^8*d^18*e^3*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i - a^18*c^7*d^16*e^5*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^19*c^6*d^14*e^7*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i - a^20*c^5*d^12*e^9*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2* d^4*e^4))^(5/2)*28i + a^21*c^4*d^10*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^22*c^3*d^8*e^13*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i + a^23*c^2*d^6*e^15*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^20*c*d*e^18*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i))/(9765625*a^9*c^21*d^32 + 481890304*a^25*c^5*e^32 + 159765625*a^10*c^20*d^30*e^2 + 1159031250*a^11*c^19*d^28*e^4 + 4879001250*a^12*c^18*d^26*e^6 + 13043411775*a^13*c^17*d^24*e^8 + 22507897839*a^14*c^16*d^22*e^10 + 23209461788*a^15*c^15*d^20*e^12 + 7790140604*a^16*c^14*d^18*e^14 - 15160518297*a^17*c^13*d^16*e^16 - 24964288057*a^18*c^12*d^14*e^18 - 11511478798*a^19*c^11*d^12*e^20 + 8613907074*a^20*c^10*d^10*e^22 + 11397074817*a^21*c^9*d^8*e^24 + 586708977*a^22*c^8*d^6*e^26 - 3576733440*a^23*c^7*d^4*e^28 - 521228288*a^24*c^6*d^2*e^30))*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(256*(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4)))^(1/2)*2i - atan(((11875*a^5*c^10*d^15*e - a^9*c^3*(72128*a^3*d*e^15 + 265655*c^3*d^7*e^9 - 76440*a*c^2*d^5*e^11 - 178585*a^2*c*d^3*e^13) + 68800*a^6*c^9*d^13*e^3 + 89403*a^7*c^8*d^11*e^5 - 126488*a^8*c^7*d^9*e^7)*(a^25*d^2*e^19*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i - a^15*c^2*e^17*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*3136i - a^11*c^10*d^19*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*25i - a^16*c^9*d^20*e*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i + a^24*c*d^4*e^17*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i + a^8*c^9*d^14*e^3*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*1250i + a^9*c^8*d^12*e^5*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 +

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 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*9900i +
 a^10*c^7*d^10*e^7*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)
^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*
c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8
 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4
))^(1/2)*31902i + a^11*c^6*d^8*e^9*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^
3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3
*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^
(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*
a^11*c^2*d^4*e^4))^(1/2)*52008i + a^12*c^5*d^6*e^11*x*(-(25*c^3*d^6*(-a^9*c
^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*
e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^
2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10
*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*42238i + a^13*c^4*d^4*e^13*x*(-(2
5*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e
 + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1
/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c
*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*10924i - a^14*c^
3*d^2*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) +
 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*
e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c
^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)
*5694i - a^12*c^9*d^17*e^2*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-
a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3
 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(
a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2
*d^4*e^4))^(3/2)*216i - a^13*c^8*d^15*e^4*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2)
- 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*
a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^
9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e
^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*700i - a^14*c^7*d^13*e^6*x*(-(25*c^3*d^6*(-
a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*808i + a^15*c^6*d^11*e^8*x*(
-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^
5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)
^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^1
2*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*778i + a^16*c
^5*d^9*e^10*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4
*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*
c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2
)*3224i + a^17*c^4*d^7*e^12*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(
-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^
3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/
(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^
2*d^4*e^4))^(3/2)*3460i + a^18*c^3*d^5*e^14*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2
) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 12
4*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-
a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6
*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*1384i - a^19*c^2*d^3*e^16*x*(-(25*c^3*d^6
*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^
7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*
a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6
+ 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*57i - a^17*c^8*d^18*e^3*x
*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*
d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^
3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a
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^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i - a^18*c^7*d^16*e^5*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^19*c^6*d^14*e^7*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i - a^20*c^5*d^12*e^9*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^21*c^4*d^10*e^11*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^22*c^3*d^8*e^13*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i + a^23*c^2*d^6*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^20*c*d*e^18*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i) - (-a^9*c^3)^(1/2)*(3125*c^9*d^16 + 21952*a^8*c*e^16 + 3000*a*c^8*d^14*e^2 - 77435*a^2*c^7*d^12*e^4 - 242104*a^3*c^6*d^10*e^6 - 127665*a^4*c^5*d^8*e^8 + 240064*a^5*c^4*d^6*e^10 + 118199*a^6*c^3*d^4*e^12 - 130368*a^7*c^2*d^2*e^14)*(a^25*d^2*e^19*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i - a^15*c^2*e^17*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*3136i - a^11*c^10*d^19*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*25i - a^16*c^9*d^20*e*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i + a^24*c*d^4*e^17*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i + a^8*c^9*d^14*e^3*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*1250i + a^9*c^8*d^12*e^5*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3* e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*9900i + a^10*c^7*d^10*e^7*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*31902i + a^11*c^6*d^8*e^9*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*52008i + a^12*c^5*d^6*e^11*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*42238i + a^13*c^4*d^4*e^13*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*10924i - a^14*c^3*d^2*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*5694i - a^12*c^9*d^17*e^2*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*216i - a^13*c^8*d^15*e^4*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*700i - a^14*c^7*d^13*e^6*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*808i + a^15*c^6*d^11*e^8*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*778i + a^16*c^5*d^9*e^10*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3224i + a^17*c^4*d^7*e^12*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3460i + a^18*c^3*d^5*e^14*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*1384i - a^19*c^2*d^3*e^16*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*57i - a^17*c^8*d^18*e^3*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*

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(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e
^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))
/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c
^2*d^4*e^4))^(5/2)*14i - a^18*c^7*d^16*e^5*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2)
 - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124
*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a
^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*
e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^19*c^6*d^14*e^7*x*(-(25*c^3*d^6*(-
a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i - a^20*c^5*d^12*e^9*x*(-
(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5
*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^
(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12
*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^21*c^4
*d^10*e^11*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) +
 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*
e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c
^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)
*28i + a^22*c^3*d^8*e^13*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^
9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 +
 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^
13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d
^4*e^4))^(5/2)*56i + a^23*c^2*d^6*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 4
9*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6
*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c
^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2
+ 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^20*c*d*e^18*x*(-(25*c^3*d^6*(-a^9*c^3)
^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5
 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e
^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^
3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i))/(9765625*a^9*c^21*d^32 + 4818
90304*a^25*c^5*e^32 + 159765625*a^10*c^20*d^30*e^2 + 1159031250*a^11*c^19*d
^28*e^4 + 4879001250*a^12*c^18*d^26*e^6 + 13043411775*a^13*c^17*d^24*e^8 +
22507897839*a^14*c^16*d^22*e^10 + 23209461788*a^15*c^15*d^20*e^12 + 7790140
604*a^16*c^14*d^18*e^14 - 15160518297*a^17*c^13*d^16*e^16 - 24964288057*a^1
8*c^12*d^14*e^18 - 11511478798*a^19*c^11*d^12*e^20 + 8613907074*a^20*c^10*d
^10*e^22 + 11397074817*a^21*c^9*d^8*e^24 + 586708977*a^22*c^8*d^6*e^26 - 35
76733440*a^23*c^7*d^4*e^28 - 521228288*a^24*c^6*d^2*e^30))*(-(25*c^3*d^6*(-
a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(256*(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^
6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4)))^(1/2)*2i - (atan((a^9*e^3*x*
(-d^3*e^9)^(5/2)*4096i - a^3*c^6*d^15*x*(-d^3*e^9)^(3/2)*26804i + c^9*d^24*
e^3*x*(-d^3*e^9)^(1/2)*625i - a^4*c^5*d^13*e^2*x*(-d^3*e^9)^(3/2)*24831i -
a^5*c^4*d^11*e^4*x*(-d^3*e^9)^(3/2)*8214i + a^6*c^3*d^9*e^6*x*(-d^3*e^9)^(3
/2)*13471i + a^7*c^2*d^7*e^8*x*(-d^3*e^9)^(3/2)*16128i + a^2*c^7*d^20*e^7*x
*(-d^3*e^9)^(1/2)*15951i + a*c^8*d^22*e^5*x*(-d^3*e^9)^(1/2)*4950i)/(4096*a
^9*d^8*e^25 + 625*c^9*d^26*e^7 + 4950*a*c^8*d^24*e^9 + 15951*a^2*c^7*d^22*e
^11 + 26804*a^3*c^6*d^20*e^13 + 24831*a^4*c^5*d^18*e^15 + 8214*a^5*c^4*d^16
*e^17 - 13471*a^6*c^3*d^14*e^19 - 16128*a^7*c^2*d^12*e^21))*(-d^3*e^9)^(1/2
)*1i)/(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.258 $\int \frac{1}{x^4(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=751

$$\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(2ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\right.}{16\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)}$$

```
[Out] -1/3/a^2/d/x^3+e/a^2/d^2/x-1/4*c^2*x*(-e*x^2+d)/a^2/(a*e^2+c*d^2)/(c*x^4+a)
+e^(11/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/(a*e^2+c*d^2)^2-1/4*c^(5/4)*(2*
a*e^2+c*d^2)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^
(11/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/4*c^(5/4)*(2*a*e^2+c*d^2)*arctan(1+c^(1/4)
*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)^2*2^(1/2)
+1/8*c^(5/4)*(2*a*e^2+c*d^2)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1
/2))*(e*a^(1/2)+d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(5/4)*(2*
a*e^2+c*d^2)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d
*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/16*c^(5/4)*arctan(-1+c^(1/4)*x
*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)*2^(1/2)-1
/16*c^(5/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^
(11/4)/(a*e^2+c*d^2)*2^(1/2)+1/32*c^(5/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)*2^(1/2)-1/
32*c^(5/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d
*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1336, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{c^2x\left(d-ex^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(2ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\left(\sqrt{a}\,e+3\sqrt{c}\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/(3*a^2*d*x^3) + e/(a^2*d^2*x) - (c^2*x*(d - e*x^2))/(4*a^2*(c*d^2 + a*e^
2)*(a + c*x^4)) + (e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2 +
a*e^2)^2) + (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - S
qrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqr
t[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) -
(c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) + (c^(5/4)*(3*Sqrt[
c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
16*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*
d^2 + 2*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(11/4)*(
c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(11/4)*(c*d^
2 + a*e^2)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \int \left(\frac{1}{a^2dx^4} - \frac{e}{a^2d^2x^2} + \frac{e^6}{d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} - \frac{c^2\left(d-ex^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)^2} - \frac{c^2\,(}{a^2\,(}\right.$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} + \frac{e^6\int\frac{1}{d+ex^2}\,dx}{d^2\left(cd^2+ae^2\right)^2} - \frac{c^2\int\frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{a\left(cd^2+ae^2\right)} - \frac{\left(c^2\left(cd^2+2ae^2\right)\right)\int\frac{d-ex^2}{a+cx^4}}{a^2\left(cd^2+ae^2\right)^2}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2} + \frac{c^2\int\frac{-3d+ex^2}{a+cx^4}}{4a^2\left(cd^2+a\right.}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2} - \frac{\left(c\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right.\right.}{8a^2\,(}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2} + \frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+\right.}{}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2} + \frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-\right.}{}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2} + \frac{c^{5/4}\left(\frac{3\sqrt{c}d}{\sqrt{a}}-\right.}{8\sqrt{2}\,}$$

Mathematica [A] time = 0.39, size = 513, normalized size = 0.68

$$\frac{1}{96}\left(\frac{3\sqrt{2}c^{5/4}\left(9a^{3/2}e^3+5\sqrt{a}cd^2e+11a\sqrt{c}de^2+7c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{11/4}\left(ae^2+cd^2\right)^2} - \frac{3\sqrt{2}c^{5/4}\left(9a^{3/2}e^3+\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-32/(a^2*d*x^3) + (96*e)/(a^2*d^2*x) - (24*c^2*x*(d - e*x^2))/(a^2*(c*d^2
+ a*e^2)*(a + c*x^4)) + (96*e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*
(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 - 5*Sqrt[a]*c*d^2*e
+ 11*a*Sqrt[c]*d*e^2 - 9*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(-7*c^(3/2)*d^3 + 5*S
qrt[a]*c*d^2*e - 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (3*Sqrt[2]*c^(5/4)*(7*c^(
3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(11/4)*(c*d^2 + a*e^2)^2
) - (3*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*
e^2 + 9*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/(a^(11/4)*(c*d^2 + a*e^2)^2))/96
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.50, size = 628, normalized size = 0.84

$$-\frac{\left(7\,\left(ac^3\right)^{\frac{1}{4}}c^3d^3+11\,\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-5\,\left(ac^3\right)^{\frac{3}{4}}cd^2e-9\,\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^3c^3d^4+2\,\sqrt{2}\,a^4c^2d^2e^2+\sqrt{2}\,a^5ce^4\right)}-\frac{\left(7\,\left(ac^3\right)^{\frac{1}{4}}c^3d^3+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] -1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 - 5*(a*c^3)^(3/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 - 5*(a*c^3)^(3/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)^(3/4)*a*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(11/2)/((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4)*sqrt(d)) + 1/4*(c^2*x^3*e - c^2*d*x)/((a^2*c*d^2 + a^3*e^2)*(c*x^4 + a)) + 1/3*(3*x^2*e - d)/(a^2*d^2*x^3)

maple [A] time = 0.02, size = 932, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x)

[Out] -1/3/a^2/d/x^3+e/a^2/d^2/x+1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^3*e^3+1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x^3*e*d^2-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x*d*e^2-1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x*d^3-11/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^2-7/16*c^3/(a*e^2+c*d^2)^2/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3-11/32*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^2-7/32*c^3/(a*e^2+c*d^2)^2/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3-11/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2-7/16*c^3/(a*e^2+c*d^2)^2/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3+9/32*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^3+5/32*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e+9/16*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^3+5/16*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e+9/16*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^3

```
+5/16*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x+1)*d^2*e+1/d^2*e^6/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x
)
```

maxima [A] time = 2.11, size = 543, normalized size = 0.72

$$\frac{e^{6}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^{2}d^{6}+2\,acd^{4}e^{2}+a^{2}d^{2}e^{4}\right)\sqrt{de}}-\frac{c^{2}\left(\frac{2\sqrt{2}\left(7c^{\frac{3}{2}}d^{3}-5\sqrt{a}cd^{2}e+11\,a\sqrt{c}de^{2}-9\,a^{\frac{3}{2}}e^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(7c^{\frac{3}{2}}d^{3}-5\sqrt{a}cd^{2}\cdots\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] e^6*arctan(e*x/sqrt(d*e))/((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4)*sqrt(d*e
)) - 1/32*c^2*(2*sqrt(2)*(7*c^(3/2)*d^3 - 5*sqrt(a)*c*d^2*e + 11*a*sqrt(c)*
d*e^2 - 9*a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^
(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*s
qrt(2)*(7*c^(3/2)*d^3 - 5*sqrt(a)*c*d^2*e + 11*a*sqrt(c)*d*e^2 - 9*a^(3/2)*
e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(7*c^(3/2)*d^
3 + 5*sqrt(a)*c*d^2*e + 11*a*sqrt(c)*d*e^2 + 9*a^(3/2)*e^3)*log(sqrt(c)*x^2
 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(7*c^(3
/2)*d^3 + 5*sqrt(a)*c*d^2*e + 11*a*sqrt(c)*d*e^2 + 9*a^(3/2)*e^3)*log(sqrt(
c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a^2*c^2*d
^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) + 1/12*(3*(5*c^2*d^2*e + 4*a*c*e^3)*x^6 - 4
*a*c*d^3 - 4*a^2*d*e^2 - (7*c^2*d^3 + 4*a*c*d*e^2)*x^4 + 12*(a*c*d^2*e + a^
2*e^3)*x^2)/((a^2*c^2*d^4 + a^3*c*d^2*e^2)*x^7 + (a^3*c*d^4 + a^4*d^2*e^2)*
x^3)
```

mupad [B] time = 5.22, size = 20828, normalized size = 27.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] atan(((x*(4917248*a^10*c^18*d^36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 230498
304*a^12*c^16*d^32*e^9 + 607559680*a^13*c^15*d^30*e^11 + 1026486272*a^14*c^
14*d^28*e^13 + 1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^
17 + 539500544*a^17*c^11*d^22*e^19 + 259409920*a^18*c^10*d^20*e^21 + 109709
312*a^19*c^9*d^18*e^23 + 34537472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^7*d^1
4*e^27) - ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 7
0*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e
^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 +
 a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)
))^(1/2)*((x*(1787297792*a^19*c^13*d^31*e^12 - 147587072*a^15*c^17*d^39*e^4
 - 698089472*a^16*c^16*d^37*e^6 - 1660157952*a^17*c^15*d^35*e^8 - 158806835
2*a^18*c^14*d^33*e^10 - 12845056*a^14*c^18*d^41*e^2 + 7839678464*a^20*c^12*
d^29*e^14 + 11879841792*a^21*c^11*d^27*e^16 + 10631249920*a^22*c^10*d^25*e^
18 + 6274940928*a^23*c^9*d^23*e^20 + 2652110848*a^24*c^8*d^21*e^22 + 891027
456*a^25*c^7*d^19*e^24 + 234881024*a^26*c^6*d^17*e^26 + 33554432*a^27*c^5*d
^15*e^28) + ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) +
 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4
*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8
```

+ a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^
4)))^(1/2)*(x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2)
+ 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d
^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e
^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*
e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 1342177280*a^21*c^15*d^40*e^5
+ 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^23*c^13*d^36*e^9 + 22145925
120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11*d^32*e^13 - 17716740096*a^2
7*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 14763950080*a^29*c^7*d^2
4*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177280*a^31*c^5*d^20*e^25 - 13
4217728*a^32*c^4*d^18*e^27) + 29360128*a^17*c^17*d^42*e^2 + 239075328*a^18*
c^16*d^40*e^4 + 708837376*a^19*c^15*d^38*e^6 + 465567744*a^20*c^14*d^36*e^8
- 2726297600*a^21*c^13*d^34*e^10 - 9084862464*a^22*c^12*d^32*e^12 - 136147
10784*a^23*c^11*d^30*e^14 - 10745806848*a^24*c^10*d^28*e^16 - 2403336192*a^
25*c^9*d^26*e^18 + 3879731200*a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22
*e^22 + 2294284288*a^28*c^6*d^20*e^24 + 603979776*a^29*c^5*d^18*e^26 + 6710
8864*a^30*c^4*d^16*e^28))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^1
1*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 -
129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/
(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a
^13*c^2*d^4*e^4)))^(1/2) + 7225344*a^12*c^18*d^39*e^3 + 76972032*a^13*c^17*
d^37*e^5 + 367607808*a^14*c^16*d^35*e^7 + 1036910592*a^15*c^15*d^33*e^9 + 1
876983808*a^16*c^14*d^31*e^11 + 2115436544*a^17*c^13*d^29*e^13 + 1052803072
*a^18*c^12*d^27*e^15 - 848429056*a^19*c^11*d^25*e^17 - 2105458688*a^20*c^10
*d^23*e^19 - 1909030912*a^21*c^9*d^21*e^21 - 959037440*a^22*c^8*d^19*e^23 -
262144000*a^23*c^7*d^17*e^25 - 30408704*a^24*c^6*d^15*e^27))*((81*a^3*e^6*
(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a
^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) -
31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*
c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*1i + (x*(49172
48*a^10*c^18*d^36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 230498304*a^12*c^16*d
^32*e^9 + 607559680*a^13*c^15*d^30*e^11 + 1026486272*a^14*c^14*d^28*e^13 +
1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^17 + 539500544*
a^17*c^11*d^22*e^19 + 259409920*a^18*c^10*d^20*e^21 + 109709312*a^19*c^9*d^
18*e^23 + 34537472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^7*d^14*e^27) - ((81*
a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e
+ 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^
(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 +
4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*((x*(1
787297792*a^19*c^13*d^31*e^12 - 147587072*a^15*c^17*d^39*e^4 - 698089472*a^
16*c^16*d^37*e^6 - 1660157952*a^17*c^15*d^35*e^8 - 1588068352*a^18*c^14*d^3
3*e^10 - 12845056*a^14*c^18*d^41*e^2 + 7839678464*a^20*c^12*d^29*e^14 + 118
79841792*a^21*c^11*d^27*e^16 + 10631249920*a^22*c^10*d^25*e^18 + 6274940928
*a^23*c^9*d^23*e^20 + 2652110848*a^24*c^8*d^21*e^22 + 891027456*a^25*c^7*d^
19*e^24 + 234881024*a^26*c^6*d^17*e^26 + 33554432*a^27*c^5*d^15*e^28) - ((8
1*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5
*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5
)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8
+ 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(293
60128*a^17*c^17*d^42*e^2 - x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-
a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^
3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2
))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 +
6*a^13*c^2*d^4*e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 1342177280*a^21
*c^15*d^40*e^5 + 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^23*c^13*d^36
*e^9 + 22145925120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11*d^32*e^13 -
17716740096*a^27*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 147639500
80*a^29*c^7*d^24*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177280*a^31*c^5
*d^20*e^25 - 134217728*a^32*c^4*d^18*e^27) + 239075328*a^18*c^16*d^40*e^4 +

708837376*a^19*c^15*d^38*e^6 + 465567744*a^20*c^14*d^36*e^8 - 2726297600*a^21*c^13*d^34*e^10 - 9084862464*a^22*c^12*d^32*e^12 - 13614710784*a^23*c^11*d^30*e^14 - 10745806848*a^24*c^10*d^28*e^16 - 2403336192*a^25*c^9*d^26*e^18 + 3879731200*a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22*e^22 + 2294284288*a^28*c^6*d^20*e^24 + 603979776*a^29*c^5*d^18*e^26 + 67108864*a^30*c^4*d^16*e^28))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) - 7225344*a^12*c^18*d^39*e^3 - 76972032*a^13*c^17*d^37*e^5 - 367607808*a^14*c^16*d^35*e^7 - 1036910592*a^15*c^15*d^33*e^9 - 1876983808*a^16*c^14*d^31*e^11 - 2115436544*a^17*c^13*d^29*e^13 - 1052803072*a^18*c^12*d^27*e^15 + 848429056*a^19*c^11*d^25*e^17 + 2105458688*a^20*c^10*d^23*e^19 + 1909030912*a^21*c^9*d^21*e^21 + 959037440*a^22*c^8*d^19*e^23 + 262144000*a^23*c^7*d^17*e^25 + 30408704*a^24*c^6*d^15*e^27))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*1i)/((x*(4917248*a^10*c^18*d^36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 230498304*a^12*c^16*d^32*e^9 + 607559680*a^13*c^15*d^30*e^11 + 1026486272*a^14*c^14*d^28*e^13 + 1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^17 + 539500544*a^17*c^11*d^22*e^19 + 259409920*a^18*c^10*d^20*e^21 + 109709312*a^19*c^9*d^18*e^23 + 34537472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^7*d^14*e^27) - ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*((x*(1787297792*a^19*c^13*d^31*e^12 - 147587072*a^15*c^17*d^39*e^4 - 698089472*a^16*c^16*d^37*e^6 - 1660157952*a^17*c^15*d^35*e^8 - 1588068352*a^18*c^14*d^33*e^10 - 12845056*a^14*c^18*d^41*e^2 + 7839678464*a^20*c^12*d^29*e^14 + 11879841792*a^21*c^11*d^27*e^16 + 10631249920*a^22*c^10*d^25*e^18 + 6274940928*a^23*c^9*d^23*e^20 + 2652110848*a^24*c^8*d^21*e^22 + 891027456*a^25*c^7*d^19*e^24 + 234881024*a^26*c^6*d^17*e^26 + 33554432*a^27*c^5*d^15*e^28) - ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(29360128*a^17*c^17*d^42*e^2 - x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 1342177280*a^21*c^15*d^40*e^5 + 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^23*c^13*d^36*e^9 + 22145925120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11*d^32*e^13 - 17716740096*a^27*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 14763950080*a^29*c^7*d^24*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177280*a^31*c^5*d^20*e^25 - 134217728*a^32*c^4*d^18*e^27) + 239075328*a^18*c^16*d^40*e^4 + 708837376*a^19*c^15*d^38*e^6 + 465567744*a^20*c^14*d^36*e^8 - 2726297600*a^21*c^13*d^34*e^10 - 9084862464*a^22*c^12*d^32*e^12 - 13614710784*a^23*c^11*d^30*e^14 - 10745806848*a^24*c^10*d^28*e^16 - 2403336192*a^25*c^9*d^26*e^18 + 3879731200*a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22*e^22 + 2294284288*a^28*c^6*d^20*e^24 + 603979776*a^29*c^5*d^18*e^26 + 67108864*a^30*c^4*d^16*e^28))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) - 7225344*a^12*c^18*d^39*e^3 - 76972032*a^13*c^17*d^37*e^5 - 367607808*a^14*c^16*d^35*e^7 - 1036910592*a^15*c^15*d^33*e^9 - 1876983808*a^16*c^14*d^31*e^11

- 2115436544*a^17*c^13*d^29*e^13 - 1052803072*a^18*c^12*d^27*e^15 + 8484290 56*a^19*c^11*d^25*e^17 + 2105458688*a^20*c^10*d^23*e^19 + 1909030912*a^21*c ^9*d^21*e^21 + 959037440*a^22*c^8*d^19*e^23 + 262144000*a^23*c^7*d^17*e^25 + 30408704*a^24*c^6*d^15*e^27))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6 *(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3 *e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) - (x*(4917248*a^10*c^18*d^36*e^5 + 50677760* a^11*c^17*d^34*e^7 + 230498304*a^12*c^16*d^32*e^9 + 607559680*a^13*c^15*d^3 0*e^11 + 1026486272*a^14*c^14*d^28*e^13 + 1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^17 + 539500544*a^17*c^11*d^22*e^19 + 259409920*a ^18*c^10*d^20*e^21 + 109709312*a^19*c^9*d^18*e^23 + 34537472*a^20*c^8*d^16* e^25 + 5308416*a^21*c^7*d^14*e^27) - ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^ 3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^ 4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c ^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^ 6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*((x*(1787297792*a^19*c^13*d^31*e^12 - 1 47587072*a^15*c^17*d^39*e^4 - 698089472*a^16*c^16*d^37*e^6 - 1660157952*a^1 7*c^15*d^35*e^8 - 1588068352*a^18*c^14*d^33*e^10 - 12845056*a^14*c^18*d^41* e^2 + 7839678464*a^20*c^12*d^29*e^14 + 11879841792*a^21*c^11*d^27*e^16 + 10 631249920*a^22*c^10*d^25*e^18 + 6274940928*a^23*c^9*d^23*e^20 + 2652110848* a^24*c^8*d^21*e^22 + 891027456*a^25*c^7*d^19*e^24 + 234881024*a^26*c^6*d^17 *e^26 + 33554432*a^27*c^5*d^15*e^28) + ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49* c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7* c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11 *c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3* d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 4 9*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^ 7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^ 11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^ 3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 134 2177280*a^21*c^15*d^40*e^5 + 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^ 23*c^13*d^36*e^9 + 22145925120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11* d^32*e^13 - 17716740096*a^27*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 14763950080*a^29*c^7*d^24*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177 280*a^31*c^5*d^20*e^25 - 134217728*a^32*c^4*d^18*e^27) + 29360128*a^17*c^17 *d^42*e^2 + 239075328*a^18*c^16*d^40*e^4 + 708837376*a^19*c^15*d^38*e^6 + 4 65567744*a^20*c^14*d^36*e^8 - 2726297600*a^21*c^13*d^34*e^10 - 9084862464*a ^22*c^12*d^32*e^12 - 13614710784*a^23*c^11*d^30*e^14 - 10745806848*a^24*c^1 0*d^28*e^16 - 2403336192*a^25*c^9*d^26*e^18 + 3879731200*a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22*e^22 + 2294284288*a^28*c^6*d^20*e^24 + 60397977 6*a^29*c^5*d^18*e^26 + 67108864*a^30*c^4*d^16*e^28))*((81*a^3*e^6*(-a^11*c^ 5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d* e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c* d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) + 7225344*a^12*c^18*d^3 9*e^3 + 76972032*a^13*c^17*d^37*e^5 + 367607808*a^14*c^16*d^35*e^7 + 103691 0592*a^15*c^15*d^33*e^9 + 1876983808*a^16*c^14*d^31*e^11 + 2115436544*a^17* c^13*d^29*e^13 + 1052803072*a^18*c^12*d^27*e^15 - 848429056*a^19*c^11*d^25* e^17 - 2105458688*a^20*c^10*d^23*e^19 - 1909030912*a^21*c^9*d^21*e^21 - 959 037440*a^22*c^8*d^19*e^23 - 262144000*a^23*c^7*d^17*e^25 - 30408704*a^24*c^ 6*d^15*e^27))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d ^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e ^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4* e^4)))^(1/2) + 4917248*a^10*c^16*d^30*e^10 + 40843264*a^11*c^15*d^28*e^12 + 147507200*a^12*c^14*d^26*e^14 + 302962688*a^13*c^13*d^24*e^16 + 387512320* a^14*c^12*d^22*e^18 + 316418048*a^15*c^11*d^20*e^20 + 161224704*a^16*c^10*d ^18*e^22 + 46909440*a^17*c^9*d^16*e^24 + 5971968*a^18*c^8*d^14*e^26))*((81*

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a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e
 + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^
(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 +
 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*2i - (
1/(3*a*d) - (e*x^2)/(a*d^2) + (x^4*(7*c^2*d^2 + 4*a*c*e^2))/(12*a^2*d*(a*e^
2 + c*d^2)) - (c*x^6*(4*a*e^3 + 5*c*d^2*e))/(4*a^2*d^2*(a*e^2 + c*d^2)))/(a
*x^3 + c*x^7) + atan(((a^11*c^5*(156627*c^2*d^6*e^12 - 245952*a^2*d^2*e^16
+ 324032*a*c*d^4*e^14) - 16807*a^5*c^13*d^18 + 46656*a^14*c^4*e^18 + 24696*
a^6*c^12*d^16*e^2 + 455609*a^7*c^11*d^14*e^4 + 856936*a^8*c^10*d^12*e^6 - 2
7429*a^9*c^9*d^10*e^8 - 805344*a^10*c^8*d^8*e^10)*(a^13*c^11*d^21*x*((49*c^
3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e +
 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1
/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14
*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*49i + a^17*c^3
*e^19*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*
a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2
*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c
^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)
*5184i + a^28*d^4*e^19*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11
*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 +
129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(
a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^
2*d^4*e^4))^(5/2)*2i - a^19*c^9*d^22*e*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 8
1*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^
7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^
11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6
*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*2i + a^27*c*d^6*e^17*x*((49*c^3*d^6*(-a^1
1*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^
3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^
2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6
+ 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*14i + a^9*c^11*d^16*e^3*x
*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5
*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11
*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8
+ 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*4802i
+ a^10*c^10*d^14*e^5*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c
^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 12
9*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^
15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*
d^4*e^4))^(1/2)*35084i + a^11*c^9*d^12*e^7*x*((49*c^3*d^6*(-a^11*c^5)^(1/2)
 - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 23
6*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*
(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3
*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*105438i + a^12*c^8*d^10*e^9*x*((49*c^
3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e +
 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1
/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14
*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*166952i + a^13
*c^7*d^8*e^11*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/
2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2
*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8
+ a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4
))^(1/2)*150174i + a^14*c^6*d^6*e^13*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*
a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*
c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11
*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e
^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*82444i + a^15*c^5*d^4*e^15*x*((49*c^3*d^6*(
-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^
8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 3
```

1*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*
e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*37058i + a^16*c^4*d^2
*e^17*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*
a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2
*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c
^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)
*18176i + a^14*c^10*d^19*e^2*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*
(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*
e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1
/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a
^13*c^2*d^4*e^4))^(3/2)*416i + a^15*c^9*d^17*e^4*x*((49*c^3*d^6*(-a^11*c^5)
^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^
5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^
2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^
12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*1268i + a^16*c^8*d^15*e^6*x*((4
9*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5
*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5
)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*
a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*1232i - a^
17*c^7*d^13*e^8*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(
1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c
^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^
8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e
^4))^(3/2)*1858i - a^18*c^6*d^11*e^10*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81
*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7
*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^1
1*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*
e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*6208i - a^19*c^5*d^9*e^12*x*((49*c^3*d^6*(
-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^
8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 3
1*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*
e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*6940i - a^20*c^4*d^7*
e^14*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a
^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*
(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^
4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*
4016i - a^21*c^3*d^5*e^16*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a
^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3
 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2)
)/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13
*c^2*d^4*e^4))^(3/2)*1479i - a^22*c^2*d^3*e^18*x*((49*c^3*d^6*(-a^11*c^5)^(
1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5
+ 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*
e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12
*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*512i - a^20*c^8*d^20*e^3*x*((49*c
^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e
+ 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(
1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^1
4*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*14i - a^21*c^
7*d^18*e^5*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2)
+ 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^
4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a
^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^
(5/2)*40i - a^22*c^6*d^16*e^7*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6
*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3
*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(
1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*
a^13*c^2*d^4*e^4))^(5/2)*56i - a^23*c^5*d^14*e^9*x*((49*c^3*d^6*(-a^11*c^5)
^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^

5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^
2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^
12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*28i + a^24*c^4*d^12*e^11*x*((49
*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*
e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)
^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a
^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*28i + a^25*
c^3*d^10*e^13*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/
2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2
*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8
+ a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4
))^(5/2)*56i + a^26*c^2*d^8*e^15*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*
e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*
d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5
)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 +
 6*a^13*c^2*d^4*e^4))^(5/2)*40i - a^23*c*d*e^20*x*((49*c^3*d^6*(-a^11*c^5)^
(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5
 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2
e^4(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^1
2*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*128i) - (-a^11*c^5)^(1/2)*(69629
*c^10*d^17*e + 286944*a*c^9*d^15*e^3 + 150336*a^8*c^2*d*e^17 + 110645*a^2*c
^8*d^13*e^5 - 770024*a^3*c^7*d^11*e^7 - 606089*a^4*c^6*d^9*e^9 + 566984*a^5
*c^5*d^7*e^11 + 157207*a^6*c^4*d^5*e^13 - 327104*a^7*c^3*d^3*e^15)*(a^13*c^
11*d^21*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 7
0*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e
^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11
*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/
2)*49i + a^17*c^3*e^19*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11
*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 +
129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(
a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^
2*d^4*e^4))^(1/2)*5184i + a^28*d^4*e^19*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) -
81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a
^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a
^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^
6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*2i - a^19*c^9*d^22*e*x*((49*c^3*d^6*(-a^
11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c
^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a
^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6
 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*2i + a^27*c*d^6*e^17*x*(
(49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d
^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c
^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 +
4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*14i + a^
9*c^11*d^16*e^3*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(
1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c
^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^
8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e
^4))^(1/2)*4802i + a^10*c^10*d^14*e^5*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81
*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7
*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^1
1*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*
e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*35084i + a^11*c^9*d^12*e^7*x*((49*c^3*d^6*
(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a
^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) +
31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2
*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*105438i + a^12*c^8*d
^10*e^9*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 7
0*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e

^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11
*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/
2)*166952i + a^13*c^7*d^8*e^11*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^
6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^
3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^
(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6
*a^13*c^2*d^4*e^4))^(1/2)*150174i + a^14*c^6*d^6*e^13*x*((49*c^3*d^6*(-a^11
*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3
*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2
*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 +
 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*82444i + a^15*c^5*d^4*e^15
x((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c
^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^
11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^
8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*3705
8i + a^16*c^4*d^2*e^17*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11
*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 +
129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(
a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^
2*d^4*e^4))^(1/2)*18176i + a^14*c^10*d^19*e^2*x*((49*c^3*d^6*(-a^11*c^5)^(1
/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 +
 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e
^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*
c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*416i + a^15*c^9*d^17*e^4*x*((49*c^
3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e +
 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1
/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14
*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*1268i + a^16*c
^8*d^15*e^6*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2)
 + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d
^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 +
a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))
^(3/2)*1232i - a^17*c^7*d^13*e^8*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*
e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*
d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5
)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 +
 6*a^13*c^2*d^4*e^4))^(3/2)*1858i - a^18*c^6*d^11*e^10*x*((49*c^3*d^6*(-a^1
1*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^
3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^
2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6
+ 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*6208i - a^19*c^5*d^9*e^12
x((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c
^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^
11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^
8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*6940
i - a^20*c^4*d^7*e^14*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*
c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 1
29*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a
^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2
*d^4*e^4))^(3/2)*4016i - a^21*c^3*d^5*e^16*x*((49*c^3*d^6*(-a^11*c^5)^(1/2)
 - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 23
6*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*
(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3
*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*1479i - a^22*c^2*d^3*e^18*x*((49*c^3*
d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 1
98*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2
) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c
*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*512i - a^20*c^8*
d^20*e^3*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) +

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70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*
e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^1
1*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5
/2)*14i - a^21*c^7*d^18*e^5*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(
-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e
^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/
2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^
13*c^2*d^4*e^4))^(5/2)*40i - a^22*c^6*d^16*e^7*x*((49*c^3*d^6*(-a^11*c^5)^(
1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5
 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*
e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12
*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*56i - a^23*c^5*d^14*e^9*x*((49*c^
3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e +
 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1
/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14
*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*28i + a^24*c^4
*d^12*e^11*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2)
 + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^
4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a
^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^
(5/2)*28i + a^25*c^3*d^10*e^13*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^
6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^
3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^
(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6
*a^13*c^2*d^4*e^4))^(5/2)*56i + a^26*c^2*d^8*e^15*x*((49*c^3*d^6*(-a^11*c^5
)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e
^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d
^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a
^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*40i - a^23*c*d*e^20*x*((49*c^3
*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e +
198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/
2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*
c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*128i))/(2824752
49*a^10*c^26*d^36 + 2176782336*a^28*c^8*e^36 + 4018066297*a^11*c^25*d^34*e^
2 + 25254299042*a^12*c^24*d^32*e^4 + 91443453570*a^13*c^23*d^30*e^6 + 20709
3177767*a^14*c^22*d^28*e^8 + 292503608847*a^15*c^21*d^26*e^10 + 22503434162
8*a^16*c^20*d^24*e^12 + 22083537020*a^17*c^19*d^22*e^14 - 108969417553*a^18
*c^18*d^20*e^16 - 43670306041*a^19*c^17*d^18*e^18 + 58023955010*a^20*c^16*d
^16*e^20 + 18862267874*a^21*c^15*d^14*e^22 - 60676266279*a^22*c^14*d^12*e^2
4 - 33348619375*a^23*c^13*d^10*e^26 + 20433166080*a^24*c^12*d^8*e^28 + 9487
311616*a^25*c^11*d^6*e^30 - 7622553600*a^26*c^10*d^4*e^32 - 349360128*a^27*
c^9*d^2*e^34))*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2
) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*
d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*
e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4
*e^4)))^(1/2)*2i + (atan((a^11*e^3*x*(-d^5*e^11)^(5/2)*4096i - a^4*c^7*d^19
*x*(-d^5*e^11)^(3/2)*73519i + c^11*d^32*e^3*x*(-d^5*e^11)^(1/2)*2401i - a^5
*c^6*d^17*e^2*x*(-d^5*e^11)^(3/2)*34182i - a^6*c^5*d^15*e^4*x*(-d^5*e^11)^(
3/2)*15521i - a^7*c^4*d^13*e^6*x*(-d^5*e^11)^(3/2)*30208i - a^8*c^3*d^11*e^
8*x*(-d^5*e^11)^(3/2)*25344i + a^2*c^9*d^28*e^7*x*(-d^5*e^11)^(1/2)*52719i
+ a^3*c^8*d^26*e^9*x*(-d^5*e^11)^(1/2)*83476i + a*c^10*d^30*e^5*x*(-d^5*e^1
1)^(1/2)*17542i)/(4096*a^11*d^13*e^30 + 2401*c^11*d^35*e^8 + 17542*a*c^10*d
^33*e^10 + 52719*a^2*c^9*d^31*e^12 + 83476*a^3*c^8*d^29*e^14 + 73519*a^4*c^
7*d^27*e^16 + 34182*a^5*c^6*d^25*e^18 + 15521*a^6*c^5*d^23*e^20 + 30208*a^7
*c^4*d^21*e^22 + 25344*a^8*c^3*d^19*e^24))*(-d^5*e^11)^(1/2)*1i)/(c^2*d^9 +
 a^2*d^5*e^4 + 2*a*c*d^7*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.259 $\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=70

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)+1/4*(x^2+1)*(cos(2*arctan(x))^
2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/
(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1318, 220, 1699, 203}

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] -ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1318

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx - \frac{1}{2}\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$$

$$= \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

Mathematica [**C**] time = 0.10, size = 40, normalized size = 0.57

$$\sqrt[4]{-1}\left(\Pi\left(-i; i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right) - F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(-EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[-I, I*ArcS
inh[(-1)^(1/4)*x], -1])
```

fricas [**F**] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}\,x^2}{x^6+x^4+x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 1)*x^2/(x^6 + x^4 + x^2 + 1), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^4+1}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

maple [**C**] time = 0.07, size = 110, normalized size = 1.57

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\,\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}} + \frac{(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\,\text{EllipticPi}\left((-1)^{\frac{1}{4}}x, i, -\sqrt{-i}\,\,(-1)\right.}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(x^4+1)^(1/2),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I
*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^4+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(x^2+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)*(x^4 + 1)^(1/2)),x)
```

```
[Out] int(x^2/((x^2 + 1)*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] Integral(x**2/((x**2 + 1)*sqrt(x**4 + 1)), x)
```

3.260 $\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

```
[Out] 1/4*arctanh(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))^
2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/
(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1318, 220, 1699, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) - ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1318

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx = -\left(\frac{1}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx$$
$$= -\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

Mathematica [C] time = 0.09, size = 36, normalized size = 0.51

$$\sqrt[4]{-1}\left(F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-\Pi\left(i;\sin^{-1}\left((-1)^{3/4}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - EllipticPi[I, ArcSin[(
-1)^(3/4)*x], -1])
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{x^4+1}\,x^2}{x^6-x^4+x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^4 + 1)*x^2/(x^6 - x^4 + x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\sqrt{x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^2/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

maple [C] time = 0.02, size = 112, normalized size = 1.60

$$-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\;\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\;\text{EllipticPi}\left((-1)^{\frac{1}{4}}x, -i, -\sqrt{-i}\;(-1)\right.}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(x^4+1)^(1/2),x)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/
2)*EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)-(-1)^(3/4)*(-I*x^2+1)^(1/2)*(
```

```
I*x^2+1)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/
4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x^2/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\left(x^2-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
[Out] -int(x^2/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(x**4+1)**(1/2),x)
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

3.261 $\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=99

$$-\frac{x(1-x^2)}{2\sqrt{1-x^4}}+\frac{\sqrt{x^2+1}\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{1-x^4}}-\frac{\sqrt{x^2+1}\sqrt{1-x^2}\,E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{1-x^4}}$$

```
[Out] -1/2*x*(-x^2+1)/(-x^4+1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1
/2)/(-x^4+1)^(1/2)+EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/
2)
```

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1256, 471, 423, 424, 248, 221}

$$-\frac{x(1-x^2)}{2\sqrt{1-x^4}}+\frac{\sqrt{x^2+1}\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{1-x^4}}-\frac{\sqrt{x^2+1}\sqrt{1-x^2}\,E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] -(x*(1 - x^2))/(2*Sqrt[1 - x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[A
rcSin[x], -1])/(2*Sqrt[1 - x^4]) + (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticF[A
rcSin[x], -1])/Sqrt[1 - x^4]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1256

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&& !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(1+x^2\right)\sqrt{1-x^4}}\,dx &= \frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \frac{x^2}{\sqrt{1-x^2}\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{1-x^4}}\\
&= -\frac{x\left(1-x^2\right)}{2\sqrt{1-x^4}}+\frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx}{2\sqrt{1-x^4}}\\
&= -\frac{x\left(1-x^2\right)}{2\sqrt{1-x^4}}-\frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{1-x^4}}+\frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1-x^2}\,\sqrt{1+x^2}}\,dx}{\sqrt{1-x^4}}\\
&= -\frac{x\left(1-x^2\right)}{2\sqrt{1-x^4}}-\frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{1-x^4}}+\frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1-x^4}}\,dx}{\sqrt{1-x^4}}\\
&= -\frac{x\left(1-x^2\right)}{2\sqrt{1-x^4}}-\frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{1-x^4}}+\frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{1-x^4}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 46, normalized size = 0.46

$$\frac{1}{2}\left(-\frac{x}{\sqrt{1-x^4}}+\frac{x^3}{\sqrt{1-x^4}}+2F\left(\sin^{-1}(x)\middle|-1\right)-E\left(\sin^{-1}(x)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] (-(x/Sqrt[1 - x^4]) + x^3/Sqrt[1 - x^4] - EllipticE[ArcSin[x], -1] + 2*Elli
pticF[ArcSin[x], -1])/2
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}\,x^2}{x^6+x^4-x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 1)*x^2/(x^6 + x^4 - x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x^4+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

maple [A] time = 0.02, size = 96, normalized size = 0.97

$$-\frac{\left(-x^{2}+1\right)x}{2\sqrt{\left(-x^{2}+1\right)\left(x^{2}+1\right)}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}\left(x,i\right)}{2\sqrt{-x^{4}+1}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\left(-\operatorname{EllipticE}\left(x,i\right)+\operatorname{EllipticF}\left(x\right.\right.}{2\sqrt{-x^{4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(-x^4+1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)-1/2*(-x^2+1)
*x/((-x^2+1)*(x^2+1))^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)
*(EllipticF(x,I)-EllipticE(x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{-x^{4}+1}\left(x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2}}{\left(x^{2}+1\right)\sqrt{1-x^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)*(1 - x^4)^(1/2)),x)
```

```
[Out] int(x^2/((x^2 + 1)*(1 - x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{-\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)}\left(x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(-x**4+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

3.262 $\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=61

$$\frac{x(x^2+1)}{2\sqrt{1-x^4}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}\,E\left(\sin^{-1}(x)|-1\right)}{2\sqrt{1-x^4}}$$

```
[Out] 1/2*x*(x^2+1)/(-x^4+1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2
)/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1256, 471, 424}

$$\frac{x(x^2+1)}{2\sqrt{1-x^4}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}\,E\left(\sin^{-1}(x)|-1\right)}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] (x*(1 + x^2))/(2*Sqrt[1 - x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[Ar
cSin[x], -1])/(2*Sqrt[1 - x^4])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1256

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{x^2}{(1-x^2)^{3/2}\sqrt{1+x^2}}\,dx}{\sqrt{1-x^4}}$$
$$= \frac{x(1+x^2)}{2\sqrt{1-x^4}} - \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{1-x^4}}$$
$$= \frac{x(1+x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}\,E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{1-x^4}}$$

Mathematica [A] time = 0.08, size = 37, normalized size = 0.61

$$\frac{-\sqrt{1-x^4}\,E\left(\sin^{-1}(x)\middle|-1\right)+x^3+x}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] (x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[1 - x^4])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}\,x^2}{x^6-x^4-x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 1)*x^2/(x^6 - x^4 - x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\sqrt{-x^4+1}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^2/(sqrt(-x^4 + 1)*(x^2 - 1)), x)
```

maple [B] time = 0.03, size = 143, normalized size = 2.34

$$-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\operatorname{EllipticF}(x,i)}{2\sqrt{-x^4+1}}-\frac{-x^3+x^2-x+1}{4\sqrt{(x+1)\left(-x^3+x^2-x+1\right)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\left(-\operatorname{EllipticE}(x,i)+\operatorname{Elli}\right.}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(-x^4+1)^(1/2),x)
```

```
[Out] -1/2*EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)-1/4*(-x^3+x
^2-x+1)/((x+1)*(-x^3+x^2-x+1))^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4
+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I))-1/4*(-x^3-x^2-x-1)/((x-1)*(-x^3-x
^2-x-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{-x^4+1}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x^2/(sqrt(-x^4 + 1)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^2}{(x^2-1)\,\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)*(1 - x^4)^(1/2)),x)
[Out] -int(x^2/((x^2 - 1)*(1 - x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{1-x^4}-\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(-x**4+1)**(1/2),x)
[Out] -Integral(x**2/(x**2*sqrt(1 - x**4) - sqrt(1 - x**4)), x)
```

3.263 $\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=113

$$-\frac{x(1-x^2)}{2\sqrt{x^4-1}}+\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\frac{\sqrt{x^2+1}\sqrt{1-x^2}\,E\left(\sin^{-1}(x)|-1\right)}{2\sqrt{x^4-1}}$$

```
[Out] -1/2*x*(-x^2+1)/(x^4-1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/
2)/(x^4-1)^(1/2)+1/2*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1/2))*(x^2-1)
^(1/2)*(x^2+1)^(1/2)*2^(1/2)/(x^4-1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1256, 471, 423, 427, 424, 253, 222}

$$-\frac{x(1-x^2)}{2\sqrt{x^4-1}}+\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\frac{\sqrt{x^2+1}\sqrt{1-x^2}\,E\left(\sin^{-1}(x)|-1\right)}{2\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] -(x*(1 - x^2))/(2*Sqrt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[
ArcSin[x], -1])/(2*Sqrt[-1 + x^4]) + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*Elliptic
F[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1256

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{\left(1+x^2\right)\sqrt{-1+x^4}}\,dx = \frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\int \frac{x^2}{\sqrt{-1+x^2}\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{-1+x^4}}$$

$$= -\frac{x\left(1-x^2\right)}{2\sqrt{-1+x^4}} - \frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\int \frac{\sqrt{-1+x^2}}{\sqrt{1+x^2}}\,dx}{2\sqrt{-1+x^4}}$$

$$= -\frac{x\left(1-x^2\right)}{2\sqrt{-1+x^4}} - \frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^2}}\,dx}{2\sqrt{-1+x^4}} + \frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\int \frac{}{\sqrt{-1+x^2}}}{\sqrt{-1+x^4}}$$

$$= -\frac{x\left(1-x^2\right)}{2\sqrt{-1+x^4}} - \frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{-1+x^4}} + \int \frac{1}{\sqrt{-1+x^4}}\,dx$$

$$= -\frac{x\left(1-x^2\right)}{2\sqrt{-1+x^4}} - \frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\right.\right.}{\sqrt{2}\,\sqrt{-1+x^4}}$$

Mathematica [A] time = 0.08, size = 54, normalized size = 0.48

$$\frac{2\sqrt{1-x^4}\,F\left(\sin^{-1}(x)\middle|-1\right)-\sqrt{1-x^4}\,E\left(\sin^{-1}(x)\middle|-1\right)+x^3-x}{2\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] (-x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1] + 2*Sqrt[1 - x^4]*Ellipt
icF[ArcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4-1}\,x^2}{x^6+x^4-x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 - 1)*x^2/(x^6 + x^4 - x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^4-1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

maple [A] time = 0.02, size = 99, normalized size = 0.88

$$\frac{\left(x^2-1\right)x}{2\sqrt{\left(x^2+1\right)\left(x^2-1\right)}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(ix,i\right)}{2\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\left(-\operatorname{EllipticE}\left(ix,i\right)+\operatorname{EllipticF}\left(ix\right.\right.}{2\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(x^4-1)^(1/2),x)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticF(I*x,I)+1/2*(x^2
-1)*x/((x^2+1)*(x^2-1))^(1/2)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1
/2)*(EllipticF(I*x,I)-EllipticE(I*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^4-1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(x^2+1\right)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)*(x^4 - 1)^(1/2)),x)
```

```
[Out] int(x^2/((x^2 + 1)*(x^4 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

3.264 $\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=57

$$\frac{x(x^2+1)}{2\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}\,E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{x^4-1}}$$

```
[Out] 1/2*x*(x^2+1)/(x^4-1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)
/(x^4-1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1256, 471, 426, 424}

$$\frac{x(x^2+1)}{2\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}\,E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] (x*(1 + x^2))/(2*Sqrt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[A
rcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1256

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \frac{\left(\sqrt{-1-x^2}\sqrt{1-x^2}\right)\int \frac{x^2}{\sqrt{-1-x^2}(1-x^2)^{3/2}}\,dx}{\sqrt{-1+x^4}}$$
$$= \frac{x(1+x^2)}{2\sqrt{-1+x^4}} + \frac{\left(\sqrt{-1-x^2}\sqrt{1-x^2}\right)\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{-1+x^4}}$$
$$= \frac{x(1+x^2)}{2\sqrt{-1+x^4}} + \frac{\left((-1-x^2)\sqrt{1-x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{-1+x^4}}$$
$$= \frac{x(1+x^2)}{2\sqrt{-1+x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}\,E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{-1+x^4}}$$

Mathematica [A] time = 0.07, size = 35, normalized size = 0.61

$$\frac{-\sqrt{1-x^4}\,E\left(\sin^{-1}(x)\middle|-1\right)+x^3+x}{2\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[-1 + x^4]),x]
[Out] (x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{x^4-1}\,x^2}{x^6-x^4-x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(x^4 - 1)*x^2/(x^6 - x^4 - x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\sqrt{x^4-1}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(-x^2/(sqrt(x^4 - 1)*(x^2 - 1)), x)
```

maple [B] time = 0.02, size = 134, normalized size = 2.35

$$\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\;\text{EllipticF}(ix,i)}{2\sqrt{x^4-1}}+\frac{x^3-x^2+x-1}{4\sqrt{(x+1)\left(x^3-x^2+x-1\right)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\;\left(-\text{EllipticE}(ix,i)+\text{Elli}\right.}{2\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(x^4-1)^(1/2),x)
```

[Out] 1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticF(I*x,I)+1/4*(x^3-x^2+x-1)/((x+1)*(x^3-x^2+x-1))^(1/2)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(EllipticF(I*x,I)-EllipticE(I*x,I))+1/4*(x^3+x^2+x+1)/((x-1)*(x^3+x^2+x+1))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{x^4-1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")

[Out] -integrate(x^2/(sqrt(x^4 - 1)*(x^2 - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^2}{\left(x^2-1\right)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^2/((x^2 - 1)*(x^4 - 1)^(1/2)),x)

[Out] -int(x^2/((x^2 - 1)*(x^4 - 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{x^4-1}-\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-x**2+1)/(x**4-1)**(1/2),x)

[Out] -Integral(x**2/(x**2*sqrt(x**4 - 1) - sqrt(x**4 - 1)), x)

3.265 $\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx$

Optimal. Leaf size=74

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{-x^4-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arctanh(x*2^(1/2)/(-x^4-1)^(1/2))*2^(1/2)+1/4*(x^2+1)*(cos(2*arctan(x)
)^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1
)/(x^2+1)^2)^(1/2)/(-x^4-1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1318, 220, 1699, 206}

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{-x^4-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*Sqrt[-1 - x^4]),x]
```

```
[Out] -ArcTanh[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/(2*Sqrt[2]) + ((1 + x^2)*Sqrt[(1 + x^4
)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1318

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(1+x^2\right)\sqrt{-1-x^4}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{-1-x^4}}\,dx - \frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{-1-x^4}}\,dx$$
$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-1-x^4}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{-1-x^4}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-1-x^4}}$$

Mathematica [C] time = 0.09, size = 60, normalized size = 0.81

$$\frac{\sqrt[4]{-1}\,\sqrt{x^4+1}\,\left(\Pi\left(-i;\,i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)\right)}{\sqrt{-x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[-1 - x^4]),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[1 + x^4]*(-EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + Ellipt
icPi[-I, I*ArcSinh[(-1)^(1/4)*x], -1]))/Sqrt[-1 - x^4]
```

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$-\frac{1}{8}\,\sqrt{2}\,\log\left(\frac{\sqrt{2}\,x+\sqrt{-x^4-1}}{x^2+1}\right)+\frac{1}{8}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}\,x-\sqrt{-x^4-1}}{x^2+1}\right)+\text{integral}\left(-\frac{\sqrt{-x^4-1}}{2\left(x^4+1\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*log((sqrt(2)*x + sqrt(-x^4 - 1))/(x^2 + 1)) + 1/8*sqrt(2)*log(
-(sqrt(2)*x - sqrt(-x^4 - 1))/(x^2 + 1)) + integral(-1/2*sqrt(-x^4 - 1)/(x^
4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x^4-1}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-x^4 - 1)*(x^2 + 1)), x)
```

maple [C] time = 0.04, size = 168, normalized size = 2.27

$$\frac{\sqrt{ix^2+1}\,\sqrt{-ix^2+1}\ \text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x, i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}} - \frac{i\sqrt{-i}\,\sqrt{ix^2+1}\,\sqrt{-ix^2+1}\ \text{EllipticPi}\left(\sqrt{-i}\,x, -i, \frac{(-1)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{2\sqrt{-x^4-1}} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(-x^4-1)^(1/2),x)
```

```
[Out] 1/(1/2*2^(1/2)-1/2*I*2^(1/2))*(I*x^2+1)^(1/2)*(-I*x^2+1)^(1/2)/(-x^4-1)^(1/
2)*EllipticF((1/2*2^(1/2)-1/2*I*2^(1/2))*x,I)-1/2*I*(-I)^(1/2)*(I*x^2+1)^(1
/2)*(-I*x^2+1)^(1/2)/(-x^4-1)^(1/2)*EllipticPi((-I)^(1/2)*x,-I,(-1)^(1/4)/(
-I)^(1/2))-1/2/(-I)^(1/2)*(I*x^2+1)^(1/2)*(-I*x^2+1)^(1/2)/(-x^4-1)^(1/2)*E
llipticPi((-I)^(1/2)*x,-I,(-1)^(1/4)/(-I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x^4-1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^4 - 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(x^2+1\right)\sqrt{-x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)*(- x^4 - 1)^(1/2)),x)
```

```
[Out] int(x^2/((x^2 + 1)*(- x^4 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2+1\right)\sqrt{-x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(-x**4-1)**(1/2),x)
```

```
[Out] Integral(x**2/((x**2 + 1)*sqrt(-x**4 - 1)), x)
```

3.266 $\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-x^4-1}}$$

```
[Out] 1/4*arctan(x*2^(1/2)/(-x^4-1)^(1/2))*2^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))^
2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/
(x^2+1)^2)^(1/2)/(-x^4-1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1318, 220, 1699, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*Sqrt[-1 - x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/(2*Sqrt[2]) - ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1318

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = -\left(\frac{1}{2}\int \frac{1}{\sqrt{-1-x^4}}\,dx\right) + \frac{1}{2}\int \frac{1+x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx$$
$$= -\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-1-x^4}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{-1-x^4}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-1-x^4}}$$

Mathematica [C] time = 0.09, size = 56, normalized size = 0.76

$$\frac{\sqrt[4]{-1}\sqrt{x^4+1}\left(F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right) - \Pi\left(i;\sin^{-1}\left((-1)^{3/4}x\right)\middle|-1\right)\right)}{\sqrt{-x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[-1 - x^4]),x]
[Out] ((-1)^(1/4)*Sqrt[1 + x^4]*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - Ellipti
cPi[I, ArcSin[(-1)^(3/4)*x], -1]))/Sqrt[-1 - x^4]
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$-\frac{1}{8}i\sqrt{2}\log\left(\frac{i\sqrt{2}x+\sqrt{-x^4-1}}{x^2-1}\right) + \frac{1}{8}i\sqrt{2}\log\left(\frac{-i\sqrt{2}x+\sqrt{-x^4-1}}{x^2-1}\right) + \text{integral}\left(\frac{\sqrt{-x^4-1}}{2\left(x^4+1\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
[Out] -1/8*I*sqrt(2)*log((I*sqrt(2)*x + sqrt(-x^4 - 1))/(x^2 - 1)) + 1/8*I*sqrt(2
)*log((-I*sqrt(2)*x + sqrt(-x^4 - 1))/(x^2 - 1)) + integral(1/2*sqrt(-x^4 -
 1)/(x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\sqrt{-x^4-1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(-x^2/(sqrt(-x^4 - 1)*(x^2 - 1)), x)
```

maple [C] time = 0.02, size = 115, normalized size = 1.55

$$-\frac{\sqrt{ix^2+1}\,\sqrt{-ix^2+1}\;\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x, i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}} + \frac{\sqrt{ix^2+1}\,\sqrt{-ix^2+1}\;\text{EllipticPi}\left(\sqrt{-i}\,x, i, \frac{(-1)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{\sqrt{-i}\,\sqrt{-x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(-x^4-1)^(1/2),x)
```

```
[Out] -1/(1/2*2^(1/2)-1/2*I*2^(1/2))*(I*x^2+1)^(1/2)*(-I*x^2+1)^(1/2)/(-x^4-1)^(1
/2)*EllipticF((1/2*2^(1/2)-1/2*I*2^(1/2))*x,I)+1/(-I)^(1/2)*(I*x^2+1)^(1/2)
*(-I*x^2+1)^(1/2)/(-x^4-1)^(1/2)*EllipticPi((-I)^(1/2)*x,I,(-1)^(1/4)/(-I)^
(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{-x^4-1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/(sqrt(-x^4 - 1)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\left(x^2-1\right)\sqrt{-x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)*(- x^4 - 1)^(1/2)),x)
```

```
[Out] -int(x^2/((x^2 - 1)*(- x^4 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{-x^4-1}-\sqrt{-x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(-x**4-1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(-x**4 - 1) - sqrt(-x**4 - 1)), x)
```

3.267 $\int x^2\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=243

$$\frac{c^2\sqrt{a^2+2abx^2+b^2x^4}\,(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}\left(a+bx^2\right)}-\frac{cx\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(bc-2ad)}{16d^2\left(a+bx^2\right)}+\frac{bx^3\sqrt{a^2+2abx^2}}{6d\left(a\right.}$$

```
[Out] 1/6*b*x^3*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+1/16*c^2*(-2*a*d+
b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/d^(5/2)/(b*x^2+
a)-1/16*c*(-2*a*d+b*c)*x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)-
1/8*(-2*a*d+b*c)*x^3*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)
```

Rubi [A] time = 0.13, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1250, 459, 279, 321, 217, 206}

$$\frac{c^2\sqrt{a^2+2abx^2+b^2x^4}\,(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}\left(a+bx^2\right)}-\frac{cx\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(bc-2ad)}{16d^2\left(a+bx^2\right)}+\frac{bx^3\sqrt{a^2+2abx^2}}{6d\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(c*(b*c - 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*d^
2*(a + b*x^2)) - ((b*c - 2*a*d)*x^3*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(8*d*(a + b*x^2)) + (b*x^3*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(6*d*(a + b*x^2)) + (c^2*(b*c - 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(5/2)*(a + b*x^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2\left(ab+b^2x^2\right)\sqrt{c+dx^2}\,dx}{ab+b^2x^2}$$

$$= \frac{bx^3\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{6d\left(a+bx^2\right)} - \frac{\left(b(bc-2ad)\sqrt{a^2+2abx^2+b\right.}{2d\left(ab+b\right.}$$

$$= -\frac{(bc-2ad)x^3\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a+bx^2\right)} + \frac{bx^3\left(c+dx^2\right)^{3/2}\sqrt{a^2\ -}{6d\left(a+b\right.}$$

$$= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\left(a+bx^2\right)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}}{8d\left(a+\right.}$$

$$= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\left(a+bx^2\right)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}}{8d\left(a+\right.}$$

$$= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\left(a+bx^2\right)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}}{8d\left(a+\right.}$$

Mathematica [A] time = 0.18, size = 142, normalized size = 0.58

$$\frac{\sqrt{\left(a+bx^2\right)^2}\sqrt{c+dx^2}\left(3c^{3/2}(bc-2ad)\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)+\sqrt{d}\,x\sqrt{\frac{dx^2}{c}+1}\left(6ad\left(c+2dx^2\right)+b\left(-3c^2+2cdx^2+8a\right.\right.\right.}{48d^{5/2}\left(a+bx^2\right)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*Sqrt[c + d*x^2]*(Sqrt[d]*x*Sqrt[1 + (d*x^2)/c]*(6*a*d*
(c + 2*d*x^2) + b*(-3*c^2 + 2*c*d*x^2 + 8*d^2*x^4)) + 3*c^(3/2)*(b*c - 2*a*
d)*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(48*d^(5/2)*(a + b*x^2)*Sqrt[1 + (d*x^2)/
c])
```

fricas [A] time = 0.63, size = 206, normalized size = 0.85

$$\left[-\frac{3\left(bc^3-2\,ac^2d\right)\sqrt{d}\,\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-2\left(8\,bd^3x^5+2\left(bcd^2+6\,ad^3\right)x^3-3\left(bc^2d-2\,acd^2\right.\right.}{96\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt
(d)*x - c) - 2*(8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 - 3*(b*c^2*d - 2*a*
c*d^2)*x)*sqrt(d*x^2 + c))/d^3, -1/48*(3*(b*c^3 - 2*a*c^2*d)*sqrt(-d)*arcta
n(sqrt(-d)*x/sqrt(d*x^2 + c)) - (8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 -
3*(b*c^2*d - 2*a*c*d^2)*x)*sqrt(d*x^2 + c))/d^3]
```

giac [A] time = 0.38, size = 156, normalized size = 0.64

$$\frac{1}{48}\left(2\left(4\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{bcd^3\mathrm{sgn}\left(bx^2+a\right)+6\,ad^4\mathrm{sgn}\left(bx^2+a\right)}{d^4}\right)x^2-\frac{3\left(bc^2d^2\mathrm{sgn}\left(bx^2+a\right)-2\,acd^3\mathrm{sgn}\left(bx\right.\right.}{d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*x^2*sgn(b*x^2 + a) + (b*c*d^3*sgn(b*x^2 + a) + 6*a*d^4*sgn(b*x
^2 + a))/d^4)*x^2 - 3*(b*c^2*d^2*sgn(b*x^2 + a) - 2*a*c*d^3*sgn(b*x^2 + a))
/d^4)*sqrt(d*x^2 + c)*x - 1/16*(b*c^3*sgn(b*x^2 + a) - 2*a*c^2*d*sgn(b*x^2
+ a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

maple [A] time = 0.01, size = 159, normalized size = 0.65

$$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(8\left(d\,x^2+c\right)^{\frac{3}{2}}b\,d^{\frac{3}{2}}x^3-6a\,c^2d\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+3b\,c^3\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)-6\sqrt{d\,x^2+c}\,\ldots\right.}{48\left(b\,x^2+a\right)d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/48*((b*x^2+a)^2)^(1/2)*(8*d^(3/2)*(d*x^2+c)^(3/2)*x^3*b+12*d^(3/2)*(d*x^2
+c)^(3/2)*x*a-6*d^(1/2)*(d*x^2+c)^(3/2)*x*b*c-6*d^(3/2)*(d*x^2+c)^(1/2)*x*a
*c+3*d^(1/2)*(d*x^2+c)^(1/2)*x*b*c^2-6*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c^2*
d+3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^3)/(b*x^2+a)/d^(5/2)
```

maxima [A] time = 0.92, size = 124, normalized size = 0.51

$$\frac{\left(dx^2+c\right)^{\frac{3}{2}}bx^3}{6\,d}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}bcx}{8\,d^2}+\frac{\sqrt{dx^2+c}\,bc^2x}{16\,d^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}ax}{4\,d}-\frac{\sqrt{dx^2+c}\,acx}{8\,d}+\frac{bc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{5}{2}}}-\frac{ac^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(d*x^2 + c)^(3/2)*b*x^3/d - 1/8*(d*x^2 + c)^(3/2)*b*c*x/d^2 + 1/16*sqrt
(d*x^2 + c)*b*c^2*x/d^2 + 1/4*(d*x^2 + c)^(3/2)*a*x/d - 1/8*sqrt(d*x^2 + c)
*a*c*x/d + 1/16*b*c^3*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/8*a*c^2*arcsinh(d*
x/sqrt(c*d))/d^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^2*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.268 $\int x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=108

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{5/2}}{5d^2\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^2\left(a+bx^2\right)}$$

```
[Out] -1/3*(-a*d+b*c)*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)+1/5*b*(d*
x^2+c)^(5/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1247, 646, 43}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{5/2}}{5d^2\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((b*c - a*d)*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^2*(a
+ b*x^2)) + (b*(c + d*x^2)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^2*(a
 + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{2}\text{Subst}\left(\int\sqrt{c+dx}\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int(ab+b^2x)\sqrt{c+dx}\,dx,x,x^2\right)}{2(ab+b^2x^2)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(-\frac{b(bc-ad)\sqrt{c+dx}}{d}+\frac{b^2(c+dx)^{3/2}}{d}\right)dx,x,x^2\right)}{2(ab+b^2x^2)}$$
$$=-\frac{(bc-ad)(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^2(a+bx^2)}+\frac{b(c+dx^2)^{5/2}\sqrt{a^2+2a}}{5d^2(a+bx^2}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.52

$$\frac{\sqrt{(a+bx^2)^2}(c+dx^2)^{3/2}(5ad-2bc+3bdx^2)}{15d^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(c + d*x^2)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x^2))/(15*d^
2*(a + b*x^2))
```

fricas [A] time = 0.93, size = 50, normalized size = 0.46

$$\frac{\left(3\,bd^2x^4-2\,bc^2+5\,acd+(bcd+5\,ad^2)x^2\right)\sqrt{dx^2+c}}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^2*x^4 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x^2)*sqrt(d*x^2 +
 c)/d^2
```

giac [A] time = 0.34, size = 68, normalized size = 0.63

$$\frac{3(dx^2+c)^{\frac{5}{2}}b\text{sgn}(bx^2+a)-5(dx^2+c)^{\frac{3}{2}}bc\text{sgn}(bx^2+a)+5(dx^2+c)^{\frac{3}{2}}ad\text{sgn}(bx^2+a)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(d*x^2 + c)^(5/2)*b*sgn(b*x^2 + a) - 5*(d*x^2 + c)^(3/2)*b*c*sgn(b*
x^2 + a) + 5*(d*x^2 + c)^(3/2)*a*d*sgn(b*x^2 + a))/d^2
```

maple [A] time = 0.00, size = 51, normalized size = 0.47

$$\frac{(d\,x^2+c)^{\frac{3}{2}}(3bd\,x^2+5ad-2bc)\sqrt{(b\,x^2+a)^2}}{15(b\,x^2+a)\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(3/2)*(3*b*d*x^2+5*a*d-2*b*c)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2
+a)
```

maxima [A] time = 0.90, size = 50, normalized size = 0.46

$$\frac{\left(dx^2+c\right)^{\frac{3}{2}}bx^2}{5\,d}-\frac{2\left(dx^2+c\right)^{\frac{3}{2}}bc}{15\,d^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}a}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(d*x^2 + c)^(3/2)*b*x^2/d - 2/15*(d*x^2 + c)^(3/2)*b*c/d^2 + 1/3*(d*x^2
 + c)^(3/2)*a/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.269 $\int \sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=178

$$-\frac{c\sqrt{a^2+2abx^2+b^2x^4}\,(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}\left(a+bx^2\right)}+\frac{bx\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{3/2}}{4d\left(a+bx^2\right)}-\frac{x\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a\right.}$$

```
[Out] 1/4*b*x*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)-1/8*c*(-4*a*d+b*c)*
arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/d^(3/2)/(b*x^2+a)-1/
8*(-4*a*d+b*c)*x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)
```

Rubi [A] time = 0.08, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1148, 388, 195, 217, 206}

$$-\frac{c\sqrt{a^2+2abx^2+b^2x^4}\,(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}\left(a+bx^2\right)}+\frac{bx\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{3/2}}{4d\left(a+bx^2\right)}-\frac{x\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((b*c - 4*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*d*(a
+ b*x^2)) + (b*x*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*d*(a
 + b*x^2)) - (c*(b*c - 4*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt
[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a + b*x^2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1148

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
```

```
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab+b^2x^2\right)\sqrt{c+dx^2}\,dx}{ab+b^2x^2}$$

$$= \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\left(a+bx^2\right)} - \frac{\left(b(bc-4ad)\sqrt{a^2+2abx^2+b^2x^4}\right)\int}{4d\left(ab+b^2x^2\right)}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2}}{4d\left(a+bx^2\right)}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2}}{4d\left(a+bx^2\right)}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2}}{4d\left(a+bx^2\right)}$$

Mathematica [A] time = 0.11, size = 121, normalized size = 0.68

$$\frac{\sqrt{\left(a+bx^2\right)^2}\sqrt{c+dx^2}\left(\sqrt{d}\,x\sqrt{\frac{dx^2}{c}+1}\left(4ad+b\left(c+2dx^2\right)\right)-\sqrt{c}\,(bc-4ad)\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right)}{8d^{3/2}\left(a+bx^2\right)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*Sqrt[c + d*x^2]*(Sqrt[d]*x*Sqrt[1 + (d*x^2)/c]*(4*a*d
+ b*(c + 2*d*x^2)) - Sqrt[c]*(b*c - 4*a*d)*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(
8*d^(3/2)*(a + b*x^2)*Sqrt[1 + (d*x^2)/c])
```

fricas [A] time = 0.98, size = 155, normalized size = 0.87

$$\left[-\frac{\left(bc^2-4\,acd\right)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-2\left(2\,bd^2x^3+\left(bcd+4\,ad^2\right)x\right)\sqrt{dx^2+c}}{16\,d^2}, \frac{\left(bc^2-4\,acd\right)\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*
x - c) - 2*(2*b*d^2*x^3 + (b*c*d + 4*a*d^2)*x)*sqrt(d*x^2 + c))/d^2, 1/8*((
b*c^2 - 4*a*c*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b*d^2*x^3
 + (b*c*d + 4*a*d^2)*x)*sqrt(d*x^2 + c))/d^2]
```

giac [A] time = 0.46, size = 109, normalized size = 0.61

$$\frac{1}{8}\left(2\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{bcd\mathrm{sgn}\left(bx^2+a\right)+4\,ad^2\mathrm{sgn}\left(bx^2+a\right)}{d^2}\right)\sqrt{dx^2+c}\,x+\frac{\left(bc^2\mathrm{sgn}\left(bx^2+a\right)-4\,acd\mathrm{sgn}\left(bx^2\right.\right.}{8\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2*sgn(b*x^2 + a) + (b*c*d*sgn(b*x^2 + a) + 4*a*d^2*sgn(b*x^2 + a
))/d^2)*sqrt(d*x^2 + c)*x + 1/8*(b*c^2*sgn(b*x^2 + a) - 4*a*c*d*sgn(b*x^2 +
 a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(3/2)
```

maple [A] time = 0.01, size = 119, normalized size = 0.67

$$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(4acd\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)-b\,c^{2}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)+4\sqrt{d\,x^{2}+c}\,a\,d^{\frac{3}{2}}x-\sqrt{d\,x^{2}+c}\,bc\sqrt{}\right.}{8\left(b\,x^{2}+a\right)d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/8*((b*x^2+a)^2)^(1/2)*(2*d^(1/2)*(d*x^2+c)^(3/2)*x*b+4*d^(3/2)*(d*x^2+c)^
(1/2)*x*a-d^(1/2)*(d*x^2+c)^(1/2)*x*b*c+4*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c
*d-ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^2)/(b*x^2+a)/d^(3/2)
```

maxima [A] time = 1.08, size = 81, normalized size = 0.46

$$\frac{1}{2}\sqrt{dx^{2}+c}\,ax+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}bx}{4\,d}-\frac{\sqrt{dx^{2}+c}\,bcx}{8\,d}-\frac{bc^{2}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}+\frac{ac\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*a*x + 1/4*(d*x^2 + c)^(3/2)*b*x/d - 1/8*sqrt(d*x^2 + c)
*b*c*x/d - 1/8*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 1/2*a*c*arcsinh(d*x/s
qrt(c*d))/sqrt(d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x^{2}+c}\,\sqrt{\left(b\,x^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx^{2}}\,\sqrt{\left(a+bx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

3.270 $\int \frac{\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$

Optimal. Leaf size=152

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{3/2}}{3d\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}}{a+bx^2}-\frac{a\sqrt{c}\,\sqrt{a^2+2abx^2+b^2x^4}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

```
[Out] 1/3*b*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)-a*arctanh((d*x^2+c)^(
1/2)/c^(1/2))*c^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a*(d*x^2+c)^(1/2)*((b*x
^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1250, 446, 80, 50, 63, 208}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,\left(c+dx^2\right)^{3/2}}{3d\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}}{a+bx^2}-\frac{a\sqrt{c}\,\sqrt{a^2+2abx^2+b^2x^4}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*(c + d
*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) - (a*Sqrt[c]
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x
^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x}\,dx}{ab+b^2x^2}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)\sqrt{c+dx}}{x}\,dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}$$

$$= \frac{b\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} + \frac{\left(ab\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\Big(}{2\left(ab+b^2x^2\right)}$$

$$= \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} +$$

$$= \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} +$$

$$= \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} -$$

Mathematica [A] time = 0.05, size = 83, normalized size = 0.55

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(\sqrt{c+dx^2}\left(3ad+b\left(c+dx^2\right)\right)-3a\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{3d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[c + d*x^2]*(3*a*d + b*(c + d*x^2)) - 3*a*Sqrt[c]
*d*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(3*d*(a + b*x^2))
```

fricas [A] time = 0.87, size = 123, normalized size = 0.81

$$\left[\frac{3\,a\sqrt{c}\,d\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\left(bdx^2+bc+3\,ad\right)\sqrt{dx^2+c}}{6\,d}, \frac{3\,a\sqrt{-c}\,d\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(bdx^2+bc+\right.}{3\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2
*(b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d, 1/3*(3*a*sqrt(-c)*d*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) + (b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d]
```

giac [A] time = 0.45, size = 84, normalized size = 0.55

$$\frac{ac \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right) \operatorname{sgn}\left(bx^2+a\right)}{\sqrt{-c}} + \frac{\left(dx^2+c\right)^{\frac{3}{2}} bd^2 \operatorname{sgn}\left(bx^2+a\right) + 3\sqrt{dx^2+c}\, ad^3 \operatorname{sgn}\left(bx^2+a\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))*sgn(b*x^2 + a)/sqrt(-c) + 1/3*((d*x^2
+ c)^(3/2)*b*d^2*sgn(b*x^2 + a) + 3*sqrt(d*x^2 + c)*a*d^3*sgn(b*x^2 + a))/d
^3
```

maple [A] time = 0.01, size = 80, normalized size = 0.53

$$-\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(3a\sqrt{c}\,d\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)-3\sqrt{d\,x^2+c}\,ad-\left(d\,x^2+c\right)^{\frac{3}{2}} b\right)}{3\left(b\,x^2+a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x)
```

```
[Out] -1/3*((b*x^2+a)^2)^(1/2)*(3*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*c^(1/2)*a*d
-b*(d*x^2+c)^(3/2)-3*(d*x^2+c)^(1/2)*a*d)/(b*x^2+a)/d
```

maxima [A] time = 1.45, size = 45, normalized size = 0.30

$$-a\sqrt{c}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\sqrt{dx^2+c}\,a+\frac{\left(dx^2+c\right)^{\frac{3}{2}} b}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -a*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) + sqrt(d*x^2 + c)*a + 1/3*(d*x^2 +
 c)^(3/2)*b/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}\,\sqrt{\left(b\,x^2+a\right)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x,x)
```

```
[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{\left(a+bx^2\right)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x, x)
```

3.271 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

Optimal. Leaf size=177

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{cx\left(a+bx^2\right)}+\frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}\,(2ad+bc)}{2c\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(2ad+bc)\,\mathrm{t}}{2\sqrt{d}\left(a+bx^2\right)}$$

```
[Out] -a*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x/(b*x^2+a)+1/2*(2*a*d+b*c)*arctan
h(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/d^(1/2)+1/2*(2*a
*d+b*c)*x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c/(b*x^2+a)
```

Rubi [A] time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1250, 453, 195, 217, 206}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{cx\left(a+bx^2\right)}+\frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}\,(2ad+bc)}{2c\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(2ad+bc)\,\mathrm{t}}{2\sqrt{d}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] ((b*c + 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a +
 b*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(c*x*(a +
b*x^2)) + ((b*c + 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x^2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x^2}\,dx}{ab+b^2x^2}$$

$$= -\frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\left(a+bx^2\right)} + -\frac{\left(\left(-b^2c-2abd\right)\sqrt{a^2+2abx^2+\cdots}\right.}{c\left(ab+b^2\cdots\right.}$$

$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx\cdots}}{cx\left(a+bx^2\right)}$$

$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx\cdots}}{cx\left(a+bx^2\right)}$$

$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx\cdots}}{cx\left(a+bx^2\right)}$$

Mathematica [A] time = 0.11, size = 122, normalized size = 0.69

$$\frac{\sqrt{\left(a+bx^2\right)^2}\sqrt{c+dx^2}\left(\sqrt{c}\sqrt{d}\left(bx^2-2a\right)\sqrt{\frac{dx^2}{c}+1}+x(2ad+bc)\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right)}{2\sqrt{c}\sqrt{d}x\left(a+bx^2\right)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*Sqrt[c + d*x^2]*(Sqrt[c]*Sqrt[d]*(-2*a + b*x^2)*Sqrt[1
 + (d*x^2)/c] + (b*c + 2*a*d)*x*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(2*Sqrt[c]*S
qrt[d]*x*(a + b*x^2)*Sqrt[1 + (d*x^2)/c])
```

fricas [A] time = 0.80, size = 134, normalized size = 0.76

$$\left[\frac{(bc+2\,ad)\sqrt{d}\,x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(bdx^2-2\,ad\right)\sqrt{dx^2+c}}{4\,dx}, -\frac{(bc+2\,ad)\sqrt{-d}\,x\arctan\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) + 2*(b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c))/(d*x), -1/2*((b*c + 2*a*d)*sqrt(-
d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c)
)/(d*x)]
```

giac [A] time = 0.47, size = 116, normalized size = 0.66

$$\frac{1}{2}\sqrt{dx^2+c}\,bx\mathrm{sgn}\left(bx^2+a\right)+\frac{2\,ac\sqrt{d}\,\mathrm{sgn}\left(bx^2+a\right)}{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c}-\frac{\left(bc\sqrt{d}\,\mathrm{sgn}\left(bx^2+a\right)+2\,ad^{\frac{3}{2}}\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(\left(\sqrt{d}\,x-\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*x*sgn(b*x^2 + a) + 2*a*c*sqrt(d)*sgn(b*x^2 + a)/((sqr
t(d)*x - sqrt(d*x^2 + c))^2 - c) - 1/4*(b*c*sqrt(d)*sgn(b*x^2 + a) + 2*a*d^
(3/2)*sgn(b*x^2 + a))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d
```

maple [A] time = 0.01, size = 128, normalized size = 0.72

$$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(2acdx\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+b\,c^2x\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+2\sqrt{d\,x^2+c}\,a\,d^{\frac{3}{2}}x^2+\sqrt{d\,x^2+c}\,bc\right.}{2\left(b\,x^2+a\right)c\sqrt{d}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x)
```

```
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(2*d^(3/2)*(d*x^2+c)^(1/2)*x^2*a+d^(1/2)*(d*x^2+c)^
(1/2)*x^2*b*c-2*d^(1/2)*(d*x^2+c)^(3/2)*a+2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x
*a*c*d+ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x*b*c^2)/(b*x^2+a)/c/x/d^(1/2)
```

maxima [A] time = 1.08, size = 59, normalized size = 0.33

$$\frac{1}{2}\sqrt{dx^2+c}\,bx+\frac{bc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}}+a\sqrt{d}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{\sqrt{dx^2+c}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*x + 1/2*b*c*arcsinh(d*x/sqrt(c*d))/sqrt(d) + a*sqrt(d
)*arcsinh(d*x/sqrt(c*d)) - sqrt(d*x^2 + c)*a/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{d\,x^2+c}\,\sqrt{\left(b\,x^2+a\right)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^2,x)
```

```
[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c+dx^2}\,\sqrt{\left(a+bx^2\right)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x**2, x)
```

3.272 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal. Leaf size=177

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{2cx^2\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}\,(ad+2bc)}{2c\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(ad+2bc}{2\sqrt{c}\left(a+bx\right.}$$

```
[Out] -1/2*a*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x^2/(b*x^2+a)-1/2*(a*d+2*b*c)*
arctanh((d*x^2+c)^(1/2)/c^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/c^(1/2)+1/2*
(a*d+2*b*c)*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c/(b*x^2+a)
```

Rubi [A] time = 0.12, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1250, 446, 78, 50, 63, 208}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{2cx^2\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}\,(ad+2bc)}{2c\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(ad+2bc}{2\sqrt{c}\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] ((2*b*c + a*d)*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a + b
*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*x^2*(a
+ b*x^2)) - ((2*b*c + a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x^2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x^3}\,dx}{ab+b^2x^2}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)\sqrt{c+dx}}{x^2}\,dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}$$

$$= -\frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\left(a+bx^2\right)} + \frac{\left(\left(b^2c+\frac{abd}{2}\right)\sqrt{a^2+2abx^2+b^2x^4}\right)}{2c\left(ab+b^2\right.}$$

$$= \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b}}{2cx^2\left(a+bx^2\right)}$$

$$= \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b}}{2cx^2\left(a+bx^2\right)}$$

$$= \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b}}{2cx^2\left(a+bx^2\right)}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.51

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(\sqrt{c}\left(a-2bx^2\right)\sqrt{c+dx^2}+x^2(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{2\sqrt{c}\,x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] -1/2*(Sqrt[(a + b*x^2)^2]*(Sqrt[c]*(a - 2*b*x^2)*Sqrt[c + d*x^2] + (2*b*c +
 a*d)*x^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(Sqrt[c]*x^2*(a + b*x^2))
```

fricas [A] time = 0.68, size = 141, normalized size = 0.80

$$\left[\frac{(2\,bc+ad)\sqrt{c}\,x^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\left(2\,bcx^2-ac\right)\sqrt{dx^2+c}}{4\,cx^2}, \frac{(2\,bc+ad)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(2\,b\right.}{2\,cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2
*c)/x^2) + 2*(2*b*c*x^2 - a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*((2*b*c + a*d)
*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x^2 - a*c)*sqrt(d*x
^2 + c))/(c*x^2)]
```

giac [A] time = 0.44, size = 100, normalized size = 0.56

$$\frac{2\sqrt{dx^2+c}\,bd\mathrm{sgn}\left(bx^2+a\right)+\frac{\left(2\,bcd\mathrm{sgn}\left(bx^2+a\right)+ad^2\mathrm{sgn}\left(bx^2+a\right)\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{dx^2+c}\,ad\mathrm{sgn}\left(bx^2+a\right)}{x^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(d*x^2 + c)*b*d*sgn(b*x^2 + a) + (2*b*c*d*sgn(b*x^2 + a) + a*d^2
*sgn(b*x^2 + a))*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - sqrt(d*x^2 + c
)*a*d*sgn(b*x^2 + a)/x^2)/d
```

maple [A] time = 0.01, size = 133, normalized size = 0.75

$$-\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(a\sqrt{c}\,d\,x^2\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)+2b\,c^{\frac{3}{2}}x^2\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)-\sqrt{d\,x^2+c}\,ad\,x^2-2\sqrt{d\,x^2+c}\,bc\right.}{2\left(b\,x^2+a\right)c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x)
```

```
[Out] -1/2*((b*x^2+a)^2)^(1/2)*(c^(1/2)*ln(2*(c+(d*x^2+c)^(1/2)*c^(1/2))/x)*x^2*a
*d+2*c^(3/2)*ln(2*(c+(d*x^2+c)^(1/2)*c^(1/2))/x)*x^2*b-(d*x^2+c)^(1/2)*x^2*
a*d-2*(d*x^2+c)^(1/2)*x^2*b*c+(d*x^2+c)^(3/2)*a)/(b*x^2+a)/c/x^2
```

maxima [A] time = 1.23, size = 83, normalized size = 0.47

$$-b\sqrt{c}\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{ad\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\sqrt{c}}+\sqrt{dx^2+c}\,b+\frac{\sqrt{dx^2+c}\,ad}{2\,c}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}a}{2\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -b*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) - 1/2*a*d*arcsinh(c/(sqrt(c*d)*abs
(x)))/sqrt(c) + sqrt(d*x^2 + c)*b + 1/2*sqrt(d*x^2 + c)*a*d/c - 1/2*(d*x^2
+ c)^(3/2)*a/(c*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}\,\sqrt{\left(b\,x^2+a\right)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^3,x)
```

```
[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

3.273 $\int x^3 \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=78

$$\frac{1}{8}x^8\left(e(ae+2bd)+cd^2\right)+\frac{1}{6}dx^6(2ae+bd)+\frac{1}{4}ad^2x^4+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{12}ce^2x^{12}$$

```
[Out] 1/4*a*d^2*x^4+1/6*d*(2*a*e+b*d)*x^6+1/8*(c*d^2+e*(a*e+2*b*d))*x^8+1/10*e*(b
*e+2*c*d)*x^10+1/12*c*e^2*x^12
```

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1251, 771}

$$\frac{1}{8}x^8\left(e(ae+2bd)+cd^2\right)+\frac{1}{6}dx^6(2ae+bd)+\frac{1}{4}ad^2x^4+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{12}ce^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^4)/4 + (d*(b*d + 2*a*e)*x^6)/6 + ((c*d^2 + e*(2*b*d + a*e))*x^8)/8
 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^12)/12
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^3\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(d+ex)^2\left(a+bx+cx^2\right)\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(ad^2x+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^3+e(2cd+b\right.\right.\\ &= \frac{1}{4}ad^2x^4+\frac{1}{6}d(bd+2ae)x^6+\frac{1}{8}\left(cd^2+e(2bd+ae)\right)x^8+\frac{1}{10}e(2cd+be)x^{10}\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.92

$$\frac{1}{120}x^4\left(15x^4\left(e(ae+2bd)+cd^2\right)+20dx^2(2ae+bd)+30ad^2+12ex^6(be+2cd)+10ce^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (x^4*(30*a*d^2 + 20*d*(b*d + 2*a*e)*x^2 + 15*(c*d^2 + e*(2*b*d + a*e))*x^4
+ 12*e*(2*c*d + b*e)*x^6 + 10*c*e^2*x^8))/120
```

fricas [A] time = 0.54, size = 79, normalized size = 1.01

$$\frac{1}{12}x^{12}e^2c+\frac{1}{5}x^{10}edc+\frac{1}{10}x^{10}e^2b+\frac{1}{8}x^8d^2c+\frac{1}{4}x^8edb+\frac{1}{8}x^8e^2a+\frac{1}{6}x^6d^2b+\frac{1}{3}x^6eda+\frac{1}{4}x^4d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^2*c + 1/5*x^10*e*d*c + 1/10*x^10*e^2*b + 1/8*x^8*d^2*c + 1/4*x^
8*e*d*b + 1/8*x^8*e^2*a + 1/6*x^6*d^2*b + 1/3*x^6*e*d*a + 1/4*x^4*d^2*a
```

giac [A] time = 0.27, size = 79, normalized size = 1.01

$$\frac{1}{12}cx^{12}e^2+\frac{1}{5}cdx^{10}e+\frac{1}{10}bx^{10}e^2+\frac{1}{8}cd^2x^8+\frac{1}{4}bdx^8e+\frac{1}{8}ax^8e^2+\frac{1}{6}bd^2x^6+\frac{1}{3}adx^6e+\frac{1}{4}ad^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*c*x^12*e^2 + 1/5*c*d*x^10*e + 1/10*b*x^10*e^2 + 1/8*c*d^2*x^8 + 1/4*b*
d*x^8*e + 1/8*a*x^8*e^2 + 1/6*b*d^2*x^6 + 1/3*a*d*x^6*e + 1/4*a*d^2*x^4
```

maple [A] time = 0.00, size = 73, normalized size = 0.94

$$\frac{ce^2x^{12}}{12}+\frac{\left(e^2b+2dec\right)x^{10}}{10}+\frac{\left(ae^2+2deb+cd^2\right)x^8}{8}+\frac{ad^2x^4}{4}+\frac{\left(2dea+d^2b\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/12*c*e^2*x^12+1/10*(b*e^2+2*c*d*e)*x^10+1/8*(a*e^2+2*b*d*e+c*d^2)*x^8+1/6
*(2*a*d*e+b*d^2)*x^6+1/4*a*d^2*x^4
```

maxima [A] time = 1.11, size = 72, normalized size = 0.92

$$\frac{1}{12}ce^2x^{12}+\frac{1}{10}\left(2\,cde+be^2\right)x^{10}+\frac{1}{8}\left(cd^2+2\,bde+ae^2\right)x^8+\frac{1}{4}ad^2x^4+\frac{1}{6}\left(bd^2+2\,ade\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*c*e^2*x^12 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/8*(c*d^2 + 2*b*d*e + a*e^
2)*x^8 + 1/4*a*d^2*x^4 + 1/6*(b*d^2 + 2*a*d*e)*x^6
```

mupad [B] time = 0.04, size = 73, normalized size = 0.94

$$x^8\left(\frac{cd^2}{8}+\frac{bde}{4}+\frac{ae^2}{8}\right)+x^6\left(\frac{bd^2}{6}+\frac{aed}{3}\right)+x^{10}\left(\frac{be^2}{10}+\frac{cde}{5}\right)+\frac{ad^2x^4}{4}+\frac{ce^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^8*((a*e^2)/8 + (c*d^2)/8 + (b*d*e)/4) + x^6*((b*d^2)/6 + (a*d*e)/3) + x^1
0*((b*e^2)/10 + (c*d*e)/5) + (a*d^2*x^4)/4 + (c*e^2*x^12)/12
```

sympy [A] time = 0.08, size = 76, normalized size = 0.97

$$\frac{ad^2x^4}{4}+\frac{ce^2x^{12}}{12}+x^{10}\left(\frac{be^2}{10}+\frac{cde}{5}\right)+x^8\left(\frac{ae^2}{8}+\frac{bde}{4}+\frac{cd^2}{8}\right)+x^6\left(\frac{ade}{3}+\frac{bd^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**4/4 + c*e**2*x**12/12 + x**10*(b*e**2/10 + c*d*e/5) + x**8*(a*e**
2/8 + b*d*e/4 + c*d**2/8) + x**6*(a*d*e/3 + b*d**2/6)
```

3.274 $\int x^2 \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=78

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

```
[Out] 1/3*a*d^2*x^3+1/5*d*(2*a*e+b*d)*x^5+1/7*(c*d^2+e*(a*e+2*b*d))*x^7+1/9*e*(b*
e+2*c*d)*x^9+1/11*c*e^2*x^11
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1261}

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7
 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx=\int\left(ad^2x^2+d(bd+2ae)x^4+\left(cd^2+e(2bd+ae)\right)x^6+e(2cd+be)x^8+ce^2x^1\right.$$
$$=\frac{1}{3}ad^2x^3+\frac{1}{5}d(bd+2ae)x^5+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7+\frac{1}{9}e(2cd+be)x^9+\frac{1}{1}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 1.00

$$\frac{1}{7}x^7\left(ae^2+2bde+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7
 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

fricas [A] time = 0.88, size = 79, normalized size = 1.01

$$\frac{1}{11}x^{11}e^2c+\frac{2}{9}x^9edc+\frac{1}{9}x^9e^2b+\frac{1}{7}x^7d^2c+\frac{2}{7}x^7edb+\frac{1}{7}x^7e^2a+\frac{1}{5}x^5d^2b+\frac{2}{5}x^5eda+\frac{1}{3}x^3d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^2*c + 2/9*x^9*e*d*c + 1/9*x^9*e^2*b + 1/7*x^7*d^2*c + 2/7*x^7*e
*d*b + 1/7*x^7*e^2*a + 1/5*x^5*d^2*b + 2/5*x^5*e*d*a + 1/3*x^3*d^2*a
```

giac [A] time = 0.39, size = 79, normalized size = 1.01

$$\frac{1}{11}\,cx^{11}e^{2}+\frac{2}{9}\,cdx^{9}e+\frac{1}{9}\,bx^{9}e^{2}+\frac{1}{7}\,cd^{2}x^{7}+\frac{2}{7}\,bdx^{7}e+\frac{1}{7}\,ax^{7}e^{2}+\frac{1}{5}\,bd^{2}x^{5}+\frac{2}{5}\,adx^{5}e+\frac{1}{3}\,ad^{2}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^2 + 2/9*c*d*x^9*e + 1/9*b*x^9*e^2 + 1/7*c*d^2*x^7 + 2/7*b*d*x
^7*e + 1/7*a*x^7*e^2 + 1/5*b*d^2*x^5 + 2/5*a*d*x^5*e + 1/3*a*d^2*x^3
```

maple [A] time = 0.00, size = 73, normalized size = 0.94

$$\frac{c\,e^{2}x^{11}}{11}+\frac{\left(e^{2}b+2dec\right)x^{9}}{9}+\frac{\left(a\,e^{2}+2deb+c\,d^{2}\right)x^{7}}{7}+\frac{a\,d^{2}x^{3}}{3}+\frac{\left(2dea+d^{2}b\right)x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/11*c*e^2*x^11+1/9*(b*e^2+2*c*d*e)*x^9+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/5*(
2*a*d*e+b*d^2)*x^5+1/3*a*d^2*x^3
```

maxima [A] time = 1.19, size = 72, normalized size = 0.92

$$\frac{1}{11}\,ce^{2}x^{11}+\frac{1}{9}\left(2\,cde+be^{2}\right)x^{9}+\frac{1}{7}\left(cd^{2}+2\,bde+ae^{2}\right)x^{7}+\frac{1}{3}\,ad^{2}x^{3}+\frac{1}{5}\left(bd^{2}+2\,ade\right)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/11*c*e^2*x^11 + 1/9*(2*c*d*e + b*e^2)*x^9 + 1/7*(c*d^2 + 2*b*d*e + a*e^2)
*x^7 + 1/3*a*d^2*x^3 + 1/5*(b*d^2 + 2*a*d*e)*x^5
```

mupad [B] time = 0.03, size = 73, normalized size = 0.94

$$x^{7}\left(\frac{c\,d^{2}}{7}+\frac{2\,b\,d\,e}{7}+\frac{a\,e^{2}}{7}\right)+x^{5}\left(\frac{b\,d^{2}}{5}+\frac{2\,a\,e\,d}{5}\right)+x^{9}\left(\frac{b\,e^{2}}{9}+\frac{2\,c\,d\,e}{9}\right)+\frac{a\,d^{2}\,x^{3}}{3}+\frac{c\,e^{2}\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^7*((a*e^2)/7 + (c*d^2)/7 + (2*b*d*e)/7) + x^5*((b*d^2)/5 + (2*a*d*e)/5) +
 x^9*((b*e^2)/9 + (2*c*d*e)/9) + (a*d^2*x^3)/3 + (c*e^2*x^11)/11
```

sympy [A] time = 0.08, size = 82, normalized size = 1.05

$$\frac{ad^{2}x^{3}}{3}+\frac{ce^{2}x^{11}}{11}+x^{9}\left(\frac{be^{2}}{9}+\frac{2cde}{9}\right)+x^{7}\left(\frac{ae^{2}}{7}+\frac{2bde}{7}+\frac{cd^{2}}{7}\right)+x^{5}\left(\frac{2ade}{5}+\frac{bd^{2}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**3/3 + c*e**2*x**11/11 + x**9*(b*e**2/9 + 2*c*d*e/9) + x**7*(a*e**
2/7 + 2*b*d*e/7 + c*d**2/7) + x**5*(2*a*d*e/5 + b*d**2/5)
```

3.275 $\int x\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=75

$$\frac{\left(d+ex^2\right)^3\left(ae^2-bde+cd^2\right)}{6e^3}-\frac{\left(d+ex^2\right)^4(2cd-be)}{8e^3}+\frac{c\left(d+ex^2\right)^5}{10e^3}$$

```
[Out] 1/6*(a*e^2-b*d*e+c*d^2)*(e*x^2+d)^3/e^3-1/8*(-b*e+2*c*d)*(e*x^2+d)^4/e^3+1/
10*c*(e*x^2+d)^5/e^3
```

Rubi [A] time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1247, 698}

$$\frac{\left(d+ex^2\right)^3\left(ae^2-bde+cd^2\right)}{6e^3}-\frac{\left(d+ex^2\right)^4(2cd-be)}{8e^3}+\frac{c\left(d+ex^2\right)^5}{10e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x^2)^3)/(6*e^3) - ((2*c*d - b*e)*(d + e*x^2
)^4)/(8*e^3) + (c*(d + e*x^2)^5)/(10*e^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx&=\frac{1}{2}\text{Subst}\left(\int(d+ex)^2\left(a+bx+cx^2\right)dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{\left(cd^2-bde+ae^2\right)(d+ex)^2}{e^2}+\frac{(-2cd+be)(d+ex)^3}{e^2}+\frac{c(d+ex}{e^2}\right.\right.\\&=\frac{\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)^3}{6e^3}-\frac{(2cd-be)\left(d+ex^2\right)^4}{8e^3}+\frac{c\left(d+ex^2\right)^5}{10e^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 0.96

$$\frac{1}{120}x^2\left(20x^4\left(e(ae+2bd)+cd^2\right)+30dx^2(2ae+bd)+60ad^2+15ex^6(be+2cd)+12ce^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (x^2*(60*a*d^2 + 30*d*(b*d + 2*a*e)*x^2 + 20*(c*d^2 + e*(2*b*d + a*e))*x^4
+ 15*e*(2*c*d + b*e)*x^6 + 12*c*e^2*x^8))/120
```

fricas [A] time = 0.83, size = 79, normalized size = 1.05

$$\frac{1}{10}x^{10}e^2c + \frac{1}{4}x^8edc + \frac{1}{8}x^8e^2b + \frac{1}{6}x^6d^2c + \frac{1}{3}x^6edb + \frac{1}{6}x^6e^2a + \frac{1}{4}x^4d^2b + \frac{1}{2}x^4eda + \frac{1}{2}x^2d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^2*c + 1/4*x^8*e*d*c + 1/8*x^8*e^2*b + 1/6*x^6*d^2*c + 1/3*x^6*e
*d*b + 1/6*x^6*e^2*a + 1/4*x^4*d^2*b + 1/2*x^4*e*d*a + 1/2*x^2*d^2*a
```

giac [A] time = 0.27, size = 79, normalized size = 1.05

$$\frac{1}{10}\,cx^{10}e^2 + \frac{1}{4}\,cdx^8e + \frac{1}{8}\,bx^8e^2 + \frac{1}{6}\,cd^2x^6 + \frac{1}{3}\,bdx^6e + \frac{1}{6}\,ax^6e^2 + \frac{1}{4}\,bd^2x^4 + \frac{1}{2}\,adx^4e + \frac{1}{2}\,ad^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/10*c*x^10*e^2 + 1/4*c*d*x^8*e + 1/8*b*x^8*e^2 + 1/6*c*d^2*x^6 + 1/3*b*d*x
^6*e + 1/6*a*x^6*e^2 + 1/4*b*d^2*x^4 + 1/2*a*d*x^4*e + 1/2*a*d^2*x^2
```

maple [A] time = 0.00, size = 73, normalized size = 0.97

$$\frac{c\,e^2x^{10}}{10} + \frac{\left(e^2b + 2dec\right)x^8}{8} + \frac{\left(a\,e^2 + 2deb + c\,d^2\right)x^6}{6} + \frac{a\,d^2x^2}{2} + \frac{\left(2dea + d^2b\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/10*c*e^2*x^10+1/8*(b*e^2+2*c*d*e)*x^8+1/6*(a*e^2+2*b*d*e+c*d^2)*x^6+1/4*(
2*a*d*e+b*d^2)*x^4+1/2*a*d^2*x^2
```

maxima [A] time = 1.21, size = 72, normalized size = 0.96

$$\frac{1}{10}\,ce^2x^{10} + \frac{1}{8}\left(2\,cde + be^2\right)x^8 + \frac{1}{6}\left(cd^2 + 2\,bde + ae^2\right)x^6 + \frac{1}{2}\,ad^2x^2 + \frac{1}{4}\left(bd^2 + 2\,ade\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/10*c*e^2*x^10 + 1/8*(2*c*d*e + b*e^2)*x^8 + 1/6*(c*d^2 + 2*b*d*e + a*e^2)
*x^6 + 1/2*a*d^2*x^2 + 1/4*(b*d^2 + 2*a*d*e)*x^4
```

mupad [B] time = 0.03, size = 73, normalized size = 0.97

$$x^6\left(\frac{c\,d^2}{6} + \frac{b\,d\,e}{3} + \frac{a\,e^2}{6}\right) + x^4\left(\frac{b\,d^2}{4} + \frac{a\,e\,d}{2}\right) + x^8\left(\frac{b\,e^2}{8} + \frac{c\,d\,e}{4}\right) + \frac{a\,d^2\,x^2}{2} + \frac{c\,e^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^6*((a*e^2)/6 + (c*d^2)/6 + (b*d*e)/3) + x^4*((b*d^2)/4 + (a*d*e)/2) + x^8
*((b*e^2)/8 + (c*d*e)/4) + (a*d^2*x^2)/2 + (c*e^2*x^10)/10
```

sympy [A] time = 0.08, size = 76, normalized size = 1.01

$$\frac{ad^2x^2}{2} + \frac{ce^2x^{10}}{10} + x^8\left(\frac{be^2}{8} + \frac{cde}{4}\right) + x^6\left(\frac{ae^2}{6} + \frac{bde}{3} + \frac{cd^2}{6}\right) + x^4\left(\frac{ade}{2} + \frac{bd^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**2/2 + c*e**2*x**10/10 + x**8*(b*e**2/8 + c*d*e/4) + x**6*(a*e**2/
6 + b*d*e/3 + c*d**2/6) + x**4*(a*d*e/2 + b*d**2/4)
```

3.276 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=73

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x+1/3*d*(2*a*e+b*d)*x^3+1/5*(c*d^2+e*(a*e+2*b*d))*x^5+1/7*e*(b*e+2*c*
d)*x^7+1/9*c*e^2*x^9
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx &= \int \left(ad^2+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^4+e(2cd+be)x^6+ce^2x^8\right)\\ &= ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5+\frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce\end{aligned}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.00

$$\frac{1}{5}x^5\left(ae^2+2bde+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

fricas [A] time = 0.55, size = 76, normalized size = 1.04

$$\frac{1}{9}x^9e^2c+\frac{2}{7}x^7edc+\frac{1}{7}x^7e^2b+\frac{1}{5}x^5d^2c+\frac{2}{5}x^5edb+\frac{1}{5}x^5e^2a+\frac{1}{3}x^3d^2b+\frac{2}{3}x^3eda+xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c + 2/7*x^7*e*d*c + 1/7*x^7*e^2*b + 1/5*x^5*d^2*c + 2/5*x^5*e*d
*b + 1/5*x^5*e^2*a + 1/3*x^3*d^2*b + 2/3*x^3*e*d*a + x*d^2*a
```

giac [A] time = 0.35, size = 76, normalized size = 1.04

$$\frac{1}{9}\,cx^9e^2 + \frac{2}{7}\,cdx^7e + \frac{1}{7}\,bx^7e^2 + \frac{1}{5}\,cd^2x^5 + \frac{2}{5}\,bdx^5e + \frac{1}{5}\,ax^5e^2 + \frac{1}{3}\,bd^2x^3 + \frac{2}{3}\,adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/7*b*x^7*e^2 + 1/5*c*d^2*x^5 + 2/5*b*d*x^5
*e + 1/5*a*x^5*e^2 + 1/3*b*d^2*x^3 + 2/3*a*d*x^3*e + a*d^2*x
```

maple [A] time = 0.00, size = 70, normalized size = 0.96

$$\frac{c\,e^2x^9}{9} + \frac{\left(e^2b + 2dec\right)x^7}{7} + \frac{\left(a\,e^2 + 2deb + c\,d^2\right)x^5}{5} + a\,d^2x + \frac{\left(2dea + d^2b\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x^5+1/3*(2*
a*d*e+b*d^2)*x^3+a*d^2*x
```

maxima [A] time = 1.17, size = 69, normalized size = 0.95

$$\frac{1}{9}\,ce^2x^9 + \frac{1}{7}\left(2\,cde + be^2\right)x^7 + \frac{1}{5}\left(cd^2 + 2\,bde + ae^2\right)x^5 + ad^2x + \frac{1}{3}\left(bd^2 + 2\,ade\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*x
^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

mupad [B] time = 0.03, size = 70, normalized size = 0.96

$$x^5\left(\frac{c\,d^2}{5} + \frac{2\,b\,d\,e}{5} + \frac{a\,e^2}{5}\right) + x^3\left(\frac{b\,d^2}{3} + \frac{2\,a\,e\,d}{3}\right) + x^7\left(\frac{b\,e^2}{7} + \frac{2\,c\,d\,e}{7}\right) + \frac{c\,e^2\,x^9}{9} + a\,d^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^5*((a*e^2)/5 + (c*d^2)/5 + (2*b*d*e)/5) + x^3*((b*d^2)/3 + (2*a*d*e)/3) +
 x^7*((b*e^2)/7 + (2*c*d*e)/7) + (c*e^2*x^9)/9 + a*d^2*x
```

sympy [A] time = 0.08, size = 78, normalized size = 1.07

$$ad^2x + \frac{ce^2x^9}{9} + x^7\left(\frac{be^2}{7} + \frac{2cde}{7}\right) + x^5\left(\frac{ae^2}{5} + \frac{2bde}{5} + \frac{cd^2}{5}\right) + x^3\left(\frac{2ade}{3} + \frac{bd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 + 2
*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

3.277 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx$

Optimal. Leaf size=74

$$\frac{1}{4}x^4\left(e(ae+2bd)+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

```
[Out] 1/2*d*(2*a*e+b*d)*x^2+1/4*(c*d^2+e*(a*e+2*b*d))*x^4+1/6*e*(b*e+2*c*d)*x^6+1
/8*c*e^2*x^8+a*d^2*ln(x)
```

Rubi [A] time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1251, 893}

$$\frac{1}{4}x^4\left(e(ae+2bd)+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + e*(2*b*d + a*e))*x^4)/4 + (e*(2*c*d + b
*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^2(a+bx+cx^2)}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(d(bd+2ae)+\frac{ad^2}{x}+\left(cd^2+e(2bd+ae)\right)x+e(2cd+be)x^2+\right.\right.\\ &= \frac{1}{2}d(bd+2ae)x^2+\frac{1}{4}\left(cd^2+e(2bd+ae)\right)x^4+\frac{1}{6}e(2cd+be)x^6+\frac{1}{8}ce^2x^8+a\end{aligned}$$

Mathematica [A] time = 0.02, size = 74, normalized size = 1.00

$$\frac{1}{4}x^4\left(ae^2+2bde+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + 2*b*d*e + a*e^2)*x^4)/4 + (e*(2*c*d + b
*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

fricas [A] time = 0.80, size = 70, normalized size = 0.95

$$\frac{1}{8}ce^2x^8 + \frac{1}{6}\left(2\,cde + be^2\right)x^6 + \frac{1}{4}\left(cd^2 + 2\,bde + ae^2\right)x^4 + ad^2\log(x) + \frac{1}{2}\left(bd^2 + 2\,ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + a*d^2*log(x) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

giac [A] time = 0.26, size = 79, normalized size = 1.07

$$\frac{1}{8}cx^8e^2 + \frac{1}{3}cdx^6e + \frac{1}{6}bx^6e^2 + \frac{1}{4}cd^2x^4 + \frac{1}{2}bdx^4e + \frac{1}{4}ax^4e^2 + \frac{1}{2}bd^2x^2 + adx^2e + \frac{1}{2}ad^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/8*c*x^8*e^2 + 1/3*c*d*x^6*e + 1/6*b*x^6*e^2 + 1/4*c*d^2*x^4 + 1/2*b*d*x^4
*e + 1/4*a*x^4*e^2 + 1/2*b*d^2*x^2 + a*d*x^2*e + 1/2*a*d^2*log(x^2)
```

maple [A] time = 0.00, size = 77, normalized size = 1.04

$$\frac{c\,e^2x^8}{8} + \frac{b\,e^2x^6}{6} + \frac{cde\,x^6}{3} + \frac{a\,e^2x^4}{4} + \frac{bde\,x^4}{2} + \frac{c\,d^2x^4}{4} + ade\,x^2 + \frac{b\,d^2x^2}{2} + a\,d^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x)
```

```
[Out] 1/8*c*e^2*x^8+1/6*x^6*b*e^2+1/3*x^6*c*d*e+1/4*x^4*a*e^2+1/2*x^4*b*d*e+1/4*x
^4*c*d^2+x^2*a*d*e+1/2*x^2*b*d^2+a*d^2*ln(x)
```

maxima [A] time = 1.12, size = 73, normalized size = 0.99

$$\frac{1}{8}ce^2x^8 + \frac{1}{6}\left(2\,cde + be^2\right)x^6 + \frac{1}{4}\left(cd^2 + 2\,bde + ae^2\right)x^4 + \frac{1}{2}ad^2\log\left(x^2\right) + \frac{1}{2}\left(bd^2 + 2\,ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + 1/2*a*d^2*log(x^2) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

mupad [B] time = 0.03, size = 70, normalized size = 0.95

$$x^4\left(\frac{c\,d^2}{4} + \frac{b\,d\,e}{2} + \frac{a\,e^2}{4}\right) + x^2\left(\frac{b\,d^2}{2} + a\,e\,d\right) + x^6\left(\frac{b\,e^2}{6} + \frac{c\,d\,e}{3}\right) + \frac{c\,e^2\,x^8}{8} + a\,d^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x)
```

```
[Out] x^4*((a*e^2)/4 + (c*d^2)/4 + (b*d*e)/2) + x^2*((b*d^2)/2 + a*d*e) + x^6*((b
*e^2)/6 + (c*d*e)/3) + (c*e^2*x^8)/8 + a*d^2*log(x)
```

sympy [A] time = 0.17, size = 73, normalized size = 0.99

$$ad^2\log(x) + \frac{ce^2x^8}{8} + x^6\left(\frac{be^2}{6} + \frac{cde}{3}\right) + x^4\left(\frac{ae^2}{4} + \frac{bde}{2} + \frac{cd^2}{4}\right) + x^2\left(ade + \frac{bd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] a*d**2*log(x) + c*e**2*x**8/8 + x**6*(b*e**2/6 + c*d*e/3) + x**4*(a*e**2/4
+ b*d*e/2 + c*d**2/4) + x**2*(a*d*e + b*d**2/2)
```

3.278 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2} dx$

Optimal. Leaf size=71

$$\frac{1}{3}x^3\left(e(ae+2bd)+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

```
[Out] -a*d^2/x+d*(2*a*e+b*d)*x+1/3*(c*d^2+e*(a*e+2*b*d))*x^3+1/5*e*(b*e+2*c*d)*x^
5+1/7*c*e^2*x^7
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1261}

$$\frac{1}{3}x^3\left(e(ae+2bd)+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + e*(2*b*d + a*e))*x^3)/3 + (e*(
2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2} dx &= \int \left(d(bd+2ae)+\frac{ad^2}{x^2}+\left(cd^2+e(2bd+ae)\right)x^2+e(2cd+be)x^4+ce^2x^6\right) dx\\ &= -\frac{ad^2}{x}+d(bd+2ae)x+\frac{1}{3}\left(cd^2+e(2bd+ae)\right)x^3+\frac{1}{5}e(2cd+be)x^5+\frac{1}{7}ce^2x^7\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.00

$$\frac{1}{3}x^3\left(ae^2+2bde+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + 2*b*d*e + a*e^2)*x^3)/3 + (e*(
2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

fricas [A] time = 0.55, size = 74, normalized size = 1.04

$$\frac{15\,ce^2x^8+21\left(2\,cde+be^2\right)x^6+35\left(cd^2+2\,bde+ae^2\right)x^4-105\,ad^2+105\left(bd^2+2\,ade\right)x^2}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*c*e^2*x^8 + 21*(2*c*d*e + b*e^2)*x^6 + 35*(c*d^2 + 2*b*d*e + a*e^
2)*x^4 - 105*a*d^2 + 105*(b*d^2 + 2*a*d*e)*x^2)/x
```

giac [A] time = 0.36, size = 74, normalized size = 1.04

$$\frac{1}{7}\,cx^7e^2 + \frac{2}{5}\,cdx^5e + \frac{1}{5}\,bx^5e^2 + \frac{1}{3}\,cd^2x^3 + \frac{2}{3}\,bdx^3e + \frac{1}{3}\,ax^3e^2 + bd^2x + 2\,adxe - \frac{ad^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e^2 + 2/5*c*d*x^5*e + 1/5*b*x^5*e^2 + 1/3*c*d^2*x^3 + 2/3*b*d*x^3
*e + 1/3*a*x^3*e^2 + b*d^2*x + 2*a*d*x*e - a*d^2/x
```

maple [A] time = 0.00, size = 75, normalized size = 1.06

$$\frac{c\,e^2x^7}{7} + \frac{b\,e^2x^5}{5} + \frac{2cde\,x^5}{5} + \frac{a\,e^2x^3}{3} + \frac{2bde\,x^3}{3} + \frac{c\,d^2x^3}{3} + 2adex + b\,d^2x - \frac{a\,d^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x)
```

```
[Out] 1/7*c*e^2*x^7+1/5*x^5*b*e^2+2/5*x^5*c*d*e+1/3*x^3*a*e^2+2/3*x^3*b*d*e+1/3*x
^3*c*d^2+2*d*e*a*x+d^2*b*x-a*d^2/x
```

maxima [A] time = 1.07, size = 69, normalized size = 0.97

$$\frac{1}{7}\,ce^2x^7 + \frac{1}{5}\left(2\,cde + be^2\right)x^5 + \frac{1}{3}\left(cd^2 + 2\,bde + ae^2\right)x^3 - \frac{ad^2}{x} + \left(bd^2 + 2\,ade\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c*e^2*x^7 + 1/5*(2*c*d*e + b*e^2)*x^5 + 1/3*(c*d^2 + 2*b*d*e + a*e^2)*x
^3 - a*d^2/x + (b*d^2 + 2*a*d*e)*x
```

mupad [B] time = 0.03, size = 70, normalized size = 0.99

$$x^3\left(\frac{c\,d^2}{3} + \frac{2\,b\,d\,e}{3} + \frac{a\,e^2}{3}\right) + x\,\left(b\,d^2 + 2\,a\,e\,d\right) + x^5\left(\frac{b\,e^2}{5} + \frac{2\,c\,d\,e}{5}\right) - \frac{a\,d^2}{x} + \frac{c\,e^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x)
```

```
[Out] x^3*((a*e^2)/3 + (c*d^2)/3 + (2*b*d*e)/3) + x*(b*d^2 + 2*a*d*e) + x^5*((b*e
^2)/5 + (2*c*d*e)/5) - (a*d^2)/x + (c*e^2*x^7)/7
```

sympy [A] time = 0.16, size = 73, normalized size = 1.03

$$-\frac{ad^2}{x} + \frac{ce^2x^7}{7} + x^5\left(\frac{be^2}{5} + \frac{2cde}{5}\right) + x^3\left(\frac{ae^2}{3} + \frac{2bde}{3} + \frac{cd^2}{3}\right) + x\left(2ade + bd^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -a*d**2/x + c*e**2*x**7/7 + x**5*(b*e**2/5 + 2*c*d*e/5) + x**3*(a*e**2/3 +
2*b*d*e/3 + c*d**2/3) + x*(2*a*d*e + b*d**2)
```

3.279 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx$

Optimal. Leaf size=74

$$\frac{1}{2}x^2\left(e(ae+2bd)+cd^2\right)+d\log(x)(2ae+bd)-\frac{ad^2}{2x^2}+\frac{1}{4}ex^4(be+2cd)+\frac{1}{6}ce^2x^6$$

```
[Out] -1/2*a*d^2/x^2+1/2*(c*d^2+e*(a*e+2*b*d))*x^2+1/4*e*(b*e+2*c*d)*x^4+1/6*c*e^
2*x^6+d*(2*a*e+b*d)*ln(x)
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1251, 893}

$$\frac{1}{2}x^2\left(e(ae+2bd)+cd^2\right)+d\log(x)(2ae+bd)-\frac{ad^2}{2x^2}+\frac{1}{4}ex^4(be+2cd)+\frac{1}{6}ce^2x^6$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -(a*d^2)/(2*x^2) + ((c*d^2 + e*(2*b*d + a*e))*x^2)/2 + (e*(2*c*d + b*e)*x^4
)/4 + (c*e^2*x^6)/6 + d*(b*d + 2*a*e)*Log[x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(d+ex)^2(a+bx+cx^2)}{x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(cd^2\left(1+\frac{e(2bd+ae)}{cd^2}\right)+\frac{ad^2}{x^2}+\frac{d(bd+2ae)}{x}+e(2cd+be)x+ce^2\right.\right.\\
&= -\frac{ad^2}{2x^2}+\frac{1}{2}\left(cd^2+e(2bd+ae)\right)x^2+\frac{1}{4}e(2cd+be)x^4+\frac{1}{6}ce^2x^6+d(bd+2ae)\log
\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.96

$$\frac{1}{12}\left(6x^2\left(e(ae+2bd)+cd^2\right)+12d\log(x)(2ae+bd)-\frac{6ad^2}{x^2}+3ex^4(be+2cd)+2ce^2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] ((-6*a*d^2)/x^2 + 6*(c*d^2 + e*(2*b*d + a*e))*x^2 + 3*e*(2*c*d + b*e)*x^4 +
 2*c*e^2*x^6 + 12*d*(b*d + 2*a*e)*Log[x])/12
```

fricas [A] time = 0.90, size = 76, normalized size = 1.03

$$\frac{2\,ce^{2}x^{8}+3\left(2\,cde+be^{2}\right)x^{6}+6\left(cd^{2}+2\,bde+ae^{2}\right)x^{4}+12\left(bd^{2}+2\,ade\right)x^{2}\log(x)-6\,ad^{2}}{12\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*c*e^2*x^8 + 3*(2*c*d*e + b*e^2)*x^6 + 6*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + 12*(b*d^2 + 2*a*d*e)*x^2*log(x) - 6*a*d^2)/x^2
```

giac [A] time = 0.38, size = 97, normalized size = 1.31

$$\frac{1}{6}\,cx^{6}e^{2}+\frac{1}{2}\,cdx^{4}e+\frac{1}{4}\,bx^{4}e^{2}+\frac{1}{2}\,cd^{2}x^{2}+bdx^{2}e+\frac{1}{2}\,ax^{2}e^{2}+\frac{1}{2}\left(bd^{2}+2\,ade\right)\log\left(x^{2}\right)-\frac{bd^{2}x^{2}+2\,adx^{2}e+ad^{2}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c*x^6*e^2 + 1/2*c*d*x^4*e + 1/4*b*x^4*e^2 + 1/2*c*d^2*x^2 + b*d*x^2*e +
 1/2*a*x^2*e^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*(b*d^2*x^2 + 2*a*d*x^
2*e + a*d^2)/x^2
```

maple [A] time = 0.01, size = 76, normalized size = 1.03

$$\frac{c\,e^{2}x^{6}}{6}+\frac{b\,e^{2}x^{4}}{4}+\frac{cde\,x^{4}}{2}+\frac{a\,e^{2}x^{2}}{2}+bde\,x^{2}+\frac{c\,d^{2}x^{2}}{2}+2ade\ln(x)+b\,d^{2}\ln(x)-\frac{a\,d^{2}}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x)
```

```
[Out] 1/6*c*e^2*x^6+1/4*x^4*b*e^2+1/2*x^4*c*d*e+1/2*x^2*a*e^2+x^2*b*d*e+1/2*x^2*c
*d^2+2*ln(x)*a*d*e+ln(x)*b*d^2-1/2*a*d^2/x^2
```

maxima [A] time = 1.09, size = 73, normalized size = 0.99

$$\frac{1}{6}\,ce^{2}x^{6}+\frac{1}{4}\left(2\,cde+be^{2}\right)x^{4}+\frac{1}{2}\left(cd^{2}+2\,bde+ae^{2}\right)x^{2}+\frac{1}{2}\left(bd^{2}+2\,ade\right)\log\left(x^{2}\right)-\frac{ad^{2}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c*e^2*x^6 + 1/4*(2*c*d*e + b*e^2)*x^4 + 1/2*(c*d^2 + 2*b*d*e + a*e^2)*x
^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*a*d^2/x^2
```

mupad [B] time = 0.04, size = 70, normalized size = 0.95

$$x^{2}\left(\frac{c\,d^{2}}{2}+b\,d\,e+\frac{a\,e^{2}}{2}\right)+x^{4}\left(\frac{b\,e^{2}}{4}+\frac{c\,d\,e}{2}\right)+\ln(x)\,\left(b\,d^{2}+2\,a\,e\,d\right)-\frac{a\,d^{2}}{2\,x^{2}}+\frac{c\,e^{2}\,x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x)
```

```
[Out] x^2*((a*e^2)/2 + (c*d^2)/2 + b*d*e) + x^4*((b*e^2)/4 + (c*d*e)/2) + log(x)*
(b*d^2 + 2*a*d*e) - (a*d^2)/(2*x^2) + (c*e^2*x^6)/6
```

sympy [A] time = 0.26, size = 71, normalized size = 0.96

$$-\frac{ad^2}{2x^2} + \frac{ce^2x^6}{6} + d(2ae + bd)\log(x) + x^4\left(\frac{be^2}{4} + \frac{cde}{2}\right) + x^2\left(\frac{ae^2}{2} + bde + \frac{cd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -a*d**2/(2*x**2) + c*e**2*x**6/6 + d*(2*a*e + b*d)*log(x) + x**4*(b*e**2/4
+ c*d*e/2) + x**2*(a*e**2/2 + b*d*e + c*d**2/2)
```

3.280 $\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2} dx$

Optimal. Leaf size=168

$$-\frac{dx\left(4cd^2-e(3bd-2ae)\right)}{e^5}+\frac{x^3\left(3cd^2-e(2bd-ae)\right)}{3e^4}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{2e^5\left(d+ex^2\right)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(9cd^2-e(7bd-5\right.}{2e^{11/2}}$$

```
[Out] -d*(4*c*d^2-e*(-2*a*e+3*b*d))*x/e^5+1/3*(3*c*d^2-e*(-a*e+2*b*d))*x^3/e^4-1/
5*(-b*e+2*c*d)*x^5/e^3+1/7*c*x^7/e^2-1/2*d^2*(a*e^2-b*d*e+c*d^2)*x/e^5/(e*x
^2+d)+1/2*d^(3/2)*(9*c*d^2-e*(-5*a*e+7*b*d))*arctan(x*e^(1/2)/d^(1/2))/e^(1
1/2)
```

Rubi [A] time = 0.23, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1257, 1810, 205}

$$\frac{x^3\left(3cd^2-e(2bd-ae)\right)}{3e^4}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{2e^5\left(d+ex^2\right)}-\frac{dx\left(4cd^2-e(3bd-2ae)\right)}{e^5}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(9cd^2-e(7bd-5a\right.}{2e^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -((d*(4*c*d^2 - e*(3*b*d - 2*a*e))*x)/e^5) + ((3*c*d^2 - e*(2*b*d - a*e))*x
^3)/(3*e^4) - ((2*c*d - b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - (d^2*(c*d^2 -
 b*d*e + a*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - e*(7*b*d - 5*a
*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^6\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}dx = -\frac{d^2\left(cd^2-bde+ae^2\right)x}{2e^5\left(d+ex^2\right)} - \frac{\int \frac{-d^2\left(cd^2-bde+ae^2\right)+2de\left(cd^2-bde+ae^2\right)x^2-2e^2\left(cd^2-bde+ae^2\right)x^4+2e^3(cd-be)x}{d+ex^2}}{2e^5}$$

$$= -\frac{d^2\left(cd^2-bde+ae^2\right)x}{2e^5\left(d+ex^2\right)} - \frac{\int \left(2d\left(4cd^2-e(3bd-2ae)\right)-2e\left(3cd^2-e(2bd-ae)\right)x^2+\right.}{2e^5}$$

$$= -\frac{d\left(4cd^2-e(3bd-2ae)\right)x}{e^5} + \frac{\left(3cd^2-e(2bd-ae)\right)x^3}{3e^4} - \frac{(2cd-be)x^5}{5e^3} + \frac{cx^7}{7e^2} - \frac{d^2\left(cd^2\right.}{2e^5}$$

$$= -\frac{d\left(4cd^2-e(3bd-2ae)\right)x}{e^5} + \frac{\left(3cd^2-e(2bd-ae)\right)x^3}{3e^4} - \frac{(2cd-be)x^5}{5e^3} + \frac{cx^7}{7e^2} - \frac{d^2\left(cd^2\right.}{2e^5}$$

Mathematica [A] time = 0.14, size = 165, normalized size = 0.98

$$-\frac{dx\left(2ae^2-3bde+4cd^2\right)}{e^5}+\frac{x^3\left(ae^2-2bde+3cd^2\right)}{3e^4}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5ae^2-7bde+9cd^2\right)}{2e^{11/2}}-\frac{x\left(ad^2e^2-bd^3e+cd^4\right.}{2e^5\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -((d*(4*c*d^2 - 3*b*d*e + 2*a*e^2)*x)/e^5) + ((3*c*d^2 - 2*b*d*e + a*e^2)*x
^3)/(3*e^4) + ((-2*c*d + b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - ((c*d^4 - b*
d^3*e + a*d^2*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - 7*b*d*e + 5
*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

fricas [A] time = 0.94, size = 426, normalized size = 2.54

$$\left[\frac{60\,ce^4x^9-12\left(9\,cde^3-7\,be^4\right)x^7+28\left(9\,cd^2e^2-7\,bde^3+5\,ae^4\right)x^5-140\left(9\,cd^3e-7\,bd^2e^2+5\,ade^3\right)x^3+105\left(9\,cd\right.}{420\left(e^6x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*c*e^4*x^9 - 12*(9*c*d*e^3 - 7*b*e^4)*x^7 + 28*(9*c*d^2*e^2 - 7*b
*d*e^3 + 5*a*e^4)*x^5 - 140*(9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 105
*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)
*x^2)*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) - 210*(9*c
*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*x)/(e^6*x^2 + d*e^5), 1/210*(30*c*e^4*x^9 -
 6*(9*c*d*e^3 - 7*b*e^4)*x^7 + 14*(9*c*d^2*e^2 - 7*b*d*e^3 + 5*a*e^4)*x^5 -
 70*(9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 105*(9*c*d^4 - 7*b*d^3*e +
5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^2)*sqrt(d/e)*arctan(e
*x*sqrt(d/e)/d) - 105*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*x)/(e^6*x^2 + d*e
^5)]
```

giac [A] time = 0.32, size = 160, normalized size = 0.95

$$\frac{\left(9\,cd^4-7\,bd^3e+5\,ad^2e^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}}{2\,\sqrt{d}}+\frac{1}{105}\left(15\,cx^7e^{12}-42\,cdx^5e^{11}+21\,bx^5e^{12}+105\,cd^2x^3e^{10}-70\,bdx^3e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-11/2)
/sqrt(d) + 1/105*(15*c*x^7*e^12 - 42*c*d*x^5*e^11 + 21*b*x^5*e^12 + 105*c*d
^2*x^3*e^10 - 70*b*d*x^3*e^11 - 420*c*d^3*x*e^9 + 35*a*x^3*e^12 + 315*b*d^2
*x*e^10 - 210*a*d*x*e^11)*e^(-14) - 1/2*(c*d^4*x - b*d^3*x*e + a*d^2*x*e^2)
*e^(-5)/(x^2*e + d)
```

maple [A] time = 0.01, size = 214, normalized size = 1.27

$$\frac{cx^7}{7e^2}+\frac{bx^5}{5e^2}-\frac{2cdx^5}{5e^3}+\frac{ax^3}{3e^2}-\frac{2bdx^3}{3e^3}+\frac{cd^2x^3}{e^4}-\frac{ad^2x}{2\left(ex^2+d\right)e^3}+\frac{5ad^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\,e^3}+\frac{bd^3x}{2\left(ex^2+d\right)e^4}-\frac{7bd^3\arctan\left(\right.}{2\sqrt{de}\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/7*c*x^7/e^2+1/5/e^2*x^5*b-2/5/e^3*x^5*c*d+1/3/e^2*x^3*a-2/3/e^3*x^3*b*d+1
/e^4*x^3*c*d^2-2/e^3*a*d*x+3/e^4*d^2*b*x-4/e^5*c*d^3*x-1/2*d^2/e^3*x/(e*x^2
+d)*a+1/2*d^3/e^4*x/(e*x^2+d)*b-1/2*d^4/e^5*x/(e*x^2+d)*c+5/2*d^2/e^3/(d*e)
^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a-7/2*d^3/e^4/(d*e)^(1/2)*arctan(1/(d*e)^(
1/2)*e*x)*b+9/2*d^4/e^5/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.49, size = 165, normalized size = 0.98

$$-\frac{\left(cd^4-bd^3e+ad^2e^2\right)x}{2\left(e^6x^2+de^5\right)}+\frac{\left(9\,cd^4-7\,bd^3e+5\,ad^2e^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,e^5}+\frac{15\,ce^3x^7-21\left(2\,cde^2-be^3\right)x^5+35\left(3\,cd^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*d^4 - b*d^3*e + a*d^2*e^2)*x/(e^6*x^2 + d*e^5) + 1/2*(9*c*d^4 - 7*b
*d^3*e + 5*a*d^2*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^5) + 1/105*(15*c*e
^3*x^7 - 21*(2*c*d*e^2 - b*e^3)*x^5 + 35*(3*c*d^2*e - 2*b*d*e^2 + a*e^3)*x^
3 - 105*(4*c*d^3 - 3*b*d^2*e + 2*a*d*e^2)*x)/e^5
```

mupad [B] time = 0.33, size = 251, normalized size = 1.49

$$x^5\left(\frac{b}{5\,e^2}-\frac{2\,c\,d}{5\,e^3}\right)-x^3\left(\frac{c\,d^2}{3\,e^4}-\frac{a}{3\,e^2}+\frac{2\,d\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{3\,e}\right)+x\left(\frac{2\,d\left(\frac{c\,d^2}{e^4}-\frac{a}{e^2}+\frac{2\,d\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e^2}\right)-\frac{x\left(\frac{c\,d^4}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

```
[Out] x^5*(b/(5*e^2) - (2*c*d)/(5*e^3)) - x^3*((c*d^2)/(3*e^4) - a/(3*e^2) + (2*d
*(b/e^2 - (2*c*d)/e^3))/(3*e)) + x*((2*d*((c*d^2)/e^4 - a/e^2 + (2*d*(b/e^2
 - (2*c*d)/e^3))/e))/e - (d^2*(b/e^2 - (2*c*d)/e^3))/e^2) - (x*((c*d^4)/2 +
 (a*d^2*e^2)/2 - (b*d^3*e)/2))/(d*e^5 + e^6*x^2) + (c*x^7)/(7*e^2) + (d^(3/
2)*atan((d^(3/2)*e^(1/2)*x*(5*a*e^2 + 9*c*d^2 - 7*b*d*e))/(9*c*d^4 + 5*a*d^
2*e^2 - 7*b*d^3*e))*(5*a*e^2 + 9*c*d^2 - 7*b*d*e))/(2*e^(11/2))
```

sympy [B] time = 1.18, size = 320, normalized size = 1.90

$$\frac{cx^7}{7e^2}+x^5\left(\frac{b}{5e^2}-\frac{2cd}{5e^3}\right)+x^3\left(\frac{a}{3e^2}-\frac{2bd}{3e^3}+\frac{cd^2}{e^4}\right)+x\left(-\frac{2ad}{e^3}+\frac{3bd^2}{e^4}-\frac{4cd^3}{e^5}\right)+\frac{x\left(-ad^2e^2+bd^3e-cd^4\right)}{2de^5+2e^6x^2}-\frac{\sqrt{-\frac{d^3}{e^{11}}}\left(5ae^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**7/(7*e**2) + x**5*(b/(5*e**2) - 2*c*d/(5*e**3)) + x**3*(a/(3*e**2) - 2
*b*d/(3*e**3) + c*d**2/e**4) + x*(-2*a*d/e**3 + 3*b*d**2/e**4 - 4*c*d**3/e*
*5) + x*(-a*d**2*e**2 + b*d**3*e - c*d**4)/(2*d*e**5 + 2*e**6*x**2) - sqrt(
-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*log(-e**5*sqrt(-d**3/e**11)*(5
*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x)/4 +
 sqrt(-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*log(e**5*sqrt(-d**3/e**1
1)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x
)/4
```

3.281 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=135

$$-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7cd^2-e(5bd-3ae)\right)}{2e^{9/2}}+\frac{x\left(3cd^2-e(2bd-ae)\right)}{e^4}+\frac{dx\left(ae^2-bde+cd^2\right)}{2e^4\left(d+ex^2\right)}-\frac{x^3(2cd-be)}{3e^3}+\frac{cx^5}{5e^2}$$

```
[Out] (3*c*d^2-e*(-a*e+2*b*d))*x/e^4-1/3*(-b*e+2*c*d)*x^3/e^3+1/5*c*x^5/e^2+1/2*d
*(a*e^2-b*d*e+c*d^2)*x/e^4/(e*x^2+d)-1/2*(7*c*d^2-e*(-3*a*e+5*b*d))*arctan(
x*e^(1/2)/d^(1/2))*d^(1/2)/e^(9/2)
```

Rubi [A] time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1257, 1810, 205}

$$\frac{dx\left(ae^2-bde+cd^2\right)}{2e^4\left(d+ex^2\right)}+\frac{x\left(3cd^2-e(2bd-ae)\right)}{e^4}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7cd^2-e(5bd-3ae)\right)}{2e^{9/2}}-\frac{x^3(2cd-be)}{3e^3}+\frac{cx^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((3*c*d^2 - e*(2*b*d - a*e))*x)/e^4 - ((2*c*d - b*e)*x^3)/(3*e^3) + (c*x^5)
/(5*e^2) + (d*(c*d^2 - b*d*e + a*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]*(7*
c*d^2 - e*(5*b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}dx = \frac{d\left(cd^2-bde+ae^2\right)x}{2e^4\left(d+ex^2\right)} - \frac{\int \frac{d\left(cd^2-bde+ae^2\right)-2e\left(cd^2-bde+ae^2\right)x^2+2e^2(cd-be)x^4-2ce^3x^6}{d+ex^2}dx}{2e^4}$$

$$= \frac{d\left(cd^2-bde+ae^2\right)x}{2e^4\left(d+ex^2\right)} - \frac{\int \left(-2\left(3cd^2-2bde+ae^2\right)+2e(2cd-be)x^2-2ce^2x^4+\frac{7cd^3-5b}{d\cdot}\right.}{2e^4}$$

$$= \frac{\left(3cd^2-e(2bd-ae)\right)x}{e^4} - \frac{(2cd-be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{d\left(cd^2-bde+ae^2\right)x}{2e^4\left(d+ex^2\right)} - \frac{\left(d\left(7cd^2-e(5b\right.\right.}{}$$

$$= \frac{\left(3cd^2-e(2bd-ae)\right)x}{e^4} - \frac{(2cd-be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{d\left(cd^2-bde+ae^2\right)x}{2e^4\left(d+ex^2\right)} - \frac{\sqrt{d}\left(7cd^2-e(5\right.}{}$$

Mathematica [A] time = 0.08, size = 133, normalized size = 0.99

$$-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3ae^2-5bde+7cd^2\right)}{2e^{9/2}}+\frac{x\left(ae^2-2bde+3cd^2\right)}{e^4}+\frac{x\left(ade^2-bd^2e+cd^3\right)}{2e^4\left(d+ex^2\right)}+\frac{x^3(be-2cd)}{3e^3}+\frac{cx^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((3*c*d^2 - 2*b*d*e + a*e^2)*x)/e^4 + ((-2*c*d + b*e)*x^3)/(3*e^3) + (c*x^5
)/(5*e^2) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]*
(7*c*d^2 - 5*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

fricas [A] time = 0.68, size = 350, normalized size = 2.59

$$\left[\frac{12\,ce^3x^7-4\left(7\,cde^2-5\,be^3\right)x^5+20\left(7\,cd^2e-5\,bde^2+3\,ae^3\right)x^3+15\left(7\,cd^3-5\,bd^2e+3\,ade^2+\left(7\,cd^2e-5\,bde^2+\right.\right.}{60\left(e^5x^2+de^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c*e^3*x^7 - 4*(7*c*d*e^2 - 5*b*e^3)*x^5 + 20*(7*c*d^2*e - 5*b*d*e
^2 + 3*a*e^3)*x^3 + 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*b*
d*e^2 + 3*a*e^3)*x^2)*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2
+ d)) + 30*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*x)/(e^5*x^2 + d*e^4), 1/30*(6*
c*e^3*x^7 - 2*(7*c*d*e^2 - 5*b*e^3)*x^5 + 10*(7*c*d^2*e - 5*b*d*e^2 + 3*a*e
^3)*x^3 - 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*b*d*e^2 + 3*
a*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 15*(7*c*d^3 - 5*b*d^2*e + 3
*a*d*e^2)*x)/(e^5*x^2 + d*e^4)]
```

giac [A] time = 0.28, size = 125, normalized size = 0.93

$$-\frac{\left(7\,cd^3-5\,bd^2e+3\,ade^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,\sqrt{d}}+\frac{1}{15}\left(3\,cx^5e^8-10\,cdx^3e^7+5\,bx^3e^8+45\,cd^2xe^6-30\,bdxe^7+15\,axe^8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/s
qrt(d) + 1/15*(3*c*x^5*e^8 - 10*c*d*x^3*e^7 + 5*b*x^3*e^8 + 45*c*d^2*x*e^6
- 30*b*d*x*e^7 + 15*a*x*e^8)*e^(-10) + 1/2*(c*d^3*x - b*d^2*x*e + a*d*x*e^2
)*e^(-4)/(x^2*e + d)
```

maple [A] time = 0.01, size = 176, normalized size = 1.30

$$\frac{c\,x^5}{5e^2}+\frac{b\,x^3}{3e^2}-\frac{2cd\,x^3}{3e^3}+\frac{adx}{2\left(e\,x^2+d\right)e^2}-\frac{3ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;e^2}-\frac{b\,d^2x}{2\left(e\,x^2+d\right)e^3}+\frac{5b\,d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;e^3}+\frac{c\,d^3x}{2\left(e\,x^2+d\right)e^4}-7c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/5*c*x^5/e^2+1/3/e^2*x^3*b-2/3/e^3*x^3*c*d+1/e^2*a*x-2/e^3*d*b*x+3/e^4*c*d
^2*x+1/2*d/e^2*x/(e*x^2+d)*a-1/2*d^2/e^3*x/(e*x^2+d)*b+1/2*d^3/e^4*x/(e*x^2
+d)*c-3/2*d/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+5/2*d^2/e^3/(d*e)^(
1/2)*arctan(1/(d*e)^(1/2)*e*x)*b-7/2*d^3/e^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/
2)*e*x)*c
```

maxima [A] time = 2.45, size = 130, normalized size = 0.96

$$\frac{\left(cd^3-bd^2e+ade^2\right)x}{2\left(e^5x^2+de^4\right)}-\frac{\left(7\,cd^3-5\,bd^2e+3\,ade^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,e^4}+\frac{3\,ce^2x^5-5\left(2\,cde-be^2\right)x^3+15\left(3\,cd^2-2\,bde\right.}{15\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*d^3 - b*d^2*e + a*d*e^2)*x/(e^5*x^2 + d*e^4) - 1/2*(7*c*d^3 - 5*b*d^
2*e + 3*a*d*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4) + 1/15*(3*c*e^2*x^5
- 5*(2*c*d*e - b*e^2)*x^3 + 15*(3*c*d^2 - 2*b*d*e + a*e^2)*x)/e^4
```

mupad [B] time = 0.32, size = 179, normalized size = 1.33

$$x^3\left(\frac{b}{3\,e^2}-\frac{2\,c\,d}{3\,e^3}\right)-x\left(\frac{c\,d^2}{e^4}-\frac{a}{e^2}+\frac{2\,d\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e}\right)+\frac{c\,x^5}{5\,e^2}+\frac{x\,\left(\frac{c\,d^3}{2}-\frac{b\,d^2\,e}{2}+\frac{a\,d\,e^2}{2}\right)}{e^5\,x^2+d\,e^4}-\frac{\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,x\,\left(7\,c\,d^2-5\,b\,d\,e+\right.}{7\,c\,d^3-5\,b\,d^2\,e+3\,a\,d}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

```
[Out] x^3*(b/(3*e^2) - (2*c*d)/(3*e^3)) - x*((c*d^2)/e^4 - a/e^2 + (2*d*(b/e^2 -
(2*c*d)/e^3))/e) + (c*x^5)/(5*e^2) + (x*((c*d^3)/2 + (a*d*e^2)/2 - (b*d^2*e
)/2))/(d*e^4 + e^5*x^2) - (d^(1/2)*atan((d^(1/2)*e^(1/2)*x*(3*a*e^2 + 7*c*d
^2 - 5*b*d*e))/(7*c*d^3 + 3*a*d*e^2 - 5*b*d^2*e))*(3*a*e^2 + 7*c*d^2 - 5*b*
d*e))/(2*e^(9/2))
```

sympy [A] time = 1.08, size = 189, normalized size = 1.40

$$\frac{cx^5}{5e^2}+x^3\left(\frac{b}{3e^2}-\frac{2cd}{3e^3}\right)+x\left(\frac{a}{e^2}-\frac{2bd}{e^3}+\frac{3cd^2}{e^4}\right)+\frac{x\left(ade^2-bd^2e+cd^3\right)}{2de^4+2e^5x^2}+\frac{\sqrt{-\frac{d}{e^9}}\left(3ae^2-5bde+7cd^2\right)\log\left(-e^4\sqrt{-\frac{d}{e^9}}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**5/(5*e**2) + x**3*(b/(3*e**2) - 2*c*d/(3*e**3)) + x*(a/e**2 - 2*b*d/e*
*3 + 3*c*d**2/e**4) + x*(a*d*e**2 - b*d**2*e + c*d**3)/(2*d*e**4 + 2*e**5*x
**2) + sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(-e**4*sqrt(-d/e**9
) + x)/4 - sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(e**4*sqrt(-d/e
**9) + x)/4
```

3.282 $\int \frac{x^2\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=106

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(5cd^2-e(3bd-ae)\right)}{2\sqrt{d}\,e^{7/2}}-\frac{x\left(ae^2-bde+cd^2\right)}{2e^3\left(d+ex^2\right)}-\frac{x(2cd-be)}{e^3}+\frac{cx^3}{3e^2}$$

```
[Out] -(-b*e+2*c*d)*x/e^3+1/3*c*x^3/e^2-1/2*(a*e^2-b*d*e+c*d^2)*x/e^3/(e*x^2+d)+1
/2*(5*c*d^2-e*(-a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)/d^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1257, 1153, 205}

$$-\frac{x\left(ae^2-bde+cd^2\right)}{2e^3\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(5cd^2-e(3bd-ae)\right)}{2\sqrt{d}\,e^{7/2}}-\frac{x(2cd-be)}{e^3}+\frac{cx^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -(((2*c*d - b*e)*x)/e^3) + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2
*e^3*(d + e*x^2)) + ((5*c*d^2 - e*(3*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*Sqrt[d]*e^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{\left(cd^2-bde+ae^2\right)x}{2e^3\left(d+ex^2\right)} - \frac{\int \frac{-cd^2+bde-ae^2+2e(cd-be)x^2-2ce^2x^4}{d+ex^2}\,dx}{2e^3}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)x}{2e^3\left(d+ex^2\right)} - \frac{\int \left(2(2cd-be)-2cex^2+\frac{-5cd^2+3bde-ae^2}{d+ex^2}\right)dx}{2e^3}$$

$$= -\frac{(2cd-be)x}{e^3} + \frac{cx^3}{3e^2} - \frac{\left(cd^2-bde+ae^2\right)x}{2e^3\left(d+ex^2\right)} - \frac{\left(-5cd^2+e(3bd-ae)\right)\int \frac{1}{d+ex^2}\,dx}{2e^3}$$

$$= -\frac{(2cd-be)x}{e^3} + \frac{cx^3}{3e^2} - \frac{\left(cd^2-bde+ae^2\right)x}{2e^3\left(d+ex^2\right)} + \frac{\left(5cd^2-e(3bd-ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{7/2}}$$

Mathematica [A] time = 0.06, size = 102, normalized size = 0.96

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(ae^2-3bde+5cd^2\right)}{2\sqrt{d}\,e^{7/2}} - \frac{x\left(ae^2-bde+cd^2\right)}{2e^3\left(d+ex^2\right)} + \frac{x(be-2cd)}{e^3} + \frac{cx^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((-2*c*d + b*e)*x)/e^3 + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*e
^3*(d + e*x^2)) + ((5*c*d^2 - 3*b*d*e + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(2*Sqrt[d]*e^(7/2))
```

fricas [A] time = 0.84, size = 302, normalized size = 2.85

$$\left[\frac{4\,cde^3x^5-4\left(5\,cd^2e^2-3\,bde^3\right)x^3-3\left(5\,cd^3-3\,bd^2e+ade^2+\left(5\,cd^2e-3\,bde^2+ae^3\right)x^2\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\,\sqrt{-de}\,x-d}{ex^2+d}\right)}{12\left(de^5x^2+d^2e^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*c*d*e^3*x^5 - 4*(5*c*d^2*e^2 - 3*b*d*e^3)*x^3 - 3*(5*c*d^3 - 3*b*d
^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2
 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(5*c*d^3*e - 3*b*d^2*e^2 + a*d*e^3)
*x)/(d*e^5*x^2 + d^2*e^4), 1/6*(2*c*d*e^3*x^5 - 2*(5*c*d^2*e^2 - 3*b*d*e^3)
*x^3 + 3*(5*c*d^3 - 3*b*d^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e^3)*x
^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(5*c*d^3*e - 3*b*d^2*e^2 + a*d*e^3)
*x)/(d*e^5*x^2 + d^2*e^4)]
```

giac [A] time = 0.34, size = 91, normalized size = 0.86

$$\frac{\left(5\,cd^2-3\,bde+ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{2\,\sqrt{d}} + \frac{1}{3}\left(cx^3e^4-6\,cdxe^3+3\,bxe^4\right)e^{(-6)} - \frac{\left(cd^2x-bdxe+axe^2\right)e^{(-3)}}{2\left(x^2e+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2 - 3*b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d)
+ 1/3*(c*x^3*e^4 - 6*c*d*x*e^3 + 3*b*x*e^4)*e^(-6) - 1/2*(c*d^2*x - b*d*x*e
 + a*x*e^2)*e^(-3)/(x^2*e + d)
```

maple [A] time = 0.01, size = 141, normalized size = 1.33

$$\frac{c\,x^3}{3e^2}-\frac{ax}{2\left(e\,x^2+d\right)e}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;e}+\frac{bdx}{2\left(e\,x^2+d\right)e^2}-\frac{3bd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;e^2}-\frac{c\,d^2x}{2\left(e\,x^2+d\right)e^3}+\frac{5c\,d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;e^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/3*c*x^3/e^2+1/e^2*b*x-2/e^3*c*d*x-1/2/e*x/(e*x^2+d)*a+1/2/e^2*x/(e*x^2+d)
*d*b-1/2/e^3*x/(e*x^2+d)*c*d^2+1/2/e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*
a-3/2/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*d*b+5/2/e^3/(d*e)^(1/2)*arc
tan(1/(d*e)^(1/2)*e*x)*c*d^2
```

maxima [A] time = 2.52, size = 95, normalized size = 0.90

$$-\frac{\left(cd^2-bde+ae^2\right)x}{2\left(e^4x^2+de^3\right)}+\frac{\left(5\,cd^2-3\,bde+ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,e^3}+\frac{cex^3-3\,(2\,cd-be)x}{3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*d^2 - b*d*e + a*e^2)*x/(e^4*x^2 + d*e^3) + 1/2*(5*c*d^2 - 3*b*d*e +
 a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^3) + 1/3*(c*e*x^3 - 3*(2*c*d - b
*e)*x)/e^3
```

mupad [B] time = 0.34, size = 95, normalized size = 0.90

$$x\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)-\frac{x\,\left(\frac{c\,d^2}{2}-\frac{b\,d\,e}{2}+\frac{a\,e^2}{2}\right)}{e^4\,x^2+d\,e^3}+\frac{c\,x^3}{3\,e^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(5\,c\,d^2-3\,b\,d\,e+a\,e^2\right)}{2\,\sqrt{d}\,e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

```
[Out] x*(b/e^2 - (2*c*d)/e^3) - (x*((a*e^2)/2 + (c*d^2)/2 - (b*d*e)/2))/(d*e^3 +
e^4*x^2) + (c*x^3)/(3*e^2) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + 5*c*d^2 -
3*b*d*e))/(2*d^(1/2)*e^(7/2))
```

sympy [A] time = 0.97, size = 162, normalized size = 1.53

$$\frac{cx^3}{3e^2}+x\left(\frac{b}{e^2}-\frac{2cd}{e^3}\right)+\frac{x\left(-ae^2+bde-cd^2\right)}{2de^3+2e^4x^2}-\frac{\sqrt{-\frac{1}{de^7}}\,\left(ae^2-3bde+5cd^2\right)\log\left(-de^3\sqrt{-\frac{1}{de^7}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{de^7}}\,\left(ae^2-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**3/(3*e**2) + x*(b/e**2 - 2*c*d/e**3) + x*(-a*e**2 + b*d*e - c*d**2)/(2
*d*e**3 + 2*e**4*x**2) - sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e + 5*c*d**2)*lo
g(-d*e**3*sqrt(-1/(d*e**7)) + x)/4 + sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e +
5*c*d**2)*log(d*e**3*sqrt(-1/(d*e**7)) + x)/4
```

3.283 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2} \, dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[Out] c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*ar
ctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}$$
$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\left(3cd^2-e(bd+ae)\right)\int \frac{1}{d+ex^2}\,dx}{2de^2}$$
$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\left(3cd^2-e(bd+ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

fricas [A] time = 0.81, size = 268, normalized size = 3.23

$$\left[\frac{4\,cd^2e^2x^3+\left(3\,cd^3-bd^2e-ade^2+\left(3\,cd^2e-bde^2-ae^3\right)x^2\right)\sqrt{-de}\,\log\left(\frac{ex^2-2\,\sqrt{-de}\,x-d}{ex^2+d}\right)+2\left(3\,cd^3e-bd^2e^2+ad\right.}{4\left(d^2e^4x^2+d^3e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

giac [A] time = 0.41, size = 75, normalized size = 0.90

$$cxe^{(-2)} - \frac{\left(3\,cd^2-bde-ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x-bdxe+axe^2\right)e^{(-2)}}{2\left(x^2e+d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

maple [A] time = 0.01, size = 118, normalized size = 1.42

$$\frac{ax}{2\left(e\,x^2+d\right)d}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;d}-\frac{bx}{2\left(e\,x^2+d\right)e}+\frac{b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;e}+\frac{cdx}{2\left(e\,x^2+d\right)e^2}-\frac{3cd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;e^2}+\frac{cx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^2+d)*c+1/2
/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+1/2/e/(d*e)^(1/2)*arctan(1/(d*e)
^(1/2)*e*x)*b-3/2/e^2*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.34, size = 84, normalized size = 1.01

$$\frac{\left(cd^2 - bde + ae^2\right)x}{2\left(de^3x^2 + d^2e^2\right)} + \frac{cx}{e^2} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*d^2 - b*d*e + a*e^2)*x/(d*e^3*x^2 + d^2*e^2) + c*x/e^2 - 1/2*(3*c*d^
2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^2)
```

mupad [B] time = 0.36, size = 77, normalized size = 0.93

$$\frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(-3\,c\,d^2 + b\,d\,e + a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{2\,d\,\left(e^3\,x^2 + d\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

sympy [B] time = 0.77, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x\left(ae^2 - bde + cd^2\right)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\,\left(ae^2 + bde - 3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\,\left(ae^2 + bde - 3cd^2\right)\log\left(d^2\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

3.284 $\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=89

$$-\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(bd-3ae)+cd^2\right)}{2d^{5/2}e^{3/2}}-\frac{a}{d^2x}$$

```
[Out] -a/d^2/x-1/2*(a*e^2-b*d*e+c*d^2)*x/d^2/e/(e*x^2+d)+1/2*(c*d^2+e*(-3*a*e+b*d
))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1259, 453, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(bd-3ae)+cd^2\right)}{2d^{5/2}e^{3/2}}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{2\left(d+ex^2\right)}-\frac{a}{d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - ((c/e - (b*d - a*e)/d^2)*x)/(2*(d + e*x^2)) + ((c*d^2 + e*(b
*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx = -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2\left(d+ex^2\right)} - \frac{\int \frac{-2ade^2-e\left(cd^2+e(bd-ae)\right)x^2}{x^2\left(d+ex^2\right)}\,dx}{2d^2e^2}$$
$$= -\frac{a}{d^2x} - \frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2\left(d+ex^2\right)} + \frac{1}{2}\left(\frac{c}{e}+\frac{bd-3ae}{d^2}\right)\int \frac{1}{d+ex^2}\,dx$$
$$= -\frac{a}{d^2x} - \frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2\left(d+ex^2\right)} + \frac{\left(cd^2+e(bd-3ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}e^{3/2}}$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.00

$$-\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e\left(d+ex^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(-3ae^2+bde+cd^2\right)}{2d^{5/2}e^{3/2}} - \frac{a}{d^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^2*e*(d + e*x^2)) + ((c*d^2
+ b*d*e - 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))
```

fricas [A] time = 0.98, size = 267, normalized size = 3.00

$$\left[-\frac{4\,ad^2e^2+2\left(cd^3e-bd^2e^2+3\,ade^3\right)x^2-\left(\left(cd^2e+bde^2-3\,ae^3\right)x^3+\left(cd^3+bd^2e-3\,ade^2\right)x\right)\sqrt{-de}\,\log\left(\frac{ex^2+2\,\sqrt{-de}}{ex^2+d}\right.}{4\left(d^3e^3x^3+d^4e^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*d^2*e^2 + 2*(c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*x^2 - ((c*d^2*e +
b*d*e^2 - 3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*sqrt(-d*e)*log((e
*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^3*x^3 + d^4*e^2*x), -1/2*(2
*a*d^2*e^2 + (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*x^2 - ((c*d^2*e + b*d*e^2 -
3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*sqrt(d*e)*arctan(sqrt(d*e)*
x/d))/(d^3*e^3*x^3 + d^4*e^2*x)]
```

giac [A] time = 0.29, size = 83, normalized size = 0.93

$$\frac{\left(cd^2+bde-3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{2\,d^{\frac{5}{2}}} - \frac{\left(cd^2x^2-bdx^2e+3\,ax^2e^2+2\,ade\right)e^{(-1)}}{2\left(x^3e+dx\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 + b*d*e - 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(5/2) -
1/2*(c*d^2*x^2 - b*d*x^2*e + 3*a*x^2*e^2 + 2*a*d*e)*e^(-1)/((x^3*e + d*x)*d
^2)
```

maple [A] time = 0.01, size = 121, normalized size = 1.36

$$-\frac{aex}{2\left(e\,x^{2}+d\right)d^{2}}-\frac{3ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;d^{2}}+\frac{bx}{2\left(e\,x^{2}+d\right)d}+\frac{b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;d}-\frac{cx}{2\left(e\,x^{2}+d\right)e}+\frac{c\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;e}-\frac{a}{d^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x)
```

```
[Out] -a/d^2/x-1/2/d^2*e*x/(e*x^2+d)*a+1/2/d*x/(e*x^2+d)*b-1/2/e*x/(e*x^2+d)*c-3/
2/d^2*e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+1/2/d/(d*e)^(1/2)*arctan(1/
(d*e)^(1/2)*e*x)*b+1/2/e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.46, size = 87, normalized size = 0.98

$$-\frac{2\,ade+\left(cd^{2}-bde+3\,ae^{2}\right)x^{2}}{2\left(d^{2}e^{2}x^{3}+d^{3}ex\right)}+\frac{\left(cd^{2}+bde-3\,ae^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}\,d^{2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*d*e + (c*d^2 - b*d*e + 3*a*e^2)*x^2)/(d^2*e^2*x^3 + d^3*e*x) + 1/
2*(c*d^2 + b*d*e - 3*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e)
```

mupad [B] time = 0.37, size = 81, normalized size = 0.91

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(c\,d^{2}+b\,d\,e-3\,a\,e^{2}\right)}{2\,d^{5/2}\,e^{3/2}}-\frac{\frac{a}{d}+\frac{x^{2}\left(c\,d^{2}-b\,d\,e+3\,a\,e^{2}\right)}{2\,d^{2}\,e}}{e\,x^{3}+d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(c*d^2 - 3*a*e^2 + b*d*e))/(2*d^(5/2)*e^(3/2)) -
 (a/d + (x^2*(3*a*e^2 + c*d^2 - b*d*e))/(2*d^2*e))/(d*x + e*x^3)
```

sympy [A] time = 1.12, size = 155, normalized size = 1.74

$$\frac{\sqrt{-\frac{1}{d^{5}e^{3}}}\left(3ae^{2}-bde-cd^{2}\right)\log\left(-d^{3}e\sqrt{-\frac{1}{d^{5}e^{3}}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{d^{5}e^{3}}}\left(3ae^{2}-bde-cd^{2}\right)\log\left(d^{3}e\sqrt{-\frac{1}{d^{5}e^{3}}}+x\right)}{4}+\frac{-2ade+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**2,x)
```

```
[Out] sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(-d**3*e*sqrt(-1/(d**5*
e**3)) + x)/4 - sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(d**3*e
*sqrt(-1/(d**5*e**3)) + x)/4 + (-2*a*d*e + x**2*(-3*a*e**2 + b*d*e - c*d**2
))/(2*d**3*e*x + 2*d**2*e**2*x**3)
```

3.285 $\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=106

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(cd^2-e(3bd-5ae)\right)}{2d^{7/2}\sqrt{e}}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}-\frac{bd-2ae}{d^3x}-\frac{a}{3d^2x^3}$$

```
[Out] -1/3*a/d^2/x^3+(2*a*e-b*d)/d^3/x+1/2*(a*e^2-b*d*e+c*d^2)*x/d^3/(e*x^2+d)+1/
2*(c*d^2-e*(-5*a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1259, 1261, 205}

$$\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(cd^2-e(3bd-5ae)\right)}{2d^{7/2}\sqrt{e}}-\frac{bd-2ae}{d^3x}-\frac{a}{3d^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

```
[Out] -a/(3*d^2*x^3) - (b*d - 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^3
*(d + e*x^2)) + ((c*d^2 - e*(3*b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
2*d^(7/2)*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx = \frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)} + \frac{\int \frac{2ad^2e^2+2de^2(bd-ae)x^2+e^2\left(cd^2-bde+ae^2\right)x^4}{x^4\left(d+ex^2\right)}\,dx}{2d^3e^2}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)} + \frac{\int \left(\frac{2ade^2}{x^4} - \frac{2e^2(-bd+2ae)}{x^2} + \frac{e^2\left(cd^2-e(3bd-5ae)\right)}{d+ex^2}\right)dx}{2d^3e^2}$$

$$= -\frac{a}{3d^2x^3} - \frac{bd-2ae}{d^3x} + \frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)} + \frac{\left(cd^2-e(3bd-5ae)\right)\int \frac{1}{d+ex^2}\,dx}{2d^3}$$

$$= -\frac{a}{3d^2x^3} - \frac{bd-2ae}{d^3x} + \frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)} + \frac{\left(cd^2-e(3bd-5ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{7/2}\sqrt{e}}$$

Mathematica [A] time = 0.06, size = 105, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5ae^2-3bde+cd^2\right)}{2d^{7/2}\sqrt{e}} + \frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)} + \frac{2ae-bd}{d^3x} - \frac{a}{3d^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

```
[Out] -1/3*a/(d^2*x^3) + (-(b*d) + 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(
2*d^3*(d + e*x^2)) + ((c*d^2 - 3*b*d*e + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(2*d^(7/2)*Sqrt[e])
```

fricas [A] time = 0.93, size = 316, normalized size = 2.98

$$\left[-\frac{4\,ad^3e-6\left(cd^3e-3\,bd^2e^2+5\,ade^3\right)x^4+4\left(3\,bd^3e-5\,ad^2e^2\right)x^2+3\left(\left(cd^2e-3\,bde^2+5\,ae^3\right)x^5+\left(cd^3-3\,bd^2e\right.\right.}{12\left(d^4e^2x^5+d^5ex^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a*d^3*e - 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*x^4 + 4*(3*b*d^3*
e - 5*a*d^2*e^2)*x^2 + 3*((c*d^2*e - 3*b*d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*
b*d^2*e + 5*a*d*e^2)*x^3)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^
2 + d)))/(d^4*e^2*x^5 + d^5*e*x^3), -1/6*(2*a*d^3*e - 3*(c*d^3*e - 3*b*d^2*
e^2 + 5*a*d*e^3)*x^4 + 2*(3*b*d^3*e - 5*a*d^2*e^2)*x^2 - 3*((c*d^2*e - 3*b*
d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*b*d^2*e + 5*a*d*e^2)*x^3)*sqrt(d*e)*arcta
n(sqrt(d*e)*x/d))/(d^4*e^2*x^5 + d^5*e*x^3)]
```

giac [A] time = 0.26, size = 94, normalized size = 0.89

$$\frac{\left(cd^2-3\,bde+5\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{7}{2}}} + \frac{cd^2x-bdxe+axe^2}{2\left(x^2e+d\right)d^3} - \frac{3\,bdx^2-6\,ax^2e+ad}{3\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 - 3*b*d*e + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(7/2)
+ 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)/((x^2*e + d)*d^3) - 1/3*(3*b*d*x^2 - 6*
a*x^2*e + a*d)/(d^3*x^3)
```

maple [A] time = 0.01, size = 146, normalized size = 1.38

$$\frac{a e^{2} x}{2\left(e x^{2}+d\right) d^{3}}+\frac{5 a e^{2} \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{2 \sqrt{d e}\, d^{3}}-\frac{b e x}{2\left(e x^{2}+d\right) d^{2}}-\frac{3 b e \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{2 \sqrt{d e}\, d^{2}}+\frac{c x}{2\left(e x^{2}+d\right) d}+\frac{c \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{2 \sqrt{d e}\, d}+\frac{2 a e}{d^{3} x}-\frac{b}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x)
```

```
[Out] -1/3*a/d^2/x^3+2/d^3/x*a*e-1/d^2/x*b+1/2/d^3*x/(e*x^2+d)*a*e^2-1/2/d^2*x/(e
*x^2+d)*e*b+1/2/d*x/(e*x^2+d)*c+5/2/d^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*
x)*a*e^2-3/2/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*e*b+1/2/d/(d*e)^(1/2
)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.39, size = 103, normalized size = 0.97

$$\frac{3\left(c d^{2}-3 b d e+5 a e^{2}\right) x^{4}-2 a d^{2}-2\left(3 b d^{2}-5 a d e\right) x^{2}}{6\left(d^{3} e x^{5}+d^{4} x^{3}\right)}+\frac{\left(c d^{2}-3 b d e+5 a e^{2}\right) \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{2 \sqrt{d e}\, d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(c*d^2 - 3*b*d*e + 5*a*e^2)*x^4 - 2*a*d^2 - 2*(3*b*d^2 - 5*a*d*e)*x^
2)/(d^3*e*x^5 + d^4*x^3) + 1/2*(c*d^2 - 3*b*d*e + 5*a*e^2)*arctan(e*x/sqrt(
d*e))/(sqrt(d*e)*d^3)
```

mupad [B] time = 0.36, size = 98, normalized size = 0.92

$$\frac{\frac{x^2\,\left(5\,a\,e-3\,b\,d\right)}{3\,d^2}-\frac{a}{3\,d}+\frac{x^4\,\left(c\,d^2-3\,b\,d\,e+5\,a\,e^2\right)}{2\,d^3}}{e\,x^5+d\,x^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2-3\,b\,d\,e+5\,a\,e^2\right)}{2\,d^{7/2}\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x)
```

```
[Out] ((x^2*(5*a*e - 3*b*d))/(3*d^2) - a/(3*d) + (x^4*(5*a*e^2 + c*d^2 - 3*b*d*e)
)/(2*d^3))/(d*x^3 + e*x^5) + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2 -
3*b*d*e))/(2*d^(7/2)*e^(1/2))
```

sympy [A] time = 1.53, size = 167, normalized size = 1.58

$$-\frac{\sqrt{-\frac{1}{d^{7} e}}\left(5 a e^{2}-3 b d e+c d^{2}\right) \log\left(-d^{4} \sqrt{-\frac{1}{d^{7} e}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^{7} e}}\left(5 a e^{2}-3 b d e+c d^{2}\right) \log\left(d^{4} \sqrt{-\frac{1}{d^{7} e}}+x\right)}{4}+\frac{-2 a d^{2}+x^{4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**2,x)
```

```
[Out] -sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(-d**4*sqrt(-1/(d**7*e)
) + x)/4 + sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(d**4*sqrt(-1
/(d**7*e)) + x)/4 + (-2*a*d**2 + x**4*(15*a*e**2 - 9*b*d*e + 3*c*d**2) + x*
*2*(10*a*d*e - 6*b*d**2))/(6*d**4*x**3 + 6*d**3*e*x**5)
```

3.286 $\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(5bd-7ae)\right)}{2d^{9/2}}-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}-\frac{cd^2-e(2bd-3ae)}{d^4x}-\frac{bd-2ae}{3d^3x^3}-\frac{a}{5d^2x^5}$$

```
[Out] -1/5*a/d^2/x^5+1/3*(2*a*e-b*d)/d^3/x^3+(-c*d^2+e*(-3*a*e+2*b*d))/d^4/x-1/2*
e*(a*e^2-b*d*e+c*d^2)*x/d^4/(e*x^2+d)-1/2*(3*c*d^2-e*(-7*a*e+5*b*d))*arctan
(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(9/2)
```

Rubi [A] time = 0.25, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1259, 1802, 205}

$$-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}-\frac{cd^2-e(2bd-3ae)}{d^4x}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(5bd-7ae)\right)}{2d^{9/2}}-\frac{bd-2ae}{3d^3x^3}-\frac{a}{5d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

```
[Out] -a/(5*d^2*x^5) - (b*d - 2*a*e)/(3*d^3*x^3) - (c*d^2 - e*(2*b*d - 3*a*e))/(d
^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e]*(3*c*d
^2 - e*(5*b*d - 7*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx = -\frac{e\left(cd^2-bde+ae^2\right)x}{2d^4\left(d+ex^2\right)} - \frac{\int \frac{-2ad^3e^2-2d^2e^2(bd-ae)x^2-2de^2\left(cd^2-bde+ae^2\right)x^4+e^3\left(cd^2-bde+ae^2\right)x^6}{x^6\left(d+ex^2\right)}dx}{2d^4e^2}$$

$$= -\frac{e\left(cd^2-bde+ae^2\right)x}{2d^4\left(d+ex^2\right)} - \frac{\int \left(-\frac{2ad^2e^2}{x^6} - \frac{2de^2(bd-2ae)}{x^4} + \frac{2e^2\left(-cd^2+e(2bd-3ae)\right)}{x^2} + \frac{e^3\left(3cd^2-e(5bd-7ae)\right)}{d+ex^2}\right) d}{2d^4e^2}$$

$$= -\frac{a}{5d^2x^5} - \frac{bd-2ae}{3d^3x^3} - \frac{cd^2-e(2bd-3ae)}{d^4x} - \frac{e\left(cd^2-bde+ae^2\right)x}{2d^4\left(d+ex^2\right)} - \frac{\left(e\left(3cd^2-e(5bd-7ae)\right)\right.}{2d^4}$$

$$= -\frac{a}{5d^2x^5} - \frac{bd-2ae}{3d^3x^3} - \frac{cd^2-e(2bd-3ae)}{d^4x} - \frac{e\left(cd^2-bde+ae^2\right)x}{2d^4\left(d+ex^2\right)} - \frac{\sqrt{e}\left(3cd^2-e(5bd-7ae\right.}{2d^{9/2}}$$

Mathematica [A] time = 0.09, size = 135, normalized size = 0.99

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7ae^2-5bde+3cd^2\right)}{2d^{9/2}}-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}+\frac{-3ae^2+2bde-cd^2}{d^4x}+\frac{2ae-bd}{3d^3x^3}-\frac{a}{5d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

```
[Out] -1/5*a/(d^2*x^5) + (-(b*d) + 2*a*e)/(3*d^3*x^3) + (-(c*d^2) + 2*b*d*e - 3*a
*e^2)/(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e
]*(3*c*d^2 - 5*b*d*e + 7*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))
```

fricas [A] time = 0.89, size = 360, normalized size = 2.65

$$\left[-\frac{30\left(3\,cd^2e-5\,bde^2+7\,ae^3\right)x^6+20\left(3\,cd^3-5\,bd^2e+7\,ade^2\right)x^4+12\,ad^3+4\left(5\,bd^3-7\,ad^2e\right)x^2-15\left(\left(3\,cd^2e-5\right.\right.}{60\left(d^4ex^7+d^5x^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(30*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 20*(3*c*d^3 - 5*b*d^2*e
+ 7*a*d*e^2)*x^4 + 12*a*d^3 + 4*(5*b*d^3 - 7*a*d^2*e)*x^2 - 15*((3*c*d^2*e
- 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*sqrt(-e
/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(d^4*e*x^7 + d^5*x^5),
 -1/30*(15*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 10*(3*c*d^3 - 5*b*d^2*e
+ 7*a*d*e^2)*x^4 + 6*a*d^3 + 2*(5*b*d^3 - 7*a*d^2*e)*x^2 + 15*((3*c*d^2*e -
 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*sqrt(e/d
)*arctan(x*sqrt(e/d)))/(d^4*e*x^7 + d^5*x^5)]
```

giac [A] time = 0.33, size = 131, normalized size = 0.96

$$-\frac{\left(3\,cd^2e-5\,bde^2+7\,ae^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{9}{2}}}-\frac{cd^2xe-bdxe^2+axe^3}{2\left(x^2e+d\right)d^4}-\frac{15\,cd^2x^4-30\,bdx^4e+45\,ax^4e^2+5\,bd^2x^2-}{15\,d^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d
^(9/2) - 1/2*(c*d^2*x*e - b*d*x*e^2 + a*x*e^3)/((x^2*e + d)*d^4) - 1/15*(15
*c*d^2*x^4 - 30*b*d*x^4*e + 45*a*x^4*e^2 + 5*b*d^2*x^2 - 10*a*d*x^2*e + 3*a
*d^2)/(d^4*x^5)
```

maple [A] time = 0.02, size = 183, normalized size = 1.35

$$-\frac{a e^3 x}{2\left(e x^2+d\right) d^4}-\frac{7 a e^3 \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{2\sqrt{d e}\, d^4}+\frac{b e^2 x}{2\left(e x^2+d\right) d^3}+\frac{5 b e^2 \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{2\sqrt{d e}\, d^3}-\frac{c e x}{2\left(e x^2+d\right) d^2}-\frac{3 c e \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{2\sqrt{d e}\, d^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x)
```

```
[Out] -1/5*a/d^2/x^5+2/3/d^3/x^3*a*e-1/3/d^2/x^3*b-3/d^4/x*a*e^2+2/d^3/x*e*b-1/d^
2/x*c-1/2*e^3/d^4*x/(e*x^2+d)*a+1/2*e^2/d^3*x/(e*x^2+d)*b-1/2*e/d^2*x/(e*x^
2+d)*c-7/2*e^3/d^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+5/2*e^2/d^3/(d*e
)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b-3/2*e/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1
/2)*e*x)*c
```

maxima [A] time = 2.46, size = 139, normalized size = 1.02

$$-\frac{15\left(3 c d^2 e-5 b d e^2+7 a e^3\right) x^6+10\left(3 c d^3-5 b d^2 e+7 a d e^2\right) x^4+6 a d^3+2\left(5 b d^3-7 a d^2 e\right) x^2}{30\left(d^4 e x^7+d^5 x^5\right)}-\frac{\left(3 c d^2 e-5 b d e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 10*(3*c*d^3 - 5*b*d^2*e +
 7*a*d*e^2)*x^4 + 6*a*d^3 + 2*(5*b*d^3 - 7*a*d^2*e)*x^2)/(d^4*e*x^7 + d^5*x
^5) - 1/2*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*arctan(e*x/sqrt(d*e))/(sqrt(d*e
)*d^4)
```

mupad [B] time = 0.38, size = 128, normalized size = 0.94

$$-\frac{\frac{a}{5 d}-\frac{x^2(7 a e-5 b d)}{15 d^2}+\frac{x^4\left(3 c d^2-5 b d e+7 a e^2\right)}{3 d^3}+\frac{e x^6\left(3 c d^2-5 b d e+7 a e^2\right)}{2 d^4}}{e x^7+d x^5}-\frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(3 c d^2-5 b d e+7 a e^2\right)}{2 d^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x)
```

```
[Out] - (a/(5*d) - (x^2*(7*a*e - 5*b*d))/(15*d^2) + (x^4*(7*a*e^2 + 3*c*d^2 - 5*b
*d*e))/(3*d^3) + (e*x^6*(7*a*e^2 + 3*c*d^2 - 5*b*d*e))/(2*d^4))/(d*x^5 + e*
x^7) - (e^(1/2)*atan((e^(1/2)*x)/d^(1/2))*(7*a*e^2 + 3*c*d^2 - 5*b*d*e))/(2
*d^(9/2))
```

sympy [B] time = 2.13, size = 284, normalized size = 2.09

$$\frac{\sqrt{-\frac{e}{d^9}}\left(7ae^2-5bde+3cd^2\right)\log\left(-\frac{d^5\sqrt{-\frac{e}{d^9}}\left(7ae^2-5bde+3cd^2\right)}{7ae^3-5bde^2+3cd^2e}+x\right)}{4}-\frac{\sqrt{-\frac{e}{d^9}}\left(7ae^2-5bde+3cd^2\right)\log\left(\frac{d^5\sqrt{-\frac{e}{d^9}}\left(7ae^2-5b\right.}{7ae^3-5bde^2+}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**2,x)
```

```
[Out] sqrt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-e/d**9)*(7*a*
e**2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c*d**2*e) + x)/4 - sq
rt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(d**5*sqrt(-e/d**9)*(7*a*e**
2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c*d**2*e) + x)/4 + (-6*a
*d**3 + x**6*(-105*a*e**3 + 75*b*d*e**2 - 45*c*d**2*e) + x**4*(-70*a*d*e**2
 + 50*b*d**2*e - 30*c*d**3) + x**2*(14*a*d**2*e - 10*b*d**3))/(30*d**5*x**5
 + 30*d**4*e*x**7)
```

3.287 $\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=167

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(5cd^2-e(7bd-9ae)\right)}{2d^{11/2}}+\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}+\frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}-\frac{bc}{5}$$

```
[Out] -1/7*a/d^2/x^7+1/5*(2*a*e-b*d)/d^3/x^5+1/3*(-c*d^2+e*(-3*a*e+2*b*d))/d^4/x^
3+e*(2*c*d^2-e*(-4*a*e+3*b*d))/d^5/x+1/2*e^2*(a*e^2-b*d*e+c*d^2)*x/d^5/(e*x
^2+d)+1/2*e^(3/2)*(5*c*d^2-e*(-9*a*e+7*b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(1
1/2)
```

Rubi [A] time = 0.33, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1259, 1802, 205}

$$\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(5cd^2-e(7bd-9ae)\right)}{2d^{11/2}}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}+\frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x}-\frac{bc}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

```
[Out] -a/(7*d^2*x^7) - (b*d - 2*a*e)/(5*d^3*x^5) - (c*d^2 - e*(2*b*d - 3*a*e))/(3
*d^4*x^3) + (e*(2*c*d^2 - e*(3*b*d - 4*a*e)))/(d^5*x) + (e^2*(c*d^2 - b*d*e
 + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - e*(7*b*d - 9*a*e))*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = \frac{e^2\left(cd^2-bde+ae^2\right)x}{2d^5\left(d+ex^2\right)} + \frac{\int \frac{2ad^4e^2+2d^3e^2(bd-ae)x^2+2d^2e^2\left(cd^2-bde+ae^2\right)x^4-2de^3\left(cd^2-bde+ae^2\right)x^6+e^4\left(cd^2-bde\right.}{x^8\left(d+ex^2\right)}}{2d^5e^2}$$

$$= \frac{e^2\left(cd^2-bde+ae^2\right)x}{2d^5\left(d+ex^2\right)} + \frac{\int \left(\frac{2ad^3e^2}{x^8} + \frac{2d^2e^2(bd-2ae)}{x^6} + \frac{2de^2\left(cd^2-e(2bd-3ae)\right)}{x^4} + \frac{2e^3\left(-2cd^2+e(3bd-4ae)\right)}{x^2} + \right.}{2d^5e^2}$$

$$= -\frac{a}{7d^2x^7} - \frac{bd-2ae}{5d^3x^5} - \frac{cd^2-e(2bd-3ae)}{3d^4x^3} + \frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x} + \frac{e^2\left(cd^2-bde+ae^2\right)}{2d^5\left(d+ex^2\right)}$$

$$= -\frac{a}{7d^2x^7} - \frac{bd-2ae}{5d^3x^5} - \frac{cd^2-e(2bd-3ae)}{3d^4x^3} + \frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x} + \frac{e^2\left(cd^2-bde+ae^2\right)}{2d^5\left(d+ex^2\right)}$$

Mathematica [A] time = 0.10, size = 166, normalized size = 0.99

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(9ae^2-7bde+5cd^2\right)}{2d^{11/2}} + \frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)} + \frac{e\left(4ae^2-3bde+2cd^2\right)}{d^5x} + \frac{-3ae^2+2bde-cd^2}{3d^4x^3} + \frac{2ae}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

```
[Out] -1/7*a/(d^2*x^7) + (-(b*d) + 2*a*e)/(5*d^3*x^5) + (-(c*d^2) + 2*b*d*e - 3*a
*e^2)/(3*d^4*x^3) + (e*(2*c*d^2 - 3*b*d*e + 4*a*e^2))/(d^5*x) + (e^2*(c*d^2
 - b*d*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - 7*b*d*e + 9*
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))
```

fricas [A] time = 0.90, size = 436, normalized size = 2.61

$$\left[\frac{210\left(5\,cd^2e^2-7\,bde^3+9\,ae^4\right)x^8+140\left(5\,cd^3e-7\,bd^2e^2+9\,ade^3\right)x^6-60\,ad^4-28\left(5\,cd^4-7\,bd^3e+9\,ad^2e^2\right)x^4-}{420\left(d^5e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(210*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 140*(5*c*d^3*e - 7*b*
d^2*e^2 + 9*a*d*e^3)*x^6 - 60*a*d^4 - 28*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2
)*x^4 - 12*(7*b*d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*
e^4)*x^9 + (5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(-e/d)*log((e*x^2
 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(d^5*e*x^9 + d^6*x^7), 1/210*(105*(5
*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 70*(5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d
*e^3)*x^6 - 30*a*d^4 - 14*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2)*x^4 - 6*(7*b*
d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^9 + (5*c*
d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(e/d)*arctan(x*sqrt(e/d)))/(d^5*e
*x^9 + d^6*x^7)]
```

giac [A] time = 0.42, size = 164, normalized size = 0.98

$$\frac{\left(5\,cd^2e^2-7\,bde^3+9\,ae^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{11}{2}}} + \frac{cd^2xe^2-bdxe^3+axe^4}{2\left(x^2e+d\right)d^5} + \frac{210\,cd^2x^6e-315\,bdx^6e^2-35\,cd^3x^4+420\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/
d^(11/2) + 1/2*(c*d^2*x*e^2 - b*d*x*e^3 + a*x*e^4)/((x^2*e + d)*d^5) + 1/10
5*(210*c*d^2*x^6*e - 315*b*d*x^6*e^2 - 35*c*d^3*x^4 + 420*a*x^6*e^3 + 70*b*
d^2*x^4*e - 105*a*d*x^4*e^2 - 21*b*d^3*x^2 + 42*a*d^2*x^2*e - 15*a*d^3)/(d^
5*x^7)
```

maple [A] time = 0.02, size = 221, normalized size = 1.32

$$\frac{a\,e^4 x}{2\left(e\,x^2+d\right)d^5}+\frac{9a\,e^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;d^5}-\frac{b\,e^3 x}{2\left(e\,x^2+d\right)d^4}-\frac{7b\,e^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;d^4}+\frac{c\,e^2 x}{2\left(e\,x^2+d\right)d^3}+\frac{5c\,e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}\;d^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x)
```

```
[Out] -1/7*a/d^2/x^7+2/5/d^3/x^5*a*e-1/5/d^2/x^5*b-1/d^4/x^3*a*e^2+2/3/d^3/x^3*e*
b-1/3/d^2/x^3*c+4*e^3/d^5/x*a-3*e^2/d^4/x*b+2*e/d^3/x*c+1/2*e^4/d^5*x/(e*x^
2+d)*a-1/2*e^3/d^4*x/(e*x^2+d)*b+1/2*e^2/d^3*x/(e*x^2+d)*c+9/2*e^4/d^5/(d*e
)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a-7/2*e^3/d^4/(d*e)^(1/2)*arctan(1/(d*e)^
(1/2)*e*x)*b+5/2*e^2/d^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.57, size = 174, normalized size = 1.04

$$\frac{105\left(5\,cd^2e^2-7\,bde^3+9\,ae^4\right)x^8+70\left(5\,cd^3e-7\,bd^2e^2+9\,ade^3\right)x^6-30\,ad^4-14\left(5\,cd^4-7\,bd^3e+9\,ad^2e^2\right)x^4-}{210\left(d^5ex^9+d^6x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/210*(105*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 70*(5*c*d^3*e - 7*b*d^
2*e^2 + 9*a*d*e^3)*x^6 - 30*a*d^4 - 14*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2)*
x^4 - 6*(7*b*d^4 - 9*a*d^3*e)*x^2)/(d^5*e*x^9 + d^6*x^7) + 1/2*(5*c*d^2*e^2
 - 7*b*d*e^3 + 9*a*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^5)
```

mupad [B] time = 0.40, size = 156, normalized size = 0.93

$$\frac{\frac{x^2\,(9\,a\,e-7\,b\,d)}{35\,d^2}-\frac{a}{7\,d}-\frac{x^4\left(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2\right)}{15\,d^3}+\frac{e\,x^6\left(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2\right)}{3\,d^4}+\frac{e^2\,x^8\left(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2\right)}{2\,d^5}}{e\,x^9+d\,x^7}+\frac{e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(5\,c\,d^2-\right.}{2\,d^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x)
```

```
[Out] ((x^2*(9*a*e - 7*b*d))/(35*d^2) - a/(7*d) - (x^4*(9*a*e^2 + 5*c*d^2 - 7*b*d
*e))/(15*d^3) + (e*x^6*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(3*d^4) + (e^2*x^8*(9
*a*e^2 + 5*c*d^2 - 7*b*d*e))/(2*d^5))/(d*x^7 + e*x^9) + (e^(3/2)*atan((e^(1
/2)*x)/d^(1/2))*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(2*d^(11/2))
```

sympy [B] time = 2.68, size = 328, normalized size = 1.96

$$-\frac{\sqrt{-\frac{e^3}{d^{11}}}\left(9ae^2-7bde+5cd^2\right)\log\left(-\frac{d^6\sqrt{-\frac{e^3}{d^{11}}}\left(9ae^2-7bde+5cd^2\right)}{9ae^4-7bde^3+5cd^2e^2}+x\right)}{4}+\frac{\sqrt{-\frac{e^3}{d^{11}}}\left(9ae^2-7bde+5cd^2\right)\log\left(\frac{d^6\sqrt{-\frac{e^3}{d^{11}}}\left(9\right.}{9ae^4-7b}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**8/(e*x**2+d)**2,x)
```

```
[Out] -sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(-d**6*sqrt(-e**3/d**
11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d**2*e**2)
 + x)/4 + sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(d**6*sqrt(-
e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d*
*2*e**2) + x)/4 + (-30*a*d**4 + x**8*(945*a*e**4 - 735*b*d*e**3 + 525*c*d**
2*e**2) + x**6*(630*a*d*e**3 - 490*b*d**2*e**2 + 350*c*d**3*e) + x**4*(-126
*a*d**2*e**2 + 98*b*d**3*e - 70*c*d**4) + x**2*(54*a*d**3*e - 42*b*d**4))/(
210*d**6*x**7 + 210*d**5*e*x**9)
```

3.288 $\int \frac{x^6\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=173

$$\frac{dx\left(17cd^2-e(13bd-9ae)\right)}{8e^5\left(d+ex^2\right)}+\frac{x\left(6cd^2-e(3bd-ae)\right)}{e^5}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(15ae^2-35bde+63cd^2\right)}{8e^{11/2}}-\frac{d^2x\left(ae^2-bde\right.}{4e^5\left(d+ex\right.}$$

```
[Out] (6*c*d^2-e*(-a*e+3*b*d))*x/e^5-1/3*(-b*e+3*c*d)*x^3/e^4+1/5*c*x^5/e^3-1/4*d
^2*(a*e^2-b*d*e+c*d^2)*x/e^5/(e*x^2+d)^2+1/8*d*(17*c*d^2-e*(-9*a*e+13*b*d))
*x/e^5/(e*x^2+d)-1/8*(15*a*e^2-35*b*d*e+63*c*d^2)*arctan(x*e^(1/2)/d^(1/2))
*d^(1/2)/e^(11/2)
```

Rubi [A] time = 0.32, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1257, 1814, 1810, 205}

$$\frac{dx\left(17cd^2-e(13bd-9ae)\right)}{8e^5\left(d+ex^2\right)}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{4e^5\left(d+ex^2\right)^2}+\frac{x\left(6cd^2-e(3bd-ae)\right)}{e^5}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(15ae^2-35bde+\right.}{8e^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((6*c*d^2 - e*(3*b*d - a*e))*x)/e^5 - ((3*c*d - b*e)*x^3)/(3*e^4) + (c*x^5)
/(5*e^3) - (d^2*(c*d^2 - b*d*e + a*e^2)*x)/(4*e^5*(d + e*x^2)^2) + (d*(17*c
*d^2 - e*(13*b*d - 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^2 - 35
*b*d*e + 15*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^6\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}dx=-\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2}-\frac{\int \frac{-d^2\left(cd^2-bde+ae^2\right)+4de\left(cd^2-bde+ae^2\right)x^2-4e^2\left(cd^2-bde+ae^2\right)x^4+4e^3(cd-be)x}{\left(d+ex^2\right)^2}}{4e^5}$$

$$=-\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2}+\frac{d\left(17cd^2-e(13bd-9ae)\right)x}{8e^5\left(d+ex^2\right)}+\frac{\int \frac{-d^2\left(15cd^2-e(11bd-7ae)\right)+8de\left(3cd\right.}{}}{}$$

$$=-\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2}+\frac{d\left(17cd^2-e(13bd-9ae)\right)x}{8e^5\left(d+ex^2\right)}+\frac{\int\left(8d\left(6cd^2-e(3bd-ae)\right)-\right.}{}$$

$$=\frac{\left(6cd^2-e(3bd-ae)\right)x}{e^5}-\frac{(3cd-be)x^3}{3e^4}+\frac{cx^5}{5e^3}-\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2}+\frac{d\left(17cd^2-e(1\right.}{8e^5\left(d\right.}$$

$$=\frac{\left(6cd^2-e(3bd-ae)\right)x}{e^5}-\frac{(3cd-be)x^3}{3e^4}+\frac{cx^5}{5e^3}-\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2}+\frac{d\left(17cd^2-e(1\right.}{8e^5\left(d\right.}$$

Mathematica [A] time = 0.11, size = 170, normalized size = 0.98

$$\frac{x\left(de(9ae-13bd)+17cd^3\right)}{8e^5\left(d+ex^2\right)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5e(3ae-7bd)+63cd^2\right)}{8e^{11/2}}+\frac{x\left(e(ae-3bd)+6cd^2\right)}{e^5}-\frac{x\left(d^2e(ae-bd)+\right.}{4e^5\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((6*c*d^2 + e*(-3*b*d + a*e))*x)/e^5 + ((-3*c*d + b*e)*x^3)/(3*e^4) + (c*x^
5)/(5*e^3) - ((c*d^4 + d^2*e*(-(b*d) + a*e))*x)/(4*e^5*(d + e*x^2)^2) + ((1
7*c*d^3 + d*e*(-13*b*d + 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^
2 + 5*e*(-7*b*d + 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

fricas [A] time = 0.97, size = 504, normalized size = 2.91

$$\left[\frac{48\,ce^4x^9-16\left(9\,cde^3-5\,be^4\right)x^7+16\left(63\,cd^2e^2-35\,bde^3+15\,ae^4\right)x^5+50\left(63\,cd^3e-35\,bd^2e^2+15\,ade^3\right)x^3+15\,(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*c*e^4*x^9 - 16*(9*c*d*e^3 - 5*b*e^4)*x^7 + 16*(63*c*d^2*e^2 - 35
*b*d*e^3 + 15*a*e^4)*x^5 + 50*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^3
+ 15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3 + 1
5*a*e^4)*x^4 + 2*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(-d/e)*l
og((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 30*(63*c*d^4 - 35*b*d^3*e
+ 15*a*d^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e^5), 1/120*(24*c*e^4*x^9 -
 8*(9*c*d*e^3 - 5*b*e^4)*x^7 + 8*(63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^5
 + 25*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^3 - 15*(63*c*d^4 - 35*b*d^
3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^4 + 2*(63*c*d
^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) +
```

```
15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e
^5)]
```

giac [A] time = 0.36, size = 160, normalized size = 0.92

$$-\frac{\left(63\,cd^3-35\,bd^2e+15\,ade^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}}{8\,\sqrt{d}}+\frac{1}{15}\left(3\,cx^5e^{12}-15\,cdx^3e^{11}+5\,bx^3e^{12}+90\,cd^2xe^{10}-45\,bdxe\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(63*c*d^3 - 35*b*d^2*e + 15*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-11/
2)/sqrt(d) + 1/15*(3*c*x^5*e^12 - 15*c*d*x^3*e^11 + 5*b*x^3*e^12 + 90*c*d^2
*x*e^10 - 45*b*d*x*e^11 + 15*a*x*e^12)*e^(-15) + 1/8*(17*c*d^3*x^3*e - 13*b
*d^2*x^3*e^2 + 15*c*d^4*x + 9*a*d*x^3*e^3 - 11*b*d^3*x*e + 7*a*d^2*x*e^2)*e
^(-5)/(x^2*e + d)^2
```

maple [A] time = 0.02, size = 239, normalized size = 1.38

$$\frac{9ad\,x^3}{8\left(e\,x^2+d\right)^2e^2}-\frac{13b\,d^2x^3}{8\left(e\,x^2+d\right)^2e^3}+\frac{17c\,d^3x^3}{8\left(e\,x^2+d\right)^2e^4}+\frac{c\,x^5}{5e^3}+\frac{7a\,d^2x}{8\left(e\,x^2+d\right)^2e^3}-\frac{11b\,d^3x}{8\left(e\,x^2+d\right)^2e^4}+\frac{b\,x^3}{3e^3}+\frac{15c\,d^4x}{8\left(e\,x^2+d\right)^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] 1/5*c*x^5/e^3+1/3/e^3*x^3*b-1/e^4*x^3*c*d+1/e^3*a*x-3/e^4*d*b*x+6/e^5*c*d^2
*x+9/8*d/e^2/(e*x^2+d)^2*x^3*a-13/8*d^2/e^3/(e*x^2+d)^2*x^3*b+17/8*d^3/e^4/
(e*x^2+d)^2*x^3*c+7/8*d^2/e^3/(e*x^2+d)^2*a*x-11/8*d^3/e^4/(e*x^2+d)^2*b*x+
15/8*d^4/e^5/(e*x^2+d)^2*c*x-15/8*d/e^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*
x)*a+35/8*d^2/e^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b-63/8*d^3/e^5/(d*e
)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.47, size = 175, normalized size = 1.01

$$\frac{\left(17\,cd^3e-13\,bd^2e^2+9\,ade^3\right)x^3+\left(15\,cd^4-11\,bd^3e+7\,ad^2e^2\right)x}{8\left(e^7x^4+2\,de^6x^2+d^2e^5\right)}-\frac{\left(63\,cd^3-35\,bd^2e+15\,ade^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}\,e^5}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((17*c*d^3*e - 13*b*d^2*e^2 + 9*a*d*e^3)*x^3 + (15*c*d^4 - 11*b*d^3*e +
 7*a*d^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e^5) - 1/8*(63*c*d^3 - 35*b*d
^2*e + 15*a*d*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^5) + 1/15*(3*c*e^2*x^
5 - 5*(3*c*d*e - b*e^2)*x^3 + 15*(6*c*d^2 - 3*b*d*e + a*e^2)*x)/e^5
```

mupad [B] time = 0.35, size = 223, normalized size = 1.29

$$x^3\left(\frac{b}{3\,e^3}-\frac{c\,d}{e^4}\right)-x\left(\frac{3\,c\,d^2}{e^5}-\frac{a}{e^3}+\frac{3\,d\left(\frac{b}{e^3}-\frac{3\,c\,d}{e^4}\right)}{e}\right)+\frac{\left(\frac{17\,c\,d^3\,e}{8}-\frac{13\,b\,d^2\,e^2}{8}+\frac{9\,a\,d\,e^3}{8}\right)x^3+\left(\frac{15\,c\,d^4}{8}-\frac{11\,b\,d^3\,e}{8}+\frac{7\,a\,d^2\,e^2}{8}\right)}{d^2\,e^5+2\,d\,e^6\,x^2+e^7\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

```
[Out] x^3*(b/(3*e^3) - (c*d)/e^4) - x*((3*c*d^2)/e^5 - a/e^3 + (3*d*(b/e^3 - (3*c
*d)/e^4))/e) + (x^3*((9*a*d*e^3)/8 - (13*b*d^2*e^2)/8 + (17*c*d^3*e)/8) + x
*((15*c*d^4)/8 + (7*a*d^2*e^2)/8 - (11*b*d^3*e)/8))/(d^2*e^5 + e^7*x^4 + 2*
d*e^6*x^2) + (c*x^5)/(5*e^3) - (d^(1/2)*atan((d^(1/2)*e^(1/2)*x*(15*a*e^2 +
 63*c*d^2 - 35*b*d*e))/(63*c*d^3 + 15*a*d*e^2 - 35*b*d^2*e))*(15*a*e^2 + 63
*c*d^2 - 35*b*d*e))/(8*e^(11/2))
```

sympy [A] time = 3.58, size = 235, normalized size = 1.36

$$\frac{cx^5}{5e^3}+x^3\left(\frac{b}{3e^3}-\frac{cd}{e^4}\right)+x\left(\frac{a}{e^3}-\frac{3bd}{e^4}+\frac{6cd^2}{e^5}\right)+\frac{\sqrt{-\frac{d}{e^{11}}}\left(15ae^2-35bde+63cd^2\right)\log\left(-e^5\sqrt{-\frac{d}{e^{11}}}+x\right)}{16}-\frac{\sqrt{-\frac{d}{e^{11}}}\left(15ae^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**5/(5*e**3) + x**3*(b/(3*e**3) - c*d/e**4) + x*(a/e**3 - 3*b*d/e**4 + 6
*c*d**2/e**5) + sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d**2)*log(-e**5
*sqrt(-d/e**11) + x)/16 - sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d**2)
*log(e**5*sqrt(-d/e**11) + x)/16 + (x**3*(9*a*d*e**3 - 13*b*d**2*e**2 + 17*
c*d**3*e) + x*(7*a*d**2*e**2 - 11*b*d**3*e + 15*c*d**4))/(8*d**2*e**5 + 16*
d*e**6*x**2 + 8*e**7*x**4)
```

3.289 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=143

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(35cd^2-3e(5bd-ae)\right)}{8\sqrt{d}\,e^{9/2}}-\frac{x\left(13cd^2-e(9bd-5ae)\right)}{8e^4\left(d+ex^2\right)}+\frac{dx\left(ae^2-bde+cd^2\right)}{4e^4\left(d+ex^2\right)^2}-\frac{x(3cd-be)}{e^4}+\frac{cx^3}{3e^3}$$

```
[Out] -(-b*e+3*c*d)*x/e^4+1/3*c*x^3/e^3+1/4*d*(a*e^2-b*d*e+c*d^2)*x/e^4/(e*x^2+d)
^2-1/8*(13*c*d^2-e*(-5*a*e+9*b*d))*x/e^4/(e*x^2+d)+1/8*(35*c*d^2-3*e*(-a*e+
5*b*d))*arctan(x*e^(1/2)/d^(1/2))/e^(9/2)/d^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1257, 1814, 1153, 205}

$$-\frac{x\left(13cd^2-e(9bd-5ae)\right)}{8e^4\left(d+ex^2\right)}+\frac{dx\left(ae^2-bde+cd^2\right)}{4e^4\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(35cd^2-3e(5bd-ae)\right)}{8\sqrt{d}\,e^{9/2}}-\frac{x(3cd-be)}{e^4}+\frac{cx^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] -(((3*c*d - b*e)*x)/e^4) + (c*x^3)/(3*e^3) + (d*(c*d^2 - b*d*e + a*e^2)*x)/
(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - e*(9*b*d - 5*a*e))*x)/(8*e^4*(d + e*x^
2)) + ((35*c*d^2 - 3*e*(5*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[
d]*e^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}dx = \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2} - \frac{\int \frac{d\left(cd^2-bde+ae^2\right)-4e\left(cd^2-bde+ae^2\right)x^2+4e^2(cd-be)x^4-4ce^3x^6}{\left(d+ex^2\right)^2}dx}{4e^4}$$

$$= \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-e(9bd-5ae)\right)x}{8e^4\left(d+ex^2\right)} + \frac{\int \frac{d\left(11cd^2-e(7bd-3ae)\right)-8de(2cd-be)x^2+8cd}{d+ex^2}}{8de^4}$$

$$= \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-e(9bd-5ae)\right)x}{8e^4\left(d+ex^2\right)} + \frac{\int\left(-8d(3cd-be)+8cdex^2+\frac{35cd^3}{}\right.}{8de^4}$$

$$= -\frac{(3cd-be)x}{e^4} + \frac{cx^3}{3e^3} + \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-e(9bd-5ae)\right)x}{8e^4\left(d+ex^2\right)} + \frac{\left(35cd^2-3e\right.}{}$$

$$= -\frac{(3cd-be)x}{e^4} + \frac{cx^3}{3e^3} + \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-e(9bd-5ae)\right)x}{8e^4\left(d+ex^2\right)} + \frac{\left(35cd^2-3e\right.}{}$$

Mathematica [A] time = 0.09, size = 141, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3ae^2-15bde+35cd^2\right)}{8\sqrt{d}\,e^{9/2}} - \frac{x\left(5ae^2-9bde+13cd^2\right)}{8e^4\left(d+ex^2\right)} + \frac{x\left(ade^2-bd^2e+cd^3\right)}{4e^4\left(d+ex^2\right)^2} + \frac{x(be-3cd)}{e^4} + \frac{cx^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((-3*c*d + b*e)*x)/e^4 + (c*x^3)/(3*e^3) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/
(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 9*b*d*e + 5*a*e^2)*x)/(8*e^4*(d + e*x^
2)) + ((35*c*d^2 - 15*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt
[d]*e^(9/2))
```

fricas [A] time = 0.68, size = 462, normalized size = 3.23

$$\left[\frac{16\,cde^4x^7 - 16\left(7\,cd^2e^3 - 3\,bde^4\right)x^5 - 10\left(35\,cd^3e^2 - 15\,bd^2e^3 + 3\,ade^4\right)x^3 - 3\left(35\,cd^4 - 15\,bd^3e + 3\,ad^2e^2 + \left(35\,c\right.\right.}{48\left(de^7x^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c*d*e^4*x^7 - 16*(7*c*d^2*e^3 - 3*b*d*e^4)*x^5 - 10*(35*c*d^3*e^2
 - 15*b*d^2*e^3 + 3*a*d*e^4)*x^3 - 3*(35*c*d^4 - 15*b*d^3*e + 3*a*d^2*e^2 +
 (35*c*d^2*e^2 - 15*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d^2*e^2 +
 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) -
 6*(35*c*d^4*e - 15*b*d^3*e^2 + 3*a*d^2*e^3)*x)/(d*e^7*x^4 + 2*d^2*e^6*x^2
+ d^3*e^5), 1/24*(8*c*d*e^4*x^7 - 8*(7*c*d^2*e^3 - 3*b*d*e^4)*x^5 - 5*(35*c
*d^3*e^2 - 15*b*d^2*e^3 + 3*a*d*e^4)*x^3 + 3*(35*c*d^4 - 15*b*d^3*e + 3*a*d
^2*e^2 + (35*c*d^2*e^2 - 15*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d
```

```
^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(35*c*d^4*e -
15*b*d^3*e^2 + 3*a*d^2*e^3)*x)/(d*e^7*x^4 + 2*d^2*e^6*x^2 + d^3*e^5)]
```

giac [A] time = 0.47, size = 125, normalized size = 0.87

$$\frac{\left(35\,cd^2-15\,bde+3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,\sqrt{d}}+\frac{1}{3}\left(cx^3e^6-9\,cdxe^5+3\,bxe^6\right)e^{(-9)}-\frac{\left(13\,cd^2x^3e-9\,bdx^3e^2+11\,cd^3x\right.}{8\left(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*c*d^2 - 15*b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt
(d) + 1/3*(c*x^3*e^6 - 9*c*d*x*e^5 + 3*b*x*e^6)*e^(-9) - 1/8*(13*c*d^2*x^3*
e - 9*b*d*x^3*e^2 + 11*c*d^3*x + 5*a*x^3*e^3 - 7*b*d^2*x*e + 3*a*d*x*e^2)*e
^(-4)/(x^2*e + d)^2
```

maple [A] time = 0.01, size = 202, normalized size = 1.41

$$-\frac{5a\,x^3}{8\left(e\,x^2+d\right)^2e}+\frac{9bd\,x^3}{8\left(e\,x^2+d\right)^2e^2}-\frac{13c\,d^2x^3}{8\left(e\,x^2+d\right)^2e^3}-\frac{3adx}{8\left(e\,x^2+d\right)^2e^2}+\frac{7b\,d^2x}{8\left(e\,x^2+d\right)^2e^3}-\frac{11c\,d^3x}{8\left(e\,x^2+d\right)^2e^4}+\frac{c\,x^3}{3e^3}+\frac{3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] 1/3*c*x^3/e^3+1/e^3*b*x-3/e^4*c*d*x-5/8/e/(e*x^2+d)^2*x^3*a+9/8/e^2/(e*x^2+
d)^2*x^3*b*d-13/8/e^3/(e*x^2+d)^2*x^3*c*d^2-3/8/e^2/(e*x^2+d)^2*a*d*x+7/8/e
^3/(e*x^2+d)^2*d^2*b*x-11/8/e^4/(e*x^2+d)^2*c*d^3*x+3/8/e^2/(d*e)^(1/2)*arc
tan(1/(d*e)^(1/2)*e*x)*a-15/8/e^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*d*b
+35/8/e^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c*d^2
```

maxima [A] time = 2.51, size = 139, normalized size = 0.97

$$-\frac{\left(13\,cd^2e-9\,bde^2+5\,ae^3\right)x^3+\left(11\,cd^3-7\,bd^2e+3\,ade^2\right)x}{8\left(e^6x^4+2\,de^5x^2+d^2e^4\right)}+\frac{\left(35\,cd^2-15\,bde+3\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}\,e^4}+\frac{cex^3-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*c*d^2*e - 9*b*d*e^2 + 5*a*e^3)*x^3 + (11*c*d^3 - 7*b*d^2*e + 3*a*
d*e^2)*x)/(e^6*x^4 + 2*d*e^5*x^2 + d^2*e^4) + 1/8*(35*c*d^2 - 15*b*d*e + 3*
a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4) + 1/3*(c*e*x^3 - 3*(3*c*d - b*
e)*x)/e^4
```

mupad [B] time = 0.34, size = 137, normalized size = 0.96

$$x\left(\frac{b}{e^3}-\frac{3\,c\,d}{e^4}\right)-\frac{\left(\frac{13\,c\,d^2\,e}{8}-\frac{9\,b\,d\,e^2}{8}+\frac{5\,a\,e^3}{8}\right)x^3+\left(\frac{11\,c\,d^3}{8}-\frac{7\,b\,d^2\,e}{8}+\frac{3\,a\,d\,e^2}{8}\right)x}{d^2\,e^4+2\,d\,e^5\,x^2+e^6\,x^4}+\frac{c\,x^3}{3\,e^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(35\,c\,d^2-15\,b\,d\,e\right.}{8\,\sqrt{d}\,e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

```
[Out] x*(b/e^3 - (3*c*d)/e^4) - (x*((11*c*d^3)/8 + (3*a*d*e^2)/8 - (7*b*d^2*e)/8)
 + x^3*((5*a*e^3)/8 - (9*b*d*e^2)/8 + (13*c*d^2*e)/8))/(d^2*e^4 + e^6*x^4 +
```

```
 2*d*e^5*x^2) + (c*x^3)/(3*e^3) + (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 35*
c*d^2 - 15*b*d*e))/(8*d^(1/2)*e^(9/2))
```

sympy [A] time = 3.37, size = 212, normalized size = 1.48

$$\frac{cx^3}{3e^3}+x\left(\frac{b}{e^3}-\frac{3cd}{e^4}\right)-\frac{\sqrt{-\frac{1}{de^9}}\left(3ae^2-15bde+35cd^2\right)\log\left(-de^4\sqrt{-\frac{1}{de^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{de^9}}\left(3ae^2-15bde+35cd^2\right)\log\left(d\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**3/(3*e**3) + x*(b/e**3 - 3*c*d/e**4) - sqrt(-1/(d*e**9))*(3*a*e**2 - 1
5*b*d*e + 35*c*d**2)*log(-d*e**4*sqrt(-1/(d*e**9)) + x)/16 + sqrt(-1/(d*e**
9))*(3*a*e**2 - 15*b*d*e + 35*c*d**2)*log(d*e**4*sqrt(-1/(d*e**9)) + x)/16
+ (x**3*(-5*a*e**3 + 9*b*d*e**2 - 13*c*d**2*e) + x*(-3*a*d*e**2 + 7*b*d**2*
e - 11*c*d**3))/(8*d**2*e**4 + 16*d*e**5*x**2 + 8*e**6*x**4)
```

3.290 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=124

$$\frac{x\left(9cd^2-e(5bd-ae)\right)}{8de^3\left(d+ex^2\right)}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(15cd^2-e(ae+3bd)\right)}{8d^{3/2}e^{7/2}}+\frac{cx}{e^3}$$

```
[Out] c*x/e^3-1/4*(a*e^2-b*d*e+c*d^2)*x/e^3/(e*x^2+d)^2+1/8*(9*c*d^2-e*(-a*e+5*b*
d))*x/d/e^3/(e*x^2+d)-1/8*(15*c*d^2-e*(a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2)
)/d^(3/2)/e^(7/2)
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1257, 1157, 388, 205}

$$\frac{x\left(9cd^2-e(5bd-ae)\right)}{8de^3\left(d+ex^2\right)}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(15cd^2-e(ae+3bd)\right)}{8d^{3/2}e^{7/2}}+\frac{cx}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 -
 e*(5*b*d - a*e))*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - e*(3*b*d + a*e))*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
```

```
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}\,dx=-\frac{\left(cd^2-bde+ae^2\right)x}{4e^3\left(d+ex^2\right)^2}-\frac{\int \frac{-cd^2+bde-ae^2+4e(cd-be)x^2-4ce^2x^4}{\left(d+ex^2\right)^2}\,dx}{4e^3}$$
$$=-\frac{\left(cd^2-bde+ae^2\right)x}{4e^3\left(d+ex^2\right)^2}+\frac{\left(9cd^2-e(5bd-ae)\right)x}{8de^3\left(d+ex^2\right)}+\frac{\int \frac{-7cd^2+e(3bd+ae)+8cdex^2}{d+ex^2}\,dx}{8de^3}$$
$$=\frac{cx}{e^3}-\frac{\left(cd^2-bde+ae^2\right)x}{4e^3\left(d+ex^2\right)^2}+\frac{\left(9cd^2-e(5bd-ae)\right)x}{8de^3\left(d+ex^2\right)}-\frac{\left(15cd^2-e(3bd+ae)\right)\int \frac{1}{d+ex^2}\,dx}{8de^3}$$
$$=\frac{cx}{e^3}-\frac{\left(cd^2-bde+ae^2\right)x}{4e^3\left(d+ex^2\right)^2}+\frac{\left(9cd^2-e(5bd-ae)\right)x}{8de^3\left(d+ex^2\right)}-\frac{\left(15cd^2-e(3bd+ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{7/2}}$$

Mathematica [A] time = 0.10, size = 122, normalized size = 0.98

$$\frac{x\left(ae^2-5bde+9cd^2\right)}{8de^3\left(d+ex^2\right)}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-3bde+15cd^2\right)}{8d^{3/2}e^{7/2}}+\frac{cx}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 -
 5*b*d*e + a*e^2)*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - 3*b*d*e - a*e^2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))
```

fricas [A] time = 0.81, size = 421, normalized size = 3.40

$$\left[\frac{16\,cd^2e^3x^5+2\left(25\,cd^3e^2-5\,bd^2e^3+ade^4\right)x^3+\left(15\,cd^4-3\,bd^3e-ad^2e^2+\left(15\,cd^2e^2-3\,bde^3-ae^4\right)x^4+2\left(15\,cd^3\right.\right.}{16\left(d^2e^6x^4+2\,d^3e^5x^2+d^4e^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*c*d^2*e^3*x^5 + 2*(25*c*d^3*e^2 - 5*b*d^2*e^3 + a*d*e^4)*x^3 + (1
5*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d^2*e^2 - 3*b*d*e^3 - a*e^4)*x^4 +
2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(
-d*e)*x - d)/(e*x^2 + d)) + 2*(15*c*d^4*e - 3*b*d^3*e^2 - a*d^2*e^3)*x)/(d^
2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4), 1/8*(8*c*d^2*e^3*x^5 + (25*c*d^3*e^2
- 5*b*d^2*e^3 + a*d*e^4)*x^3 - (15*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d^
2*e^2 - 3*b*d*e^3 - a*e^4)*x^4 + 2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (15*c*d^4*e - 3*b*d^3*e^2 - a*d^2*e^3)*
x)/(d^2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4)]
```

giac [A] time = 0.26, size = 107, normalized size = 0.86

$$cxe^{(-3)}-\frac{\left(15\,cd^2-3\,bde-ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{8\,d^{\frac{3}{2}}}+\frac{\left(9\,cd^2x^3e-5\,bdx^3e^2+7\,cd^3x+ax^3e^3-3\,bd^2xe-adxe^2\right)e^{(-3)}}{8\left(x^2e+d\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] c*x*e^(-3) - 1/8*(15*c*d^2 - 3*b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(
-7/2)/d^(3/2) + 1/8*(9*c*d^2*x^3*e - 5*b*d*x^3*e^2 + 7*c*d^3*x + a*x^3*e^3
- 3*b*d^2*x*e - a*d*x*e^2)*e^(-3)/((x^2*e + d)^2*d)
```

maple [A] time = 0.01, size = 179, normalized size = 1.44

$$\frac{a\,x^{3}}{8\left(e\,x^{2}+d\right)^{2} d}-\frac{5b\,x^{3}}{8\left(e\,x^{2}+d\right)^{2} e}+\frac{9cd\,x^{3}}{8\left(e\,x^{2}+d\right)^{2} e^{2}}-\frac{ax}{8\left(e\,x^{2}+d\right)^{2} e}-\frac{3bdx}{8\left(e\,x^{2}+d\right)^{2} e^{2}}+\frac{7c\,d^{2}x}{8\left(e\,x^{2}+d\right)^{2} e^{3}}+\frac{a\arctan\left(\cdots\right)}{8\sqrt{de}\,de\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] c*x/e^3+1/8/(e*x^2+d)^2/d*x^3*a-5/8/e/(e*x^2+d)^2*x^3*b+9/8/e^2/(e*x^2+d)^2
*x^3*c*d-1/8/e/(e*x^2+d)^2*a*x-3/8/e^2/(e*x^2+d)^2*d*b*x+7/8/e^3/(e*x^2+d)^
2*c*d^2*x+1/8/e/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+3/8/e^2/(d*e)^(1/
2)*arctan(1/(d*e)^(1/2)*e*x)*b-15/8/e^3*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*
e*x)*c
```

maxima [A] time = 2.46, size = 126, normalized size = 1.02

$$\frac{\left(9\,cd^{2}e-5\,bde^{2}+ae^{3}\right)x^{3}+\left(7\,cd^{3}-3\,bd^{2}e-ade^{2}\right)x}{8\left(de^{5}x^{4}+2\,d^{2}e^{4}x^{2}+d^{3}e^{3}\right)}+\frac{cx}{e^{3}}-\frac{\left(15\,cd^{2}-3\,bde-ae^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}\,de^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*c*d^2*e - 5*b*d*e^2 + a*e^3)*x^3 + (7*c*d^3 - 3*b*d^2*e - a*d*e^2)*
x)/(d*e^5*x^4 + 2*d^2*e^4*x^2 + d^3*e^3) + c*x/e^3 - 1/8*(15*c*d^2 - 3*b*d*
e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^3)
```

mupad [B] time = 0.39, size = 118, normalized size = 0.95

$$\frac{c\,x}{e^{3}}-\frac{x\left(-\frac{7\,c\,d^{2}}{8}+\frac{3\,b\,d\,e}{8}+\frac{a\,e^{2}}{8}\right)-\frac{x^{3}\left(9\,c\,d^{2}\,e-5\,b\,d\,e^{2}+a\,e^{3}\right)}{8\,d}}{d^{2}\,e^{3}+2\,d\,e^{4}\,x^{2}+e^{5}\,x^{4}}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(-15\,c\,d^{2}+3\,b\,d\,e+a\,e^{2}\right)}{8\,d^{3/2}\,e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

```
[Out] (c*x)/e^3 - (x*((a*e^2)/8 - (7*c*d^2)/8 + (3*b*d*e)/8) - (x^3*(a*e^3 - 5*b*
d*e^2 + 9*c*d^2*e))/(8*d))/(d^2*e^3 + e^5*x^4 + 2*d*e^4*x^2) + (atan((e^(1/
2)*x)/d^(1/2))*(a*e^2 - 15*c*d^2 + 3*b*d*e))/(8*d^(3/2)*e^(7/2))
```

sympy [A] time = 2.62, size = 201, normalized size = 1.62

$$\frac{cx}{e^{3}}-\frac{\sqrt{-\frac{1}{d^{3}e^{7}}}\left(ae^{2}+3bde-15cd^{2}\right)\log\left(-d^{2}e^{3}\sqrt{-\frac{1}{d^{3}e^{7}}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^{3}e^{7}}}\left(ae^{2}+3bde-15cd^{2}\right)\log\left(d^{2}e^{3}\sqrt{-\frac{1}{d^{3}e^{7}}}+x\cdots\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x/e**3 - sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e - 15*c*d**2)*log(-d**2*e*
*3*sqrt(-1/(d**3*e**7)) + x)/16 + sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e -
15*c*d**2)*log(d**2*e**3*sqrt(-1/(d**3*e**7)) + x)/16 + (x**3*(a*e**3 - 5*b
*d*e**2 + 9*c*d**2*e) + x*(-a*d*e**2 - 3*b*d**2*e + 7*c*d**3))/(8*d**3*e**3
 + 16*d**2*e**4*x**2 + 8*d*e**5*x**4)
```

3.291 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=115

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

```
[Out] 1/4*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^2-1/8*(5*c*d^2-e*(3*a*e+b*d))*x/d^2/
e^2/(e*x^2+d)+1/8*(3*c*d^2+e*(3*a*e+b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)
/e^(5/2)
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 385, 205}

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d + 3*
a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2} - \frac{\int \frac{-3a+\frac{d(cd-be)}{e^2}-\frac{4cdx^2}{e}}{\left(d+ex^2\right)^2}\,dx}{4d}$$

$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2} - \frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)} - \frac{\left(-\frac{4cd^2}{e}+e\left(-3a+\frac{d(cd-be)}{e^2}\right)\right)\int \frac{1}{d+ex^2}\,dx}{8d^2e}$$

$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2} - \frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)} + \frac{\left(3cd^2+e(bd+3ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Mathematica [A] time = 0.10, size = 110, normalized size = 0.96

$$\frac{x\left(e\left(ae\left(5d+3ex^2\right)+bd\left(ex^2-d\right)\right)-cd^2\left(3d+5ex^2\right)\right)}{8d^2e^2\left(d+ex^2\right)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2))))
/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

fricas [A] time = 0.77, size = 391, normalized size = 3.40

$$\left[-\frac{2\left(5\,cd^3e^2-bd^2e^3-3\,ade^4\right)x^3+\left(3\,cd^4+bd^3e+3\,ad^2e^2+\left(3\,cd^2e^2+bde^3+3\,ae^4\right)x^4+2\left(3\,cd^3e+bd^2e^2+3\right.\right.}{16\left(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 + (3*c*d^4 + b*d^3*e +
3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*
e^2 + 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 +
d)) + 2*(3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x
^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 - (3*c*d^4 +
 b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3
*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^4
*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3)]
```

giac [A] time = 0.31, size = 101, normalized size = 0.88

$$\frac{\left(3\,cd^2+bde+3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}} - \frac{\left(5\,cd^2x^3e-bdx^3e^2+3\,cd^3x-3\,ax^3e^3+bd^2xe-5\,adxe^2\right)e^{(-2)}}{8\left(x^2e+d\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2)
- 1/8*(5*c*d^2*x^3*e - b*d*x^3*e^2 + 3*c*d^3*x - 3*a*x^3*e^3 + b*d^2*x*e -
5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2)
```

maple [A] time = 0.01, size = 131, normalized size = 1.14

$$\frac{3a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^2}+\frac{b\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,de}+\frac{3c\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,e^2}+\frac{\frac{\left(3a\,e^2+deb-5c\,d^2\right)x^3}{8d^2e}+\frac{\left(5a\,e^2-deb-3c\,d^2\right)x}{8d\,e^2}}{\left(e\,x^2+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] (1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/e^2/d*x)
/(e*x^2+d)^2+3/8/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+1/8/d/e/(d*e)^
(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b+3/8/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*
e*x)*c
```

maxima [A] time = 2.46, size = 121, normalized size = 1.05

$$-\frac{\left(5\,cd^2e-bde^2-3\,ae^3\right)x^3+\left(3\,cd^3+bd^2e-5\,ade^2\right)x}{8\left(d^2e^4x^4+2\,d^3e^3x^2+d^4e^2\right)}+\frac{\left(3\,cd^2+bde+3\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}\,d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*c*d^2*e - b*d*e^2 - 3*a*e^3)*x^3 + (3*c*d^3 + b*d^2*e - 5*a*d*e^2)
*x)/(d^2*e^4*x^4 + 2*d^3*e^3*x^2 + d^4*e^2) + 1/8*(3*c*d^2 + b*d*e + 3*a*e^
2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^2)
```

mupad [B] time = 0.38, size = 112, normalized size = 0.97

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(3\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}}-\frac{\frac{x\left(3\,c\,d^2+b\,d\,e-5\,a\,e^2\right)}{8\,d\,e^2}-\frac{x^3\left(-5\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^2\,e}}{d^2+2\,d\,e\,x^2+e^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^3,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 3*c*d^2 + b*d*e))/(8*d^(5/2)*e^(5/2))
 - ((x*(3*c*d^2 - 5*a*e^2 + b*d*e))/(8*d*e^2) - (x^3*(3*a*e^2 - 5*c*d^2 + b
*d*e))/(8*d^2*e))/(d^2 + e^2*x^4 + 2*d*e*x^2)
```

sympy [A] time = 1.50, size = 196, normalized size = 1.70

$$-\frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2+bde+3cd^2\right)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2+bde+3cd^2\right)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{x^3\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1/
(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*l
og(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 - 5*
c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e*
*3*x**2 + 8*d**2*e**4*x**4)
```

3.292 $\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3} \, dx$

Optimal. Leaf size=127

$$-\frac{x\left(ae^2-bde+cd^2\right)}{4d^2e\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3e(bd-5ae)+cd^2\right)}{8d^{7/2}e^{3/2}}+\frac{x\left(e(3bd-7ae)+cd^2\right)}{8d^3e\left(d+ex^2\right)}-\frac{a}{d^3x}$$

```
[Out] -a/d^3/x-1/4*(a*e^2-b*d*e+c*d^2)*x/d^2/e/(e*x^2+d)^2+1/8*(c*d^2+e*(-7*a*e+3
*b*d))*x/d^3/e/(e*x^2+d)+1/8*(c*d^2+3*e*(-5*a*e+b*d))*arctan(x*e^(1/2)/d^(1
/2))/d^(7/2)/e^(3/2)
```

Rubi [A] time = 0.20, antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1259, 456, 453, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3e(bd-5ae)+cd^2\right)}{8d^{7/2}e^{3/2}}+\frac{x\left(e(3bd-7ae)+cd^2\right)}{8d^3e\left(d+ex^2\right)}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{4\left(d+ex^2\right)^2}-\frac{a}{d^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] -(a/(d^3*x)) - ((c/e - (b*d - a*e)/d^2)*x)/(4*(d + e*x^2)^2) + ((c*d^2 + e*
(3*b*d - 7*a*e))*x)/(8*d^3*e*(d + e*x^2)) + ((c*d^2 + 3*e*(b*d - 5*a*e))*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*e^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
```

```
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx = -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2} - \frac{\int \frac{-4ade^2-e\left(cd^2+3e(bd-ae)\right)x^2}{x^2\left(d+ex^2\right)^2}\,dx}{4d^2e^2}$$

$$= -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2} + \frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)} + \frac{\int \frac{8ae^2+e\left(cd+e\left(3b-\frac{7ae}{d}\right)\right)x^2}{x^2\left(d+ex^2\right)}\,dx}{8d^2e^2}$$

$$= -\frac{a}{d^3x} - \frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2} + \frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)} + \frac{\left(cd^2+3e(bd-5ae)\right)\int \frac{1}{d+ex^2}\,dx}{8d^3e}$$

$$= -\frac{a}{d^3x} - \frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2} + \frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)} + \frac{\left(cd^2+3e(bd-5ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}e^{3/2}}$$

Mathematica [A] time = 0.13, size = 124, normalized size = 0.98

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3e(bd-5ae)+cd^2\right)}{e^{3/2}} + \frac{\sqrt{d}\left(dx^2\left(be\left(5d+3ex^2\right)+cd\left(ex^2-d\right)\right)-ae\left(8d^2+25dex^2+15e^2x^4\right)\right)}{ex\left(d+ex^2\right)^2}}{8d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] ((Sqrt[d]*(-(a*e*(8*d^2 + 25*d*e*x^2 + 15*e^2*x^4)) + d*x^2*(c*d*(-d + e*x^
2) + b*e*(5*d + 3*e*x^2))))/(e*x*(d + e*x^2)^2) + ((c*d^2 + 3*e*(b*d - 5*a*
e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2))/(8*d^(7/2))
```

fricas [A] time = 0.70, size = 421, normalized size = 3.31

$$\left[-\frac{16\,ad^3e^2 - 2\left(cd^3e^2 + 3\,bd^2e^3 - 15\,ade^4\right)x^4 + 2\left(cd^4e - 5\,bd^3e^2 + 25\,ad^2e^3\right)x^2 - \left(\left(cd^2e^2 + 3\,bde^3 - 15\,ae^4\right)x^5 + 2\right.}{16\left(d^4e^4x^5 + 2\,d^5e^3x^3 + d^6e^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a*d^3*e^2 - 2*(c*d^3*e^2 + 3*b*d^2*e^3 - 15*a*d*e^4)*x^4 + 2*(c*
d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*x^2 - ((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^
4)*x^5 + 2*(c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e -
15*a*d^2*e^2)*x)*sqrt(-d*e)*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/
(d^4*e^4*x^5 + 2*d^5*e^3*x^3 + d^6*e^2*x), -1/8*(8*a*d^3*e^2 - (c*d^3*e^2 +
 3*b*d^2*e^3 - 15*a*d*e^4)*x^4 + (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*x^2
 - ((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*x^5 + 2*(c*d^3*e + 3*b*d^2*e^2 - 15*
a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*x)*sqrt(d*e)*arctan(sqrt(
d*e)*x/d))/(d^4*e^4*x^5 + 2*d^5*e^3*x^3 + d^6*e^2*x)]
```

giac [A] time = 0.32, size = 110, normalized size = 0.87

$$\frac{\left(cd^2+3\,bde-15\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{8\,d^{\frac{7}{2}}}-\frac{a}{d^3x}+\frac{\left(cd^2x^3e+3\,bdx^3e^2-cd^3x-7\,ax^3e^3+5\,bd^2xe-9\,adxe^2\right)e^{(-1)}}{8\left(x^2e+d\right)^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(c*d^2 + 3*b*d*e - 15*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(7/2)
 - a/(d^3*x) + 1/8*(c*d^2*x^3*e + 3*b*d*x^3*e^2 - c*d^3*x - 7*a*x^3*e^3 + 5
*b*d^2*x*e - 9*a*d*x*e^2)*e^(-1)/((x^2*e + d)^2*d^3)
```

maple [A] time = 0.01, size = 182, normalized size = 1.43

$$-\frac{7a\,e^2x^3}{8\left(e\,x^2+d\right)^2d^3}+\frac{3be\,x^3}{8\left(e\,x^2+d\right)^2d^2}+\frac{c\,x^3}{8\left(e\,x^2+d\right)^2d}-\frac{9aex}{8\left(e\,x^2+d\right)^2d^2}+\frac{5bx}{8\left(e\,x^2+d\right)^2d}-\frac{cx}{8\left(e\,x^2+d\right)^2e}-\frac{15ae\,\mathrm{arct}}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x)
```

```
[Out] -a/d^3/x-7/8/d^3/(e*x^2+d)^2*x^3*a*e^2+3/8/d^2/(e*x^2+d)^2*x^3*b*e+1/8/d/(e
*x^2+d)^2*x^3*c-9/8/d^2/(e*x^2+d)^2*e*a*x+5/8/d/(e*x^2+d)^2*b*x-1/8/(e*x^2+
d)^2/e*x*c-15/8/d^3*e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+3/8/d^2/(d*e)
^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b+1/8/d/e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)
*e*x)*c
```

maxima [A] time = 2.65, size = 129, normalized size = 1.02

$$\frac{\left(cd^2e+3\,bde^2-15\,ae^3\right)x^4-8\,ad^2e-\left(cd^3-5\,bd^2e+25\,ade^2\right)x^2}{8\left(d^3e^3x^5+2\,d^4e^2x^3+d^5ex\right)}+\frac{\left(cd^2+3\,bde-15\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}\,d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((c*d^2*e + 3*b*d*e^2 - 15*a*e^3)*x^4 - 8*a*d^2*e - (c*d^3 - 5*b*d^2*e
+ 25*a*d*e^2)*x^2)/(d^3*e^3*x^5 + 2*d^4*e^2*x^3 + d^5*e*x) + 1/8*(c*d^2 + 3
*b*d*e - 15*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3*e)
```

mupad [B] time = 0.39, size = 118, normalized size = 0.93

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(c\,d^2+3\,b\,d\,e-15\,a\,e^2\right)}{8\,d^{7/2}\,e^{3/2}}-\frac{\frac{a}{d}-\frac{x^4\left(c\,d^2+3\,b\,d\,e-15\,a\,e^2\right)}{8\,d^3}+\frac{x^2\left(c\,d^2-5\,b\,d\,e+25\,a\,e^2\right)}{8\,d^2\,e}}{d^2\,x+2\,d\,e\,x^3+e^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(c*d^2 - 15*a*e^2 + 3*b*d*e))/(8*d^(7/2)*e^(3/2)
) - (a/d - (x^4*(c*d^2 - 15*a*e^2 + 3*b*d*e))/(8*d^3) + (x^2*(25*a*e^2 + c*
d^2 - 5*b*d*e))/(8*d^2*e))/(d^2*x + e^2*x^5 + 2*d*e*x^3)
```

sympy [A] time = 2.14, size = 202, normalized size = 1.59

$$\frac{\sqrt{-\frac{1}{d^7e^3}}\left(15ae^2-3bde-cd^2\right)\log\left(-d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{d^7e^3}}\left(15ae^2-3bde-cd^2\right)\log\left(d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{16}+\frac{-8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log(-d**4*e*sqrt(-1/(d*
*7*e**3)) + x)/16 - sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log
(d**4*e*sqrt(-1/(d**7*e**3)) + x)/16 + (-8*a*d**2*e + x**4*(-15*a*e**3 + 3*
b*d*e**2 + c*d**2*e) + x**2*(-25*a*d*e**2 + 5*b*d**2*e - c*d**3))/(8*d**5*e
*x + 16*d**4*e**2*x**3 + 8*d**3*e**3*x**5)
```

3.293 $\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^3} dx$

Optimal. Leaf size=142

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35ae^2-15bde+3cd^2\right)}{8d^{9/2}\sqrt{e}}+\frac{x\left(3cd^2-e(7bd-11ae)\right)}{8d^4\left(d+ex^2\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2}-\frac{bd-3ae}{d^4x}-\frac{a}{3d^3x^3}$$

```
[Out] -1/3*a/d^3/x^3+(3*a*e-b*d)/d^4/x+1/4*(a*e^2-b*d*e+c*d^2)*x/d^3/(e*x^2+d)^2+
1/8*(3*c*d^2-e*(-11*a*e+7*b*d))*x/d^4/(e*x^2+d)+1/8*(35*a*e^2-15*b*d*e+3*c*
d^2)*arctan(x*e^(1/2)/d^(1/2))/d^(9/2)/e^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1259, 1261, 205}

$$\frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35ae^2-15bde+3cd^2\right)}{8d^{9/2}\sqrt{e}}+\frac{x\left(3cd^2-e(7bd-11ae)\right)}{8d^4\left(d+ex^2\right)}-\frac{bd-3ae}{d^4x}-\frac{a}{3d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] -a/(3*d^3*x^3) - (b*d - 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(4*d^3
*(d + e*x^2)^2) + ((3*c*d^2 - e*(7*b*d - 11*a*e))*x)/(8*d^4*(d + e*x^2)) +
((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(9/2)*Sq
rt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^3}\,dx = \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\int \frac{4ad^2e^2+4de^2(bd-ae)x^2+3e^2\left(cd^2-bde+ae^2\right)x^4}{x^4\left(d+ex^2\right)^2}dx}{4d^3e^2}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\left(3cd^2-e(7bd-11ae)\right)x}{8d^4\left(d+ex^2\right)} + \frac{\int \frac{8ad^4e^4+8d^3e^4(bd-2ae)x^2+d^2e^4\left(3cd^2-e(7bd-11ae)\right)}{x^4\left(d+ex^2\right)}}{8d^6e^4}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\left(3cd^2-e(7bd-11ae)\right)x}{8d^4\left(d+ex^2\right)} + \frac{\int \left(\frac{8ad^3e^4}{x^4} + \frac{8d^2e^4(bd-3ae)}{x^2} + \frac{d^2e^4\left(3cd^2-15bde+3\right.}{d+ex^2}\right.}{8d^6e^4}$$

$$= -\frac{a}{3d^3x^3} - \frac{bd-3ae}{d^4x} + \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\left(3cd^2-e(7bd-11ae)\right)x}{8d^4\left(d+ex^2\right)} + \frac{\left(3cd^2-15bde+3\right.}{8d^4}$$

$$= -\frac{a}{3d^3x^3} - \frac{bd-3ae}{d^4x} + \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\left(3cd^2-e(7bd-11ae)\right)x}{8d^4\left(d+ex^2\right)} + \frac{\left(3cd^2-15bde+3\right.}{8d^{9/2}}$$

Mathematica [A] time = 0.09, size = 141, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35ae^2-15bde+3cd^2\right)}{8d^{9/2}\sqrt{e}}+\frac{x\left(11ae^2-7bde+3cd^2\right)}{8d^4\left(d+ex^2\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2}+\frac{3ae-bd}{d^4x}-\frac{a}{3d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] -1/3*a/(d^3*x^3) + (-(b*d) + 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(
4*d^3*(d + e*x^2)^2) + ((3*c*d^2 - 7*b*d*e + 11*a*e^2)*x)/(8*d^4*(d + e*x^2
)) + ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(9/
2)*Sqrt[e])
```

fricas [A] time = 0.88, size = 476, normalized size = 3.35

$$\left[\frac{6\left(3\,cd^3e^2-15\,bd^2e^3+35\,ade^4\right)x^6-16\,ad^4e+10\left(3\,cd^4e-15\,bd^3e^2+35\,ad^2e^3\right)x^4-16\left(3\,bd^4e-7\,ad^3e^2\right)x^2-3\left(\right.}{48\left(d^5e^3x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(6*(3*c*d^3*e^2 - 15*b*d^2*e^3 + 35*a*d*e^4)*x^6 - 16*a*d^4*e + 10*(3
*c*d^4*e - 15*b*d^3*e^2 + 35*a*d^2*e^3)*x^4 - 16*(3*b*d^4*e - 7*a*d^3*e^2)*
x^2 - 3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c*d^3*e - 15*b*d^
2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35*a*d^2*e^2)*x^3)*sqrt(-
d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^5*e^3*x^7 + 2*d^6*e^
2*x^5 + d^7*e*x^3), 1/24*(3*(3*c*d^3*e^2 - 15*b*d^2*e^3 + 35*a*d*e^4)*x^6 -
 8*a*d^4*e + 5*(3*c*d^4*e - 15*b*d^3*e^2 + 35*a*d^2*e^3)*x^4 - 8*(3*b*d^4*e
 - 7*a*d^3*e^2)*x^2 + 3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c
*d^3*e - 15*b*d^2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35*a*d^2*
e^2)*x^3)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d^5*e^3*x^7 + 2*d^6*e^2*x^5 + d
^7*e*x^3)]
```

giac [A] time = 0.34, size = 128, normalized size = 0.90

$$\frac{\left(3\,cd^2-15\,bde+35\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d^{\frac{9}{2}}}+\frac{3\,cd^2x^3e-7\,bdx^3e^2+5\,cd^3x+11\,ax^3e^3-9\,bd^2xe+13\,adxe^2}{8\,\left(x^2e+d\right)^2d^4}-3\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(9
/2) + 1/8*(3*c*d^2*x^3*e - 7*b*d*x^3*e^2 + 5*c*d^3*x + 11*a*x^3*e^3 - 9*b*d
^2*x*e + 13*a*d*x*e^2)/((x^2*e + d)^2*d^4) - 1/3*(3*b*d*x^2 - 9*a*x^2*e + a
*d)/(d^4*x^3)
```

maple [A] time = 0.02, size = 207, normalized size = 1.46

$$\frac{11a\,e^3x^3}{8\left(e\,x^2+d\right)^2d^4}-\frac{7b\,e^2x^3}{8\left(e\,x^2+d\right)^2d^3}+\frac{3ce\,x^3}{8\left(e\,x^2+d\right)^2d^2}+\frac{13a\,e^2x}{8\left(e\,x^2+d\right)^2d^3}-\frac{9bex}{8\left(e\,x^2+d\right)^2d^2}+\frac{5cx}{8\left(e\,x^2+d\right)^2d}+\frac{35a\,e^2\,\mathrm{a}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x)
```

```
[Out] -1/3*a/d^3/x^3+3/d^4/x*a*e-1/d^3/x*b+11/8/d^4/(e*x^2+d)^2*x^3*a*e^3-7/8/d^3
/(e*x^2+d)^2*x^3*b*e^2+3/8/d^2/(e*x^2+d)^2*x^3*c*e+13/8/d^3/(e*x^2+d)^2*a*e
^2*x-9/8/d^2/(e*x^2+d)^2*b*e*x+5/8/d/(e*x^2+d)^2*c*x+35/8/d^4/(d*e)^(1/2)*a
rctan(1/(d*e)^(1/2)*e*x)*a*e^2-15/8/d^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*
x)*e*b+3/8/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.59, size = 147, normalized size = 1.04

$$\frac{3\left(3\,cd^2e-15\,bde^2+35\,ae^3\right)x^6+5\left(3\,cd^3-15\,bd^2e+35\,ade^2\right)x^4-8\,ad^3-8\left(3\,bd^3-7\,ad^2e\right)x^2}{24\left(d^4e^2x^7+2\,d^5ex^5+d^6x^3\right)}+\frac{\left(3\,cd^2-15\,bd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(3*c*d^2*e - 15*b*d*e^2 + 35*a*e^3)*x^6 + 5*(3*c*d^3 - 15*b*d^2*e +
 35*a*d*e^2)*x^4 - 8*a*d^3 - 8*(3*b*d^3 - 7*a*d^2*e)*x^2)/(d^4*e^2*x^7 + 2*
d^5*e*x^5 + d^6*x^3) + 1/8*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*arctan(e*x/sqrt(
d*e))/(sqrt(d*e)*d^4)
```

mupad [B] time = 0.40, size = 138, normalized size = 0.97

$$\frac{\frac{x^2\,(7\,a\,e-3\,b\,d)}{3\,d^2}-\frac{a}{3\,d}+\frac{5\,x^4\,\left(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2\right)}{24\,d^3}+\frac{e\,x^6\,\left(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2\right)}{8\,d^4}}{d^2\,x^3+2\,d\,e\,x^5+e^2\,x^7}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2\right)}{8\,d^{9/2}\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x)
```

```
[Out] ((x^2*(7*a*e - 3*b*d))/(3*d^2) - a/(3*d) + (5*x^4*(35*a*e^2 + 3*c*d^2 - 15*
b*d*e))/(24*d^3) + (e*x^6*(35*a*e^2 + 3*c*d^2 - 15*b*d*e))/(8*d^4))/(d^2*x^
3 + e^2*x^7 + 2*d*e*x^5) + (atan((e^(1/2)*x)/d^(1/2))*(35*a*e^2 + 3*c*d^2 -
 15*b*d*e))/(8*d^(9/2)*e^(1/2))
```

sympy [A] time = 2.92, size = 214, normalized size = 1.51

$$-\frac{\sqrt{-\frac{1}{d^9e}}\left(35ae^2-15bde+3cd^2\right)\log\left(-d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^9e}}\left(35ae^2-15bde+3cd^2\right)\log\left(d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{16}+\frac{-8a\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-1/(d**
9*e)) + x)/16 + sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(d**
5*sqrt(-1/(d**9*e)) + x)/16 + (-8*a*d**3 + x**6*(105*a*e**3 - 45*b*d*e**2 +
 9*c*d**2*e) + x**4*(175*a*d*e**2 - 75*b*d**2*e + 15*c*d**3) + x**2*(56*a*d
**2*e - 24*b*d**3))/(24*d**6*x**3 + 48*d**5*e*x**5 + 24*d**4*e**2*x**7)
```

3.294 $\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=171

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(63ae^2-35bde+15cd^2\right)}{8d^{11/2}}-\frac{6ae^2-3bde+cd^2}{d^5x}-\frac{ex\left(7cd^2-e(11bd-15ae)\right)}{8d^5\left(d+ex^2\right)}-\frac{ex\left(ae^2-bde+c\right.}{4d^4\left(d+ex^2\right)^2}$$

```
[Out] -1/5*a/d^3/x^5+1/3*(3*a*e-b*d)/d^4/x^3+(-6*a*e^2+3*b*d*e-c*d^2)/d^5/x-1/4*e
*(a*e^2-b*d*e+c*d^2)*x/d^4/(e*x^2+d)^2-1/8*e*(7*c*d^2-e*(-15*a*e+11*b*d))*x
/d^5/(e*x^2+d)-1/8*(63*a*e^2-35*b*d*e+15*c*d^2)*arctan(x*e^(1/2)/d^(1/2))*e
^(1/2)/d^(11/2)
```

Rubi [A] time = 0.37, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1259, 1805, 1802, 205}

$$-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}-\frac{6ae^2-3bde+cd^2}{d^5x}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(63ae^2-35bde+15cd^2\right)}{8d^{11/2}}-\frac{ex\left(7cd^2-e(11bd-15a\right.}{8d^5\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] -a/(5*d^3*x^5) - (b*d - 3*a*e)/(3*d^4*x^3) - (c*d^2 - 3*b*d*e + 6*a*e^2)/(d
^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - (e*(7*c*d^2 -
 e*(11*b*d - 15*a*e))*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*d^2 - 35*b*d*
e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx = -\frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{\int \frac{-4ad^3e^2-4d^2e^2(bd-ae)x^2-4de^2\left(cd^2-bde+ae^2\right)x^4+3e^3\left(cd^2-bde+ae^2\right)x^6}{x^6\left(d+ex^2\right)^2}\,dx}{4d^4e^2}$$

$$= -\frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae)\right)x}{8d^5\left(d+ex^2\right)} + \frac{\int \frac{8ad^3e^2+8d^2e^2(bd-2ae)x^2+8de^2\left(cd^2-e(2bd-\right.}{x^6\left(d+ex^2\right.}}{8d^5e^2}$$

$$= -\frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae)\right)x}{8d^5\left(d+ex^2\right)} + \frac{\int \left(\frac{8ad^2e^2}{x^6}+\frac{8de^2(bd-3ae)}{x^4}+\frac{8e^2\left(cd^2-3bde\right.}{x^2}\right.}{8d^5e}$$

$$= -\frac{a}{5d^3x^5} - \frac{bd-3ae}{3d^4x^3} - \frac{cd^2-3bde+6ae^2}{d^5x} - \frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae\right)}{8d^5\left(d+ex^2\right)}$$

$$= -\frac{a}{5d^3x^5} - \frac{bd-3ae}{3d^4x^3} - \frac{cd^2-3bde+6ae^2}{d^5x} - \frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae\right)}{8d^5\left(d+ex^2\right)}$$

Mathematica [A] time = 0.11, size = 173, normalized size = 1.01

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(63ae^2-35bde+15cd^2\right)}{8d^{11/2}}+\frac{-6ae^2+3bde-cd^2}{d^5x}-\frac{x\left(15ae^3-11bde^2+7cd^2e\right)}{8d^5\left(d+ex^2\right)}-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] -1/5*a/(d^3*x^5) + (-(b*d) + 3*a*e)/(3*d^4*x^3) + (-(c*d^2) + 3*b*d*e - 6*a
*e^2)/(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - ((7*c
*d^2*e - 11*b*d*e^2 + 15*a*e^3)*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*d^2
 - 35*b*d*e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))
```

fricas [A] time = 0.95, size = 514, normalized size = 3.01

$$\left[-\frac{30\left(15\,cd^2e^2-35\,bde^3+63\,ae^4\right)x^8+50\left(15\,cd^3e-35\,bd^2e^2+63\,ade^3\right)x^6+48\,ad^4+16\left(15\,cd^4-35\,bd^3e+63\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(30*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 50*(15*c*d^3*e - 3
5*b*d^2*e^2 + 63*a*d*e^3)*x^6 + 48*a*d^4 + 16*(15*c*d^4 - 35*b*d^3*e + 63*a
*d^2*e^2)*x^4 + 16*(5*b*d^4 - 9*a*d^3*e)*x^2 - 15*((15*c*d^2*e^2 - 35*b*d*e
^3 + 63*a*e^4)*x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (15*c
*d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^5)*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-
```

```
e/d) - d)/(e*x^2 + d)))/(d^5*e^2*x^9 + 2*d^6*e*x^7 + d^7*x^5), -1/120*(15*(
15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 25*(15*c*d^3*e - 35*b*d^2*e^2 +
 63*a*d*e^3)*x^6 + 24*a*d^4 + 8*(15*c*d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^4
+ 8*(5*b*d^4 - 9*a*d^3*e)*x^2 + 15*((15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*
x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (15*c*d^4 - 35*b*d^3
*e + 63*a*d^2*e^2)*x^5)*sqrt(e/d)*arctan(x*sqrt(e/d)))/(d^5*e^2*x^9 + 2*d^6
*e*x^7 + d^7*x^5)]
```

giac [A] time = 0.35, size = 164, normalized size = 0.96

$$-\frac{\left(15\,cd^2e-35\,bde^2+63\,ae^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d^{\frac{11}{2}}}-\frac{7\,cd^2x^3e^2-11\,bdx^3e^3+9\,cd^3xe+15\,ax^3e^4-13\,bd^2xe^2+17}{8\left(x^2e+d\right)^2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*c*d^2*e - 35*b*d*e^2 + 63*a*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2
)/d^(11/2) - 1/8*(7*c*d^2*x^3*e^2 - 11*b*d*x^3*e^3 + 9*c*d^3*x*e + 15*a*x^3
*e^4 - 13*b*d^2*x*e^2 + 17*a*d*x*e^3)/((x^2*e + d)^2*d^5) - 1/15*(15*c*d^2*
x^4 - 45*b*d*x^4*e + 90*a*x^4*e^2 + 5*b*d^2*x^2 - 15*a*d*x^2*e + 3*a*d^2)/(
d^5*x^5)
```

maple [A] time = 0.02, size = 245, normalized size = 1.43

$$-\frac{15a\,e^4x^3}{8\left(e\,x^2+d\right)^2d^5}+\frac{11b\,e^3x^3}{8\left(e\,x^2+d\right)^2d^4}-\frac{7c\,e^2x^3}{8\left(e\,x^2+d\right)^2d^3}-\frac{17a\,e^3x}{8\left(e\,x^2+d\right)^2d^4}+\frac{13b\,e^2x}{8\left(e\,x^2+d\right)^2d^3}-\frac{9cex}{8\left(e\,x^2+d\right)^2d^2}-\frac{63a\,e^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x)
```

```
[Out] -1/5*a/d^3/x^5+1/d^4/x^3*a*e-1/3/d^3/x^3*b-6/d^5/x*a*e^2+3/d^4/x*e*b-1/d^3/
x*c-15/8*e^4/d^5/(e*x^2+d)^2*x^3*a+11/8*e^3/d^4/(e*x^2+d)^2*x^3*b-7/8*e^2/d
^3/(e*x^2+d)^2*x^3*c-17/8*e^3/d^4/(e*x^2+d)^2*a*x+13/8*e^2/d^3/(e*x^2+d)^2*
b*x-9/8*e/d^2/(e*x^2+d)^2*c*x-63/8*e^3/d^5/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)
*e*x)*a+35/8*e^2/d^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b-15/8*e/d^3/(d*
e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c
```

maxima [A] time = 2.45, size = 183, normalized size = 1.07

$$-\frac{15\left(15\,cd^2e^2-35\,bde^3+63\,ae^4\right)x^8+25\left(15\,cd^3e-35\,bd^2e^2+63\,ade^3\right)x^6+24\,ad^4+8\left(15\,cd^4-35\,bd^3e+63\right.}{120\left(d^5e^2x^9+2\,d^6ex^7+d^7x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 25*(15*c*d^3*e - 35
*b*d^2*e^2 + 63*a*d*e^3)*x^6 + 24*a*d^4 + 8*(15*c*d^4 - 35*b*d^3*e + 63*a*d
^2*e^2)*x^4 + 8*(5*b*d^4 - 9*a*d^3*e)*x^2)/(d^5*e^2*x^9 + 2*d^6*e*x^7 + d^7
*x^5) - 1/8*(15*c*d^2*e - 35*b*d*e^2 + 63*a*e^3)*arctan(e*x/sqrt(d*e))/(sqr
t(d*e)*d^5)
```

mupad [B] time = 0.41, size = 168, normalized size = 0.98

$$-\frac{\frac{a}{5\,d}-\frac{x^2\,\left(9\,a\,e-5\,b\,d\right)}{15\,d^2}+\frac{x^4\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{15\,d^3}+\frac{5\,e\,x^6\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{24\,d^4}+\frac{e^2\,x^8\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{8\,d^5}}{d^2\,x^5+2\,d\,e\,x^7+e^2\,x^9}-\frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x)
```

```
[Out] - (a/(5*d) - (x^2*(9*a*e - 5*b*d))/(15*d^2) + (x^4*(63*a*e^2 + 15*c*d^2 - 3
5*b*d*e))/(15*d^3) + (5*e*x^6*(63*a*e^2 + 15*c*d^2 - 35*b*d*e))/(24*d^4) +
(e^2*x^8*(63*a*e^2 + 15*c*d^2 - 35*b*d*e))/(8*d^5))/(d^2*x^5 + e^2*x^9 + 2*
d*e*x^7) - (e^(1/2)*atan((e^(1/2)*x)/d^(1/2))*(63*a*e^2 + 15*c*d^2 - 35*b*d
*e))/(8*d^(11/2))
```

sympy [B] time = 3.76, size = 330, normalized size = 1.93

$$\frac{\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)\log\left(-\frac{d^6\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)}{63ae^3-35bde^2+15cd^2e}+x\right)}{16}-\frac{\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)\log\left(\frac{d^6\sqrt{-}}{6}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(-d**6*sqrt(-e/d**11)*
(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c*d**2*e)
+ x)/16 - sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(d**6*sqrt(-
e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c
*d**2*e) + x)/16 + (-24*a*d**4 + x**8*(-945*a*e**4 + 525*b*d*e**3 - 225*c*d
**2*e**2) + x**6*(-1575*a*d*e**3 + 875*b*d**2*e**2 - 375*c*d**3*e) + x**4*(
-504*a*d**2*e**2 + 280*b*d**3*e - 120*c*d**4) + x**2*(72*a*d**3*e - 40*b*d*
*4))/(120*d**7*x**5 + 240*d**6*e*x**7 + 120*d**5*e**2*x**9)
```

3.295 $\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=230

$$-\frac{\left(a^2ce-ab^2e-2abcd+b^3d\right)\log\left(a+bx^2+cx^4\right)}{4c^3\left(ae^2-bde+cd^2\right)}-\frac{\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}$$

```
[Out] -1/2*(b*e+c*d)*x^2/c^2/e^2+1/4*x^4/c/e+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2-b*d*e
+c*d^2)-1/4*(a^2*c*e-a*b^2*e-2*a*b*c*d+b^3*d)*ln(c*x^4+b*x^2+a)/c^3/(a*e^2-
b*d*e+c*d^2)-1/2*(3*a^2*b*c*e+2*a^2*c^2*d-a*b^3*e-4*a*b^2*c*d+b^4*d)*arctan
h((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2
)
```

Rubi [A] time = 0.49, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1251, 1628, 634, 618, 206, 628}

$$-\frac{\left(a^2ce-ab^2e-2abcd+b^3d\right)\log\left(a+bx^2+cx^4\right)}{4c^3\left(ae^2-bde+cd^2\right)}-\frac{\left(3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((c*d + b*e)*x^2)/(2*c^2*e^2) + x^4/(4*c*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^
2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^4*Log[d + e*x^2])/(2
*e^3*(c*d^2 - b*d*e + a*e^2)) - ((b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*Lo
g[a + b*x^2 + c*x^4])/(4*c^3*(c*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-cd-be}{c^2e^2} + \frac{x}{ce} + \frac{d^4}{e^2(cd^2-bde+ae^2)(d+ex)} + \frac{-a(b^2d-acd-\ldots}{c^2(cd^2\ldots}\right.\right.$$
$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{-a(b^2d-acd-abe)-(b^3d-2abc\ldots}{a+bx+cx^2}\ldots\right.}{2c^2(cd^2-bde+\ldots}$$
$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} - \frac{(b^3d-2abcd-ab^2e+a^2ce)\,\text{Sub}\ldots}{4c^3(cd^2-bde\ldots}$$
$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} - \frac{(b^3d-2abcd-ab^2e+a^2ce)\log\ldots}{4c^3(cd^2-bde+a\ldots}$$
$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} - \frac{(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4a\ldots}}\right.}{2c^3\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}$$

Mathematica [A] time = 0.24, size = 228, normalized size = 0.99

$$\frac{1}{4}\left(\frac{(-a^2ce+ab^2e+2abcd+b^3(-d))\log(a+bx^2+cx^4)}{c^3(e(ae-bd)+cd^2)} - \frac{2(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d)\tan^{-1}\left(\frac{b+2\ldots}{\sqrt{4ac\ldots}}\right.}{c^3\sqrt{4ac-b^2}\,(e(bd-ae)-cd^2)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*(c*d + b*e)*x^2)/(c^2*e^2) + x^4/(c*e) - (2*(b^4*d - 4*a*b^2*c*d + 2*a
^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])
/(c^3*Sqrt[-b^2 + 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) + (2*d^4*Log[d + e*x^2
])/(e^3*(c*d^2 + e*(-(b*d) + a*e))) + ((-(b^3*d) + 2*a*b*c*d + a*b^2*e - a^
2*c*e)*Log[a + b*x^2 + c*x^4])/(c^3*(c*d^2 + e*(-(b*d) + a*e))))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.73, size = 236, normalized size = 1.03

$$\frac{d^4\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e^3-bde^4+ae^5\right)}-\frac{\left(b^3d-2\,abcd-ab^2e+a^2ce\right)\log\left(cx^4+bx^2+a\right)}{4\left(c^4d^2-bc^3de+ac^3e^2\right)}+\frac{\left(b^4d-4\,ab^2cd+2\,a^2c^2d-ab^3e+3\right.}{2\left(c^4d^2-bc^3de+ac^3e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(x^2*e + d))/(c*d^2*e^3 - b*d*e^4 + a*e^5) - 1/4*(b^3*d - 2*
a*b*c*d - a*b^2*e + a^2*c*e)*log(c*x^4 + b*x^2 + a)/(c^4*d^2 - b*c^3*d*e +
a*c^3*e^2) + 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e
)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^4*d^2 - b*c^3*d*e + a*c^3*e^
2)*sqrt(-b^2 + 4*a*c)) + 1/4*(c*x^4*e - 2*c*d*x^2 - 2*b*x^2*e)*e^(-2)/c^2
```

maple [B] time = 0.02, size = 538, normalized size = 2.34

$$\frac{3a^2be\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;c^2}+\frac{a^2d\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;c}-\frac{a\,b^3e\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;c^3}-\frac{2a}{\left(a\,e^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/c/e*x^4-1/2/c^2/e*x^2*b-1/2/c*d/e^2*x^2-1/4/(a*e^2-b*d*e+c*d^2)/c^2*ln(
c*x^4+b*x^2+a)*a^2*e+1/4/(a*e^2-b*d*e+c*d^2)/c^3*ln(c*x^4+b*x^2+a)*a*b^2*e+
1/2/(a*e^2-b*d*e+c*d^2)/c^2*ln(c*x^4+b*x^2+a)*a*b*d-1/4/(a*e^2-b*d*e+c*d^2)
/c^3*ln(c*x^4+b*x^2+a)*b^3*d+3/2/(a*e^2-b*d*e+c*d^2)/c^2/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b*e+1/(a*e^2-b*d*e+c*d^2)/c/(4*a*
c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*d-2/(a*e^2-b*d*e+c*d
^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*d-1/2
/(a*e^2-b*d*e+c*d^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(
1/2))*b^3*a*e+1/2/(a*e^2-b*d*e+c*d^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*b^4*d+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 69.94, size = 7024, normalized size = 30.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d^4*log(d + e*x^2))/(2*a*e^5 + 2*c*d^2*e^3 - 2*b*d*e^4) + (log((x^2*(a^7*e
^7 + b^7*d^7 - 2*a^3*b*c^3*d^7 - a^4*c^3*d^6*e - 2*a^6*c*d^2*e^5 + 7*a^2*b^
3*c^2*d^7 + 3*a^2*b^5*d^5*e^2 + 4*a^3*b^4*d^4*e^3 + 4*a^4*b^3*d^3*e^4 + 3*a
```

```
^5*b^2*d^2*e^5 + 2*a^5*c^2*d^4*e^3 - 5*a*b^5*c*d^7 + 2*a*b^6*d^6*e + 2*a^6*
b*d*e^6 - 8*a^2*b^4*c*d^6*e - 6*a^5*b*c*d^3*e^4 + 8*a^3*b^2*c^2*d^6*e - 9*a
^3*b^3*c*d^5*e^2 + 5*a^4*b*c^2*d^5*e^2 - 9*a^4*b^2*c*d^4*e^3))/(c^4*e^4) +
(a*d*(a^3*e^3 + b^3*d^3 - 2*a*b*c*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 - a^2*c*d
^2*e)^2)/(c^4*e^4) + (((x^2*(4*a^2*c^6*d^8 + 6*a^4*b^4*e^8 + 18*a^6*c^2*e^8
 + 6*b^4*c^4*d^8 + 6*b^8*d^4*e^4 - 16*a*b^2*c^5*d^8 - 26*a^5*b^2*c*e^8 + 8*
a*b^7*d^3*e^5 + 8*a^3*b^5*d*e^7 - 2*b^5*c^3*d^7*e - 2*b^7*c*d^5*e^3 + 8*a^2
*b^6*d^2*e^6 - 20*a^3*c^5*d^6*e^2 + 40*a^4*c^4*d^4*e^4 - 36*a^5*c^3*d^2*e^6
 + 2*b^6*c^2*d^6*e^2 + 42*a^2*b^2*c^4*d^6*e^2 - 28*a^2*b^3*c^3*d^5*e^3 + 80
*a^2*b^4*c^2*d^4*e^4 - 64*a^3*b^2*c^3*d^4*e^4 + 80*a^3*b^3*c^2*d^3*e^5 + 48
*a^4*b^2*c^2*d^2*e^6 + 18*a*b^3*c^4*d^7*e - 40*a*b^6*c*d^4*e^4 - 26*a^2*b*c
^5*d^7*e - 32*a^4*b^3*c*d*e^7 + 12*a^5*b*c^2*d*e^7 - 16*a*b^4*c^3*d^6*e^2 +
 10*a*b^5*c^2*d^5*e^3 - 48*a^2*b^5*c*d^3*e^5 + 46*a^3*b*c^4*d^5*e^3 - 40*a^
3*b^4*c*d^2*e^6 - 48*a^4*b*c^3*d^3*e^5))/(c^4*e^4) + (((x^2*(8*a*b^8*e^9 +
8*b*c^8*d^9 + 8*b^9*d*e^8 + 120*a^5*c^4*e^9 - 72*a^2*b^6*c*e^9 - 8*b^2*c^7*
d^8*e - 8*b^8*c*d^2*e^7 + 212*a^3*b^4*c^2*e^9 - 240*a^4*b^2*c^3*e^9 - 112*a
^2*c^7*d^6*e^3 + 240*a^3*c^6*d^4*e^5 - 228*a^4*c^5*d^2*e^7 + 4*b^3*c^6*d^7*
e^2 - 24*b^4*c^5*d^6*e^3 + 32*b^5*c^4*d^5*e^4 - 24*b^6*c^3*d^4*e^5 + 4*b^7*
c^2*d^3*e^6 + 32*a*c^8*d^8*e - 56*a*b^7*c*d*e^8 - 428*a^2*b^2*c^5*d^4*e^5 +
 108*a^2*b^3*c^4*d^3*e^6 - 216*a^2*b^4*c^3*d^2*e^7 + 424*a^3*b^2*c^4*d^2*e^
7 - 16*a*b*c^7*d^7*e^2 + 8*a^4*b*c^4*d*e^8 + 88*a*b^2*c^6*d^6*e^3 - 116*a*b
^3*c^5*d^5*e^4 + 188*a*b^4*c^4*d^4*e^5 - 36*a*b^5*c^3*d^3*e^6 + 60*a*b^6*c^
2*d^2*e^7 + 40*a^2*b*c^6*d^5*e^4 + 100*a^2*b^5*c^2*d*e^8 - 72*a^3*b*c^5*d^3
*e^6 - 4*a^3*b^3*c^3*d*e^8))/(c^4*e^4) - (((x^2*(32*a*b^6*c^3*e^10 - 352*a^
4*c^6*e^10 + 128*a*c^9*d^6*e^4 + 32*b*c^9*d^7*e^3 + 32*b^7*c^3*d*e^9 - 256*
a^2*b^4*c^4*e^10 + 600*a^3*b^2*c^5*e^10 - 464*a^2*c^8*d^4*e^6 + 592*a^3*c^7
*d^2*e^8 - 64*b^2*c^8*d^6*e^4 + 56*b^3*c^7*d^5*e^5 - 48*b^4*c^6*d^4*e^6 + 5
6*b^5*c^5*d^3*e^7 - 64*b^6*c^4*d^2*e^8 - 688*a^2*b^2*c^6*d^2*e^8 - 192*a*b*
c^8*d^5*e^5 - 224*a*b^5*c^4*d*e^9 - 72*a^3*b*c^6*d*e^9 + 272*a*b^2*c^7*d^4*
e^6 - 200*a*b^3*c^6*d^3*e^7 + 360*a*b^4*c^5*d^2*e^8 + 136*a^2*b*c^7*d^3*e^7
 + 424*a^2*b^3*c^5*d*e^9))/(c^4*e^4) + (32*a*d*(2*b^6*e^6 + 2*c^6*d^6 - 15*
a^3*c^3*e^6 - 10*a*c^5*d^4*e^2 + 29*a^2*b^2*c^2*e^6 + 17*a^2*c^4*d^2*e^4 +
3*b^2*c^4*d^4*e^2 - b^3*c^3*d^3*e^3 + 3*b^4*c^2*d^2*e^4 - 14*a*b^4*c*e^6 -
2*b*c^5*d^5*e - 2*b^5*c*d*e^5 + 2*a*b*c^4*d^3*e^3 + 6*a*b^3*c^2*d*e^5 + a^2
*b*c^3*d*e^5 - 13*a*b^2*c^3*d^2*e^4))/(c*e) - (8*e^2*(b^2*e^2 + c^2*d^2 - 3
*a*c*e^2 - b*c*d*e)*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^
4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2
*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) +
3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2))*(2*a*c^2*d^3 + a*b^2*e^3*x^2 + b*c^2*d^3*x
^2 - 4*a^2*c*e^3*x^2 + b^3*d*e^2*x^2 + 2*a*b^2*d*e^2 - 6*a^2*c*d*e^2 + 4*a*
c^2*d^2*e*x^2 - 2*b^2*c*d^2*e*x^2 - 2*a*b*c*d^2*e - 3*a*b*c*d*e^2*x^2))/(c*
(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2)
- 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2
*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b
^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*
(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(4*b^8*e^8 + 4*c^8*d^8 + 37*a^4*c^4*e^8 -
 16*a*c^7*d^6*e^2 + 84*a^2*b^4*c^2*e^8 - 84*a^3*b^2*c^3*e^8 + 40*a^2*c^6*d^
4*e^4 - 56*a^3*c^5*d^2*e^6 + 4*b^2*c^6*d^6*e^2 - 4*b^3*c^5*d^5*e^3 + 13*b^4
*c^4*d^4*e^4 - 4*b^5*c^3*d^3*e^5 + 4*b^6*c^2*d^2*e^6 - 32*a*b^6*c*e^8 + 98*
a^2*b^2*c^4*d^2*e^6 - 8*a*b^5*c^2*d*e^7 - 4*a^3*b*c^4*d*e^7 - 52*a*b^2*c^5*
d^4*e^4 + 20*a*b^3*c^4*d^3*e^5 - 36*a*b^4*c^3*d^2*e^6 - 16*a^2*b*c^5*d^3*e^
5 + 28*a^2*b^3*c^3*d*e^7))/(c^4*e^4))*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) -
4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b
*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2
 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a
*e^2 + c*d^2 - b*d*e)) + (4*a*d*(2*a^3*b^4*e^7 + 5*a^5*c^2*e^7 + 2*b^3*c^4*
d^7 + 2*b^7*d^3*e^4 - 8*a^4*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 -
 2*a^2*c^5*d^6*e + 6*a^3*c^4*d^4*e^3 - 9*a^4*c^3*d^2*e^5 + b^5*c^2*d^5*e^2
- 4*a*b*c^5*d^7 - a^2*b^2*c^3*d^4*e^3 + 20*a^2*b^3*c^2*d^3*e^4 + 12*a^3*b^2
```

```
*c^2*d^2*e^5 + 2*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 8*a^3*b^3*c*d*e^6 +
 3*a^4*b*c^2*d*e^6 - 6*a*b^3*c^3*d^5*e^2 - a*b^4*c^2*d^4*e^3 + 10*a^2*b*c^4
*d^5*e^2 - 10*a^2*b^4*c*d^2*e^5 - 12*a^3*b*c^3*d^3*e^4))/(c^4*e^4))*(b^5*d
+ b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e
*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4
*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(
1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5*d + b^4*d*(b^2 -
 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)
^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) -
4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a
*c^5*d^2 + 4*a^2*c^4*e^2 - b^2*c^4*d^2 - a*b^2*c^3*e^2 + b^3*c^3*d*e - 4*a*
b*c^4*d*e)) - (log((x^2*(a^7*e^7 + b^7*d^7 - 2*a^3*b*c^3*d^7 - a^4*c^3*d^6*
e - 2*a^6*c*d^2*e^5 + 7*a^2*b^3*c^2*d^7 + 3*a^2*b^5*d^5*e^2 + 4*a^3*b^4*d^4
*e^3 + 4*a^4*b^3*d^3*e^4 + 3*a^5*b^2*d^2*e^5 + 2*a^5*c^2*d^4*e^3 - 5*a*b^5*
c*d^7 + 2*a*b^6*d^6*e + 2*a^6*b*d*e^6 - 8*a^2*b^4*c*d^6*e - 6*a^5*b*c*d^3*e
^4 + 8*a^3*b^2*c^2*d^6*e - 9*a^3*b^3*c*d^5*e^2 + 5*a^4*b*c^2*d^5*e^2 - 9*a^
4*b^2*c*d^4*e^3))/(c^4*e^4) + (a*d*(a^3*e^3 + b^3*d^3 - 2*a*b*c*d^3 + a*b^2
*d^2*e + a^2*b*d*e^2 - a^2*c*d^2*e)^2)/(c^4*e^4) - (((x^2*(4*a^2*c^6*d^8 +
6*a^4*b^4*e^8 + 18*a^6*c^2*e^8 + 6*b^4*c^4*d^8 + 6*b^8*d^4*e^4 - 16*a*b^2*c
^5*d^8 - 26*a^5*b^2*c*e^8 + 8*a*b^7*d^3*e^5 + 8*a^3*b^5*d*e^7 - 2*b^5*c^3*d
^7*e - 2*b^7*c*d^5*e^3 + 8*a^2*b^6*d^2*e^6 - 20*a^3*c^5*d^6*e^2 + 40*a^4*c^
4*d^4*e^4 - 36*a^5*c^3*d^2*e^6 + 2*b^6*c^2*d^6*e^2 + 42*a^2*b^2*c^4*d^6*e^2
 - 28*a^2*b^3*c^3*d^5*e^3 + 80*a^2*b^4*c^2*d^4*e^4 - 64*a^3*b^2*c^3*d^4*e^4
 + 80*a^3*b^3*c^2*d^3*e^5 + 48*a^4*b^2*c^2*d^2*e^6 + 18*a*b^3*c^4*d^7*e - 4
0*a*b^6*c*d^4*e^4 - 26*a^2*b*c^5*d^7*e - 32*a^4*b^3*c*d*e^7 + 12*a^5*b*c^2*
d*e^7 - 16*a*b^4*c^3*d^6*e^2 + 10*a*b^5*c^2*d^5*e^3 - 48*a^2*b^5*c*d^3*e^5
+ 46*a^3*b*c^4*d^5*e^3 - 40*a^3*b^4*c*d^2*e^6 - 48*a^4*b*c^3*d^3*e^5))/(c^4
*e^4) - (((x^2*(8*a*b^8*e^9 + 8*b*c^8*d^9 + 8*b^9*d*e^8 + 120*a^5*c^4*e^9 -
 72*a^2*b^6*c*e^9 - 8*b^2*c^7*d^8*e - 8*b^8*c*d^2*e^7 + 212*a^3*b^4*c^2*e^9
 - 240*a^4*b^2*c^3*e^9 - 112*a^2*c^7*d^6*e^3 + 240*a^3*c^6*d^4*e^5 - 228*a^
4*c^5*d^2*e^7 + 4*b^3*c^6*d^7*e^2 - 24*b^4*c^5*d^6*e^3 + 32*b^5*c^4*d^5*e^4
 - 24*b^6*c^3*d^4*e^5 + 4*b^7*c^2*d^3*e^6 + 32*a*c^8*d^8*e - 56*a*b^7*c*d*e
^8 - 428*a^2*b^2*c^5*d^4*e^5 + 108*a^2*b^3*c^4*d^3*e^6 - 216*a^2*b^4*c^3*d^
2*e^7 + 424*a^3*b^2*c^4*d^2*e^7 - 16*a*b*c^7*d^7*e^2 + 8*a^4*b*c^4*d*e^8 +
88*a*b^2*c^6*d^6*e^3 - 116*a*b^3*c^5*d^5*e^4 + 188*a*b^4*c^4*d^4*e^5 - 36*a
*b^5*c^3*d^3*e^6 + 60*a*b^6*c^2*d^2*e^7 + 40*a^2*b*c^6*d^5*e^4 + 100*a^2*b^
5*c^2*d*e^8 - 72*a^3*b*c^5*d^3*e^6 - 4*a^3*b^3*c^3*d*e^8))/(c^4*e^4) + (((x
^2*(32*a*b^6*c^3*e^10 - 352*a^4*c^6*e^10 + 128*a*c^9*d^6*e^4 + 32*b*c^9*d^7
*e^3 + 32*b^7*c^3*d*e^9 - 256*a^2*b^4*c^4*e^10 + 600*a^3*b^2*c^5*e^10 - 464
*a^2*c^8*d^4*e^6 + 592*a^3*c^7*d^2*e^8 - 64*b^2*c^8*d^6*e^4 + 56*b^3*c^7*d^
5*e^5 - 48*b^4*c^6*d^4*e^6 + 56*b^5*c^5*d^3*e^7 - 64*b^6*c^4*d^2*e^8 - 688*
a^2*b^2*c^6*d^2*e^8 - 192*a*b*c^8*d^5*e^5 - 224*a*b^5*c^4*d*e^9 - 72*a^3*b*
c^6*d*e^9 + 272*a*b^2*c^7*d^4*e^6 - 200*a*b^3*c^6*d^3*e^7 + 360*a*b^4*c^5*d
^2*e^8 + 136*a^2*b*c^7*d^3*e^7 + 424*a^2*b^3*c^5*d*e^9))/(c^4*e^4) + (32*a*
d*(2*b^6*e^6 + 2*c^6*d^6 - 15*a^3*c^3*e^6 - 10*a*c^5*d^4*e^2 + 29*a^2*b^2*c
^2*e^6 + 17*a^2*c^4*d^2*e^4 + 3*b^2*c^4*d^4*e^2 - b^3*c^3*d^3*e^3 + 3*b^4*c
^2*d^2*e^4 - 14*a*b^4*c*e^6 - 2*b*c^5*d^5*e - 2*b^5*c*d*e^5 + 2*a*b*c^4*d^3
*e^3 + 6*a*b^3*c^2*d*e^5 + a^2*b*c^3*d*e^5 - 13*a*b^2*c^3*d^2*e^4))/(c*e) +
 (8*e^2*(b^2*e^2 + c^2*d^2 - 3*a*c*e^2 - b*c*d*e)*(b^4*d*(b^2 - 4*a*c)^(1/2
) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/
2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*
b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2))*(2*a*c^2*d^3
 + a*b^2*e^3*x^2 + b*c^2*d^3*x^2 - 4*a^2*c*e^3*x^2 + b^3*d*e^2*x^2 + 2*a*b^
2*d*e^2 - 6*a^2*c*d*e^2 + 4*a*c^2*d^2*e*x^2 - 2*b^2*c*d^2*e*x^2 - 2*a*b*c*d
^2*e - 3*a*b*c*d*e^2*x^2))/(c*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*
d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3
*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 -
 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)
^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(4*b^8*e^8
```

```
+ 4*c^8*d^8 + 37*a^4*c^4*e^8 - 16*a*c^7*d^6*e^2 + 84*a^2*b^4*c^2*e^8 - 84*a
^3*b^2*c^3*e^8 + 40*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 4*b^2*c^6*d^6*e^
2 - 4*b^3*c^5*d^5*e^3 + 13*b^4*c^4*d^4*e^4 - 4*b^5*c^3*d^3*e^5 + 4*b^6*c^2*
d^2*e^6 - 32*a*b^6*c*e^8 + 98*a^2*b^2*c^4*d^2*e^6 - 8*a*b^5*c^2*d*e^7 - 4*a
^3*b*c^4*d*e^7 - 52*a*b^2*c^5*d^4*e^4 + 20*a*b^3*c^4*d^3*e^5 - 36*a*b^4*c^3
*d^2*e^6 - 16*a^2*b*c^5*d^3*e^5 + 28*a^2*b^3*c^3*d*e^7))/(c^4*e^4))*(b^4*d*
(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3*e
*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4
*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(
1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(2*a^3*b^4*e^
7 + 5*a^5*c^2*e^7 + 2*b^3*c^4*d^7 + 2*b^7*d^3*e^4 - 8*a^4*b^2*c*e^7 + 2*a*b
^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 2*a^2*c^5*d^6*e + 6*a^3*c^4*d^4*e^3 - 9*a^4*
c^3*d^2*e^5 + b^5*c^2*d^5*e^2 - 4*a*b*c^5*d^7 - a^2*b^2*c^3*d^4*e^3 + 20*a^
2*b^3*c^2*d^3*e^4 + 12*a^3*b^2*c^2*d^2*e^5 + 2*a*b^2*c^4*d^6*e - 12*a*b^5*c
*d^3*e^4 - 8*a^3*b^3*c*d*e^6 + 3*a^4*b*c^2*d*e^6 - 6*a*b^3*c^3*d^5*e^2 - a*
b^4*c^2*d^4*e^3 + 10*a^2*b*c^4*d^5*e^2 - 10*a^2*b^4*c*d^2*e^5 - 12*a^3*b*c^
3*d^3*e^4))/(c^4*e^4))*(b^4*d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a
*b^4*e + 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*
b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2)
 + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 -
b*d*e)))*(b^4*d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b
^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^
2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e
*(b^2 - 4*a*c)^(1/2)))/(4*(4*a*c^5*d^2 + 4*a^2*c^4*e^2 - b^2*c^4*d^2 - a*b^
2*c^3*e^2 + b^3*c^3*d*e - 4*a*b*c^4*d*e)) + x^4/(4*c*e) - (x^2*(b*e + c*d))
/(2*c^2*e^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.296 $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=189

$$\frac{(2a^2ce-ab^2e-3abcd+b^3d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{(-abe-acd+b^2d)\log(a+bx^2+cx^4)}{4c^2(ae^2-bde+cd^2)}-\frac{d^3\log(d+ex^2)}{2e^2(ae^2-bde+cd^2}$$

```
[Out] 1/2*x^2/c/e-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2-b*d*e+c*d^2)+1/4*(-a*b*e-a*c*d+b
^2*d)*ln(c*x^4+b*x^2+a)/c^2/(a*e^2-b*d*e+c*d^2)+1/2*(2*a^2*c*e-a*b^2*e-3*a*
b*c*d+b^3*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(a*e^2-b*d*e+c*d^2
)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 1628, 634, 618, 206, 628}

$$\frac{(2a^2ce-ab^2e-3abcd+b^3d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{(-abe-acd+b^2d)\log(a+bx^2+cx^4)}{4c^2(ae^2-bde+cd^2)}-\frac{d^3\log(d+ex^2)}{2e^2(ae^2-bde+cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x^2/(2*c*e) + ((b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) -
 (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a
*b*e)*Log[a + b*x^2 + c*x^4])/(4*c^2*(c*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ce} - \frac{d^3}{e(cd^2-bde+ae^2)(d+ex)} + \frac{a(bd-ae)+(b^2d-acd-ab}{c(cd^2-bde+ae^2)(a+bx+}\right.\right.$$
$$= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{a(bd-ae)+(b^2d-acd-abe)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c(cd^2-bde+ae^2)}$$
$$= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)} + \frac{(b^2d-acd-abe)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2(cd^2-bde+ae^2)} -$$
$$= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)} + \frac{(b^2d-acd-abe)\log(a+bx^2+cx^4)}{4c^2(cd^2-bde+ae^2)} + \frac{(b^3d-}{}$$
$$= \frac{x^2}{2ce} + \frac{(b^3d-3abcd-ab^2e+2a^2ce)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)}$$

Mathematica [A] time = 0.19, size = 186, normalized size = 0.98

$$\frac{2e^2(2a^2ce-ab^2e-3abcd+b^3d)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)+\sqrt{4ac-b^2}\,\left(e\left(e\left(abe+acd+b^2(-d)\right)\log\left(a+bx^2+cx^4\right)-2c\right.\right.}{4c^2e^2\sqrt{4ac-b^2}\,\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*e^2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[
-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(2*c^2*d^3*Log[d + e*x^2] + e*(-2*c*(c*
d^2 - b*d*e + a*e^2)*x^2 + e*(-(b^2*d) + a*c*d + a*b*e)*Log[a + b*x^2 + c*x
^4])))/(4*c^2*Sqrt[-b^2 + 4*a*c]*e^2*(-(c*d^2) + e*(b*d - a*e)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 2.19, size = 194, normalized size = 1.03

$$-\frac{d^3\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e^2-bde^3+ae^4\right)}+\frac{x^2e^{(-1)}}{2\,c}+\frac{\left(b^2d-acd-abe\right)\log\left(cx^4+bx^2+a\right)}{4\left(c^3d^2-bc^2de+ac^2e^2\right)}-\frac{\left(b^3d-3\,abcd-ab^2e+2\,a^2ce\right)\arctan}{2\left(c^3d^2-bc^2de+ac^2e^2\right)\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*d^3*log(abs(x^2*e + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4) + 1/2*x^2*e^(-1)
/c + 1/4*(b^2*d - a*c*d - a*b*e)*log(c*x^4 + b*x^2 + a)/(c^3*d^2 - b*c^2*d*
e + a*c^2*e^2) - 1/2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*arctan((2*c*
x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-b^2 +
 4*a*c))
```

maple [B] time = 0.01, size = 408, normalized size = 2.16

$$-\frac{a^2e\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;c}+\frac{a\,b^2e\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;c^2}+\frac{3abd\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;c}-\frac{}{2\left(a\,e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/c/e*x^2-1/4/(a*e^2-b*d*e+c*d^2)/c^2*ln(c*x^4+b*x^2+a)*a*b*e-1/4/(a*e^2-
b*d*e+c*d^2)/c*ln(c*x^4+b*x^2+a)*a*d+1/4/(a*e^2-b*d*e+c*d^2)/c^2*ln(c*x^4+b
*x^2+a)*b^2*d-1/(a*e^2-b*d*e+c*d^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*a^2*e+3/2/(a*e^2-b*d*e+c*d^2)/c/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*d+1/2/(a*e^2-b*d*e+c*d^2)/c^2/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*a*e-1/2/(a*e^2-b*d*e+c*d
^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*d-1/2*d
^3*ln(e*x^2+d)/e^2/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 15.21, size = 2304, normalized size = 12.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] x^2/(2*c*e) - (d^3*log(d + e*x^2))/(2*(a*e^4 + c*d^2*e^2 - b*d*e^3)) - (log
(a*e^5*x^2*(b^2 - 4*a*c)^(7/2) - 128*a^5*c^3*e^5 - 8*c^3*d^5*(b^2 - 4*a*c)^
(5/2) - 512*a^3*c^5*d^4*e + 8*b^2*c^3*d^5*(b^2 - 4*a*c)^(3/2) + 6*b^3*d^2*e
^3*(b^2 - 4*a*c)^(5/2) - 3*b^5*d^2*e^3*(b^2 - 4*a*c)^(3/2) + 32*a^4*b^2*c^2
*e^5 + 384*a^4*c^4*d^2*e^3 + 256*a^2*c^6*d^5*x^2 + 16*b^4*c^4*d^5*x^2 + 3*a
*d*e^4*(b^2 - 4*a*c)^(7/2) - 3*b*d^2*e^3*(b^2 - 4*a*c)^(7/2) - 3*c*d^3*e^2*
```

```
(b^2 - 4*a*c)^(7/2) - 16*a^2*b^3*c^3*d^3*e^2 + 48*a^2*b^4*c^2*d^2*e^3 - 288
*a^3*b^2*c^3*d^2*e^3 + 16*a^3*b^3*c^2*e^5*x^2 - 384*a^3*c^5*d^3*e^2*x^2 + 1
6*b^6*c^2*d^3*e^2*x^2 - 6*a*b^2*d*e^4*(b^2 - 4*a*c)^(5/2) + 3*a*b^4*d*e^4*(
b^2 - 4*a*c)^(3/2) + 8*b*c^2*d^4*e*(b^2 - 4*a*c)^(5/2) - 32*a*b^4*c^3*d^4*e
 + 192*a^4*b*c^3*d*e^4 - 2*b^2*c*d^3*e^2*(b^2 - 4*a*c)^(5/2) - 8*b^3*c^2*d^
4*e*(b^2 - 4*a*c)^(3/2) + 5*b^4*c*d^3*e^2*(b^2 - 4*a*c)^(3/2) - 2*a*b^2*e^5
*x^2*(b^2 - 4*a*c)^(5/2) + a*b^4*e^5*x^2*(b^2 - 4*a*c)^(3/2) + 16*b*c^4*d^5
*x^2*(b^2 - 4*a*c)^(3/2) - 3*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 16*c^3*d^4
*e*x^2*(b^2 - 4*a*c)^(5/2) + 256*a^2*b^2*c^4*d^4*e + 64*a^3*b*c^4*d^3*e^2 -
 48*a^3*b^3*c^2*d*e^4 - 128*a*b^2*c^5*d^5*x^2 - 64*a^4*b*c^3*e^5*x^2 + 384*
a^4*c^4*d*e^4*x^2 - 32*b^5*c^3*d^4*e*x^2 + 480*a^2*b^2*c^4*d^3*e^2*x^2 + 48
*a^2*b^3*c^3*d^2*e^3*x^2 + 256*a*b^3*c^4*d^4*e*x^2 - 512*a^2*b*c^5*d^4*e*x^
2 + 8*b*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(5/2) + 6*b^2*c*d^2*e^3*x^2*(b^2 - 4*
a*c)^(5/2) - 16*b^2*c^3*d^4*e*x^2*(b^2 - 4*a*c)^(3/2) - 3*b^4*c*d^2*e^3*x^2
*(b^2 - 4*a*c)^(3/2) - 160*a*b^4*c^3*d^3*e^2*x^2 - 192*a^3*b*c^4*d^2*e^3*x^
2 - 96*a^3*b^2*c^3*d*e^4*x^2 + 8*b^3*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(
b^4*d - b^3*d*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d - a*b^3*e - 5*a*b^2*c*d + 4
*a^2*b*c*e + a*b^2*e*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*e*(b^2 - 4*a*c)^(1/2) +
3*a*b*c*d*(b^2 - 4*a*c)^(1/2)))/(4*(4*a*c^4*d^2 + 4*a^2*c^3*e^2 - b^2*c^3*d
^2 - a*b^2*c^2*e^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) - (log(8*c^3*d^5*(b^2 -
4*a*c)^(5/2) - 128*a^5*c^3*e^5 - a*e^5*x^2*(b^2 - 4*a*c)^(7/2) - 512*a^3*c^
5*d^4*e - 8*b^2*c^3*d^5*(b^2 - 4*a*c)^(3/2) - 6*b^3*d^2*e^3*(b^2 - 4*a*c)^(
5/2) + 3*b^5*d^2*e^3*(b^2 - 4*a*c)^(3/2) + 32*a^4*b^2*c^2*e^5 + 384*a^4*c^4
*d^2*e^3 + 256*a^2*c^6*d^5*x^2 + 16*b^4*c^4*d^5*x^2 - 3*a*d*e^4*(b^2 - 4*a*
c)^(7/2) + 3*b*d^2*e^3*(b^2 - 4*a*c)^(7/2) + 3*c*d^3*e^2*(b^2 - 4*a*c)^(7/2
) - 16*a^2*b^3*c^3*d^3*e^2 + 48*a^2*b^4*c^2*d^2*e^3 - 288*a^3*b^2*c^3*d^2*e
^3 + 16*a^3*b^3*c^2*e^5*x^2 - 384*a^3*c^5*d^3*e^2*x^2 + 16*b^6*c^2*d^3*e^2*
x^2 + 6*a*b^2*d*e^4*(b^2 - 4*a*c)^(5/2) - 3*a*b^4*d*e^4*(b^2 - 4*a*c)^(3/2)
 - 8*b*c^2*d^4*e*(b^2 - 4*a*c)^(5/2) - 32*a*b^4*c^3*d^4*e + 192*a^4*b*c^3*d
*e^4 + 2*b^2*c*d^3*e^2*(b^2 - 4*a*c)^(5/2) + 8*b^3*c^2*d^4*e*(b^2 - 4*a*c)^
(3/2) - 5*b^4*c*d^3*e^2*(b^2 - 4*a*c)^(3/2) + 2*a*b^2*e^5*x^2*(b^2 - 4*a*c)
^(5/2) - a*b^4*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^4*d^5*x^2*(b^2 - 4*a*c)
^(3/2) + 3*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 16*c^3*d^4*e*x^2*(b^2 - 4*a*
c)^(5/2) + 256*a^2*b^2*c^4*d^4*e + 64*a^3*b*c^4*d^3*e^2 - 48*a^3*b^3*c^2*d*
e^4 - 128*a*b^2*c^5*d^5*x^2 - 64*a^4*b*c^3*e^5*x^2 + 384*a^4*c^4*d*e^4*x^2
- 32*b^5*c^3*d^4*e*x^2 + 480*a^2*b^2*c^4*d^3*e^2*x^2 + 48*a^2*b^3*c^3*d^2*e
^3*x^2 + 256*a*b^3*c^4*d^4*e*x^2 - 512*a^2*b*c^5*d^4*e*x^2 - 8*b*c^2*d^3*e^
2*x^2*(b^2 - 4*a*c)^(5/2) - 6*b^2*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 16*b^
2*c^3*d^4*e*x^2*(b^2 - 4*a*c)^(3/2) + 3*b^4*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(3/
2) - 160*a*b^4*c^3*d^3*e^2*x^2 - 192*a^3*b*c^4*d^2*e^3*x^2 - 96*a^3*b^2*c^3
*d*e^4*x^2 - 8*b^3*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(b^4*d + b^3*d*(b^2
 - 4*a*c)^(1/2) + 4*a^2*c^2*d - a*b^3*e - 5*a*b^2*c*d + 4*a^2*b*c*e - a*b^2
*e*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*e*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*d*(b^2 - 4
*a*c)^(1/2)))/(4*(4*a*c^4*d^2 + 4*a^2*c^3*e^2 - b^2*c^3*d^2 - a*b^2*c^2*e^2
 + b^3*c^2*d*e - 4*a*b*c^3*d*e))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.297 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=158

$$-\frac{(-abe-2acd+b^2d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{d^2\log(d+ex^2)}{2e(ae^2-bde+cd^2)}-\frac{(bd-ae)\log(a+bx^2+cx^4)}{4c(ae^2-bde+cd^2)}$$

```
[Out] 1/2*d^2*ln(e*x^2+d)/e/(a*e^2-b*d*e+c*d^2)-1/4*(-a*e+b*d)*ln(c*x^4+b*x^2+a)/
c/(a*e^2-b*d*e+c*d^2)-1/2*(-a*b*e-2*a*c*d+b^2*d)*arctanh((2*c*x^2+b)/(-4*a*
c+b^2)^(1/2))/c/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 1628, 634, 618, 206, 628}

$$-\frac{(-abe-2acd+b^2d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{d^2\log(d+ex^2)}{2e(ae^2-bde+cd^2)}-\frac{(bd-ae)\log(a+bx^2+cx^4)}{4c(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*
Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^2*Log[d + e*x^2])/(2*e*(c*d
^2 - b*d*e + a*e^2)) - ((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*c*(c*d^2 - b
*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2}{(cd^2-bde+ae^2)(d+ex)} + \frac{-ad-(bd-ae)x}{(cd^2-bde+ae^2)(a+bx+cx^2)}\right) d\right.$$

$$= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{-ad-(bd-ae)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)}$$

$$= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c(cd^2-bde+ae^2)} + \frac{(b^2d-2acd-a}{4c}$$

$$= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\log(a+bx^2+cx^4)}{4c(cd^2-bde+ae^2)} - \frac{(b^2d-2acd-abe)\,\text{Sub}}{2c(cd^2}$$

$$= -\frac{(b^2d-2acd-abe)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\log(}{4c(cd^2-}$$

Mathematica [A] time = 0.11, size = 139, normalized size = 0.88

$$-\frac{\sqrt{4ac-b^2}\left(e(bd-ae)\log(a+bx^2+cx^4)-2cd^2\log(d+ex^2)\right)+2e\left(abe+2acd+b^2(-d)\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{4ce\sqrt{4ac-b^2}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*(2*e*(-(b^2*d) + 2*a*c*d + a*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a
*c]] + Sqrt[-b^2 + 4*a*c]*(-2*c*d^2*Log[d + e*x^2] + e*(b*d - a*e)*Log[a +
b*x^2 + c*x^4]))/(c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*d) + a*e)))
```

fricas [A] time = 144.36, size = 421, normalized size = 2.66

$$\left[\frac{2(b^2c-4ac^2)d^2\log(ex^2+d)+(abe^2-(b^2-2ac)de)\sqrt{b^2-4ac}\,\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-((b}{4((b^2c^2-4ac^3)d^2e-(b^3c-4abc^2)de^2+(ab^2c-4a^2c^2)e^3)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*d^2*log(e*x^2 + d) + (a*b*e^2 - (b^2 - 2*a*c)*d*e
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b
```

```
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^3 - 4*a*b*c)*d*e - (a*b^2 -
 4*a^2*c)*e^2)*log(c*x^4 + b*x^2 + a))/((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c
- 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/4*(2*(b^2*c - 4*a*c^2)*d
^2*log(e*x^2 + d) + 2*(a*b*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arct
an(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*d*e
- (a*b^2 - 4*a^2*c)*e^2)*log(c*x^4 + b*x^2 + a))/((b^2*c^2 - 4*a*c^3)*d^2*e
 - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

giac [A] time = 1.84, size = 157, normalized size = 0.99

$$\frac{d^2\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e-bde^2+ae^3\right)}-\frac{(bd-ae)\log\left(cx^4+bx^2+a\right)}{4\left(c^2d^2-bcde+ace^2\right)}+\frac{\left(b^2d-2\,acd-abe\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(c^2d^2-bcde+ace^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^2*log(abs(x^2*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) - 1/4*(b*d - a*e)*l
og(c*x^4 + b*x^2 + a)/(c^2*d^2 - b*c*d*e + a*c*e^2) + 1/2*(b^2*d - 2*a*c*d
- a*b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e + a*c
*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [A] time = 0.01, size = 289, normalized size = 1.83

$$-\frac{abe\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;c}-\frac{ad\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}}+\frac{b^2d\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;c}+\frac{ae\ln\left(c\,x\right.}{4\left(a\,e^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/(a*e^2-b*d*e+c*d^2)/c*ln(c*x^4+b*x^2+a)*a*e-1/4/(a*e^2-b*d*e+c*d^2)/c*l
n(c*x^4+b*x^2+a)*b*d-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*a*d-1/2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b/c*a*e+1/2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2/c*d+1/2*d^2*ln(e*x^2+d)/e
/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 11.05, size = 1853, normalized size = 11.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d^2*log(d + e*x^2))/(2*a*e^3 - 2*b*d*e^2 + 2*c*d^2*e) + (log(4*a^2*e^4*(b^
2 - 4*a*c)^(5/2) + 8*c^2*d^4*(b^2 - 4*a*c)^(5/2) + 5*d^2*e^2*(b^2 - 4*a*c)^
```

```
(7/2) + 3*d*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 16*a^3*b^3*c*e^4 + 64*a^4*b*c^2*e
^4 + 640*a^3*c^4*d^3*e - 384*a^4*c^3*d*e^3 - 4*a^2*b^2*e^4*(b^2 - 4*a*c)^(3
/2) - 8*b^2*c^2*d^4*(b^2 - 4*a*c)^(3/2) - 6*b^2*d^2*e^2*(b^2 - 4*a*c)^(5/2)
 + b^4*d^2*e^2*(b^2 - 4*a*c)^(3/2) - 256*a^2*c^5*d^4*x^2 - 128*a^4*c^3*e^4*
x^2 - 16*b^4*c^3*d^4*x^2 + 80*a^2*b^3*c^2*d^2*e^2 + 96*a^3*b^2*c^2*e^4*x^2
+ 640*a^3*c^4*d^2*e^2*x^2 + 4*b^3*c*d^3*e*(b^2 - 4*a*c)^(3/2) + 4*a*b*e^4*x
^2*(b^2 - 4*a*c)^(5/2) + 48*a*b^4*c^2*d^3*e - 16*a*b^5*c*d^2*e^2 - 4*a*b^3*
e^4*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^3*d^4*x^2*(b^2 - 4*a*c)^(3/2) - 6*b^2*
d*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 3*b^4*d*e^3*x^2*(b^2 - 4*a*c)^(3/2) + 20*c^
2*d^3*e*x^2*(b^2 - 4*a*c)^(5/2) - 352*a^2*b^2*c^3*d^3*e - 64*a^3*b*c^3*d^2*
e^2 + 96*a^3*b^2*c^2*d*e^3 + 128*a*b^2*c^4*d^4*x^2 - 16*a^2*b^4*c*e^4*x^2 +
 32*b^5*c^2*d^3*e*x^2 - 16*b^6*c*d^2*e^2*x^2 - 4*b*c*d^3*e*(b^2 - 4*a*c)^(5
/2) - 480*a^2*b^2*c^3*d^2*e^2*x^2 - 12*b*c*d^2*e^2*x^2*(b^2 - 4*a*c)^(5/2)
- 240*a*b^3*c^3*d^3*e*x^2 + 448*a^2*b*c^4*d^3*e*x^2 - 192*a^3*b*c^3*d*e^3*x
^2 + 12*b^2*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(3/2) - 4*b^3*c*d^2*e^2*x^2*(b^2 -
4*a*c)^(3/2) + 144*a*b^4*c^2*d^2*e^2*x^2 + 48*a^2*b^3*c^2*d*e^3*x^2)*((b^3*
d)/4 + e*(a^2*c - (a*b^2)/4 + (a*b*(b^2 - 4*a*c)^(1/2))/4) - (b^2*d*(b^2 -
4*a*c)^(1/2))/4 + (a*c*d*(b^2 - 4*a*c)^(1/2))/2 - a*b*c*d))/(4*a*c^3*d^2 +
4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e) - (l
og(4*a^2*e^4*(b^2 - 4*a*c)^(5/2) + 8*c^2*d^4*(b^2 - 4*a*c)^(5/2) + 5*d^2*e^
2*(b^2 - 4*a*c)^(7/2) + 3*d*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 16*a^3*b^3*c*e^4
- 64*a^4*b*c^2*e^4 - 640*a^3*c^4*d^3*e + 384*a^4*c^3*d*e^3 - 4*a^2*b^2*e^4*
(b^2 - 4*a*c)^(3/2) - 8*b^2*c^2*d^4*(b^2 - 4*a*c)^(3/2) - 6*b^2*d^2*e^2*(b^
2 - 4*a*c)^(5/2) + b^4*d^2*e^2*(b^2 - 4*a*c)^(3/2) + 256*a^2*c^5*d^4*x^2 +
128*a^4*c^3*e^4*x^2 + 16*b^4*c^3*d^4*x^2 - 80*a^2*b^3*c^2*d^2*e^2 - 96*a^3*
b^2*c^2*e^4*x^2 - 640*a^3*c^4*d^2*e^2*x^2 + 4*b^3*c*d^3*e*(b^2 - 4*a*c)^(3/
2) + 4*a*b*e^4*x^2*(b^2 - 4*a*c)^(5/2) - 48*a*b^4*c^2*d^3*e + 16*a*b^5*c*d^
2*e^2 - 4*a*b^3*e^4*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^3*d^4*x^2*(b^2 - 4*a*c
)^(3/2) - 6*b^2*d*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 3*b^4*d*e^3*x^2*(b^2 - 4*a*
c)^(3/2) + 20*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(5/2) + 352*a^2*b^2*c^3*d^3*e + 6
4*a^3*b*c^3*d^2*e^2 - 96*a^3*b^2*c^2*d*e^3 - 128*a*b^2*c^4*d^4*x^2 + 16*a^2
*b^4*c*e^4*x^2 - 32*b^5*c^2*d^3*e*x^2 + 16*b^6*c*d^2*e^2*x^2 - 4*b*c*d^3*e*
(b^2 - 4*a*c)^(5/2) + 480*a^2*b^2*c^3*d^2*e^2*x^2 - 12*b*c*d^2*e^2*x^2*(b^2
 - 4*a*c)^(5/2) + 240*a*b^3*c^3*d^3*e*x^2 - 448*a^2*b*c^4*d^3*e*x^2 + 192*a
^3*b*c^3*d*e^3*x^2 + 12*b^2*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(3/2) - 4*b^3*c*d^2
*e^2*x^2*(b^2 - 4*a*c)^(3/2) - 144*a*b^4*c^2*d^2*e^2*x^2 - 48*a^2*b^3*c^2*d
*e^3*x^2)*(e*((a*b^2)/4 - a^2*c + (a*b*(b^2 - 4*a*c)^(1/2))/4) - (b^3*d)/4
- (b^2*d*(b^2 - 4*a*c)^(1/2))/4 + (a*c*d*(b^2 - 4*a*c)^(1/2))/2 + a*b*c*d))
/(4*a*c^3*d^2 + 4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a
*b*c^2*d*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.298 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=132

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2-bde+cd^2\right)}+\frac{d\log\left(a+bx^2+cx^4\right)}{4\left(ae^2-bde+cd^2\right)}$$

```
[Out] -1/2*d*ln(e*x^2+d)/(a*e^2-b*d*e+c*d^2)+1/4*d*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e
+c*d^2)+1/2*(-2*a*e+b*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d
*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 800, 634, 618, 206, 628}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2-bde+cd^2\right)}+\frac{d\log\left(a+bx^2+cx^4\right)}{4\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)) - (d*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e^2)
) + (d*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{de}{(cd^2-bde+ae^2)(d+ex)} + \frac{ae+cdx}{(cd^2-bde+ae^2)(a+bx+cx^2)}\right)\right.$$

$$= -\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{ae+cdx}{a+bx+cx^2}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)}$$

$$= -\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4(cd^2-bde+ae^2)} - \frac{(bd-2ae)\,\text{Subst}\left(\int \frac{}{a+}\right.}{4(cd^2-bde}$$

$$= -\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)} + \frac{(bd-2ae)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,d\right.}{2(cd^2-bde+ae}$$

$$= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)}$$

Mathematica [A] time = 0.07, size = 114, normalized size = 0.86

$$\frac{2(bd-2ae)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right) + d\sqrt{4ac-b^2}\left(2\log(d+ex^2) - \log(a+bx^2+cx^4)\right)}{4\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*(b*d - 2*a*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a
*c]*d*(2*Log[d + e*x^2] - Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*(-
(c*d^2) + e*(b*d - a*e)))
```

fricas [A] time = 27.80, size = 321, normalized size = 2.43

$$\left[\frac{(b^2-4ac)d\log(cx^4+bx^2+a) - 2(b^2-4ac)d\log(ex^2+d) - \sqrt{b^2-4ac}\,(bd-2ae)\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-(2c}{cx^4+bx^2+a}\right.}{4\left((b^2c-4ac^2)d^2-(b^3-4abc)de+(ab^2-4a^2c)e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2 - 4*a*c)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(e*x^2
+ d) - sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c - 4*a
*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), 1/4*((b^2 - 4*a*c
```

```
)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(e*x^2 + d) + 2*sqrt(-b^2
 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)
]
```

giac [A] time = 1.72, size = 133, normalized size = 1.01

$$-\frac{de\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e-bde^2+ae^3\right)}+\frac{d\log\left(cx^4+bx^2+a\right)}{4\left(cd^2-bde+ae^2\right)}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(cd^2-bde+ae^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*d*e*log(abs(x^2*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/4*d*log(c*x^4
+ b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) - 1/2*(b*d - 2*a*e)*arctan((2*c*x^2 +
b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [A] time = 0.01, size = 176, normalized size = 1.33

$$\frac{ae\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}}-\frac{bd\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(ae^2-deb+cd^2\right)\sqrt{4ac-b^2}}-\frac{d\ln\left(ex^2+d\right)}{2\left(ae^2-deb+cd^2\right)}+\frac{d\ln\left(cx^4+bx^2+a\right)}{4ae^2-4deb+4cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*d*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+c*d^2)+1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*e-1/2/(a*e^2-b*d*e+c*d^2)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*d-1/2*d*ln(e*x^2+
d)/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 9.75, size = 3704, normalized size = 28.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(76*d^3*e^3*(b^2 - 4*a*c)^(9/2) - 64*a^3*b^6*e^6 - 4608*a^3*c^6*d^6 + 5
12*a^6*c^3*e^6 - 320*a*b^4*c^4*d^6 + 512*a^4*b^4*c*e^6 - 64*a*b^8*d^2*e^4 -
 128*a^2*b^7*d*e^5 + 32*a^3*b^3*e^6*(b^2 - 4*a*c)^(3/2) - 48*b^3*c^3*d^6*(b
^2 - 4*a*c)^(3/2) - 68*b^2*d^3*e^3*(b^2 - 4*a*c)^(7/2) - 28*b^4*d^3*e^3*(b^
2 - 4*a*c)^(5/2) + 20*b^6*d^3*e^3*(b^2 - 4*a*c)^(3/2) + 4*a^2*e^6*x^2*(b^2
- 4*a*c)^(7/2) + 144*c^4*d^6*x^2*(b^2 - 4*a*c)^(5/2) + 39*d^2*e^4*x^2*(b^2
- 4*a*c)^(9/2) + 2432*a^2*b^2*c^5*d^6 - 1152*a^5*b^2*c^2*e^6 + 40448*a^4*c^
5*d^4*e^2 - 19968*a^5*c^4*d^2*e^4 - 64*a^2*b^7*e^6*x^2 - 64*b^5*c^4*d^6*x^2
```

- 64*b^9*d^2*e^4*x^2 + 32*a^3*b*e^6*(b^2 - 4*a*c)^(5/2) + 48*b*c^3*d^6*(b^2 - 4*a*c)^(5/2) + 40*a^2*d*e^5*(b^2 - 4*a*c)^(7/2) + 168*c^2*d^5*e*(b^2 - 4*a*c)^(7/2) + 40*a^2*b^2*e^6*x^2*(b^2 - 4*a*c)^(5/2) + 20*a^2*b^4*e^6*x^2*(b^2 - 4*a*c)^(3/2) - 80*b^2*c^4*d^6*x^2*(b^2 - 4*a*c)^(3/2) + 155*b^2*d^2*e^4*x^2*(b^2 - 4*a*c)^(7/2) - 155*b^4*d^2*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 25*b^6*d^2*e^4*x^2*(b^2 - 4*a*c)^(3/2) + 316*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(7/2) + 5120*a^2*b^4*c^3*d^4*e^2 - 4096*a^2*b^5*c^2*d^3*e^3 - 24448*a^3*b^2*c^4*d^4*e^2 + 21760*a^3*b^3*c^3*d^3*e^3 - 9920*a^3*b^4*c^2*d^2*e^4 + 26240*a^4*b^2*c^3*d^2*e^4 - 1600*a^4*b^3*c^2*e^6*x^2 + 38912*a^4*c^5*d^3*e^3*x^2 - 384*b^7*c^2*d^4*e^2*x^2 + 212*a*b*d^2*e^4*(b^2 - 4*a*c)^(7/2) - 176*b*c*d^4*e^2*(b^2 - 4*a*c)^(7/2) + 256*a*b^5*c^3*d^5*e + 256*a*b^7*c*d^3*e^3 + 2560*a^3*b*c^5*d^5*e + 1664*a^3*b^5*c*d*e^5 + 8704*a^5*b*c^3*d*e^5 - 128*a*b^8*d*e^5*x^2 - 168*a*b^3*d^2*e^4*(b^2 - 4*a*c)^(5/2) + 20*a*b^5*d^2*e^4*(b^2 - 4*a*c)^(3/2) + 144*a^2*b^2*d*e^5*(b^2 - 4*a*c)^(5/2) - 56*a^2*b^4*d*e^5*(b^2 - 4*a*c)^(3/2) - 272*b^2*c^2*d^5*e*(b^2 - 4*a*c)^(5/2) + 256*b^3*c*d^4*e^2*(b^2 - 4*a*c)^(5/2) + 104*b^4*c^2*d^5*e*(b^2 - 4*a*c)^(3/2) - 80*b^5*c*d^4*e^2*(b^2 - 4*a*c)^(3/2) - 384*a*b^6*c^2*d^4*e^2 - 1664*a^2*b^3*c^4*d^5*e + 1408*a^2*b^6*c*d^2*e^4 - 37888*a^4*b*c^4*d^3*e^3 - 6784*a^4*b^3*c^2*d*e^5 + 448*a*b^3*c^5*d^6*x^2 - 768*a^2*b*c^6*d^6*x^2 + 576*a^3*b^5*c*e^6*x^2 + 1280*a^5*b*c^3*e^6*x^2 - 21504*a^3*c^6*d^5*e*x^2 - 5120*a^5*c^4*d*e^5*x^2 + 256*b^6*c^3*d^5*e*x^2 + 256*b^8*c*d^3*e^3*x^2 - 26560*a^2*b^3*c^4*d^4*e^2*x^2 + 25600*a^2*b^4*c^3*d^3*e^3*x^2 - 11264*a^2*b^5*c^2*d^2*e^4*x^2 - 58880*a^3*b^2*c^4*d^3*e^3*x^2 + 34880*a^3*b^3*c^3*d^2*e^4*x^2 + 80*a*b^3*d*e^5*x^2*(b^2 - 4*a*c)^(5/2) - 40*a*b^5*d*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 448*b*c*d^3*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 416*b*c^3*d^5*e*x^2*(b^2 - 4*a*c)^(5/2) - 3200*a*b^4*c^4*d^5*e*x^2 + 1472*a*b^7*c*d^2*e^4*x^2 + 1792*a^2*b^6*c*d*e^5*x^2 + 192*b^3*c*d^3*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 160*b^3*c^3*d^5*e*x^2*(b^2 - 4*a*c)^(3/2) + 5504*a*b^5*c^3*d^4*e^2*x^2 - 4352*a*b^6*c^2*d^3*e^3*x^2 + 14080*a^2*b^2*c^5*d^5*e*x^2 + 42752*a^3*b*c^5*d^4*e^2*x^2 - 8320*a^3*b^4*c^2*d*e^5*x^2 - 37120*a^4*b*c^4*d^2*e^4*x^2 + 14080*a^4*b^2*c^3*d*e^5*x^2 + 88*a*b*d*e^5*x^2*(b^2 - 4*a*c)^(7/2) + 168*b^2*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(5/2) - 100*b^4*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(d*((b*(b^2 - 4*a*c)^(1/2))/4 - a*c + b^2/4) - (a*e*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) - (log(76*d^3*e^3*(b^2 - 4*a*c)^(9/2) + 64*a^3*b^6*e^6 + 4608*a^3*c^6*d^6 - 512*a^6*c^3*e^6 + 320*a*b^4*c^4*d^6 - 512*a^4*b^4*c*e^6 + 64*a*b^8*d^2*e^4 + 128*a^2*b^7*d*e^5 + 32*a^3*b^3*e^6*(b^2 - 4*a*c)^(3/2) - 48*b^3*c^3*d^6*(b^2 - 4*a*c)^(3/2) - 68*b^2*d^3*e^3*(b^2 - 4*a*c)^(7/2) - 28*b^4*d^3*e^3*(b^2 - 4*a*c)^(5/2) + 20*b^6*d^3*e^3*(b^2 - 4*a*c)^(3/2) + 4*a^2*e^6*x^2*(b^2 - 4*a*c)^(7/2) + 144*c^4*d^6*x^2*(b^2 - 4*a*c)^(5/2) + 39*d^2*e^4*x^2*(b^2 - 4*a*c)^(9/2) - 2432*a^2*b^2*c^5*d^6 + 1152*a^5*b^2*c^2*e^6 - 40448*a^4*c^5*d^4*e^2 + 19968*a^5*c^4*d^2*e^4 + 64*a^2*b^7*e^6*x^2 + 64*b^5*c^4*d^6*x^2 + 64*b^9*d^2*e^4*x^2 + 32*a^3*b*e^6*(b^2 - 4*a*c)^(5/2) + 48*b*c^3*d^6*(b^2 - 4*a*c)^(5/2) + 40*a^2*d*e^5*(b^2 - 4*a*c)^(7/2) + 168*c^2*d^5*e*(b^2 - 4*a*c)^(7/2) + 40*a^2*b^2*e^6*x^2*(b^2 - 4*a*c)^(5/2) + 20*a^2*b^4*e^6*x^2*(b^2 - 4*a*c)^(3/2) - 80*b^2*c^4*d^6*x^2*(b^2 - 4*a*c)^(3/2) + 155*b^2*d^2*e^4*x^2*(b^2 - 4*a*c)^(7/2) - 155*b^4*d^2*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 25*b^6*d^2*e^4*x^2*(b^2 - 4*a*c)^(3/2) + 316*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(7/2) - 5120*a^2*b^4*c^3*d^4*e^2 + 4096*a^2*b^5*c^2*d^3*e^3 + 24448*a^3*b^2*c^4*d^4*e^2 - 21760*a^3*b^3*c^3*d^3*e^3 + 9920*a^3*b^4*c^2*d^2*e^4 - 26240*a^4*b^2*c^3*d^2*e^4 + 1600*a^4*b^3*c^2*e^6*x^2 - 38912*a^4*c^5*d^3*e^3*x^2 + 384*b^7*c^2*d^4*e^2*x^2 + 212*a*b*d^2*e^4*(b^2 - 4*a*c)^(7/2) - 176*b*c*d^4*e^2*(b^2 - 4*a*c)^(7/2) - 256*a*b^5*c^3*d^5*e - 256*a*b^7*c*d^3*e^3 - 2560*a^3*b*c^5*d^5*e - 1664*a^3*b^5*c*d*e^5 - 8704*a^5*b*c^3*d*e^5 + 128*a*b^8*d*e^5*x^2 - 168*a*b^3*d^2*e^4*(b^2 - 4*a*c)^(5/2) + 20*a*b^5*d^2*e^4*(b^2 - 4*a*c)^(3/2) + 144*a^2*b^2*d*e^5*(b^2 - 4*a*c)^(5/2) - 56*a^2*b^4*d*e^5*(b^2 - 4*a*c)^(3/2) - 272*b^2*c^2*d^5*e*(b^2 - 4*a*c)^(5/2) + 256*b^3*c*d^4*e^2*(b^2 - 4*a*c)^(5/2) + 104*b^4*c^2*d^5*e*(b^2 - 4*a*c)^(3/2) - 80*b^5*c*d^4*e^2*(b^2 - 4*a*c)^(3/2) + 384*a*b^6*c^2*d^4*e^2 + 1664*a^2*b^3*c^4*d^5*e - 1408*a^2*b

```
^6*c*d^2*e^4 + 37888*a^4*b*c^4*d^3*e^3 + 6784*a^4*b^3*c^2*d*e^5 - 448*a*b^3
*c^5*d^6*x^2 + 768*a^2*b*c^6*d^6*x^2 - 576*a^3*b^5*c*e^6*x^2 - 1280*a^5*b*c
^3*e^6*x^2 + 21504*a^3*c^6*d^5*e*x^2 + 5120*a^5*c^4*d*e^5*x^2 - 256*b^6*c^3
*d^5*e*x^2 - 256*b^8*c*d^3*e^3*x^2 + 26560*a^2*b^3*c^4*d^4*e^2*x^2 - 25600*
a^2*b^4*c^3*d^3*e^3*x^2 + 11264*a^2*b^5*c^2*d^2*e^4*x^2 + 58880*a^3*b^2*c^4
*d^3*e^3*x^2 - 34880*a^3*b^3*c^3*d^2*e^4*x^2 + 80*a*b^3*d*e^5*x^2*(b^2 - 4*
a*c)^(5/2) - 40*a*b^5*d*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 448*b*c*d^3*e^3*x^2*(
b^2 - 4*a*c)^(7/2) - 416*b*c^3*d^5*e*x^2*(b^2 - 4*a*c)^(5/2) + 3200*a*b^4*c
^4*d^5*e*x^2 - 1472*a*b^7*c*d^2*e^4*x^2 - 1792*a^2*b^6*c*d*e^5*x^2 + 192*b^
3*c*d^3*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 160*b^3*c^3*d^5*e*x^2*(b^2 - 4*a*c)^(
3/2) - 5504*a*b^5*c^3*d^4*e^2*x^2 + 4352*a*b^6*c^2*d^3*e^3*x^2 - 14080*a^2*
b^2*c^5*d^5*e*x^2 - 42752*a^3*b*c^5*d^4*e^2*x^2 + 8320*a^3*b^4*c^2*d*e^5*x^
2 + 37120*a^4*b*c^4*d^2*e^4*x^2 - 14080*a^4*b^2*c^3*d*e^5*x^2 + 88*a*b*d*e^
5*x^2*(b^2 - 4*a*c)^(7/2) + 168*b^2*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(5/2) - 1
00*b^4*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(d*(a*c + (b*(b^2 - 4*a*c)^(1/2
))/4 - b^2/4) - (a*e*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4*a*c^2*d^2 - 4*
a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) - (d*log(d + e*x^2))/(2*(a*e
^2 + c*d^2 - b*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.299 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=133

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,\left(ae^2-bde+cd^2\right)}+\frac{e\log\left(d+ex^2\right)}{2\left(ae^2-bde+cd^2\right)}-\frac{e\log\left(a+bx^2+cx^4\right)}{4\left(ae^2-bde+cd^2\right)}$$

```
[Out] 1/2*e*ln(e*x^2+d)/(a*e^2-b*d*e+c*d^2)-1/4*e*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+
c*d^2)-1/2*(-b*e+2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*
e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1247, 705, 31, 634, 618, 206, 628}

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,\left(ae^2-bde+cd^2\right)}+\frac{e\log\left(d+ex^2\right)}{2\left(ae^2-bde+cd^2\right)}-\frac{e\log\left(a+bx^2+cx^4\right)}{4\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[b^2 - 4*a
*c]*(c*d^2 - b*d*e + a*e^2)) + (e*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e^2
)) - (e*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{cd-be-cex}{a+bx+cx^2}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)} + \frac{e^2\,\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)}\\
&= \frac{e\log(d+ex^2)}{2(cd^2-bde+ae^2)} - \frac{e\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4(cd^2-bde+ae^2)} + \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{a\ldots}\right.}{4(cd^2-bde\ldots}\\
&= \frac{e\log(d+ex^2)}{2(cd^2-bde+ae^2)} - \frac{e\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\ldots\right.}{2(cd^2-bde+ae\ldots}\\
&= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{e\log(d+ex^2)}{2(cd^2-bde+ae^2)} - \frac{e\log(a+bx^2+cx\ldots}{4(cd^2-bde+ae^2\ldots}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 112, normalized size = 0.84

$$\frac{(2be-4cd)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)+e\sqrt{4ac-b^2}\left(\log\left(a+bx^2+cx^4\right)-2\log\left(d+ex^2\right)\right)}{4\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*c*d + 2*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*
a*c]*e*(-2*Log[d + e*x^2] + Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*
(-(c*d^2) + e*(b*d - a*e)))
```

fricas [A] time = 19.13, size = 321, normalized size = 2.41

$$\left[-\frac{(b^2-4\,ac)e\log\left(cx^4+bx^2+a\right)-2\left(b^2-4\,ac\right)e\log\left(ex^2+d\right)+\sqrt{b^2-4\,ac}\,(2\,cd-be)\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,a\ldots}{cx^4+b\ldots}\right.}{4\left(\left(b^2c-4\,ac^2\right)d^2-\left(b^3-4\,abc\right)de+\left(ab^2-4\,a^2c\right)e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 4*a*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(e*x^2
 + d) + sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
```

```
2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c - 4*
a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), -1/4*((b^2 - 4*a
*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(e*x^2 + d) + 2*sqrt(-b
^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^
2)]
```

giac [A] time = 1.91, size = 134, normalized size = 1.01

$$-\frac{e\log\left(cx^4+bx^2+a\right)}{4\left(cd^2-bde+ae^2\right)}+\frac{e^2\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e-bde^2+ae^3\right)}+\frac{(2\,cd-be)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(cd^2-bde+ae^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*e*log(c*x^4 + b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) + 1/2*e^2*log(abs(x^2
*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/2*(2*c*d - b*e)*arctan((2*c*x^2 +
b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [A] time = 0.01, size = 176, normalized size = 1.32

$$-\frac{be\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}}+\frac{cd\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}}+\frac{e\ln\left(e\,x^2+d\right)}{2a\,e^2-2deb+2c\,d^2}-\frac{e\ln\left(c\,x^4+b\,x^2+a\right)}{4\left(a\,e^2-deb+c\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*e*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+c*d^2)-1/2/(a*e^2-b*d*e+c*d^2)/(4*a*c
-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*e+1/(a*e^2-b*d*e+c*d^2)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*d+1/2*e*ln(e*x^2
+d)/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.71, size = 2434, normalized size = 18.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (e*log(d + e*x^2))/(2*a*e^2 + 2*c*d^2 - 2*b*d*e) - (log(36*a^4*c^3*e^5 - 4*
a*b^6*e^5 - 4*b^7*e^5*x^2 + 32*a^2*b^4*c*e^5 + 36*a^2*c^5*d^4*e - 4*a*c^6*d
^5*x^2 - 4*b^6*e^5*x^2*(b^2 - 4*a*c)^(1/2) - 73*a^3*b^2*c^2*e^5 - 184*a^3*c
^4*d^2*e^3 + b^2*c^5*d^5*x^2 - 4*a*b^5*e^5*(b^2 - 4*a*c)^(1/2) + 2*a*c^5*d^
5*(b^2 - 4*a*c)^(1/2) + 16*a*b^5*c*d*e^4 - 60*a^2*c^4*d^3*e^2*(b^2 - 4*a*c)
^(1/2) + 18*a^3*c^3*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 146*a^2*b^2*c^3*d^2*e^3 -
```

```
101*a^2*b^3*c^2*e^5*x^2 + 120*a^2*c^5*d^3*e^2*x^2 + 19*b^4*c^3*d^3*e^2*x^2
 - 25*b^5*c^2*d^2*e^3*x^2 - 9*a*b^2*c^4*d^4*e + 184*a^3*b*c^3*d*e^4 + 36*a*
b^5*c*e^5*x^2 + 16*b^6*c*d*e^4*x^2 + 24*a^2*b^3*c*e^5*(b^2 - 4*a*c)^(1/2) -
 33*a^3*b*c^2*e^5*(b^2 - 4*a*c)^(1/2) + 66*a^3*c^3*d*e^4*(b^2 - 4*a*c)^(1/2
) + b*c^5*d^5*x^2*(b^2 - 4*a*c)^(1/2) + 18*a*b^3*c^3*d^3*e^2 - 25*a*b^4*c^2
*d^2*e^3 - 72*a^2*b*c^4*d^3*e^2 - 110*a^2*b^3*c^2*d*e^4 + 84*a^3*b*c^3*e^5*
x^2 - 132*a^3*c^4*d*e^4*x^2 - 7*b^3*c^4*d^4*e*x^2 + 28*a*b^4*c*e^5*x^2*(b^2
 - 4*a*c)^(1/2) + 18*a*c^5*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) + 16*b^5*c*d*e^4*x
^2*(b^2 - 4*a*c)^(1/2) - 126*a*b^4*c^2*d*e^4*x^2 + 20*a*b^2*c^3*d^3*e^2*(b^
2 - 4*a*c)^(1/2) - 25*a*b^3*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 90*a^2*b*c^3*
d^2*e^3*(b^2 - 4*a*c)^(1/2) - 78*a^2*b^2*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) - 7*
b^2*c^4*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) - 106*a*b^2*c^4*d^3*e^2*x^2 + 168*a*b
^3*c^3*d^2*e^3*x^2 - 272*a^2*b*c^4*d^2*e^3*x^2 + 281*a^2*b^2*c^3*d*e^4*x^2
 - 5*a*b*c^4*d^4*e*(b^2 - 4*a*c)^(1/2) + 16*a*b^4*c*d*e^4*(b^2 - 4*a*c)^(1/2
) - 53*a^2*b^2*c^2*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 28*a*b*c^5*d^4*e*x^2 - 92*
a^2*c^4*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) + 19*b^3*c^3*d^3*e^2*x^2*(b^2 - 4*a
*c)^(1/2) - 25*b^4*c^2*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) + 118*a*b^2*c^3*d^2*
e^3*x^2*(b^2 - 4*a*c)^(1/2) - 66*a*b*c^4*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) -
94*a*b^3*c^2*d*e^4*x^2*(b^2 - 4*a*c)^(1/2) + 125*a^2*b*c^3*d*e^4*x^2*(b^2 -
 4*a*c)^(1/2))*(e*((b*(b^2 - 4*a*c)^(1/2))/4 - a*c + b^2/4) - (c*d*(b^2 - 4
*a*c)^(1/2))/2))/(a*b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d
*e + 4*a*b*c*d*e) + (log(4*a*b^6*e^5 - 36*a^4*c^3*e^5 + 4*b^7*e^5*x^2 - 32*
a^2*b^4*c*e^5 - 36*a^2*c^5*d^4*e + 4*a*c^6*d^5*x^2 - 4*b^6*e^5*x^2*(b^2 - 4
*a*c)^(1/2) + 73*a^3*b^2*c^2*e^5 + 184*a^3*c^4*d^2*e^3 - b^2*c^5*d^5*x^2 -
4*a*b^5*e^5*(b^2 - 4*a*c)^(1/2) + 2*a*c^5*d^5*(b^2 - 4*a*c)^(1/2) - 16*a*b^
5*c*d*e^4 - 60*a^2*c^4*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 18*a^3*c^3*e^5*x^2*(b^
2 - 4*a*c)^(1/2) - 146*a^2*b^2*c^3*d^2*e^3 + 101*a^2*b^3*c^2*e^5*x^2 - 120*
a^2*c^5*d^3*e^2*x^2 - 19*b^4*c^3*d^3*e^2*x^2 + 25*b^5*c^2*d^2*e^3*x^2 + 9*a
*b^2*c^4*d^4*e - 184*a^3*b*c^3*d*e^4 - 36*a*b^5*c*e^5*x^2 - 16*b^6*c*d*e^4*
x^2 + 24*a^2*b^3*c*e^5*(b^2 - 4*a*c)^(1/2) - 33*a^3*b*c^2*e^5*(b^2 - 4*a*c)
^(1/2) + 66*a^3*c^3*d*e^4*(b^2 - 4*a*c)^(1/2) + b*c^5*d^5*x^2*(b^2 - 4*a*c)
^(1/2) - 18*a*b^3*c^3*d^3*e^2 + 25*a*b^4*c^2*d^2*e^3 + 72*a^2*b*c^4*d^3*e^2
 + 110*a^2*b^3*c^2*d*e^4 - 84*a^3*b*c^3*e^5*x^2 + 132*a^3*c^4*d*e^4*x^2 + 7
*b^3*c^4*d^4*e*x^2 + 28*a*b^4*c*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 18*a*c^5*d^4*
e*x^2*(b^2 - 4*a*c)^(1/2) + 16*b^5*c*d*e^4*x^2*(b^2 - 4*a*c)^(1/2) + 126*a*
b^4*c^2*d*e^4*x^2 + 20*a*b^2*c^3*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 25*a*b^3*c^2
*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 90*a^2*b*c^3*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 7
8*a^2*b^2*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) - 7*b^2*c^4*d^4*e*x^2*(b^2 - 4*a*c)
^(1/2) + 106*a*b^2*c^4*d^3*e^2*x^2 - 168*a*b^3*c^3*d^2*e^3*x^2 + 272*a^2*b*
c^4*d^2*e^3*x^2 - 281*a^2*b^2*c^3*d*e^4*x^2 - 5*a*b*c^4*d^4*e*(b^2 - 4*a*c)
^(1/2) + 16*a*b^4*c*d*e^4*(b^2 - 4*a*c)^(1/2) - 53*a^2*b^2*c^2*e^5*x^2*(b^2
 - 4*a*c)^(1/2) - 28*a*b*c^5*d^4*e*x^2 - 92*a^2*c^4*d^2*e^3*x^2*(b^2 - 4*a*
c)^(1/2) + 19*b^3*c^3*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) - 25*b^4*c^2*d^2*e^3*
x^2*(b^2 - 4*a*c)^(1/2) + 118*a*b^2*c^3*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) - 6
6*a*b*c^4*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) - 94*a*b^3*c^2*d*e^4*x^2*(b^2 - 4
*a*c)^(1/2) + 125*a^2*b*c^3*d*e^4*x^2*(b^2 - 4*a*c)^(1/2))*(e*(a*c + (b*(b^
2 - 4*a*c)^(1/2))/4 - b^2/4) - (c*d*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4
*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.300 $\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=167

$$\frac{\left(2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2-bde+cd^2\right)}-\frac{(cd-be)\log\left(a+bx^2+cx^4\right)}{4a\left(ae^2-bde+cd^2\right)}+\frac{\log(x)}{ad}$$

```
[Out] ln(x)/a/d-1/2*e^2*ln(e*x^2+d)/d/(a*e^2-b*d*e+c*d^2)-1/4*(-b*e+c*d)*ln(c*x^4
+b*x^2+a)/a/(a*e^2-b*d*e+c*d^2)+1/2*(2*a*c*e-b^2*e+b*c*d)*arctanh((2*c*x^2+
b)/(-4*a*c+b^2)^(1/2))/a/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1251, 893, 634, 618, 206, 628}

$$\frac{\left(2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2-bde+cd^2\right)}-\frac{(cd-be)\log\left(a+bx^2+cx^4\right)}{4a\left(ae^2-bde+cd^2\right)}+\frac{\log(x)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*S
qrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + Log[x]/(a*d) - (e^2*Log[d + e*x
^2])/(2*d*(c*d^2 - b*d*e + a*e^2)) - ((c*d - b*e)*Log[a + b*x^2 + c*x^4])/(
4*a*(c*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int\left(\frac{1}{adx}-\frac{e^3}{d\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{-bcd+b^2e-ace-c(}{a\left(cd^2-bde+ae^2\right)\left(a\right.}\right.\right.$$

$$= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}+\frac{\text{Subst}\left(\int\frac{-bcd+b^2e-ace-c(cd-be)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a\left(cd^2-bde+ae^2\right)}$$

$$= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(cd-be)\,\text{Subst}\left(\int\frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a\left(cd^2-bde+ae^2\right)}-\frac{(}{}$$

$$= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(cd-be)\log\left(a+bx^2+cx^4\right)}{4a\left(cd^2-bde+ae^2\right)}+\frac{\left(bcd-b^2\right.}{}$$

$$= \frac{\left(bcd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}+\frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(}{}$$

Mathematica [A] time = 0.32, size = 242, normalized size = 1.45

$$\frac{4\log(x)\sqrt{b^2-4ac}\left(e(ae-bd)+cd^2\right)-2ae^2\sqrt{b^2-4ac}\log\left(d+ex^2\right)-d\left(cd\sqrt{b^2-4ac}-be\sqrt{b^2-4ac}+2ace\right.}{4ad\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*Log[x] - d*(b*c*d + c*Sqrt[
b^2 - 4*a*c]*d - b^2*e + 2*a*c*e - b*Sqrt[b^2 - 4*a*c]*e)*Log[b - Sqrt[b^2
- 4*a*c] + 2*c*x^2] + d*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e +
b*Sqrt[b^2 - 4*a*c]*e)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2] - 2*a*Sqrt[b^2
- 4*a*c]*e^2*Log[d + e*x^2])/(4*a*Sqrt[b^2 - 4*a*c]*d*(c*d^2 + e*(-(b*d) +
a*e)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

[Out] Timed out

giac [A] time = 1.92, size = 172, normalized size = 1.03

$$-\frac{(cd-be)\log\left(cx^4+bx^2+a\right)}{4\left(acd^2-abde+a^2e^2\right)}-\frac{e^3\log\left(\left|x^2e+d\right|\right)}{2\left(cd^3e-bd^2e^2+ade^3\right)}-\frac{\left(bcd-b^2e+2\,ace\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(acd^2-abde+a^2e^2\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(x^2\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(c*d - b*e)*log(c*x^4 + b*x^2 + a)/(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/2
*e^3*log(abs(x^2*e + d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3) - 1/2*(b*c*d - b^2
*e + 2*a*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - a*b*d*e
+ a^2*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*log(x^2)/(a*d)
```

maple [A] time = 0.01, size = 298, normalized size = 1.78

$$\frac{b^2e\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;a}-\frac{bcd\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;a}-\frac{ce\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}}+\frac{be\ln\left(c\,x^4+\right.}{4\left(a\,e^2-deb\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a/d*ln(x)+1/4/(a*e^2-b*d*e+c*d^2)/a*ln(c*x^4+b*x^2+a)*b*e-1/4/(a*e^2-b*d*
e+c*d^2)/a*c*ln(c*x^4+b*x^2+a)*d-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*e+1/2/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*e-1/2/(a*e^2-b*d*e+c*d^2
)/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*d-1/2*e^2*l
n(e*x^2+d)/d/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 17.20, size = 6285, normalized size = 37.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(256*a^4*e^8*(4*a*c - b^2)^4 - 80*c^4*d^8*(4*a*c - b^2)^4 - 61*d^4*e^4*
(4*a*c - b^2)^6 + 160*b^3*c^4*d^8*(b^2 - 4*a*c)^(5/2) + 16*b^5*c^4*d^8*(b^2
 - 4*a*c)^(3/2) - 184*b^3*d^4*e^4*(b^2 - 4*a*c)^(9/2) + 370*b^5*d^4*e^4*(b^
2 - 4*a*c)^(7/2) + 128*b^7*d^4*e^4*(b^2 - 4*a*c)^(5/2) + 5*b^9*d^4*e^4*(b^2
 - 4*a*c)^(3/2) + 128*a^3*e^8*x^2*(b^2 - 4*a*c)^(9/2) + 160*c^5*d^8*x^2*(b^
2 - 4*a*c)^(7/2) - 256*a^4*b^2*e^8*(4*a*c - b^2)^3 + 32*b^2*c^4*d^8*(4*a*c
- b^2)^3 + 112*b^4*c^4*d^8*(4*a*c - b^2)^2 - 144*a^2*d^2*e^6*(4*a*c - b^2)^
5 + 544*b^2*d^4*e^4*(4*a*c - b^2)^5 + 382*b^4*d^4*e^4*(4*a*c - b^2)^4 - 152
```

*b^6*d^4*e^4*(4*a*c - b^2)^3 + 71*b^8*d^4*e^4*(4*a*c - b^2)^2 + 200*c^2*d^6
e^2(4*a*c - b^2)^5 - 13*d^3*e^5*x^2*(4*a*c - b^2)^6 + 512*a^4*b*e^8*(b^2
- 4*a*c)^(7/2) - 176*b*c^4*d^8*(b^2 - 4*a*c)^(7/2) - 26*a*d^3*e^5*(b^2 - 4*
a*c)^(11/2) + 352*a^3*d*e^7*(b^2 - 4*a*c)^(9/2) - 319*b*d^4*e^4*(b^2 - 4*a*
c)^(11/2) + 148*c*d^5*e^3*(b^2 - 4*a*c)^(11/2) + 168*c^3*d^7*e*(b^2 - 4*a*c
)^(9/2) - 768*a*b^3*d^3*e^5*(4*a*c - b^2)^4 - 368*a*b^5*d^3*e^5*(4*a*c - b^
2)^3 + 128*a^3*b^3*d*e^7*(4*a*c - b^2)^3 - 32*a*b^7*d^3*e^5*(4*a*c - b^2)^2
 - 672*a^2*b^3*d^2*e^6*(b^2 - 4*a*c)^(7/2) - 272*a^2*b^5*d^2*e^6*(b^2 - 4*a
*c)^(5/2) + 408*b^3*c*d^5*e^3*(4*a*c - b^2)^4 + 256*b^3*c^3*d^7*e*(4*a*c -
b^2)^3 + 792*b^5*c*d^5*e^3*(4*a*c - b^2)^3 - 352*b^5*c^3*d^7*e*(4*a*c - b^2
)^2 - 248*b^7*c*d^5*e^3*(4*a*c - b^2)^2 - 328*b^3*c^2*d^6*e^2*(b^2 - 4*a*c)
^(7/2) + 1064*b^5*c^2*d^6*e^2*(b^2 - 4*a*c)^(5/2) + 40*b^7*c^2*d^6*e^2*(b^2
 - 4*a*c)^(3/2) + 384*a^3*b*e^8*x^2*(4*a*c - b^2)^4 + 384*a^3*b^2*e^8*x^2*(
b^2 - 4*a*c)^(7/2) - 512*b*c^5*d^8*x^2*(4*a*c - b^2)^3 + 576*b^2*c^5*d^8*x^
2*(b^2 - 4*a*c)^(5/2) + 32*b^4*c^5*d^8*x^2*(b^2 - 4*a*c)^(3/2) - 176*a^2*d*
e^7*x^2*(4*a*c - b^2)^5 - 800*b^3*d^3*e^5*x^2*(b^2 - 4*a*c)^(9/2) + 158*b^5
*d^3*e^5*x^2*(b^2 - 4*a*c)^(7/2) + 56*b^7*d^3*e^5*x^2*(b^2 - 4*a*c)^(5/2) -
 b^9*d^3*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 336*c^4*d^7*e*x^2*(4*a*c - b^2)^4 +
400*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(9/2) - 608*a^2*b^2*d^2*e^6*(4*a*c - b^2)
^4 + 560*a^2*b^4*d^2*e^6*(4*a*c - b^2)^3 - 1096*b^2*c^2*d^6*e^2*(4*a*c - b^
2)^4 - 872*b^4*c^2*d^6*e^2*(4*a*c - b^2)^3 + 424*b^6*c^2*d^6*e^2*(4*a*c - b
^2)^2 - 128*a^3*b^3*e^8*x^2*(4*a*c - b^2)^3 + 256*b^3*c^5*d^8*x^2*(4*a*c -
b^2)^2 + 584*b^2*d^3*e^5*x^2*(4*a*c - b^2)^5 - 410*b^4*d^3*e^5*x^2*(4*a*c -
 b^2)^4 - 256*b^6*d^3*e^5*x^2*(4*a*c - b^2)^3 - 17*b^8*d^3*e^5*x^2*(4*a*c -
 b^2)^2 + 296*c^2*d^5*e^3*x^2*(4*a*c - b^2)^5 + 336*a*b*d^3*e^5*(4*a*c - b^
2)^5 + 384*a^3*b*d*e^7*(4*a*c - b^2)^4 - 832*a*b^2*d^3*e^5*(b^2 - 4*a*c)^(9
/2) - 52*a*b^4*d^3*e^5*(b^2 - 4*a*c)^(7/2) + 144*a*b^6*d^3*e^5*(b^2 - 4*a*c
)^(5/2) - 2*a*b^8*d^3*e^5*(b^2 - 4*a*c)^(3/2) - 80*a^2*b*d^2*e^6*(b^2 - 4*a
*c)^(9/2) - 192*a^3*b^2*d*e^7*(b^2 - 4*a*c)^(7/2) + 96*a^3*b^4*d*e^7*(b^2 -
 4*a*c)^(5/2) - 632*b*c*d^5*e^3*(4*a*c - b^2)^5 + 608*b*c^3*d^7*e*(4*a*c -
b^2)^4 - 776*b*c^2*d^6*e^2*(b^2 - 4*a*c)^(9/2) + 920*b^2*c*d^5*e^3*(b^2 - 4
*a*c)^(9/2) + 584*b^2*c^3*d^7*e*(b^2 - 4*a*c)^(7/2) - 384*b^4*c*d^5*e^3*(b^
2 - 4*a*c)^(7/2) - 712*b^4*c^3*d^7*e*(b^2 - 4*a*c)^(5/2) - 664*b^6*c*d^5*e^
3*(b^2 - 4*a*c)^(5/2) - 40*b^6*c^3*d^7*e*(b^2 - 4*a*c)^(3/2) - 20*b^8*c*d^5
e^3(b^2 - 4*a*c)^(3/2) + 72*a*d^2*e^6*x^2*(b^2 - 4*a*c)^(11/2) - 181*b*d^
3*e^5*x^2*(b^2 - 4*a*c)^(11/2) + 122*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(11/2) + 3
68*a^2*b*d*e^7*x^2*(b^2 - 4*a*c)^(9/2) - 1552*b*c^4*d^7*e*x^2*(b^2 - 4*a*c)
^(7/2) - 3400*b^2*c^2*d^5*e^3*x^2*(4*a*c - b^2)^4 - 4800*b^3*c^3*d^6*e^2*x^
2*(4*a*c - b^2)^3 + 3448*b^4*c^2*d^5*e^3*x^2*(4*a*c - b^2)^3 + 928*b^5*c^3*
d^6*e^2*x^2*(4*a*c - b^2)^2 - 536*b^6*c^2*d^5*e^3*x^2*(4*a*c - b^2)^2 - 32*
a*b*d^2*e^6*x^2*(4*a*c - b^2)^5 - 344*a*b^2*d^2*e^6*x^2*(b^2 - 4*a*c)^(9/2)
 - 616*a*b^4*d^2*e^6*x^2*(b^2 - 4*a*c)^(7/2) - 136*a*b^6*d^2*e^6*x^2*(b^2 -
 4*a*c)^(5/2) - 160*a^2*b^3*d*e^7*x^2*(b^2 - 4*a*c)^(7/2) + 48*a^2*b^5*d*e^
7*x^2*(b^2 - 4*a*c)^(5/2) - 760*b*c*d^4*e^4*x^2*(4*a*c - b^2)^5 - 1560*b*c^
2*d^5*e^3*x^2*(b^2 - 4*a*c)^(9/2) + 1848*b^2*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(9
/2) - 2208*b^3*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(5/2) + 1452*b^4*c*d^4*e^4*x^2*(
b^2 - 4*a*c)^(7/2) - 80*b^5*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(3/2) + 408*b^6*c*d
^4*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 10*b^8*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(3/2) -
 640*a*b^3*d^2*e^6*x^2*(4*a*c - b^2)^4 + 96*a^2*b^2*d*e^7*x^2*(4*a*c - b^2)
^4 + 416*a*b^5*d^2*e^6*x^2*(4*a*c - b^2)^3 + 16*a^2*b^4*d*e^7*x^2*(4*a*c -
b^2)^3 + 1952*b*c^3*d^6*e^2*x^2*(4*a*c - b^2)^4 + 2216*b^3*c*d^4*e^4*x^2*(4
*a*c - b^2)^4 + 2720*b^2*c^4*d^7*e*x^2*(4*a*c - b^2)^3 - 712*b^5*c*d^4*e^4*
x^2*(4*a*c - b^2)^3 - 784*b^4*c^4*d^7*e*x^2*(4*a*c - b^2)^2 + 152*b^7*c*d^4
*e^4*x^2*(4*a*c - b^2)^2 + 4144*b^2*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(7/2) - 4
216*b^3*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 3056*b^4*c^3*d^6*e^2*x^2*(b^2
 - 4*a*c)^(5/2) - 1864*b^5*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 80*b^6*c^3
*d^6*e^2*x^2*(b^2 - 4*a*c)^(3/2) - 40*b^7*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(3/
2))*(d*((b^2*c)/4 - a*c^2 + (b*c*(b^2 - 4*a*c)^(1/2))/4) - (b^3*e)/4 - (b^2
e(b^2 - 4*a*c)^(1/2))/4 + (a*c*e*(b^2 - 4*a*c)^(1/2))/2 + a*b*c*e))/(4*a^

3*c*e^2 - a^2*b^2*e^2 + 4*a^2*c^2*d^2 + a*b^3*d*e - a*b^2*c*d^2 - 4*a^2*b*c *d*e) - (log(80*c^4*d^8*(4*a*c - b^2)^4 - 256*a^4*e^8*(4*a*c - b^2)^4 + 61* d^4*e^4*(4*a*c - b^2)^6 + 160*b^3*c^4*d^8*(b^2 - 4*a*c)^(5/2) + 16*b^5*c^4* d^8*(b^2 - 4*a*c)^(3/2) - 184*b^3*d^4*e^4*(b^2 - 4*a*c)^(9/2) + 370*b^5*d^4 *e^4*(b^2 - 4*a*c)^(7/2) + 128*b^7*d^4*e^4*(b^2 - 4*a*c)^(5/2) + 5*b^9*d^4* e^4*(b^2 - 4*a*c)^(3/2) + 128*a^3*e^8*x^2*(b^2 - 4*a*c)^(9/2) + 160*c^5*d^8 *x^2*(b^2 - 4*a*c)^(7/2) + 256*a^4*b^2*e^8*(4*a*c - b^2)^3 - 32*b^2*c^4*d^8 *(4*a*c - b^2)^3 - 112*b^4*c^4*d^8*(4*a*c - b^2)^2 + 144*a^2*d^2*e^6*(4*a*c - b^2)^5 - 544*b^2*d^4*e^4*(4*a*c - b^2)^5 - 382*b^4*d^4*e^4*(4*a*c - b^2) ^4 + 152*b^6*d^4*e^4*(4*a*c - b^2)^3 - 71*b^8*d^4*e^4*(4*a*c - b^2)^2 - 200 *c^2*d^6*e^2*(4*a*c - b^2)^5 + 13*d^3*e^5*x^2*(4*a*c - b^2)^6 + 512*a^4*b*e ^8*(b^2 - 4*a*c)^(7/2) - 176*b*c^4*d^8*(b^2 - 4*a*c)^(7/2) - 26*a*d^3*e^5*(b^2 - 4*a*c)^(11/2) + 352*a^3*d*e^7*(b^2 - 4*a*c)^(9/2) - 319*b*d^4*e^4*(b^ 2 - 4*a*c)^(11/2) + 148*c*d^5*e^3*(b^2 - 4*a*c)^(11/2) + 168*c^3*d^7*e*(b^2 - 4*a*c)^(9/2) + 768*a*b^3*d^3*e^5*(4*a*c - b^2)^4 + 368*a*b^5*d^3*e^5*(4* a*c - b^2)^3 - 128*a^3*b^3*d*e^7*(4*a*c - b^2)^3 + 32*a*b^7*d^3*e^5*(4*a*c - b^2)^2 - 672*a^2*b^3*d^2*e^6*(b^2 - 4*a*c)^(7/2) - 272*a^2*b^5*d^2*e^6*(b ^2 - 4*a*c)^(5/2) - 408*b^3*c*d^5*e^3*(4*a*c - b^2)^4 - 256*b^3*c^3*d^7*e*(4*a*c - b^2)^3 - 792*b^5*c*d^5*e^3*(4*a*c - b^2)^3 + 352*b^5*c^3*d^7*e*(4*a *c - b^2)^2 + 248*b^7*c*d^5*e^3*(4*a*c - b^2)^2 - 328*b^3*c^2*d^6*e^2*(b^2 - 4*a*c)^(7/2) + 1064*b^5*c^2*d^6*e^2*(b^2 - 4*a*c)^(5/2) + 40*b^7*c^2*d^6* e^2*(b^2 - 4*a*c)^(3/2) - 384*a^3*b*e^8*x^2*(4*a*c - b^2)^4 + 384*a^3*b^2*e ^8*x^2*(b^2 - 4*a*c)^(7/2) + 512*b*c^5*d^8*x^2*(4*a*c - b^2)^3 + 576*b^2*c^ 5*d^8*x^2*(b^2 - 4*a*c)^(5/2) + 32*b^4*c^5*d^8*x^2*(b^2 - 4*a*c)^(3/2) + 17 6*a^2*d*e^7*x^2*(4*a*c - b^2)^5 - 800*b^3*d^3*e^5*x^2*(b^2 - 4*a*c)^(9/2) + 158*b^5*d^3*e^5*x^2*(b^2 - 4*a*c)^(7/2) + 56*b^7*d^3*e^5*x^2*(b^2 - 4*a*c) ^(5/2) - b^9*d^3*e^5*x^2*(b^2 - 4*a*c)^(3/2) + 336*c^4*d^7*e*x^2*(4*a*c - b ^2)^4 + 400*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(9/2) + 608*a^2*b^2*d^2*e^6*(4*a* c - b^2)^4 - 560*a^2*b^4*d^2*e^6*(4*a*c - b^2)^3 + 1096*b^2*c^2*d^6*e^2*(4* a*c - b^2)^4 + 872*b^4*c^2*d^6*e^2*(4*a*c - b^2)^3 - 424*b^6*c^2*d^6*e^2*(4 *a*c - b^2)^2 + 128*a^3*b^3*e^8*x^2*(4*a*c - b^2)^3 - 256*b^3*c^5*d^8*x^2*(4*a*c - b^2)^2 - 584*b^2*d^3*e^5*x^2*(4*a*c - b^2)^5 + 410*b^4*d^3*e^5*x^2* (4*a*c - b^2)^4 + 256*b^6*d^3*e^5*x^2*(4*a*c - b^2)^3 + 17*b^8*d^3*e^5*x^2* (4*a*c - b^2)^2 - 296*c^2*d^5*e^3*x^2*(4*a*c - b^2)^5 - 336*a*b*d^3*e^5*(4* a*c - b^2)^5 - 384*a^3*b*d*e^7*(4*a*c - b^2)^4 - 832*a*b^2*d^3*e^5*(b^2 - 4 *a*c)^(9/2) - 52*a*b^4*d^3*e^5*(b^2 - 4*a*c)^(7/2) + 144*a*b^6*d^3*e^5*(b^2 - 4*a*c)^(5/2) - 2*a*b^8*d^3*e^5*(b^2 - 4*a*c)^(3/2) - 80*a^2*b*d^2*e^6*(b ^2 - 4*a*c)^(9/2) - 192*a^3*b^2*d*e^7*(b^2 - 4*a*c)^(7/2) + 96*a^3*b^4*d*e^ 7*(b^2 - 4*a*c)^(5/2) + 632*b*c*d^5*e^3*(4*a*c - b^2)^5 - 608*b*c^3*d^7*e*(4*a*c - b^2)^4 - 776*b*c^2*d^6*e^2*(b^2 - 4*a*c)^(9/2) + 920*b^2*c*d^5*e^3* (b^2 - 4*a*c)^(9/2) + 584*b^2*c^3*d^7*e*(b^2 - 4*a*c)^(7/2) - 384*b^4*c*d^5 *e^3*(b^2 - 4*a*c)^(7/2) - 712*b^4*c^3*d^7*e*(b^2 - 4*a*c)^(5/2) - 664*b^6* c*d^5*e^3*(b^2 - 4*a*c)^(5/2) - 40*b^6*c^3*d^7*e*(b^2 - 4*a*c)^(3/2) - 20*b ^8*c*d^5*e^3*(b^2 - 4*a*c)^(3/2) + 72*a*d^2*e^6*x^2*(b^2 - 4*a*c)^(11/2) - 181*b*d^3*e^5*x^2*(b^2 - 4*a*c)^(11/2) + 122*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(1 1/2) + 368*a^2*b*d*e^7*x^2*(b^2 - 4*a*c)^(9/2) - 1552*b*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(7/2) + 3400*b^2*c^2*d^5*e^3*x^2*(4*a*c - b^2)^4 + 4800*b^3*c^3*d^ 6*e^2*x^2*(4*a*c - b^2)^3 - 3448*b^4*c^2*d^5*e^3*x^2*(4*a*c - b^2)^3 - 928* b^5*c^3*d^6*e^2*x^2*(4*a*c - b^2)^2 + 536*b^6*c^2*d^5*e^3*x^2*(4*a*c - b^2) ^2 + 32*a*b*d^2*e^6*x^2*(4*a*c - b^2)^5 - 344*a*b^2*d^2*e^6*x^2*(b^2 - 4*a* c)^(9/2) - 616*a*b^4*d^2*e^6*x^2*(b^2 - 4*a*c)^(7/2) - 136*a*b^6*d^2*e^6*x^ 2*(b^2 - 4*a*c)^(5/2) - 160*a^2*b^3*d*e^7*x^2*(b^2 - 4*a*c)^(7/2) + 48*a^2* b^5*d*e^7*x^2*(b^2 - 4*a*c)^(5/2) + 760*b*c*d^4*e^4*x^2*(4*a*c - b^2)^5 - 1 560*b*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(9/2) + 1848*b^2*c*d^4*e^4*x^2*(b^2 - 4 *a*c)^(9/2) - 2208*b^3*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(5/2) + 1452*b^4*c*d^4*e ^4*x^2*(b^2 - 4*a*c)^(7/2) - 80*b^5*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(3/2) + 408 *b^6*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 10*b^8*c*d^4*e^4*x^2*(b^2 - 4*a*c) ^(3/2) + 640*a*b^3*d^2*e^6*x^2*(4*a*c - b^2)^4 - 96*a^2*b^2*d*e^7*x^2*(4*a* c - b^2)^4 - 416*a*b^5*d^2*e^6*x^2*(4*a*c - b^2)^3 - 16*a^2*b^4*d*e^7*x^2*(

```
4*a*c - b^2)^3 - 1952*b*c^3*d^6*e^2*x^2*(4*a*c - b^2)^4 - 2216*b^3*c*d^4*e^
4*x^2*(4*a*c - b^2)^4 - 2720*b^2*c^4*d^7*e*x^2*(4*a*c - b^2)^3 + 712*b^5*c*
d^4*e^4*x^2*(4*a*c - b^2)^3 + 784*b^4*c^4*d^7*e*x^2*(4*a*c - b^2)^2 - 152*b
^7*c*d^4*e^4*x^2*(4*a*c - b^2)^2 + 4144*b^2*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(
7/2) - 4216*b^3*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 3056*b^4*c^3*d^6*e^2*
x^2*(b^2 - 4*a*c)^(5/2) - 1864*b^5*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 80
*b^6*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(3/2) - 40*b^7*c^2*d^5*e^3*x^2*(b^2 - 4*
a*c)^(3/2))*((b^3*e)/4 + d*(a*c^2 - (b^2*c)/4 + (b*c*(b^2 - 4*a*c)^(1/2))/4
) - (b^2*e*(b^2 - 4*a*c)^(1/2))/4 + (a*c*e*(b^2 - 4*a*c)^(1/2))/2 - a*b*c*e
))/(4*a^3*c*e^2 - a^2*b^2*e^2 + 4*a^2*c^2*d^2 + a*b^3*d*e - a*b^2*c*d^2 - 4
*a^2*b*c*d*e) - (e^2*log(d + e*x^2))/(2*c*d^3 + 2*a*d*e^2 - 2*b*d^2*e) + lo
g(x)/(a*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.301 $\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=205

$$\frac{\left(ace+b^2(-e)+bcd\right)\log\left(a+bx^2+cx^4\right)}{4a^2\left(ae^2-bde+cd^2\right)}-\frac{\left(3abce-2ac^2d+b^3(-e)+b^2cd\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{\log(x)(ae+bd)}{a^2d^2}+-$$

```
[Out] -1/2/a/d/x^2-(a*e+b*d)*ln(x)/a^2/d^2+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2-b*d*e+c
*d^2)+1/4*(a*c*e-b^2*e+b*c*d)*ln(c*x^4+b*x^2+a)/a^2/(a*e^2-b*d*e+c*d^2)-1/2
*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2)
)/a^2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 893, 634, 618, 206, 628}

$$-\frac{\left(3abce-2ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{\left(ace+b^2(-e)+bcd\right)\log\left(a+bx^2+cx^4\right)}{4a^2\left(ae^2-bde+cd^2\right)}-\frac{\log(x)(ae+bd)}{a^2d^2}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*a*d*x^2) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)
) - ((b*d + a*e)*Log[x])/(a^2*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 - b
*d*e + a*e^2)) + ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^2 + c*x^4])/(4*a^2*(c
*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\mathrm{Subst}\left(\int\left(\frac{1}{adx^2}+\frac{-bd-ae}{a^2d^2x}+\frac{e^4}{d^2\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{b^2cd-\cdots}{\cdots}\right.\right.\\
&= -\frac{1}{2adx^2}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\mathrm{Subst}\left(\int \frac{b^2cd-ac^2d-\cdots}{\cdots}\right.}{2a^2\cdots}\\
&= -\frac{1}{2adx^2}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\left(bcd-b^2e+ace\right)\mathrm{S}\cdots}{4a^2\left(c\cdots\right.}\\
&= -\frac{1}{2adx^2}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\left(bcd-b^2e+ace\right)\mathrm{l}\cdots}{4a^2\left(cd^2-\cdots\right.}\\
&= -\frac{1}{2adx^2}-\frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}-\frac{(bd+ae)\log\cdots}{a^2d^2}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 331, normalized size = 1.61

$$\frac{1}{4}\left(\frac{\left(b^2\left(e\sqrt{b^2-4ac}-cd\right)-bc\left(d\sqrt{b^2-4ac}+3ae\right)+ac\left(2cd-e\sqrt{b^2-4ac}\right)+b^3e\right)\log\left(-\sqrt{b^2-4ac}+b+2c\cdots\right.}{a^2\sqrt{b^2-4ac}\left(e(bd-ae)-cd^2\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-2/(a*d*x^2) - (4*(b*d + a*e)*Log[x])/(a^2*d^2) + ((b^3*e - b*c*(Sqrt[b^2
- 4*a*c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqr
t[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*
a*c]*(-(c*d^2) + e*(b*d - a*e))) + ((-(b^3*e) + b*c*(-(Sqrt[b^2 - 4*a*c]*d)
 + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*c*(2*c*d + Sqrt[b^2 - 4*a*c
]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*a*c]*(-(c*d^2
) + e*(b*d - a*e))) + (2*e^3*Log[d + e*x^2])/(c*d^4 + d^2*e*(-(b*d) + a*e))
)/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.96, size = 237, normalized size = 1.16

$$\frac{\left(bcd-b^2e+ace\right)\log\left(cx^4+bx^2+a\right)}{4\left(a^2cd^2-a^2bde+a^3e^2\right)}+\frac{e^4\log\left(\left|x^2e+d\right|\right)}{2\left(cd^4e-bd^3e^2+ad^2e^3\right)}+\frac{\left(b^2cd-2\,ac^2d-b^3e+3\,abce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2cd^2-a^2bde+a^3e^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*c*d - b^2*e + a*c*e)*log(c*x^4 + b*x^2 + a)/(a^2*c*d^2 - a^2*b*d*e +
 a^3*e^2) + 1/2*e^4*log(abs(x^2*e + d))/(c*d^4*e - b*d^3*e^2 + a*d^2*e^3) +
 1/2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b
^2 + 4*a*c))/((a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(
b*d + a*e)*log(x^2)/(a^2*d^2) + 1/2*(b*d*x^2 + a*x^2*e - a*d)/(a^2*d^2*x^2)
```

maple [B] time = 0.02, size = 430, normalized size = 2.10

$$\frac{3bce\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;a}-\frac{c^2d\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;a}-\frac{b^3e\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(a\,e^2-deb+c\,d^2\right)\sqrt{4ac-b^2}\;a^2}+\frac{b^2cd}{2\left(a\,e^2-d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a/d/x^2-1/a/d^2*e*ln(x)-1/d/a^2*ln(x)*b+1/4/(a*e^2-b*d*e+c*d^2)/a*c*ln
(c*x^4+b*x^2+a)*e-1/4/(a*e^2-b*d*e+c*d^2)/a^2*ln(c*x^4+b*x^2+a)*b^2*e+1/4/(
a*e^2-b*d*e+c*d^2)/a^2*c*ln(c*x^4+b*x^2+a)*b*d+3/2/(a*e^2-b*d*e+c*d^2)/a/(4
*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*e-1/(a*e^2-b*d*e+
c*d^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^2*d-1/2/
(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2))*b^3*e+1/2/(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*b^2*c*d+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2-b*d*e+c*d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 62.95, size = 5368, normalized size = 26.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log((((((4*c^2*e^2*(a*c^6*d^7 - 4*a^2*b^5*e^7 - 4*b^2*c^5*d^7 - 4*b^7*d^2*
e^5 + 28*a^3*b^3*c*e^7 - 48*a^4*b*c^2*e^7 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e
```

```
^4 - 16*a^2*c^5*d^5*e^2 + 16*a^3*c^4*d^3*e^4 - 4*b^4*c^3*d^5*e^2 - 4*b^5*c^
2*d^4*e^3 - 7*a*b^6*d*e^6 - 20*a*b*c^5*d^6*e + 56*a^2*b^2*c^3*d^3*e^4 - 76*
a^2*b^3*c^2*d^2*e^5 + 32*a*b^5*c*d^2*e^5 + 46*a^2*b^4*c*d*e^6 + 20*a*b^2*c^
4*d^5*e^2 + 6*a*b^3*c^3*d^4*e^3 - 44*a*b^4*c^2*d^3*e^4 + 22*a^2*b*c^4*d^4*e
^3 + 48*a^3*b*c^3*d^2*e^5 - 75*a^3*b^2*c^2*d*e^6))/(a^2*d^2) + (((16*c^2*e^
2*(a^3*b^4*e^7 + 16*a^5*c^2*e^7 + b^3*c^4*d^7 + b^7*d^3*e^4 - 8*a^4*b^2*c*e
^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 4*a^2*c^5*d^6*e - 4*b^4*c^3*d^6*e
- 4*b^6*c*d^4*e^3 + 20*a^3*c^4*d^4*e^3 - 32*a^4*c^3*d^2*e^5 + 6*b^5*c^2*d^5
*e^2 - a*b*c^5*d^7 - 52*a^2*b^2*c^3*d^4*e^3 + 45*a^2*b^3*c^2*d^3*e^4 + 48*a
^3*b^2*c^2*d^2*e^5 + 11*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 15*a^3*b^3*c
*d*e^6 + 28*a^4*b*c^2*d*e^6 - 27*a*b^3*c^3*d^5*e^2 + 27*a*b^4*c^2*d^4*e^3 +
 27*a^2*b*c^4*d^5*e^2 - 18*a^2*b^4*c*d^2*e^5 - 52*a^3*b*c^3*d^3*e^4))/(a*d)
 + (8*c^2*e^2*x^2*(10*a*c^6*d^7 + a^2*b^5*e^7 + b^2*c^5*d^7 + b^7*d^2*e^5 -
 11*a^3*b^3*c*e^7 + 28*a^4*b*c^2*e^7 - 88*a^4*c^3*d*e^6 - 6*b^3*c^4*d^6*e -
 6*b^6*c*d^3*e^4 + 26*a^2*c^5*d^5*e^2 + 88*a^3*c^4*d^3*e^4 + 5*b^4*c^3*d^5*
e^2 + 5*b^5*c^2*d^4*e^3 + 12*a*b^6*d*e^6 - 3*a*b*c^5*d^6*e - 110*a^2*b^2*c^
3*d^3*e^4 + 155*a^2*b^3*c^2*d^2*e^5 - 28*a*b^5*c*d^2*e^5 - 93*a^2*b^4*c*d*e
^6 - 10*a*b^2*c^4*d^5*e^2 - 27*a*b^3*c^3*d^4*e^3 + 46*a*b^4*c^2*d^3*e^4 + 1
5*a^2*b*c^4*d^4*e^3 - 236*a^3*b*c^3*d^2*e^5 + 202*a^3*b^2*c^2*d*e^6))/(a*d)
 + (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^
2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4
*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^
(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(a*b^3
*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12*a^3*c*e^
4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2 + a*b*c^2
*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 - 3*a^2*b*c
*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 +
22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^2*(4*a*c - b^2)*(a*e^2 + c
*d^2 - b*d*e)))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d
+ 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - b^2*c*d*(b^2
- 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*(a*e^
2 + c*d^2 - b*d*e)) - (4*c^2*e^2*x^2*(6*a*b^6*e^7 + 6*b*c^6*d^7 + 6*b^7*d*e
^6 - 16*a^4*c^3*e^7 - 44*a^2*b^4*c*e^7 - 8*b^2*c^5*d^6*e - 8*b^6*c*d^2*e^5
+ 84*a^3*b^2*c^2*e^7 + 30*a^2*c^5*d^4*e^3 - 2*b^3*c^4*d^5*e^2 + 8*b^4*c^3*d
^4*e^3 - 2*b^5*c^2*d^3*e^4 + 11*a*c^6*d^6*e - 47*a*b^5*c*d*e^6 - 96*a^2*b^2
*c^3*d^2*e^5 + 14*a*b*c^5*d^5*e^2 - 94*a^3*b*c^3*d*e^6 - 35*a*b^2*c^4*d^4*e
^3 + 7*a*b^3*c^3*d^3*e^4 + 56*a*b^4*c^2*d^2*e^5 - 17*a^2*b*c^4*d^3*e^4 + 11
7*a^2*b^3*c^2*d*e^6))/(a^2*d^2))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2
*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2
) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4
*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(b^7*e^7 + c^7*d^7 -
6*a^3*b*c^3*e^7 + 2*a*c^6*d^5*e^2 - 4*a^3*c^4*d*e^6 + 14*a^2*b^3*c^2*e^7 +
6*a^2*c^5*d^3*e^4 + b^3*c^4*d^4*e^3 + b^4*c^3*d^3*e^4 - 7*a*b^5*c*e^7 + 2*a
*b^4*c^2*d*e^6 - 6*a*b^2*c^4*d^3*e^4 + 3*a*b^3*c^3*d^2*e^5 - 9*a^2*b*c^4*d^
2*e^5 - 5*a^2*b^2*c^3*d*e^6))/(a^3*d^3) + (4*c^2*e^2*(a*e + b*d)*(b^3*e^3 +
 c^3*d^3 - 3*a*b*c*e^3)^2)/(a^3*d^3))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) +
4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)
^(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a
^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) - (2*c^5*e^5*x^2*(b^3*e^3 + c^3*d
^3 - 3*a*b*c*e^3))/(a^3*d^3))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^
2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) -
 b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a^4*c*
e^2 - a^3*b^2*e^2 + 4*a^3*c^2*d^2 - a^2*b^2*c*d^2 + a^2*b^3*d*e - 4*a^3*b*c
*d*e)) + (log((((((4*c^2*e^2*(a*c^6*d^7 - 4*a^2*b^5*e^7 - 4*b^2*c^5*d^7 - 4
*b^7*d^2*e^5 + 28*a^3*b^3*c*e^7 - 48*a^4*b*c^2*e^7 + 8*b^3*c^4*d^6*e + 8*b^
6*c*d^3*e^4 - 16*a^2*c^5*d^5*e^2 + 16*a^3*c^4*d^3*e^4 - 4*b^4*c^3*d^5*e^2 -
 4*b^5*c^2*d^4*e^3 - 7*a*b^6*d*e^6 - 20*a*b*c^5*d^6*e + 56*a^2*b^2*c^3*d^3*
e^4 - 76*a^2*b^3*c^2*d^2*e^5 + 32*a*b^5*c*d^2*e^5 + 46*a^2*b^4*c*d*e^6 + 20
*a*b^2*c^4*d^5*e^2 + 6*a*b^3*c^3*d^4*e^3 - 44*a*b^4*c^2*d^3*e^4 + 22*a^2*b*
```

```
c^4*d^4*e^3 + 48*a^3*b*c^3*d^2*e^5 - 75*a^3*b^2*c^2*d*e^6))/(a^2*d^2) + (((
16*c^2*e^2*(a^3*b^4*e^7 + 16*a^5*c^2*e^7 + b^3*c^4*d^7 + b^7*d^3*e^4 - 8*a^
4*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 4*a^2*c^5*d^6*e - 4*b^4*c
^3*d^6*e - 4*b^6*c*d^4*e^3 + 20*a^3*c^4*d^4*e^3 - 32*a^4*c^3*d^2*e^5 + 6*b^
5*c^2*d^5*e^2 - a*b*c^5*d^7 - 52*a^2*b^2*c^3*d^4*e^3 + 45*a^2*b^3*c^2*d^3*e
^4 + 48*a^3*b^2*c^2*d^2*e^5 + 11*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 15*
a^3*b^3*c*d*e^6 + 28*a^4*b*c^2*d*e^6 - 27*a*b^3*c^3*d^5*e^2 + 27*a*b^4*c^2*
d^4*e^3 + 27*a^2*b*c^4*d^5*e^2 - 18*a^2*b^4*c*d^2*e^5 - 52*a^3*b*c^3*d^3*e^
4))/(a*d) + (8*c^2*e^2*x^2*(10*a*c^6*d^7 + a^2*b^5*e^7 + b^2*c^5*d^7 + b^7*
d^2*e^5 - 11*a^3*b^3*c*e^7 + 28*a^4*b*c^2*e^7 - 88*a^4*c^3*d*e^6 - 6*b^3*c^
4*d^6*e - 6*b^6*c*d^3*e^4 + 26*a^2*c^5*d^5*e^2 + 88*a^3*c^4*d^3*e^4 + 5*b^4
*c^3*d^5*e^2 + 5*b^5*c^2*d^4*e^3 + 12*a*b^6*d*e^6 - 3*a*b*c^5*d^6*e - 110*a
^2*b^2*c^3*d^3*e^4 + 155*a^2*b^3*c^2*d^2*e^5 - 28*a*b^5*c*d^2*e^5 - 93*a^2*
b^4*c*d*e^6 - 10*a*b^2*c^4*d^5*e^2 - 27*a*b^3*c^3*d^4*e^3 + 46*a*b^4*c^2*d^
3*e^4 + 15*a^2*b*c^4*d^4*e^3 - 236*a^3*b*c^3*d^2*e^5 + 202*a^3*b^2*c^2*d*e^
6))/(a*d) + (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4
*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^4*e - b^3*e*(b^2 - 4*a*c)^
(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2
- 4*a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2
))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12
*a^3*c*e^4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2
+ a*b*c^2*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 -
3*a^2*b*c*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e
^2*x^2 + 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^2*(4*a*c - b^2)*(
a*e^2 + c*d^2 - b*d*e)))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e -
 b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) + b^2*
c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b
^2)*(a*e^2 + c*d^2 - b*d*e)) - (4*c^2*e^2*x^2*(6*a*b^6*e^7 + 6*b*c^6*d^7 +
6*b^7*d*e^6 - 16*a^4*c^3*e^7 - 44*a^2*b^4*c*e^7 - 8*b^2*c^5*d^6*e - 8*b^6*c
*d^2*e^5 + 84*a^3*b^2*c^2*e^7 + 30*a^2*c^5*d^4*e^3 - 2*b^3*c^4*d^5*e^2 + 8*
b^4*c^3*d^4*e^3 - 2*b^5*c^2*d^3*e^4 + 11*a*c^6*d^6*e - 47*a*b^5*c*d*e^6 - 9
6*a^2*b^2*c^3*d^2*e^5 + 14*a*b*c^5*d^5*e^2 - 94*a^3*b*c^3*d*e^6 - 35*a*b^2*
c^4*d^4*e^3 + 7*a*b^3*c^3*d^3*e^4 + 56*a*b^4*c^2*d^2*e^5 - 17*a^2*b*c^4*d^3
*e^4 + 117*a^2*b^3*c^2*d*e^6))/(a^2*d^2))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2
) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*
a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/
(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(b^7*e^7 + c
^7*d^7 - 6*a^3*b*c^3*e^7 + 2*a*c^6*d^5*e^2 - 4*a^3*c^4*d*e^6 + 14*a^2*b^3*c
^2*e^7 + 6*a^2*c^5*d^3*e^4 + b^3*c^4*d^4*e^3 + b^4*c^3*d^3*e^4 - 7*a*b^5*c*
e^7 + 2*a*b^4*c^2*d*e^6 - 6*a*b^2*c^4*d^3*e^4 + 3*a*b^3*c^3*d^2*e^5 - 9*a^2
*b*c^4*d^2*e^5 - 5*a^2*b^2*c^3*d*e^6))/(a^3*d^3) + (4*c^2*e^2*(a*e + b*d)*(
b^3*e^3 + c^3*d^3 - 3*a*b*c*e^3)^2)/(a^3*d^3))*(b^4*e - b^3*e*(b^2 - 4*a*c)
^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2
 - 4*a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/
2)))/(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) - (2*c^5*e^5*x^2*(b^3*e^
3 + c^3*d^3 - 3*a*b*c*e^3))/(a^3*d^3))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) +
 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*a*c
)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*
(4*a^4*c*e^2 - a^3*b^2*e^2 + 4*a^3*c^2*d^2 - a^2*b^2*c*d^2 + a^2*b^3*d*e -
4*a^3*b*c*d*e)) + (e^3*log(d + e*x^2))/(2*c*d^4 + 2*a*d^2*e^2 - 2*b*d^3*e)
- 1/(2*a*d*x^2) - (log(x)*(a*e + b*d))/(a^2*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.302 $\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=268

$$\frac{\log(x)\left(abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3d^3}-\frac{\left(2abce-ac^2d+b^3(-e)+b^2cd\right)\log\left(a+bx^2+cx^4\right)}{4a^3\left(ae^2-bde+cd^2\right)}+\frac{ae+bd}{2a^2d^2x^2}+\frac{\left(-2a^2c^2e\right.}{}$$

```
[Out] -1/4/a/d/x^4+1/2*(a*e+b*d)/a^2/d^2/x^2+(b^2*d^2+a*b*d*e-a*(-a*e^2+c*d^2))*l
n(x)/a^3/d^3-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2-b*d*e+c*d^2)-1/4*(2*a*b*c*e-a*c
^2*d-b^3*e+b^2*c*d)*ln(c*x^4+b*x^2+a)/a^3/(a*e^2-b*d*e+c*d^2)+1/2*(-2*a^2*c
^2*e+4*a*b^2*c*e-3*a*b*c^2*d-b^4*e+b^3*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2
)^(1/2))/a^3/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 893, 634, 618, 206, 628}

$$-\frac{\left(2abce-ac^2d+b^2cd+b^3(-e)\right)\log\left(a+bx^2+cx^4\right)}{4a^3\left(ae^2-bde+cd^2\right)}+\frac{\left(-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)\right)\tanh^{-1}\left(\right.}{2a^3\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*a*d*x^4) + (b*d + a*e)/(2*a^2*d^2*x^2) + ((b^3*c*d - 3*a*b*c^2*d - b^
4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*a^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d^2 + a*b*d*e - a
*(c*d^2 - a*e^2))*Log[x])/(a^3*d^3) - (e^4*Log[d + e*x^2])/(2*d^3*(c*d^2 -
b*d*e + a*e^2)) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Log[a + b*x^2 +
c*x^4])/(4*a^3*(c*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(d+ex)\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{1}{adx^3}+\frac{-bd-ae}{a^2d^2x^2}+\frac{b^2d^2+abde-a\left(cd^2-ae^2\right)}{a^3d^3x}-\frac{}{d^3\left(cd^2-bd\right.}\right.\right.$$

$$= -\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{\left(b^2d^2+abde-a\left(cd^2-ae^2\right)\right)\log(x)}{a^3d^3}-\frac{e^4\log\left(d+\right.}{2d^3\left(cd^2-bde\right.}$$

$$= -\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{\left(b^2d^2+abde-a\left(cd^2-ae^2\right)\right)\log(x)}{a^3d^3}-\frac{e^4\log\left(d+\right.}{2d^3\left(cd^2-bde\right.}$$

$$= -\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{\left(b^2d^2+abde-a\left(cd^2-ae^2\right)\right)\log(x)}{a^3d^3}-\frac{e^4\log\left(d+\right.}{2d^3\left(cd^2-bde\right.}$$

$$= -\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{\left(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e\right)\tanh^{-1}\left(\frac{b+2c}{\sqrt{b^2-}}\right.}{2a^3\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] time = 0.43, size = 426, normalized size = 1.59

$$\frac{1}{4}\left(\frac{4\log(x)\left(abde+a\left(ae^2-cd^2\right)+b^2d^2\right)}{a^3d^3}-\frac{\left(ac^2\left(d\sqrt{b^2-4ac}+2ae\right)-b^2c\left(d\sqrt{b^2-4ac}+4ae\right)+abc\left(3cd-2e\sqrt{}\right.\right.}{a^3\sqrt{b^2-4ac}\,\left(\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(1/(a*d*x^4)) + (2*(b*d + a*e))/(a^2*d^2*x^2) + (4*(b^2*d^2 + a*b*d*e + a
*(-(c*d^2) + a*e^2))*Log[x])/(a^3*d^3) - ((b^4*e + a*c^2*(Sqrt[b^2 - 4*a*c]
*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + a*b*c*(3*c*d - 2*Sqrt[b
^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2])/(a^3*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) - ((-(b^
4*e) + a*c^2*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d)
+ 4*a*e) + b^3*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*
a*c]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^3*Sqrt[b^2 - 4*a*c]*(-(c*
d^2) + e*(b*d - a*e))) - (2*e^4*Log[d + e*x^2])/(c*d^5 + d^3*e*(-(b*d) + a*
e)))/4
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 1.46, size = 332, normalized size = 1.24

$$-\frac{\left(b^2cd - ac^2d - b^3e + 2\,abce\right)\log\left(cx^4 + bx^2 + a\right)}{4\left(a^3cd^2 - a^3bde + a^4e^2\right)} - \frac{e^5\log\left(\left|x^2e + d\right|\right)}{2\left(cd^5e - bd^4e^2 + ad^3e^3\right)} - \frac{\left(b^3cd - 3\,abc^2d - b^4e + 4\,ab^2ce -\right.}{2\left(a^3cd^2 - a^3bde + a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*log(c*x^4 + b*x^2 + a)/(a^3*c*
d^2 - a^3*b*d*e + a^4*e^2) - 1/2*e^5*log(abs(x^2*e + d))/(c*d^5*e - b*d^4*e
^2 + a*d^3*e^3) - 1/2*(b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*
c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*c*d^2 - a^3*b*d*e + a
^4*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(b^2*d^2 - a*c*d^2 + a*b*d*e + a^2*e^2)*l
og(x^2)/(a^3*d^3) - 1/4*(3*b^2*d^2*x^4 - 3*a*c*d^2*x^4 + 3*a*b*d*x^4*e + 3*
a^2*x^4*e^2 - 2*a*b*d^2*x^2 - 2*a^2*d*x^2*e + a^2*d^2)/(a^3*d^3*x^4)
```

maple [**B**] time = 0.02, size = 584, normalized size = 2.18

$$\frac{c^2e\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(ae^2 - deb + cd^2\right)\sqrt{4ac-b^2}\;a} - \frac{2b^2ce\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(ae^2 - deb + cd^2\right)\sqrt{4ac-b^2}\;a^2} + \frac{3bc^2d\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(ae^2 - deb + cd^2\right)\sqrt{4ac-b^2}\;a^2} + \frac{b}{2\left(ae^2 -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4/a/d/x^4+1/2/a/d^2*e/x^2+1/2/d/a^2/x^2*b+1/a/d^3*e^2*ln(x)+1/d^2/a^2*ln
(x)*b*e-1/a^2*c/d*ln(x)+1/d/a^3*ln(x)*b^2-1/2/(a*e^2-b*d*e+c*d^2)/a^2*c*ln(
c*x^4+b*x^2+a)*b*e+1/4/(a*e^2-b*d*e+c*d^2)/a^2*c^2*ln(c*x^4+b*x^2+a)*d+1/4/
(a*e^2-b*d*e+c*d^2)/a^3*ln(c*x^4+b*x^2+a)*b^3*e-1/4/(a*e^2-b*d*e+c*d^2)/a^3
*c*ln(c*x^4+b*x^2+a)*b^2*d+1/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*e*c^2-2/(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)
^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*c*e+3/2/(a*e^2-b*d*e+c*d^2
)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*d*b*c^2+1/2/(
a*e^2-b*d*e+c*d^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2))*b^4*e-1/2/(a*e^2-b*d*e+c*d^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*b^3*c*d-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2-b*d*e+c*d^2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 144.76, size = 10300, normalized size = 38.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log((c^8*e^8*(a^2*e^2 + b^2*d^2 - a*c*d^2 + a*b*d*e))/(a^6*d^6) - (c^9*e^9
*x^2)/(a^5*d^5) - (((c^5*e^5*(4*a^3*b^3*e^6 + 4*b^3*c^3*d^6 + 4*b^6*d^3*e^3
 + 8*a*b^5*d^2*e^4 + 8*a^2*b^4*d*e^5 + 4*a^2*c^4*d^5*e + 16*a^4*c^2*d*e^5 -
 19*a^3*c^3*d^3*e^3 - 4*a*b*c^4*d^6 - 12*a^4*b*c*e^6 + 36*a^2*b^2*c^2*d^3*e
^3 - 24*a*b^4*c*d^3*e^3 - 32*a^3*b^2*c*d*e^5 - 36*a^2*b^3*c*d^2*e^4 + 28*a^
3*b*c^2*d^2*e^4))/(a^6*d^6) - (((4*a^4*b^6*c^2*e^12 - 24*a^5*b^4*c^3*e^12 +
 36*a^6*b^2*c^4*e^12 - 4*a^3*c^9*d^8*e^4 + 64*a^4*c^8*d^6*e^6 - 144*a^5*c^7
*d^4*e^8 + 96*a^6*c^6*d^2*e^10 + 4*b^4*c^8*d^10*e^2 + 8*b^7*c^5*d^7*e^5 + 4
*b^10*c^2*d^4*e^8 + 64*a^2*b^3*c^7*d^7*e^5 - 8*a^2*b^4*c^6*d^6*e^6 - 8*a^2*
b^5*c^5*d^5*e^7 + 172*a^2*b^6*c^4*d^4*e^8 - 112*a^2*b^7*c^3*d^3*e^9 + 16*a^
2*b^8*c^2*d^2*e^10 - 72*a^3*b^2*c^7*d^6*e^6 + 56*a^3*b^3*c^6*d^5*e^7 - 312*
a^3*b^4*c^5*d^4*e^8 + 348*a^3*b^5*c^4*d^3*e^9 - 132*a^3*b^6*c^3*d^2*e^10 +
324*a^4*b^2*c^6*d^4*e^8 - 428*a^4*b^3*c^5*d^3*e^9 + 344*a^4*b^4*c^4*d^2*e^1
0 - 300*a^5*b^2*c^5*d^2*e^10 - 96*a^6*b*c^5*d*e^11 - 4*a*b^2*c^9*d^10*e^2 -
 4*a*b^3*c^8*d^9*e^3 - 48*a*b^5*c^6*d^7*e^5 + 8*a*b^6*c^5*d^6*e^6 - 44*a*b^
8*c^3*d^4*e^8 + 12*a*b^9*c^2*d^3*e^9 + 8*a^2*b*c^9*d^9*e^3 - 24*a^3*b*c^8*d
^7*e^5 + 12*a^3*b^7*c^2*d*e^11 - 88*a^4*b*c^7*d^5*e^7 - 88*a^4*b^5*c^3*d*e^
11 + 228*a^5*b*c^6*d^3*e^9 + 188*a^5*b^3*c^4*d*e^11)/(a^6*d^6) + (x^2*(32*a
^6*c^6*d*e^11 - 24*a^6*b*c^5*e^12 + 4*a^3*b^7*c^2*e^12 - 28*a^4*b^5*c^3*e^1
2 + 56*a^5*b^3*c^4*e^12 + 2*a^3*c^9*d^7*e^5 + 104*a^4*c^8*d^5*e^7 - 156*a^5
*c^7*d^3*e^9 + 4*b^3*c^9*d^10*e^2 + 4*b^6*c^6*d^7*e^5 + 4*b^7*c^5*d^6*e^6 +
 4*b^10*c^2*d^3*e^9 + 8*a^2*b^2*c^8*d^7*e^5 + 40*a^2*b^3*c^7*d^6*e^6 - 12*a
^2*b^5*c^5*d^4*e^8 + 180*a^2*b^6*c^4*d^3*e^9 - 116*a^2*b^7*c^3*d^2*e^10 - 9
2*a^3*b^2*c^7*d^5*e^7 + 84*a^3*b^3*c^6*d^4*e^8 - 350*a^3*b^4*c^5*d^3*e^9 +
388*a^3*b^5*c^4*d^2*e^10 + 348*a^4*b^2*c^6*d^3*e^9 - 524*a^4*b^3*c^5*d^2*e^
10 - 4*a*b^2*c^9*d^9*e^3 - 20*a*b^4*c^7*d^7*e^5 - 20*a*b^5*c^6*d^6*e^6 + 4*
a*b^6*c^5*d^5*e^7 - 44*a*b^8*c^3*d^3*e^9 + 12*a*b^9*c^2*d^2*e^10 + 8*a^2*b*
c^9*d^8*e^4 + 12*a^2*b^8*c^2*d*e^11 - 36*a^3*b*c^8*d^6*e^6 - 100*a^3*b^6*c^
3*d*e^11 - 132*a^4*b*c^7*d^4*e^8 + 264*a^4*b^4*c^4*d*e^11 + 264*a^5*b*c^6*d
^2*e^10 - 224*a^5*b^2*c^5*d*e^11))/(a^6*d^6) + (((192*a^6*b*c^4*e^11 - 256*
a^6*c^5*d*e^10 + 16*a^4*b^5*c^2*e^11 - 112*a^5*b^3*c^3*e^11 + 60*a^3*c^8*d^
7*e^4 - 320*a^4*c^7*d^5*e^6 + 480*a^5*c^6*d^3*e^8 + 16*b^4*c^7*d^9*e^2 - 32
*b^5*c^6*d^8*e^3 + 16*b^6*c^5*d^7*e^4 + 16*b^7*c^4*d^6*e^5 - 32*b^8*c^3*d^5
*e^6 + 16*b^9*c^2*d^4*e^7 + 16*a^2*b^2*c^7*d^7*e^4 + 120*a^2*b^3*c^6*d^6*e^
5 - 816*a^2*b^4*c^5*d^5*e^6 + 880*a^2*b^5*c^4*d^4*e^7 - 424*a^2*b^6*c^3*d^3
*e^8 + 56*a^2*b^7*c^2*d^2*e^9 + 832*a^3*b^2*c^6*d^5*e^6 - 1424*a^3*b^3*c^5*
d^4*e^7 + 1340*a^3*b^4*c^4*d^3*e^8 - 464*a^3*b^5*c^3*d^2*e^9 - 1512*a^4*b^2
*c^5*d^3*e^8 + 1144*a^4*b^3*c^4*d^2*e^9 - 20*a*b^2*c^8*d^9*e^2 + 96*a*b^3*c
^7*d^8*e^3 - 64*a*b^4*c^6*d^7*e^4 - 88*a*b^5*c^5*d^6*e^5 + 288*a*b^6*c^4*d^
5*e^6 - 208*a*b^7*c^3*d^4*e^7 + 44*a*b^8*c^2*d^3*e^8 - 40*a^2*b*c^8*d^8*e^3
 - 88*a^3*b*c^7*d^6*e^5 + 44*a^3*b^6*c^2*d*e^10 + 704*a^4*b*c^6*d^4*e^7 - 3
28*a^4*b^4*c^3*d*e^10 - 736*a^5*b*c^5*d^2*e^9 + 684*a^5*b^2*c^4*d*e^10)/(a^
4*d^4) + (((256*a^6*c^4*e^10 + 16*a^4*b^4*c^2*e^10 - 128*a^5*b^2*c^3*e^10 -
 192*a^3*c^7*d^6*e^4 + 448*a^4*c^6*d^4*e^6 - 512*a^5*c^5*d^2*e^8 + 16*b^4*c
^6*d^8*e^2 - 64*b^5*c^5*d^7*e^3 + 96*b^6*c^4*d^6*e^4 - 64*b^7*c^3*d^5*e^5 +
 16*b^8*c^2*d^4*e^6 + 768*a^2*b^2*c^6*d^6*e^4 - 1200*a^2*b^3*c^5*d^5*e^5 +
896*a^2*b^4*c^4*d^4*e^6 - 320*a^2*b^5*c^3*d^3*e^7 + 32*a^2*b^6*c^2*d^2*e^8
- 1392*a^3*b^2*c^5*d^4*e^6 + 1024*a^3*b^3*c^4*d^3*e^7 - 288*a^3*b^4*c^3*d^2
*e^8 + 768*a^4*b^2*c^4*d^2*e^8 + 448*a^5*b*c^4*d*e^9 - 32*a*b^2*c^7*d^8*e^2
 + 240*a*b^3*c^6*d^7*e^3 - 528*a*b^4*c^5*d^6*e^4 + 496*a*b^5*c^4*d^5*e^5 -
208*a*b^6*c^3*d^4*e^6 + 32*a*b^7*c^2*d^3*e^7 - 176*a^2*b*c^7*d^7*e^3 + 848*
a^3*b*c^6*d^5*e^5 + 32*a^3*b^5*c^2*d*e^9 - 1024*a^4*b*c^5*d^3*e^7 - 240*a^4
```

*b^3*c^3*d*e^9)/(a^2*d^2) + (8*c^2*e^2*x^2*(a^3*b^5*e^8 + b^3*c^5*d^8 + b^8*d^3*e^5 - 11*a^4*b^3*c*e^8 + 28*a^5*b*c^2*e^8 + 8*a*b^7*d^2*e^6 + 8*a^2*b^6*d*e^7 - 30*a^2*c^6*d^7*e - 24*a^5*c^3*d*e^7 - 6*b^4*c^4*d^7*e - 6*b^7*c*d^4*e^4 - 18*a^3*c^5*d^5*e^3 + 180*a^4*c^4*d^3*e^5 + 5*b^5*c^3*d^6*e^2 + 5*b^6*c^2*d^5*e^3 + 5*a*b*c^6*d^8 + 13*a^2*b^2*c^4*d^5*e^3 - 82*a^2*b^3*c^3*d^4*e^4 + 110*a^2*b^4*c^2*d^3*e^5 - 277*a^3*b^2*c^3*d^3*e^5 + 328*a^3*b^3*c^2*d^2*e^6 + 15*a*b^2*c^5*d^7*e - 17*a*b^6*c*d^3*e^5 - 57*a^3*b^4*c*d*e^7 - 27*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 40*a*b^5*c^2*d^4*e^4 + 67*a^2*b*c^5*d^6*e^2 - 92*a^2*b^5*c*d^2*e^6 + 72*a^3*b*c^4*d^4*e^4 - 352*a^4*b*c^3*d^2*e^6 + 106*a^4*b^2*c^2*d*e^7))/(a^2*d^2) - (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12*a^3*c*e^4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2 + a*b*c^2*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 - 3*a^2*b*c*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 + 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(6*a^3*b^6*e^9 - 16*a^6*c^3*e^9 + 6*b^3*c^6*d^9 + 6*b^9*d^3*e^6 - 44*a^4*b^4*c*e^9 + 13*a*b^8*d^2*e^7 + 13*a^2*b^7*d*e^8 + 5*a^2*c^7*d^8*e - 8*b^4*c^5*d^8*e - 8*b^8*c*d^4*e^5 + 84*a^5*b^2*c^2*e^9 + 2*a^3*c^6*d^6*e^3 - 160*a^4*c^5*d^4*e^5 + 124*a^5*c^4*d^2*e^7 - 2*b^5*c^4*d^7*e^2 + 8*b^6*c^3*d^6*e^3 - 2*b^7*c^2*d^5*e^4 - 5*a*b*c^7*d^9 + 40*a^2*b^2*c^5*d^6*e^3 - 45*a^2*b^3*c^4*d^5*e^4 - 220*a^2*b^4*c^3*d^4*e^5 + 316*a^2*b^5*c^2*d^3*e^6 + 264*a^3*b^2*c^4*d^4*e^5 - 546*a^3*b^3*c^3*d^3*e^6 + 388*a^3*b^4*c^2*d^2*e^7 - 447*a^4*b^2*c^3*d^2*e^7 + 12*a*b^2*c^6*d^8*e - 74*a*b^7*c*d^3*e^6 - 111*a^3*b^5*c*d*e^8 - 210*a^5*b*c^3*d*e^8 + 18*a*b^3*c^5*d^7*e^2 - 43*a*b^4*c^4*d^6*e^3 + 19*a*b^5*c^3*d^5*e^4 + 72*a*b^6*c^2*d^4*e^5 - 20*a^2*b*c^6*d^7*e^2 - 123*a^2*b^6*c*d^2*e^7 + 31*a^3*b*c^5*d^5*e^4 + 328*a^4*b*c^4*d^3*e^6 + 290*a^4*b^3*c^2*d*e^8))/(a^4*d^4))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (2*c^5*e^5*x^2*(a^2*b^4*e^6 + 2*a^4*c^2*e^6 + b^2*c^4*d^6 + b^6*d^2*e^4 - 4*a^3*b^2*c*e^6 + 3*a^2*c^4*d^4*e^2 - 10*a^3*c^3*d^2*e^4 + 2*a*b^5*d*e^5 - a*b*c^4*d^5*e + 16*a^2*b^2*c^2*d^2*e^4 - 7*a*b^4*c*d^2*e^4 - 11*a^2*b^3*c*d*e^5 + 13*a^3*b*c^2*d*e^5))/(a^6*d^6))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a^5*c*e^2 - a^4*b^2*e^2 + 4*a^4*c^2*d^2 - a^3*b^2*c*d^2 + a^3*b^3*d*e - 4*a^4*b*c*d*e)) - (log((c^8*e^8*(a^2*e^2 + b^2*d^2 - a*c*d^2 + a*b*d*e))/(a^6*d^6) - (c^9*e^9*x^2)/(a^5*d^5) + (((((4*a^4*b^6*c^2*e^12 - 24*a^5*b^4*c^3*e^12 + 36*a^6*b^2*c^4*e^12 - 4*a^3*c^9*d^8*e^4 + 64*a^4*c^8*d^6*e^6 - 144*a^5*c^7*d^4*e^8 + 96*a^6*c^6*d^2*e^10 + 4*b^4*c^8*d^10*e^2 + 8*b^7*c^5*d^7*e^5 + 4*b^10*c^2*d^4*e^8 + 64*a^2*b^3*c^7*d^7*e^5 - 8*a^2*b^4*c^6* d^6*e^6 - 8*a^2*b^5*c^5*d^5*e^7 + 172*a^2*b^6*c^4*d^4*e^8 - 112*a^2*b^7*c^3*d^3*e^9 + 16*a^2*b^8*c^2*d^2*e^10 - 72*a^3*b^2*c^7*d^6*e^6 + 56*a^3*b^3*c^6*d^5*e^7 - 312*a^3*b^4*c^5*d^4*e^8 + 348*a^3*b^5*c^4*d^3*e^9 - 132*a^3*b^6*c^3*d^2*e^10 + 324*a^4*b^2*c^6*d^4*e^8 - 428*a^4*b^3*c^5*d^3*e^9 + 344*a^4*b^4*c^4*d^2*e^10 - 300*a^5*b^2*c^5*d^2*e^10 - 96*a^6*b*c^5*d*e^11 - 4*a*b^2*c^9*d^10*e^2 - 4*a*b^3*c^8*d^9*e^3 - 48*a*b^5*c^6*d^7*e^5 + 8*a*b^6*c^5*d^6*e^6 - 44*a*b^8*c^3*d^4*e^8 + 12*a*b^9*c^2*d^3*e^9 + 8*a^2*b*c^9*d^9*e^3 - 24*a^3*b*c^8*d^7*e^5 + 12*a^3*b^7*c^2*d*e^11 - 88*a^4*b*c^7*d^5*e^7 - 88*a^4*b^5*c^3*d*e^11 + 228*a^5*b*c^6*d^3*e^9 + 188*a^5*b^3*c^4*d*e^11)/(a^6*d^6) + (x^2*(32*a^6*c^6*d*e^11 - 24*a^6*b*c^5*e^12 + 4*a^3*b^7*c^2*e^12 - 28*a^4*b^5*c^3*e^12 + 56*a^5*b^3*c^4*e^12 + 2*a^3*c^9*d^7*e^5 + 104*a^4*c^8*d^5*e^7 - 156*a^5*c^7*d^3*e^9 + 4*b^3*c^9*d^10*e^2 + 4*b^6*c^6*d^7*e^5 + 4*b^7*c^5*d^6*e^6 + 4*b^10*c^2*d^3*e^9 + 8*a^2*b^2*c^8*d^7*e^5 + 40*a^2*b^3*c^7*d^6*e^6 - 12*a^2*b^5*c^5*d^4*e^8 + 180*a^2*b^6*c^4*d^3*e^9 - 116*a^2*b^7*c^3*d^2*e^10 - 92*a^3*b^2*c^7*d^5*e^7 + 84*a^3*b^3*c^6*d^4*e^8 - 350*a^3*b^4*c^5*d^3*e^9 + 388*a^3*b^5*c^4*d^2*e^10 + 348*a^4*b^2*c^6*d^3*e^9 - 524*a^4*b^3*c^5*d^2*e^10 - 4*a*b^2*c^9*d^9*e^3 - 20*a*b^4*c^7*d^7*e^5 - 20*a*b^5*c^6*d^6*e^6 + 4*a*b^6*c^5*d^5*e^7 - 44*a*b^8*c^3*d^3*e^9 + 12*a*b^9*c^2*d^2*e^10 + 8*a^2*b*c^9*d^8*e^4 + 12*a^2*b^8*c^2*d*e^11 - 36*a^3*b*c^8*d^6*e^6 - 100*a^3*b^6*c^3*d*e^11 - 132*a^4*b*c^7*d^4*e^8 + 264*a^4*b^4*c^4*d*e^11 + 264*a^5*b*c^6*d^2*e^10 - 224*a^5*b^2*c^5*d*e^11))/(a^6*d^6) - (((192*a^6*b*c^4*e^11 - 256*a^6*c^5*d*e^10 + 16*a^4*b^5*c^2*e^11 - 112*a^5*b^3*c^3*e^11 + 60*a^3*c^8*d^7*e^4 - 320*a^4*c^7*d^5*e^6 + 480*a^5*c^6*d^3*e^8 + 16*b^4*c^7*d^9*e^2 - 32*b^5*c^6*d^8*e^3 + 16*b^6*c^5*d^7*e^4 + 16*b^7*c^4*d^6*e^5 - 32*b^8*c^3*d^5*e^6 + 16*b^9*c^2*d^4*e^7 + 16*a^2*b^2*c^7*d^7*e^4 + 120*a^2*b^3*c^6*d^6*e^5 - 816*a^2*b^4*c^5*d^5*e^6 + 880*a^2*b^5*c^4*d^4*e^7 - 424*a^2*b^6*c^3*d^3*e^8 + 56*a^2*b^7*c^2*d^2*e^9 + 832*a^3*b^2*c^6*d^5*e^6 - 1424*a^3*b^3*c^5*d^4*e^7 + 1340*a^3*b^4*c^4*d^3*e^8 - 464*a^3*b^5*c^3*d^2*e^9 - 1512*a^4*b^2*c^5*d^3*e^8 + 1144*a^4*b^3*c^4*d^2*e^9 - 20*a*b^2*c^8*d^9*e^2 + 96*a*b^3*c^7*d^8*e^3 - 64*a*b^4*c^6*d^7*e^4 - 88*a*b^5*c^5*d^6*e^5 + 288*a*b^6*c^4*d^5*e^6 - 208*a*b^7*c^3*d^4*e^7 + 44*a*b^8*c^2*d^3*e^8 - 40*a^2*b*c^8*d^8*e^3 - 88*a^3*b*c^7*d^6*e^5 + 44*a^3*b^6*c^2*d*e^10 + 704*a^4*b*c^6*d^4*e^7 - 328*a^4*b^4*c^3*d*e^10 - 736*a^5*b*c^5*d^2*e^9 + 684*a^5*b^2*c^4*d*e^10)/(a^4*d^4) - (((256*a^6*c^4*e^10 + 16*a^4*b^4*c^2*e^10 - 128*a^5*b^2*c^3*e^10 - 192*a^3*c^7*d^6*e^4 + 448*a^4*c^6*d^4*e^6 - 512*a^5*c^5*d^2*e^8 + 16*b^4*c^6*d^8*e^2 - 64*b^5*c^5*d^7*e^3 + 96*b^6*c^4*d^6*e^4 - 64*b^7*c^3*d^5*e^5 + 16*b^8*c^2*d^4*e^6 + 768*a^2*b^2*c^6*d^6*e^4 - 1200*a^2*b^3*c^5*d^5*e^5 + 896*a^2*b^4*c^4*d^4*e^6 - 320*a^2*b^5*c^3*d^3*e^7 + 32*a^2*b^6*c^2*d^2*e^8 - 1392*a^3*b^2*c^5*d^4*e^6 + 1024*a^3*b^3*c^4*d^3*e^7 - 288*a^3*b^4*c^3*d^2*e^8 + 768*a^4*b^2*c^4*d^2*e^8 + 448*a^5*b*c^4*d*e^9 - 32*a*b^2*c^7*d^8*e^2 + 240*a*b^3*c^6*d^7*e^3 - 528*a*b^4*c^5*d^6*e^4 + 496*a*b^5*c^4*d^5*e^5 - 208*a*b^6*c^3*d^4*e^6 + 32*a*b^7*c^2*d^3*e^7 - 176*a^2*b*c^7*d^7*e^3 + 848*a^3*b*c^6*d^5*e^5 + 32*a^3*b^5*c^2*d*e^9 - 1024*a^4*b*c^5*d^3*e^7 - 240*a^4*b^3*c^3*d*e^9)/(a^2*d^2) + (8*c^2*e^2*x^2*(a^3*b^5*e^8 + b^3*c^5*d^8 + b^8*d^3*e^5 - 11*a^4*b^3*c*e^8 + 28*a^5*b*c^2*e^8 + 8*a*b^7*d^2*e^6 + 8*a^2*b^6*d*e^7 - 30*a^2*c^6*d^7*e - 24*a^5*c^3*d*e^7 - 6*b^4*c^4*d^7*e - 6*b^7*c*d^4*e^4 - 18*a^3*c^5*d^5*e^3 + 180*a^4*c^4*d^3*e^5 + 5*b^5*c^3*d^6*e^2 + 5*b^6*c^2*d^5*e^3 + 5*a*b*c^6*d^8 + 13*a^2*b^2*c^4*d^5*e^3 - 82*a^2*b^3*c^3*d^4*e^4 + 110*a^2*b^4*c^2*d^3*e^5 - 277*a^3*b^2*c^3*d^3*e^5 + 328*a^3*b^3*c^2*d^2*e^6 + 15*a*b^2*c^5*d^7*e - 17*a*b^6*c*d^3*e^5 - 57*a^3*b^4*c*d*e^7 - 27*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 40*a*b^5*c^2*d^4*e^4 + 67*a^2*b*c^5*d^6*e^2 - 92*a^2*b^5*c*d^2*e^6 + 72*a^3*b*c^4*d^4*e^4 - 352*a^4*b*c^3*d^2*e^6 + 106*a^4*b^2*c^2*d*e^7))/(a^2*d^2) + (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12*a^3*c*e^4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*

```
c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2 + a*b*c^2*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d
^3*e - 14*a^2*c^2*d^2*e^2*x^2 - 3*a^2*b*c*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b
^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 + 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*
d*e^3*x^2))/(a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5*e + b^4*e*(b^
2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a
*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2)
 + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a
^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(6*a^3*b^6*e^9 -
 16*a^6*c^3*e^9 + 6*b^3*c^6*d^9 + 6*b^9*d^3*e^6 - 44*a^4*b^4*c*e^9 + 13*a*b
^8*d^2*e^7 + 13*a^2*b^7*d*e^8 + 5*a^2*c^7*d^8*e - 8*b^4*c^5*d^8*e - 8*b^8*c
*d^4*e^5 + 84*a^5*b^2*c^2*e^9 + 2*a^3*c^6*d^6*e^3 - 160*a^4*c^5*d^4*e^5 + 1
24*a^5*c^4*d^2*e^7 - 2*b^5*c^4*d^7*e^2 + 8*b^6*c^3*d^6*e^3 - 2*b^7*c^2*d^5*
e^4 - 5*a*b*c^7*d^9 + 40*a^2*b^2*c^5*d^6*e^3 - 45*a^2*b^3*c^4*d^5*e^4 - 220
*a^2*b^4*c^3*d^4*e^5 + 316*a^2*b^5*c^2*d^3*e^6 + 264*a^3*b^2*c^4*d^4*e^5 -
546*a^3*b^3*c^3*d^3*e^6 + 388*a^3*b^4*c^2*d^2*e^7 - 447*a^4*b^2*c^3*d^2*e^7
 + 12*a*b^2*c^6*d^8*e - 74*a*b^7*c*d^3*e^6 - 111*a^3*b^5*c*d*e^8 - 210*a^5*
b*c^3*d*e^8 + 18*a*b^3*c^5*d^7*e^2 - 43*a*b^4*c^4*d^6*e^3 + 19*a*b^5*c^3*d^
5*e^4 + 72*a*b^6*c^2*d^4*e^5 - 20*a^2*b*c^6*d^7*e^2 - 123*a^2*b^6*c*d^2*e^7
 + 31*a^3*b*c^5*d^5*e^4 + 328*a^4*b*c^4*d^3*e^6 + 290*a^4*b^3*c^2*d*e^8))/(
a^4*d^4))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*
b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a
^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*
e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5
*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*
c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2
- 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c
)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (c^5*e^5*(4*a^3*b
^3*e^6 + 4*b^3*c^3*d^6 + 4*b^6*d^3*e^3 + 8*a*b^5*d^2*e^4 + 8*a^2*b^4*d*e^5
+ 4*a^2*c^4*d^5*e + 16*a^4*c^2*d*e^5 - 19*a^3*c^3*d^3*e^3 - 4*a*b*c^4*d^6 -
 12*a^4*b*c*e^6 + 36*a^2*b^2*c^2*d^3*e^3 - 24*a*b^4*c*d^3*e^3 - 32*a^3*b^2*
c*d*e^5 - 36*a^2*b^3*c*d^2*e^4 + 28*a^3*b*c^2*d^2*e^4))/(a^6*d^6) + (2*c^5*
e^5*x^2*(a^2*b^4*e^6 + 2*a^4*c^2*e^6 + b^2*c^4*d^6 + b^6*d^2*e^4 - 4*a^3*b^
2*c*e^6 + 3*a^2*c^4*d^4*e^2 - 10*a^3*c^3*d^2*e^4 + 2*a*b^5*d*e^5 - a*b*c^4*
d^5*e + 16*a^2*b^2*c^2*d^2*e^4 - 7*a*b^4*c*d^2*e^4 - 11*a^2*b^3*c*d*e^5 + 1
3*a^3*b*c^2*d*e^5))/(a^6*d^6))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c
^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d
+ 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*
c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 +
c*d^2 - b*d*e)))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d
 - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*
e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a
*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a^5*c*e^2 - a^4*b^2*e^2 + 4*a^4*c^2*d^
2 - a^3*b^2*c*d^2 + a^3*b^3*d*e - 4*a^4*b*c*d*e)) - (1/(4*a*d) - (x^2*(a*e
+ b*d))/(2*a^2*d^2))/x^4 - (e^4*log(d + e*x^2))/(2*(c*d^5 + a*d^3*e^2 - b*d
^4*e)) + (log(x)*(a^2*e^2 + b^2*d^2 - a*c*d^2 + a*b*d*e))/(a^3*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.303 $\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=387

$$-\frac{\left(-\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a\right.}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2}}}$$

```
[Out] -(b*e+c*d)*x/c^2/e^2+1/3*x^3/c/e+d^(7/2)*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)/
(a*e^2-b*d*e+c*d^2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*(b^3*d-2*a*b*c*d-a*b^2*e+a^2*c*e+(-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*
b^2*c*d-b^4*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(
-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)^(1/2))*(b^3*d-2*a*b*c*d-a*b^2*e+a^2*c*e+(3*a^2*b*c*e+2*a^2*c^2*d-a*b^3*e-
4*a*b^2*c*d+b^4*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 4.03, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$-\frac{\left(-\frac{3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d}{\sqrt{b^2-4ac}}+a\right.}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) - ((b^3*d - 2*a*b*c*d - a*b^2*e
+ a^2*c*e - (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - ((b^3
*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a
*b^3*e + 3*a^2*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 -
 b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^8}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{-cd-be}{c^2e^2}+\frac{x^2}{ce}+\frac{d^4}{e^2\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}+\frac{-a\left(b^2d-acd-abe\right)}{c^2\left(cd^2-bde\ldots\right.}\right.$$

$$= -\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}+\frac{\int \frac{-a\left(b^2d-acd-abe\right)+\left(-b^3d+2abcd+ab^2e-a^2ce\right)x^2}{a+bx^2+cx^4}\,dx}{c^2\left(cd^2-bde+ae^2\right)}+\frac{d^4\int \ldots}{e^2\left(cd^2-\ldots\right.}$$

$$= -\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2-bde+ae^2\right)}-\frac{\left(b^3d-2abcd-ab^2e+a^2ce-\ldots\right.}{2\ldots}$$

$$= -\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}-\frac{\left(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2b\ldots}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+\ldots\right.}$$

Mathematica [A] time = 0.61, size = 463, normalized size = 1.20

$$\frac{\left(a^2c\left(e\sqrt{b^2-4ac}-2cd\right)+ab^2\left(4cd-e\sqrt{b^2-4ac}\right)-abc\left(2d\sqrt{b^2-4ac}+3ae\right)+b^3\left(d\sqrt{b^2-4ac}+ae\right)+b^4(-\ldots\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) + ((-(b^4*d) + b^3*(Sqrt[b^2 - 4
*a*c]*d + a*e) - a*b*c*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(4*c*d - Sqr
t[b^2 - 4*a*c]*e) + a^2*c*(-2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^4*d + b^3*(Sq
rt[b^2 - 4*a*c]*d - a*e) + a*b*c*(-2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a^2*c*(
2*c*d + Sqrt[b^2 - 4*a*c]*e) - a*b^2*(4*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + (d^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 12.65, size = 12506, normalized size = 32.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e^2 - b*d*e^3 + a*e^4) +
1/8*((2*b^7*c^8 - 16*a*b^5*c^9 + 36*a^2*b^3*c^10 - 16*a^3*b*c^11 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 - 18*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^8 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^3*c^9 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 8*(b^2 - 4*a*c)*
a*b^3*c^9 - 4*(b^2 - 4*a*c)*a^2*b*c^10)*d^5 - (4*b^8*c^7 - 30*a*b^6*c^8 + 5
8*a^2*b^4*c^9 - 8*a^3*b^2*c^10 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^8*c^5 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^6*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^7*c^6 - 29*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^4*c^7 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^5*c^7 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^6*c^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^2*c^8 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^3*c^8 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^9 - 4*(b^2 - 4*a*c)*b^6*c^7 + 14*(b^2 - 4*a*c)*a*b^4*c^8 - 2*(b^2 - 4*a*c
)*a^2*b^2*c^9)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4
 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 - 2*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 + 2*a*b^6*c^5 + 24*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^3*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^6 - 18*a^2*
b^4*c^6 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^7 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^7 + 48*a^3*b^2*c^7 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*c^8 - 32*a^4*c^8 - 2*(b^2 - 4*a*c)*a*b^4*c^5 + 10*(b^2 - 4*a*c)*
a^2*b^2*c^6 - 8*(b^2 - 4*a*c)*a^3*c^7)*d^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2
*e^2) + (2*b^9*c^6 - 8*a*b^7*c^7 - 24*a^2*b^5*c^8 + 104*a^3*b^3*c^9 - 32*a^
4*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^
4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 12*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 52*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 24*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 12*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 2*(b^2 - 4*a*c)*b
^7*c^6 + 24*(b^2 - 4*a*c)*a^2*b^3*c^8 - 8*(b^2 - 4*a*c)*a^3*b*c^9)*d^3*e^2
+ 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 8*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^6*c^4 + 2*a*b^7*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 - 16*a^2*b^5*c^5 - 4*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 + 32*a^3*b^3*c^6 - 2*(b^2 - 4*a*c
)*a*b^5*c^4 + 8*(b^2 - 4*a*c)*a^2*b^3*c^5)*d^2*abs(-c^3*d^2 + b*c^2*d*e - a
*c^2*e^2)*e - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4
```

*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*
c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2
)^2*d - (6*a*b^8*c^6 - 42*a^2*b^6*c^7 + 68*a^3*b^4*c^8 + 16*a^4*b^2*c^9 - 3
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^4 + 21*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 + 6*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 - 34*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 - 18*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 9*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 6*(b^2 - 4*a*c
)*a*b^6*c^6 + 18*(b^2 - 4*a*c)*a^2*b^4*c^7 + 4*(b^2 - 4*a*c)*a^3*b^2*c^8)*d
^2*e^3 - 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 17*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^5*c^4 + 4*a^2*b^6*c^4 + 40*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*b^2*c^5 + 18*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b^3*c^5 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 - 34*a^3*b
^4*c^5 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^6 - 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*b^2*c^6 + 80*a^4*b^2*c^6 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^4*c^7 - 32*a^5*c^7 - 4*(b^2 - 4*a*c)*a^2*b^4*c^4 + 18*(b^2 - 4*a*c)
*a^3*b^2*c^5 - 8*(b^2 - 4*a*c)*a^4*c^6)*d*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*
e^2)*e^2 + (2*a*b^6*c^2 - 18*a^2*b^4*c^3 + 48*a^3*b^2*c^4 - 32*a^4*c^5 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 + 9*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 24*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 10*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 10*(b^2 -
 4*a*c)*a^2*b^2*c^3 - 8*(b^2 - 4*a*c)*a^3*c^4)*(c^3*d^2 - b*c^2*d*e + a*c^2
*e^2)^2*e + (6*a^2*b^7*c^6 - 44*a^3*b^5*c^7 + 84*a^4*b^3*c^8 - 16*a^5*b*c^9
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^4
+ 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5
+ 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 -
 42*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 -
 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 -
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 + 10*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 6*(b^2
- 4*a*c)*a^2*b^5*c^6 + 20*(b^2 - 4*a*c)*a^3*b^3*c^7 - 4*(b^2 - 4*a*c)*a^4*b
*c^8)*d*e^4 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^3 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 + 2*a^3*b^5*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 - 16*a^4*b^3*
c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 + 32*a^5*b*c^6 -

```
2*(b^2 - 4*a*c)*a^3*b^3*c^4 + 8*(b^2 - 4*a*c)*a^4*b*c^5)*abs(-c^3*d^2 + b*c
^2*d*e - a*c^2*e^2)*e^3 - (2*a^3*b^6*c^6 - 14*a^4*b^4*c^7 + 24*a^5*b^2*c^8
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 7
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 12*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 + 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 2*(b^2 -
 4*a*c)*a^3*b^4*c^6 + 6*(b^2 - 4*a*c)*a^4*b^2*c^7)*e^5)*arctan(2*sqrt(1/2)*
x/sqrt((b*c^3*d^2 - b^2*c^2*d*e + a*b*c^2*e^2 + sqrt((b*c^3*d^2 - b^2*c^2*d
*e + a*b*c^2*e^2)^2 - 4*(a*c^3*d^2 - a*b*c^2*d*e + a^2*c^2*e^2)*(c^4*d^2 -
b*c^3*d*e + a*c^3*e^2)))/(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)))/((a*b^4*c^7 -
8*a^2*b^2*c^8 - 2*a*b^3*c^8 + 16*a^3*c^9 + 8*a^2*b*c^9 + a*b^2*c^9 - 4*a^2*
c^10)*d^4*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c) - 2*(a*b^5*c^6 - 8*a
^2*b^3*c^7 - 2*a*b^4*c^7 + 16*a^3*b*c^8 + 8*a^2*b^2*c^8 + a*b^3*c^8 - 4*a^2
*b*c^9)*d^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)*e + (a*b^6*c^5 - 6
*a^2*b^4*c^6 - 2*a*b^5*c^6 + 4*a^2*b^3*c^7 + a*b^4*c^7 + 32*a^4*c^8 + 16*a^
3*b*c^8 - 2*a^2*b^2*c^8 - 8*a^3*c^9)*d^2*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e
^2)*abs(c)*e^2 - 2*(a^2*b^5*c^5 - 8*a^3*b^3*c^6 - 2*a^2*b^4*c^6 + 16*a^4*b*
c^7 + 8*a^3*b^2*c^7 + a^2*b^3*c^7 - 4*a^3*b*c^8)*d*abs(-c^3*d^2 + b*c^2*d*e
 - a*c^2*e^2)*abs(c)*e^3 + (a^3*b^4*c^5 - 8*a^4*b^2*c^6 - 2*a^3*b^3*c^6 + 1
6*a^5*c^7 + 8*a^4*b*c^7 + a^3*b^2*c^7 - 4*a^4*c^8)*abs(-c^3*d^2 + b*c^2*d*e
 - a*c^2*e^2)*abs(c)*e^4) - 1/8*((2*b^7*c^8 - 16*a*b^5*c^9 + 36*a^2*b^3*c^1
0 - 16*a^3*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^7*c^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c
^7 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
^8 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^8 + 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 + 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^9 - 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^10 - 2*(b^2 - 4*a*c
)*b^5*c^8 + 8*(b^2 - 4*a*c)*a*b^3*c^9 - 4*(b^2 - 4*a*c)*a^2*b*c^10)*d^5 - (
4*b^8*c^7 - 30*a*b^6*c^8 + 58*a^2*b^4*c^9 - 8*a^3*b^2*c^10 - 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 15*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 29*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 14*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 4*(b^2 - 4*a*c)*b^6*c^7 + 14*(b^2 - 4*a*c
)*a*b^4*c^8 - 2*(b^2 - 4*a*c)*a^2*b^2*c^9)*d^4*e + 2*(sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^4*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 - 2*a*b^6*c
^5 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 10*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c^6 + 18*a^2*b^4*c^6 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*c^7 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^7 - 5*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^7 - 48*a^3*b^2*c^7 + 4*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^8 + 32*a^4*c^8 + 2*(b^2 - 4*a*c)*a
*b^4*c^5 - 10*(b^2 - 4*a*c)*a^2*b^2*c^6 + 8*(b^2 - 4*a*c)*a^3*c^7)*d^3*abs(
-c^3*d^2 + b*c^2*d*e - a*c^2*e^2) + (2*b^9*c^6 - 8*a*b^7*c^7 - 24*a^2*b^5*c
^8 + 104*a^3*b^3*c^9 - 32*a^4*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
```

(b^2 - 4*a*c)*c)*a*b^7*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^8*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^5*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^7*c^6 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b^3*c^7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^4*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b*c^8 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^2*c^8 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c^8 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^9 - 2*(b^2 - 4*a*c)*b^7*c^6 + 24*(b^2 - 4*a*c)*a^2*b^3*c^8 - 8*(b^2
- 4*a*c)*a^3*b*c^9)*d^3*e^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^7*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 2*a*b^7*c^4 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 + 1
6*a^2*b^5*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 - 32*
a^3*b^3*c^6 + 2*(b^2 - 4*a*c)*a*b^5*c^4 - 8*(b^2 - 4*a*c)*a^2*b^3*c^5)*d^2*
abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*e - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^
2*b^3*c^4 - 64*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6
*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a
*c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*(c^3
*d^2 - b*c^2*d*e + a*c^2*e^2)^2*d - (6*a*b^8*c^6 - 42*a^2*b^6*c^7 + 68*a^3*
b^4*c^8 + 16*a^4*b^2*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^8*c^4 + 21*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^6*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^7*c^5 - 34*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^4*c^6 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^5*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^6*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^2*c^7 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^3*c^7 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^4*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^2*c^8 - 6*(b^2 - 4*a*c)*a*b^6*c^6 + 18*(b^2 - 4*a*c)*a^2*b^4*c^7 + 4*
(b^2 - 4*a*c)*a^3*b^2*c^8)*d^2*e^3 + 2*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^6*c^3 - 17*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4
 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 4*a^2*b^6*c^4 +
40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + 18*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^4*c^5 + 34*a^3*b^4*c^5 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*c^6 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 9*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 80*a^4*b^2*c^6 + 4*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^7 + 32*a^5*c^7 + 4*(b^2 - 4*a*c)*a^
2*b^4*c^4 - 18*(b^2 - 4*a*c)*a^3*b^2*c^5 + 8*(b^2 - 4*a*c)*a^4*c^6)*d*abs(-
c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*e^2 + (2*a*b^6*c^2 - 18*a^2*b^4*c^3 + 48*a
^3*b^2*c^4 - 32*a^4*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 +

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8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 5*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 2*(b^2 -
4*a*c)*a*b^4*c^2 + 10*(b^2 - 4*a*c)*a^2*b^2*c^3 - 8*(b^2 - 4*a*c)*a^3*c^4)*
(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)^2*e + (6*a^2*b^7*c^6 - 44*a^3*b^5*c^7 + 8
4*a^4*b^3*c^8 - 16*a^5*b*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^7*c^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^5*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^6*c^5 - 42*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^4*b^3*c^6 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^4*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^5*c^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b*c^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*b^2*c^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^3*c^7 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b*c^8 - 6*(b^2 - 4*a*c)*a^2*b^5*c^6 + 20*(b^2 - 4*a*c)*a^3*b^3
*c^7 - 4*(b^2 - 4*a*c)*a^4*b*c^8)*d*e^4 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c
^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 2*a^3*b^5*c^4
+ 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^5 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b^3*c^5 + 16*a^4*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^4*b*c^6 - 32*a^5*b*c^6 + 2*(b^2 - 4*a*c)*a^3*b^3*c^4 - 8*(b^2 - 4*a*c)*a
^4*b*c^5)*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*e^3 - (2*a^3*b^6*c^6 - 14*a
^4*b^4*c^7 + 24*a^5*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^6*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c^5 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^5*b^2*c^6 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^4*c^6 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*b^2*c^7 - 2*(b^2 - 4*a*c)*a^3*b^4*c^6 + 6*(b^2 - 4*a*c)*a^4*b^2*c
^7)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3*d^2 - b^2*c^2*d*e + a*b*c^2*e^2 -
 sqrt((b*c^3*d^2 - b^2*c^2*d*e + a*b*c^2*e^2)^2 - 4*(a*c^3*d^2 - a*b*c^2*d*
e + a^2*c^2*e^2)*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)))/(c^4*d^2 - b*c^3*d*e +
 a*c^3*e^2)))/((a*b^4*c^7 - 8*a^2*b^2*c^8 - 2*a*b^3*c^8 + 16*a^3*c^9 + 8*a^
2*b*c^9 + a*b^2*c^9 - 4*a^2*c^10)*d^4*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)
*abs(c) - 2*(a*b^5*c^6 - 8*a^2*b^3*c^7 - 2*a*b^4*c^7 + 16*a^3*b*c^8 + 8*a^2
*b^2*c^8 + a*b^3*c^8 - 4*a^2*b*c^9)*d^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^
2)*abs(c)*e + (a*b^6*c^5 - 6*a^2*b^4*c^6 - 2*a*b^5*c^6 + 4*a^2*b^3*c^7 + a*
b^4*c^7 + 32*a^4*c^8 + 16*a^3*b*c^8 - 2*a^2*b^2*c^8 - 8*a^3*c^9)*d^2*abs(-c
^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)*e^2 - 2*(a^2*b^5*c^5 - 8*a^3*b^3*c^6
 - 2*a^2*b^4*c^6 + 16*a^4*b*c^7 + 8*a^3*b^2*c^7 + a^2*b^3*c^7 - 4*a^3*b*c^8
)*d*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)*e^3 + (a^3*b^4*c^5 - 8*a^4
*b^2*c^6 - 2*a^3*b^3*c^6 + 16*a^5*c^7 + 8*a^4*b*c^7 + a^3*b^2*c^7 - 4*a^4*c
^8)*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)*e^4) + 1/3*(c^2*x^3*e^2 -
3*c^2*d*x*e - 3*b*c*x*e^2)*e^(-3)/c^3
```

maple [B] time = 0.04, size = 1449, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3/c/e*x^3-1/c^2/e*b*x-1/c*d/e^2*x+1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*a^2*e-1/2/(a*e^2-b*d*e+c*d^2)/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*e-1/
(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
```

```
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*d+1/2/(a*e^2-b*d*e+c*d^2)/c
^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)*b^3*d-3/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*a^2*b*e-1/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*a^2*d+1/2/(a*e^2-b*d*e+c*d^2)/c^2/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*a*b^3*e+2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*a*b^2*d-1/2/(a*e^2-b*d*e+c*d^2)/c^2/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*b^4*d-1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*e
+1/2/(a*e^2-b*d*e+c*d^2)/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*e+1/(a*e^2-b*d*e+c*d^
2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*a*b*d-1/2/(a*e^2-b*d*e+c*d^2)/c^2*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*b^3*d-3/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2
*b*e-1/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*d+1/2/
(a*e^2-b*d*e+c*d^2)/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^3*e+2/(a*
e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*d-1/2/(a*e^2
-b*d*e+c*d^2)/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*d+1/e^2*d^4/(a*
e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{d^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2e^2 - bde^3 + ae^4\right)\sqrt{de}} - \frac{-\int \frac{a^2be - \left(\left(b^3 - 2\,abc\right)d - \left(ab^2 - a^2c\right)e\right)x^2 - \left(ab^2 - a^2c\right)d}{cx^4 + bx^2 + a}\,dx}{c^3d^2 - bc^2de + ac^2e^2} + \frac{cex^3 - 3\,(cd + be)x}{3\,c^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] d^4*arctan(e*x/sqrt(d*e))/((c*d^2*e^2 - b*d*e^3 + a*e^4)*sqrt(d*e)) - integ
rate(-(a^2*b*e - ((b^3 - 2*a*b*c)*d - (a*b^2 - a^2*c)*e)*x^2 - (a*b^2 - a^2
*c)*d)/(c*x^4 + b*x^2 + a), x)/(c^3*d^2 - b*c^2*d*e + a*c^2*e^2) + 1/3*(c*e
*x^3 - 3*(c*d + b*e)*x)/(c^2*e^2)
```

mupad [B] time = 7.13, size = 41755, normalized size = 107.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9
- 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7
+ 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9
- 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^9*d^
2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a
^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3
*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c
```

- b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^

3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*1i - (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) + (2*x*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42

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*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2
) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(
-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e
+ 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3
*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^
5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^
6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e
 - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)
 - (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^
10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9
*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2
*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*
c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^
4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d
^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d
^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7
+ 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b
^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5
*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d
*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e +
42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1
/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e
*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*
e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b
^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*
b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 +
b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3
*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/
2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8
+ 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*
e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*
b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3
*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*
a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*
c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 -
 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a
^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c
*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b
*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*
e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^
4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*
a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a
^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d
^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^
4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5
*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16
*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*
a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^
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2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a
^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3
*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) +
 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*
a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)
^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*
a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5
*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*
c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6
*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*1i)/((((((192*a^3*c^8*d^
6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5
+ 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7
+ 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 -
 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2
*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c
^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^
2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*
b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^
5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2
*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4
 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4
+ 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d
*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a
^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256
*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^
2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*
b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^
3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b
*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^
8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2
*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e
^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2
*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c
^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^
2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*
b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^
5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2
*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4
 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4
+ 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d
*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a
^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*
d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b
^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*
d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4
*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5
*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^
5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^
2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^
9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a
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*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7
*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^
2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b
*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*
e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^
4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*
a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a
^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d
^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^
4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5
*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16
*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d
^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7
*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*
a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*
b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a
^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a
*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^
8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*
a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*
c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*
d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9
*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a
^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*
c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2)
 + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e -
3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^
3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 1
6*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c
^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^
3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 -
 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(a^8*e^8 + b^8*
d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d
^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2)
 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42
*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2
) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(
-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e
+ 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3
*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^
5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^
6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e
 - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)
 + (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 4
8*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 9
6*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 3
84*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) + (2*x*(-(b^9*d^2 +
 a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*
b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^
3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b
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^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20
*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3
*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1
/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4
*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^
4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7
*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*
b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*
a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^
4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96
*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^
8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2
*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*
d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*
e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11
 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 +
 a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*
b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^
3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b
^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20
*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3
*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1
/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4
*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^
4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7
*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*
b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(4*a^3*b^7*e^10 + 4*
b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a
*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^
4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 +
16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^1
0 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e
^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*
e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2
*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*
b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^
7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d
*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2
 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^
3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*
b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) +
20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a
^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^
(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a
^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*
e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c
^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*
a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b
^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b
^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 6
4*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5
*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b
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^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*(a^4*b^3*d^7 + a^7*d^4*e^3 + a^5*b^2*d^6*e + a^6*b*d^5*e^2 - 2*a^5*b*c*d^7 - a^6*c*d^6*e))/(c^3*e^3)))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*2i + atan(((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4

*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8
*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32
*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3
 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b
^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5
*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^
10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*
c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^
7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^1
0*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^
5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^1
0 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 -
 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e
^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-
(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*
c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^
4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2
*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8
*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32
*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3
 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b
^3*c^6*d*e^3)))^(1/2) + (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*
e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d
*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*
e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5
*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5
 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^
5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^
8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e
- 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6
*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d
^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^
8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3
*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*
(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^
3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*
c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b
^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 -
 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 +
32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e
^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2
*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e
^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4
*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*
c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*
c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b
^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5
*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 +
4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*
b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*
d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2

```
+ a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3
*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b
^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2
0*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^
3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^
4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e
^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^
7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a
*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(a^8*e^8 + b^8*d^8
+ 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))
/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 2
8*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2
*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) +
a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76
*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d
*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^
6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^
5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 3
2*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*1i
- (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48
*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96
*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 38
4*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) + (2*x*(-(b^9*d^2 +
a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b
^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3
*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 1
1*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*
a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*
b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*
c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4
 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*
d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b
^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a
^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4
*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*
b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8
*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*
c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d
^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e
^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11
- 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 +
a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b
^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3
*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 1
1*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*
a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*
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b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*
c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4
 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*
d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b
^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(4*a^3*b^7*e^10 + 4*b
^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*
b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4
*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 1
6*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10
 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^
7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e
^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*
e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b
^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7
*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*
e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2
+ a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3
*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b
^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2
0*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^
3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^
4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e
^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^
7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a
*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^
4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^
2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64
*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*
c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^
4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*
b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*
e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^
2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*
c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^
7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a
^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e +
76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c
*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*
c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*
c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e -
 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) -
 (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*
c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e
^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-
(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*
c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^
4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2
```

*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8
*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32
*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3
- 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b
^3*c^6*d*e^3)))^(1/2)*1i)/((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 +
192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48
*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48
*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*
e^3) - (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 2
8*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2
*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) +
a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76
*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d
e(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^
6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^
5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 3
2*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(12
8*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*
d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 +
64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*
d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^
6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7
*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4
*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7
+ 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))
/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 2
8*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2
*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) +
a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76
*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d
e(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^
6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^
5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 3
2*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (
2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 -
80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e -
56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^1
0 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*
d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^
6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e
^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e
^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48
*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c
^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4
*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9
))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) +
28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a
^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2)

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+ a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e +
76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c
*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*
c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*
c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e -
 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) -
 (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*
a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7
- 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*
c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3
*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*
b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^
6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*
a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b
*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2
*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3
*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*
(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^
3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*
c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b
^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 -
 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 +
32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e
^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2
*b^3*c^6*d*e^3)))^(1/2) + (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*
b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 +
a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b
^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3
*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 1
1*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*
a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*
b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*
c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4
 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*
d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b
^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (((((192*a^3*c^8*d^6*e^5 +
 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^
2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^
3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^
4*b*c^6*d^3*e^8)/(c^3*e^3) + (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a
*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2
- 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*
a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2
*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d
*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^
2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 6
6*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*
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c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a

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^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^
5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*
c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a
^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b
^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c
^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4
- 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d
^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^
4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(a^8*e^8 + b^8*d^8 + 2
*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c
^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a
^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^
5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3
*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c
 - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^
4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*
(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16
*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d
^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d
^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a
^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*(
a^4*b^3*d^7 + a^7*d^4*e^3 + a^5*b^2*d^6*e + a^6*b*d^5*e^2 - 2*a^5*b*c*d^7 -
 a^6*c*d^6*e))/(c^3*e^3)))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b
^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b
^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 -
a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*
a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b
^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8
*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d
^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*
b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e
^3)))^(1/2)*2i - (log(a^9*d^4*e^26 - b^9*d^13*e^17 + 2*a*b^8*d^12*e^18 - 2*
a^8*b*d^5*e^25 + 2*a^8*c*d^6*e^24 - a^2*b^7*d^11*e^19 + a^7*b^2*d^6*e^24 +
16*a^2*c^7*d^18*e^12 + 16*a^5*c^4*d^12*e^18 + a^7*c^2*d^8*e^22 + b^4*c^5*d^
18*e^12 + 16*a^2*c^7*x*(-d^7*e^5)^(5/2) + b^4*c^5*x*(-d^7*e^5)^(5/2) + a^9*
e^24*x*(-d^7*e^5)^(1/2) - 8*a*b^2*c^6*x*(-d^7*e^5)^(5/2) - 42*a^2*b^5*c^2*d
^13*e^17 + 63*a^3*b^3*c^3*d^13*e^17 + 66*a^3*b^4*c^2*d^12*e^18 - 76*a^4*b^2
*c^3*d^12*e^18 - 25*a^4*b^3*c^2*d^11*e^19 + a^2*b^7*e^12*x*(-d^7*e^5)^(3/2)
 + b^9*d^2*e^10*x*(-d^7*e^5)^(3/2) + 11*a*b^7*c*d^13*e^17 - 2*a^7*b*c*d^7*e
^23 - 8*a*b^2*c^6*d^18*e^12 - 20*a^2*b^6*c*d^12*e^18 + 9*a^3*b^5*c*d^11*e^1
9 - 28*a^4*b*c^4*d^13*e^17 + 20*a^5*b*c^3*d^11*e^19 + 25*a^4*b^3*c^2*e^12*x
*(-d^7*e^5)^(3/2) + a^7*b^2*d^2*e^22*x*(-d^7*e^5)^(1/2) + a^7*c^2*d^4*e^20*
x*(-d^7*e^5)^(1/2) - 2*a*b^8*d*e^11*x*(-d^7*e^5)^(3/2) - 2*a^8*b*d*e^23*x*(
-d^7*e^5)^(1/2) - 9*a^3*b^5*c*e^12*x*(-d^7*e^5)^(3/2) - 20*a^5*b*c^3*e^12*x
*(-d^7*e^5)^(3/2) - 16*a^5*c^4*d*e^11*x*(-d^7*e^5)^(3/2) + 2*a^8*c*d^2*e^22
*x*(-d^7*e^5)^(1/2) - 11*a*b^7*c*d^2*e^10*x*(-d^7*e^5)^(3/2) + 20*a^2*b^6*c
*d*e^11*x*(-d^7*e^5)^(3/2) - 2*a^7*b*c*d^3*e^21*x*(-d^7*e^5)^(1/2) - 66*a^3
*b^4*c^2*d*e^11*x*(-d^7*e^5)^(3/2) + 28*a^4*b*c^4*d^2*e^10*x*(-d^7*e^5)^(3/
2) + 76*a^4*b^2*c^3*d*e^11*x*(-d^7*e^5)^(3/2) + 42*a^2*b^5*c^2*d^2*e^10*x*(
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-d^7*e^5)^(3/2) - 63*a^3*b^3*c^3*d^2*e^10*x*(-d^7*e^5)^(3/2))*(-d^7*e^5)^(1
/2))/(2*(a*e^7 + c*d^2*e^5 - b*d*e^6)) + (log(a^9*d^4*e^26 - b^9*d^13*e^17
+ 2*a*b^8*d^12*e^18 - 2*a^8*b*d^5*e^25 + 2*a^8*c*d^6*e^24 - a^2*b^7*d^11*e^
19 + a^7*b^2*d^6*e^24 + 16*a^2*c^7*d^18*e^12 + 16*a^5*c^4*d^12*e^18 + a^7*c
^2*d^8*e^22 + b^4*c^5*d^18*e^12 - 16*a^2*c^7*x*(-d^7*e^5)^(5/2) - b^4*c^5*x
*(-d^7*e^5)^(5/2) - a^9*e^24*x*(-d^7*e^5)^(1/2) + 8*a*b^2*c^6*x*(-d^7*e^5)^
(5/2) - 42*a^2*b^5*c^2*d^13*e^17 + 63*a^3*b^3*c^3*d^13*e^17 + 66*a^3*b^4*c^
2*d^12*e^18 - 76*a^4*b^2*c^3*d^12*e^18 - 25*a^4*b^3*c^2*d^11*e^19 - a^2*b^7
*e^12*x*(-d^7*e^5)^(3/2) - b^9*d^2*e^10*x*(-d^7*e^5)^(3/2) + 11*a*b^7*c*d^1
3*e^17 - 2*a^7*b*c*d^7*e^23 - 8*a*b^2*c^6*d^18*e^12 - 20*a^2*b^6*c*d^12*e^1
8 + 9*a^3*b^5*c*d^11*e^19 - 28*a^4*b*c^4*d^13*e^17 + 20*a^5*b*c^3*d^11*e^19
 - 25*a^4*b^3*c^2*e^12*x*(-d^7*e^5)^(3/2) - a^7*b^2*d^2*e^22*x*(-d^7*e^5)^(
1/2) - a^7*c^2*d^4*e^20*x*(-d^7*e^5)^(1/2) + 2*a*b^8*d*e^11*x*(-d^7*e^5)^(3
/2) + 2*a^8*b*d*e^23*x*(-d^7*e^5)^(1/2) + 9*a^3*b^5*c*e^12*x*(-d^7*e^5)^(3/
2) + 20*a^5*b*c^3*e^12*x*(-d^7*e^5)^(3/2) + 16*a^5*c^4*d*e^11*x*(-d^7*e^5)^
(3/2) - 2*a^8*c*d^2*e^22*x*(-d^7*e^5)^(1/2) + 11*a*b^7*c*d^2*e^10*x*(-d^7*e
^5)^(3/2) - 20*a^2*b^6*c*d*e^11*x*(-d^7*e^5)^(3/2) + 2*a^7*b*c*d^3*e^21*x*(
-d^7*e^5)^(1/2) + 66*a^3*b^4*c^2*d*e^11*x*(-d^7*e^5)^(3/2) - 28*a^4*b*c^4*d
^2*e^10*x*(-d^7*e^5)^(3/2) - 76*a^4*b^2*c^3*d*e^11*x*(-d^7*e^5)^(3/2) - 42*
a^2*b^5*c^2*d^2*e^10*x*(-d^7*e^5)^(3/2) + 63*a^3*b^3*c^3*d^2*e^10*x*(-d^7*e
^5)^(3/2))*(-d^7*e^5)^(1/2))/(2*a*e^7 + 2*c*d^2*e^5 - 2*b*d*e^6) + x^3/(3*c
*e) - (x*(b*e + c*d))/(c^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.304 $\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=323

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\cdots\right.}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+c\cdots\right.}$$

```
[Out] x/c/e-d^(5/2)*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)/(a*e^2-b*d*e+c*d^2)+1/2*arc
tan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(-2*
a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(a*e^2-b*d*e+c
*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+
(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(2*a^2*c*e-a*b^2*e-3*a*b*c*d+
b^3*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.37, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1287, 205, 1166}

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\cdots\right.}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+c\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x/(c*e) + ((b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) +
 ((b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (d^(5/2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 - b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^6}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{ce} - \frac{d^3}{e\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{a(bd-ae)+\left(b^2d-acd-abe\right)x^2}{c\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)}\right) d$$

$$= \frac{x}{ce} + \frac{\int \frac{a(bd-ae)+\left(b^2d-acd-abe\right)x^2}{a+bx^2+cx^4}\,dx}{c\left(cd^2-bde+ae^2\right)} - \frac{d^3\int \frac{1}{d+ex^2}\,dx}{e\left(cd^2-bde+ae^2\right)}$$

$$= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-}}}{2c\left(cd^2-bde+ae^2\right)}$$

$$= \frac{x}{ce} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)} + \frac{\left(b^2d-acd\right.}{\sqrt{2}}$$

Mathematica [A] time = 0.52, size = 385, normalized size = 1.19

$$\frac{\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)+ab\left(e\sqrt{b^2-4ac}-3cd\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(e(bd-ae)-cd^2\right)} + \frac{\left(b^2\left(d\sqrt{\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x/(c*e) + ((b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c
]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^3*d + b^2*(Sqrt[b^2
- 4*a*c]*d - a*e) + a*c*(-(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) - a*b*(3*c*d + Sqr
t[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*
(-(b*d) + a*e))) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 -
b*d*e + a*e^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 13.87, size = 11030, normalized size = 34.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e - b*d*e^2 + a*e^3) - 1
/8*((2*b^6*c^6 - 14*a*b^4*c^7 + 24*a^2*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*
```

```
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^4*c^6 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 6*(b^2 - 4*a*c)*a*b^2*c^7)*d
^5 - (4*b^7*c^5 - 26*a*b^5*c^6 + 36*a^2*b^3*c^7 + 16*a^3*b*c^8 - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 13*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 - 18*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 10*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 4*(b^2 - 4*a*c)*b^5*c^5 + 10*(b^2 - 4*a
*c)*a*b^3*c^6 + 4*(b^2 - 4*a*c)*a^2*b*c^7)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*a*b^5*c
^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^5 - 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^6 + 32*a^3*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^4 + 8*(b^2 - 4*a*c)*a^
2*b*c^5)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) + (2*b^8*c^4 - 6*a*b^6*c^5 -
 28*a^2*b^4*c^6 + 80*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^8*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^7*c^3 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^6*c^4 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^2*c^5 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c
^5 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c
^6 - 2*(b^2 - 4*a*c)*b^6*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^5 + 20*(b^2 - 4*a*c)
*a^2*b^2*c^6)*d^3*e^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^
2 - 7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*a*b^6*c^3 + 8*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 14*a^2*b
^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^5 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^5 + 16*a^3*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*c^6 + 32*a^4*c^6 - 2*(b^2 - 4*a*c)*a*b^4*c^3 + 6*(b^2 - 4*a*c)*a^
2*b^2*c^4 + 8*(b^2 - 4*a*c)*a^3*c^5)*d^2*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*
e - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^
2 - 4*a*c)*a^2*c^4)*(c^2*d^2 - b*c*d*e + a*c*e^2)^2*d - (6*a*b^7*c^4 - 36*a
```

```
^2*b^5*c^5 + 40*a^3*b^3*c^6 + 32*a^4*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b*c^6 - 6*(b^2 - 4*a*c)*a*b^5*c^4 + 12*(b^2 - 4*a*c)*a
^2*b^3*c^5 + 8*(b^2 - 4*a*c)*a^3*b*c^6)*d^2*e^3 - 4*(sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 2*a^2*
b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 8*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^3*c^4 - 16*a^3*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^5 + 32*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*b^3*c^3 + 8*(b^2 -
4*a*c)*a^3*b*c^4)*d*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*e^2 + (2*a*b^5*c^2 -
16*a^2*b^3*c^3 + 32*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^
3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*(c^2*d^2 - b*c*d
*e + a*c*e^2)^2*e + (6*a^2*b^6*c^4 - 38*a^3*b^4*c^5 + 56*a^4*b^2*c^6 - 3*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 19*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 6*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 - 28*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 14*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 3*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 7*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 - 6*(b^2 -
 4*a*c)*a^2*b^4*c^4 + 14*(b^2 - 4*a*c)*a^3*b^2*c^5)*d*e^4 + 2*(sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^
3 + 2*a^3*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^4 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 16*a^4*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*c^5 + 32*a^5*c^5 - 2*(b^2 - 4*a*c)*a^3*b^2*c^3 + 8*(b^2
- 4*a*c)*a^4*c^4)*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*e^3 - (2*a^3*b^5*c^4 -
12*a^4*b^3*c^5 + 16*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^4*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^5*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^3*b^3*c^4 + 4*(b^2 - 4*a*c)*a^4*b*c^5)*e
^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c^2*d^2 - b^2*c*d*e + a*b*c*e^2 + sqrt((b*
c^2*d^2 - b^2*c*d*e + a*b*c*e^2)^2 - 4*(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2)*
(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)))/(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)))/((a
*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^3*c^6 + 16*a^3*c^7 + 8*a^2*b*c^7 + a*b^2*c
^7 - 4*a^2*c^8)*d^4*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c) - 2*(a*b^5*c^4
 - 8*a^2*b^3*c^5 - 2*a*b^4*c^5 + 16*a^3*b*c^6 + 8*a^2*b^2*c^6 + a*b^3*c^6 -
 4*a^2*b*c^7)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c)*e + (a*b^6*c^3 -
 6*a^2*b^4*c^4 - 2*a*b^5*c^4 + 4*a^2*b^3*c^5 + a*b^4*c^5 + 32*a^4*c^6 + 16*
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a^3*b*c^6 - 2*a^2*b^2*c^6 - 8*a^3*c^7)*d^2*abs(-c^2*d^2 + b*c*d*e - a*c*e^2
)*abs(c)*e^2 - 2*(a^2*b^5*c^3 - 8*a^3*b^3*c^4 - 2*a^2*b^4*c^4 + 16*a^4*b*c^
5 + 8*a^3*b^2*c^5 + a^2*b^3*c^5 - 4*a^3*b*c^6)*d*abs(-c^2*d^2 + b*c*d*e - a
*c*e^2)*abs(c)*e^3 + (a^3*b^4*c^3 - 8*a^4*b^2*c^4 - 2*a^3*b^3*c^4 + 16*a^5*
c^5 + 8*a^4*b*c^5 + a^3*b^2*c^5 - 4*a^4*c^6)*abs(-c^2*d^2 + b*c*d*e - a*c*e
^2)*abs(c)*e^4) + 1/8*((2*b^6*c^6 - 14*a*b^4*c^7 + 24*a^2*b^2*c^8 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 7*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 12*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 6*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^6 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 6*(b^2 -
4*a*c)*a*b^2*c^7)*d^5 - (4*b^7*c^5 - 26*a*b^5*c^6 + 36*a^2*b^3*c^7 + 16*a^3
*b*c^8 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^
3 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 - 18*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 10*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 5*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 4*(b^2 - 4*a*c)*b^5*
c^5 + 10*(b^2 - 4*a*c)*a*b^3*c^6 + 4*(b^2 - 4*a*c)*a^2*b*c^7)*d^4*e + 2*(sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^4*c^4 - 2*a*b^5*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^
5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 32*a^3*b*c^6 + 2*(b^2 - 4*a*c)*a*b^3*c^4 -
 8*(b^2 - 4*a*c)*a^2*b*c^5)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) + (2*b^8*
c^4 - 6*a*b^6*c^5 - 28*a^2*b^4*c^6 + 80*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^6*c^4 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^2*c^5 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^5 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^5
 + 20*(b^2 - 4*a*c)*a^2*b^2*c^6)*d^3*e^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^6*c^2 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^
3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 2*a*b^6*c^3 + 8*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 6*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c^4 + 14*a^2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 3*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 16*a^3*b^2*c^5 - 4*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^6 - 32*a^4*c^6 + 2*(b^2 - 4*a*c)*a*b^4*c^3 -
 6*(b^2 - 4*a*c)*a^2*b^2*c^4 - 8*(b^2 - 4*a*c)*a^3*c^5)*d^2*abs(-c^2*d^2 +
b*c*d*e - a*c*e^2)*e - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*
c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 -
```

4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*(c^2*d^2 - b*c*d*e + a*c*e^2)^2*d - (6*a*b^7*c^4 - 36*a^2*b^5*c^5 + 40*a^3*b^3*c^6 + 32*a^4*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 6*(b^2 - 4*a*c)*a*b^5*c^4 + 12*(b^2 - 4*a*c)*a^2*b^3*c^5 + 8*(b^2 - 4*a*c)*a^3*b*c^6)*d^2*e^3 + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*a^2*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + 16*a^3*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 32*a^4*b*c^5 + 2*(b^2 - 4*a*c)*a^2*b^3*c^3 - 8*(b^2 - 4*a*c)*a^3*b*c^4)*d*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*e^2 + (2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 32*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*(c^2*d^2 - b*c*d*e + a*c*e^2)^2*e + (6*a^2*b^6*c^4 - 38*a^3*b^4*c^5 + 56*a^4*b^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 - 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 - 6*(b^2 - 4*a*c)*a^2*b^4*c^4 + 14*(b^2 - 4*a*c)*a^3*b^2*c^5)*d*e^4 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 2*a^3*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 16*a^4*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^5 - 32*a^5*c^5 + 2*(b^2 - 4*a*c)*a^3*b^2*c^3 - 8*(b^2 - 4*a*c)*a^4*c^4)*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*e^3 - (2*a^3*b^5*c^4 - 12*a^4*b^3*c^5 + 16*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^3*b^3*c^4 + 4*(b^2 - 4*a*c)*a^4*b*c^5)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c^2*d^2 - b^2*c*d*e + a*b*c*e^2 - sqrt((b*c^2*d^2 - b^2*c*d*e + a*b*c*e^2)^2 - 4*(a*c^2*d^2 - a*b*

```
c*d*e + a^2*c*e^2)*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)))/(c^3*d^2 - b*c^2*d*e
 + a*c^2*e^2)))/((a*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^3*c^6 + 16*a^3*c^7 + 8*
a^2*b*c^7 + a*b^2*c^7 - 4*a^2*c^8)*d^4*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*ab
s(c) - 2*(a*b^5*c^4 - 8*a^2*b^3*c^5 - 2*a*b^4*c^5 + 16*a^3*b*c^6 + 8*a^2*b^
2*c^6 + a*b^3*c^6 - 4*a^2*b*c^7)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(
c)*e + (a*b^6*c^3 - 6*a^2*b^4*c^4 - 2*a*b^5*c^4 + 4*a^2*b^3*c^5 + a*b^4*c^5
 + 32*a^4*c^6 + 16*a^3*b*c^6 - 2*a^2*b^2*c^6 - 8*a^3*c^7)*d^2*abs(-c^2*d^2
+ b*c*d*e - a*c*e^2)*abs(c)*e^2 - 2*(a^2*b^5*c^3 - 8*a^3*b^3*c^4 - 2*a^2*b^
4*c^4 + 16*a^4*b*c^5 + 8*a^3*b^2*c^5 + a^2*b^3*c^5 - 4*a^3*b*c^6)*d*abs(-c^
2*d^2 + b*c*d*e - a*c*e^2)*abs(c)*e^3 + (a^3*b^4*c^3 - 8*a^4*b^2*c^4 - 2*a^
3*b^3*c^4 + 16*a^5*c^5 + 8*a^4*b*c^5 + a^3*b^2*c^5 - 4*a^4*c^6)*abs(-c^2*d^
2 + b*c*d*e - a*c*e^2)*abs(c)*e^4) + x*e^(-1)/c
```

maple [B] time = 0.04, size = 1098, normalized size = 3.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/c/e*x+1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*e+1/2/(a*e^2-b*
d*e+c*d^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*d-1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*b^2*d+1/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a^2*e-1/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*a*b^2*e-3/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
a*b*d+1/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^
3*d-1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*e-1/2/(a*e^2-b*d*e+c*d
^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*a*d+1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2
*d+1/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*e-1/2/(a
*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*e-3/2/(a*e^
2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*d+1/2/(a*e^2-b*d*e
+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d-1/e*d^3/(a*e^2-b*d*e+
c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2e - bde^2 + ae^3\right)\sqrt{de}} - \frac{-\int \frac{abd - a^2e - \left(abe - \left(b^2 - ac\right)d\right)x^2}{cx^4 + bx^2 + a}\,dx}{c^2d^2 - bcde + ace^2} + \frac{x}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -d^3*arctan(e*x/sqrt(d*e))/((c*d^2*e - b*d*e^2 + a*e^3)*sqrt(d*e)) - integr
ate(-(a*b*d - a^2*e - (a*b*e - (b^2 - a*c)*d)*x^2)/(c*x^4 + b*x^2 + a), x)/
(c^2*d^2 - b*c*d*e + a*c*e^2) + x/(c*e)
```

mupad [B] time = 6.45, size = 33892, normalized size = 104.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] x/(c*e) - atan((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^
3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d
^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6
*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 -
 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(b^7*d^2 + a^2
*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*
c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e
 + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*
d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*
d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*
a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*
a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6
*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*
b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*
e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^
2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9
- 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 4
8*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2
*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(
b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2
 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*
a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4
 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e
^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^
6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3))
)^(1/2) + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*
c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d
^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5
*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*
c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*
d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^
7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a
*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5
*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*
d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 1
6*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3
*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4
 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4
+ 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d
*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a
```

^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d
^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4
*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^
2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*
d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^
2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e
))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*
e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c
^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6
*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^
2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d
*e^3)))^(1/2) + (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6
 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^
5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^
4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a
^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 +
b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b
^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*
a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2
*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*1i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^
6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*
d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*
d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^
5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*
e) + (2*x*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*
a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4
*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^
2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*
a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*
a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3
 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^
3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*
c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8
 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4
*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d
^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*
e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 -
 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*
a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*
b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*
(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c
*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c
*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*
b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4
*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*
c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b

^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^
2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4
*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^
2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7
*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4
*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3
*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2
*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 2
8*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*
b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*
d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c
^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*
d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^
2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16
*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^
3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3
*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 -
6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16
*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a
^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5
 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*
b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4
*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 +
 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2
*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e -
4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4
 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^
3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e -
 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*
d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c
^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^
2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 +
 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a
^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c -
 b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c
^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e +
 a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2
 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c
^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*1i)/((((((64
*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5
*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4
*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5
*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 -
 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^
2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 -
 a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) +
 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2
*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*

c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4
 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*
d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b
^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a
^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*
e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^
5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d
^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5
*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4
 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 1
6*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^
3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 -
 b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12
*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*
b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 3
6*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*
d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^
2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 +
16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*
d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(4*a^3*b
^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^
8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7
 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e
^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^
4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^
3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^
5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a
*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c
^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d
^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*
a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a
*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*
c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3
*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*
b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 +
b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3
*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/
2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^
6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d
^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*
a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^
3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^
4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5
*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^
2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 2
5*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*
c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*
e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*
e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*
b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(a
^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e
))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c

```
^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*
e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c
^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6
*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^
2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d
*e^3)))^(1/2) + (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^
3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d
^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6
*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 -
 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) + (2*x*(-(b^7*d^2 + a^2
*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*
c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e
 + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*
d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*
d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*
a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*
a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6
*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*
b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*
e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^
2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9
- 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 4
8*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2
*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(
b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2
 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*
a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4
 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e
^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^
6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3))
)^(1/2) - (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*
c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d
^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5
*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*
c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*
d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^
7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a
*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5
*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*
d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 1
6*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3
*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4
 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4
```

+ 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d
*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a
^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d
^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4
*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^
2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*
d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^
2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e
))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*
e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c
^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6
*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^
2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d
*e^3)))^(1/2) - (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6
 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^
5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^
4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a
^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 +
b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b
^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*
a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2
*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*(a^3*b^2*d^5 - a^4*c*d^5 + a^5*d^
3*e^2 + a^4*b*d^4*e))/(c*e)))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^
3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^
2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e -
 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^
4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a
^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e
- 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4
*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*2i - atan((((((64*a^5*c^4*d*e^8 +
 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^
2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^
3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*
c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*
d*e^8)/(c*e) - (2*x*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^
2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*
e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^
5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4
*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*
c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 +
 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10
- 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^
6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 +
64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2

*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*
b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6
*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*
e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7
- 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 -
a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^
2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) -
9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16
*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*
e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5
*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 -
8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3
*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*
c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c
^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*
e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7
*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d
^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a
^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a
^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c
*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4
+ 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*
a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2
+ b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 +
12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^
2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c -
b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^
7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e +
a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2
+ 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^
5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^
3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d
^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c
^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*
e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2
*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2
*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2
*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^
2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*
c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a
^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*
e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c
^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*
a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(a^6*e^6 + b^6*d^6
- 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 +
a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b
^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*
d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*
(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c
^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c

^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*1i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) + (2*x*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^

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3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2
 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^
3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b
^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^
2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d
*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^
5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^
6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e
 - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)
 - (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*
d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) -
20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*
a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a
*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 -
 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 +
32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e
^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2
*b^3*c^4*d*e^3)))^(1/2)*1i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 1
28*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*
a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*
a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*
c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(
b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2
 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*
a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4
 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e
^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^
6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3))
)^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 25
6*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*
d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16
*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a
^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4
*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^
5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5
*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9
))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*
a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4
*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^
2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*
a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*
a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3
- 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^
3*c^4*d*e^3)))^(1/2) + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5
- 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 -
 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 -
8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*
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e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12
*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48
*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d
^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2
 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108
*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*
c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2
e^2(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*
b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*
b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4
*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4
+ b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3
*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e -
2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d
^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^
7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^
4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*
d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e
 + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3
*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*
d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2)
 - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 +
16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e -
3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^
4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4
 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*
d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*
a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^
2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2
*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^
2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*
c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a
^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*
e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c
^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*
a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (((((64*a^5*c^4*d*e^8 +
 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^
2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^
3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*
c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*
d*e^8)/(c*e) + (2*x*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^
2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*
e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^
5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4
*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*
c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 +
 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10
- 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^

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6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 +
64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2
*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*
b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6
*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*
e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7
 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 -
 a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^
2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16
*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*
e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5
*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 -
8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3
*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*
c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(4*a^3*b^5*e^8 + 4*b^3*c
^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*
e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7
*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d
^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a
^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a
^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c
*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4
 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*
a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2
 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 +
12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^
2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c -
b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^
7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e +
a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2
+ 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^
5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^
3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d
^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c
^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*
e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2
*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2
*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2
*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^
2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*
c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a
^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*
e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c
^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*
a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(a^6*e^6 + b^6*d^6
 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 +
a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b
^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*
d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*
(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
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*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c
^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c
^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e -
32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) +
(2*(a^3*b^2*d^5 - a^4*c*d^5 + a^5*d^3*e^2 + a^4*b*d^4*e))/(c*e)))*(-(b^7*d^
2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a
^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*
b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c
^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*
d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b
^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b
^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*
e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2
)*2i - (log(b^7*d^10*e^10 - a^7*d^3*e^17 - 2*a*b^6*d^9*e^11 + 2*a^6*b*d^4*e
^16 - 2*a^6*c*d^5*e^15 + a^2*b^5*d^8*e^12 - a^5*b^2*d^5*e^15 - 16*a^2*c^5*d
^13*e^7 + 16*a^4*c^3*d^9*e^11 - a^5*c^2*d^7*e^13 - b^4*c^3*d^13*e^7 + 16*a^
2*c^5*x*(-d^5*e^3)^(5/2) + b^4*c^3*x*(-d^5*e^3)^(5/2) + a^7*e^16*x*(-d^5*e^
3)^(1/2) - 8*a*b^2*c^4*x*(-d^5*e^3)^(5/2) + 25*a^2*b^3*c^2*d^10*e^10 - 36*a
^3*b^2*c^2*d^9*e^11 + a^2*b^5*e^8*x*(-d^5*e^3)^(3/2) + b^7*d^2*e^6*x*(-d^5*
e^3)^(3/2) - 9*a*b^5*c*d^10*e^10 + 2*a^5*b*c*d^6*e^14 + 8*a*b^2*c^4*d^13*e^
7 + 16*a^2*b^4*c*d^9*e^11 - 20*a^3*b*c^3*d^10*e^10 - 7*a^3*b^3*c*d^8*e^12 +
 12*a^4*b*c^2*d^8*e^12 + a^5*b^2*d^2*e^14*x*(-d^5*e^3)^(1/2) + a^5*c^2*d^4*
e^12*x*(-d^5*e^3)^(1/2) - 2*a*b^6*d*e^7*x*(-d^5*e^3)^(3/2) - 2*a^6*b*d*e^15
*x*(-d^5*e^3)^(1/2) - 7*a^3*b^3*c*e^8*x*(-d^5*e^3)^(3/2) + 12*a^4*b*c^2*e^8
*x*(-d^5*e^3)^(3/2) + 16*a^4*c^3*d*e^7*x*(-d^5*e^3)^(3/2) + 2*a^6*c*d^2*e^1
4*x*(-d^5*e^3)^(1/2) - 9*a*b^5*c*d^2*e^6*x*(-d^5*e^3)^(3/2) + 16*a^2*b^4*c*
d*e^7*x*(-d^5*e^3)^(3/2) - 2*a^5*b*c*d^3*e^13*x*(-d^5*e^3)^(1/2) - 20*a^3*b
*c^3*d^2*e^6*x*(-d^5*e^3)^(3/2) - 36*a^3*b^2*c^2*d*e^7*x*(-d^5*e^3)^(3/2) +
 25*a^2*b^3*c^2*d^2*e^6*x*(-d^5*e^3)^(3/2))*(-d^5*e^3)^(1/2))/(2*(a*e^5 + c
*d^2*e^3 - b*d*e^4)) + (log(a^7*d^3*e^17 - b^7*d^10*e^10 + 2*a*b^6*d^9*e^11
 - 2*a^6*b*d^4*e^16 + 2*a^6*c*d^5*e^15 - a^2*b^5*d^8*e^12 + a^5*b^2*d^5*e^1
5 + 16*a^2*c^5*d^13*e^7 - 16*a^4*c^3*d^9*e^11 + a^5*c^2*d^7*e^13 + b^4*c^3*
d^13*e^7 + 16*a^2*c^5*x*(-d^5*e^3)^(5/2) + b^4*c^3*x*(-d^5*e^3)^(5/2) + a^7
*e^16*x*(-d^5*e^3)^(1/2) - 8*a*b^2*c^4*x*(-d^5*e^3)^(5/2) - 25*a^2*b^3*c^2*
d^10*e^10 + 36*a^3*b^2*c^2*d^9*e^11 + a^2*b^5*e^8*x*(-d^5*e^3)^(3/2) + b^7*
d^2*e^6*x*(-d^5*e^3)^(3/2) + 9*a*b^5*c*d^10*e^10 - 2*a^5*b*c*d^6*e^14 - 8*a
*b^2*c^4*d^13*e^7 - 16*a^2*b^4*c*d^9*e^11 + 20*a^3*b*c^3*d^10*e^10 + 7*a^3*
b^3*c*d^8*e^12 - 12*a^4*b*c^2*d^8*e^12 + a^5*b^2*d^2*e^14*x*(-d^5*e^3)^(1/2
) + a^5*c^2*d^4*e^12*x*(-d^5*e^3)^(1/2) - 2*a*b^6*d*e^7*x*(-d^5*e^3)^(3/2)
- 2*a^6*b*d*e^15*x*(-d^5*e^3)^(1/2) - 7*a^3*b^3*c*e^8*x*(-d^5*e^3)^(3/2) +
12*a^4*b*c^2*e^8*x*(-d^5*e^3)^(3/2) + 16*a^4*c^3*d*e^7*x*(-d^5*e^3)^(3/2) +
 2*a^6*c*d^2*e^14*x*(-d^5*e^3)^(1/2) - 9*a*b^5*c*d^2*e^6*x*(-d^5*e^3)^(3/2)
 + 16*a^2*b^4*c*d*e^7*x*(-d^5*e^3)^(3/2) - 2*a^5*b*c*d^3*e^13*x*(-d^5*e^3)^
(1/2) - 20*a^3*b*c^3*d^2*e^6*x*(-d^5*e^3)^(3/2) - 36*a^3*b^2*c^2*d*e^7*x*(-
d^5*e^3)^(3/2) + 25*a^2*b^3*c^2*d^2*e^6*x*(-d^5*e^3)^(3/2))*(-d^5*e^3)^(1/2
))/(2*a*e^5 + 2*c*d^2*e^3 - 2*b*d*e^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.305 $\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=280

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\ldots}{\sqrt{d}\ldots}\right.}{\sqrt{e}\left(ae^2-bde+\ldots\right.}$$

```
[Out] d^(3/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)/e^(1/2)-1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^2*d
)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d
-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)
/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\ldots}{\sqrt{d}\ldots}\right.}{\sqrt{e}\left(ae^2-bde+\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - ((b*d - a*e + (b^2*d - 2*a*c*d - a
*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)
) + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{d^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{-ad-(bd-ae)x^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right)dx$$

$$= \frac{\int \frac{-ad+(-bd+ae)x^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{d^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\,(cd^2-bde+ae^2)} - \frac{(bd-}{}$$

$$= -\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}\,(c}$$

Mathematica [A] time = 0.33, size = 323, normalized size = 1.15

$$\frac{\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-ae^2+bde-cd^2)} + \frac{\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}-ab\right.}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*
e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c
]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))
 + ((b^2*d - 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*
e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c
]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))
 + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

fricas [B] time = 9.38, size = 15553, normalized size = 55.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*
d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a
*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*
a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*
e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^
2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*
d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b
^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 -
24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 +
2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4
*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d
^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*
e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-
2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x + sqrt(1/2)*((b^4 - 5*a*b
^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d
*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d
```

```
^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^
2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2
*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^
2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b
^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a
*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*
e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*
b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b
^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^
2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2
 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e
+ (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e
^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2
*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)
*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b
^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^
2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^
4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c
^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*
e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^
2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3
*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) - sqrt(1/2)*(c*d^2
 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2
*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c
 - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^
2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c
 + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2
)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 -
 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)
*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 -
4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*
a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a
^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*
b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a
^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e
^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3
- 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c^3 - 4*a
*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^
3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2
*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*s
qrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3
- a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8
- 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*
d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b
^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3
- 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^
2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8
)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^
2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2
 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^
3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^
4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 -
4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5
- 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^
6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 -
 a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 -
8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4
*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e +
 (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^
```

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3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt
(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 -
4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2
*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e
^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a
*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)
*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*
c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 +
8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3
*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^
4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3
)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c
- 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2
*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d
^2)*x + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c
)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c
^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*
d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c -
4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 -
a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(
3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c
^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 -
 a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^
4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^
5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2
 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 +
 (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 -
 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2
- 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a
^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c
)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*
c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 -
 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 -
42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*
b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4
*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2
*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2
- 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3
*c^2)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 -
3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*
c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b
^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e
^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e
- 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*
a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*
c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b
^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d
^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^
3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4
*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*
c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^
4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^
4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4
*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a
^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c -
 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2
*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b
^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2
*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c
```

^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + d*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)))/(c*d^2 - b*d*e + a*e^2), 1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 +

(a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b
^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2
*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c
^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*
c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^
4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 -
10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7
+ (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 -
4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c -
4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a
^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*
d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c^3 -
4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*
a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)
*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^
5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*
b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*
d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c
^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8
*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*
c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4
)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)
*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e +
((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2
*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c -
4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2
)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^
6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*
c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 +
(b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c
^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^
3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2
- 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3
*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*
d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*
sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^
3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8
*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^
2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 +
4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*
c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*
a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^
2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2
*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^
4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5
*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^
4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 +
(a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*
c)*d^2)*x + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2
*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b
^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c
^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*
c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^
3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e -
2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a
*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c
^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^
2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^
3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3

```
*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e
^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d
^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*
e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-
(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2
*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(
b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e
^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^
3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*
a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6
 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/(
(b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*
c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4
*a^3*c^2)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^
3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(
b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*
(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b
*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^
3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5
 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(
b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a
^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^
4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^
3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3
 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*
a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2
)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*
((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2
 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 -
 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4
*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4
 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2
*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)
*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b
^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^
2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^
4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c
^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*
e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d
^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*
b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a
^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e
^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2
*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d
^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^
3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 2
4*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2
*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*
a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^
4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e
^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + 2*d
*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d))/(c*d^2 - b*d*e + a*e^2)]
```

giac [B] time = 11.24, size = 8658, normalized size = 30.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2 - b*d*e + a*e^2) + 1/8*((
2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*d^5 - (4*b^6*c^
3 - 22*a*b^4*c^4 + 24*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^6*c + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^5*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^3 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^4*c^3 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^4 - 4*(b^2 - 4*a*c)*b^4*c^3 + 6*(b^2 - 4*a*c)*a*b^2*c^4)*d^4*e - 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^
4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^
2 - 4*a*c)*a^2*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + (2*b^7*c^2 - 4*a*b^5*c
^3 - 24*a^2*b^3*c^4 + 32*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^
2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3
- 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 -
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^
2 - 4*a*c)*b^5*c^2 - 4*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)
*d^3*e^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^2 - 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 16*a^2*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 32*a^3*b*c^4 + 2*(b^2 - 4*a*
c)*a*b^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e
- (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*(c*d^2 - b*d*e + a*
e^2)^2*d - (6*a*b^6*c^2 - 28*a^2*b^4*c^3 + 16*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 + 14*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 6*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*(b^2 - 4*a*c)*a*b^4*c^2 + 4*(b^2
- 4*a*c)*a^2*b^2*c^3)*d^2*e^3 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
```

a^2*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*a^2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*a^3*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 32*a^4*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 + (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*(c*d^2 - b*d*e + a*e^2)^2*e + (6*a^2*b^5*c^2 - 28*a^3*b^3*c^3 + 16*a^4*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 6*(b^2 - 4*a*c)*a^2*b^3*c^2 + 4*(b^2 - 4*a*c)*a^3*b*c^3)*d*e^4 - (2*a^3*b^4*c^2 - 8*a^4*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*(b^2 - 4*a*c)*a^3*b^2*c^2)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 + sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2*b*c^5)*d^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6*c - 6*a^2*b^4*c^2 - 2*a*b^5*c^2 + 4*a^2*b^3*c^3 + a*b^4*c^3 + 32*a^4*c^4 + 16*a^3*b*c^4 - 2*a^2*b^2*c^4 - 8*a^3*c^5)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5*c - 8*a^3*b^3*c^2 - 2*a^2*b^4*c^2 + 16*a^4*b*c^3 + 8*a^3*b^2*c^3 + a^2*b^3*c^3 - 4*a^3*b*c^4)*d*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4*c - 8*a^4*b^2*c^2 - 2*a^3*b^3*c^2 + 16*a^5*c^3 + 8*a^4*b*c^3 + a^3*b^2*c^3 - 4*a^4*c^4)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4) - 1/8*((2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*d^5 - (4*b^6*c^3 - 22*a*b^4*c^4 + 24*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*(b^2 - 4*a*c)*b^4*c^3 + 6*(b^2 - 4*a*c)*a*b^2*c^4)*d^4*e + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sq

rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*
c)*a^2*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + (2*b^7*c^2 - 4*a*b^5*c^3 - 24*
a^2*b^3*c^4 + 32*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 8*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*
c)*b^5*c^2 - 4*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*d^3*e^2
- 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^2 + 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 16*a^2*b^3*c^3 - 4*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 32*a^3*b*c^4 - 2*(b^2 - 4*a*c)*a*b^3
c^2 + 8(b^2 - 4*a*c)*a^2*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (2*b^5
*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(
b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*(c*d^2 - b*d*e + a*e^2)^2*d
- (6*a*b^6*c^2 - 28*a^2*b^4*c^3 + 16*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*(b^2 - 4*a*c)*a*b^4*c^2 + 4*(b^2 - 4*a*c)
*a^2*b^2*c^3)*d^2*e^3 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*
c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 2*a^2*b^4*c^2 + 16*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 16*a^3*
b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 32*a^4*c^4 -
2*(b^2 - 4*a*c)*a^2*b^2*c^2 + 8*(b^2 - 4*a*c)*a^3*c^3)*d*abs(c*d^2 - b*d*e
+ a*e^2)*e^2 + (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c
^3)*(c*d^2 - b*d*e + a*e^2)^2*e + (6*a^2*b^5*c^2 - 28*a^3*b^3*c^3 + 16*a^4*
b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5
+ 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c +
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 8*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 - 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 3*sqrt

```
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 6*(b^2 - 4
*a*c)*a^2*b^3*c^2 + 4*(b^2 - 4*a*c)*a^3*b*c^3)*d*e^4 - (2*a^3*b^4*c^2 - 8*a
^4*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*(
b^2 - 4*a*c)*a^3*b^2*c^2)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e
 + a*b*e^2 - sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e +
a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((
a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*
c^5 - 4*a^2*c^6)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c^2 - 8*a
^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2
*b*c^5)*d^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6*c - 6*a^2*b^4*c^2
- 2*a*b^5*c^2 + 4*a^2*b^3*c^3 + a*b^4*c^3 + 32*a^4*c^4 + 16*a^3*b*c^4 - 2*a
^2*b^2*c^4 - 8*a^3*c^5)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*
b^5*c - 8*a^3*b^3*c^2 - 2*a^2*b^4*c^2 + 16*a^4*b*c^3 + 8*a^3*b^2*c^3 + a^2*
b^3*c^3 - 4*a^3*b*c^4)*d*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4*c
 - 8*a^4*b^2*c^2 - 2*a^3*b^3*c^2 + 16*a^5*c^3 + 8*a^4*b*c^3 + a^3*b^2*c^3 -
 4*a^4*c^4)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4)
```

maple [B] time = 0.03, size = 764, normalized size = 2.73

$$\frac{\sqrt{2}\, abe \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2 - deb + c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} + \frac{\sqrt{2}\, abe \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2 - deb + c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e+1/2/(a*e^2-b*d*e+c*d^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b*d+1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*a*b*e+1/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*a*d-1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*b^2*d+1/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e-1/2/(a*e^2-b*d*e
+c*d^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*a*b*e+1/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*a*d-1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*b^2*d+d^2/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{d^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^2 - bde + ae^2\right)\sqrt{de}} + \frac{-\int \frac{(bd-ae)x^2+ad}{cx^4+bx^2+a}\,dx}{cd^2 - bde + ae^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] d^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + integrate(-
((b*d - a*e)*x^2 + a*d)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

mupad [B] time = 5.80, size = 25202, normalized size = 90.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2
 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a
^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*
a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3
*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e
^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4
*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*
b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2
*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 -
 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^
5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^
3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64
*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e
- 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3
*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4
- 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2
- 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*
d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*c^6*d^6
*e^2 - x*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12
*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 1
6*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d
^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d
*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b
^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^
3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5
 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5
*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d
^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^
4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e
^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6
+ 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 64
0*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4
*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4
*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*
d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^
5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2
+ a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*
b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c
^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2
*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*
```

e^3)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24*a^2*b^2*c^2*d^3*e^3 - 4*a*b^2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*d*e^5 - 4*a*b^3*c^2*d^4*e^2 + 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 - 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i + ((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) - (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(x*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*

a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) - 16*
a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3 - 24*a^2*b^2*c^2*d^3*e
^3 + 4*a*b^2*c^3*d^5*e + 4*a*b^4*c*d^3*e^3 + 4*a^3*b^2*c*d*e^5 + 4*a*b^3*c^
2*d^4*e^2 - 20*a^2*b*c^3*d^4*e^2 - 8*a^2*b^3*c*d^2*e^4 + 16*a^3*b*c^2*d^2*e
^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e)
)*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^
2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c
^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e +
b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 +
16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*
d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i)/(((-(b^5*d^2 + a^2*b^3*e^2 + a^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*
c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e
^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c
^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)
))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^
3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 -
8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d
^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64
*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*
c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a
^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*
d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3
*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 -
8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3
*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2
*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*c^6*d^6*e^2 - x*(-(b^5*d^2 + a^2*b^3*e^2 +
a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c
- b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^
2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2
*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b
*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^
3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 51
2*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*
d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 -
 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 1
92*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 64
0*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*
b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*
c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^
3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^
4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e
^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*
a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^
3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^
3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e
 + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^
4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*
c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^

5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24*a^2*b^2*c^2*d^3*e^3 - 4*a*b^2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*d*e^5 - 4*a*b^3*c^2*d^4*e^2 + 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 - 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) - ((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) - (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(x*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2

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*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*
c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 +
 16*a^2*b^3*c^2*d*e^3)))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^
3*c^3*d^3*e^3 - 24*a^2*b^2*c^2*d^3*e^3 + 4*a*b^2*c^3*d^5*e + 4*a*b^4*c*d^3*
e^3 + 4*a^3*b^2*c*d*e^5 + 4*a*b^3*c^2*d^4*e^2 - 20*a^2*b*c^3*d^4*e^2 - 8*a^
2*b^3*c*d^2*e^4 + 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e
+ 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^
2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*
b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4
+ a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*
a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e
 - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)
 + 2*a^3*c*d^2*e^2 + 2*a^2*b*c*d^3*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d
^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3
*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4
 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32
*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*
e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2
)*2i + atan((((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^
3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e
 + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^
4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*
c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^
5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 -
6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 -
32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 -
112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^
4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 -
 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3
*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e
^5 + 64*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b
^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 -
 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4
*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d
^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3
*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*
c^6*d^6*e^2 - x*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*
b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d
*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*
d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^
5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*
b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3
- 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9
- 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*
d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 -
192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^
2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3
*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^
6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d
^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e
^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 +
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64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*
a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b
^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2
*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b
^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4
 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^
3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e -
 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3
*c^2*d*e^3)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e
^3 + 24*a^2*b^2*c^2*d^3*e^3 - 4*a*b^2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3
*b^2*c*d*e^5 - 4*a*b^3*c^2*d^4*e^2 + 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2
*e^4 - 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d
^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4
*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 1
6*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(
16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c
*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2
*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b
*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i + ((-(b
^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*
e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4
 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*
d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b
^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^
2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 -
 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^
2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5
*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5
 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^
4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^
2*d*e^6) - (-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^
2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c
*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e +
12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 +
 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2
*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c
*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a
*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(x*(-(b^5*d^2 + a^2*b^3*e^
2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*
a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2
*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a
^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*
d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9
+ 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*
c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e
^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6
 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2
+ 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 67
2*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^
2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^

3*c^3*d*e^8) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^
6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^
6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 1
6*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*
b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d
^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3
*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4
 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32
*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*
e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2
) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3 - 24*a^2*b^2*c^
2*d^3*e^3 + 4*a*b^2*c^3*d^5*e + 4*a*b^4*c*d^3*e^3 + 4*a^3*b^2*c*d*e^5 + 4*a
*b^3*c^2*d^4*e^2 - 20*a^2*b*c^3*d^4*e^2 - 8*a^2*b^3*c*d^2*e^4 + 16*a^3*b*c^
2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2
*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12
*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 1
6*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d
^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d
*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b
^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i)/(((-(b^5*d^2 + a^2*b^3*e
^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8
*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^
2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*
a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2
*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6
 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3
*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^
2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e
^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64
*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) + (-(b^5*
d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e +
 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 +
b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2
*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*
c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 +
 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*c^6*d^6*e^2 - x*(-(b^5*d^2 + a^2*b^3
*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 -
 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*
b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 3
2*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c
^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e
^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b
^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^
4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*
e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e
^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 -
 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152

*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3
*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4
*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^
3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^
3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16
*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^
(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e -
7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*
c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 -
2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 -
2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d
*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) + 16*a^2*c^4*d^5
*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24*a^2*b^2*c^2*d^3*e^3 - 4*a*b^
2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*d*e^5 - 4*a*b^3*c^2*d^4*e^2 +
20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 - 16*a^3*b*c^2*d^2*e^4) + x*(2*
a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^
2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2
*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^
4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e
^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^
3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 1
6*a^2*b^3*c^2*d*e^3)))^(1/2) - ((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*
a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^
2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*
b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^
4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*
a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(
8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e +
8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*
e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2
*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5
*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 +
64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) - (-(b^5*d^2 + a^2*b^3*e^2 - a
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2
*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*
e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*
c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3
)))^(1/2)*(x*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3
*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e
+ 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4
+ 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c
^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5
*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6
*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 3
2*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4
*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192
*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c
^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^
2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d
^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*

```
e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8
- 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 64*a^2*c^6*d^6*e^2 + 128
*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3
*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*
c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*
e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*
(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*
c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3
*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^
6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16
*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^
2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5
+ 60*a^3*c^3*d^3*e^3 - 24*a^2*b^2*c^2*d^3*e^3 + 4*a*b^2*c^3*d^5*e + 4*a*b^4
*c*d^3*e^3 + 4*a^3*b^2*c*d*e^5 + 4*a*b^3*c^2*d^4*e^2 - 20*a^2*b*c^3*d^4*e^2
 - 8*a^2*b^3*c*d^2*e^4 + 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3
*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2
*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c
^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^
4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^
4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3))
)^(1/2) + 2*a^3*c*d^2*e^2 + 2*a^2*b*c*d^3*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*
c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e
^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c
^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)
))^(1/2)*2i - (log(a^5*d^2*e^8 - b^5*d^7*e^3 + 2*a*b^4*d^6*e^4 - 2*a^4*b*d^
3*e^7 + 2*a^4*c*d^4*e^6 + b^4*c*d^8*e^2 + 16*a^2*c^3*x*(-d^3*e)^(5/2) + a^5
*e^8*x*(-d^3*e)^(1/2) - a^2*b^3*d^5*e^5 + a^3*b^2*d^4*e^6 + 16*a^2*c^3*d^8*
e^2 + 17*a^3*c^2*d^6*e^4 + b^4*c*x*(-d^3*e)^(5/2) + a^2*b^3*e^4*x*(-d^3*e)^
(3/2) + b^5*d^2*e^2*x*(-d^3*e)^(3/2) + 7*a*b^3*c*d^7*e^3 + 2*a^3*b*c*d^5*e^
5 - 8*a*b^2*c^2*x*(-d^3*e)^(5/2) - 8*a*b^2*c^2*d^8*e^2 - 12*a^2*b*c^2*d^7*e
^3 - 12*a^2*b^2*c*d^6*e^4 - 2*a^3*b*c*e^4*x*(-d^3*e)^(3/2) - 2*a*b^4*d*e^3*
x*(-d^3*e)^(3/2) - 2*a^4*b*d*e^7*x*(-d^3*e)^(1/2) - 17*a^3*c^2*d*e^3*x*(-d^
3*e)^(3/2) + 2*a^4*c*d^2*e^6*x*(-d^3*e)^(1/2) + a^3*b^2*d^2*e^6*x*(-d^3*e)^
(1/2) + 12*a^2*b*c^2*d^2*e^2*x*(-d^3*e)^(3/2) - 7*a*b^3*c*d^2*e^2*x*(-d^3*e
)^(3/2) + 12*a^2*b^2*c*d*e^3*x*(-d^3*e)^(3/2))*(-d^3*e)^(1/2))/(2*(a*e^3 -
b*d*e^2 + c*d^2*e)) + (log(a^5*d^2*e^8 - b^5*d^7*e^3 + 2*a*b^4*d^6*e^4 - 2*
a^4*b*d^3*e^7 + 2*a^4*c*d^4*e^6 + b^4*c*d^8*e^2 - 16*a^2*c^3*x*(-d^3*e)^(5/
2) - a^5*e^8*x*(-d^3*e)^(1/2) - a^2*b^3*d^5*e^5 + a^3*b^2*d^4*e^6 + 16*a^2*
c^3*d^8*e^2 + 17*a^3*c^2*d^6*e^4 - b^4*c*x*(-d^3*e)^(5/2) - a^2*b^3*e^4*x*(
-d^3*e)^(3/2) - b^5*d^2*e^2*x*(-d^3*e)^(3/2) + 7*a*b^3*c*d^7*e^3 + 2*a^3*b*
c*d^5*e^5 + 8*a*b^2*c^2*x*(-d^3*e)^(5/2) - 8*a*b^2*c^2*d^8*e^2 - 12*a^2*b*c
^2*d^7*e^3 - 12*a^2*b^2*c*d^6*e^4 + 2*a^3*b*c*e^4*x*(-d^3*e)^(3/2) + 2*a*b^
4*d*e^3*x*(-d^3*e)^(3/2) + 2*a^4*b*d*e^7*x*(-d^3*e)^(1/2) + 17*a^3*c^2*d*e^
3*x*(-d^3*e)^(3/2) - 2*a^4*c*d^2*e^6*x*(-d^3*e)^(1/2) - a^3*b^2*d^2*e^6*x*(
-d^3*e)^(1/2) - 12*a^2*b*c^2*d^2*e^2*x*(-d^3*e)^(3/2) + 7*a*b^3*c*d^2*e^2*x
*(-d^3*e)^(3/2) - 12*a^2*b^2*c*d*e^3*x*(-d^3*e)^(3/2))*(-d^3*e)^(1/2))/(2*a
*e^3 - 2*b*d*e^2 + 2*c*d^2*e)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.306 $\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=251

$$\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

```
[Out] -arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/(a*e^2-b*d*e+c*d^2)+1/2*arctan(x
*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(d+(2*a*e-b*d)/(-4*a
*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2
*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(d+(-2*a*e+
b*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))
^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 -
b*d*e + a*e^2)) + (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(c*d^2 - b*d*e + a*e^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(-\frac{de}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{ae+cdx^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{ae+cdx^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} - \frac{(de)\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2-bde+ae^2} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2(cd^2-bde+ae^2)} + \frac{\left(c\left(d+\frac{bd-2a}{\sqrt{b^2-4}}\right.\right.}{2(c}$$

$$= \frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right.}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2}$$

Mathematica [A] time = 0.50, size = 277, normalized size = 1.10

$$-\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}+2ae-bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-ae^2+bde-cd^2)} - \frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(-ae^2+bde-cd^2)} - \frac{\sqrt{d}\sqrt{e}}{ae^2-}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))) - (Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c
]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2
)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 - b*d*e + a*e^2)
```

fricas [B] time = 3.76, size = 12269, normalized size = 48.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^
2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2
*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)
*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4
*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6
*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42
*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d
^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*
a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b
^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3
- 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x
 + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3
 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 -
4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 -
3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4
)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b
```

^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3
*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4
 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4
*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4
*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d
^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d
^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b
*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c -
 a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24
*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2
*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 +
(a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)
*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^
3 + (a^2*b^2 - 4*a^3*c)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b
*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*
c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*
d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(
(b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10
*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*
e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 -
a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2
)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^
2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a
^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*l
og(-2*(c^2*d^2 - a*c*e^2)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 -
 4*a^2*c)*e^3 - (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e
+ 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b
*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^
3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d
^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^
4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*
c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*
c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^
3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*
e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4
 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2
- 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^
5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^
2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*
b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^
3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^
3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*
d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 -
 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) + sqrt(1/2)*(c*d^
2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c
^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4
)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b
^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3
*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4
 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4
*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3
*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 +
(a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b^2*c
- 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 + (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(
b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 -
(b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c
^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*

```
e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^
2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3
)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*
b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a
^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8))
)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*
c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4
*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 +
a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^
4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b
*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4
*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c -
 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e
^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b
^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*
c)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e +
a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2
*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*
a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d
^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^
4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*
c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*
c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^
3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4
- 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(
a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*
e^2)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 + (2*(b
^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*
c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*
b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2
*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a
*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c
 - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 -
24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a
^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7
+ (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c
^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*
c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt(
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 -
 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b
^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^
2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*
(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*
e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*
b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c
)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) + sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)
*x - d)/(e*x^2 + d)))/(c*d^2 - b*d*e + a*e^2), 1/2*(sqrt(1/2)*(c*d^2 - b*d*
e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4
- 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(
a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d
^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e
+ 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 -
 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^
4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^
3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4
*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^
4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2
 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b^2*c - 4*a*c^
2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2
```

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- 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2
*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^
4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b
^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a
*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^
3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^
2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*
d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-
(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*
b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c
)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)
/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 -
10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^
5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5
- a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c
^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((
b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8
*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))
) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2
+ ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c
 - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e
^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(
b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e
^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a
^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3
*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^
4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3
*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 -
4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x -
 sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3 -
 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*
a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*
a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2
*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*
d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3
*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c
^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 -
 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b
^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a
*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2
*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4
 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c
^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a
*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a
^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b
^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a
^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d
^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3
+ (a^2*b^2 - 4*a^3*c)*e^4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c
*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^
2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*
e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b
^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a
*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^
3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^
2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*
d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*
c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log
```

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(-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4
*a^2*c)*e^3 + (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e +
4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c
^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*
b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8
 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)
*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c
- 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^
2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3
- 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e
+ a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 -
 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 -
4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)
*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*
c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^
4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*
b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*
b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^
4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2
*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) - sqrt(1/2)*(c*d^2
- b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3
)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2
 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2
*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*
d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3
*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c
^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 -
 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b
^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e
 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a
^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x - sqrt(1/2)*((b^2*c -
4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 + (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^
3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b
^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2
)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^
4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2
- 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*
d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^
5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4
*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*
sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c
- 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a
^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*
c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c
^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(
a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8
*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8
)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2
*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)
*e^4))) - 2*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(c*d^2 - b*d*e + a*e^2)]
```

giac [B] time = 8.45, size = 6921, normalized size = 27.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c*d^2 - b*d*e + a*e^2) - 1/8*((
```

2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^5 - 2*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^4*e + (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^3*e^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^2)^2*d - 4*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^3)*d^2*e^3 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 + 5*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*d*e^4 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2

*c^2 + 16*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 -
32*a^4*c^3 + 2*(b^2 - 4*a*c)*a^2*b^2*c - 8*(b^2 - 4*a*c)*a^3*c^2)*abs(c*d^
2 - b*d*e + a*e^2)*e^3 - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b*c^2)*e^5)*arcta
n(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 + sqrt((b*c*d^2 - b^2*d*e
+ a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^
2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*
c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 - b*d
*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^
3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2)
*abs(c)*e + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2 +
32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*abs(c*d^2 - b*d*
e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*b*c
^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*abs(c*d^2 - b*d*e + a*e^2
)*abs(c)*e^3 + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*
c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4) + 1/8
*((2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2
c^4 - 2(b^2 - 4*a*c)*b^2*c^4)*d^5 - 2*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*
c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^
2 - 4*a*c)*a*b*c^4)*d^4*e + (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*
a*c)*a*b^2*c^3)*d^3*e^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*
c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^
3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2
- 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^
2)*e - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3
- 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^2)^
2*d - 4*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr

```
t(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^3
)*d^2*e^3 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^
3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 + 5*(2*a^
2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*d*e^4 + 2*(sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*a^2*b^4*c + 1
6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^2 - 16*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^
3 + 32*a^4*c^3 - 2*(b^2 - 4*a*c)*a^2*b^2*c + 8*(b^2 - 4*a*c)*a^3*c^2)*abs(c
*d^2 - b*d*e + a*e^2)*e^3 - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b*c^2)*e^5)*ar
ctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 - sqrt((b*c*d^2 - b^2*
d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c
*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b
^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 -
b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b
*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*abs(c*d^2 - b*d*e + a*e
^2)*abs(c)*e + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2
 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*abs(c*d^2 - b
*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*
b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*abs(c*d^2 - b*d*e + a*
e^2)*abs(c)*e^3 + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4
*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4)
```

maple [B] time = 0.02, size = 478, normalized size = 1.90

$$-\frac{\sqrt{2}\, ace \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\left(a e^{2}-deb+c d^{2}\right)\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\sqrt{2}\, ace \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\left(a e^{2}-deb+c d^{2}\right)\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d-1/(a*e^2-b*d*e+c*d^2)*c/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*d-1/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
```

```
a*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-d*e
/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -d*e*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + integrate(
(c*d*x^2 + a*e)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

mupad [B] time = 4.96, size = 19401, normalized size = 77.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(b^4*d^3*e^5 - a*b^3*d^2*e^6 + a*c^3*d^5*e^3 - b^3*c*d^4*e^4 + 2*a^2*c^
2*d^3*e^5 + a^3*c*d*e^7 + b^4*e^3*x*(-d*e)^(5/2) + a*b^3*e^5*x*(-d*e)^(3/2)
 + a^3*c*e^7*x*(-d*e)^(1/2) + 2*a*b*c^2*d^4*e^4 - 3*a*b^2*c*d^3*e^5 + 2*a^2
*b*c*d^2*e^6 + 2*a^2*c^2*e^3*x*(-d*e)^(5/2) - a*c^3*d*x*(-d*e)^(7/2) + b^3*
c*e*x*(-d*e)^(7/2) - 2*a*b*c^2*e*x*(-d*e)^(7/2) - 3*a*b^2*c*e^3*x*(-d*e)^(5
/2) - 2*a^2*b*c*e^5*x*(-d*e)^(3/2))*(-d*e)^(1/2))/(2*a*e^2 + 2*c*d^2 - 2*b*
d*e) - atan(((x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(
a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d
*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d
^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d
*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^
3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^
3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*
c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8
*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c
^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^
2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^
4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^
2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2
*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a
^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) +
b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2
+ 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4
*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 +
32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6
*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d
*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 +
 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c
^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^
5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6
- 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 +
 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672
*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2
*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3
*c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5
 - 96*a^2*b^2*c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 -
 96*a*b^3*c^4*d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384*
```

```
a^3*b*c^4*d^2*e^6 + 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c -
b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2
- 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*
c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*
a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b
^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3
- 32*a^3*b*c^2*d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a*
b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^
4))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*
b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4
+ b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*
a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a
^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*1i + (x*(
2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 - a*e^2
*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e
^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*
c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^
3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*
b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*
d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b
^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5
- 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a
^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d
^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^
2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^
4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*
c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*
c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))
^(1/2)*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e
- 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^
2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^
2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2
- 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(25
6*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*
e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*
b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^
3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3
*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*
e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^
5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 +
32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 192
*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5 + 96*a^2*b^2*c^4
*d^3*e^5 + 96*a^2*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^3 + 96*a*b^3*c^4*d^4
*e^4 - 48*a*b^4*c^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 384*a^3*b*c^4*d^2*e^6
 - 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b
^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 +
 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*
c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 3
2*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*
a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*
e^3)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8*a*b*c^4*d^3*e^3 + 4
*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2
- a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^
2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8
*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b
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^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e +
16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*1i)/((x*(2*a^2*c^3*e^5 - 4
*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^
3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a
^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d
^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b
^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^
2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*
a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3
*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 +
8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^
3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6)
 + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b
^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 +
 b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a
*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^
2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^
3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/
(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e
^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3
 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^
3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9
 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6
*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 -
 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b
^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^
3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c
^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*
d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*
e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 192*a^4*c^4*d*e^7 -
192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2*c^4*d^3*e^5 - 96*a^2
*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4*d^4*e^4 + 48*a*b^4*c
^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2*e^6 + 48*a^3*b^2*c^3
*d*e^7))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e -
 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2
*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2
 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 -
 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - 4*
a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5
+ 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c
- b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^
2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^
2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 -
8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a
*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^
3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - (x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b
^2*c^3*d^2*e^3) - (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2
 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*
c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4
+ b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^
3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*
d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(
1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b
^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 +
8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3
*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*
```

e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^
4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b
^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c
*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a
^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a
*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2
*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16
*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4
 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 1
6*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d
*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^
9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c
^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4
 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288
*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 +
128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*
b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c
^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*
d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3
+ 384*a^3*c^5*d^3*e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2*b^3*c^3*d^2*e^6 - 4
8*a*b^2*c^5*d^5*e^3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c^3*d^3*e^5 - 384*a^2
*b*c^5*d^4*e^4 - 384*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^
2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(
a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 -
 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2
*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e
+ 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a
^2*c^4*d^2*e^4 - 8*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5
 - 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^
3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 +
16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c
^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32
*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a
*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e
^3)))^(1/2) + 2*a*c^3*d*e^3))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e
^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*
a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4
 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e
- 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^
2*d*e^3)))^(1/2)*2i - (log(b^4*d^3*e^5 - a*b^3*d^2*e^6 + a*c^3*d^5*e^3 - b^
3*c*d^4*e^4 + 2*a^2*c^2*d^3*e^5 + a^3*c*d*e^7 - b^4*e^3*x*(-d*e)^(5/2) - a*
b^3*e^5*x*(-d*e)^(3/2) - a^3*c*e^7*x*(-d*e)^(1/2) + 2*a*b*c^2*d^4*e^4 - 3*a
*b^2*c*d^3*e^5 + 2*a^2*b*c*d^2*e^6 - 2*a^2*c^2*e^3*x*(-d*e)^(5/2) + a*c^3*d
x(-d*e)^(7/2) - b^3*c*e*x*(-d*e)^(7/2) + 2*a*b*c^2*e*x*(-d*e)^(7/2) + 3*a
*b^2*c*e^3*x*(-d*e)^(5/2) + 2*a^2*b*c*e^5*x*(-d*e)^(3/2))*(-d*e)^(1/2))/(2*
(a*e^2 + c*d^2 - b*d*e)) - atan(((x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^
2*c^3*d^2*e^3) - (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2
- c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c
^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 +
 b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3
*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d
^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1
/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^
3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8
*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*
d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 + a*e

```
^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4
*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^
2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*
d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^
2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 + a*e^2*(-(4*a*
c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*
d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*
a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4
- 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16
*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*
e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9
 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^
5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4
+ 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*
a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 1
28*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b
^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^
2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d
^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 -
 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2*c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48
*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4*d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*
b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2*e^6 + 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2
 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a
^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 -
8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*
b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e +
 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^
2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5
+ 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3
*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 1
6*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^
2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*
a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*
b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^
3)))^(1/2)*1i + (x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) -
(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*
c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^
6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^
5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b
*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b
*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a
^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4
- 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*
b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4
*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4
*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3
*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*
c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 3
2*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e
^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*
a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4
 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e
- 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^
2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^
```

9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^
3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4
*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e
^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^
2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 -
672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*
a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*
b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*
e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^
3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 3
84*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d
^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a
^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 -
 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*
a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e
^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8
*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2
e^4))(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-
(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4
*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d
^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 -
 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 3
2*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*1i)/((
x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 + a*
e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^
4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b
^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c
*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a
^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c
^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 -
8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e
^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 2
4*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*
c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16
*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2
*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a
^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b
^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3
)))^(1/2)*(x*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d
*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4
*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2
*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e
^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*
(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d
^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 1
28*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2
*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*
c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4
*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4
*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4
 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) -
192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2*
c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4*
d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2*
e^6 + 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^

2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a
^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4
+ 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e -
6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2
*d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3
- 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e
^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*
(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2
- 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 -
2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e
+ 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - (x*(2*a^2*c^3*e^5 - 4
*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^
3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a
^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d
^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b
^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^
2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*
a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3
*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 +
8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^
3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6)
+ (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b
^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 +
b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a
*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^
2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^
3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/
(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e
^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3
- 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^
3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9
- 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6
*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 -
192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b
^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^
3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c
^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*
d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*
e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 +
192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2
*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c
^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 384*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3
*d*e^7))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e -
4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2
*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2
- 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 -
32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 4*
a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5
- 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c
- b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^
2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^
2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 -
8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a
*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^
3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 2*a*c^3*d*e^3))*(-(a*b^3*e^2 + a*e^2*(-(4

```
*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c
^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 +
16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d
^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e +
 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c
*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.307 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=254

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)}$$

```
[Out] e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)/d^(1/2)-1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(b*e-2*c*d)/(-4*a*
c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(-b*e+2*c
*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1170, 205, 1166}

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{e^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{cd-be-cex^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{cd-be-cex^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\,(cd^2-bde+ae^2)} - \frac{\left(c\left(e+\frac{2c}{\sqrt{b^2}}\right.\right.}{2}$$

$$= -\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{}}{\sqrt{b+\sqrt{b^2}}}\right.}{\sqrt{2}\,\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde}$$

Mathematica [A] time = 0.25, size = 274, normalized size = 1.08

$$\frac{\sqrt{c}\left(e\sqrt{b^2-4ac}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,(-ae^2+bde-cd^2)} + \frac{\sqrt{c}\left(e\sqrt{b^2-4ac}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}\,(-ae^2+bde-cd^2)} + \frac{e^{3/}}{\sqrt{d}\,(}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) +
(e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 9.13, size = 7650, normalized size = 30.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*
c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
```

```
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e +
2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^3*c^3 - 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^2*c^4 + 16*a*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*c^5 - 32*a^2*c^5 + 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^
4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2
- 4*a*c)*a*b*c^4)*d^3*e^2 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a
*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (
2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^2*e^3 + 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 2*
b^6*c + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 6*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^4*c^2 + 14*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 3*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*b^4*c -
6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*d*abs(c*d^2 - b*d*e +
a*e^2)*e^2 - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a
*e^2)^2*e + 2*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2
*b*c^3)*d*e^4 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt
```

```
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*
a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(c*d^2 - b*d*e + a*e^2)*e^3 - (2*a^
2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*
c*d^2 - b^2*d*e + a*b*e^2 + sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d
^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e
+ a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2
*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(
a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^
3*c^3 - 4*a^2*b*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6 - 6*a
^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^
3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 -
2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2
*b^3*c^2 - 4*a^3*b*c^3)*d*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4
- 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^
4*c^3)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4) - 1/8*(2*(2*b^3*c^5 - 8*a*b*c
^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5
)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 2*b^4*c^3 + 16*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 1
6*a*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^5 + 32*a^2*c^5
- 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*d^3*abs(c*d^2 - b*d*e +
a*e^2) + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^3*e^2 +
 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 +
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*
c)*a*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (2*b^6*c^2 + 4*a*b^4*c^3 - 4
8*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4
*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^2*e^3 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^6 - 7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sq
```

```
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c + 8*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 14*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*b^4*c + 6*(b^2 - 4*a*c)*a*b^2*c^2 +
 8*(b^2 - 4*a*c)*a^2*c^3)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 - (2*b^4*c^2 - 1
6*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*
c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 -
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2
*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^2)^2*e + 2*(2*a*b^5*c^2
- 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*
c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 -
 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^3)*d*e^4 + 2*(sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*
b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a
^2*b*c^2)*abs(c*d^2 - b*d*e + a*e^2)*e^3 - (2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)
*a^2*b^2*c^2)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 -
 sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^
2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 -
8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*
c^5)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2
*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*ab
s(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^
2*b^3*c^2 + a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c
^4)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5 - 8*a^3*b^3*c -
2*a^2*b^4*c + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*a
bs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c
 + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c*d^2 - b*d*e +
a*e^2)*abs(c)*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*d^2 - b*d*e + a*
e^2)*sqrt(d))
```

maple [B] time = 0.03, size = 480, normalized size = 1.89

$$\frac{\sqrt{2}\, bce \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2 - deb + c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} + \frac{\sqrt{2}\, bce \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(a e^2 - deb + c d^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/2/(a*e^2-b*d*e+c*d^2)*c
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d-1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+e^2/
(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] e^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) - integrate((
c*e*x^2 - c*d + b*e)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

mupad [B] time = 5.61, size = 23640, normalized size = 93.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2
 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a
*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3
*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2
 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3
*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*
b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d
^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2
*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*
b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 +
b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^
2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 -
 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^
3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^
2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*
c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) -
16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*
c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^
2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*
b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*
b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 +
512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^
```

5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i + ((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128

```
*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3
*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*
d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3
*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128
*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d
^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*
b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*
c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3
*e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7
))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7
*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2
*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4
*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2
*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e -
 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*
c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 -
4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^
5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d
^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12
*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a
^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e
^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d
^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^
2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^5*e^2 + b^3*c^2*d
^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a
*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^
2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*
a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d
^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 +
192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3
 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*
d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2
*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4
*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c
^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32
*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(
-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b
^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2
*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4
 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2
*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*
a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*
d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*
e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4
*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e
^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 2
88*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7
+ 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*
a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6
*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^
```

5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - ((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96

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*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^
4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384*a^3*b*c^4*d*e^7
- 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4*c^3*d^2*e^6 - 12
8*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^
2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d
^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3
*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)
))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^
5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^
2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*
b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2
*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a
^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^
2*e^2)))^(1/2)))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a
*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*
d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b
^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a
*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3
 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*2i + atan((((-(b^5*e
^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^
2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e +
2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a
^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*
e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c
^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e -
 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*
b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32
*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5
 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c
^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*
d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^
3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2
)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c
^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e
+ 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 +
16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d
^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5
*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 1
6*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 3
2*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4
*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192
*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c
^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^
2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d
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^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*
e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8
- 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c
^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4
 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c
^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*
e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*
a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*
b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*
(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*
e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4
- 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 3
2*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e
^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/
2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16
*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^
2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d
^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3
*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)
))^(1/2)*1i + ((-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b
*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*
e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4
+ 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6
*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b
^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 +
 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 +
16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 -
 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e
^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^
5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c
*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e
 + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 1
6*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5
*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*
b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*
e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*
d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 +
a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b
^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16
*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*
d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^
9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^
3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4
*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e
^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^
2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 -
672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*
a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*
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b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^
8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3
*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6
 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384
*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4
*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 +
 b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 +
a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*
c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c
^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3
- 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d
^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a
^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4
 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5
*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*
e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e
+ 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16
*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*
d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b
*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e
 - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^
2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 -
8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*
a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)
((x(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e
^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d
*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a
*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*
a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3
*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e
^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*
b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*
a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^
3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3
*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e
^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*
a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e
 + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b
^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c
^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 -
32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^
3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*
d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d
^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e
^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 -
1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288
*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e
^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b
^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e

^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - ((-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3

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*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b
^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b
^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3
*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a
^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^
3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*
e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2
 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8
*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a
^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
- 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2))
)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*
a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*
c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*
d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*
a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e -
32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c
^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*2i - (log(b^5*d*(-d*e^3)^(5/2) - b^
5*d^3*e^8*x + c^5*d^8*e^3*x + 2*a*c^4*d^5*(-d*e^3)^(3/2) - 16*a^3*c^2*e*(-d
*e^3)^(5/2) - c^5*d^8*e*(-d*e^3)^(1/2) + b^2*c^3*d^5*(-d*e^3)^(3/2) - a*b^4
*e*(-d*e^3)^(5/2) - 7*a*b^3*c*d*(-d*e^3)^(5/2) + 17*a^2*c^3*d^3*e^2*(-d*e^3
)^(3/2) + a*b^4*d^2*e^9*x + 2*a*c^4*d^6*e^5*x - 2*b*c^4*d^7*e^4*x + 2*b^4*c
*d^4*e^7*x + 12*a^2*b*c^2*d*(-d*e^3)^(5/2) + 8*a^2*b^2*c*e*(-d*e^3)^(5/2) +
 17*a^2*c^3*d^4*e^7*x + 16*a^3*c^2*d^2*e^9*x + b^2*c^3*d^6*e^5*x - b^3*c^2*
d^5*e^6*x - b^3*c^2*d^4*e*(-d*e^3)^(3/2) + 2*b^4*c*d^3*e^2*(-d*e^3)^(3/2) +
 2*b*c^4*d^7*e^2*(-d*e^3)^(1/2) - 12*a*b^2*c^2*d^4*e^7*x - 12*a^2*b*c^2*d^3
*e^8*x - 8*a^2*b^2*c*d^2*e^9*x - 12*a*b^2*c^2*d^3*e^2*(-d*e^3)^(3/2) + 2*a*
b*c^3*d^5*e^6*x + 7*a*b^3*c*d^3*e^8*x + 2*a*b*c^3*d^4*e*(-d*e^3)^(3/2))*(-d
*e^3)^(1/2))/(2*(c*d^3 + a*d*e^2 - b*d^2*e)) + (log(b^5*d*(-d*e^3)^(5/2) +
b^5*d^3*e^8*x - c^5*d^8*e^3*x + 2*a*c^4*d^5*(-d*e^3)^(3/2) - 16*a^3*c^2*e*(
-d*e^3)^(5/2) - c^5*d^8*e*(-d*e^3)^(1/2) + b^2*c^3*d^5*(-d*e^3)^(3/2) - a*b
^4*e*(-d*e^3)^(5/2) - 7*a*b^3*c*d*(-d*e^3)^(5/2) + 17*a^2*c^3*d^3*e^2*(-d*e
^3)^(3/2) - a*b^4*d^2*e^9*x - 2*a*c^4*d^6*e^5*x + 2*b*c^4*d^7*e^4*x - 2*b^4
*c*d^4*e^7*x + 12*a^2*b*c^2*d*(-d*e^3)^(5/2) + 8*a^2*b^2*c*e*(-d*e^3)^(5/2)
 - 17*a^2*c^3*d^4*e^7*x - 16*a^3*c^2*d^2*e^9*x - b^2*c^3*d^6*e^5*x + b^3*c^
2*d^5*e^6*x - b^3*c^2*d^4*e*(-d*e^3)^(3/2) + 2*b^4*c*d^3*e^2*(-d*e^3)^(3/2)
 + 2*b*c^4*d^7*e^2*(-d*e^3)^(1/2) + 12*a*b^2*c^2*d^4*e^7*x + 12*a^2*b*c^2*d
^3*e^8*x + 8*a^2*b^2*c*d^2*e^9*x - 12*a*b^2*c^2*d^3*e^2*(-d*e^3)^(3/2) - 2*
a*b*c^3*d^5*e^6*x - 7*a*b^3*c*d^3*e^8*x + 2*a*b*c^3*d^4*e*(-d*e^3)^(3/2))*(
-d*e^3)^(1/2))/(2*c*d^3 + 2*a*d*e^2 - 2*b*d^2*e)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.308 $\int \frac{1}{x^2\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=298

$$-\frac{\sqrt{c}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)}-\frac{e^{5/2}\,\mathrm{t}}{d^{3/2}\left(ae\right.}$$

```
[Out] -1/a/d/x-e^(5/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)-1/2*
arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(c*d-b*e+(2*
a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))/a/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2))*c^(1/2)*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))/a/(a*e^2
-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.96, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$-\frac{\sqrt{c}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)}-\frac{e^{5/2}\,\mathrm{t}}{d^{3/2}\left(ae\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(c*d - b*e
- (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 -
 b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{adx^2} - \frac{e^3}{d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{-bcd+b^2e-ace-c(cd-b}{a\left(cd^2-bde+ae^2\right)\left(a+bx^2\right.}\right.$$

$$= -\frac{1}{adx} + \frac{\int \frac{-bcd+b^2e-ace-c(cd-be)x^2}{a+bx^2+cx^4}\,dx}{a\left(cd^2-bde+ae^2\right)} - \frac{e^3\int \frac{1}{d+ex^2}\,dx}{d\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2-bde+ae^2\right)} - \frac{\left(c\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}}}{2a\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{1}{adx} - \frac{\sqrt{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,\left(cd^2-bde+ae^2\right)} - \frac{\sqrt{c}\left(cd-be-\right.}{\sqrt{2}\,a\sqrt{b+}}$$

Mathematica [A] time = 0.40, size = 340, normalized size = 1.14

$$-\frac{\sqrt{c}\left(cd\sqrt{b^2-4ac}-be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,\left(e(ae-bd)+cd^2\right)} + \frac{\sqrt{c}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-\right.}{\sqrt{2}\,a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e -
b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-
(b*d) + a*e))) + (Sqrt[c]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e
+ b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*
(-(b*d) + a*e))) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 -
b*d*e + a*e^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 12.82, size = 10058, normalized size = 33.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*((2*a^2*b^4*c^5 - 8*a^3*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 2*(b^2 - 4*a*c)*a^2*b^2*c^5)*d^5 - (6*a^2*b^
```

5*c^4 - 28*a^3*b^3*c^5 + 16*a^4*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^5 - 6*(b^2 - 4*a*c)*a^2*b^3*c^4 + 4*(b^2 - 4*a*c)
*a^3*b*c^5)*d^4*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 -
8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^
4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*
(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*d^3*abs(a*c*d^2 - a*b*
d*e + a^2*e^2) + (6*a^2*b^6*c^3 - 28*a^3*b^4*c^4 + 16*a^4*b^2*c^5 - 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 14*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 6*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 3*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 6*(b^2 - 4*a*c)
*a^2*b^4*c^3 + 4*(b^2 - 4*a*c)*a^3*b^2*c^4)*d^3*e^2 - 2*(2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 17*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 4*a*b
^6*c^2 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 18*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^4*c^3 + 34*a^2*b^4*c^3 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 -
 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 80*a^3*b^2*c^4 + 4
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 32*a^4*c^5 + 4*(b^2 - 4*
a*c)*a*b^4*c^2 - 18*(b^2 - 4*a*c)*a^2*b^2*c^3 + 8*(b^2 - 4*a*c)*a^3*c^4)*d^
2*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*e + (2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 -
4*a*c)*a*c^4)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*d - (2*a^2*b^7*c^2 - 4*a^3*b^
5*c^3 - 24*a^4*b^3*c^4 + 32*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^5*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 - 4*(b^2 - 4*a*c)*a^3*b^3*c^3 +
 8*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*e^3 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^7 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*b^7*c + 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 16*
a^2*b^5*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 32*a^

3*b^3*c^3 + 2*(b^2 - 4*a*c)*a*b^5*c - 8*(b^2 - 4*a*c)*a^2*b^3*c^2)*d*abs(a*
c*d^2 - a*b*d*e + a^2*e^2)*e^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*
c)*a*b*c^3)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*e + (4*a^3*b^6*c^2 - 22*a^4*b^4
*c^3 + 24*a^5*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^6 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^4*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^5*b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^4*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^2*c^3 - 4*(b^2 - 4*a*c)*a^3*b^4*c^2 + 6*(b^2 - 4*a*c)*a^4*b^2*c^3)*d*e^4
- 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6 - 9*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^5*c - 2*a^2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b^2*c^2 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 18*a^3*b^4*c^2 - 16*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^4*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 -
48*a^4*b^2*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 32*a^
5*c^4 + 2*(b^2 - 4*a*c)*a^2*b^4*c - 10*(b^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 -
4*a*c)*a^4*c^3)*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*e^3 - (2*a^4*b^5*c^2 - 12
*a^5*b^3*c^3 + 16*a^6*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^5*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^4*b^4*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^6*b*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^5*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b
c^3 - 2(b^2 - 4*a*c)*a^4*b^3*c^2 + 4*(b^2 - 4*a*c)*a^5*b*c^3)*e^5)*arctan
(2*sqrt(1/2)*x/sqrt((a*b*c*d^2 - a*b^2*d*e + a^2*b*e^2 + sqrt((a*b*c*d^2 -
a*b^2*d*e + a^2*b*e^2)^2 - 4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*(a*c^2*d^2 -
a*b*c*d*e + a^2*c*e^2)))/(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2)))/((a^3*b^4*c
^2 - 8*a^4*b^2*c^3 - 2*a^3*b^3*c^3 + 16*a^5*c^4 + 8*a^4*b*c^4 + a^3*b^2*c^4
- 4*a^4*c^5)*d^4*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c) - 2*(a^3*b^5*c -
8*a^4*b^3*c^2 - 2*a^3*b^4*c^2 + 16*a^5*b*c^3 + 8*a^4*b^2*c^3 + a^3*b^3*c^3
- 4*a^4*b*c^4)*d^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e + (a^3*b^6 - 6
*a^4*b^4*c - 2*a^3*b^5*c + 4*a^4*b^3*c^2 + a^3*b^4*c^2 + 32*a^6*c^3 + 16*a^
5*b*c^3 - 2*a^4*b^2*c^3 - 8*a^5*c^4)*d^2*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*a
bs(c)*e^2 - 2*(a^4*b^5 - 8*a^5*b^3*c - 2*a^4*b^4*c + 16*a^6*b*c^2 + 8*a^5*b
^2*c^2 + a^4*b^3*c^2 - 4*a^5*b*c^3)*d*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(
c)*e^3 + (a^5*b^4 - 8*a^6*b^2*c - 2*a^5*b^3*c + 16*a^7*c^2 + 8*a^6*b*c^2 +
a^5*b^2*c^2 - 4*a^6*c^3)*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e^4) + 1/8
*((2*a^2*b^4*c^5 - 8*a^3*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^5 - 2*(b^2 - 4*a*c)*a^2*b^2*c^5)*d^5 - (6*a^2*b^5*c^
4 - 28*a^3*b^3*c^5 + 16*a^4*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(

b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^5 - 6*(b^2 - 4*a*c)*a^2*b^3*c^4 + 4*(b^2 - 4*a*c)*a^3
*b*c^5)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 -
4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2
 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*d^3*abs(a*c*d^2 - a*b*d*e
+ a^2*e^2) + (6*a^2*b^6*c^3 - 28*a^3*b^4*c^4 + 16*a^4*b^2*c^5 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 14*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 6*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 6*(b^2 - 4*a*c)*a^2
*b^4*c^3 + 4*(b^2 - 4*a*c)*a^3*b^2*c^4)*d^3*e^2 + 2*(2*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^6*c - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^4*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 4*a*b^6*c
^2 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 18*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^4*c^3 - 34*a^2*b^4*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^4*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 9*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 80*a^3*b^2*c^4 + 4*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 32*a^4*c^5 - 4*(b^2 - 4*a*c)
*a*b^4*c^2 + 18*(b^2 - 4*a*c)*a^2*b^2*c^3 - 8*(b^2 - 4*a*c)*a^3*c^4)*d^2*ab
s(a*c*d^2 - a*b*d*e + a^2*e^2)*e + (2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*
c)*a*c^4)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*d - (2*a^2*b^7*c^2 - 4*a^3*b^5*c^
3 - 24*a^4*b^3*c^4 + 32*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^5*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 - 4*(b^2 - 4*a*c)*a^3*b^3*c^3 + 8*(
b^2 - 4*a*c)*a^4*b*c^4)*d^2*e^3 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^7 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c + 2*a*b^7*c + 16*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 16*a^2*
b^5*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 32*a^3*b^
3*c^3 - 2*(b^2 - 4*a*c)*a*b^5*c + 8*(b^2 - 4*a*c)*a^2*b^3*c^2)*d*abs(a*c*d^
2 - a*b*d*e + a^2*e^2)*e^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*

```
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a
*b*c^3)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*e + (4*a^3*b^6*c^2 - 22*a^4*b^4*c^3
 + 24*a^5*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^3*b^6 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*
b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b
^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2
*c^3 - 4*(b^2 - 4*a*c)*a^3*b^4*c^2 + 6*(b^2 - 4*a*c)*a^4*b^2*c^3)*d*e^4 + 2
*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6 - 9*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^5*c + 2*a^2*b^6*c + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*
c^2 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 18*a^3*b^4*c^2 - 16*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*b*c^3 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 48*
a^4*b^2*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 32*a^5*c^
4 - 2*(b^2 - 4*a*c)*a^2*b^4*c + 10*(b^2 - 4*a*c)*a^3*b^2*c^2 - 8*(b^2 - 4*a
*c)*a^4*c^3)*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*e^3 - (2*a^4*b^5*c^2 - 12*a^5
*b^3*c^3 + 16*a^6*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^4*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
6*b*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b
^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*
c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3
 - 2*(b^2 - 4*a*c)*a^4*b^3*c^2 + 4*(b^2 - 4*a*c)*a^5*b*c^3)*e^5)*arctan(2*s
qrt(1/2)*x/sqrt((a*b*c*d^2 - a*b^2*d*e + a^2*b*e^2 - sqrt((a*b*c*d^2 - a*b^
2*d*e + a^2*b*e^2)^2 - 4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*(a*c^2*d^2 - a*b
*c*d*e + a^2*c*e^2)))/(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2)))/((a^3*b^4*c^2 -
 8*a^4*b^2*c^3 - 2*a^3*b^3*c^3 + 16*a^5*c^4 + 8*a^4*b*c^4 + a^3*b^2*c^4 - 4
*a^4*c^5)*d^4*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c) - 2*(a^3*b^5*c - 8*a^
4*b^3*c^2 - 2*a^3*b^4*c^2 + 16*a^5*b*c^3 + 8*a^4*b^2*c^3 + a^3*b^3*c^3 - 4*
a^4*b*c^4)*d^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e + (a^3*b^6 - 6*a^4
*b^4*c - 2*a^3*b^5*c + 4*a^4*b^3*c^2 + a^3*b^4*c^2 + 32*a^6*c^3 + 16*a^5*b*
c^3 - 2*a^4*b^2*c^3 - 8*a^5*c^4)*d^2*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c
)*e^2 - 2*(a^4*b^5 - 8*a^5*b^3*c - 2*a^4*b^4*c + 16*a^6*b*c^2 + 8*a^5*b^2*c
^2 + a^4*b^3*c^2 - 4*a^5*b*c^3)*d*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e
^3 + (a^5*b^4 - 8*a^6*b^2*c - 2*a^5*b^3*c + 16*a^7*c^2 + 8*a^6*b*c^2 + a^5*
b^2*c^2 - 4*a^6*c^3)*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e^4) - arctan(
x*e^(1/2)/sqrt(d))*e^(5/2)/((c*d^3 - b*d^2*e + a*d*e^2)*sqrt(d)) - 1/(a*d*x
)
```

maple [B] time = 0.03, size = 817, normalized size = 2.74

$$-\frac{\sqrt{2}\, b^{2} c e \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{2 \left(a \,e^{2}-d e b+c \,d^{2}\right) \sqrt{-4 a c+b^{2}}\, \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, a}-\frac{\sqrt{2}\, b^{2} c e \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{2 \left(a \,e^{2}-d e b+c \,d^{2}\right) \sqrt{-4 a c+b^{2}}\, \sqrt{\left(b+\sqrt{-4ac\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/d/x-1/2/(a*e^2-b*d*e+c*d^2)/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
```

```
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e+1/2/(a*e^2-b
*d*e+c*d^2)/a*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/(a*e^2-b*d*e+c*d^2)*c^2/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-1/2/(a*e^2-b*d*e+c*d^2)/a*c/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e+1/2/(a*e^2-b*d*e+c*d^2)/a*c^2/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+1/2/(a*e^2-b*d*e+c*d^2)/a*c*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*b*e-1/2/(a*e^2-b*d*e+c*d^2)/a*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/(a*e^2-b*d
*e+c*d^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-1/2/(a*e^2-b*d*e+c*d^
2)/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e+1/2/(a*e^2-b*d*e+c*d^2)/
a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-1/d*e^3/(a*e^2-b*d*e+c*d^2)
/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{e^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(cd^3-bd^2e+ade^2\right)\sqrt{de}}+\frac{-\int\frac{bcd+\left(c^2d-bce\right)x^2-\left(b^2-ac\right)e}{cx^4+bx^2+a}\,dx}{acd^2-abde+a^2e^2}-\frac{1}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -e^3*arctan(e*x/sqrt(d*e))/((c*d^3 - b*d^2*e + a*d*e^2)*sqrt(d*e)) + integr
ate(-(b*c*d + (c^2*d - b*c*e)*x^2 - (b^2 - a*c)*e)/(c*x^4 + b*x^2 + a), x)/
(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/(a*d*x)
```

mupad [B] time = 5.89, size = 33644, normalized size = 112.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^
3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*
b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^
4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6
*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^
6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 1
6*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c
*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^
2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^
2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3
*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*
```

```
c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e
- 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*
c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4
*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e
+ 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e
 - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d
*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d
^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a
^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a
^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14
*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9
*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 64
0*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^1
3*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*
c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a
^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^
8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^1
6*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4
*d^10*e^8) + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d
^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4
*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2
*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^
9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 +
80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*
e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^
2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c
^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) + x*(112*a^
10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*
d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*
d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2
*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*
c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b
^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*
b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10
*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*
e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-
(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^
2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^
5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^
2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^
5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 +
16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 +
4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^
6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^
2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) + x*(2*a^7*c^7
*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2
*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^
2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 2
5*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e -
2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b
^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^
```

3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*1i - ((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9

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*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*
c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9
*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b
^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e
+ 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4
+ 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*
a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e
- 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3
*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*
d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*
e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*
e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) - x*(2*a^7*c^7*d^9*e^5 -
 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*
c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*
e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^
2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 1
6*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^
4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^
3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^
3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^7*e^2 +
b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b
*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*
d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2
*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b
^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c
^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 -
32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*((
(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d
^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e
^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e +
 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*
e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e
^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b
^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^
2)))^(1/2)*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2
 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c
^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c
^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d
^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c
*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*
a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 5
12*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^
9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 +
128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*
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e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*
c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a
^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8
 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*
d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*
c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 128*a^1
1*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*
c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b
^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*
a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 +
 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11
*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c
^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12
*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^
11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) + x*(112*a^10*c^6*d^10*e^6 - 32
*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*
c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5
*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^
3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^
9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a
^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32
*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*
a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^
2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c
^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^
3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b
*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e
 - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 -
 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a
^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^
7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) + x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6
*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 +
a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
 - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c
 - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*
d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*
d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4
 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^
3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4
*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + ((-(b^7*e^2 + b^5*c^2*d^2 +
 b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*
c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*
b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*
c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3
6*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4
 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^
3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 -
2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*
d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 +
b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b
*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*

d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2
*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b
^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c
^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 -
32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(x
*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*
d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*
e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e
+ 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c
*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*
e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*
b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e
^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d
^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*
d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7
*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 67
2*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e
^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c
^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^
12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 +
128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^
6 + 640*a^13*b*c^4*d^10*e^8) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^
5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2
+ 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e
^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d
^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c
^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^
10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 -
 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*
e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11
*e^6 + 256*a^12*b*c^4*d^9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12
*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2
- 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5
 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^
5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^1
0*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^
7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^
13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^
7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b
^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3
*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)
^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c
^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*
c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*
c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^
6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b
^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e +
16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*
c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*
d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^
8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*
e^3 - 32*a^9*b*c^5*d^8*e^7) - x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*
a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)

^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*
e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1
/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c
^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*
c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c
^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*
b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*
e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 +
a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
 - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c
 - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*
d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*
d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4
- 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^
3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4
*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*2i + atan((((-(b^7*e^2 + b^5*
c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4
*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e
+ 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e
 + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d
*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d
^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a
^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b
^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 +
 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 -
 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*
a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4
- 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 +
 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c
*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))
^(1/2)*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a
^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c
- b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d
*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d
^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 -
 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3
*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*
b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a
^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^
3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*
a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3
+ 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*
d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*
b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 3
84*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*
e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*

d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) + x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) + x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*1i - ((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^

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5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^
3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 3
2*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(x*
(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d
^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e
^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e +
 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*
e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e
^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b
^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^
2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^
11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d
^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*
c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672
*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^
7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^
3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^1
2*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 1
28*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6
 + 640*a^13*b*c^4*d^10*e^8) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^5
 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 +
 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^
5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^
13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^
2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^1
0*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 -
128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e
^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*
e^6 + 256*a^12*b*c^4*d^9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*
e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 -
 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5
- 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5
 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10
*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7
*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^1
3*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7
*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^
3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*
b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^
4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6
*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^
6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 1
6*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c
*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d
^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8
*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e
^3 - 32*a^9*b*c^5*d^8*e^7) - x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a
^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^
```

2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c
^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^
3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b
*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e
 - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2
 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c
^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c
^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d
^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c
*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*
a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2
 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 +
a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^
2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^
3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e
^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 +
a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2
 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^
2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(192*a^10*c^7
*d^14*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) -
20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16
*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*
a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4
 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 +
 a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^
3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^
4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 5
12*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128
*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5
 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*
d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b
^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 1
92*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11
*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c
^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*
b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 128*a^11*c^6*d^12*e^5 - 320*a^1
2*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^
4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8
*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 3
52*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7
 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*
d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b
^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^
9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*
a^12*b*c^4*d^9*e^8) + x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a
^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^
3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b
^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8
*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208
*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*

a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 12
8*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(
b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2
+ 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2
- 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16
*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4
- 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2
+ 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*
c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2))
)^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4
*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*
a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^
9*b*c^5*d^8*e^7) + x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5
*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*
a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*
a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^
3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b
^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8
*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^
3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e
+ 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b
^4*c*d^2*e^2)))^(1/2) + ((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5
*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)
^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*
b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a
^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^
2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a
^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d
^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2
- a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c
^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c
^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d
^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c
*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*
a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^7*e^2 + b^5*c^2*d^
2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 +
a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a
^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b
^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*
e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 +
a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2
- 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c
^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^
7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d
^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c

^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) - x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*2i - (log(c^6*d^11*(-d^3*e^5)^(1/2) + b^6*d^6*e^9*x +

```
 c^6*d^12*e^3*x + b^5*c*d^3*(-d^3*e^5)^(3/2) - b^6*d^2*e*(-d^3*e^5)^(3/2) -
 a^2*b^4*e^3*(-d^3*e^5)^(3/2) - 16*a^4*c^2*e^3*(-d^3*e^5)^(3/2) - 7*a*b^3*c
^2*d^3*(-d^3*e^5)^(3/2) + 12*a^2*b*c^3*d^3*(-d^3*e^5)^(3/2) + 8*a^3*b^2*c*e
^3*(-d^3*e^5)^(3/2) + 16*a^3*c^3*d^2*e*(-d^3*e^5)^(3/2) + a*c^5*d^9*e^2*(-d
^3*e^5)^(1/2) + a*b^5*d^5*e^10*x + a*c^5*d^10*e^5*x - b*c^5*d^11*e^4*x - b^
5*c*d^7*e^8*x + a^2*b^4*d^4*e^11*x - 16*a^3*c^3*d^6*e^9*x + 16*a^4*c^2*d^4*
e^11*x - a*b^5*d*e^2*(-d^3*e^5)^(3/2) - b*c^5*d^10*e*(-d^3*e^5)^(1/2) - 24*
a^2*b^2*c^2*d^2*e*(-d^3*e^5)^(3/2) + 7*a*b^3*c^2*d^7*e^8*x - 12*a^2*b*c^3*d
^7*e^8*x - 8*a^2*b^3*c*d^5*e^10*x + 16*a^3*b*c^2*d^5*e^10*x - 8*a^3*b^2*c*d
^4*e^11*x + 9*a*b^4*c*d^2*e*(-d^3*e^5)^(3/2) + 24*a^2*b^2*c^2*d^6*e^9*x + 8
*a^2*b^3*c*d*e^2*(-d^3*e^5)^(3/2) - 16*a^3*b*c^2*d*e^2*(-d^3*e^5)^(3/2) - 9
*a*b^4*c*d^6*e^9*x)*(-d^3*e^5)^(1/2))/(2*(c*d^5 + a*d^3*e^2 - b*d^4*e)) + (
log(b^6*d^6*e^9*x - c^6*d^11*(-d^3*e^5)^(1/2) + c^6*d^12*e^3*x - b^5*c*d^3*
(-d^3*e^5)^(3/2) + b^6*d^2*e*(-d^3*e^5)^(3/2) + a^2*b^4*e^3*(-d^3*e^5)^(3/2
) + 16*a^4*c^2*e^3*(-d^3*e^5)^(3/2) + 7*a*b^3*c^2*d^3*(-d^3*e^5)^(3/2) - 12
*a^2*b*c^3*d^3*(-d^3*e^5)^(3/2) - 8*a^3*b^2*c*e^3*(-d^3*e^5)^(3/2) - 16*a^3
*c^3*d^2*e*(-d^3*e^5)^(3/2) - a*c^5*d^9*e^2*(-d^3*e^5)^(1/2) + a*b^5*d^5*e^
10*x + a*c^5*d^10*e^5*x - b*c^5*d^11*e^4*x - b^5*c*d^7*e^8*x + a^2*b^4*d^4*
e^11*x - 16*a^3*c^3*d^6*e^9*x + 16*a^4*c^2*d^4*e^11*x + a*b^5*d*e^2*(-d^3*e
^5)^(3/2) + b*c^5*d^10*e*(-d^3*e^5)^(1/2) + 24*a^2*b^2*c^2*d^2*e*(-d^3*e^5)
^(3/2) + 7*a*b^3*c^2*d^7*e^8*x - 12*a^2*b*c^3*d^7*e^8*x - 8*a^2*b^3*c*d^5*e
^10*x + 16*a^3*b*c^2*d^5*e^10*x - 8*a^3*b^2*c*d^4*e^11*x - 9*a*b^4*c*d^2*e*
(-d^3*e^5)^(3/2) + 24*a^2*b^2*c^2*d^6*e^9*x - 8*a^2*b^3*c*d*e^2*(-d^3*e^5)^
(3/2) + 16*a^3*b*c^2*d*e^2*(-d^3*e^5)^(3/2) - 9*a*b^4*c*d^6*e^9*x)*(-d^3*e^
5)^(1/2))/(2*c*d^5 + 2*a*d^3*e^2 - 2*b*d^4*e) - 1/(a*d*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.309 $\int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=348

$$\frac{\sqrt{c}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-\right.}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}\left(a\right.}$$

```
[Out] -1/3/a/d/x^3+(a*e+b*d)/a^2/d^2/x+e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/
(a*e^2-b*d*e+c*d^2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*c^(1/2)*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+
b^2)^(1/2))/a^2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/
2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*c*d-b^2
*e+a*c*e+(-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/a^2/(a*e^
2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.55, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1287, 205, 1166}

$$\frac{\sqrt{c}\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-\right.}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(b*c*d - b^2*e + a*c*e
+ (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2
 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(b*c*d - b^2*e + a*c*e - (b^
2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(
5/2)*(c*d^2 - b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx=\int\left(\frac{1}{adx^4}+\frac{-bd-ae}{a^2d^2x^2}+\frac{e^4}{d^2\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}+\frac{b^2cd-ac^2d-b^3e+\ldots}{a^2\left(cd^2-bd\ldots\right.}\right.$$
$$=-\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{\int\frac{b^2cd-ac^2d-b^3e+2abce+c\left(bcd-b^2e+ace\right)x^2}{a+bx^2+cx^4}\,dx}{a^2\left(cd^2-bde+ae^2\right)}+\frac{e^4\int\frac{1}{d+ex^2}\ldots}{d^2\left(cd^2-bde\ldots\right.}$$
$$=-\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2-bde+ae^2\right)}+\frac{\left(c\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2\ldots}{\sqrt{b^2\ldots}}\right.\right.}{2a^2\left(cd^2\ldots\right.}$$
$$=-\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{\sqrt{c}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt{b-\ldots}}\right.}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] time = 0.52, size = 410, normalized size = 1.18

$$\frac{\sqrt{c}\left(b^2\left(cd-e\sqrt{b^2-4ac}\right)+bc\left(d\sqrt{b^2-4ac}+3ae\right)+ac\left(e\sqrt{b^2-4ac}-2cd\right)+b^3(-e)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(e(ae-bd)+cd^2\right)}+\frac{\sqrt{c}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*1/(a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(-(b^3*e) + b*c*(Sqrt
[b^2 - 4*a*c]*d + 3*a*e) + b^2*(c*d - Sqrt[b^2 - 4*a*c]*e) + a*c*(-2*c*d +
Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(
-(b*d) + a*e))) + (Sqrt[c]*(b^3*e + b*c*(Sqrt[b^2 - 4*a*c]*d - 3*a*e) - b^2
*(c*d + Sqrt[b^2 - 4*a*c]*e) + a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a
*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))) + (e^(7/2)*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 12.00, size = 12268, normalized size = 35.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*((2*a^4*b^5*c^5 - 12*a^5*b^3*c^6 + 16*a^6*b*c^7 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a
```

```
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^6 - 2*(b^2 - 4*a*c)*a^4*b^3*c^5 + 4*(b^2
 - 4*a*c)*a^5*b*c^6)*d^5 - (6*a^4*b^6*c^4 - 38*a^5*b^4*c^5 + 56*a^6*b^2*c^6
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2
+ 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3
+ 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 -
 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 -
 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 -
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4 +
7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - 6
*(b^2 - 4*a*c)*a^4*b^4*c^4 + 14*(b^2 - 4*a*c)*a^5*b^2*c^5)*d^4*e + 2*(sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 9*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^5*c^3 - 2*a^2*b^6*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
^2*c^4 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 18*a^3*b^4*c^4 - 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^4*b*c^5 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 -
48*a^4*b^2*c^5 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 + 32*a^5
*c^6 + 2*(b^2 - 4*a*c)*a^2*b^4*c^3 - 10*(b^2 - 4*a*c)*a^3*b^2*c^4 + 8*(b^2
- 4*a*c)*a^4*c^5)*d^3*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) + (6*a^4*b^7*c^3
 - 36*a^5*b^5*c^4 + 40*a^6*b^3*c^5 + 32*a^7*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c + 18*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 6*(b^2 - 4*a*c)*a^4*b^5*c^3 + 12*(b
^2 - 4*a*c)*a^5*b^3*c^4 + 8*(b^2 - 4*a*c)*a^6*b*c^5)*d^3*e^2 - 2*(2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 19*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^5*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
6*c^2 - 4*a^2*b^7*c^2 + 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*
c^3 + 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 2*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 38*a^3*b^5*c^3 - 48*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b^2*c^4 - 11*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c
^4 - 112*a^4*b^3*c^4 + 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5
 + 96*a^5*b*c^5 + 4*(b^2 - 4*a*c)*a^2*b^5*c^2 - 22*(b^2 - 4*a*c)*a^3*b^3*c^
3 + 24*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*e
+ (2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*(a^2*c*d^2 -
a^2*b*d*e + a^3*e^2)^2*d - (2*a^4*b^8*c^2 - 6*a^5*b^6*c^3 - 28*a^6*b^4*c^4
+ 80*a^7*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^4*b^8 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5
*b^6*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^
```

7*c + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*
c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c
^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2
- 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3
- 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 + 1
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 2
*(b^2 - 4*a*c)*a^4*b^6*c^2 - 2*(b^2 - 4*a*c)*a^5*b^4*c^3 + 20*(b^2 - 4*a*c)
*a^6*b^2*c^4)*d^2*e^3 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8
- 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*a^2*b^8*c + 23*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^5*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 18*a^3*b
^6*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 6*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^4*c^3 - 46*a^4*b^4*c^3 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^6*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 +
 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 16*a^5*b^2*c^4 + 4
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 32*a^6*c^5 + 2*(b^2 - 4*
a*c)*a^2*b^6*c - 10*(b^2 - 4*a*c)*a^3*b^4*c^2 + 6*(b^2 - 4*a*c)*a^4*b^2*c^3
 + 8*(b^2 - 4*a*c)*a^5*c^4)*d*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*e^2 - (2
*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*
a*c)*a^2*c^4)*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)^2*e + (4*a^5*b^7*c^2 - 26*a
^6*b^5*c^3 + 36*a^7*b^3*c^4 + 16*a^8*b*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^8*b*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^6*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^7*b*c^4 - 4*(b^2 - 4*a*c)*a^5*b^5*c^2 + 10*(b^2 - 4*a*c)*a^
6*b^3*c^3 + 4*(b^2 - 4*a*c)*a^7*b*c^4)*d*e^4 - 2*(sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^7 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5
*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*a^3*b^7*c + 32
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 + 12*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^5*c^2 + 20*a^4*b^5*c^2 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^6*b*c^3 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 6*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 64*a^5*b^3*c^3 + 8*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 64*a^6*b*c^4 + 2*(b^2 - 4*a*
c)*a^3*b^5*c - 12*(b^2 - 4*a*c)*a^4*b^3*c^2 + 16*(b^2 - 4*a*c)*a^5*b*c^3)*a
bs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*e^3 - (2*a^6*b^6*c^2 - 14*a^7*b^4*c^3 +
 24*a^8*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^6*b^6 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*
b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5
*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c

```
^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^
2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 +
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 -
2*(b^2 - 4*a*c)*a^6*b^4*c^2 + 6*(b^2 - 4*a*c)*a^7*b^2*c^3)*e^5)*arctan(2*sq
rt(1/2)*x/sqrt((a^2*b*c*d^2 - a^2*b^2*d*e + a^3*b*e^2 + sqrt((a^2*b*c*d^2 -
 a^2*b^2*d*e + a^3*b*e^2)^2 - 4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*(a^2*c^2*
d^2 - a^2*b*c*d*e + a^3*c*e^2)))/(a^2*c^2*d^2 - a^2*b*c*d*e + a^3*c*e^2)))/
((a^5*b^4*c^2 - 8*a^6*b^2*c^3 - 2*a^5*b^3*c^3 + 16*a^7*c^4 + 8*a^6*b*c^4 +
a^5*b^2*c^4 - 4*a^6*c^5)*d^4*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c) -
2*(a^5*b^5*c - 8*a^6*b^3*c^2 - 2*a^5*b^4*c^2 + 16*a^7*b*c^3 + 8*a^6*b^2*c^3
 + a^5*b^3*c^3 - 4*a^6*b*c^4)*d^3*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(
c)*e + (a^5*b^6 - 6*a^6*b^4*c - 2*a^5*b^5*c + 4*a^6*b^3*c^2 + a^5*b^4*c^2 +
 32*a^8*c^3 + 16*a^7*b*c^3 - 2*a^6*b^2*c^3 - 8*a^7*c^4)*d^2*abs(a^2*c*d^2 -
 a^2*b*d*e + a^3*e^2)*abs(c)*e^2 - 2*(a^6*b^5 - 8*a^7*b^3*c - 2*a^6*b^4*c +
 16*a^8*b*c^2 + 8*a^7*b^2*c^2 + a^6*b^3*c^2 - 4*a^7*b*c^3)*d*abs(a^2*c*d^2
- a^2*b*d*e + a^3*e^2)*abs(c)*e^3 + (a^7*b^4 - 8*a^8*b^2*c - 2*a^7*b^3*c +
16*a^9*c^2 + 8*a^8*b*c^2 + a^7*b^2*c^2 - 4*a^8*c^3)*abs(a^2*c*d^2 - a^2*b*d
*e + a^3*e^2)*abs(c)*e^4) - 1/8*((2*a^4*b^5*c^5 - 12*a^5*b^3*c^6 + 16*a^6*b
*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^
3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4
- 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^5 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^6 - 2*(b^2 -
 4*a*c)*a^4*b^3*c^5 + 4*(b^2 - 4*a*c)*a^5*b*c^6)*d^5 - (6*a^4*b^6*c^4 - 38*
a^5*b^4*c^5 + 56*a^6*b^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^5*b^4*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*b^5*c^3 - 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^6*b^2*c^4 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^5*b^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^4*b^4*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^5*b^2*c^5 - 6*(b^2 - 4*a*c)*a^4*b^4*c^4 + 14*(b^2 - 4*a*c)*a
^5*b^2*c^5)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2
- 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 2*a^2*b^6*c^3 + 24*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^3*b^3*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 18
*a^3*b^4*c^4 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^5 - 8*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 5*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b^2*c^5 + 48*a^4*b^2*c^5 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^4*c^6 - 32*a^5*c^6 - 2*(b^2 - 4*a*c)*a^2*b^4*c^3 + 10*(b^2 -
4*a*c)*a^3*b^2*c^4 - 8*(b^2 - 4*a*c)*a^4*c^5)*d^3*abs(a^2*c*d^2 - a^2*b*d*e
 + a^3*e^2) + (6*a^4*b^7*c^3 - 36*a^5*b^5*c^4 + 40*a^6*b^3*c^5 + 32*a^7*b*c
^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c
+ 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2
+ 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 -
 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 -
 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 -
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 8*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 + 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 6*(b^2
- 4*a*c)*a^4*b^5*c^3 + 12*(b^2 - 4*a*c)*a^5*b^3*c^4 + 8*(b^2 - 4*a*c)*a^6*b
*c^5)*d^3*e^2 + 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 19
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 4*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 4*a^2*b^7*c^2 + 56*sqrt(2)*sqrt(b*c - s
```

```
qrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^4*c^3 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 - 38*
a^3*b^5*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 24*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 11*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 112*a^4*b^3*c^4 + 12*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 96*a^5*b*c^5 - 4*(b^2 - 4*a*c)*a^2*b^5*c^2 +
 22*(b^2 - 4*a*c)*a^3*b^3*c^3 - 24*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*abs(a^2*c*d
^2 - a^2*b*d*e + a^3*e^2)*e + (2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4
*a*c)*a*b*c^4)*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)^2*d - (2*a^4*b^8*c^2 - 6*a
^5*b^6*c^3 - 28*a^6*b^4*c^4 + 80*a^7*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^6*c^2 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^7*b^2*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^6*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^5*b^4*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^6*b^2*c^4 - 2*(b^2 - 4*a*c)*a^4*b^6*c^2 - 2*(b^2 - 4*a*c)*a^
5*b^4*c^3 + 20*(b^2 - 4*a*c)*a^6*b^2*c^4)*d^2*e^3 - 2*(sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^8 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b^6*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c + 2*a^2*b^8*c
+ 23*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + 10*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^6*c^2 - 18*a^3*b^6*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b^2*c^3 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 5
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 46*a^4*b^4*c^3 - 16*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b*c^4 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^2*c^4 - 16*a^5*b^2*c^4 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^
5 - 32*a^6*c^5 - 2*(b^2 - 4*a*c)*a^2*b^6*c + 10*(b^2 - 4*a*c)*a^3*b^4*c^2 -
 6*(b^2 - 4*a*c)*a^4*b^2*c^3 - 8*(b^2 - 4*a*c)*a^5*c^4)*d*abs(a^2*c*d^2 - a
^2*b*d*e + a^3*e^2)*e^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a
^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*
a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)
^2*e + (4*a^5*b^7*c^2 - 26*a^6*b^5*c^3 + 36*a^7*b^3*c^4 + 16*a^8*b*c^5 - 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 13*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 18*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 10*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 8*sqrt(2)*sqr
```

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t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b*c^3 - 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 5*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 4*(b^2 - 4*a*c)*a^5*b
^5*c^2 + 10*(b^2 - 4*a*c)*a^6*b^3*c^3 + 4*(b^2 - 4*a*c)*a^7*b*c^4)*d*e^4 +
2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7 - 10*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^6*c + 2*a^3*b^7*c + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^
3*c^2 + 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 20*a^4*b^5*c^2 - 32*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^5*b^2*c^3 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^
3 + 64*a^5*b^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 -
64*a^6*b*c^4 - 2*(b^2 - 4*a*c)*a^3*b^5*c + 12*(b^2 - 4*a*c)*a^4*b^3*c^2 - 1
6*(b^2 - 4*a*c)*a^5*b*c^3)*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*e^3 - (2*a^
6*b^6*c^2 - 14*a^7*b^4*c^3 + 24*a^8*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^6 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^8*b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^6*b^4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^7*b^2*c^3 - 2*(b^2 - 4*a*c)*a^6*b^4*c^2 + 6*(b^2 - 4*a*c)*a
^7*b^2*c^3)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c*d^2 - a^2*b^2*d*e + a^3
*b*e^2 - sqrt((a^2*b*c*d^2 - a^2*b^2*d*e + a^3*b*e^2)^2 - 4*(a^3*c*d^2 - a^
3*b*d*e + a^4*e^2)*(a^2*c^2*d^2 - a^2*b*c*d*e + a^3*c*e^2)))/(a^2*c^2*d^2 -
 a^2*b*c*d*e + a^3*c*e^2)))/((a^5*b^4*c^2 - 8*a^6*b^2*c^3 - 2*a^5*b^3*c^3 +
 16*a^7*c^4 + 8*a^6*b*c^4 + a^5*b^2*c^4 - 4*a^6*c^5)*d^4*abs(a^2*c*d^2 - a^
2*b*d*e + a^3*e^2)*abs(c) - 2*(a^5*b^5*c - 8*a^6*b^3*c^2 - 2*a^5*b^4*c^2 +
16*a^7*b*c^3 + 8*a^6*b^2*c^3 + a^5*b^3*c^3 - 4*a^6*b*c^4)*d^3*abs(a^2*c*d^2
 - a^2*b*d*e + a^3*e^2)*abs(c)*e + (a^5*b^6 - 6*a^6*b^4*c - 2*a^5*b^5*c + 4
*a^6*b^3*c^2 + a^5*b^4*c^2 + 32*a^8*c^3 + 16*a^7*b*c^3 - 2*a^6*b^2*c^3 - 8*
a^7*c^4)*d^2*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c)*e^2 - 2*(a^6*b^5 -
 8*a^7*b^3*c - 2*a^6*b^4*c + 16*a^8*b*c^2 + 8*a^7*b^2*c^2 + a^6*b^3*c^2 - 4
*a^7*b*c^3)*d*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c)*e^3 + (a^7*b^4 -
8*a^8*b^2*c - 2*a^7*b^3*c + 16*a^9*c^2 + 8*a^8*b*c^2 + a^7*b^2*c^2 - 4*a^8*
c^3)*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c)*e^4) + arctan(x*e^(1/2)/sq
rt(d))*e^(7/2)/((c*d^4 - b*d^3*e + a*d^2*e^2)*sqrt(d)) + 1/3*(3*b*d*x^2 + 3
*a*x^2*e - a*d)/(a^2*d^2*x^3)
```

maple [B] time = 0.04, size = 1160, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3/a/d/x^3+1/a/d^2*e/x+1/d/a^2/x*b-1/2/(a*e^2-b*d*e+c*d^2)/a*c^2*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*e+1/2/(a*e^2-b*d*e+c*d^2)/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e-1
/2/(a*e^2-b*d*e+c*d^2)/a^2*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-3/2/(a*e^2-b*d*e+c
*d^2)/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e+1/(a*e^2-b*d*e+c*d
^2)/a*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/2/(a*e^2-b*d*e+c*d^2
)/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e-1/2/(a*e^2-b*d*e+c*d
^2)/a^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
```

```
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+1/2/(a*e^2-b*d*e
+c*d^2)/a*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-1/2/(a*e^2-b*d*e+c*d^2)/a^2*c*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*b^2*e+1/2/(a*e^2-b*d*e+c*d^2)/a^2*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-3/2/(
a*e^2-b*d*e+c*d^2)/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e+1/(a*e^2
-b*d*e+c*d^2)/a*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/2/(a*e^2-b*d*
e+c*d^2)/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e-1/2/(a*e^2-b*d*e
+c*d^2)/a^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+1/d^2*e^4/(a*e^
2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c*d^4 - b*d^3*e + a*d^2*e^2)*sqrt(d*e)) + integ
rate(((b*c^2*d - (b^2*c - a*c^2)*e)*x^2 + (b^2*c - a*c^2)*d - (b^3 - 2*a*b*
c)*e)/(c*x^4 + b*x^2 + a), x)/(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) + 1/3*(3*(b
*d + a*e)*x^2 - a*d)/(a^2*d^2*x^3)
```

mupad [B] time = 6.73, size = 42882, normalized size = 123.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(c^9*d^27*e^6 - b^9*d^18*e^15 + 2*a*c^8*d^25*e^8 - 2*b*c^8*d^26*e^7 + 2
*b^8*c*d^19*e^14 + a^5*b^4*d^13*e^20 + a^2*c^7*d^23*e^10 + 16*a^4*c^5*d^19*
e^14 + 16*a^7*c^2*d^13*e^20 + b^2*c^7*d^25*e^8 - b^7*c^2*d^20*e^13 - 25*a^2
*b^3*c^4*d^20*e^13 + 66*a^2*b^4*c^3*d^19*e^14 - 42*a^2*b^5*c^2*d^18*e^15 -
76*a^3*b^2*c^4*d^19*e^14 + 63*a^3*b^3*c^3*d^18*e^15 - a^5*b^4*e^3*x*(-d^5*e
^7)^(5/2) + a^2*c^7*d^15*x*(-d^5*e^7)^(3/2) - 16*a^7*c^2*e^3*x*(-d^5*e^7)^(
5/2) - b^9*d^10*e^5*x*(-d^5*e^7)^(3/2) - c^9*d^24*e^3*x*(-d^5*e^7)^(1/2) -
2*a*b*c^7*d^24*e^9 + 11*a*b^7*c*d^18*e^15 + 9*a*b^5*c^3*d^20*e^13 - 20*a*b^
6*c^2*d^19*e^14 + 20*a^3*b*c^5*d^20*e^13 - 28*a^4*b*c^4*d^18*e^15 - 8*a^6*b
^2*c*d^13*e^20 + 16*a^4*c^5*d^11*e^4*x*(-d^5*e^7)^(3/2) - b^7*c^2*d^12*e^3*
x*(-d^5*e^7)^(3/2) - b^2*c^7*d^22*e^5*x*(-d^5*e^7)^(1/2) + 8*a^6*b^2*c*e^3*
x*(-d^5*e^7)^(5/2) - 2*a*c^8*d^22*e^5*x*(-d^5*e^7)^(1/2) + 2*b^8*c*d^11*e^4
*x*(-d^5*e^7)^(3/2) + 2*b*c^8*d^23*e^4*x*(-d^5*e^7)^(1/2) + 11*a*b^7*c*d^10
*e^5*x*(-d^5*e^7)^(3/2) + 2*a*b*c^7*d^21*e^6*x*(-d^5*e^7)^(1/2) + 9*a*b^5*c
^3*d^12*e^3*x*(-d^5*e^7)^(3/2) - 20*a*b^6*c^2*d^11*e^4*x*(-d^5*e^7)^(3/2) +
 20*a^3*b*c^5*d^12*e^3*x*(-d^5*e^7)^(3/2) - 28*a^4*b*c^4*d^10*e^5*x*(-d^5*e
^7)^(3/2) - 25*a^2*b^3*c^4*d^12*e^3*x*(-d^5*e^7)^(3/2) + 66*a^2*b^4*c^3*d^1
1*e^4*x*(-d^5*e^7)^(3/2) - 42*a^2*b^5*c^2*d^10*e^5*x*(-d^5*e^7)^(3/2) - 76*
a^3*b^2*c^4*d^11*e^4*x*(-d^5*e^7)^(3/2) + 63*a^3*b^3*c^3*d^10*e^5*x*(-d^5*e
^7)^(3/2))*(-d^5*e^7)^(1/2))/(2*c*d^7 + 2*a*d^5*e^2 - 2*b*d^6*e) - atan((((
-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^
2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2
- a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^
3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e
```

```
*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*
a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*
d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c -
 b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4
+ 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5
*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2
*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d
*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b
^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*
c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2
*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c
^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b
^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2
*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*
d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4
- 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^
3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6
*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 -
 b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28
*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*
a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2
 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*
c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9
*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*
c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7
*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3
*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^2
2*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^2
5*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^
6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 105
6*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*
e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c
^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^
21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9
+ 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^2
3*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) - 64*a^18*c^8*d^
24*e^2 + 128*a^19*c^7*d^22*e^4 + 192*a^20*c^6*d^20*e^6 - 256*a^21*c^5*d^18*
e^8 - 256*a^22*c^4*d^16*e^10 - 16*a^16*b^4*c^6*d^24*e^2 + 64*a^16*b^5*c^5*d
^23*e^3 - 96*a^16*b^6*c^4*d^22*e^4 + 64*a^16*b^7*c^3*d^21*e^5 - 16*a^16*b^8
*c^2*d^20*e^6 + 80*a^17*b^2*c^7*d^24*e^2 - 368*a^17*b^3*c^6*d^23*e^3 + 608*
a^17*b^4*c^5*d^22*e^4 - 416*a^17*b^5*c^4*d^21*e^5 + 80*a^17*b^6*c^3*d^20*e^
6 + 16*a^17*b^7*c^2*d^19*e^7 - 928*a^18*b^2*c^6*d^22*e^4 + 640*a^18*b^3*c^5
*d^21*e^5 + 32*a^18*b^4*c^4*d^20*e^6 - 128*a^18*b^5*c^3*d^19*e^7 - 432*a^19
*b^2*c^5*d^20*e^6 + 304*a^19*b^3*c^4*d^19*e^7 - 16*a^19*b^4*c^3*d^18*e^8 +
16*a^19*b^5*c^2*d^17*e^9 + 128*a^20*b^2*c^4*d^18*e^8 - 128*a^20*b^3*c^3*d^1
7*e^9 - 16*a^20*b^4*c^2*d^16*e^10 + 128*a^21*b^2*c^3*d^16*e^10 + 448*a^18*b
*c^7*d^23*e^3 - 192*a^20*b*c^5*d^19*e^7 + 256*a^21*b*c^4*d^17*e^9) - x*(16*
a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20
*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^1
4*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8
*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^
3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*
```

d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b ^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 5 76*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16* e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^ 4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17 *b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^2 0*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^ 2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2 *d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6* c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3* d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b ^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1 /2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 3 2*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d *e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) - 4*a^15*c^9*d^21*e^3 - 4*a^16*c^8*d^19*e^5 + 48*a^18*c^6*d^15*e^9 - 4 *a^14*b^2*c^8*d^21*e^3 - 4*a^14*b^7*c^3*d^16*e^8 + 4*a^14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d^16*e^8 - 44*a^15*b^6*c^3*d^15*e^9 + 4*a^15*b^7*c^2*d^14 *e^10 - 100*a^16*b^3*c^5*d^16*e^8 + 160*a^16*b^4*c^4*d^15*e^9 - 32*a^16*b^5 *c^3*d^14*e^10 - 204*a^17*b^2*c^5*d^15*e^9 + 76*a^17*b^3*c^4*d^14*e^10 + 4* a^14*b*c^9*d^22*e^2 + 8*a^15*b*c^8*d^20*e^4 + 80*a^17*b*c^6*d^16*e^8 - 48*a ^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^1 4*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 - b ^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a ^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^ 2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a* c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2) ^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4* a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^ 3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c ^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^ 3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b *c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*1i + ((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(- (4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4* e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/ 2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^ 3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c ^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^ 2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a ^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3* e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6* b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2) ^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c *d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b ^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^ 4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a *b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4

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*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) -
8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e
^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^
2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^
3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2
)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) -
9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^
2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 -
 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e
 + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 7
6*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*
d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b
^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c
^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 -
32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(5
12*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*
a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 -
192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^2
1*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b
^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512
*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*
e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c
^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^
19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 64
0*a^22*b*c^4*d^19*e^8) + 64*a^18*c^8*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192
*a^20*c^6*d^20*e^6 + 256*a^21*c^5*d^18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^
16*b^4*c^6*d^24*e^2 - 64*a^16*b^5*c^5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 -
 64*a^16*b^7*c^3*d^21*e^5 + 16*a^16*b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24
*e^2 + 368*a^17*b^3*c^6*d^23*e^3 - 608*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5
*c^4*d^21*e^5 - 80*a^17*b^6*c^3*d^20*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a
^18*b^2*c^6*d^22*e^4 - 640*a^18*b^3*c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6
 + 128*a^18*b^5*c^3*d^19*e^7 + 432*a^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4
*d^19*e^7 + 16*a^19*b^4*c^3*d^18*e^8 - 16*a^19*b^5*c^2*d^17*e^9 - 128*a^20*
b^2*c^4*d^18*e^8 + 128*a^20*b^3*c^3*d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 -
128*a^21*b^2*c^3*d^16*e^10 - 448*a^18*b*c^7*d^23*e^3 + 192*a^20*b*c^5*d^19*
e^7 - 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21
*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23
*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*
d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2
*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^
15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7
- 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d
^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*
b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*
a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e
^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^1
8*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7
*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^
5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3
*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a
*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2
)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c
^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2)
```

+ 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 4*a^15*c^9*d^21*e^3 + 4*a^16*c^8*d^19*e^5 - 48*a^18*c^6*d^15*e^9 + 4*a^14*b^2*c^8*d^21*e^3 + 4*a^14*b^7*c^3*d^16*e^8 - 4*a^14*b^8*c^2*d^15*e^9 - 36*a^15*b^5*c^4*d^16*e^8 + 44*a^15*b^6*c^3*d^15*e^9 - 4*a^15*b^7*c^2*d^14*e^10 + 100*a^16*b^3*c^5*d^16*e^8 - 160*a^16*b^4*c^4*d^15*e^9 + 32*a^16*b^5*c^3*d^14*e^10 + 204*a^17*b^2*c^5*d^15*e^9 - 76*a^17*b^3*c^4*d^14*e^10 - 4*a^14*b*c^9*d^22*e^2 - 8*a^15*b*c^8*d^20*e^4 - 80*a^17*b*c^6*d^16*e^8 + 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2

```
*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*
a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b
^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*
a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*
d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*
d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*
d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18
*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 +
1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^
21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^
3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384
*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e
^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*
d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) + 64*a^18*c^8
*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192*a^20*c^6*d^20*e^6 + 256*a^21*c^5*d^
18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^16*b^4*c^6*d^24*e^2 - 64*a^16*b^5*c^
5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 - 64*a^16*b^7*c^3*d^21*e^5 + 16*a^16*
b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24*e^2 + 368*a^17*b^3*c^6*d^23*e^3 - 6
08*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5*c^4*d^21*e^5 - 80*a^17*b^6*c^3*d^20
*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a^18*b^2*c^6*d^22*e^4 - 640*a^18*b^3*
c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6 + 128*a^18*b^5*c^3*d^19*e^7 + 432*a
^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4*d^19*e^7 + 16*a^19*b^4*c^3*d^18*e^8
 - 16*a^19*b^5*c^2*d^17*e^9 - 128*a^20*b^2*c^4*d^18*e^8 + 128*a^20*b^3*c^3*
d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 - 128*a^21*b^2*c^3*d^16*e^10 - 448*a^1
8*b*c^7*d^23*e^3 + 192*a^20*b*c^5*d^19*e^7 - 256*a^21*b*c^4*d^17*e^9) - x*(
16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a
^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*
a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6
+ 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22
*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c
^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^1
6*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7
+ 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^
16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2
*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a
^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*
a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d
*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5
*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*
c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b
^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*
a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c
^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*
a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4
 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2
+ 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*
c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2))
)^(1/2) + 4*a^15*c^9*d^21*e^3 + 4*a^16*c^8*d^19*e^5 - 48*a^18*c^6*d^15*e^9
+ 4*a^14*b^2*c^8*d^21*e^3 + 4*a^14*b^7*c^3*d^16*e^8 - 4*a^14*b^8*c^2*d^15*e
^9 - 36*a^15*b^5*c^4*d^16*e^8 + 44*a^15*b^6*c^3*d^15*e^9 - 4*a^15*b^7*c^2*d
^14*e^10 + 100*a^16*b^3*c^5*d^16*e^8 - 160*a^16*b^4*c^4*d^15*e^9 + 32*a^16*
b^5*c^3*d^14*e^10 + 204*a^17*b^2*c^5*d^15*e^9 - 76*a^17*b^3*c^4*d^14*e^10 -
 4*a^14*b*c^9*d^22*e^2 - 8*a^15*b*c^8*d^20*e^4 - 80*a^17*b*c^6*d^16*e^8 + 4
8*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*
a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2
- b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 2
8*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c -
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b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) - ((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) - 64*a^18*c^8*d^24*e^2 + 128*a^19*c^7*d^22*e^4 + 192 *a^20*c^6*d^20*e^6 - 256*a^21*c^5*d^18*e^8 - 256*a^22*c^4*d^16*e^10 - 16*a^16*b^4*c^6*d^24*e^2 + 64*a^16*b^5*c^5*d^23*e^3 - 96*a^16*b^6*c^4*d^22*e^4 + 64*a^16*b^7*c^3*d^21*e^5 - 16*a^16*b^8*c^2*d^20*e^6 + 80*a^17*b^2*c^7*d^24*e^2 - 368*a^17*b^3*c^6*d^23*e^3 + 608*a^17*b^4*c^5*d^22*e^4 - 416*a^17*b^5*c^4*d^21*e^5 + 80*a^17*b^6*c^3*d^20*e^6 + 16*a^17*b^7*c^2*d^19*e^7 - 928*a^18*b^2*c^6*d^22*e^4 + 640*a^18*b^3*c^5*d^21*e^5 + 32*a^18*b^4*c^4*d^20*e^6 - 128*a^18*b^5*c^3*d^19*e^7 - 432*a^19*b^2*c^5*d^20*e^6 + 304*a^19*b^3*c^4*d^19*e^7 - 16*a^19*b^4*c^3*d^18*e^8 + 16*a^19*b^5*c^2*d^17*e^9 + 128*a^20*b^2*c^4*d^18*e^8 - 128*a^20*b^3*c^3*d^17*e^9 - 16*a^20*b^4*c^2*d^16*e^10 + 128*a^21*b^2*c^3*d^16*e^10 + 448*a^18*b*c^7*d^23*e^3 - 192*a^20*b*c^5*d^19*e^7 + 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) - 4*a^15*c^9*d^21*e^3 - 4*a^16*c^8*d^19*e^5 + 48*a^18*c^6*d^15*e^9 - 4*a^14*b^2*c^8*d^21*e^3 - 4*a^14*b^7*c^3*d^16*e^8 + 4*a^14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d^16*e^8 - 44*a^15*b^6*c^3*d^15*e^9 + 4*a^15*b^7*c^2*d^14*e^10 - 100*a^16*b^3*c^5*d^16*e^8 + 160*a^16*b^4*c^4*d^15*e^9 - 32*a^16*b^5*c^3*d^14*e^10 - 204*a^17*b^2*c^5*d^15*e^9 + 76*a^17*b^3*c^4*d^14*e^10 + 4*a^14*b*c^9*d^22*e^2 + 8*a^15*b*c^8*d^20*e^4 + 80*a^17*b*c^6*d^16*e^8 - 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 2*a^14*c^8*d^14*e^8))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2

d^2(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*
c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*
d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b
^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 -
2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 3
2*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d
*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(
1/2)*2i - (1/(3*a*d) - (x^2*(a*e + b*d))/(a^2*d^2))/x^3 - (log(c^9*d^27*e^6
 - b^9*d^18*e^15 + 2*a*c^8*d^25*e^8 - 2*b*c^8*d^26*e^7 + 2*b^8*c*d^19*e^14
+ a^5*b^4*d^13*e^20 + a^2*c^7*d^23*e^10 + 16*a^4*c^5*d^19*e^14 + 16*a^7*c^2
*d^13*e^20 + b^2*c^7*d^25*e^8 - b^7*c^2*d^20*e^13 - 25*a^2*b^3*c^4*d^20*e^1
3 + 66*a^2*b^4*c^3*d^19*e^14 - 42*a^2*b^5*c^2*d^18*e^15 - 76*a^3*b^2*c^4*d^
19*e^14 + 63*a^3*b^3*c^3*d^18*e^15 + a^5*b^4*e^3*x*(-d^5*e^7)^(5/2) - a^2*c
^7*d^15*x*(-d^5*e^7)^(3/2) + 16*a^7*c^2*e^3*x*(-d^5*e^7)^(5/2) + b^9*d^10*e
^5*x*(-d^5*e^7)^(3/2) + c^9*d^24*e^3*x*(-d^5*e^7)^(1/2) - 2*a*b*c^7*d^24*e^
9 + 11*a*b^7*c*d^18*e^15 + 9*a*b^5*c^3*d^20*e^13 - 20*a*b^6*c^2*d^19*e^14 +
 20*a^3*b*c^5*d^20*e^13 - 28*a^4*b*c^4*d^18*e^15 - 8*a^6*b^2*c*d^13*e^20 -
16*a^4*c^5*d^11*e^4*x*(-d^5*e^7)^(3/2) + b^7*c^2*d^12*e^3*x*(-d^5*e^7)^(3/2
) + b^2*c^7*d^22*e^5*x*(-d^5*e^7)^(1/2) - 8*a^6*b^2*c*e^3*x*(-d^5*e^7)^(5/2
) + 2*a*c^8*d^22*e^5*x*(-d^5*e^7)^(1/2) - 2*b^8*c*d^11*e^4*x*(-d^5*e^7)^(3/
2) - 2*b*c^8*d^23*e^4*x*(-d^5*e^7)^(1/2) - 11*a*b^7*c*d^10*e^5*x*(-d^5*e^7)
^(3/2) - 2*a*b*c^7*d^21*e^6*x*(-d^5*e^7)^(1/2) - 9*a*b^5*c^3*d^12*e^3*x*(-d
^5*e^7)^(3/2) + 20*a*b^6*c^2*d^11*e^4*x*(-d^5*e^7)^(3/2) - 20*a^3*b*c^5*d^1
2*e^3*x*(-d^5*e^7)^(3/2) + 28*a^4*b*c^4*d^10*e^5*x*(-d^5*e^7)^(3/2) + 25*a^
2*b^3*c^4*d^12*e^3*x*(-d^5*e^7)^(3/2) - 66*a^2*b^4*c^3*d^11*e^4*x*(-d^5*e^7
)^(3/2) + 42*a^2*b^5*c^2*d^10*e^5*x*(-d^5*e^7)^(3/2) + 76*a^3*b^2*c^4*d^11*
e^4*x*(-d^5*e^7)^(3/2) - 63*a^3*b^3*c^3*d^10*e^5*x*(-d^5*e^7)^(3/2))*(-d^5*
e^7)^(1/2))/(2*(c*d^7 + a*d^5*e^2 - b*d^6*e)) - atan((((-(b^9*e^2 + b^7*c^2
*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^
2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*
a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2
*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7
*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)
^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*
a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 +
16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^
6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e -
32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c
^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*
c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b
^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a
^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^
2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^
4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4
 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*
d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*
a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*
b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*
a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2)

+ b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) - 64*a^18*c^8*d^24*e^2 + 128*a^19*c^7*d^22*e^4 + 192*a^20*c^6*d^20*e^6 - 256*a^21*c^5*d^18*e^8 - 256*a^22*c^4*d^16*e^10 - 16*a^16*b^4*c^6*d^24*e^2 + 64*a^16*b^5*c^5*d^23*e^3 - 96*a^16*b^6*c^4*d^22*e^4 + 64*a^16*b^7*c^3*d^21*e^5 - 16*a^16*b^8*c^2*d^20*e^6 + 80*a^17*b^2*c^7*d^24*e^2 - 368*a^17*b^3*c^6*d^23*e^3 + 608*a^17*b^4*c^5*d^22*e^4 - 416*a^17*b^5*c^4*d^21*e^5 + 80*a^17*b^6*c^3*d^20*e^6 + 16*a^17*b^7*c^2*d^19*e^7 - 928*a^18*b^2*c^6*d^22*e^4 + 640*a^18*b^3*c^5*d^21*e^5 + 32*a^18*b^4*c^4*d^20*e^6 - 128*a^18*b^5*c^3*d^19*e^7 - 432*a^19*b^2*c^5*d^20*e^6 + 304*a^19*b^3*c^4*d^19*e^7 - 16*a^19*b^4*c^3*d^18*e^8 + 16*a^19*b^5*c^2*d^17*e^9 + 128*a^20*b^2*c^4*d^18*e^8 - 128*a^20*b^3*c^3*d^17*e^9 - 16*a^20*b^4*c^2*d^16*e^10 + 128*a^21*b^2*c^3*d^16*e^10 + 448*a^18*b*c^7*d^23*e^3 - 192*a^20*b*c^5*d^19*e^7 + 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11)) *(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) - 4*a^15*c^9*d^21*e^3 - 4*a^16*c^8*d^19*e^5 + 48*a^18*c^6*d^15*e^9 - 4*a^14*b^2*c^8*d^21*e^3 - 4*a^14*b^7*c^3*d^16*e^8 + 4*a^14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d^16*e^8 - 44*a^15*b^6*c^3*d^15*e^9 + 4*a^15*b^7*c^2*d^14*e^10 - 100*a^16*b^3*c^5*d^16*e^8 + 160*a^16*b^4*c^4*d^15*e^9 - 32*a^16*b^5*c^3*d^14*e^10 - 204*a^17*b^2*c^5*d^15*e^9 + 76*a^17*b^3*c^4*d^14*e^10 + 4*a^14*b*c^9*d^22*e^2

+ 8*a^15*b*c^8*d^20*e^4 + 80*a^17*b*c^6*d^16*e^8 - 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b ^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^ 8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^ 2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 2 0*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2* (-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2* b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^ 2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2* c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2 *e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7 *b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2* e^2)))^(1/2)*1i + ((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1 /2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2 *e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2* d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c ^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d *e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^ 3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/ 2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2 *a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32 *a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d* e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1 /2)*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5 *c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c ^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3 *b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^ 5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/ 2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b ^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b ^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^ 3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2* e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8* b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9 *e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 2 0*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2 *c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1 /2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4 *a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4 *c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2) ^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16* a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4* c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5* b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^ 3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^ 23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^ 2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 9 6*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*

e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c
^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^
21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 -
1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e
^8) + 64*a^18*c^8*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192*a^20*c^6*d^20*e^6
+ 256*a^21*c^5*d^18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^16*b^4*c^6*d^24*e^2
- 64*a^16*b^5*c^5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 - 64*a^16*b^7*c^3*d^
21*e^5 + 16*a^16*b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24*e^2 + 368*a^17*b^3
*c^6*d^23*e^3 - 608*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5*c^4*d^21*e^5 - 80*
a^17*b^6*c^3*d^20*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a^18*b^2*c^6*d^22*e^
4 - 640*a^18*b^3*c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6 + 128*a^18*b^5*c^3
*d^19*e^7 + 432*a^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4*d^19*e^7 + 16*a^19
*b^4*c^3*d^18*e^8 - 16*a^19*b^5*c^2*d^17*e^9 - 128*a^20*b^2*c^4*d^18*e^8 +
128*a^20*b^3*c^3*d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 - 128*a^21*b^2*c^3*d^
16*e^10 - 448*a^18*b*c^7*d^23*e^3 + 192*a^20*b*c^5*d^19*e^7 - 256*a^21*b*c^
4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7
*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*
c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*
b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*
a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5
+ 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d
^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*
b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 +
8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e
^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8
*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c
^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^
4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(
1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4
*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*
b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b
^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4
- 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 +
a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^
3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^
6*b^4*c*d^2*e^2)))^(1/2) + 4*a^15*c^9*d^21*e^3 + 4*a^16*c^8*d^19*e^5 - 48*a
^18*c^6*d^15*e^9 + 4*a^14*b^2*c^8*d^21*e^3 + 4*a^14*b^7*c^3*d^16*e^8 - 4*a^
14*b^8*c^2*d^15*e^9 - 36*a^15*b^5*c^4*d^16*e^8 + 44*a^15*b^6*c^3*d^15*e^9 -
4*a^15*b^7*c^2*d^14*e^10 + 100*a^16*b^3*c^5*d^16*e^8 - 160*a^16*b^4*c^4*d^
15*e^9 + 32*a^16*b^5*c^3*d^14*e^10 + 204*a^17*b^2*c^5*d^15*e^9 - 76*a^17*b^
3*c^4*d^14*e^10 - 4*a^14*b*c^9*d^22*e^2 - 8*a^15*b*c^8*d^20*e^4 - 80*a^17*b
*c^6*d^16*e^8 + 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*
c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e
^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*
a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c
^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 -
a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a
*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c
e^2(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3
*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3
)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^
7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^
2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^
5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 +

16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) + 64*a^18*c^8*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192*a^20*c^6*d^20*e^6 + 256*a^21*c^5*d^18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^16*b^4*c^6*d^24*e^2 - 64*a^16*b^5*c^5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 - 64*a^16*b^7*c^3*d^21*e^5 + 16*a^16*b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24*e^2 + 368*a^17*b^3*c^6*d^23*e^3 - 608*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5*c^4*d^21*e^5 - 80*a^17*b^6*c^3*d^20*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a^18*b^2*c^6*d^22*e^4 - 640*a^18*b^3*c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6 + 128*a^18*b^5*c^3*d^19*e^7 + 432*a^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4*d^19*e^7 + 16*a^19*b^4*c^3*d^18*e^8 - 16*a^19*b^5*c^2*d^17*e^9 - 128*a^20*b^2*c^4*d^18*e^8 + 128*a^20*b^3*c^3*d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 - 128*a^21*b^2*c^3*d^16*e^10 - 448*a^18*b*c^7*d^23*e^3 +

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192*a^20*b*c^5*d^19*e^7 - 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^
2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 +
 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e
^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^
18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*
c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^
15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5
- 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*
d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^
4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56
*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5
+ 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^1
1))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c
^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4
*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b
^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*
c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2
*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*
a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4
*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3
+ a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^
2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*
c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 4*a^15*c^
9*d^21*e^3 + 4*a^16*c^8*d^19*e^5 - 48*a^18*c^6*d^15*e^9 + 4*a^14*b^2*c^8*d^
21*e^3 + 4*a^14*b^7*c^3*d^16*e^8 - 4*a^14*b^8*c^2*d^15*e^9 - 36*a^15*b^5*c^
4*d^16*e^8 + 44*a^15*b^6*c^3*d^15*e^9 - 4*a^15*b^7*c^2*d^14*e^10 + 100*a^16
*b^3*c^5*d^16*e^8 - 160*a^16*b^4*c^4*d^15*e^9 + 32*a^16*b^5*c^3*d^14*e^10 +
 204*a^17*b^2*c^5*d^15*e^9 - 76*a^17*b^3*c^4*d^14*e^10 - 4*a^14*b*c^9*d^22*
e^2 - 8*a^15*b*c^8*d^20*e^4 - 80*a^17*b*c^6*d^16*e^8 + 48*a^18*b*c^5*d^14*e
^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e
^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2
*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42
*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2
) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e
+ 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a
^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b
^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*
d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*
a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d
^2*e^2)))^(1/2) - ((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e +
 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2
*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c
^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d
*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^
3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2
*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32
*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*
e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1
/2)*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5
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*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c
^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3
*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^
5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b
^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(
4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b
^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^
3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*
e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*
b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9
*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 2
0*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2
*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2
 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4
*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e -
 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)
^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*
a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*
c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*
b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3
 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^
3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 -
 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^
23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^
2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 9
6*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*
e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c
^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^
21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 -
 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e
^8) - 64*a^18*c^8*d^24*e^2 + 128*a^19*c^7*d^22*e^4 + 192*a^20*c^6*d^20*e^6
 - 256*a^21*c^5*d^18*e^8 - 256*a^22*c^4*d^16*e^10 - 16*a^16*b^4*c^6*d^24*e^2
 + 64*a^16*b^5*c^5*d^23*e^3 - 96*a^16*b^6*c^4*d^22*e^4 + 64*a^16*b^7*c^3*d^
21*e^5 - 16*a^16*b^8*c^2*d^20*e^6 + 80*a^17*b^2*c^7*d^24*e^2 - 368*a^17*b^3
*c^6*d^23*e^3 + 608*a^17*b^4*c^5*d^22*e^4 - 416*a^17*b^5*c^4*d^21*e^5 + 80*
a^17*b^6*c^3*d^20*e^6 + 16*a^17*b^7*c^2*d^19*e^7 - 928*a^18*b^2*c^6*d^22*e^
4 + 640*a^18*b^3*c^5*d^21*e^5 + 32*a^18*b^4*c^4*d^20*e^6 - 128*a^18*b^5*c^3
*d^19*e^7 - 432*a^19*b^2*c^5*d^20*e^6 + 304*a^19*b^3*c^4*d^19*e^7 - 16*a^19
*b^4*c^3*d^18*e^8 + 16*a^19*b^5*c^2*d^17*e^9 + 128*a^20*b^2*c^4*d^18*e^8 -
128*a^20*b^3*c^3*d^17*e^9 - 16*a^20*b^4*c^2*d^16*e^10 + 128*a^21*b^2*c^3*d^
16*e^10 + 448*a^18*b*c^7*d^23*e^3 - 192*a^20*b*c^5*d^19*e^7 + 256*a^21*b*c^
4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7
*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*
c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*
b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*
a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5
 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d
^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*
b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 +
8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e
^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8
*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c
^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^
4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(
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1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4
*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*
b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b
^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4
- 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 +
 a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^
3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^
6*b^4*c*d^2*e^2)))^(1/2) - 4*a^15*c^9*d^21*e^3 - 4*a^16*c^8*d^19*e^5 + 48*a
^18*c^6*d^15*e^9 - 4*a^14*b^2*c^8*d^21*e^3 - 4*a^14*b^7*c^3*d^16*e^8 + 4*a^
14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d^16*e^8 - 44*a^15*b^6*c^3*d^15*e^9 +
 4*a^15*b^7*c^2*d^14*e^10 - 100*a^16*b^3*c^5*d^16*e^8 + 160*a^16*b^4*c^4*d^
15*e^9 - 32*a^16*b^5*c^3*d^14*e^10 - 204*a^17*b^2*c^5*d^15*e^9 + 76*a^17*b^
3*c^4*d^14*e^10 + 4*a^14*b*c^9*d^22*e^2 + 8*a^15*b*c^8*d^20*e^4 + 80*a^17*b
*c^6*d^16*e^8 - 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*
c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e
^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*
a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c
^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 -
 a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a
*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3
*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3
)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^
7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^
2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^
5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 +
16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 2*a^14*c^8*d^14*e^8))
*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*
d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^
2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*
c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d
*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^
4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c
 - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^
4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a
^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 -
 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2
*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.310 $\int \frac{1}{\sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=866

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)e^{7/4}}{\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}+1\right)e^{7/4}}{\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}-\frac{\log\left(\sqrt{e}\sqrt{f}\,x+\sqrt{d}\sqrt{f}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\cdots$$

```
[Out] -1/2*e^(7/4)*arctan(1-e^(1/4)*2^(1/2)*(f*x)^(1/2)/d^(1/4)/f^(1/2))/d^(3/4)/
(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)+1/2*e^(7/4)*arctan(1+e^(1/4)*2^(1/2)*(f
*x)^(1/2)/d^(1/4)/f^(1/2))/d^(3/4)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)-1/4*
e^(7/4)*ln(d^(1/2)*f^(1/2)+x*e^(1/2)*f^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(f*x)^
(1/2))/d^(3/4)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)+1/4*e^(7/4)*ln(d^(1/2)*f
^(1/2)+x*e^(1/2)*f^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(f*x)^(1/2))/d^(3/4)/(a*e^
2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)+1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*(f*x)^(1/
2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/f^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))*
2^(3/4)/(a*e^2-b*d*e+c*d^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2
)/f^(1/2)+1/2*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*(f*x)^(1/2)/(-b-(-4*a*c+b^2)^
(1/2))^(1/4)/f^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))*2^(3/4)/(a*e^2-b*d*e
+c*d^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)/f^(1/2)-1/2*c^(3/4
)*arctan(2^(1/4)*c^(1/4)*(f*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/f^(1/2))
*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))*2^(3/4)/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^
(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)/f^(1/2)-1/2*c^(3/4)*arctanh(2^(1/4)*c^(
1/4)*(f*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/f^(1/2))*(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))*2^(3/4)/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^
2)^(1/2))^(3/4)/f^(1/2)
```

Rubi [A] time = 2.51, antiderivative size = 866, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {1269, 1424, 211, 1165, 628, 1162, 617, 204, 1422, 212, 208, 205}

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)e^{7/4}}{\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}+1\right)e^{7/4}}{\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}-\frac{\log\left(\sqrt{e}\sqrt{f}\,x+\sqrt{d}\sqrt{f}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}\,d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f
*x])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
 + Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*ArcTan[1
 - (Sqrt[2]*e^(1/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[f*x])/
(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (c^
(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x
])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
+ Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt
[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*Log[Sqrt[d
]*Sqrt[f] + Sqrt[e]*Sqrt[f]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt
```

```
[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*Log[Sqrt[d]*Sqrt[f]
 + Sqrt[e]*Sqrt[f]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt[2]*d^(3/
4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1269

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/f, Subst[I
nt[x^(k*(m + 1) - 1)*(d + (e*x^(2*k))/f^2)^q*(a + (b*x^(2*k))/f^k + (c*x^(4
*k))/f^4)^p, x], x, (f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, p, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1424

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\left(d+\frac{ex^4}{f^2}\right)\left(a+\frac{bx^4}{f^2}+\frac{cx^8}{f^4}\right)}dx, x, \sqrt{fx}\right)}{f}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{e^2f^2}{\left(cd^2-bde+ae^2\right)\left(df^2+ex^4\right)} + \frac{cdf^4-bef^4-cef^2x^4}{\left(cd^2-bde+ae^2\right)\left(af^4+bf^2x^4+cx^8\right)}\right)dx, x, \sqrt{\right.}{f}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{cdf^4-bef^4-cef^2x^4}{af^4+bf^2x^4+cx^8}dx, x, \sqrt{fx}\right)}{\left(cd^2-bde+ae^2\right)f} + \frac{\left(2e^2f\right)\text{Subst}\left(\int \frac{1}{df^2+ex^4}dx, \right.}{cd^2-bde+ae^2}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{\sqrt{d}f-\sqrt{e}x^2}{df^2+ex^4}dx, x, \sqrt{fx}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)} + \frac{e^2\,\text{Subst}\left(\int \frac{\sqrt{d}f+\sqrt{e}x^2}{df^2+ex^4}dx, x, \sqrt{fx}\right.}{\sqrt{d}\left(cd^2-bde+ae^2\right)}$$

$$= \frac{e^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}f}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{f}x}{\sqrt[4]{e}}+x^2}dx, x, \sqrt{fx}\right)}{2\sqrt{d}\left(cd^2-bde+ae^2\right)} + \frac{e^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}f}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}}{\sqrt[4]{e}}}\right.}{2\sqrt{d}\left(cd^2-b\right.}$$

$$= \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} - \frac{c^{3/4}\left(2cd\right.}{\sqrt[4]{2}\sqrt{b^2-}}$$

$$= \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} - \frac{c^{3/4}\left(2cd\right.}{\sqrt[4]{2}\sqrt{b^2-}}$$

Mathematica [C] time = 0.38, size = 267, normalized size = 0.31

$$\frac{\sqrt{x}\left(\sqrt{2}e^{7/4}\left(-\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)+\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{}}{\sqrt[4]{d}}\right.\right.\right.}{4d^{3/4}\sqrt{fx}\left(e(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[x]*(Sqrt[2]*e^(7/4)*(-2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)]
 + 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - Log[Sqrt[d] - Sqrt[2]*
d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] + Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[x] + Sqrt[e]*x]) - 2*d^(3/4)*RootSum[a + b*#1^4 + c*#1^8 & , (-(c*d*
Log[Sqrt[x] - #1]) + b*e*Log[Sqrt[x] - #1] + c*e*Log[Sqrt[x] - #1]*#1^4)/(b
*#1^3 + 2*c*#1^7) & ]))/(4*d^(3/4)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[f*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage2
```

maple [C] time = 0.10, size = 336, normalized size = 0.39

$$\frac{f\left(-\mathrm{RootOf}\left(c\_Z^8+b f^2\_Z^4+a f^4\right)^4 ce-be f^2+cd f^2\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b f^2\_Z^4+a f^4\right)+\sqrt{fx}\right)}{2\left(a e^2-deb+c d^2\right)\left(2\,\mathrm{RootOf}\left(c\_Z^8+b f^2\_Z^4+a f^4\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b f^2\_Z^4+a f^4\right)^3 b f^2\right)}+\frac{\left(\frac{d f^2}{e}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x)
```

```
[Out] 1/2*f/(a*e^2-b*d*e+c*d^2)*sum((-_R^4*c*e-b*e*f^2+c*d*f^2)/(2*_R^7*c+_R^3*b*
f^2)*ln((f*x)^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b*f^2+a*f^4))+1/4/f*e^2/(a*e^
2-b*d*e+c*d^2)*(d*f^2/e)^(1/4)/d*2^(1/2)*ln((f*x+(d*f^2/e)^(1/4)*(f*x)^(1/2
)*2^(1/2)+(d*f^2/e)^(1/2))/(f*x-(d*f^2/e)^(1/4)*(f*x)^(1/2)*2^(1/2)+(d*f^2/
e)^(1/2)))+1/2/f*e^2/(a*e^2-b*d*e+c*d^2)*(d*f^2/e)^(1/4)/d*2^(1/2)*arctan(2
^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2)+1)+1/2/f*e^2/(a*e^2-b*d*e+c*d^2)*(d*f^2/
e)^(1/4)/d*2^(1/2)*arctan(2^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2 e^2\sqrt{x}}{cd^3\sqrt{f}-bd^2e\sqrt{f}+ade^2\sqrt{f}}+\frac{\frac{2\sqrt{2}e^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}d^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{e}\sqrt{x}\right)}{2\sqrt{\sqrt{d}\sqrt{e}}}\right)}{\sqrt{d}\sqrt{\sqrt{d}\sqrt{e}}}+\frac{2\sqrt{2}e^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}d^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{e}\sqrt{x}\right)}{2\sqrt{\sqrt{d}\sqrt{e}}}\right)}{\sqrt{d}\sqrt{\sqrt{d}\sqrt{e}}}+\frac{\sqrt{2}e^{\frac{7}{4}}\log\left(\sqrt{2}d^{\frac{1}{4}}e\right.}{d}}{4\left(cd^2\sqrt{f}-bde\sqrt{f}+ae^2\sqrt{f}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^2*sqrt(x)/(c*d^3*sqrt(f) - b*d^2*e*sqrt(f) + a*d*e^2*sqrt(f)) + 1/4*(2
*sqrt(2)*e^2*arctan(1/2*sqrt(2)*(sqrt(2)*d^(1/4)*e^(1/4) + 2*sqrt(e)*sqrt(x
))/sqrt(sqrt(d)*sqrt(e)))/(sqrt(d)*sqrt(sqrt(d)*sqrt(e))) + 2*sqrt(2)*e^2*a
rctan(-1/2*sqrt(2)*(sqrt(2)*d^(1/4)*e^(1/4) - 2*sqrt(e)*sqrt(x))/sqrt(sqrt(
d)*sqrt(e)))/(sqrt(d)*sqrt(sqrt(d)*sqrt(e))) + sqrt(2)*e^(7/4)*log(sqrt(2)*
d^(1/4)*e^(1/4)*sqrt(x) + sqrt(e)*x + sqrt(d))/d^(3/4) - sqrt(2)*e^(7/4)*lo
g(-sqrt(2)*d^(1/4)*e^(1/4)*sqrt(x) + sqrt(e)*x + sqrt(d))/d^(3/4))/(c*d^2*s
qrt(f) - b*d*e*sqrt(f) + a*e^2*sqrt(f)) + 2*sqrt(x)/(a*d*sqrt(f)) + integra
te(-((c^2*d - b*c*e)*x^(7/2) + (b*c*d - b^2*e + a*c*e)*x^(3/2))/(a^3*e^2*sq
rt(f) + (a^2*c*e^2*sqrt(f) + (c^2*d^2*sqrt(f) - b*c*d*e*sqrt(f))*a)*x^4 + (
c*d^2*sqrt(f) - b*d*e*sqrt(f))*a^2 + (a^2*b*e^2*sqrt(f) + (b*c*d^2*sqrt(f)
- b^2*d*e*sqrt(f))*a)*x^2), x)
```

mupad [B] time = 6.84, size = 43112, normalized size = 49.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] symsum(log(-root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^
7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*
a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 3
00941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^
6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d
^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*
b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 1468
00640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h
^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*
e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3
*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 10276044
8*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 +
 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6
*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*
e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^
6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^
6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 3407
8720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^1
2 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*
f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d
^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*
b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 166297
60*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 -
 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^
6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*
e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^
2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*
b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 43581
44*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 +
4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*
h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^1
5*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d
^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^
2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^
13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 2293
76*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 +
 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h
^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9
*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d
^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*
c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^1
4*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*
a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 314572
8*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 1802
240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 52
4288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 16
3840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 6
5536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 32
768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 587
20256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 29
360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 8
388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 10
48576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 2293
76*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*
a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10
*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d
```

^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d
^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f
^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 +
262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*
a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*
c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4
*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b
^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5
*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^
3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176
*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656
*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880
*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080
*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968
*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 51609
6*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 3932
16*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 3665
92*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 3563
52*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 3440
64*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 210
944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 144
640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 131
072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 10
4448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 91
904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 716
80*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864
*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*
a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a
^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*
b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^
7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d
^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^1
0*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^
14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^
4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8
+ 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 -
9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^
2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3
*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6
*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8
- 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 13107
2*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9
*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10
*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 +
65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^
2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e
*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 10
24*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*
d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 655
36*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*
h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^
3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2
*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d
^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*
f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4
+ 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a
^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^
6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h
^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9

*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4
- 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^
3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4
480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7
*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4
+ 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*
e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*
f^2*h^4 - c^7*e^7, h, k)*(root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 5138022
40*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^1
2 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^
9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c
^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*
a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 +
166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f
^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*
d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9
*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117
440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h
^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e
^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^
4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6
*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370
176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12
- 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f
^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^
5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9
*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*
a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 +
18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6
*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*
e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c
^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6
*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 56197
12*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12
- 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^
6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e
^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5
*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b
^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240
*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1
048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^
12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f
^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*
e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*
d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*
b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 17616076
8*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58
720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12
+ 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h
^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6
*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f
^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10
*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e
*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^
8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7
*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*
f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6
*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^
4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2

*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d

^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f
^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 19
2*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5
*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^
4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^
8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2
032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e
^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4
- 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d
^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 6
4*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 -
16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)*(root(8388608*a^7*b*c^11*d^18*e*f^6*
h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^
9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b
^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601
280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^1
2 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e
^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*
c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544
*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 +
119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*
f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*
d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11
*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065
600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12
- 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^
6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6
*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*
c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a
^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22
118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^
12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e
^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^
7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5
*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 1048
5760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12
+ 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^
6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13
*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c
^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*
b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 389939
2*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 +
3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*
h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^
8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d
^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*
c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*
b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688
*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 2
93601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h
^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5
*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^1
6*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^1
0*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*
c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^1
1*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b
^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*
b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*
b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b

^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^

9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^
2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8
*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4
 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 76
80*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7
*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^
8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 +
 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*
d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 +
 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e
^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64
*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3
*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4
 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^
10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*
d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)^3*(root(8388608*a^7*b*
c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664
*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 -
 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f
^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*
d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^
7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146
800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*
h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^1
3*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b
^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760
448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12
 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*
f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^
9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7
*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*
a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 3
4078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h
^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e
^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4
*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7
*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 183
50080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^1
2 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14
*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d
^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3
*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a
^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 55
70560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^1
2 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f
^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16
*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c
^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*
b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 68812
8*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 -
286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^
12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f
^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*
e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^
8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^
9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 5872025
6*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 838
8608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 -
3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 -

2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^1
2 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^
12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^
12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^
12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12
 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^1
2 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^
12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^
12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^1
2 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 +
 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 11
4688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*
a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^1
6*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c
^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19
*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h
^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328
*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a
^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632
*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 157286
4*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261
568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 11
10016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 -
 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 -
 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 +
714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 +
 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 +
 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8
+ 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8
+ 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8
 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8
- 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8
 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^
8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8
 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^
8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h
^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^
8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8
+ 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 -
 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 +
909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651
264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*
a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^
9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*
c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*
c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*
d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^
6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^
4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8
 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 1792
0*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*
c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^
2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^
8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 983
04*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^
10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^1
2*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8
 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*
b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*

e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 409
6*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d
^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*
b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*
h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 585
6*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2
*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7
*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e
^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*
h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*
a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d
^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h
^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d
^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 -
16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*
e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 +
3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^
6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4
+ 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^
2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)*(root(
8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^
12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*
e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*
c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 29360128
0*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12
- 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*
f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9
*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a
^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 11
7440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6
*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^1
3*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4
*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*
a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40
370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^
12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^
3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*
d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*
b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 221184
00*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12
- 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*
f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^
7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^
6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*
a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 83
88608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^
12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9
*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^
8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*
c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^
4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523
136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12
+ 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^
6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^
16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d
^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*
c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*
b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 29360
1280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12

+ 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8

- 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)^3*(root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 22937

6*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 +
163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h^
12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9*
f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d^
8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*c
^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^14
*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*a
^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 3145728
*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 18022
40*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 524
288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 163
840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 65
536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 327
68*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 5872
0256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 293
60128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 83
88608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 104
8576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 22937
6*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*a
^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10*
b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d^
3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d^
19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f^
6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 +
262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*a
^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*c
^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4*
c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b^
5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5*
b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^3
*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176*
a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656*
a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880*
a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080*
a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968*
a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 516096
*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 39321
6*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 36659
2*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 35635
2*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 34406
4*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 2109
44*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 1446
40*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 1310
72*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 104
448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 919
04*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 7168
0*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864*
a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*a
^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a^
9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*b
*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^7
*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d^
13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^10
*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^1
4*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^4
*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8
+ 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 - 9
472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^2
*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3*

d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*
f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 -
360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072
*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*
d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*
f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 6
5536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2
*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*
f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 102
4*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d
^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 6553
6*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h
^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3
*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*
c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^
4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f
^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4
+ 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^
3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6
*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^
4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*
d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 -
16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3
*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 44
80*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*
c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4
+ 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e
*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f
^2*h^4 - c^7*e^7, h, k)*(4697620480*a^9*c^11*d^7*e^13*f^55 - 1879048192*a^6
*c^14*d^13*e^7*f^55 - 2818572288*a^7*c^13*d^11*e^9*f^55 - 402653184*a^5*c^1
5*d^15*e^5*f^55 + 5637144576*a^10*c^10*d^5*e^15*f^55 + 2818572288*a^11*c^9*
d^3*e^17*f^55 + 536870912*a^12*c^8*d*e^19*f^55 + 2097152*a*b^7*c^12*d^16*e^
4*f^55 - 16777216*a*b^8*c^11*d^15*e^5*f^55 + 58720256*a*b^9*c^10*d^14*e^6*f
^55 - 117440512*a*b^10*c^9*d^13*e^7*f^55 + 146800640*a*b^11*c^8*d^12*e^8*f^
55 - 117440512*a*b^12*c^7*d^11*e^9*f^55 + 58720256*a*b^13*c^6*d^10*e^10*f^5
5 - 16777216*a*b^14*c^5*d^9*e^11*f^55 + 2097152*a*b^15*c^4*d^8*e^12*f^55 -
134217728*a^4*b*c^15*d^16*e^4*f^55 + 2147483648*a^5*b*c^14*d^14*e^6*f^55 +
10066329600*a^6*b*c^13*d^12*e^8*f^55 + 13421772800*a^7*b*c^12*d^10*e^10*f^5
5 + 671088640*a^8*b*c^11*d^8*e^12*f^55 + 2097152*a^8*b^8*c^4*d*e^19*f^55 -
12884901888*a^9*b*c^10*d^6*e^14*f^55 - 33554432*a^9*b^6*c^5*d*e^19*f^55 - 1
0603200512*a^10*b*c^9*d^4*e^16*f^55 + 201326592*a^10*b^4*c^6*d*e^19*f^55 -
2684354560*a^11*b*c^8*d^2*e^18*f^55 - 536870912*a^11*b^2*c^7*d*e^19*f^55 -
25165824*a^2*b^5*c^13*d^16*e^4*f^55 + 207618048*a^2*b^6*c^12*d^15*e^5*f^55
- 738197504*a^2*b^7*c^11*d^14*e^6*f^55 + 1468006400*a^2*b^8*c^10*d^13*e^7*f
^55 - 1761607680*a^2*b^9*c^9*d^12*e^8*f^55 + 1262485504*a^2*b^10*c^8*d^11*e
^9*f^55 - 469762048*a^2*b^11*c^7*d^10*e^10*f^55 + 25165824*a^2*b^12*c^6*d^9
*e^11*f^55 + 41943040*a^2*b^13*c^5*d^8*e^12*f^55 - 10485760*a^2*b^14*c^4*d^
7*e^13*f^55 + 100663296*a^3*b^3*c^14*d^16*e^4*f^55 - 880803840*a^3*b^4*c^13
*d^15*e^5*f^55 + 3221225472*a^3*b^5*c^12*d^14*e^6*f^55 - 6312427520*a^3*b^6
*c^11*d^13*e^7*f^55 + 6889144320*a^3*b^7*c^10*d^12*e^8*f^55 - 3548381184*a^
3*b^8*c^9*d^11*e^9*f^55 - 304087040*a^3*b^9*c^8*d^10*e^10*f^55 + 1371537408
*a^3*b^10*c^7*d^9*e^11*f^55 - 597688320*a^3*b^11*c^6*d^8*e^12*f^55 + 419430
40*a^3*b^12*c^5*d^7*e^13*f^55 + 18874368*a^3*b^13*c^4*d^6*e^14*f^55 + 13757
31712*a^4*b^2*c^14*d^15*e^5*f^55 - 5368709120*a^4*b^3*c^13*d^14*e^6*f^55 +
9982443520*a^4*b^4*c^12*d^13*e^7*f^55 - 7507804160*a^4*b^5*c^11*d^12*e^8*f^
55 - 3412066304*a^4*b^6*c^10*d^11*e^9*f^55 + 10955522048*a^4*b^7*c^9*d^10*e
^10*f^55 - 7748976640*a^4*b^8*c^8*d^9*e^11*f^55 + 1468006400*a^4*b^9*c^7*d^
8*e^12*f^55 + 618659840*a^4*b^10*c^6*d^7*e^13*f^55 - 218103808*a^4*b^11*c^5
*d^6*e^14*f^55 - 10485760*a^4*b^12*c^4*d^5*e^15*f^55 - 2348810240*a^5*b^2*c

^13*d^13*e^7*f^55 - 7549747200*a^5*b^3*c^12*d^12*e^8*f^55 + 24570232832*a^5*b^4*c^11*d^11*e^9*f^55 - 27111981056*a^5*b^5*c^10*d^10*e^10*f^55 + 9638510592*a^5*b^6*c^9*d^9*e^11*f^55 + 4854906880*a^5*b^7*c^8*d^8*e^12*f^55 - 4697620480*a^5*b^8*c^7*d^7*e^13*f^55 + 742391808*a^5*b^9*c^6*d^6*e^14*f^55 + 167772160*a^5*b^10*c^5*d^5*e^15*f^55 - 10485760*a^5*b^11*c^4*d^4*e^16*f^55 - 18824036352*a^6*b^2*c^12*d^11*e^9*f^55 + 9395240960*a^6*b^3*c^11*d^10*e^10*f^55 + 14596177920*a^6*b^4*c^10*d^9*e^11*f^55 - 22825402368*a^6*b^5*c^9*d^8*e^12*f^55 + 10328473600*a^6*b^6*c^8*d^7*e^13*f^55 + 150994944*a^6*b^7*c^7*d^6*e^14*f^55 - 1170210816*a^6*b^8*c^6*d^5*e^15*f^55 + 142606336*a^6*b^9*c^5*d^4*e^16*f^55 + 18874368*a^6*b^10*c^4*d^3*e^17*f^55 - 24830279680*a^7*b^2*c^11*d^9*e^11*f^55 + 20971520000*a^7*b^3*c^10*d^8*e^12*f^55 - 4487905280*a^7*b^4*c^9*d^7*e^13*f^55 - 5972688896*a^7*b^5*c^8*d^6*e^14*f^55 + 4559208448*a^7*b^6*c^7*d^5*e^15*f^55 - 538968064*a^7*b^7*c^6*d^4*e^16*f^55 - 293601280*a^7*b^8*c^5*d^3*e^17*f^55 - 10485760*a^7*b^9*c^4*d^2*e^18*f^55 - 6207569920*a^8*b^2*c^10*d^7*e^13*f^55 + 13690208256*a^8*b^3*c^9*d^6*e^14*f^55 - 9479127040*a^8*b^4*c^8*d^5*e^15*f^55 - 511705088*a^8*b^5*c^7*d^4*e^16*f^55 + 1667235840*a^8*b^6*c^6*d^3*e^17*f^55 + 167772160*a^8*b^7*c^5*d^2*e^18*f^55 + 6241124352*a^9*b^2*c^9*d^5*e^15*f^55 + 6878658560*a^9*b^3*c^8*d^4*e^16*f^55 - 3900702720*a^9*b^4*c^7*d^3*e^17*f^55 - 1006632960*a^9*b^5*c^6*d^2*e^18*f^55 + 2181038080*a^10*b^2*c^8*d^3*e^17*f^55 + 2684354560*a^10*b^3*c^7*d^2*e^18*f^55) + (f*x)^(1/2)*(268435456*a^11*c^8*e^19*f^54 + 1048576*a^7*b^8*c^4*e^19*f^54 - 16777216*a^8*b^6*c^5*e^19*f^54 + 100663296*a^9*b^4*c^6*e^19*f^54 - 268435456*a^10*b^2*c^7*e^19*f^54 - 134217728*a^4*c^15*d^14*e^5*f^54 - 402653184*a^5*c^14*d^12*e^7*f^54 - 268435456*a^6*c^13*d^10*e^9*f^54 + 536870912*a^7*c^12*d^8*e^11*f^54 + 1476395008*a^8*c^11*d^6*e^13*f^54 + 1744830464*a^9*c^10*d^4*e^15*f^54 + 1073741824*a^10*c^9*d^2*e^17*f^54 + 1048576*b^7*c^12*d^15*e^4*f^54 - 8388608*b^8*c^11*d^14*e^5*f^54 + 29360128*b^9*c^10*d^13*e^6*f^54 - 58720256*b^10*c^9*d^12*e^7*f^54 + 73400320*b^11*c^8*d^11*e^8*f^54 - 58720256*b^12*c^7*d^10*e^9*f^54 + 29360128*b^13*c^6*d^9*e^10*f^54 - 8388608*b^14*c^5*d^8*e^11*f^54 + 1048576*b^15*c^4*d^7*e^12*f^54 - 1073741824*a^10*b*c^8*d*e^18*f^54 - 11534336*a*b^5*c^13*d^15*e^4*f^54 + 96468992*a*b^6*c^12*d^14*e^5*f^54 - 348127232*a*b^7*c^11*d^13*e^6*f^54 + 704643072*a*b^8*c^10*d^12*e^7*f^54 - 866123776*a*b^9*c^9*d^11*e^8*f^54 + 645922816*a*b^10*c^8*d^10*e^9*f^54 - 264241152*a*b^11*c^7*d^9*e^10*f^54 + 33554432*a*b^12*c^6*d^8*e^11*f^54 + 13631488*a*b^13*c^5*d^7*e^12*f^54 - 4194304*a*b^14*c^4*d^6*e^13*f^54 - 50331648*a^3*b*c^15*d^15*e^4*f^54 + 838860800*a^4*b*c^14*d^13*e^6*f^54 + 2667577344*a^5*b*c^13*d^11*e^8*f^54 + 2348810240*a^6*b*c^12*d^9*e^10*f^54 - 4194304*a^6*b^9*c^4*d*e^18*f^54 - 889192448*a^7*b*c^11*d^7*e^12*f^54 + 67108864*a^7*b^7*c^5*d*e^18*f^54 - 3724541952*a^8*b*c^10*d^5*e^14*f^54 - 402653184*a^8*b^5*c^6*d*e^18*f^54 - 3338665984*a^9*b*c^9*d^3*e^16*f^54 + 1073741824*a^9*b^3*c^7*d*e^18*f^54 + 41943040*a^2*b^3*c^14*d^15*e^4*f^54 - 377487360*a^2*b^4*c^13*d^14*e^5*f^54 + 1428160512*a^2*b^5*c^12*d^13*e^6*f^54 - 2927624192*a^2*b^6*c^11*d^12*e^7*f^54 + 3435134976*a^2*b^7*c^10*d^11*e^8*f^54 - 2113929216*a^2*b^8*c^9*d^10*e^9*f^54 + 293601280*a^2*b^9*c^8*d^9*e^10*f^54 + 427819008*a^2*b^10*c^7*d^8*e^11*f^54 - 239075328*a^2*b^11*c^6*d^7*e^12*f^54 + 25165824*a^2*b^12*c^5*d^6*e^13*f^54 + 6291456*a^2*b^13*c^4*d^5*e^14*f^54 + 536870912*a^3*b^2*c^14*d^14*e^5*f^54 - 2231369728*a^3*b^3*c^13*d^13*e^6*f^54 + 4605345792*a^3*b^4*c^12*d^12*e^7*f^54 - 4530896896*a^3*b^5*c^11*d^11*e^8*f^54 + 528482304*a^3*b^6*c^10*d^10*e^9*f^54 + 3258974208*a^3*b^7*c^9*d^9*e^10*f^54 - 2993684480*a^3*b^8*c^8*d^8*e^11*f^54 + 812646400*a^3*b^9*c^7*d^7*e^12*f^54 + 144703488*a^3*b^10*c^6*d^6*e^13*f^54 - 77594624*a^3*b^11*c^5*d^5*e^14*f^54 - 3145728*a^3*b^12*c^4*d^4*e^15*f^54 - 1543503872*a^4*b^2*c^13*d^12*e^7*f^54 - 864026624*a^4*b^3*c^12*d^11*e^8*f^54 + 7029653504*a^4*b^4*c^11*d^10*e^9*f^54 - 9953083392*a^4*b^5*c^10*d^9*e^10*f^54 + 5167382528*a^4*b^6*c^9*d^8*e^11*f^54 + 592445440*a^4*b^7*c^8*d^7*e^12*f^54 - 1488977920*a^4*b^8*c^7*d^6*e^13*f^54 + 304087040*a^4*b^9*c^6*d^5*e^14*f^54 + 54525952*a^4*b^10*c^5*d^4*e^15*f^54 - 3145728*a^4*b^11*c^4*d^3*e^16*f^54 - 6442450944*a^5*b^2*c^12*d^10*e^9*f^54 + 5872025600*a^5*b^3*c^11*d^9*e^10*f^54 + 1459617792*a^5*b^4*c^10*d^8*e^11*f^54 - 6489636864*a^5*b^5

*c^9*d^7*e^12*f^54 + 3837788160*a^5*b^6*c^8*d^6*e^13*f^54 - 150994944*a^5*b
^7*c^7*d^5*e^14*f^54 - 396361728*a^5*b^8*c^6*d^4*e^15*f^54 + 38797312*a^5*b
^9*c^5*d^3*e^16*f^54 + 6291456*a^5*b^10*c^4*d^2*e^17*f^54 - 6576668672*a^6*
b^2*c^11*d^8*e^11*f^54 + 7642021888*a^6*b^3*c^10*d^7*e^12*f^54 - 2625634304
*a^6*b^4*c^9*d^6*e^13*f^54 - 1809842176*a^6*b^5*c^8*d^5*e^14*f^54 + 1501560
832*a^6*b^6*c^7*d^4*e^15*f^54 - 111149056*a^6*b^7*c^6*d^3*e^16*f^54 - 96468
992*a^6*b^8*c^5*d^2*e^17*f^54 - 1610612736*a^7*b^2*c^10*d^6*e^13*f^54 + 454
6625536*a^7*b^3*c^9*d^5*e^14*f^54 - 2810183680*a^7*b^4*c^8*d^4*e^15*f^54 -
376438784*a^7*b^5*c^7*d^3*e^16*f^54 + 536870912*a^7*b^6*c^6*d^2*e^17*f^54 +
 1409286144*a^8*b^2*c^9*d^4*e^15*f^54 + 2441084928*a^8*b^3*c^8*d^3*e^16*f^5
4 - 1207959552*a^8*b^4*c^7*d^2*e^17*f^54 + 536870912*a^9*b^2*c^8*d^2*e^17*f
^54)) + 8388608*a^7*c^9*e^16*f^53 - 131072*a^2*b^10*c^4*e^16*f^53 + 1966080
*a^3*b^8*c^5*e^16*f^53 - 11141120*a^4*b^6*c^6*e^16*f^53 + 28835840*a^5*b^4*
c^7*e^16*f^53 - 31457280*a^6*b^2*c^8*e^16*f^53 + 2097152*a^2*c^14*d^10*e^6*
f^53 + 3145728*a^3*c^13*d^8*e^8*f^53 - 14680064*a^4*c^12*d^6*e^10*f^53 - 24
641536*a^5*c^11*d^4*e^12*f^53 - 131072*b^2*c^14*d^12*e^4*f^53 + 655360*b^3*
c^13*d^11*e^5*f^53 - 1310720*b^4*c^12*d^10*e^6*f^53 + 1310720*b^5*c^11*d^9*
e^7*f^53 - 655360*b^6*c^10*d^8*e^8*f^53 + 262144*b^7*c^9*d^7*e^9*f^53 - 655
360*b^8*c^8*d^6*e^10*f^53 + 1310720*b^9*c^7*d^5*e^11*f^53 - 1310720*b^10*c^
6*d^4*e^12*f^53 + 655360*b^11*c^5*d^3*e^13*f^53 - 131072*b^12*c^4*d^2*e^14*
f^53 + 524288*a*c^15*d^12*e^4*f^53 - 2621440*a*b*c^14*d^11*e^5*f^53 + 26214
4*a*b^11*c^4*d*e^15*f^53 + 27262976*a^6*b*c^9*d*e^15*f^53 + 4718592*a*b^2*c
^13*d^10*e^6*f^53 - 3145728*a*b^3*c^12*d^9*e^7*f^53 - 524288*a*b^4*c^11*d^8
*e^8*f^53 + 131072*a*b^5*c^10*d^7*e^9*f^53 + 7208960*a*b^6*c^9*d^6*e^10*f^5
3 - 16252928*a*b^7*c^8*d^5*e^11*f^53 + 16515072*a*b^8*c^7*d^4*e^12*f^53 - 7
733248*a*b^9*c^6*d^3*e^13*f^53 + 917504*a*b^10*c^5*d^2*e^14*f^53 - 8388608*
a^2*b*c^13*d^9*e^7*f^53 - 3538944*a^2*b^9*c^5*d*e^15*f^53 - 15728640*a^3*b*
c^12*d^7*e^9*f^53 + 16908288*a^3*b^7*c^6*d*e^15*f^53 + 60817408*a^4*b*c^11*
d^5*e^11*f^53 - 30801920*a^4*b^5*c^7*d*e^15*f^53 + 98041856*a^5*b*c^10*d^3*
e^13*f^53 + 5242880*a^5*b^3*c^8*d*e^15*f^53 + 11796480*a^2*b^2*c^12*d^8*e^8
*f^53 - 786432*a^2*b^3*c^11*d^7*e^9*f^53 - 31719424*a^2*b^4*c^10*d^6*e^10*f
^53 + 71958528*a^2*b^5*c^9*d^5*e^11*f^53 - 73269248*a^2*b^6*c^8*d^4*e^12*f^
53 + 28835840*a^2*b^7*c^7*d^3*e^13*f^53 + 3145728*a^2*b^8*c^6*d^2*e^14*f^53
 + 57147392*a^3*b^2*c^11*d^6*e^10*f^53 - 126877696*a^3*b^3*c^10*d^5*e^11*f^
53 + 126877696*a^3*b^4*c^9*d^4*e^12*f^53 - 21102592*a^3*b^5*c^8*d^3*e^13*f^
53 - 42336256*a^3*b^6*c^7*d^2*e^14*f^53 - 50462720*a^4*b^2*c^10*d^4*e^12*f^
53 - 74317824*a^4*b^3*c^9*d^3*e^13*f^53 + 120586240*a^4*b^4*c^8*d^2*e^14*f^
53 - 106954752*a^5*b^2*c^9*d^2*e^14*f^53) + (f*x)^(1/2)*(131072*b^11*c^4*e^
15*f^52 + 131072*c^15*d^11*e^4*f^52 + 11272192*a^2*b^7*c^6*e^15*f^52 - 3027
7632*a^3*b^5*c^7*e^15*f^52 + 36700160*a^4*b^3*c^8*e^15*f^52 + 786432*a^2*c^
13*d^7*e^8*f^52 + 524288*a^3*c^12*d^5*e^10*f^52 - 16646144*a^4*c^11*d^3*e^1
2*f^52 + 786432*b^2*c^13*d^9*e^6*f^52 - 524288*b^3*c^12*d^8*e^7*f^52 + 1310
72*b^4*c^11*d^7*e^8*f^52 + 131072*b^7*c^8*d^4*e^11*f^52 - 524288*b^8*c^7*d^
3*e^12*f^52 + 786432*b^9*c^6*d^2*e^13*f^52 - 1966080*a*b^9*c^5*e^15*f^52 -
14680064*a^5*b*c^9*e^15*f^52 + 524288*a*c^14*d^9*e^6*f^52 + 16777216*a^5*c^
10*d*e^14*f^52 - 524288*b*c^14*d^10*e^5*f^52 - 524288*b^10*c^5*d*e^14*f^52
- 1572864*a*b*c^13*d^8*e^7*f^52 + 7340032*a*b^8*c^6*d*e^14*f^52 + 1572864*a
*b^2*c^12*d^7*e^8*f^52 - 524288*a*b^3*c^11*d^6*e^9*f^52 - 1441792*a*b^5*c^9
*d^4*e^11*f^52 + 6291456*a*b^6*c^8*d^3*e^12*f^52 - 10223616*a*b^7*c^7*d^2*e
^13*f^52 - 1572864*a^2*b*c^12*d^6*e^9*f^52 - 38273024*a^2*b^6*c^7*d*e^14*f^
52 - 6815744*a^3*b*c^11*d^4*e^11*f^52 + 89128960*a^3*b^4*c^8*d*e^14*f^52 +
62914560*a^4*b*c^10*d^2*e^13*f^52 - 83886080*a^4*b^2*c^9*d*e^14*f^52 + 7864
32*a^2*b^2*c^11*d^5*e^10*f^52 + 5242880*a^2*b^3*c^10*d^4*e^11*f^52 - 262144
00*a^2*b^4*c^9*d^3*e^12*f^52 + 47972352*a^2*b^5*c^8*d^2*e^13*f^52 + 4194304
0*a^3*b^2*c^10*d^3*e^12*f^52 - 94371840*a^3*b^3*c^9*d^2*e^13*f^52)) + 8192*
b^3*c^9*e^12*f^51 + 8192*c^12*d^3*e^9*f^51 - 32768*a*b*c^10*e^12*f^51 + 409
60*a*c^11*d*e^11*f^51 - 8192*b*c^11*d^2*e^10*f^51 - 8192*b^2*c^10*d*e^11*f^
51) + 12288*c^11*e^11*f^50*(f*x)^(1/2)))*root(8388608*a^7*b*c^11*d^18*e*f^6
*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d

^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*
b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 29360
1280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^
12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*
e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2
*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 12478054
4*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12
+ 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11
*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4
*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^1
1*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 7106
5600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12
 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f
^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^
6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7
*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*
a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 2
2118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h
^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*
e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c
^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^
5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 104
85760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^1
2 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f
^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^1
3*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*
c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5
*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 38993
92*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 +
 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6
*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e
^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*
d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12
*c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3
*b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 11468
8*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 +
293601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*
h^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^
5*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^
16*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^
10*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9
*c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^
11*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*
b^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4
*b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4
*b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*
b^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^1
1*c^8*d^11*e^8*f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^
10*c^9*d^13*e^6*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a
^9*c^10*d^15*e^4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^
8*c^11*d^17*e^2*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*
b^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^1
3*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d
^9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e
^9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*
h^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^
12 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 104
8576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^
6*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e

^8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*
e^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^
8*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d
^10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5
*d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c
^5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6
*c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7
*c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*
c^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4
*c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3
*c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^
5*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^
2*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b
^8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^
5*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b
^4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*
b^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b
^2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*
b^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*
b^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b
^9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^1
1*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10
*c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^
6*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d
^11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e
^14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*
f^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^
4*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4
*h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8
+ 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 235
52*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a
^4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*
c^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11
*e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^
4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425
984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10
*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^1
2*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4
*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768
*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^
4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*
f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 102
4*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^1
5*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 2
56*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^
4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c
^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d
^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^
8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^
4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7
680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^
7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e
^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4
+ 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2
*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4
+ 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*
e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 6
4*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^
3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^

```
4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e
^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11
*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k), k, 1, 12)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{fx}\,\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)/(f*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(f*x)*(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.311 $\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=272

$$-\frac{\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4}+\frac{\sqrt{a+bx^2+cx^4}\,\left((2cd-be)(b\right.}{16c^2}$$

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c/e-1/32*(16*c^3*d^3-b^3*e^3-2*b*c*e^2*(-2*a*e+b*
d)-8*c^2*d*e*(-a*e+b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1
/2))/c^(5/2)/e^4+1/2*d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*
d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^4+1/16*
((-b*e+2*c*d)*(b*e+4*c*d)-2*c*e*(b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2/
e^3
```

Rubi [A] time = 0.57, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1251, 1653, 814, 843, 621, 206, 724}

$$-\frac{\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4}+\frac{\sqrt{a+bx^2+cx^4}\,\left((2cd-be)(b\right.}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (((2*c*d - b*e)*(4*c*d + b*e) - 2*c*e*(2*c*d + b*e)*x^2)*Sqrt[a + b*x^2 + c
*x^4])/(16*c^2*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*c*e) - ((16*c^3*d^3 - b^
3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) - 8*c^2*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2)*e^4) + (d^2*Sqrt[c*d^
2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2
- b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
```

```
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{3}{2}bde-\frac{3}{2}e(2cd+be)x\right)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{6ce^2}$$

$$= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} - \text{Subs}\ldots$$

$$= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} + \frac{\left(d^2\left(\ldots\right.\right.}{\ldots}$$

$$= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} - \frac{\left(d^2\left(\ldots\right.\right.}{\ldots}$$

$$= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} - \frac{\left(16c\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.43, size = 267, normalized size = 0.98

$$\frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4}\left(2ce\left(4ae-3bd+bex^2\right)-3b^2e^2+4c^2\left(6d^2-3dex^2+2e^2x^4\right)\right)+24c^2d^2\sqrt{e(ae-bd)+c}\right.}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (-3*(16*c^3*d^3 - b^3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) + 8*c^2*d*e*(-(b*d) + a
*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]
*(e*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2*e^2 + 2*c*e*(-3*b*d + 4*a*e + b*e*x^2)
+ 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)) + 24*c^2*d^2*Sqrt[c*d^2 + e*(-(b*d
) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*x^2))/(2*Sqrt[c*d^2 + e*(-
(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])]))/(96*c^(5/2)*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.04, size = 1049, normalized size = 3.86

$$-\frac{a\,d^2\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\,e^3}-\frac{\sqrt{c\,x^4+b\,x^2+a}\,b\,x^2}{8ce}+\frac{b\,d^3}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)
```

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c/e-1/8/e*b/c*x^2*(c*x^4+b*x^2+a)^(1/2)-1/16/e*b^
2/c^2*(c*x^4+b*x^2+a)^(1/2)-1/8/e*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4
+b*x^2+a)^(1/2))*a+1/32/e*b^3/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2
+a)^(1/2))-1/4/e^2*d*(c*x^4+b*x^2+a)^(1/2)*x^2-1/8/e^2*d/c*(c*x^4+b*x^2+a)^
(1/2)*b-1/4/e^2*d/c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a
+1/16/e^2*d/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2+1/2
*d^2/e^3*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)+1/4*d^2/e^3*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(
b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/2*d^3/e^
4*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(
x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)-1/2*d^2/e^3/((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*(
```

```
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a+1/2*d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2))/(x^2+d/e))*b-1/2*d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x^2+d/e))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

```
[Out] int((x^5*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \sqrt{a + b x^{2} + c x^{4}}}{d + e x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

3.312 $\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=208

$$\frac{\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3}-\frac{d\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3}-$$

```
[Out] 1/16*(8*c^2*d^2-b^2*e^2-4*c*e*(-a*e+b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(
c*x^4+b*x^2+a)^(1/2))/c^(3/2)/e^3-1/2*d*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)
*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^
(1/2)/e^3-1/8*(-2*c*e*x^2-b*e+4*c*d)*(c*x^4+b*x^2+a)^(1/2)/c/e^2
```

Rubi [A] time = 0.31, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3}-\frac{d\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] -((4*c*d - b*e - 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c*e^2) + ((8*c^2*d^
2 - b^2*e^2 - 4*c*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(16*c^(3/2)*e^3) - (d*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh
[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
b*x^2 + c*x^4])])/(2*e^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)$$
$$= -\frac{\left(4cd-be-2cex^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2} - \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}d\left(4bcd-b^2e-4ace\right)-\frac{1}{2}\left(8c^2d^2-b^2e^2-4ce(bd-ae\right.}{(d+ex)\sqrt{a+bx+cx^2}}\right.}{8ce^2}$$
$$= -\frac{\left(4cd-be-2cex^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2} - \frac{\left(d\left(cd^2-bde+ae^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\right.}{2e^3}$$
$$= -\frac{\left(4cd-be-2cex^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2} + \frac{\left(d\left(cd^2-bde+ae^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\right.}{e^3}$$
$$= -\frac{\left(4cd-be-2cex^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2} + \frac{\left(8c^2d^2-b^2e^2-4ce(bd-ae)\right)\tanh^{-1}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.}{16c^{3/2}e^3}$$

Mathematica [A] time = 0.26, size = 205, normalized size = 0.99

$$\frac{\left(4ce(ae-bd)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+2\sqrt{c}\left(4cd\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+c}}\right.\right.}{16c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] ((8*c^2*d^2 - b^2*e^2 + 4*c*e*(-(b*d) + a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*(-4*c*d + b*e + 2*c*e*x^2)*Sqr
t[a + b*x^2 + c*x^4] + 4*c*d*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(-(b*d) +
2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2
+ c*x^4])]))/(16*c^(3/2)*e^3)
```

fricas [A] time = 46.63, size = 1231, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/32*(8*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (
b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*
c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sq
rt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d
*e*x^2 + d^2)) + (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(c)*log(-8
*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c
) - 4*a*c) + 4*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a
))/(c^2*e^3), -1/32*(16*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d*arctan(-1/2*sqrt
(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d -
 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 +
(b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c
)*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)
*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*s
qrt(c*x^4 + b*x^2 + a))/(c^2*e^3), 1/16*(4*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*
d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2
*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*
x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*
x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (8*c^2*d^2 - 4*b*c*d*e -
 (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c
*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3), -1/16*(8*sqrt(-c*d^2 + b*d*e - a*
e^2)*c^2*d*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)
*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c
*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (8*c^2*d^2
 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 +
a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*c^2*e^2*x^2 - 4
*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 887, normalized size = 4.26

$$\frac{a d \ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a e^2-2deb+2c d^2}{e^2}+2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-deb+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\, e^2}-\frac{b d^2 \ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a e^2-2deb+2c \cdots}{e^2}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)
```

```
[Out] 1/4/e*(c*x^4+b*x^2+a)^(1/2)*x^2+1/8/e/c*(c*x^4+b*x^2+a)^(1/2)*b+1/4/e/c^(1/
2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a-1/16/e/c^(3/2)*ln((c*x
^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2-1/2*d/e^2*((x^2+d/e)^2*c+(b*e-
2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/4*d/e^2*ln(((x^2+d/e)*c
+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b+1/2*d^2/e^3*ln(((x^2+d/e)*c+1/2*(b*e-2*c*
```

```
d)/e)/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))*c^(1/2)+1/2*d/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)
*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d
/e))*a-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e
)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^
2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b+1/
2*d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*
e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-
2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

```
[Out] int((x^3*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

3.313 $\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,e^2}+\frac{\sqrt{a+bx^2+cx^4}}{2e}$$

```
[Out] -1/4*(-b*e+2*c*d)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^
2/c^(1/2)+1/2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^
(1/2)/(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^2+1/2*(c*x^4+b*x^2
+a)^(1/2)/e
```

Rubi [A] time = 0.22, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1247, 734, 843, 621, 206, 724}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,e^2}+\frac{\sqrt{a+bx^2+cx^4}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*e) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e
^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{\text{Subst}\left(\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4e}$$
$$= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4e^2} + \frac{\left(cd^2-bde+ae^2\right)\text{Subst}\left(\right.}{2}$$
$$= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2e^2} - \frac{\left(cd^2-bde+ae^2\right)\text{Sub}}{}$$
$$= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,e^2} + \frac{\sqrt{cd^2-bde+ae^2}\,\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.}{2e^2}$$

Mathematica [A] time = 0.13, size = 167, normalized size = 0.99

$$\frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4} - \sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)\right) + (be-2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right.}{4\sqrt{c}\,e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] ((-2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]
+ 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4] - Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqr
t[a + b*x^2 + c*x^4])]))/(4*Sqrt[c]*e^2)
```

fricas [A] time = 3.69, size = 1050, normalized size = 6.25

$$\left[\frac{4\sqrt{cx^4+bx^2+a}\,ce - (2cd-be)\sqrt{c}\log\left(-8c^2x^4 - 8bcx^2 - b^2 - 4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c} - 4ac\right) + 2\sqrt{c}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4
- 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c
) + 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4
*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 +
 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^
2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
+ d^2)))/(c*e^2), 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*c*e + (2*c*d - b*e)*sqrt(-
c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c
*x^2 + a*c)) + sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-((8*c^2*d^2 - 8*b*c*d*e +
 (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*
b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*
sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2
*d*e*x^2 + d^2)))/(c*e^2), 1/8*(4*sqrt(c*x^4 + b*x^2 + a)*c*e + 4*sqrt(-c*d
^2 + b*d*e - a*e^2)*c*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d
*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2
)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) -
 (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*
x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(c*e^2), 1/4*(2*sqrt(c*x^4 + b*x^2
 + a)*c*e + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sqrt(c*x^4 + b*x^2
 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*
d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2
*d*e + a*b*e^2)*x^2)) + (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^
2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.00, size = 757, normalized size = 4.51

$$-\frac{a\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a e^2-2deb+2c d^2}{e^2}+2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-deb+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\,e}+\frac{bd\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a e^2-2deb+2c d^2}{e^2}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)
```

```
[Out] 1/2/e*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
+1/4/e*ln(((x^2+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d
)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/2/e^2*ln(((x^2+d/
e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(x^2+d/e))*a+1/2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c
*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^
2+d/e))*d*b-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/
e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)
^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c*d
^2
```

maxima [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more details)Is a*e^2-b*d*e                            +c*d^2 zero or nonze
ro?
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

3.314 $\int \frac{\sqrt{a+bx^2+cx^4}}{x\left(d+ex^2\right)}\,dx$

Optimal. Leaf size=186

$$-\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)/d+1/2*a
rctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*c^(1/2)/e-1/2*arctanh
(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/d/e
```

Rubi [A] time = 0.26, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 895, 724, 206, 843, 621}

$$-\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] -(Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d)
 + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*
e) - (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)
/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\left(d+ex^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)}\,dx, x, x^2\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{-bd+ae-cdx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e} - \frac{1}{2}\left(-b+\frac{cd}{e}+\ldots\right.$$

$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{c\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{e} - \left(b-\frac{cd}{e}-\frac{ae}{d}\right)\text{Sub}\ldots$$

$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e} - \frac{\sqrt{cd^2-bde+ae^2}\,\tanh^{-1}\ldots}{2\ldots}$$

Mathematica [A] time = 0.15, size = 179, normalized size = 0.96

$$-\frac{\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)-\sqrt{c}\,d\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx\ldots}}\right.}{2de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] -1/2*(Sqrt[a]*e*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]
- Sqrt[c]*d*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + Sq
rt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(d*e)
```

fricas [A] time = 162.64, size = 2367, normalized size = 12.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*e*log(-((b^2 + 4*a*c)*x^4 + 8*a*
```

```
b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + sqr
t(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*
x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2
- (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e +
 a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)))/(d
*e), -1/4*(2*sqrt(-c)*d*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sq
rt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(a)*e*log(-((b^2 + 4*a*c)*x^4 + 8*a
*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - sq
rt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)
*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2
 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e
+ a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)))/(
d*e), 1/4*(sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^
2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*e*log(-((b^2 + 4*a*c)*x^4 +
 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4)
- 2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-
c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*
e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e
^2)*x^2)))/(d*e), -1/4*(2*sqrt(-c)*d*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*
c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(a)*e*log(-((b^2 + 4*a
*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a
^2)/x^4) + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 +
a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2
 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*
e + a*b*e^2)*x^2)))/(d*e), 1/4*(2*sqrt(-a)*e*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + sqrt(c)*d*log(-8*c
^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c)
- 4*a*c) + sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2
+ 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^
2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c
*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x
^2 + d^2)))/(d*e), 1/4*(2*sqrt(-a)*e*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*
x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*sqrt(-c)*d*arctan(1/2*sq
rt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + s
qrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2
)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^
2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e
 + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)))/
(d*e), 1/4*(2*sqrt(-a)*e*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*s
qrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 -
 b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 2*sqrt(-c
*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d
*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2
)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)))/
(d*e), 1/2*(sqrt(-a)*e*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqr
t(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - sqrt(-c)*d*arctan(1/2*sqrt(c*x^4 + b*x^2
 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(-c*d^2 + b*d
*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2
)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*
c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)))/(d*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 851, normalized size = 4.58

$$\frac{a\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a e^2-2deb+2c d^2}{e^2}+2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-deb+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\, d}-\frac{b\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a e^2-2deb+2c d^2}{e^2}+2\sqrt{\frac{a}{}}}{}\right)}{2\sqrt{\frac{a}{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x)
```

```
[Out] 1/2/d*(c*x^4+b*x^2+a)^(1/2)+1/4/d*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))/c^(1/2)-1/2/d*a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2
))/x^2)-1/2/d*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)-1/4/d*ln(((x^2+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x^2+d/e)^2*c+(b
*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b+1/2/e*ln(((
x^2+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/
e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)+1/2/d/((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(x^2+d/e))*a-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2
*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(
x^2+d/e))*b+1/2/e^2*d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+
d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/
e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2+a}}{(ex^2+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(x*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x*(d + e*x**2)), x)
```

3.315 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\,\sqrt{ae^2-bde+cd^2}}\right)}{2d^2}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2d^2}-\frac{be\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,d^2}$$

```
[Out] -1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/a^(1/2)+1/2
*e*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)/d^2-1/4*b
*e*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/c^(1/2)-1/4*(
-b*e+2*c*d)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/c^(1
/2)+1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*c^(1/2)/d+1/
2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/d^2-1/2*(c*x^4+b*x^2+a)^(1/2)/d/
x^2
```

Rubi [A] time = 0.51, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1251, 960, 732, 843, 621, 206, 724, 734}

$$\frac{\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\,\sqrt{ae^2-bde+cd^2}}\right)}{2d^2}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2d^2}-\frac{be\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*d*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])])/(4*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (Sqrt[c]*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) - (b*e*ArcTanh[(b + 2*c*x^2)
/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) - ((2*c*d - b*e)*Arc
Tanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) +
(Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2} - \frac{e\sqrt{a+bx+cx^2}}{d^2x} + \frac{e^2\sqrt{a+bx+cx^2}}{d^2(d+ex)}\right)dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^2\right)}{2d} - \frac{e\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{e^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx+}}{d+e}\right.}{2d^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} + \frac{\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4d} + \frac{e\,\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right.}{4d^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4d} + \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right.}{2d}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{c\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+}{\sqrt{a+b}}\right.}{d}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} - \frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\,d} + \frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} + \frac{\sqrt{c}}{}$$

Mathematica [A] time = 0.32, size = 165, normalized size = 0.46

$$\frac{2\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right) + \frac{(2ae-bd)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}} - \frac{2d\sqrt{a+bx^2+cx^4}}{x^2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] ((-2*d*Sqrt[a + b*x^2 + c*x^4])/x^2 + ((-(b*d) + 2*a*e)*ArcTanh[(2*a + b*x^
2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a] + 2*Sqrt[c*d^2 - b*d*e + a
*e^2]*ArcTanh[(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a
*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^2)
```

fricas [A] time = 1.49, size = 1094, normalized size = 3.03

$$\left[\frac{2\sqrt{cd^2-bde+ae^2}\,ax^2\log\left(-\frac{\left(8c^2d^2-8bcde+\left(b^2+4ac\right)e^2\right)x^4-8abde+8a^2e^2+\left(b^2+4ac\right)d^2+2\left(4bcd^2+4abe^2-\left(3b^2+4ac\right)de\right)x^2+4\sqrt{cx^4+}}{e^2x^4+2dex^2+d^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b
^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c
*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqr
t(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*
e*x^2 + d^2)) - (b*d - 2*a*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt
(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2), 1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*a
*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c
*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 -
```

```
 a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (b*d - 2*a*e)*sq
rt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*
x^2), 1/4*((b*d - 2*a*e)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + sqrt(c*d^2 - b*d*e + a*e^
2)*a*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e
+ 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c
)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d
- b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 2*sqrt(c*x^4 + b*
x^2 + a)*a*d)/(a*d^2*x^2), 1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a*x^2*arctan
(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x
^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e +
a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (b*d - 2*a*e)*sqrt(-a)*x^2*
arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^
2 + a^2)) - 2*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2)]
```

giac [A] time = 0.52, size = 216, normalized size = 0.60

$$\frac{\left(cd^2-bde+ae^2\right)\arctan\left(-\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}\,d^2}+\frac{(bd-2\,ae)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,d^2}+\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+b\cdots}\right.}{2\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e
+ sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^
2) + 1/2*(b*d - 2*a*e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt
(-a))/(sqrt(-a)*d^2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a
*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*d)
```

maple [B] time = 0.02, size = 1009, normalized size = 2.80

$$-\frac{ae\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,d^2}+\frac{\sqrt{c\,x^4+b\,x^2+a}\;c\,x^2}{2ad}+\frac{b\ln\left(\frac{(be-2\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x)
```

```
[Out] -1/2/d^2*e*(c*x^4+b*x^2+a)^(1/2)-1/4/d^2*e*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))/c^(1/2)+1/2/d^2*e*a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)
^(1/2)*a^(1/2))/x^2)+1/2*e/d^2*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)+1/4*e/d^2*ln(((x^2+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1
/2)+((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
c^(1/2)*b-1/2/d*ln(((x^2+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x^2+d/e)^2*c+(
b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)-1/2*e/d^2/((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*
d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+
d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a+1/2/d/((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2
```

```
)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x^2+d/e))*c-1/2/d/a/x^2*(c*x^4+b*x^2+a)^(3/2)+1/2/d*b/a*(c*x^4+b*
x^2+a)^(1/2)-1/4/d*b/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))
/x^2)+1/2/d/a*c*(c*x^4+b*x^2+a)^(1/2)*x^2+1/2/d*c^(1/2)*ln((c*x^2+1/2*b)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{(ex^2 + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{x^3\,\left(e\,x^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(x^3*(d + e*x^2)),x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(x^3*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^3\left(d + ex^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**3/(e*x**2+d),x)
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x**3*(d + e*x**2)), x)
```

3.316 $\int \frac{x^4 \sqrt{1+2x^2+2x^4}}{3+2x^2} dx$

Optimal. Leaf size=424

$$-\frac{1}{60}\left(13-6x^2\right)\sqrt{2x^4+2x^2+1}\,x+\frac{109\sqrt{2x^4+2x^2+1}\,x}{60\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}+\frac{3}{16}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\left(263\sqrt{2}-70\right)\left(\sqrt{2}\,x\right.}{60}$$

```
[Out] 3/16*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/60*x*(-6*x^2+1
3)*(2*x^4+2*x^2+1)^(1/2)+109/120*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(
1/2))-109/120*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*E
llipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((
2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+15/32
*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin
(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*
(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1
/2))/(2*x^4+2*x^2+1)^(1/2)+1/120*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*a
rctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))
*(-70+263*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2
)*2^(1/4)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 619, normalized size of antiderivative = 1.46, number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1335, 1091, 1197, 1103, 1195, 1116, 1208, 1216, 1706}

$$\frac{1}{30}\left(3x^2+1\right)\sqrt{2x^4+2x^2+1}\,x+\frac{109\sqrt{2x^4+2x^2+1}\,x}{60\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}-\frac{1}{4}\sqrt{2x^4+2x^2+1}\,x+\frac{3}{16}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{45}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] -(x*Sqrt[1 + 2*x^2 + 2*x^4])/4 + (x*(1 + 3*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/30
 + (109*x*Sqrt[1 + 2*x^2 + 2*x^4])/(60*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sqrt
[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/16 - (109*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/
4)*x], (2 - Sqrt[2])/4])/(60*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (139*(1 - S
qrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(240*2^(1/4)*Sqrt[1 + 2*x^2 +
 2*x^4]) - ((1 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(4*2^(3/4)*
Sqrt[1 + 2*x^2 + 2*x^4]) + (45*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*
x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[
2])/4])/(112*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (15*(3 + Sqrt[2])^2*(1 + Sq
rt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 1
1*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(224*2^(1/4)*Sqrt[1 +
 2*x^2 + 2*x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1116

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*(2*b*p + c*(m + 4*p - 1)*x^
2))/(c*(m + 4*p + 1)*(m + 4*p - 1)), x] - Dist[(2*p*d^2)/(c*(m + 4*p + 1)*(
m + 4*p - 1)), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
```

```
(c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \left(-\frac{3}{4}\sqrt{1+2x^2+2x^4}+\frac{1}{2}x^2\sqrt{1+2x^2+2x^4}+\frac{9\sqrt{1+2x^2+2x^4}}{4\left(3+2x^2\right)}\right)dx$$

$$= \frac{1}{2}\int x^2\sqrt{1+2x^2+2x^4}\,dx - \frac{3}{4}\int \sqrt{1+2x^2+2x^4}\,dx + \frac{9}{4}\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$$

$$= -\frac{1}{4}x\sqrt{1+2x^2+2x^4}+\frac{1}{30}x\left(1+3x^2\right)\sqrt{1+2x^2+2x^4}-\frac{1}{60}\int \frac{2-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \ldots$$

$$= -\frac{1}{4}x\sqrt{1+2x^2+2x^4}+\frac{1}{30}x\left(1+3x^2\right)\sqrt{1+2x^2+2x^4}-\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{15\sqrt{2}}+\frac{\int \frac{1-\sqrt{2}\ldots}{\sqrt{1+2x^2+\ldots}}}{2\sqrt{\ldots}}$$

$$= -\frac{1}{4}x\sqrt{1+2x^2+2x^4}+\frac{1}{30}x\left(1+3x^2\right)\sqrt{1+2x^2+2x^4}+\frac{109x\sqrt{1+2x^2+2x^4}}{60\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{3}{16}\sqrt{\ldots}$$

Mathematica [C] time = 0.26, size = 209, normalized size = 0.49

$$\frac{48x^7-56x^5-80x^3-(199-417i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-218i\sqrt{1-i}\sqrt{1+\ldots}}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (-52*x - 80*x^3 - 56*x^5 + 48*x^7 - (218*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^
2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (199 - 41
7*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*Ar
cSinh[Sqrt[1 - I]*x], I] + 225*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 +
 (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(240*Sqrt
[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}\,x^4}{2\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}\,x^4}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

maple [C] time = 0.10, size = 528, normalized size = 1.25

$$\frac{\sqrt{2x^4+2x^2+1}\,x^3}{10}-\frac{13\sqrt{2x^4+2x^2+1}\,x}{60}+\frac{9\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\operatorname{EllipticE}\left(\sqrt{-1+i}\,x,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{8\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x)
```

```
[Out] 1/10*x^3*(2*x^4+2*x^2+1)^(1/2)-13/60*x*(2*x^4+2*x^2+1)^(1/2)-8/15/(-1+I)^(1
/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF
(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(13/60-13/60*I)/(-1+I)^(1/2)*(1+
(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1
+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1
/2*I*2^(1/2)))-9/4/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2
*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+9/8
*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+9/8/(-1+I)^(1/2)*(-
I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-
1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-9/8*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/
2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2
^(1/2)+1/2*I*2^(1/2))+15/8/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^
(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2
)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}\,x^4}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3),x)
```

```
[Out] int((x^4*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] Integral(x**4*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

3.317 $\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal. Leaf size=417

$$-\frac{7\sqrt{2x^4+2x^2+1}\,x}{6\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}+\frac{1}{6}\sqrt{2x^4+2x^2+1}\,x-\frac{1}{8}\sqrt{15}\,\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(17\sqrt{2}-4\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}}{6\,2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2}}$$

```
[Out] -1/8*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/6*x*(2*x^4+2*x
^2+1)^(1/2)-7/12*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))+7/12*(cos(
2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arct
an(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x
^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-5/16*(cos(2*arctan(2^(1/
4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)
),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)
^(1/2)-1/12*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ell
ipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(-4+17*2^(1/2))*(1+x
^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^(1/2)
)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 591, normalized size of antiderivative = 1.42, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1335, 1091, 1197, 1103, 1195, 1208, 1216, 1706}

$$-\frac{7\sqrt{2x^4+2x^2+1}\,x}{6\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}+\frac{1}{6}\sqrt{2x^4+2x^2+1}\,x-\frac{1}{8}\sqrt{15}\,\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{15\left(3+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}}{56\sqrt[4]{2}\sqrt{2x^4+}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/6 - (7*x*Sqrt[1 + 2*x^2 + 2*x^4])/(6*Sqrt[2]*(1
 + Sqrt[2]*x^2)) - (Sqrt[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])
/8 + (7*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(6*2^(3/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) + (3*(1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(1/4)*
Sqrt[1 + 2*x^2 + 2*x^4]) + ((1 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(6*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (15*(3 + Sqrt[2])*(1 + Sqrt[2]*x
^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4
)*x], (2 - Sqrt[2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt
[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(112*2^
(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \left(\frac{1}{2}\sqrt{1+2x^2+2x^4} - \frac{3\sqrt{1+2x^2+2x^4}}{2\left(3+2x^2\right)}\right) dx$$
$$= \frac{1}{2}\int \sqrt{1+2x^2+2x^4}\,dx - \frac{3}{2}\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$$
$$= \frac{1}{6}x\sqrt{1+2x^2+2x^4} + \frac{1}{6}\int \frac{2+2x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{3}{8}\int \frac{2-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{15}{4}\int \frac{}{(3}$$
$$= \frac{1}{6}x\sqrt{1+2x^2+2x^4} - \frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{3\sqrt{2}} + \frac{3\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}} + \frac{1}{4}\left(3\left(1-\sqrt{2}\right)\right)\int \frac{}{\sqrt{1+}}$$
$$= \frac{1}{6}x\sqrt{1+2x^2+2x^4} - \frac{7x\sqrt{1+2x^2+2x^4}}{6\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{1}{8}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) + \frac{7\left(1+\right.}{}$$

Mathematica [C] time = 0.20, size = 204, normalized size = 0.49

$$\frac{8x^5+8x^3+(13-27i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+14i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1}}{24\sqrt{2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (4*x + 8*x^3 + 8*x^5 + (14*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1
 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (13 - 27*I)*Sqrt[1 - I]
*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I
]*x], I] - 15*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Ell
ipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(24*Sqrt[1 + 2*x^2 + 2*x^4
])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}\,x^2}{2\,x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}\,x^2}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

maple [C] time = 0.01, size = 509, normalized size = 1.22

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{6} - \frac{3\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\operatorname{EllipticE}\left(\sqrt{-1+i}\,x, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{4\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}} + \frac{3i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+}}{4\sqrt{-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x)
```

```
[Out] 1/6*(2*x^4+2*x^2+1)^(1/2)*x+1/3/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2
+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*
2^(1/2))+(-1/6+1/6*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/
(2*x^4+2*x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-
EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))+3/2/(-1+I)^(1/2)*(-I*x
^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^
(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-3/4*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*
(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1
/2)+1/2*I*2^(1/2))-3/4/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2
)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))
+3/4*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1
)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-5/4/(-1+I)^(1/2
)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi
((-1+I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^4 + 2\,x^2 + 1}\,x^2}{2\,x^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{2\,x^4 + 2\,x^2 + 1}}{2\,x^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3),x)
```

```
[Out] int((x^2*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{2x^4 + 2x^2 + 1}}{2x^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] Integral(x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

3.318 $\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal. Leaf size=381

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}+\frac{1}{4}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{2^{3/4}\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}-$$

```
[Out] 1/12*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/2*x*(2*x^4+2*x
^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/2*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/
cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))
^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(
2*x^4+2*x^2+1)^(1/2)+5/24*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2
^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(
1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2
)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+2^(3/4)*(cos(2*arctan(2
^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)
*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2)
)^2)^(1/2)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 470, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1208, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}+\frac{1}{4}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\right.\right.}{28\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(3 + 2*x^2),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[5/3]*ArcTan
[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/4 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2
*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt
[2])/4])/(2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(4*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2
])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elliptic
F[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(28*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4
]) - (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqr
t[2])/4])/(168*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
```

```
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = -\left(\frac{1}{4}\int \frac{2-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx\right) + \frac{5}{2}\int \frac{1}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\int \frac{1-\sqrt{2}\,x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{\sqrt{2}} - \frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{14}\left(5\left(3+\sqrt{2}\right)\right)\int \frac{}{\sqrt{1+2}}$$

$$= \frac{x\sqrt{1+2x^2+2x^4}}{\sqrt{2}\left(1+\sqrt{2}\,x^2\right)} + \frac{1}{4}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,E\left(2\,\mathrm{t}\right.}{2^{3/4}\sqrt{1+2x}}$$

Mathematica [C] time = 0.11, size = 127, normalized size = 0.33

$$-\frac{\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(-(3+6i)F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+(3+3i)E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+5i\Pi\left(\frac{1}{3}+\right.\right.}{6\sqrt{1-i}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(3 + 2*x^2),x]
```

```
[Out] -1/6*(Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*((3 + 3*I)*EllipticE[I*Ar
cSinh[Sqrt[1 - I]*x], I] - (3 + 6*I)*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
 + (5*I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(Sqrt[1 - I]*
Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

maple [C] time = 0.01, size = 341, normalized size = 0.90

$$\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\;\text{EllipticE}\left(\sqrt{-1+i}\,x,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\;\text{EllipticE}\left(\sqrt{-1}\right.}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x)
```

```
[Out] -1/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+1/2*I/(-1+I)^(1/2)*
(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-
1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+1/2/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/
2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2
^(1/2)+1/2*I*2^(1/2))-1/2*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)
^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(
1/2))+5/6/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^
2+1)^(1/2)*EllipticPi((-1+I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 2*x^4 + 1)^(1/2)/(2*x^2 + 3),x)
[Out] int((2*x^2 + 2*x^4 + 1)^(1/2)/(2*x^2 + 3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

3.319 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx$

Optimal. Leaf size=399

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\right.}{21\sqrt[4]{2}\sqrt{2x^4+2x^2}}$$

```
[Out] -1/18*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/3*(2*x^4+2*x^
2+1)^(1/2)/x+1/3*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/3*(cos(2
*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arcta
n(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^
2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/42*(cos(2*arctan(2^(1/4
)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),
1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*
2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+5/504*(cos(2*arctan(2^(1/4)
*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),
1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1311, 1281, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\right.}{21\sqrt[4]{2}\sqrt{2x^4+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^2*(3 + 2*x^2)),x]
```

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) - (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])
/6 - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^
2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x
^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(21*2^(1/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^
2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2
^(1/4)*x], (2 - Sqrt[2])/4])/(252*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1311

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[((f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2\left(3+2x^2\right)}\,dx = \frac{1}{6}\int \frac{2+6x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx - \frac{5}{3}\int \frac{1}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} - \frac{1}{6}\int \frac{-6-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{21}\left(5\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} - \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+}{(1+\sqrt{2}\,x}}}{42\sqrt[4]{2}\sqrt{1-}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)} - \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\sqrt[4]{2}\left(1+\right.}{}$$

Mathematica [C] time = 0.19, size = 208, normalized size = 0.52

$$\frac{-12x^4-12x^2+(9-3i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,xF\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-6i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{}}{18x\sqrt{2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^2*(3 + 2*x^2)),x]
```

```
[Out] (-6 - 12*x^2 - 12*x^4 - (6*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 +
(1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (9 - 3*I)*Sqrt[1 - I]
*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 -
 I]*x], I] - 5*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*
EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(18*x*Sqrt[1 + 2*x^2 +
2*x^4])
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^4+3\,x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^4 + 3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{\left(2\,x^2+3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2), x)
```

maple [C] time = 0.01, size = 511, normalized size = 1.28

$$-\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\ \text{EllipticE}\left(\sqrt{-1+i}\,x, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}} + \frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\ \text{EllipticE}\left(\sqrt{-}\right.}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x+2/3/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^
2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I
*2^(1/2))+(-2/3+2/3*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)
/(2*x^4+2*x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))
-EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))+2/3/(-1+I)^(1/2)*(-I*
x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)
^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-1/3*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)
*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(
1/2)+1/2*I*2^(1/2))-1/3/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)
)+1/3*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+
1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-5/9/(-1+I)^(1/
2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticP
i((-1+I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{\left(2\,x^2+3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{x^2\,\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^2*(2*x^2 + 3)),x)
```

```
[Out] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^2*(2*x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^2\left(2x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**2/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**2*(2*x**2 + 3)), x)
```

3.320 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^4(3+2x^2)}\,dx$

Optimal. Leaf size=360

$$\frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}x^2+1\right.}{}$$

```
[Out] 1/27*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/9*(2*x^4+2*x^2
+1)^(1/2)/x^3-1/18*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*
x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2
))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+
5/126*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF
(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2)
)*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-5
/756*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi
(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/
2))^2*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*
x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1309, 1281, 12, 1103, 1216, 1706}

$$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}+\frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\right.\right.}{63\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^4*(3 + 2*x^2)),x]
```

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(9*x^3) + (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/9 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*2^(1/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(63*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt
[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(378*2^(1/4)*Sqrt[1 + 2*x^2
 + 2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[((f*x)^(m + 4)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\left(3+2x^2\right)}\,dx = \frac{1}{9}\int \frac{3+4x^2}{x^4\sqrt{1+2x^2+2x^4}}\,dx + \frac{10}{9}\int \frac{1}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} - \frac{1}{27}\int \frac{6}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{63}\left(10\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2 \cdots}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) + \frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2\cdots}{(1+\cdots}}}{63\sqrt[4]{2}\cdots}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,F\left(2\,\mathrm{t}\cdots\right.}{9\sqrt[4]{2}\,\sqrt{1+2\cdots}}$$

Mathematica [C] time = 0.17, size = 154, normalized size = 0.43

$$-\frac{6x^4+6x^2+3(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,x^3F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-5(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(}}{27x^3\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^4*(3 + 2*x^2)),x]
```

```
[Out] -1/27*(3 + 6*x^2 + 6*x^4 + 3*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1
 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 5*(1 - I)^(3/2)*x^
3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSi
nh[Sqrt[1 - I]*x], I])/(x^3*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^6+3\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^6 + 3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{2\,x^4+2\,x^2+1}}{\left(2\,x^2+3\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4), x)
```

maple [C] time = 0.02, size = 448, normalized size = 1.24

$$\frac{2\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\ \text{EllipticE}\left(\sqrt{-1+i}\,x,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{2i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\ \text{EllipticE}\left(\sqrt{}\right.}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x)
```

```
[Out] (2/9-2/9*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x
^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(
(-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))-4/9/(-1+I)^(1/2)*(-I*x^2+x^2+1)^
(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/
2*2^(1/2)+1/2*I*2^(1/2))+2/9*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2
+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*
2^(1/2))+2/9/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-2/9*I/(-1
+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*El
lipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+10/27/(-1+I)^(1/2)*(-I*x^
2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi((-1+I)^
(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))-1/9*(2*x^4+2*x^2+1)^(1/2)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{2\,x^4+2\,x^2+1}}{\left(2\,x^2+3\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{x^4\,\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^4*(2*x^2 + 3)),x)
```

```
[Out] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^4*(2*x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^4\left(2x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**4/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**4*(2*x**2 + 3)), x)
```

3.321 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\left(3+2x^2\right)}\,dx$

Optimal. Leaf size=546

$$\frac{4\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{45\left(\sqrt{2}\,x^2+1\right)}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}-\frac{2}{27}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\sqrt[4]{2}\left(19-2\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+\cdots}{\left(\sqrt{2}\,x\cdots\right.}}}{135\sqrt{2x^4\cdots}}$$

```
[Out] -2/81*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/15*(2*x^4+2*x
^2+1)^(1/2)/x^5+4/135*(2*x^4+2*x^2+1)^(1/2)/x^3-4/45*(2*x^4+2*x^2+1)^(1/2)/
x+4/45*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-4/45*(cos(2*arctan(2
^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)
*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2)
)^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+5/189*2^(1/4)*(cos(2*arctan(2^(1/4
)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),
1/2*(2-2^(1/2))^(1/2))*(5-3*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^
2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)-1/135*2^(1/4)*(cos(2*arctan(2^(1/
4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x))
,1/2*(2-2^(1/2))^(1/2))*(19-2*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+
x^2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)+5/1134*(cos(2*arctan(2^(1/4)*x)
)^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2
-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(1/2))*((2*x^4
+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1309, 1281, 1197, 1103, 1195, 1329, 1714, 1708, 1706}

$$\frac{4\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{45\left(\sqrt{2}\,x^2+1\right)}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}+\frac{4\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}-\frac{2}{27}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^6*(3 + 2*x^2)),x]
```

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(15*x^5) + (4*Sqrt[1 + 2*x^2 + 2*x^4])/(135*x^3) -
 (4*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x) + (4*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])
/(45*(1 + Sqrt[2]*x^2)) - (2*Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2
+ 2*x^4]])/27 - (4*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(45*Sqrt[1
 + 2*x^2 + 2*x^4]) + (5*2^(1/4)*(5 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sq
rt[2])/4])/(189*Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(1/4)*(19 - 2*Sqrt[2])*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(135*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqr
t[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(567*2
^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[((f*x)^(m + 4)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1329

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[(x^(m + 1)*Sqrt[a + b*x^2 + c*x^4])/(a*d*(m + 1)), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)*Simp[a*e*(m + 1) + b*d*(m + 2) + (
b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e*(m + 3)*x^4, x])/((d + e*x^2)*Sqrt[a +
 b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
```

```
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\left(3+2x^2\right)}\,dx = \frac{1}{9}\int \frac{3+4x^2}{x^6\sqrt{1+2x^2+2x^4}}\,dx + \frac{10}{9}\int \frac{1}{x^2\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} - \frac{10\sqrt{1+2x^2+2x^4}}{27x} - \frac{1}{45}\int \frac{4+18x^2}{x^4\sqrt{1+2x^2+2x^4}}\,dx + \frac{10}{27}\int \frac{-2}{\left(3+2x^2\right.}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{10\sqrt{1+2x^2+2x^4}}{27x} + \frac{1}{135}\int \frac{-38+8x^2}{x^2\sqrt{1+2x^2+2x}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \frac{10\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{27\left(1+\sqrt{2}\,x^2\right)} -$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \frac{10\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{27\left(1+\sqrt{2}\,x^2\right)} -$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \frac{4\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{45\left(1+\sqrt{2}\,x^2\right)} -$$

Mathematica [C] time = 0.24, size = 224, normalized size = 0.41

$$-\frac{72x^8+48x^6+66x^4+42x^2-(12+24i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,x^5F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+36i\sqrt{1}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^6*(3 + 2*x^2)),x]
```

```
[Out] -1/405*(27 + 42*x^2 + 66*x^4 + 48*x^6 + 72*x^8 + (36*I)*Sqrt[1 - I]*x^5*Sqr
t[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x]
, I] - (12 + 24*I)*Sqrt[1 - I]*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 50*(1 - I)^(3/2)*x^5*Sqrt[1 +
```

```
(1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 -
 I]*x], I])/(x^5*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^8+3\,x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^8 + 3*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{\left(2\,x^2+3\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6), x)
```

maple [C] time = 0.02, size = 549, normalized size = 1.01

$$\frac{4i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\operatorname{EllipticE}\left(\sqrt{-1+i}\,x,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{27\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{4\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\operatorname{EllipticE}}{27\sqrt{-1+i}\,\sqrt{2x^4+2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x)
```

```
[Out] -4/45*(2*x^4+2*x^2+1)^(1/2)/x-4/45/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*
x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2
*I*2^(1/2))+4/27*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2
*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+8/2
7/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/
2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-4/27*I/(-1+I)^(1/2)*
(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-
1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-4/27/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1
/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*
2^(1/2)+1/2*I*2^(1/2))+(-32/135+32/135*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*
((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2^(
1/2)+1/2*I*2^(1/2))-EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))-20
/81/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(
1/2)*EllipticPi((-1+I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))+4/135*(
2*x^4+2*x^2+1)^(1/2)/x^3-1/15*(2*x^4+2*x^2+1)^(1/2)/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{\left(2\,x^2+3\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{x^6\,\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^6*(2*x^2 + 3)),x)
[Out] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^6*(2*x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^6\left(2x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**6/(2*x**2+3),x)
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**6*(2*x**2 + 3)), x)
```

3.322 $\int \frac{x^5\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=482

$$-\frac{\left(16bc^2e^3\left(3a^2e^2-3abde+b^2d^2\right)+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+3b^5e^5+256c^5\right.}{512c^{7/2}e^6}$$

```
[Out] 1/96*(16*c^2*d^2-6*b*c*d*e-3*b^2*e^2-6*c*e*(b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)
^(3/2)/c^2/e^3+1/10*(c*x^4+b*x^2+a)^(5/2)/c/e-1/512*(256*c^5*d^5+3*b^5*e^5+
6*b^3*c*e^4*(-4*a*e+b*d)-384*c^4*d^3*e*(-a*e+b*d)+96*c^3*d*e^2*(-a*e+b*d)^2
+16*b*c^2*e^3*(3*a^2*e^2-3*a*b*d*e+b^2*d^2))*arctanh(1/2*(2*c*x^2+b)/c^(1/2
)/(c*x^4+b*x^2+a)^(1/2))/c^(7/2)/e^6+1/2*d^2*(a*e^2-b*d*e+c*d^2)^(3/2)*arct
anh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2
+a)^(1/2))/e^6+1/256*(128*c^4*d^4+3*b^4*e^4-32*c^3*d^2*e*(-4*a*e+5*b*d)+8*b
*c^2*d*e^2*(-3*a*e+2*b*d)+6*b^2*c*e^3*(-2*a*e+b*d)-2*c*e*(32*c^3*d^3-3*b^3*
e^3-8*c^2*d*e*(-3*a*e+2*b*d)-6*b*c*e^2*(-2*a*e+b*d))*x^2)*(c*x^4+b*x^2+a)^(
1/2)/c^3/e^5
```

Rubi [A] time = 1.10, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 1653, 814, 843, 621, 206, 724}

$$-\frac{\left(16bc^2e^3\left(3a^2e^2-3abde+b^2d^2\right)+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+3b^5e^5+256c^5\right.}{512c^{7/2}e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((128*c^4*d^4 + 3*b^4*e^4 - 32*c^3*d^2*e*(5*b*d - 4*a*e) + 8*b*c^2*d*e^2*(2
*b*d - 3*a*e) + 6*b^2*c*e^3*(b*d - 2*a*e) - 2*c*e*(32*c^3*d^3 - 3*b^3*e^3 -
 8*c^2*d*e*(2*b*d - 3*a*e) - 6*b*c*e^2*(b*d - 2*a*e))*x^2)*Sqrt[a + b*x^2 +
 c*x^4])/(256*c^3*e^5) + ((16*c^2*d^2 - 6*b*c*d*e - 3*b^2*e^2 - 6*c*e*(2*c*
d + b*e)*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(96*c^2*e^3) + (a + b*x^2 + c*x^4)
^(5/2)/(10*c*e) - ((256*c^5*d^5 + 3*b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*e) - 3
84*c^4*d^3*e*(b*d - a*e) + 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3*(b^2*d
^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(512*c^(7/2)*e^6) + (d^2*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
 b*x^2 + c*x^4])])/(2*e^6)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^5 \left(a + bx^2 + cx^4\right)^{3/2}}{d + ex^2} dx = \frac{1}{2} \text{Subst}\left(\int \frac{x^2 \left(a + bx + cx^2\right)^{3/2}}{d + ex} dx, x, x^2\right)$$

$$= \frac{\left(a + bx^2 + cx^4\right)^{5/2}}{10ce} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{5}{2}bde - \frac{5}{2}e(2cd+be)x\right)\left(a+bx+cx^2\right)^{3/2}}{d+ex} dx, x, x^2\right)}{10ce^2}$$

$$= \frac{\left(16c^2d^2 - 6bcde - 3b^2e^2 - 6ce(2cd + be)x^2\right)\left(a + bx^2 + cx^4\right)^{3/2}}{96c^2e^3} + \frac{\left(a + bx^2 + cx^4\right.}{10ce}$$

$$= \frac{\left(128c^4d^4 + 3b^4e^4 - 32c^3d^2e(5bd - 4ae) + 8bc^2de^2(2bd - 3ae) + 6b^2ce^3(bd - 2ae)\right.}{256}$$

$$= \frac{\left(128c^4d^4 + 3b^4e^4 - 32c^3d^2e(5bd - 4ae) + 8bc^2de^2(2bd - 3ae) + 6b^2ce^3(bd - 2ae)\right.}{256}$$

$$= \frac{\left(128c^4d^4 + 3b^4e^4 - 32c^3d^2e(5bd - 4ae) + 8bc^2de^2(2bd - 3ae) + 6b^2ce^3(bd - 2ae)\right.}{256}$$

$$= \frac{\left(128c^4d^4 + 3b^4e^4 - 32c^3d^2e(5bd - 4ae) + 8bc^2de^2(2bd - 3ae) + 6b^2ce^3(bd - 2ae)\right.}{256}$$

Mathematica [A] time = 0.98, size = 545, normalized size = 1.13

$$-\frac{90de\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\right)}{c^{5/2}} - \frac{240d^2\left((2cd-be)\left(4ce(3ae-2bd)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2}{2\sqrt{c}\sqrt{a+}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (1280*d^2*(a + b*x^2 + c*x^4)^(3/2) - (480*d*e*(b + 2*c*x^2)*(a + b*x^2 + c
*x^4)^(3/2))/c + (768*e^2*(a + b*x^2 + c*x^4)^(5/2))/c - (90*(b^2 - 4*a*c)*
d*e*(-2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/c^(5/2) + (15*b*e^2
*(-16*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2) + 3*(b^2 - 4*a*c)*((2*(b + 2*
c*x^2)*Sqrt[a + b*x^2 + c*x^4])/c + ((-b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^2)/(
2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/c^(3/2))))/c^2 - (240*d^2*((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4]*(-
(b^2*e^2) + 4*c^2*d*(-2*d + e*x^2) - 2*c*e*(-5*b*d + 4*a*e + b*e*x^2)) + 8*
c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*
e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])))/(c^(3
/2)*e^3))/(7680*e^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.06, size = 2068, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] 11/80/e*b*x^6*(c*x^4+b*x^2+a)^(1/2)-5/8*d^3/e^4*b*(c*x^4+b*x^2+a)^(1/2)+1/2
*d^4/e^5*c*(c*x^4+b*x^2+a)^(1/2)+2/3*d^2/e^3*a*(c*x^4+b*x^2+a)^(1/2)-1/2*d^
5/e^6*c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/10/e*a^2/c*
(c*x^4+b*x^2+a)^(1/2)-3/512/e*b^5/c^(7/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b
*x^2+a)^(1/2))+1/5/e*a*x^4*(c*x^4+b*x^2+a)^(1/2)+3/256/e*b^4/c^3*(c*x^4+b*x
^2+a)^(1/2)+1/10/e*c*x^8*(c*x^4+b*x^2+a)^(1/2)-3/32/e*a^2*b/c^(3/2)*ln((c*x
^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/160/e*b^2*x^4/c*(c*x^4+b*x^2+a)^
(1/2)-1/128/e*b^3/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)-5/64/e*a*b^2/c^2*(c*x^4+b*x
^2+a)^(1/2)+3/64/e*a*b^3/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(
1/2))-3/16/e^2*d*a^2*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2
)-5/16/e^2*d*a*x^2*(c*x^4+b*x^2+a)^(1/2)+3/128/e^2*d*b^3/c^2*(c*x^4+b*x^2+a
)^(1/2)-3/256/e^2*d*b^4/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1
/2))-3/16/e^2*d*b*x^4*(c*x^4+b*x^2+a)^(1/2)-1/8/e^2*d*c*x^6*(c*x^4+b*x^2+a)
^(1/2)+1/6*d^2/e^3*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24*d^2/e^3*b*x^2*(c*x^4+b*
x^2+a)^(1/2)+1/16*d^2/e^3/c*b^2*(c*x^4+b*x^2+a)^(1/2)-1/32*d^2/e^3*b^3/c^(3
/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/4*d^3/e^4*a*c^(1/2)*l
n((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/16*d^3/e^4*b^2*ln((c*x^2+1
/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/4*d^4/e^5*b*c^(1/2)*ln((c*x^
2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*d^3/e^4*x^2*c*(c*x^4+b*x^2+a)^(
1/2)-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/
e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*
c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2-1/
2*d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*
e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-
2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2-1/2*d^6/e
^7/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d
*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*
(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2+7/160/e*a*b*x^2/
c*(c*x^4+b*x^2+a)^(1/2)-1/64/e^2*d*b^2*x^2/c*(c*x^4+b*x^2+a)^(1/2)-5/32/e^2
*d*a*b/c*(c*x^4+b*x^2+a)^(1/2)+3/32/e^2*d*a*b^2/c^(3/2)*ln((c*x^2+1/2*b)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/8*d^2/e^3*a*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))/c^(1/2)+d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-
2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x^2+d/e))*a*b-d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2
+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d
/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*
a*c+d^5/e^6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(
a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*
e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left(c\,x^4+b\,x^2+a\right)^{3/2}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

```
[Out] int((x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**5*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

3.323 $\int \frac{x^3\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=360

$$\frac{\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}e^5}-\frac{\sqrt{a+bx^2}}{}$$

```
[Out] -1/48*(-6*c*e*x^2-3*b*e+8*c*d)*(c*x^4+b*x^2+a)^(3/2)/c/e^2+1/256*(128*c^4*d
^4+3*b^4*e^4+8*b^2*c*e^3*(-3*a*e+b*d)-192*c^3*d^2*e*(-a*e+b*d)+48*c^2*e^2*(
-a*e+b*d)^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)
/e^5-1/2*d*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^
2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^5-1/128*(64*c^3*d^3+3
*b^3*e^3-16*c^2*d*e*(-4*a*e+5*b*d)+4*b*c*e^2*(-3*a*e+2*b*d)-2*c*e*(16*c^2*d
^2-3*b^2*e^2-4*c*e*(-3*a*e+2*b*d))*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e^4
```

Rubi [A] time = 0.70, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 814, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)+3b^3e^3+\right.}{128c^2e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] -((64*c^3*d^3 + 3*b^3*e^3 - 16*c^2*d*e*(5*b*d - 4*a*e) + 4*b*c*e^2*(2*b*d -
 3*a*e) - 2*c*e*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*x^2)*Sqrt[
a + b*x^2 + c*x^4])/(128*c^2*e^4) - ((8*c*d - 3*b*e - 6*c*e*x^2)*(a + b*x^2
 + c*x^4)^(3/2))/(48*c*e^2) + ((128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d
- 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b
 + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*e^5) - (d*(c
*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
```

```
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx,x,x^2\right)$$

$$= -\frac{\left(8cd-3be-6cex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{48ce^2} - \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}d\left(4ace-2b\left(4cd-\frac{3be}{2}\right)\right)-\frac{1}{2}\left(16c^2d^2-\right.\right.}{d+}\right.}{16c}$$

$$= -\frac{\left(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce\left(16c^2d^2-3b^2e^2\right.\right.}{128c^2e^4}$$

$$= -\frac{\left(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce\left(16c^2d^2-3b^2e^2\right.\right.}{128c^2e^4}$$

$$= -\frac{\left(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce\left(16c^2d^2-3b^2e^2\right.\right.}{128c^2e^4}$$

$$= -\frac{\left(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce\left(16c^2d^2-3b^2e^2\right.\right.}{128c^2e^4}$$

Mathematica [A] time = 0.60, size = 344, normalized size = 0.96

$$\frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4}\left(8c^2e\left(ae\left(15ex^2-32d\right)+b\left(30d^2-14dex^2+9e^2x^4\right)\right)+6bce^2\left(10ae-4bd+bex^2\right)-9b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (3*(128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d - 3*a*e) - 192*c^3*d^2*e*(b*
d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4]*(-9*b^3*e^3 + 6
*b*c*e^2*(-4*b*d + 10*a*e + b*e*x^2) - 16*c^3*(12*d^3 - 6*d^2*e*x^2 + 4*d*e
^2*x^4 - 3*e^3*x^6) + 8*c^2*e*(a*e*(-32*d + 15*e*x^2) + b*(30*d^2 - 14*d*e*
x^2 + 9*e^2*x^4))) + 192*c^2*d*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(
b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[
a + b*x^2 + c*x^4])]))/(768*c^(5/2)*e^5)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1696, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] 1/64/e*b^2*x^2/c*(c*x^4+b*x^2+a)^(1/2)+5/32/e*a*b/c*(c*x^4+b*x^2+a)^(1/2)+1
/2*d^4/e^5*c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+5/8*d^2/
e^3*b*(c*x^4+b*x^2+a)^(1/2)-1/2*d^3/e^4*c*(c*x^4+b*x^2+a)^(1/2)-2/3*d/e^2*a
*(c*x^4+b*x^2+a)^(1/2)+3/16/e*a^2*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^
(1/2))/c^(1/2)+5/16/e*a*x^2*(c*x^4+b*x^2+a)^(1/2)-3/128/e*b^3/c^2*(c*x^4+b*
x^2+a)^(1/2)+3/256/e*b^4/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(
1/2))+3/16/e*b*x^4*(c*x^4+b*x^2+a)^(1/2)+1/8/e*c*x^6*(c*x^4+b*x^2+a)^(1/2)+
1/2*d^5/e^6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(
a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*
e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2-1/6*d/e
^2*c*x^4*(c*x^4+b*x^2+a)^(1/2)-7/24*d/e^2*b*x^2*(c*x^4+b*x^2+a)^(1/2)-3/4*d
^3/e^4*b*c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/4*d^2/e^
3*x^2*c*(c*x^4+b*x^2+a)^(1/2)+1/2*d/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(
((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2))/(x^2+d/e))*a^2+1/2*d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2
*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(
x^2+d/e))*b^2-1/16*d/e^2/c*b^2*(c*x^4+b*x^2+a)^(1/2)+1/32*d/e^2*b^3/c^(3/2)
*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/4*d^2/e^3*a*c^(1/2)*ln((
c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16*d^2/e^3*b^2*ln((c*x^2+1/2*
b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-3/32/e*a*b^2/c^(3/2)*ln((c*x^2+1/
2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/8*d/e^2*a*b*ln((c*x^2+1/2*b)/c^(1/2)+
```

```
(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((
(b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))/(x^2+d/e))*a*b+d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)
*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d
/e))*a*c-d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/
e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*
c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left(c\,x^4+b\,x^2+a\right)^{3/2}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

```
[Out] int((x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(a + b x^{2} + c x^{4}\right)^{\frac{3}{2}}}{d + e x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

3.324 $\int \frac{x\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)}{32c^{3/2}e^4}$$

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/e-1/32*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2-4*c*e*(-3*
a*e+2*b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/
e^4+1/2*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/
(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^4+1/16*(8*c^2*d^2+b^2*e^
2-2*c*e*(-4*a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c/e^3
```

Rubi [A] time = 0.46, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1247, 734, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)}{32c^{3/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sq
rt[a + b*x^2 + c*x^4])/(16*c*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*e) - ((2*c
*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2)*e^4) + ((c*d^2 - b*d*e
 + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b
*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
```

```
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6e} - \frac{\text{Subst}\left(\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{4e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2\right)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{\left(a+bx^2+c\right.}{6e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2\right)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{\left(a+bx^2+c\right.}{6e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2\right)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{\left(a+bx^2+c\right.}{6e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2\right)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{\left(a+bx^2+c\right.}{6e}$$

Mathematica [A] time = 0.35, size = 255, normalized size = 0.95

$$\frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4}\left(2ce\left(16ae-15bd+7bex^2\right)+3b^2e^2+4c^2\left(6d^2-3dex^2+2e^2x^4\right)\right)-24c\left(e(ae-bd)+cd^2\right)\right.}{96c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (-3*(2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b
 + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*
x^2 + c*x^4]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x^2) + 4*c^2*(6*d
^2 - 3*d*e*x^2 + 2*e^2*x^4)) - 24*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*
Sqrt[a + b*x^2 + c*x^4])]))/(96*c^(3/2)*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1411, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] 1/6/e*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24/e*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/e
/c*b^2*(c*x^4+b*x^2+a)^(1/2)-5/8/e^2*b*(c*x^4+b*x^2+a)^(1/2)*d+1/2/e^3*c*(c
*x^4+b*x^2+a)^(1/2)*d^2-1/32/e*b^3/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+
b*x^2+a)^(1/2))-3/4/e^2*a*d*c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))-3/16/e^2*b^2*d*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(
1/2)+3/4/e^3*b*c^(1/2)*d^2*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+
2/3/e*a*(c*x^4+b*x^2+a)^(1/2)-1/4/e^2*x^2*c*(c*x^4+b*x^2+a)^(1/2)*d+3/8/e*a
*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2/e^4*c^(3/2)*
d^3*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2/e/((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(x^2+d/e))*a*b*d-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-
2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
```

```
(x^2+d/e))*a*c*d^2-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*
(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x
^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/
e))*b^2*d^2+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)
/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2
*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c*d
^3-1/2/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a
*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e
-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2*d^4
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more details)Is a*e^2-b*d*e                            +c*d^2 zero or nonze
ro?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(c\,x^4+b\,x^2+a\right)^{3/2}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

3.325 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x\left(d+ex^2\right)} dx$

Optimal. Leaf size=350

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}\,de^3}-\frac{\sqrt{a+bx^2+}}{}$$

```
[Out] -1/2*a^(3/2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d-1/2*(
a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*
d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/e^3+1/4*a*b*arctanh(1/2*(2*c*x^2+
b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/c^(1/2)+1/16*(8*c^2*d^3+b*e^2*(-4*a*e+3
*b*d)-12*c*d*e*(-a*e+b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^
(1/2))/d/e^3/c^(1/2)+1/2*a*(c*x^4+b*x^2+a)^(1/2)/d-1/8*(4*c*d^2-e*(-4*a*e+5
*b*d)-2*c*d*e*x^2)*(c*x^4+b*x^2+a)^(1/2)/d/e^2
```

Rubi [A] time = 0.57, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1251, 895, 734, 843, 621, 206, 724, 814}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}\,de^3}-\frac{\sqrt{a+bx^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

```
[Out] (a*Sqrt[a + b*x^2 + c*x^4])/(2*d) - ((4*c*d^2 - e*(5*b*d - 4*a*e) - 2*c*d*e
*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d*e^2) - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) + (a*b*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d) + ((8*c^2*d^3 + b*e^2*(3*b
*d - 4*a*e) - 12*c*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
 + b*x^2 + c*x^4])])/(16*Sqrt[c]*d*e^3) - ((c*d^2 - b*d*e + a*e^2)^(3/2)*Ar
cTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x^2 + c*x^4])])/(2*d*e^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x\left(d+ex^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x(d+ex)}\,dx,x,x^2\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{(-bd+ae-cdx)\sqrt{a+bx+cx^2}}{d+ex}\,dx,x,x^2\right)}{2d} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx,x,x^2\right)}{2d}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{\left(4cd^2-e(5bd-4ae)-2cdex^2\right)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a\,\text{Subst}\left(\int \frac{-2}{x\sqrt{a\cdots}}\right.}{4\cdots}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{\left(4cd^2-e(5bd-4ae)-2cdex^2\right)\sqrt{a+bx^2+cx^4}}{8de^2} + \frac{a^2\,\text{Subst}\left(\int \frac{\cdots}{x\sqrt{\cdots}}\right.}{\cdots}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{\left(4cd^2-e(5bd-4ae)-2cdex^2\right)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{4a-\cdots}\right.}{\cdots}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{\left(4cd^2-e(5bd-4ae)-2cdex^2\right)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a^{3/2}\tanh^{-1}\left(\frac{\cdots}{2\sqrt{\cdots}}\right.}{2d\cdots}$$

Mathematica [A] time = 0.53, size = 251, normalized size = 0.72

$$\frac{1}{16}\left(-\frac{8a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{\left(12ce(ae-bd)+3b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}\,e^3} + \frac{2\left(4\left(e(ae-bd)\cdots\right.\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

```
[Out] ((-8*a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/d
+ ((8*c^2*d^2 + 3*b^2*e^2 + 12*c*e*(-(b*d) + a*e))*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[c]*e^3) + (2*(d*e*(-4*c*d + 5*b*e
 + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4] + 4*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*
ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) +
a*e)]*Sqrt[a + b*x^2 + c*x^4])]))/(d*e^3))/16
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.04, size = 1270, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x)
```

```
[Out] 5/8/e*b*(c*x^4+b*x^2+a)^(1/2)-1/2/d*a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)
^(1/2)*a^(1/2))/x^2)-1/2/e^2*d*c*(c*x^4+b*x^2+a)^(1/2)+3/4/e*a*c^(1/2)*ln((
c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16/e*b^2*ln((c*x^2+1/2*b)/c^(
1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/4/e*x^2*c*(c*x^4+b*x^2+a)^(1/2)+1/2/e
^3*d^2*c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/2/d/((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)
/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)
/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2+1/e^2*d/((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*c-1/e^3*d^2/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))/(x^2+d/e))*b*c-3/4/e^2*d*b*c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+
(c*x^4+b*x^2+a)^(1/2))-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*
(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x
^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/
e))*a*b+1/2/e^2*d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)
/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2
*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2+1
/2/e^4*d^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a
*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e
-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{\left(ex^2 + d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4 + b\,x^2 + a\right)^{3/2}}{x\,\left(e\,x^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x\left(d + ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x*(d + e*x**2)), x)
```

3.326 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^3\left(d+ex^2\right)}\,dx$

Optimal. Leaf size=562

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}+\frac{be\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2}+\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2\right.}{16cd^2e}$$

```
[Out] -1/2*(c*x^4+b*x^2+a)^(3/2)/d/x^2+1/2*a^(3/2)*e*arctanh(1/2*(b*x^2+2*a)/a^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/d^2+1/32*b*(-12*a*c+b^2)*e*arctanh(1/2*(2*c*x^2+
b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/d^2-1/32*(-b*e+2*c*d)*(8*c^2*d^2-
b^2*e^2-4*c*e*(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+
a)^(1/2))/c^(3/2)/d^2/e^2+1/2*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*
a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/
e^2-3/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)/d+
3/16*(4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/c
^(1/2)+3/8*(2*c*x^2+3*b)*(c*x^4+b*x^2+a)^(1/2)/d-1/16*e*(2*b*c*x^2+8*a*c+b^
2)*(c*x^4+b*x^2+a)^(1/2)/c/d^2+1/16*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e+5*b*d)
-2*c*e*(-b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c/d^2/e
```

Rubi [A] time = 0.92, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1251, 960, 732, 814, 843, 621, 206, 724, 734}

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}+\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16cd^2e}-\frac{(2cd-be}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d) - (e*(b^2 + 8*a*c + 2*b*c
*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^2) + ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(
5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^
2*e) - (a + b*x^2 + c*x^4)^(3/2)/(2*d*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*d) + (a^(3/2)*e*ArcTanh[(2*a
+ b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (3*(b^2 + 4*a*c)*A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[c]*d) +
 (b*(b^2 - 12*a*c)*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(32*c^(3/2)*d^2) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d
 - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*
c^(3/2)*d^2*e^2) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2
*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*d^2*e^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^3\left(d+ex^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2(d+ex)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx^2} - \frac{e\left(a+bx+cx^2\right)^{3/2}}{d^2x} + \frac{e^2\left(a+bx+cx^2\right)^{3/2}}{d^2(d+ex)}\right) dx, x,$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx, x, x^2\right)}{2d} - \frac{e\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{e^2\,\text{Subst}\left(\int \frac{(}{}\right.}{}$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2dx^2} + \frac{3\,\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)}{4d} + \frac{e\,\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{}}{x}\right.}{4d^2}$$

$$= \frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{\left(8c^2d^2+b\right.}{}$$

$$= \frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{\left(8c^2d^2+b\right.}{}$$

$$= \frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{\left(8c^2d^2+b\right.}{}$$

$$= \frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{\left(8c^2d^2+b\right.}{}$$

Mathematica [A] time = 0.50, size = 240, normalized size = 0.43

$$\frac{1}{4}\left(-\frac{2\left(x^2\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)+de\sqrt{a+bx^2+cx^4}\left(ae-cdx^2\right)\right)}{d^2e^2x^2} + \frac{\sqrt{a}\,(2ae}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

```
[Out] ((Sqrt[a]*(-3*b*d + 2*a*e)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2
+ c*x^4])])/d^2 - (Sqrt[c]*(2*c*d - 3*b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]
*Sqrt[a + b*x^2 + c*x^4])])/e^2 - (2*(d*e*(a*e - c*d*x^2)*Sqrt[a + b*x^2 +
c*x^4] + (c*d^2 + e*(-(b*d) + a*e))^(3/2)*x^2*ArcTanh[(-(b*d) + 2*a*e - 2*c
*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4]
)]))/(d^2*e^2*x^2))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [**B**] time = 0.03, size = 1207, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x)
```

```
[Out] 1/2/e*c*(c*x^4+b*x^2+a)^(1/2)-3/4/d*a^(1/2)*b*ln((b*x^2+2*a+2*(c*x^4+b*x^2+
a)^(1/2)*a^(1/2))/x^2)-1/2/d*a/x^2*(c*x^4+b*x^2+a)^(1/2)+3/4/e*b*c^(1/2)*ln
((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2/e^2*d*c^(3/2)*ln((c*x^2+1
/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x^2+d/e))*b^2+1/2/d^2*e*a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(
1/2)*a^(1/2))/x^2)-1/2*e/d^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d
)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(
(x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+
d/e))*a^2+1/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2
*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(
b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*b-1/e/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c
*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2
+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*c-1/2/e^3*d^2/((a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e
^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2+1/e^2*d/((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{\left(ex^2 + d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x^3), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x^3\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^3\left(d + ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**3/(e*x**2+d),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x**3*(d + e*x**2)), x)
```

3.327 $\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

Optimal. Leaf size=463

$$-\frac{1}{14}\left(2x^4+2x^2+1\right)^{3/2}x-\frac{2211\sqrt{2x^4+2x^2+1}\,x}{140\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}-\frac{213}{140}\sqrt{2x^4+2x^2+1}\,x+\frac{17}{16}\sqrt{51}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{3(5}{}$$

```
[Out] -1/14*x*(2*x^4+2*x^2+1)^(3/2)+17/16*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^
(1/2))*51^(1/2)-213/140*x*(2*x^4+2*x^2+1)^(1/2)-27/70*x^3*(2*x^4+2*x^2+1)^(
1/2)-2211/280*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))+2211/280*(cos
(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arc
tan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+
x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/32*(cos(2*arctan(2^
(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)
*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(1+x^2*2^(1/2))*(
(2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2+3*2^(1/2))/(2*x^4+2*x^2
+1)^(1/2)-3/280*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))
*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(514+2717*2^(1/2
))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2+3*2
^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 875, normalized size of antiderivative = 1.89, number of steps used = 19, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1335, 1091, 1176, 1197, 1103, 1195, 1208, 1216, 1706}

$$-\frac{1}{14}x\left(2x^4+2x^2+1\right)^{3/2}-\frac{3}{35}x\left(x^2+2\right)\sqrt{2x^4+2x^2+1}-\frac{3}{20}x\left(2x^2+9\right)\sqrt{2x^4+2x^2+1}-\frac{6\sqrt{2}\,x\sqrt{2x^4+2x^2+1}}{35\left(\sqrt{2}\,x^2+1\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + 2*x^2 + 2*x^4)^(3/2))/(3 - 2*x^2),x]
```

```
[Out] (-3*x*(2 + x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/35 - (3*x*(9 + 2*x^2)*Sqrt[1 + 2*x
^2 + 2*x^4])/20 - (309*x*Sqrt[1 + 2*x^2 + 2*x^4])/(20*Sqrt[2]*(1 + Sqrt[2]*
x^2)) - (6*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(35*(1 + Sqrt[2]*x^2)) - (x*(
1 + 2*x^2 + 2*x^4)^(3/2))/14 + (17*Sqrt[51]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/16 + (309*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
 Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(20*2^(3/
4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (6*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(35*Sqrt[1 + 2*x^2 + 2*x^4]) + (867*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(112*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (51*(5 + Sqrt[2])*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(16*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
- (3*(3 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2
]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(70*2^(1/4)*Sqrt
[1 + 2*x^2 + 2*x^4]) - (3*(9 + 8*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(20*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt
[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*
Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(224*2^(1/4)*Sqrt[1 + 2
*x^2 + 2*x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
```

```
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx = \int \left(-\frac{1}{2}\left(1+2x^2+2x^4\right)^{3/2}+\frac{3\left(1+2x^2+2x^4\right)^{3/2}}{2\left(3-2x^2\right)}\right)dx$$

$$= -\left(\frac{1}{2}\int \left(1+2x^2+2x^4\right)^{3/2}\,dx\right)+\frac{3}{2}\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx$$

$$= -\frac{1}{14}x\left(1+2x^2+2x^4\right)^{3/2}-\frac{3}{14}\int \left(2+2x^2\right)\sqrt{1+2x^2+2x^4}\,dx-\frac{3}{8}\int \left(10+4x^2\right)\sqrt{}$$

$$= -\frac{3}{35}x\left(2+x^2\right)\sqrt{1+2x^2+2x^4}-\frac{3}{20}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4}-\frac{1}{14}x\left(1+2x^2+\right.$$

$$= -\frac{3}{35}x\left(2+x^2\right)\sqrt{1+2x^2+2x^4}-\frac{3}{20}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4}-\frac{1}{14}x\left(1+2x^2+\right.$$

$$= -\frac{3}{35}x\left(2+x^2\right)\sqrt{1+2x^2+2x^4}-\frac{3}{20}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4}-\frac{309x\sqrt{1+2x^2+}}{20\sqrt{2}\left(1+\sqrt{2}\right.}$$

Mathematica [C] time = 0.27, size = 214, normalized size = 0.46

$$\frac{-160x^9-752x^7-2456x^5-2080x^3-(9669-5247i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\right|i}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + 2*x^2 + 2*x^4)^(3/2))/(3 - 2*x^2),x]
```

```
[Out] (-892*x - 2080*x^3 - 2456*x^5 - 752*x^7 - 160*x^9 + (4422*I)*Sqrt[1 - I]*Sq
rt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x
], I] - (9669 - 5247*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*
x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 10115*(1 - I)^(3/2)*Sqrt[1 +
(1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1
- I]*x], I])/(560*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.32, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(2\,x^6+2\,x^4+x^2\right)\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="fricas")
[Out] integral(-(2*x^6 + 2*x^4 + x^2)*sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}x^2}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="giac")
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3), x)
```

maple [C] time = 0.04, size = 547, normalized size = 1.18

$$-\frac{\sqrt{2x^4+2x^2+1}\,x^5}{7}-\frac{37\sqrt{2x^4+2x^2+1}\,x^3}{70}-\frac{223\sqrt{2x^4+2x^2+1}\,x}{140}-\frac{309\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{Ellipt}}{40\sqrt{-1+i}\,\sqrt{2x^4+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x)
[Out] -1/7*x^5*(2*x^4+2*x^2+1)^(1/2)-37/70*(2*x^4+2*x^2+1)^(1/2)*x^3-223/140*(2*x
^4+2*x^2+1)^(1/2)*x-9/35/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)
)+(6/35-6/35*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4
+2*x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-Ellipt
icE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))-531/20/(-1+I)^(1/2)*(-I*x^2+
x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/
2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-309/40*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*
(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1
/2)+1/2*I*2^(1/2))-309/40/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(
1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/
2))+309/40*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+289/8/(-1
+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*El
lipticPi((-1+I)^(1/2)*x,-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}x^2}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="maxima")
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^2\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(2*x^2 + 2*x^4 + 1)^(3/2))/(2*x^2 - 3),x)
```

```
[Out] -int((x^2*(2*x^2 + 2*x^4 + 1)^(3/2))/(2*x^2 - 3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^6\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

```
[Out] -Integral(x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**4
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**6*sqrt(2*x**4 +
 2*x**2 + 1)/(2*x**2 - 3), x)
```

3.328 $\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

Optimal. Leaf size=428

$$-\frac{1}{10}\left(2x^2+9\right)\sqrt{2x^4+2x^2+1}\,x-\frac{103\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{17}{8}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(66+383\sqrt{2}\right)(\cdots}{\cdots}$$

```
[Out] 17/24*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)-1/10*x*(2*x^2+
9)*(2*x^4+2*x^2+1)^(1/2)-103/20*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1
/2))+103/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ell
ipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/48*
(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(
2*arctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(
1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2+3*2^(1/
2))/(2*x^4+2*x^2+1)^(1/2)-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arc
tan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(
66+383*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2
^(1/4)/(2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 602, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1208, 1176, 1197, 1103, 1195, 1216, 1706}

$$-\frac{1}{10}\left(2x^2+9\right)\sqrt{2x^4+2x^2+1}\,x-\frac{103\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{17}{8}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(9+8\sqrt{2}\right)\left(\sqrt{2}x\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(3 - 2*x^2),x]
```

```
[Out] -(x*(9 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/10 - (103*x*Sqrt[1 + 2*x^2 + 2*x^4
])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)/S
qrt[1 + 2*x^2 + 2*x^4]])/8 + (103*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4
)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10
*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sq
rt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x],
(2 - Sqrt[2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2])*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) -
 ((9 + 8*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x
^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1
+ 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^
(1/4)*x], (2 - Sqrt[2])/4])/(336*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx = -\left(\frac{1}{4}\int \left(10+4x^2\right)\sqrt{1+2x^2+2x^4}\,dx\right)+\frac{17}{2}\int \frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx$$

$$= -\frac{1}{10}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4} - \frac{1}{120}\int \frac{192+216x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{17}{8}\int \frac{10+4x^2}{\sqrt{1+2x^2+}}$$

$$= -\frac{1}{10}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4} + \frac{9\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{5\sqrt{2}} + \frac{17\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}} - \frac{1}{28}\Big($$

$$= -\frac{1}{10}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4} - \frac{103x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{17}{8}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{}{\sqrt{1+}}\right.$$

Mathematica [C] time = 0.15, size = 209, normalized size = 0.49

$$-48x^7-264x^5-240x^3-(1371-753i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+618i\sqrt{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(3 - 2*x^2),x]
```

```
[Out] (-108*x - 240*x^3 - 264*x^5 - 48*x^7 + (618*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)
*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (1371
- 753*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[
I*ArcSinh[Sqrt[1 - I]*x], I] + 1445*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqr
t[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(12
0*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{2\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

maple [C] time = 0.01, size = 377, normalized size = 0.88

$$-\frac{\sqrt{2x^4+2x^2+1}\,x^3}{5}-\frac{9\sqrt{2x^4+2x^2+1}\,x}{10}-\frac{103\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticE}\left(\sqrt{-1+i}\,x,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right.}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x)
```

```
[Out] -1/5*(2*x^4+2*x^2+1)^(1/2)*x^3-9/10*(2*x^4+2*x^2+1)^(1/2)*x-177/10/(-1+I)^(
1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipti
cF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-103/20*I/(-1+I)^(1/2)*(-I*x^2+
x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/
2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-103/20/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I
*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2
)+1/2*I*2^(1/2))+103/20*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(
1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/
2))+289/12/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x
^2+1)^(1/2)*EllipticPi((-1+I)^(1/2)*x,-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{3/2}}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(2*x^2 - 3),x)
```

```
[Out] -int((2*x^2 + 2*x^4 + 1)^(3/2)/(2*x^2 - 3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**2*sqrt
(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**4*sqrt(2*x**4 + 2*x*
*2 + 1)/(2*x**2 - 3), x)
```

3.329 $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^2\left(3-2x^2\right)}\,dx$

Optimal. Leaf size=722

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}\,x^2+1\right)}-\frac{17\sqrt{2x^4+2x^2+1}\,x}{3\sqrt{2}\,\left(\sqrt{2}\,x^2+1\right)}-\frac{\left(x^2+1\right)\sqrt{2x^4+2x^2+1}}{3x}+\frac{17}{12}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)-\cdots$$

```
[Out] 17/36*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)-1/3*(x^2+1)*(2
*x^4+2*x^2+1)^(1/2)/x-5/2*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))+5
/2*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(si
n(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2
+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/6*(cos(2*arcta
n(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1
/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1
/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/1008*(cos(2*arctan(2^(1/4)*
x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1
/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(11-6*2^(1/2))*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+289/168
*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(
2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(1+x^2*2^(1/2))*((2
*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-17/24*
(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2
*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(5+2^(1/2))*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 722, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {1311, 1271, 12, 1139, 1103, 1195, 1208, 1197, 1216, 1706}

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}\,x^2+1\right)}-\frac{17\sqrt{2x^4+2x^2+1}\,x}{3\sqrt{2}\,\left(\sqrt{2}\,x^2+1\right)}-\frac{\left(x^2+1\right)\sqrt{2x^4+2x^2+1}}{3x}+\frac{17}{12}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^2*(3 - 2*x^2)),x]
```

```
[Out] -((1 + x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(3*x) - (17*x*Sqrt[1 + 2*x^2 + 2*x^4])
/(3*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*
x^4]])/12 + (17*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2
)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1 + 2
*x^2 + 2*x^4]) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 +
 2*x^2 + 2*x^4]) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1
 + 2*x^2 + 2*x^4]) + (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(84*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*
Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*El
lipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(504*
2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1311

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[((f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^2\left(3-2x^2\right)}\,dx = -\left(\frac{1}{6}\int \frac{\left(-2+6x^2\right)\sqrt{1+2x^2+2x^4}}{x^2}\,dx\right)+\frac{17}{3}\int \frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx$$

$$= -\frac{\left(1+x^2\right)\sqrt{1+2x^2+2x^4}}{3x}+\frac{1}{18}\int \frac{12x^2}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{17}{12}\int \frac{10+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\left(1+x^2\right)\sqrt{1+2x^2+2x^4}}{3x}+\frac{2}{3}\int \frac{x^2}{\sqrt{1+2x^2+2x^4}}\,dx+\frac{17\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{3\sqrt{2}}-\frac{1}{42}\Big($$

$$= -\frac{\left(1+x^2\right)\sqrt{1+2x^2+2x^4}}{3x}-\frac{17x\sqrt{1+2x^2+2x^4}}{3\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{17}{12}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{1+2x^2+2}}\right.$$

$$= -\frac{\left(1+x^2\right)\sqrt{1+2x^2+2x^4}}{3x}-\frac{17x\sqrt{1+2x^2+2x^4}}{3\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}+\frac{17}{12}\sqrt{}$$

Mathematica [C] time = 0.21, size = 213, normalized size = 0.30

$$-24x^6-48x^4-36x^2-(255-165i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,xF\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+90i\sqrt{1-i}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^2*(3 - 2*x^2)),x]
```

```
[Out] (-12 - 36*x^2 - 48*x^4 - 24*x^6 + (90*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2
]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (255 - 165
```

```
*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*A
rcSinh[Sqrt[1 - I]*x], I] + 289*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[
1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(36*x
*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{2\,x^4-3\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^4 - 3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{\left(2\,x^2-3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2), x)
```

maple [C] time = 0.02, size = 528, normalized size = 0.73

$$-\frac{\sqrt{2x^4+2x^2+1}\,x}{3}-\frac{103\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\operatorname{EllipticE}\left(\sqrt{-1+i}\,x,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{30\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{103i\sqrt{-ix^2+x^2+1}\,\sqrt{}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)*x-59/5/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+
x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2
*I*2^(1/2))-103/30*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/
(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-1
03/30/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)
^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+103/30*I/(-1+I)^
(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipt
icE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+289/18/(-1+I)^(1/2)*(-I*x^2+x
^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi((-1+I)^(1/
2)*x,-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))-1/3*(2*x^4+2*x^2+1)^(1/2)/x+16/1
5/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+(-14/15+14/15*I)/(-1+
I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(Ell
ipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE((-1+I)^(1/2)*x,1
/2*2^(1/2)+1/2*I*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{\left(2\,x^2-3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{3/2}}{x^2\,\left(2\,x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(x^2*(2*x^2 - 3)),x)
```

```
[Out] -int((2*x^2 + 2*x^4 + 1)^(3/2)/(x^2*(2*x^2 - 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**2/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x) - Integral(2*x**2
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x) - Integral(2*x**4*sqrt(2*x
**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x)
```

3.330 $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^4\left(3-2x^2\right)}\,dx$

Optimal. Leaf size=625

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{9\left(\sqrt{2}\,x^2+1\right)}-\frac{2\sqrt{2x^4+2x^2+1}}{x}+\frac{17}{18}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\sqrt[4]{2}\left(9+5\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+}{(\sqrt{2}}}}{9\sqrt{2x^4}}$$

```
[Out] 17/54*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)-2*(2*x^4+2*x^2
+1)^(1/2)/x-1/9*(-8*x^2+1)*(2*x^4+2*x^2+1)^(1/2)/x^3+1/9*x*(2*x^4+2*x^2+1)^
(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/9*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*
arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2)
)*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+
2*x^2+1)^(1/2)-289/1512*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(
1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/
2))^(1/2))*(11-6*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^
2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+289/252*(cos(2*arctan(2^(1/4)*x))^2)
^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2
^(1/2))^(1/2))*(3-2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))
^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-17/36*(cos(2*arctan(2^(1/4)*x))^2)^
(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^
(1/2))^(1/2))*(5+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^
2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+1/9*2^(1/4)*(cos(2*arctan(2^(1/4)*x)
)^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*
(2-2^(1/2))^(1/2))*(9+5*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^
(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1309, 1271, 1281, 1197, 1103, 1195, 1208, 1216, 1706}

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{9\left(\sqrt{2}\,x^2+1\right)}-\frac{2\sqrt{2x^4+2x^2+1}}{x}-\frac{\left(1-8x^2\right)\sqrt{2x^4+2x^2+1}}{9x^3}+\frac{17}{18}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\sqrt[4]{2}\left(9\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^4*(3 - 2*x^2)),x]
```

```
[Out] (-2*Sqrt[1 + 2*x^2 + 2*x^4])/x - ((1 - 8*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(9*x
^3) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(9*(1 + Sqrt[2]*x^2)) + (17*Sqrt[
17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/18 - (2^(1/4)*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) + (289*(3 - Sqr
t[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellip
ticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(126*2^(1/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(18*2^(1/4
)*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(1/4)*(9 + 5*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 +
11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(756*2^(1/4)*Sqrt[1
+ 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[((f*x)^(m + 4)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^4\left(3-2x^2\right)}\,dx = \frac{1}{9}\int \frac{\left(3+8x^2\right)\sqrt{1+2x^2+2x^4}}{x^4}\,dx + \frac{34}{9}\int \frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx$$

$$= -\frac{\left(1-8x^2\right)\sqrt{1+2x^2+2x^4}}{9x^3} - \frac{1}{27}\int \frac{-54-60x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx - \frac{17}{18}\int \frac{10+4x^2}{\sqrt{1+2x^2+2x^4}}\,d$$

$$= -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{\left(1-8x^2\right)\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{27}\int \frac{60+108x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{9}\Big(17$$

$$= -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{\left(1-8x^2\right)\sqrt{1+2x^2+2x^4}}{9x^3} - \frac{17\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{9\left(1+\sqrt{2}\,x^2\right)} + \frac{17}{18}\sqrt{\frac{17}{3}}$$

$$= -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{\left(1-8x^2\right)\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{9\left(1+\sqrt{2}\,x^2\right)} + \frac{17}{18}\sqrt{\frac{17}{3}}\,\mathrm{ta}$$

Mathematica [C] time = 0.22, size = 219, normalized size = 0.35

$$-120x^6-132x^4-72x^2-(195-201i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,x^3F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|\,i\right)-6i\sqrt{1-i}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^4*(3 - 2*x^2)),x]
```

```
[Out] (-6 - 72*x^2 - 132*x^4 - 120*x^6 - (6*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x
^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (195 - 2
01*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF
[I*ArcSinh[Sqrt[1 - I]*x], I] + 289*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]
*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])
/(54*x^3*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{2\,x^6-3\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^6 - 3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{\left(2\,x^2-3\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4), x)
```

maple [C] time = 0.02, size = 530, normalized size = 0.85

$$\frac{103i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticE}\left(\sqrt{-1+i}\,x,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{45\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{103\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{Ellip}}{45\sqrt{-1+i}\,\sqrt{2x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x)
```

```
[Out] -118/15/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+
1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+103/45*I/(-1+I
)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Elli
pticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-103/45/(-1+I)^(1/2)*(-I*x^2
+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1
/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-103/45*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)
*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(
1/2)+1/2*I*2^(1/2))+289/27/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^
(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi((-1+I)^(1/2)*x,-1/3-1/3*I,(-1-I)^(1/
2)/(-1+I)^(1/2))-10/9*(2*x^4+2*x^2+1)^(1/2)/x+44/15/(-1+I)^(1/2)*((1-I)*x^2
+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*
x,1/2*2^(1/2)+1/2*I*2^(1/2))+(-12/5+12/5*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2
)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2
^(1/2)+1/2*I*2^(1/2))-EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))-
1/9*(2*x^4+2*x^2+1)^(1/2)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{\left(2\,x^2-3\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{3/2}}{x^4\,\left(2\,x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(x^4*(2*x^2 - 3)),x)
```

```
[Out] -int((2*x^2 + 2*x^4 + 1)^(3/2)/(x^4*(2*x^2 - 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**4/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x) - Integral(2*x**2
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x) - Integral(2*x**4*sqrt(2*x
**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x)
```

3.331 $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^6\left(3-2x^2\right)}\,dx$

Optimal. Leaf size=553

$$\frac{262\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{135\left(\sqrt{2}\,x^2+1\right)}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}+\frac{17}{27}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{2^{3/4}\left(37+23\sqrt{2}\right)\left(\sqrt{2}\,x^2\right.}{}$$

```
[Out] 17/81*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)+74/135*(2*x^4+
2*x^2+1)^(1/2)/x^3-262/135*(2*x^4+2*x^2+1)^(1/2)/x-1/45*(40*x^2+3)*(2*x^4+2
*x^2+1)^(1/2)/x^5+262/135*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-2
62/135*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Elliptic
E(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2
*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/2268*(co
s(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*a
rctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(11-6*2^(1/2))*(
1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x
^2+1)^(1/2)+85/189*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*
x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(
1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x
^2+1)^(1/2)+1/135*2^(3/4)*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2
^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(37+23
*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(2*x^4+
2*x^2+1)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1309, 1271, 1281, 1197, 1103, 1195, 1311, 1216, 1706}

$$\frac{262\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{135\left(\sqrt{2}\,x^2+1\right)}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}+\frac{74\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{\left(40x^2+3\right)\sqrt{2x^4+2x^2+1}}{45x^5}+\frac{17}{27}\sqrt{\frac{17}{3}}\tan$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^6*(3 - 2*x^2)),x]
```

```
[Out] (74*Sqrt[1 + 2*x^2 + 2*x^4])/(135*x^3) - (262*Sqrt[1 + 2*x^2 + 2*x^4])/(135
*x) - ((3 + 40*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x^5) + (262*Sqrt[2]*x*Sqrt
[1 + 2*x^2 + 2*x^4])/(135*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt
[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/27 - (262*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(135*Sqrt[1 + 2*x^2 + 2*x^4]) + (85*2^(3/4)*(3 - Sqrt[2])*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(189*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(3
/4)*(37 + 23*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(135*Sqrt[1 + 2
*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/
4)*x], (2 - Sqrt[2])/4])/(1134*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
```

```
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[((f*x)^(m + 4)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1311

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[((f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^6\left(3-2x^2\right)}\,dx = \frac{1}{9}\int \frac{\left(3+8x^2\right)\sqrt{1+2x^2+2x^4}}{x^6}\,dx + \frac{34}{9}\int \frac{\sqrt{1+2x^2+2x^4}}{x^2\left(3-2x^2\right)}\,dx$$

$$= -\frac{\left(3+40x^2\right)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{1}{45}\int \frac{-74-68x^2}{x^4\sqrt{1+2x^2+2x^4}}\,dx - \frac{17}{27}\int \frac{-2+6x^2}{x^2\sqrt{1+2x^2+}}$$

$$= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{34\sqrt{1+2x^2+2x^4}}{27x} - \frac{\left(3+40x^2\right)\sqrt{1+2x^2+2x^4}}{45x^5} - \frac{1}{135}\int$$

$$= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{\left(3+40x^2\right)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{17}{27}\sqrt{\frac{17}{3}}$$

$$= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{\left(3+40x^2\right)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{34\sqrt{2}\,x}{27}$$

$$= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{\left(3+40x^2\right)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{262\sqrt{2}}{135}$$

Mathematica [C] time = 0.24, size = 224, normalized size = 0.41

$$-\frac{1572x^8+1848x^6+1116x^4+192x^2+(543-1329i)\sqrt{1-i}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,x^5F\left(i\sinh^{-1}\left(\sqrt{1-}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^6*(3 - 2*x^2)),x]
```

```
[Out] -1/405*(27 + 192*x^2 + 1116*x^4 + 1848*x^6 + 1572*x^8 + (786*I)*Sqrt[1 - I]
*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1
 - I]*x], I] + (543 - 1329*I)*Sqrt[1 - I]*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1
```

```
+ (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 1445*(1 - I)^(3/2)*
x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*Ar
cSinh[Sqrt[1 - I]*x], I])/(x^5*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{2\,x^8-3\,x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^8 - 3*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{\left(2\,x^2-3\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6), x)
```

maple [C] time = 0.02, size = 549, normalized size = 0.99

$$\frac{206i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\operatorname{EllipticE}\left(\sqrt{-1+i}\,x,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{135\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{206\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{Elliptic}}{135\sqrt{-1+i}\,\sqrt{2x^4+2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x)
```

```
[Out] -236/45/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+
1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+206/135*I/(-1+
I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ell
ipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-206/135/(-1+I)^(1/2)*(-I*x
^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^
(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-206/135*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1
/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*
2^(1/2)+1/2*I*2^(1/2))+578/81/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+
1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi((-1+I)^(1/2)*x,-1/3-1/3*I,(-1-I)^
(1/2)/(-1+I)^(1/2))-262/135*(2*x^4+2*x^2+1)^(1/2)/x+184/45/(-1+I)^(1/2)*((1
-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)
^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+(-52/15+52/15*I)/(-1+I)^(1/2)*((1-I)*x^
2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2
)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2
^(1/2)))-46/135*(2*x^4+2*x^2+1)^(1/2)/x^3-1/15*(2*x^4+2*x^2+1)^(1/2)/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{\left(2\,x^2-3\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{3/2}}{x^6\,\left(2\,x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(x^6*(2*x^2 - 3)),x)
```

```
[Out] -int((2*x^2 + 2*x^4 + 1)^(3/2)/(x^6*(2*x^2 - 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**6/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x) - Integral(2*x**2
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x) - Integral(2*x**4*sqrt(2*x
**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x)
```

3.332 $\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=173

$$-\frac{(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}+\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}}+\frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

```
[Out] -1/4*(b*e+2*c*d)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(
3/2)/e^2+1/2*d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^
2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(1/2)+1/2*(c*x^4+b*
x^2+a)^(1/2)/c/e
```

Rubi [A] time = 0.31, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 1653, 843, 621, 206, 724}

$$-\frac{(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}+\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}}+\frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c*e) - ((2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)*e^2) + (d^2*ArcTanh[(b*d - 2*a
*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x
^4])])/(2*e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}bde-\frac{1}{2}e(2cd+be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2ce^2}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e^2} - \frac{(2cd+be)\,\text{Subs}}{}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{e^2} - $$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{(2cd+be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2} + \frac{d^2\tanh^{-1}\left(\frac{bd-}{2\sqrt{cd^2-b}}\right)}{2e^2\sqrt{cd^2-}}$$

Mathematica [A] time = 0.37, size = 171, normalized size = 0.99

$$\frac{2\sqrt{c}\left(\frac{cd^2\tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}+e\sqrt{a+bx^2+cx^4}\right)-(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-((2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]
) + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4] + (c*d^2*ArcTanh[(b*d - 2*a*e + 2*
c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])]
)/Sqrt[c*d^2 - b*d*e + a*e^2]))/(4*c^(3/2)*e^2)
```

fricas [B] time = 54.25, size = 1364, normalized size = 7.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^2*log(-((8*c^2*d^2 - 8*b*c*d*e +
(b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b
*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*s
qrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*
d*e*x^2 + d^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*s
qrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^
2 + b)*sqrt(c) - 4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 +
b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/8*(4*sqrt(-c*d^2 + b
*d*e - a*e^2)*c^2*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d
*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2
)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) +
 (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(c)*log(-8*c^2
*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) -
4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 + b*x^2 + a))/(c^3*
d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^
2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2
*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*
x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*
x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + (2*c^2*d^3 - b*c*d^2*e +
 a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)
*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(c^2*d^2*e - b*c*d*e
^2 + a*c*e^3)*sqrt(c*x^4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e
^4), 1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d^2*arctan(-1/2*sqrt(c*x^4 + b
*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((
c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
 b^2*d*e + a*b*e^2)*x^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c
)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)
/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^
4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.02, size = 267, normalized size = 1.54

$$-\frac{d^2 \ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a e^2-2deb+2c d^2}{e^2}+2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-deb+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\, e^3}-\frac{b \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{4c^{\frac{3}{2}} e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)/c/e-1/4/e*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))-1/2/e^2*d*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2)
)/c^(1/2)-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+
d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/
e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(e\,x^2 + d\right)\,\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x**5/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.333 $\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=137

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\,e}-\frac{d\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e/c^(1/2)-1/2*d*
arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b
*x^2+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\,e}-\frac{d\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c]*e) -
(d*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x^2 + c*x^4])])/(2*e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e} - \frac{d\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2a}{\sqrt{a}}\right)}{e}$$
$$= \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\,e} - \frac{d\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A] time = 0.12, size = 133, normalized size = 0.97

$$\frac{\frac{d\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[c] + (d*Ar
cTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*
e)]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/(2*e)
```

fricas [B] time = 3.57, size = 1084, normalized size = 7.91

$$\left[\frac{\sqrt{cd^2-bde+ae^2}\,cd\log\left(-\frac{\left(8c^2d^2-8bcde+\left(b^2+4ac\right)e^2\right)x^4-8abde+8a^2e^2+\left(b^2+4ac\right)d^2+2\left(4bcd^2+4abe^2-\left(3b^2+4ac\right)de\right)x^2-4\sqrt{cx^4+bx^2}}{e^2x^4+2dex^2+d^2}\right.}{4\left(c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*c*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*
d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^
2 + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^
2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(c^2*d^2*e -
b*c*d*e^2 + a*c*e^3), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arctan(-1/2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b
*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^
2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*sqrt(c)*l
og(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*s
qrt(c) - 4*a*c))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), 1/4*(sqrt(c*d^2 - b*d*e
 + a*e^2)*c*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b
```

```
*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 +
4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2
*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 2*(c*d^2 - b
*d*e + a*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -1/2*(
sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-
c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*
e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e
^2)*x^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c^2*d^2*e - b*c*d*
e^2 + a*c*e^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.01, size = 204, normalized size = 1.49

$$\frac{d \ln \left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a e^2-2deb+2c d^2}{e^2}+2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-deb+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\, e^2}+\frac{\ln \left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{2\sqrt{c}\, e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2/e*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/2*d/e^2/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d
^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d
/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^4+bx^2+a}\,\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.334 $\int \frac{x}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

```
[Out] 1/2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{1247, 724, 206\}$

$$\frac{\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sq
rt[a + b*x^2 + c*x^4])]/(2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{-bd+2ae-(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^2-bde+ae^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 87, normalized size = 1.01

$$-\frac{\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{2\sqrt{e(ae-bd)+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*
d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[c*d^2 + e*(-(b*d) + a*e)]
```

fricas [B] time = 1.32, size = 357, normalized size = 4.15

$$\left[\frac{\log\left(-\frac{\left(8c^2d^2-8bcde+\left(b^2+4ac\right)e^2\right)x^4-8abde+8a^2e^2+\left(b^2+4ac\right)d^2+2\left(4bcd^2+4abe^2-\left(3b^2+4ac\right)de\right)x^2+4\sqrt{cx^4+bx^2+a}\sqrt{cd^2-bde+ae^2}\left((2cd-b\right.}{e^2x^4+2dex^2+d^2}\right.}{4\sqrt{cd^2-bde+ae^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*
a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*
e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*
e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2))/sqrt(c*d^2 - b*d*e + a*
e^2), 1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*
sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 -
b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e +
 a*b*e^2)*x^2))/(c*d^2 - b*d*e + a*e^2)]
```

giac [A] time = 0.49, size = 75, normalized size = 0.87

$$\frac{\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2
 + b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

maple [B] time = 0.01, size = 165, normalized size = 1.92

$$-\frac{\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2
-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c
*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(e\,x^2 + d\right)\,\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.335 $\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=138

$$-\frac{e\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/a^(1/2)-1/2*e
*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+
b*x^2+a)^(1/2))/d/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1251, 960, 724, 206}

$$-\frac{e\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a]*d) -
 (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{dx\sqrt{a+bx+cx^2}}-\frac{e}{d(d+ex)\sqrt{a+bx+cx^2}}\right)dx,x,x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d} - \frac{e\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{e\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2a}{\sqrt{a}}\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d} - \frac{e\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A] time = 0.14, size = 134, normalized size = 0.97

$$-\frac{\frac{e\tanh^{-1}\left(\frac{-2ae+b\left(d-ex^2\right)+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}+\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*(ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a] +
(e*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*x^2))/(2*Sqrt[c*d^2 + e*(-(b*d) +
 a*e)]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/d
```

fricas [B] time = 1.39, size = 1097, normalized size = 7.95

$$\left[\frac{\sqrt{cd^2-bde+ae^2}\,ae\log\left(-\frac{\left(8\,c^2d^2-8\,bcde+\left(b^2+4\,ac\right)e^2\right)x^4-8\,abde+8\,a^2e^2+\left(b^2+4\,ac\right)d^2+2\left(4\,bcd^2+4\,abe^2-\left(3\,b^2+4\,ac\right)de\right)x^2-4\sqrt{cx^4+bx^2+a}\,\sqrt{}}{e^2x^4+2\,dex^2+d^2}\right.}{4\left(acd^3-abd^2e+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a*e*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*
d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^
2 + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b
*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c*
d^3 - a*b*d^2*e + a^2*d*e^2), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a*e*arct
an(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)
*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e
+ a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*s
qrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x
^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c*d^3 - a*b*d^2*e + a^2*d*e^2), 1/4*(sq
rt(c*d^2 - b*d*e + a*e^2)*a*e*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*
e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b
*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*
d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)
```

```
) + 2*(c*d^2 - b*d*e + a*e^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*c*d^3 - a*b*d^2*e + a^
2*d*e^2), -1/2*(sqrt(-c*d^2 + b*d*e - a*e^2)*a*e*arctan(-1/2*sqrt(c*x^4 + b
*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((
c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
 b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*sqrt(-a)*arctan(1/2*sqr
t(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*
c*d^3 - a*b*d^2*e + a^2*d*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.01, size = 207, normalized size = 1.50

$$\frac{\ln\left(\frac{\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{2a\,e^{2}-2deb+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}-deb+c\,d^{2}}{e^{2}}}\,\sqrt{\left(x^{2}+\frac{d}{e}\right)^{2}c+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}-deb+c\,d^{2}}{e^{2}}}}{x^{2}+\frac{d}{e}}\right)}{2\sqrt{\frac{a\,e^{2}-deb+c\,d^{2}}{e^{2}}}\,d}-\frac{\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{a}}{x^{2}}\right)}{2\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/d/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/2/d/((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*
d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+
d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{4}+bx^{2}+a}\left(ex^{2}+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(e\,x^{2}+d\right)\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(d+ex^{2}\right)\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.336 $\int \frac{1}{x^3(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=218

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

```
[Out] 1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)/d+1/2*
e*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/a^(1/2)+1/2*e^
2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))/d^2/(a*e^2-b*d*e+c*d^2)^(1/2)-1/2*(c*x^4+b*x^2+a)^(1/2)/a/
d/x^2
```

Rubi [A] time = 0.27, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 960, 730, 724, 206}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*d*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x^2)/(2*Sqrt
[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*d^2) + (e^2*ArcTanh[(b*d - 2*a*e
+ (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]
)])/(2*d^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{dx^2\sqrt{a+bx+cx^2}}-\frac{e}{d^2x\sqrt{a+bx+cx^2}}+\frac{e^2}{d^2(d+ex)\sqrt{a+bx}}\right.\right.$$
$$=\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d}-\frac{e\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d^2}+\frac{e^2\,\text{Sub}}{}$$
$$=-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}-\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4ad}+\frac{e\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,\right.}{d^2}$$
$$=-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d^2}+\frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-}{2\sqrt{cd^2-bde+ae^2}\sqrt{a}}\right.}{2d^2\sqrt{cd^2-bde+a}}$$
$$=-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}+\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d^2}$$

Mathematica [A] time = 0.37, size = 175, normalized size = 0.80

$$\frac{2\sqrt{a}\left(\frac{ae^2\tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}-\frac{d\sqrt{a+bx^2+cx^4}}{x^2}\right)+(2ae+bd)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((b*d + 2*a*e)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] +
 2*Sqrt[a]*(-((d*Sqrt[a + b*x^2 + c*x^4])/x^2) + (a*e^2*ArcTanh[(b*d - 2*a*
e + 2*c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*
x^4])])/Sqrt[c*d^2 - b*d*e + a*e^2]))/(4*a^(3/2)*d^2)
```

fricas [A] time = 2.93, size = 1414, normalized size = 6.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e^2*x^2*log(-((8*c^2*d^2 - 8*b*c*d*
e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*
```

```
(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 +
a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4
+ 2*d*e*x^2 + d^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*
e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 +
 a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*c*d^3 - a*b*d^2*e + a^2*d*e^
2)*sqrt(c*x^4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2),
1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*a^2*e^2*x^2*arctan(-1/2*sqrt(c*x^4 + b*
x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c
^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
b^2*d*e + a*b*e^2)*x^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)
*d^2*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*c*d^3 - a*b*d^2*e + a^2
*d*e^2)*sqrt(c*x^4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x
^2), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e^2*x^2*log(-((8*c^2*d^2 - 8*b*c*
d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 +
2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2
+ a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^
4 + 2*d*e*x^2 + d^2)) - (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^
2*e)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)
/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^
4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2), 1/4*(2*sqrt(
-c*d^2 + b*d*e - a*e^2)*a^2*e^2*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqr
t(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c
*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*
b*e^2)*x^2)) - (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*e)*sqrt
(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4
 + a*b*x^2 + a^2)) - 2*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^4 + b*x^2
 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2)]
```

giac [A] time = 0.49, size = 208, normalized size = 0.95

$$\frac{\arctan\left(-\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2+bde-ae^2}}\right)e^2}{\sqrt{-cd^2+bde-ae^2}\,d^2}-\frac{(bd+2\,ae)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,ad^2}+\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)b+2}{2\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2
 + b*d*e - a*e^2))*e^2/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^2) - 1/2*(b*d + 2*a*
e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*d^
2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a*d)
```

maple [A] time = 0.01, size = 276, normalized size = 1.27

$$-\frac{e\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\;d^2}+\frac{e\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{2\sqrt{a}\;d^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2/d^2*e/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-1/2*e
/d^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b
```

```
*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d
)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))-1/2*(c*x^4+b*x^2+a
)^(1/2)/a/d/x^2+1/4/d*b/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/
2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.337 $\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=418

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{2\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{3\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{4\left(2-3\sqrt{2}\right)}+\frac{\left(1-3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\right.}{2\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] -3/40*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*30^(1/2)*(3-2^(1/2))/(2-
3*2^(1/2))+1/4*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/4*(cos(2*a
rctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(
2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*
2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/4*(cos(2*arctan(2^(1/4)*x
))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2
*(2-2^(1/2))^(1/2))*(1-3*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2
^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+3/16*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^
4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1325, 1103, 1195, 1706}

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{2\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{3\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{4\left(2-3\sqrt{2}\right)}+\frac{\left(1-3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\right.}{2\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(2*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (3*Sqrt[3/10]*(
3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(4*(2 - 3*Sqrt[
2])) - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(3/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) + ((1 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(3/4)*
(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*(3 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2
])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/4)*(2 - 3*Sqrt[2])*Sq
rt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
```

```
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1325

```
Int[(x_)^4/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, -Dist[(2*c*d - a*e*q)/(c*e*(e - d*q))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + (-Dist[1/(e*q), Int[(1 - q*x^2)/S
qrt[a + b*x^2 + c*x^4], x], x] + Dist[d^2/(e*(e - d*q)), Int[(1 + q*x^2)/((
d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x])] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}} + \frac{9\int \frac{1+\sqrt{2}x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx}{2\left(2-3\sqrt{2}\right)} - \frac{\left(12-2\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx}{4\left(2-3\sqrt{2}\right)}$$

$$= \frac{x\sqrt{1+2x^2+2x^4}}{2\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{3\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{4\left(2-3\sqrt{2}\right)} - \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{(1+\cdots}}}{2\cdots}$$

Mathematica [C] time = 0.21, size = 127, normalized size = 0.30

$$-\frac{\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(-(1+4i)F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+(1+i)E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+3i\Pi\left(\frac{1}{3}+\frac{i}{3}\cdots\right.\right.}{4\sqrt{1-i}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -1/4*(Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*((1 + I)*EllipticE[I*ArcS
inh[Sqrt[1 - I]*x], I] - (1 + 4*I)*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] +
 (3*I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(Sqrt[1 - I]*Sq
rt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2x^4+2x^2+1}\,x^4}{4x^6+10x^4+8x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

maple [C] time = 0.02, size = 222, normalized size = 0.53

$$-\frac{3\sqrt{(1-i)\,x^2+1}\;\sqrt{(1+i)\,x^2+1}\;\mathrm{EllipticF}\left(\sqrt{-1+i}\;x,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{4\sqrt{-1+i}\;\sqrt{2x^4+2x^2+1}}+\frac{3\sqrt{-ix^2+x^2+1}\;\sqrt{ix^2+x^2+1}\;\mathrm{EllipticI}}{4\sqrt{-1+i}\;\sqrt{2x^4+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
[Out] (-1/4+1/4*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*
x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE
((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))-3/4/(-1+I)^(1/2)*((1-I)*x^2+1)^
(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/
2*2^(1/2)+1/2*I*2^(1/2))+3/4/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1
)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi((-1+I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1
/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(2\,x^2+3\right)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
[Out] int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
[Out] Integral(x**4/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

3.338 $\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=247

$$-\frac{1}{4}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{\left(3+\sqrt{2}\right)^2}{}$$

```
[Out] -1/20*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/28*(cos(2*arc
tan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^
(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+
1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/112*(cos(2*arct
an(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^
(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/
2)
```

Rubi [A] time = 0.13, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1319, 1103, 1706}

$$-\frac{1}{4}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{\left(3+\sqrt{2}\right)^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -(Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/4 - ((3 + Sqrt[2
])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elliptic
F[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(14*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4
]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[
2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1319

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, -Dist[(a*(e + d*q))/(c*d^2 - a*e^2),
Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(a*d*(e + d*q))/(c*d^2 - a*e^2
), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
 && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
```

```
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx = -\left(\frac{1}{14}\left(2+3\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx\right)+\frac{1}{14}\left(3\left(2+3\sqrt{2}\right)\right)\int \frac{1}{\left(3+2x^2\right)}$$

$$= -\frac{1}{4}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)-\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,F\left(2\right.}{14\ 2^{3/4}\sqrt{1+2x^2+}}$$

Mathematica [C] time = 0.12, size = 99, normalized size = 0.40

$$\frac{(1-i)^{3/2}\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)\right)}{4\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*(EllipticF[I*Arc
Sinh[Sqrt[1 - I]*x], I] - EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I
]))/(4*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.38, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}\,x^2}{4\,x^6+10\,x^4+8\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}\,\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

maple [C] time = 0.01, size = 134, normalized size = 0.54

$$\frac{\sqrt{(1-i)\,x^2+1}\,\sqrt{(1+i)\,x^2+1}\ \mathrm{EllipticF}\left(\sqrt{-1+i}\,x,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\ \mathrm{EllipticPi}\left(\right.}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] 1/2/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-1/2/(-1+I)^(1/2)*(-
I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi((-1
+I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}\,\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(2\,x^2+3\right)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
```

```
[Out] int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

3.339 $\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=245

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{15}}+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right.}{}$$

```
[Out] 1/30*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/28*(cos(2*arct
an(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(
1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1
)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-1/168*(cos(2*arcta
n(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(
1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(1
/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2
)
```

Rubi [A] time = 0.11, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1216, 1103, 1706}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{15}}+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]/(2*Sqrt[15]) + ((3 + Sqrt[2])
*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[
2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(14*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
 - ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2]
)/4])/(84*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
```

```
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx = \frac{1}{7}\left(3+\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{7}\left(2+3\sqrt{2}\right)\int \frac{1+\sqrt{2}\,x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)}{2\sqrt{15}} + \frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{2}\,x\right)\middle|\frac{1}{4}\right.}{14\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] time = 0.06, size = 80, normalized size = 0.33

$$-\frac{i\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{3\sqrt{1-i}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((-1/3*I)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3,
 I*ArcSinh[Sqrt[1 - I]*x], I])/(Sqrt[1 - I]*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{4\,x^6+10\,x^4+8\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

maple [C] time = 0.01, size = 70, normalized size = 0.29

$$\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\;\mathrm{EllipticPi}\left(\sqrt{-1+i}\,x,\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] 1/3/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(
1/2)*EllipticPi((-1+I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^{4}+2\,x^{2}+1}\left(2\,x^{2}+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^{2}+3\right)\,\sqrt{2\,x^{4}+2\,x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
```

```
[Out] int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^{2}+3\right)\sqrt{2x^{4}+2x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

3.340 $\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=399

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{3\sqrt{15}}+\frac{\left(5-3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}\right.\right.}{21\,2^{3/4}\sqrt{2x^4+2x^2+1}}$$

```
[Out] -1/45*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/3*(2*x^4+2*x^
2+1)^(1/2)/x+1/3*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/3*(cos(2
*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arcta
n(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^
2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/42*2^(1/4)*(cos(2*arcta
n(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1
/4)*x)),1/2*(2-2^(1/2))^(1/2))*(5-3*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+
1)/(1+x^2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)+1/252*(cos(2*arctan(2^(1/
4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)
),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(1/2))*((
2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1329, 1714, 1195, 1708, 1103, 1706}

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{3\sqrt{15}}+\frac{\left(5-3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}\right.\right.}{21\,2^{3/4}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) - ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]/(3*Sqrt[15]
) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2
]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^
4]) + ((5 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(21*2^(3/4)*Sq
rt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^
(1/4)*x], (2 - Sqrt[2])/4])/(126*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1329

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[(x^(m + 1)*Sqrt[a + b*x^2 + c*x^4])/(a*d*(m + 1)), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)*Simp[a*e*(m + 1) + b*d*(m + 2) + (
b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e*(m + 3)*x^4, x])/((d + e*x^2)*Sqrt[a +
 b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{x^2 \left(3+2x^2\right) \sqrt{1+2x^2+2x^4}}\,dx = -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{1}{3}\int \frac{-2+6x^2+4x^4}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{1}{12}\int \frac{-8+12\sqrt{2}+\left(24-4\left(6-2\sqrt{2}\right)\right)x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{3}\sqrt{2}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)} - \frac{\sqrt[4]{2}\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,E}{3\sqrt{1+2x}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)}{3\sqrt{15}} - \frac{\sqrt[4]{2}\left(1+\right.}{}$$

Mathematica [C] time = 0.22, size = 147, normalized size = 0.37

$$-\frac{i\left(\sqrt{1-i}\,x\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(-3F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+3E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(1+i)\Pi\left(\frac{1}{3}\right.\right.\right.}{9x\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((-1/9*I)*((-3*I)*(1 + 2*x^2 + 2*x^4) + Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]
*Sqrt[1 + (1 + I)*x^2]*(3*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - 3*Ellipt
icF[I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + I)*EllipticPi[1/3 + I/3, I*ArcSinh[
Sqrt[1 - I]*x], I])))/(x*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{4\,x^8+10\,x^6+8\,x^4+3\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^8 + 10*x^6 + 8*x^4 + 3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2), x)
```

maple [C] time = 0.02, size = 178, normalized size = 0.45

$$-\frac{2\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\operatorname{EllipticPi}\left(\sqrt{-1+i}\,x,\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}} - \frac{\sqrt{2x^4+2x^2+1}}{3x} + \frac{\left(-\frac{1}{3}+\frac{i}{3}\right)\sqrt{(1-i)\,x^2+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x+(-1/3+1/3*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*
((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2^(
1/2)+1/2*I*2^(1/2))-EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))-2/
9/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/
2)*EllipticPi((-1+I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}\,\left(2\,x^2+3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(2\,x^2+3\right)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\left(2x^{2}+3\right)\sqrt{2x^{4}+2x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

3.341 $\int \frac{1}{x^4\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=422

$$-\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}x^2+1\right)}+\frac{2\sqrt{2x^4+2x^2+1}}{3x}+\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{9\sqrt{15}}-\frac{\left(1+19\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan\right.}{63\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$

```
[Out] 2/135*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/9*(2*x^4+2*x^
2+1)^(1/2)/x^3+2/3*(2*x^4+2*x^2+1)^(1/2)/x-2/3*x*(2*x^4+2*x^2+1)^(1/2)*2^(1
/2)/(1+x^2*2^(1/2))+2/3*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(
1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2
^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(
1/2)-1/378*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Elli
pticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3
+2^(1/2))^2*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/
4)/(2*x^4+2*x^2+1)^(1/2)-1/126*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arc
tan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(
1+19*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(
3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1329, 1683, 1714, 1195, 1708, 1103, 1706}

$$-\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}x^2+1\right)}+\frac{2\sqrt{2x^4+2x^2+1}}{3x}-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}+\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{9\sqrt{15}}-\frac{\left(1+19\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{}}{63\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(9*x^3) + (2*Sqrt[1 + 2*x^2 + 2*x^4])/(3*x) - (2*S
qrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1 + Sqrt[2]*x^2)) + (2*ArcTan[(Sqrt[5
/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(9*Sqrt[15]) + (2*2^(1/4)*(1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*
x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 + 19*Sqrt[2])*(1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcT
an[2^(1/4)*x], (2 - Sqrt[2])/4])/(63*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3
 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^
2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/
(189*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
```

```
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1329

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[(x^(m + 1)*Sqrt[a + b*x^2 + c*x^4])/(a*d*(m + 1)), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)*Simp[a*e*(m + 1) + b*d*(m + 2) + (
b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e*(m + 3)*x^4, x])/((d + e*x^2)*Sqrt[a +
 b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1683

```
Int[((Px_)*(x_)^(m_))/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 2], C
 = Coeff[Px, x, 4]}, Simp[(A*x^(m + 1)*Sqrt[a + b*x^2 + c*x^4])/(a*d*(m + 1
)), x] + Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 +
 c*x^4]))*Simp[a*B*d*(m + 1) - A*(a*e*(m + 1) + b*d*(m + 2)) + (a*C*d*(m +
1) - A*(b*e*(m + 2) + c*d*(m + 3)))*x^2 - A*c*e*(m + 3)*x^4, x], x], x]] /;
 FreeQ[{a, b, c, d, e}, x] && PolyQ[Px, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && I
LtQ[m/2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{1}{9}\int \frac{-18-14x^2-4x^4}{x^2\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{1}{27}\int \frac{6+120x^2+72x^4}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,d$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{1}{108}\int \frac{24+216\sqrt{2}+\left(480-72\right.}{\left(3+2x^2\right)\sqrt{1+2x^2}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)}+\frac{2\sqrt[4]{2}\left(1\right.}{}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)}+\frac{2\tan^{-1}\left(\right.}{9}$$

Mathematica [C] time = 0.18, size = 219, normalized size = 0.52

$$\frac{36x^6+30x^4+12x^2-(3+15i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,x^3F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+18i\sqrt{1-i}\sqrt{1+(}{27}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] (-3 + 12*x^2 + 30*x^4 + 36*x^6 + (18*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^
2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (3 + 15*I
)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*A
rcSinh[Sqrt[1 - I]*x], I] + 2*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[
1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(27*x^
3*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.36, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{4\,x^{10}+10\,x^8+8\,x^6+3\,x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^10 + 10*x^8 + 8*x^6 + 3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4), x)
```

maple [C] time = 0.01, size = 260, normalized size = 0.62

$$-\frac{2\sqrt{(1-i)\,x^2+1}\,\sqrt{(1+i)\,x^2+1}\,\operatorname{EllipticF}\left(\sqrt{-1+i}\,x,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{4\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\operatorname{EllipticI}}{27\sqrt{-1+i}\,\sqrt{2x^4+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] 2/3*(2*x^4+2*x^2+1)^(1/2)/x+(2/3-2/3*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((
1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2^(1/
2)+1/2*I*2^(1/2))-EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))+4/27
/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticPi((-1+I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))-1/9*(2*x^4
+2*x^2+1)^(1/2)/x^3-2/9/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2
)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}\,\left(2\,x^2+3\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(2\,x^2+3\right)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^4*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

3.342 $\int \frac{x^7}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=236

$$\frac{x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e}-\frac{d^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)\cdots}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-\cdots}}\right)}{2e\left(ae^2-bde+cd^2\right)^{3/\cdots}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/e-1/2*d^
3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(3/2)+(a*(-a*b*e-2*a*c*d+b^2*d)+(2*a
^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)*x^2)/c/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*
x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 1646, 843, 621, 206, 724}

$$\frac{x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e}-\frac{d^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)\cdots}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-\cdots}}\right)}{2e\left(ae^2-bde+cd^2\right)^{3/\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*(b^2*d - 2*a*c*d - a*b*e) + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*x^
2)/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + ArcT
anh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*c^(3/2)*e) - (d^3
*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x^2 + c*x^4])])/(2*e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{a\left(b^2d-2acd-abe\right)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{\frac{\left(b^2-4a\ldots\right.}{2c\left(cd^2\ldots\right.}}{(d+\ldots}\right.}{\ldots}$$

$$= \frac{a\left(b^2d-2acd-abe\right)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\ldots}}\right.}{2c\ldots}$$

$$= \frac{a\left(b^2d-2acd-abe\right)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\ldots\right.}{\ldots}$$

$$= \frac{a\left(b^2d-2acd-abe\right)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{\tanh^{-1}\left(\frac{b+\ldots}{2\sqrt{c}\sqrt{a\ldots}}\right.}{2c^{3/2}e\ldots}$$

Mathematica [A] time = 0.82, size = 271, normalized size = 1.15

$$\frac{1}{2}\left(\frac{2\left(a^2\left(be+2c\left(d-ex^2\right)\right)+ab\left(-bd+bex^2+3cdx^2\right)+b^3(-d)x^2\right)}{c\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}\left(e(ae-bd)+cd^2\right)}+\frac{\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{c^{3/2}e}+\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*(-(b^3*d*x^2) + a*b*(-(b*d) + 3*c*d*x^2 + b*e*x^2) + a^2*(b*e + 2*c*(d
- e*x^2))))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c
```

```
*x^4]) - (d^3*Log[d + e*x^2])/(e*(c*d^2 + e*(-(b*d) + a*e))^(3/2)) + Log[b
+ 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]/(c^(3/2)*e) + (d^3*Log[-(b*d
) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x^2 + c*x^4]])/(e*(c*d^2 + e*(-(b*d) + a*e))^(3/2)))/2
```

fricas [B] time = 151.80, size = 4901, normalized size = 20.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b
^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^
3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d
^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2
)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2
*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*
(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(c)*log(-
8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(
c) - 4*a*c) + ((b^2*c^3 - 4*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2
+ (a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2
 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*
c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4
 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)
)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 4*(a^3*b*c*e^4 - (a*b^2*c^2 - 2*a^2*c^3)*d
^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c^2)*d*e^3 - ((b^
3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3)*d^2*e^2 + (2*a
*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)*x^2)*sqrt(c*x^4
+ b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*
d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^3 - 2*(a^2*b^3*c^2
- 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5 + ((b^2*c^5 - 4*a*c^6)
*d^4*e - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e^2 + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c
^5)*d^2*e^3 - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^4 + (a^2*b^2*c^3 - 4*a^3*c^4)
*e^5)*x^4 + ((b^3*c^4 - 4*a*b*c^5)*d^4*e - 2*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e^
2 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c^4)*d^2*e^3 - 2*(a*b^4*c^2 - 4*a^2*b^
2*c^3)*d*e^4 + (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^5)*x^2), -1/4*(2*((b^2*c^3 - 4
*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d
^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-
c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*
e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e
^2)*x^2)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e
+ (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3
 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*
c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2
*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d
^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^
2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(c)
*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)
*sqrt(c) - 4*a*c) + 4*(a^3*b*c*e^4 - (a*b^2*c^2 - 2*a^2*c^3)*d^3*e + (a*b^3
*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c^2)*d*e^3 - ((b^3*c^2 - 3*a*b
*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3)*d^2*e^2 + (2*a*b^3*c - 5*a^
2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))
/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e^2 + (a*
b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^3 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3
)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5 + ((b^2*c^5 - 4*a*c^6)*d^4*e - 2*(b
^3*c^4 - 4*a*b*c^5)*d^3*e^2 + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*e^3 -
 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^4 + (a^2*b^2*c^3 - 4*a^3*c^4)*e^5)*x^4 + (
(b^3*c^4 - 4*a*b*c^5)*d^4*e - 2*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e^2 + (b^5*c^2
- 2*a*b^3*c^3 - 8*a^2*b*c^4)*d^2*e^3 - 2*(a*b^4*c^2 - 4*a^2*b^2*c^3)*d*e^4
```

```
+ (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^5)*x^2), -1/4*(2*((a*b^2*c^2 - 4*a^2*c^3)*d
^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^
2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3
 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*
a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2
)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (
b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a
^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(
2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - ((b^2*c^3 - 4*a*c^4)*d^3
*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(c*
d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4
- 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3
*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e
^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 4*(a^
3*b*c*e^4 - (a*b^2*c^2 - 2*a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 -
 2*(a^2*b^2*c - a^3*c^2)*d*e^3 - ((b^3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2*
a*b^2*c^2 - 2*a^2*c^3)*d^2*e^2 + (2*a*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2
*c - 2*a^3*c^2)*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)
*d^4*e - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 -
 8*a^3*c^4)*d^2*e^3 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 -
4*a^4*c^3)*e^5 + ((b^2*c^5 - 4*a*c^6)*d^4*e - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e
^2 + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*e^3 - 2*(a*b^3*c^3 - 4*a^2*b*c
^4)*d*e^4 + (a^2*b^2*c^3 - 4*a^3*c^4)*e^5)*x^4 + ((b^3*c^4 - 4*a*b*c^5)*d^4
*e - 2*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e^2 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c
^4)*d^2*e^3 - 2*(a*b^4*c^2 - 4*a^2*b^2*c^3)*d*e^4 + (a^2*b^3*c^2 - 4*a^3*b*
c^3)*e^5)*x^2), -1/2*(((b^2*c^3 - 4*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*
d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(
-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^
2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a
^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^4
 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*
e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 -
 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^
2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*
e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^
5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2
*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*
c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(a^3*b*c*e^4 - (a*b^2*c^
2 - 2*a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c
^2)*d*e^3 - ((b^3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3
)*d^2*e^2 + (2*a*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)*
x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c^3
 - 4*a^2*b*c^4)*d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^3 -
 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5 + ((b^
2*c^5 - 4*a*c^6)*d^4*e - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e^2 + (b^4*c^3 - 2*a*b
^2*c^4 - 8*a^2*c^5)*d^2*e^3 - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^4 + (a^2*b^2*
c^3 - 4*a^3*c^4)*e^5)*x^4 + ((b^3*c^4 - 4*a*b*c^5)*d^4*e - 2*(b^4*c^3 - 4*a
*b^2*c^4)*d^3*e^2 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c^4)*d^2*e^3 - 2*(a*b^
4*c^2 - 4*a^2*b^2*c^3)*d*e^4 + (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^5)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.06, size = 720, normalized size = 3.05

$$-\frac{2c\,d^{3}\ln\left(\frac{\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{2a\,e^{2}-2deb+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}-deb+c\,d^{2}}{e^{2}}}\,\sqrt{\left(x^{2}+\frac{d}{e}\right)^{2}c+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}-deb+c\,d^{2}}{e^{2}}}}{x^{2}+\frac{d}{e}}\right)}{\left(-be+2cd+\sqrt{-4ac+b^{2}}\,e\right)\left(be-2cd+\sqrt{-4ac+b^{2}}\,e\right)\sqrt{\frac{a\,e^{2}-deb+c\,d^{2}}{e^{2}}}\,e^{2}}+\frac{b^{2}x^{2}}{2\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/e*x^2/c/(c*x^4+b*x^2+a)^(1/2)+1/4/e*b/c^2/(c*x^4+b*x^2+a)^(1/2)+1/2/e*
b^2/c/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+1/4/e*b^3/c^2/(4*a*c-b^2)/(c*x^
4+b*x^2+a)^(1/2)+1/2/e/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/
2))+d/e^2/(c*x^4+b*x^2+a)^(1/2)*(b*x^2+2*a)/(4*a*c-b^2)+d^2/e^3*(2*c*x^2+b)
/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-2*d^3/e^3*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d)/(-4*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*(c*(x^2+1/2*(b+(-4
*a*c+b^2)^(1/2))/c)^2-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)-2*d^3/e^2*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a
*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e
-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))+2*d^3/e^3*c/
(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)
^(1/2)/c)*(c*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2+(-4*a*c+b^2)^(1/2)*(x^2-
1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}\left(ex^{2}+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7}}{\left(e\,x^{2}+d\right)\,\left(c\,x^{4}+b\,x^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(d+ex^{2}\right)\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.343 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=167

$$\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{x^2\left(-abe-2acd+b^2d\right)+a(bd-2ae)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

```
[Out] 1/2*d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+(-a*(-2*a*e+b*d)-(-a*b*e-2
*a*c*d+b^2*d)*x^2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 1646, 12, 724, 206}

$$\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{x^2\left(-abe-2acd+b^2d\right)+a(bd-2ae)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (d^2*ArcTanh[(b*d - 2*a*e + (2*
c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(
2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
```

```
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int -\frac{\left(b^2-4ac\right)}{2\left(cd^2-bde+ae^2\right)(d+e\ldots}\right.}{b^2-4a\ldots}$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\ldots\right.}{2\left(cd^2-bde+ae^2\ldots\right.}$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x\ldots}\right.}{cd^2-bde\ldots}$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{d^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-b\ldots}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+\ldots}}\right.}{2\left(cd^2-bde+ae^2\right)^{3\ldots}}$$

Mathematica [A] time = 0.62, size = 204, normalized size = 1.22

$$\frac{1}{2}\left(\frac{2\left(-2a^2e+ab\left(d-ex^2\right)-2acdx^2+b^2dx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(e(bd-ae)-cd^2\right)} - \frac{d^2\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2\ldots\right.}{\left(e(ae-bd)+cd^2\right)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*(-2*a^2*e + b^2*d*x^2 - 2*a*c*d*x^2 + a*b*(d - e*x^2)))/((b^2 - 4*a*c)*
(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (d^2*Log[d + e*x^2])/
(c*d^2 + e*(-(b*d) + a*e))^(3/2) - (d^2*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*
e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]])/(c*d^2 + e*
(-(b*d) + a*e))^(3/2))/2
```

fricas [B] time = 1.97, size = 1381, normalized size = 8.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3 - 4*a*b*c)*d^2*x^2 + (a*b^2 - 4*a^2
*c)*d^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d
```

```
^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
 + d^2)) - 4*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2
*e - (a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 -
a^2*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 -
2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2
 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*
a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c
^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4
)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^
3 - 4*a^3*b*c)*e^4)*x^2), 1/2*(((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3 - 4*a*b*c)
*d^2*x^2 + (a*b^2 - 4*a^2*c)*d^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b
*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^
2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2*
a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e - (a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b
^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/
((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2
*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2
- 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e +
 (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^
3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*
c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4
 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
```

giac [B] time = 0.72, size = 458, normalized size = 2.74

$$\frac{d^2 \arctan\left(-\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}-\frac{\left(b^2cd^3-2\,ac^2d^3-b^3d^2e+abcd^2e+2\,ab^2de^2-2\,a^2cde^2-a^2be^3\right)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] d^2*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2
)) - ((b^2*c*d^3 - 2*a*c^2*d^3 - b^3*d^2*e + a*b*c*d^2*e + 2*a*b^2*d*e^2 -
2*a^2*c*d*e^2 - a^2*b*e^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e +
 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*
a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (a*b*c*d^3 - a
*b^2*d^2*e - 2*a^2*c*d^2*e + 3*a^2*b*d*e^2 - 2*a^3*e^3)/(b^2*c^2*d^4 - 4*a*
c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2
 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^
3*c*e^4))/sqrt(c*x^4 + b*x^2 + a)
```

maple [B] time = 0.02, size = 613, normalized size = 3.67

$$\frac{2c\,d^2 \ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(-be+2cd+\sqrt{-4ac+b^2}\,e\right)\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\,e}-\frac{b\,x^2}{\sqrt{c\,x^4+b\,x^2+a}\,\left(4ac-b^2\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/e/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)*x^2*b-2/e/(c*x^4+b*x^2+a)^(1/2)/(4*a
*c-b^2)*a-2/e^2*d/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*c*x^2-1/e^2*d/(4*a*c-b^
2)/(c*x^4+b*x^2+a)^(1/2)*b+2*d^2/e^2*c/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4
*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*((x^2+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)+
2*d^2/e*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e
)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*
e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(
x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))-2*d^2/e^2*c/(-b*e+2*c
*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c
)*((x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*c+(-4*a*c+b^2)^(1/2)*(x^2-1/2*(-b+
(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.344 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=159

$$\frac{cx^2(bd-2ae)+a(2cd-be)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}-\frac{de\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}$$

```
[Out] -1/2*d*e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+(a*(-b*e+2*c*d)+c*(-2*a*e
+b*d)*x^2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 822, 12, 724, 206}

$$\frac{cx^2(bd-2ae)+a(2cd-be)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}-\frac{de\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)*Sqrt[a + b*x^2 + c*x^4]) - (d*e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^
2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*(c*d^2 - b*
d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{(b^2-4ac)de}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx, \ldots\right)}{(b^2-4ac)(cd^2-bde+a\ldots}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{(de)\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}} \ldots\right)}{2(cd^2-bde+ae^2)}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x\ldots} \ldots\right)}{cd^2-bde\ldots}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{de\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)\ldots}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+b\ldots}}\right)}{2(cd^2-bde+ae^2)^{3/2}}$$

Mathematica [A] time = 0.17, size = 162, normalized size = 1.02

$$\frac{a(be-2cd+2cex^2)-bcdx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(e(bd-ae)-cd^2)} + \frac{de\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{2(e(ae-bd)+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(b*c*d*x^2) + a*(-2*c*d + b*e + 2*c*e*x^2))/((b^2 - 4*a*c)*(-(c*d^2) + e*
(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (d*e*ArcTanh[(-(b*d) + 2*a*e - 2*c*
d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])
])/(2*(c*d^2 + e*(-(b*d) + a*e))^(3/2))
```

fricas [B] time = 2.14, size = 1349, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*c)*d*e*x^2 + (a*b^2 - 4*a^2
*c)*d*e)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
```

```
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d
^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
 + d^2)) + 4*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d
*e^2 + (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*
x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4
*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3
- 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2
*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 -
2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*
c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8
*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)
*e^4)*x^2), -1/2*(((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*c)*d*e*x^2 + (a
*b^2 - 4*a^2*c)*d*e)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 +
b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(
(c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2
- b^2*d*e + a*b*e^2)*x^2)) - 2*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (
a*b^2 + 2*a^2*c)*d*e^2 + (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c
+ 2*a*c^2)*d^2*e)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^
4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2
*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3
- 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a
^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)
*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b
^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^
2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
```

giac [B] time = 0.64, size = 441, normalized size = 2.77

$$-\frac{d\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)e}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}+\frac{\left(bc^2d^3-b^2cd^2e-2ac^2d^2e+3abcde^2-2a^2ce^3\right)x^2}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -d*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))*e/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^
2)) + ((b*c^2*d^3 - b^2*c*d^2*e - 2*a*c^2*d^2*e + 3*a*b*c*d*e^2 - 2*a^2*c*e
^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*
d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c
*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (2*a*c^2*d^3 - 3*a*b*c*d^2*e + a*b^2*
d*e^2 + 2*a^2*c*d*e^2 - a^2*b*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3
*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2
- 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sqrt(c*x^4
+ b*x^2 + a)
```

maple [B] time = 0.01, size = 506, normalized size = 3.18

$$-\frac{2cd\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2ae^2-2deb+2cd^2}{e^2}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(-be+2cd+\sqrt{-4ac+b^2}\,e\right)\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\sqrt{\frac{ae^2-deb+cd^2}{e^2}}}-\frac{2\sqrt{\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)^2c-\cdots}}{\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

[Out] 1/e*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-2*d/e*c/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)-2*d*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))+2*d/e*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*((x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*c+(-4*a*c+b^2)^(1/2)*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")

[Out] integrate(x^3/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)

[Out] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)

[Out] Integral(x**3/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)

3.345 $\int \frac{x}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=166

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

```
[Out] 1/2*e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+(-b*c*d+b^2*e-2*a*c*e-c*(-
b*e+2*c*d)*x^2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1247, 740, 12, 724, 206}

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b
*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*
d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*
(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\mathrm{Subst}\left(\int -\frac{\left(b^2-4ac\right)e^2}{2(d+ex)\sqrt{a+bx+cx^2}}\right.}{\left(b^2-4ac\right)\left(cd^2-bde+\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{e^2\,\mathrm{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\right.}{2\left(cd^2-bde+ae^2\right.}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{e^2\,\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\right.}{cd^2-bde}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-b}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+}}\right.}{2\left(cd^2-bde+ae^2\right)^{3/}}$$

Mathematica [A] time = 0.15, size = 167, normalized size = 1.01

$$-\frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(e(ae-bd)+cd^2\right)} - \frac{e^2\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{2\left(e(ae-bd)+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 + e
*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x^4])) - (e^2*ArcTanh[(-(b*d) + 2*a*e -
 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*
x^4])])/(2*(c*d^2 + e*(-(b*d) + a*e))^(3/2))
```

fricas [B] time = 2.16, size = 1379, normalized size = 8.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*e^2*x^4 + (b^3 - 4*a*b*c)*e^2*x^2 + (a*b^2 - 4*a^2
*c)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d
^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
 + d^2)) - 4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (
a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*
a*c^2)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 -
```

```
2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2
 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*
a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c
^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4
)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^
3 - 4*a^3*b*c)*e^4)*x^2), 1/2*(((b^2*c - 4*a*c^2)*e^2*x^4 + (b^3 - 4*a*b*c)
*e^2*x^2 + (a*b^2 - 4*a^2*c)*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b
*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^
2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*
d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*
d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/
((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2
*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2
- 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e +
 (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^
3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*
c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4
 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
```

giac [B] time = 0.62, size = 454, normalized size = 2.73

$$-\frac{\frac{\left(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2+2\,ac^2de^2-abce^3\right)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{bc^2d^3-2\,b^2\cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3\cdots}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)*x^2
/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2
 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3
+ a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e +
 b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*c^3*
d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8
*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*
e^4))/sqrt(c*x^4 + b*x^2 + a) + arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^2/((c*d^2 - b*d*e + a*
e^2)*sqrt(-c*d^2 + b*d*e - a*e^2))
```

maple [B] time = 0.01, size = 454, normalized size = 2.73

$$\frac{2ce\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\,\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(-be+2cd+\sqrt{-4ac+b^2}\,e\right)\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}}+\frac{2\sqrt{\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)^2c-\sqrt{\cdots}}}{\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 2*c/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1
/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)+2*c*e/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)
/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-
2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
```

```
/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x^2+d/e))-2*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2-1/2*(-b+
(-4*a*c+b^2)^(1/2))/c)*((x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*c+(-4*a*c+b^2
)^(1/2)*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(e\,x^2 + d\right)\,\left(c\,x^4 + b\,x^2 + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(d + ex^2\right)\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.346 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d}+\frac{e\left(2ace+b^2(-e)+cx^2(2cd-be)+bcd\right)}{d\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{-2ac+b^2+bcx^2}{ad\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{e^3 \text{t}}{}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)/d-1/2*e
^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2))/d/(a*e^2-b*d*e+c*d^2)^(3/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c
+b^2)/d/(c*x^4+b*x^2+a)^(1/2)+e*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x^2)/(-
4*a*c+b^2)/d/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 960, 740, 12, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d}+\frac{e\left(2ace+b^2(-e)+cx^2(2cd-be)+bcd\right)}{d\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{-2ac+b^2+bcx^2}{ad\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{e^3 \text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x^2 + c*x^4]) + (e*(b
*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d*(c*d^2 - b*
d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])]/(2*a^(3/2)*d) - (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*
(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
```

```
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}} - \frac{e}{d(d+ex)\left(a+bx+cx^2\right)^{3/2}}\right) dx, x, x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)}{2d} - \frac{e\operatorname{Subst}\left(\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)}{2d}$$
$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}} + \frac{e\left(bcd-b^2e+2ace+c(2cd-be)x^2\right)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+}}$$
$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}} + \frac{e\left(bcd-b^2e+2ace+c(2cd-be)x^2\right)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+}}$$
$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}} + \frac{e\left(bcd-b^2e+2ace+c(2cd-be)x^2\right)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+}}$$
$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}} + \frac{e\left(bcd-b^2e+2ace+c(2cd-be)x^2\right)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+}}$$

Mathematica [A] time = 0.73, size = 236, normalized size = 0.89

$$-\frac{\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{3/2}} + \frac{2d\left(bc\left(3ae+cdx^2\right)-2ac^2\left(d-ex^2\right)+b^3(-e)+b^2c\left(d-ex^2\right)\right)}{a\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}\left(e(ae-bd)+cd^2\right)} + \frac{e^3\tanh^{-1}\left(\frac{-2ae+b\left(d-ex^2\right)+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/2*((2*d*(-(b^3*e) + b*c*(3*a*e + c*d*x^2) + b^2*c*(d - e*x^2) - 2*a*c^2*
(d - e*x^2)))/(a*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 +
 c*x^4]) + ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/a^(3/
2) + (e^3*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*x^2))/(2*Sqrt[c*d^2 + e*(-
(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/
d
```

fricas [B] time = 9.53, size = 4909, normalized size = 18.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a
^3*b^2 - 4*a^4*c)*e^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c
*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 +
 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2
 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x
^4 + 2*d*e*x^2 + d^2)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*
b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a
^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3
*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*
b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 -
 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*
b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)
*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 +
a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (
2*a*b^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2
- (a^2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*
e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqr
t(c*x^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4
*b*c^2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 -
4*a^5*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)
*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8
*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^
5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3
*b^2*c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^
4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e^4)*x^2), -1/4*(2*((a^2
*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*
a^4*c)*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a
)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2
- b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e
 + a*b*e^2)*x^2)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2
)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*
c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2
- 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c
 - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*
b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2
)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)
*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (2*a*b
^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2 - (a^
2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*e + (
a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqrt(c*x
^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4*b*c^
2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 - 4*a^5
*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)*d^5
```

```
- 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8*a^4*
c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^5*c^2
)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3*b^2*
c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^4 - 4
*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e^4)*x^2), 1/4*(2*((a*b^2*c^2
 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c
- 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*
e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2
*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^
2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2
*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^
2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4
 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + ((a^2*b^2
*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*a^4*
c)*e^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4
*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 +
 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^
2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
+ d^2)) + 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e
+ (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 +
 (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e
^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2
*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^
4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 -
 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b
*c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*
c - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c
^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3*b^2*c^2)*d^4*e + (a^2*b^5 - 2*
a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4
*b^3 - 4*a^5*b*c)*d*e^4)*x^2), -1/2*(((a^2*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^
2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*a^4*c)*e^3)*sqrt(-c*d^2 + b*d*e -
 a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((
2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^
2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - ((a*b^2*c^2 -
 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c -
8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^
4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a
*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*
c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c
^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*
c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*((a*b^2*c
^2 - 2*a^2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*
c - 2*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*
b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a
^3*c^2)*d*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5
 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*
d^3*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a
^2*b^2*c^3 - 4*a^3*c^4)*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^
4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*
e^3 + (a^4*b^2*c - 4*a^5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5
 - 2*(a^2*b^4*c - 4*a^3*b^2*c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c
^2)*d^3*e^2 - 2*(a^3*b^4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e
^4)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 612, normalized size = 2.30

$$-\frac{2c\,e^2\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(-be+2cd+\sqrt{-4ac+b^2}\,e\right)\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\,d}-\frac{bc\,x^2}{\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)

[Out] 1/2/d/a/(c*x^4+b*x^2+a)^(1/2)-1/d*b/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*c*x ^2-1/2/d*b^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2/d/a^(3/2)*ln((b*x^2+2* a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-2*e/d*c/(b*e-2*c*d+(-4*a*c+b^2)^(1/ 2)*e)/(-4*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*((x^2+1/2*(b+(-4* a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)-2*e^2/d*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a *e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e -2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))+2*e/d*c/(-b* e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1 /2)/c)*((x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*c+(-4*a*c+b^2)^(1/2)*(x^2-1/2 *(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)

[Out] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)

[Out] Integral(1/(x*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)

3.347 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=419

$$\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d}-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2dx^2(b^2-4ac)}-\frac{e^2(2ace+b^2(-e)+cx^2(}{d^2(b^2-4ac)\sqrt{a+bx^2+c}}$$

```
[Out] 3/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5/2)/d+1/2*
e*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)/d^2+1/2*e^
4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))/d^2/(a*e^2-b*d*e+c*d^2)^(3/2)-e*(b*c*x^2-2*a*c+b^2)/a/(-4*
a*c+b^2)/d^2/(c*x^4+b*x^2+a)^(1/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/d/x^2
/(c*x^4+b*x^2+a)^(1/2)-e^2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x^2)/(-4*a*c
+b^2)/d^2/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*(-8*a*c+3*b^2)*(c*x
^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)/d/x^2
```

Rubi [A] time = 0.56, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 960, 740, 806, 724, 206, 12}

$$-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2dx^2(b^2-4ac)}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d}-\frac{e^2(2ace+b^2(-e)+cx}{d^2(b^2-4ac)\sqrt{a+bx^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((e*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*d^2*Sqrt[a + b*x^2 + c*x^4])
) + (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*x^2*Sqrt[a + b*x^2 + c*x^4])
 - (e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d^2
*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a
 + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*d*x^2) + (3*b*ArcTanh[(2*a + b*x^2)
/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*d) + (e*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)*d^2) + (e^4*ArcTanh[(
b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x^2 + c*x^4])])/(2*d^2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{3/2}} dx = \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^2(d+ex) (a+bx+cx^2)^{3/2}} dx, x, x^2\right)$$

$$= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{dx^2 (a+bx+cx^2)^{3/2}} - \frac{e}{d^2 x (a+bx+cx^2)^{3/2}} + \frac{}{d^2(d+ex) (}\right.\right.$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^{3/2}} dx, x, x^2\right)}{2d} - \frac{e\,\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^{3/2}} dx, x, x^2\right)}{2d^2} + -$$

$$= -\frac{e(b^2-2ac+bcx^2)}{a(b^2-4ac) d^2 \sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a(b^2-4ac) dx^2 \sqrt{a+bx^2+cx^4}} - \frac{}{(b^2}$$

$$= -\frac{e(b^2-2ac+bcx^2)}{a(b^2-4ac) d^2 \sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a(b^2-4ac) dx^2 \sqrt{a+bx^2+cx^4}} - \frac{}{(b^2}$$

$$= -\frac{e(b^2-2ac+bcx^2)}{a(b^2-4ac) d^2 \sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a(b^2-4ac) dx^2 \sqrt{a+bx^2+cx^4}} - \frac{}{(b^2}$$

$$= -\frac{e(b^2-2ac+bcx^2)}{a(b^2-4ac) d^2 \sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a(b^2-4ac) dx^2 \sqrt{a+bx^2+cx^4}} - \frac{}{(b^2}$$

Mathematica [A] time = 1.48, size = 350, normalized size = 0.84

$$\frac{\frac{(2ae+3bd)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{5/2}} + \frac{2d\left(-4a^3ce^2+a^2\left(b^2e^2+4bce\left(d-ex^2\right)-4c^2\left(d^2+dex^2+e^2x^4\right)\right)+a\left(b^3e\left(ex^2-d\right)+b^2c\left(d^2+12dex^2+e^2x^4\right)-10bc^2dx^2\left(d-\right.\right.\right.}{a^2x^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(e(bd-ae)-cd^2\right)}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*d*(-4*a^3*c*e^2 + 3*b^2*d*(c*d - b*e)*x^2*(b + c*x^2) + a^2*(b^2*e^2 +
4*b*c*e*(d - e*x^2) - 4*c^2*(d^2 + d*e*x^2 + e^2*x^4)) + a*(-8*c^3*d^2*x^4
- 10*b*c^2*d*x^2*(d - e*x^2) + b^3*e*(-d + e*x^2) + b^2*c*(d^2 + 12*d*e*x^2
 + e^2*x^4))))/(a^2*(b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*x^2*Sqrt[a + b
*x^2 + c*x^4]) + ((3*b*d + 2*a*e)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
 b*x^2 + c*x^4])])/a^(5/2) + (2*e^4*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*
x^2))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])/(c*d^2 +
 e*(-(b*d) + a*e))^(3/2))/(4*d^2)
```

fricas [B] time = 22.07, size = 6486, normalized size = 15.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*((a^3*b^2*c - 4*a^4*c^2)*e^4*x^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 +
(a^4*b^2 - 4*a^5*c)*e^4*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 -
 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)
```

```
*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/
(e^2*x^4 + 2*d*e*x^2 + d^2)) + ((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*c^2
 - 13*a*b^2*c^3 + 4*a^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c^3)
*d^3*e^2 - 4*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c - 4
*a^3*b*c^2)*d*e^4 + 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4*a*
b^2*c^3)*d^5 - 2*(3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 - 10
*a*b^4*c - 8*a^2*b^2*c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d
^2*e^3 - (a^2*b^4 - 4*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^4 +
 (3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c
^3)*d^4*e + (3*a*b^5 - 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 5
*a^3*b^2*c + 4*a^4*c^2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b^2
- 4*a^5*c)*e^5)*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c
*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((a^2*b^2*c^2 - 4
*a^3*c^3)*d^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 - 2*a^3*b^2*c
- 8*a^4*c^2)*d^3*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b^2 - 4*a^5*c
)*d*e^4 + ((3*a*b^2*c^3 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2*b*c^3)*d^4*
e + 3*(a*b^4*c - 2*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*b^3*c - 7*a^
3*b*c^2)*d^2*e^3 + (a^3*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b^3*c^2 - 10*
a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e + (3*a*b^
5 - 8*a^2*b^3*c - 10*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^2*c + a^4*c^
2)*d^2*e^3 + (a^3*b^3 - 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(((
a^3*b^2*c^3 - 4*a^4*c^4)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^5*e + (a^3*b
^4*c - 2*a^4*b^2*c^2 - 8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^3
*e^3 + (a^5*b^2*c - 4*a^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4*a^4*b*c^3)*
d^6 - 2*(a^3*b^4*c - 4*a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^3*c - 8*a^5*
b*c^2)*d^4*e^2 - 2*(a^4*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 - 4*a^6*b*c)*
d^2*e^4)*x^4 + ((a^4*b^2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c - 4*a^5*b*c^2)
*d^5*e + (a^4*b^4 - 2*a^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b^3 - 4*a^6*b
*c)*d^3*e^3 + (a^6*b^2 - 4*a^7*c)*d^2*e^4)*x^2), 1/8*(4*((a^3*b^2*c - 4*a^4
*c^2)*e^4*x^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 + (a^4*b^2 - 4*a^5*c)*e^4*x^2
)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*
d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e
+ a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2
)*x^2)) + ((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*c^2 - 13*a*b^2*c^3 + 4*a
^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c
 - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 + 2
*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4*a*b^2*c^3)*d^5 - 2*(3*b
^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 - 10*a*b^4*c - 8*a^2*b^2*
c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^3 - (a^2*b^4 - 4
*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^4 + (3*(a*b^3*c^2 - 4*a^
2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^4*e + (3*a*b^5
- 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^
2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b^2 - 4*a^5*c)*e^5)*x^2)*
sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*
x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((a^2*b^2*c^2 - 4*a^3*c^3)*d^5 - 2*(a^
2*b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^3*e^2
- 2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b^2 - 4*a^5*c)*d*e^4 + ((3*a*b^2*c
^3 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2*b*c^3)*d^4*e + 3*(a*b^4*c - 2*a^
2*b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*b^3*c - 7*a^3*b*c^2)*d^2*e^3 + (a
^3*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b^3*c^2 - 10*a^2*b*c^3)*d^5 - 2*(3
*a*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e + (3*a*b^5 - 8*a^2*b^3*c - 10*
a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^2*c + a^4*c^2)*d^2*e^3 + (a^3*b^3
 - 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(((a^3*b^2*c^3 - 4*a^4*c
^4)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^5*e + (a^3*b^4*c - 2*a^4*b^2*c^2
- 8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^3*e^3 + (a^5*b^2*c - 4
*a^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4*a^4*b*c^3)*d^6 - 2*(a^3*b^4*c -
4*a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^3*c - 8*a^5*b*c^2)*d^4*e^2 - 2*(a
^4*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 - 4*a^6*b*c)*d^2*e^4)*x^4 + ((a^4*
```

```
b^2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^5*e + (a^4*b^4 - 2
*a^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b^3 - 4*a^6*b*c)*d^3*e^3 + (a^6*b^
2 - 4*a^7*c)*d^2*e^4)*x^2), -1/4*(((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*
c^2 - 13*a*b^2*c^3 + 4*a^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c
^3)*d^3*e^2 - 4*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c
 - 4*a^3*b*c^2)*d*e^4 + 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4
*a*b^2*c^3)*d^5 - 2*(3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 -
 10*a*b^4*c - 8*a^2*b^2*c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2
)*d^2*e^3 - (a^2*b^4 - 4*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^
4 + (3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^
3*c^3)*d^4*e + (3*a*b^5 - 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4
 - 5*a^3*b^2*c + 4*a^4*c^2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b
^2 - 4*a^5*c)*e^5)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - ((a^3*b^2*c - 4*a^4*c^2)*e^4*x
^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 + (a^4*b^2 - 4*a^5*c)*e^4*x^2)*sqrt(c*d^
2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 -
8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b
^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2
)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 2*((a^2
*b^2*c^2 - 4*a^3*c^3)*d^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 -
2*a^3*b^2*c - 8*a^4*c^2)*d^3*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b
^2 - 4*a^5*c)*d*e^4 + ((3*a*b^2*c^3 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2
*b*c^3)*d^4*e + 3*(a*b^4*c - 2*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*
b^3*c - 7*a^3*b*c^2)*d^2*e^3 + (a^3*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b
^3*c^2 - 10*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4
*e + (3*a*b^5 - 8*a^2*b^3*c - 10*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^
2*c + a^4*c^2)*d^2*e^3 + (a^3*b^3 - 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x
^2 + a))/(((a^3*b^2*c^3 - 4*a^4*c^4)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^
5*e + (a^3*b^4*c - 2*a^4*b^2*c^2 - 8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^
5*b*c^2)*d^3*e^3 + (a^5*b^2*c - 4*a^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4
*a^4*b*c^3)*d^6 - 2*(a^3*b^4*c - 4*a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^
3*c - 8*a^5*b*c^2)*d^4*e^2 - 2*(a^4*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 -
 4*a^6*b*c)*d^2*e^4)*x^4 + ((a^4*b^2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c -
4*a^5*b*c^2)*d^5*e + (a^4*b^4 - 2*a^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b
^3 - 4*a^6*b*c)*d^3*e^3 + (a^6*b^2 - 4*a^7*c)*d^2*e^4)*x^2), 1/4*(2*((a^3*b
^2*c - 4*a^4*c^2)*e^4*x^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 + (a^4*b^2 - 4*a^
5*c)*e^4*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 +
 a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^
2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d
*e + a*b*e^2)*x^2)) - ((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*c^2 - 13*a*b
^2*c^3 + 4*a^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c^3)*d^3*e^2
 - 4*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c - 4*a^3*b*c^
2)*d*e^4 + 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4*a*b^2*c^3)*
d^5 - 2*(3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 - 10*a*b^4*c
 - 8*a^2*b^2*c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^3 -
(a^2*b^4 - 4*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^4 + (3*(a*b^
3*c^2 - 4*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^4*e
 + (3*a*b^5 - 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 5*a^3*b^2*
c + 4*a^4*c^2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b^2 - 4*a^5*c
)*e^5)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(
-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*((a^2*b^2*c^2 - 4*a^3*c^3)*d^5 - 2*(a^2*
b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^3*e^2 -
2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b^2 - 4*a^5*c)*d*e^4 + ((3*a*b^2*c^3
 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2*b*c^3)*d^4*e + 3*(a*b^4*c - 2*a^2*
b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*b^3*c - 7*a^3*b*c^2)*d^2*e^3 + (a^3
*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b^3*c^2 - 10*a^2*b*c^3)*d^5 - 2*(3*a
*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e + (3*a*b^5 - 8*a^2*b^3*c - 10*a^
3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^2*c + a^4*c^2)*d^2*e^3 + (a^3*b^3 -
 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(((a^3*b^2*c^3 - 4*a^4*c^4
```

```
)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^5*e + (a^3*b^4*c - 2*a^4*b^2*c^2 -
8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^3*e^3 + (a^5*b^2*c - 4*a
^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4*a^4*b*c^3)*d^6 - 2*(a^3*b^4*c - 4*
a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^3*c - 8*a^5*b*c^2)*d^4*e^2 - 2*(a^4
*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 - 4*a^6*b*c)*d^2*e^4)*x^4 + ((a^4*b^
2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^5*e + (a^4*b^4 - 2*a
^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b^3 - 4*a^6*b*c)*d^3*e^3 + (a^6*b^2
- 4*a^7*c)*d^2*e^4)*x^2)]
```

giac [B] time = 2.74, size = 762, normalized size = 1.82

$$-\frac{\frac{\left(a^{2}b^{2}c^{3}d^{3}-2\,a^{3}c^{4}d^{3}-2\,a^{2}b^{3}c^{2}d^{2}e+5\,a^{3}bc^{3}d^{2}e+a^{2}b^{4}cde^{2}-2\,a^{3}b^{2}c^{2}de^{2}-2\,a^{4}c^{3}de^{2}-a^{3}b^{3}ce^{3}+3\,a^{4}bc^{2}e^{3}\right)x^{2}}{a^{4}b^{2}c^{2}d^{4}-4\,a^{5}c^{3}d^{4}-2\,a^{4}b^{3}cd^{3}e+8\,a^{5}bc^{2}d^{3}e+a^{4}b^{4}d^{2}e^{2}-2\,a^{5}b^{2}cd^{2}e^{2}-8\,a^{6}c^{2}d^{2}e^{2}-2\,a^{5}b^{3}de^{3}+8\,a^{6}bcde^{3}+a^{6}b^{2}e^{4}-4\,a^{7}ce^{4}}+\frac{a^{2}b^{3}c^{2}d^{3}-3\,a^{3}bc^{3}d^{3}-2\,\cdots}{a^{4}b^{2}c^{2}d^{4}-4\,a^{5}c^{3}d^{4}-2\,a^{4}\cdots}}{\sqrt{cx^{4}+bx^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^2*b^2*c^3*d^3 - 2*a^3*c^4*d^3 - 2*a^2*b^3*c^2*d^2*e + 5*a^3*b*c^3*d^2*
e + a^2*b^4*c*d*e^2 - 2*a^3*b^2*c^2*d*e^2 - 2*a^4*c^3*d*e^2 - a^3*b^3*c*e^3
 + 3*a^4*b*c^2*e^3)*x^2/(a^4*b^2*c^2*d^4 - 4*a^5*c^3*d^4 - 2*a^4*b^3*c*d^3*
e + 8*a^5*b*c^2*d^3*e + a^4*b^4*d^2*e^2 - 2*a^5*b^2*c*d^2*e^2 - 8*a^6*c^2*d
^2*e^2 - 2*a^5*b^3*d*e^3 + 8*a^6*b*c*d*e^3 + a^6*b^2*e^4 - 4*a^7*c*e^4) + (
a^2*b^3*c^2*d^3 - 3*a^3*b*c^3*d^3 - 2*a^2*b^4*c*d^2*e + 7*a^3*b^2*c^2*d^2*e
 - 2*a^4*c^3*d^2*e + a^2*b^5*d*e^2 - 3*a^3*b^3*c*d*e^2 - a^4*b*c^2*d*e^2 -
a^3*b^4*e^3 + 4*a^4*b^2*c*e^3 - 2*a^5*c^2*e^3)/(a^4*b^2*c^2*d^4 - 4*a^5*c^3
*d^4 - 2*a^4*b^3*c*d^3*e + 8*a^5*b*c^2*d^3*e + a^4*b^4*d^2*e^2 - 2*a^5*b^2*
c*d^2*e^2 - 8*a^6*c^2*d^2*e^2 - 2*a^5*b^3*d*e^3 + 8*a^6*b*c*d*e^3 + a^6*b^2
*e^4 - 4*a^7*c*e^4))/sqrt(c*x^4 + b*x^2 + a) + arctan(-((sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^4/((c*d
^4 - b*d^3*e + a*d^2*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/2*(3*b*d + 2*a*
e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2*
d^2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a^2*d)
```

maple [B] time = 0.03, size = 863, normalized size = 2.06

$$\frac{2c\,e^{3}\ln\left(\frac{\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{2a\,e^{2}-2deb+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}-deb+c\,d^{2}}{e^{2}}}\,\sqrt{\left(x^{2}+\frac{d}{e}\right)^{2}c+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}-deb+c\,d^{2}}{e^{2}}}}{x^{2}+\frac{d}{e}}\right)}{\left(-be+2cd+\sqrt{-4ac+b^{2}}\,e\right)\left(be-2cd+\sqrt{-4ac+b^{2}}\,e\right)\sqrt{\frac{a\,e^{2}-deb+c\,d^{2}}{e^{2}}}\,d^{2}}+\frac{bce\,x^{2}}{\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/d^2*e/a/(c*x^4+b*x^2+a)^(1/2)+1/d^2*e*b/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^
(1/2)*c*x^2+1/2/d^2*e*b^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2/d^2*e/a^(
3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+2*e^2/d^2*c/(b*e-2
*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)
/c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2)+2*e^3/d^2*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)
/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-
2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x^2+d/e))-2*e^2/d^2*c/(-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+
```

```
1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*((x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*c+
(-4*a*c+b^2)^(1/2)*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)-1/2/d/a/x^2/(
c*x^4+b*x^2+a)^(1/2)-3/4/d*b/a^2/(c*x^4+b*x^2+a)^(1/2)+3/2/d*b^2/a^2/(4*a*c
-b^2)/(c*x^4+b*x^2+a)^(1/2)*c*x^2+3/4/d*b^3/a^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)
^(1/2)+3/4/d*b/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-
4/d*c^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2-2/d*c/a/(4*a*c-b^2)/(c*x^4+
b*x^2+a)^(1/2)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.348 $\int \frac{x^8}{\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=449

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}+\frac{1}{20}\sqrt{2x^4+2x^2+1}\,x+\frac{27}{80}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\left(7\sqrt{2}-2\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right.}}}{8\,2^{3/4}\left(3\sqrt{2}-2\right)}$$

```
[Out] 27/400*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/20*x^3*(-2*x
^2+1)/(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^
2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/
cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))
^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(
2*x^4+2*x^2+1)^(1/2)+27/160*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan
(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2
^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))
^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+1/16*(cos(2*arctan(2^
(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*
x)),1/2*(2-2^(1/2))^(1/2))*(-2+7*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/
(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 566, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1313, 1275, 1279, 1197, 1103, 1195, 1325, 1706}

$$\frac{\left(1-2x^2\right)x^3}{20\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}+\frac{1}{20}\sqrt{2x^4+2x^2+1}\,x-\frac{27\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)}{40\left(2-3\sqrt{2}\right)}-\frac{\left(7+\sqrt{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x^3*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4]
)/20 + (x*Sqrt[1 + 2*x^2 + 2*x^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (27*Sqr
t[3/10]*(3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(40*(2
 - 3*Sqrt[2])) - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x
^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1
+ 2*x^2 + 2*x^4]) + (9*(1 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(20*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - ((7 + Sqrt[2])*(1
+ Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*Ar
cTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(40*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (
27*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^
2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(80*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> -Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[(d^2*f
^4)/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^(m - 4)*(a + b*x^2 + c*x^4)^(p + 1)
)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1325

```
Int[(x_)^4/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, -Dist[(2*c*d - a*e*q)/(c*e*(e - d*q))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + (-Dist[1/(e*q), Int[(1 - q*x^2)/S
qrt[a + b*x^2 + c*x^4], x], x] + Dist[d^2/(e*(e - d*q)), Int[(1 + q*x^2)/((
d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x])] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
```

```
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\left(\frac{1}{10}\int \frac{x^4(3+4x^2)}{(1+2x^2+2x^4)^{3/2}}\,dx\right) + \frac{9}{10}\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{40}\int \frac{x^2(-6+12x^2)}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{9\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{20\sqrt{2}} + \cdots$$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{20}x\sqrt{1+2x^2+2x^4} + \frac{9x\sqrt{1+2x^2+2x^4}}{20\sqrt{2}(1+\sqrt{2}x^2)} - \cdots$$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{20}x\sqrt{1+2x^2+2x^4} + \frac{9x\sqrt{1+2x^2+2x^4}}{20\sqrt{2}(1+\sqrt{2}x^2)} - \cdots$$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{20}x\sqrt{1+2x^2+2x^4} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}(1+\sqrt{2}x^2)} - \cdots$$

Mathematica [C] time = 0.28, size = 199, normalized size = 0.44

$$\frac{12x^3-(29-33i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-4i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(\cdots}{80\sqrt{2x^4\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (4*x + 12*x^3 - (4*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^
2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (29 - 33*I)*Sqrt[1 - I]*Sqrt[1
+ (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
 + 27*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[
1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(80*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2x^4+2x^2+1}\,x^8}{8x^{10}+28x^8+40x^6+32x^4+14x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^8/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

maple [C] time = 0.05, size = 603, normalized size = 1.34

$$\frac{27x^3}{16\sqrt{2x^4+2x^2+1}}+\frac{x}{8\sqrt{2x^4+2x^2+1}}-\frac{243i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\operatorname{EllipticE}\left(\sqrt{-1+i}\,x,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{160\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{24}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
[Out] 1/8*x/(2*x^4+2*x^2+1)^(1/2)-11/4/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^
2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I
*2^(1/2))-243/160*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(
2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+3*
(1/8*x^3+1/8*x)/(2*x^4+2*x^2+1)^(1/2)+243/160*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)
^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1
/2*2^(1/2)+1/2*I*2^(1/2))-9/2*(-1/4*x^3-1/8*x)/(2*x^4+2*x^2+1)^(1/2)+(47/32
-47/32*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2
+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE((-
1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))+27/16*x^3/(2*x^4+2*x^2+1)^(1/2)-81
/4*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)+81/160/(-1+I)^(1/2)*(-I*x^2+x^2+
1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x
,1/2*2^(1/2)+1/2*I*2^(1/2))+243/160/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^
2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1
/2*I*2^(1/2))+27/40/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(
2*x^4+2*x^2+1)^(1/2)*EllipticPi((-1+I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8}{\left(2\,x^2+3\right)\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
[Out] int(x^8/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(2x^2+3)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**8/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

3.349 $\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=423

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{\left(1-2x^2\right)x}{20\sqrt{2x^4+2x^2+1}}-\frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(\sqrt[4]{2}+2^{3/4}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F}{8\left(3\sqrt{2}-2\right)\sqrt{2x^4}}$$

```
[Out] -9/200*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/20*x*(-2*x^2
+1)/(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/
2))-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ellipt
icE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4
+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-9/80*(cos(
2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arc
tan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2
*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(
2*x^4+2*x^2+1)^(1/2)-1/8*(2^(1/4)+2^(3/4))*(cos(2*arctan(2^(1/4)*x))^2)^(1/
2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/
2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(-2+3*
2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 503, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1313, 1275, 1197, 1103, 1195, 1319, 1706}

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{\left(1-2x^2\right)x}{20\sqrt{2x^4+2x^2+1}}-\frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{9\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F}{140\,2^{3/4}\sqrt{2x^4+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4])/
(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (9*Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/40 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*S
qrt[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(40*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (9*(3 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(140*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (9*(3 + Sqrt
[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(560*2^
(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
```

```
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> -Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[(d^2*f
^4)/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^(m - 4)*(a + b*x^2 + c*x^4)^(p + 1)
)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1319

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, -Dist[(a*(e + d*q))/(c*d^2 - a*e^2),
Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(a*d*(e + d*q))/(c*d^2 - a*e^2
), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
 && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = -\left(\frac{1}{10}\int \frac{x^2\left(3+4x^2\right)}{\left(1+2x^2+2x^4\right)^{3/2}}\,dx\right)+\frac{9}{10}\int \frac{x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{x\left(1-2x^2\right)}{20\sqrt{1+2x^2+2x^4}}+\frac{1}{40}\int \frac{-2+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{1}{140}\left(9\left(2+3\sqrt{2}\right)\right)\int \frac{}{\sqrt{1}}$$

$$= \frac{x\left(1-2x^2\right)}{20\sqrt{1+2x^2+2x^4}}-\frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)-\frac{9\left(3+\sqrt{2}\right)\left(1+\sqrt{}\right.}{}$$

$$= \frac{x\left(1-2x^2\right)}{20\sqrt{1+2x^2+2x^4}}+\frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)}-\frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right.$$

Mathematica [C] time = 0.21, size = 199, normalized size = 0.47

$$\frac{-4x^3+(8-6i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|\,i\right)-2i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i}{40\sqrt{2x^4+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (2*x - 4*x^3 - (2*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2
]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (8 - 6*I)*Sqrt[1 - I]*Sqrt[1 + (
1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] -
9*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3
+ I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(40*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}\,x^6}{8\,x^{10}+28\,x^8+40\,x^6+32\,x^4+14\,x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^6/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

maple [C] time = 0.01, size = 586, normalized size = 1.39

$$-\frac{9x^3}{8\sqrt{2x^4+2x^2+1}}-\frac{81\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\ \text{EllipticE}\left(\sqrt{-1+i}\,x, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{80\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{81i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2}}{80\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] -2*(1/8*x^3+1/8*x)/(2*x^4+2*x^2+1)^(1/2)+7/4/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/
2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2
^(1/2)+1/2*I*2^(1/2))+(-17/16+17/16*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1
+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2
)+1/2*I*2^(1/2))-EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))+3*(-1
/4*x^3-1/8*x)/(2*x^4+2*x^2+1)^(1/2)-9/8/(2*x^4+2*x^2+1)^(1/2)*x^3+27/2*(3/2
0*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)-27/80/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)
*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(
1/2)+1/2*I*2^(1/2))-81/80*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)
^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(
1/2))-81/80/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*
x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+81/80*I/(-
1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*E
llipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-9/20/(-1+I)^(1/2)*(-I*x^
2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi((-1+I)^
(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(2\,x^2+3\right)\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

```
[Out] int(x^6/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(2x^2+3\right)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**6/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

3.350 $\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=422

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}-\frac{\left(x^2+2\right)x}{10\sqrt{2x^4+2x^2+1}}+\frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\left(2+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\right.}{4\,2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^{\cdot}}}$$

```
[Out] 3/100*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/10*x*(x^2+2)/
(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-
1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(
sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x
^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+3/40*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^
4+2*x^2+1)^(1/2)+1/8*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4
)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(2+2^(1/2))
*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^
(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 501, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1313, 1178, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}-\frac{\left(x^2+2\right)x}{10\sqrt{2x^4+2x^2+1}}+\frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{9\left(3+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F}{140\sqrt[4]{2}\,\sqrt{2x^4+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -(x*(2 + x^2))/(10*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4])/(
10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
2*x^2 + 2*x^4]])/20 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sq
rt[1 + 2*x^2 + 2*x^4]) + ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(20*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (9*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*
x], (2 - Sqrt[2])/4])/(140*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (3*(3 + Sqrt[
2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellip
ticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(280*2^(
1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
```

```
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> -Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[(d^2*f
^4)/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^(m - 4)*(a + b*x^2 + c*x^4)^(p + 1)
)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\left(\frac{1}{10}\int \frac{3+4x^2}{(1+2x^2+2x^4)^{3/2}}\,dx\right) + \frac{9}{10}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} - \frac{1}{40}\int \frac{4-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{70}\left(9\left(3+\sqrt{2}\right)\right)\int \frac{}{\sqrt{1}}$$

$$= -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} + \frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) + \frac{9\left(3+\sqrt{2}\right)\left(1+\right.}{}$$

$$= -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right.$$

Mathematica [C] time = 0.21, size = 199, normalized size = 0.47

$$-\frac{2x^3+(1-2i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)}}{20\sqrt{2x^4+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -1/20*(4*x + 2*x^3 + I*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (1 - 2*I)*Sqrt[1 - I]*Sqrt[1 +
 (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
- 3*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/
3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/Sqrt[1 + 2*x^2 + 2*x^4]
```

fricas [F] time = 1.38, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}\,x^4}{8\,x^{10}+28\,x^8+40\,x^6+32\,x^4+14\,x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

maple [C] time = 0.01, size = 561, normalized size = 1.33

$$\frac{3x^3}{4\sqrt{2x^4+2x^2+1}} + \frac{27\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\ \mathrm{EllipticE}\left(\sqrt{-1+i}\,x, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{40\sqrt{-1+i}\sqrt{2x^4+2x^2+1}} - \frac{27i\sqrt{-ix^2+x^2+1}\sqrt{ix^2}}{40\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)

[Out] -2*(-1/4*x^3-1/8*x)/(2*x^4+2*x^2+1)^(1/2)-1/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+(5/8-5/8*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))+3/4/(2*x^4+2*x^2+1)^(1/2)*x^3-9*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)+9/40/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+27/40*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+27/40/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-27/40*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+3/10/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi((-1+I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")

[Out] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(2\,x^2+3\right)\,\left(2\,x^4+2\,x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)

[Out] int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2x^2+3\right)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)

[Out] Integral(x**4/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)

3.351 $\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=423

$$-\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{5\left(\sqrt{2}\,x^2+1\right)}+\frac{\left(4x^2+3\right)x}{10\sqrt{2x^4+2x^2+1}}-\frac{1}{10}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(\sqrt[4]{2}+2^{3/4}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2}{\left(\sqrt{2}\,x^2+\right.}}}{4\left(3\sqrt{2}-2\right)\sqrt{}}$$

```
[Out] -1/50*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/10*x*(4*x^2+3
)/(2*x^4+2*x^2+1)^(1/2)-1/5*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))
+1/5*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(
sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x
^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-1/20*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^
4+2*x^2+1)^(1/2)-1/4*(2^(1/4)+2^(3/4))*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/c
os(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^
(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(-2+3*2^(1
/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 503, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1315, 1178, 1197, 1103, 1195, 1216, 1706}

$$-\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{5\left(\sqrt{2}\,x^2+1\right)}+\frac{\left(4x^2+3\right)x}{10\sqrt{2x^4+2x^2+1}}-\frac{1}{10}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(1+2\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+}{\left(\sqrt{2}\,x^2+1\right)}}}{20\sqrt[4]{2}\sqrt{2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x*(3 + 4*x^2))/(10*Sqrt[1 + 2*x^2 + 2*x^4]) - (Sqrt[2]*x*Sqrt[1 + 2*x^2 +
2*x^4])/(5*(1 + Sqrt[2]*x^2)) - (Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*
x^2 + 2*x^4]])/10 + (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*Sqrt[
1 + 2*x^2 + 2*x^4]) - (3*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(70*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)
*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x
], (2 - Sqrt[2])/4])/(20*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])^
2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticP
i[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(140*2^(1/4)
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
```

```
c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1315

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[(d*e*f^2)/(c*d^
2 - b*d*e + a*e^2), Int[((f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1))/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1] && GtQ[m, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \frac{1}{10}\int \frac{2+6x^2}{\left(1+2x^2+2x^4\right)^{3/2}}\,dx - \frac{3}{5}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} + \frac{1}{40}\int \frac{-4-16x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{35}\left(3\left(3+\sqrt{2}\right)\right)\int \frac{}{\sqrt{1+}}$$

$$= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} - \frac{1}{10}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{3\left(3+\sqrt{2}\right)\left(1+\sqrt{}\right.}{}$$

$$= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} - \frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{5\left(1+\sqrt{2}\,x^2\right)} - \frac{1}{10}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^{}}}\right.$$

Mathematica [C] time = 0.18, size = 199, normalized size = 0.47

$$\frac{8x^3-(1+3i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+4i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x}}{20\sqrt{2x^4+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (6*x + 8*x^3 + (4*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2
]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + 3*I)*Sqrt[1 - I]*Sqrt[1 + (
1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] -
2*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3
+ I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(20*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}\,x^2}{8\,x^{10}+28\,x^8+40\,x^6+32\,x^4+14\,x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

maple [C] time = 0.01, size = 536, normalized size = 1.27

$$-\frac{x^3}{2\sqrt{2x^4+2x^2+1}} - \frac{9\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\;\mathrm{EllipticE}\left(\sqrt{-1+i}\,x, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}} + \frac{9i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+}}{20\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] -1/2/(2*x^4+2*x^2+1)^(1/2)*x^3+1/2/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*
x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2
*I*2^(1/2))+(-1/4+1/4*I)/(-1+I)^(1/2)*((1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2
))-EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2)))+6*(3/20*x^3+1/20*x)
/(2*x^4+2*x^2+1)^(1/2)-3/20/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)
^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(
1/2))-9/20*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-9/20/(-1+
I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ell
ipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+9/20*I/(-1+I)^(1/2)*(-I*x^
2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(
1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-1/5/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*
x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi((-1+I)^(1/2)*x,1/3+1/3*I,
(-1-I)^(1/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(2\,x^2+3\right)\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

```
[Out] int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(2x^{2} + 3\right) \left(2x^{4} + 2x^{2} + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

3.352 $\int \frac{1}{\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=422

$$\frac{3\sqrt{2x^4+2x^2+1}\,x}{5\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(3x^2+1\right)x}{5\sqrt{2x^4+2x^2+1}}+\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{5\sqrt{15}}+\frac{\left(2+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\right.}{2\,2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] 1/75*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/5*x*(3*x^2+1)/
(2*x^4+2*x^2+1)^(1/2)+3/10*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-
3/10*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(
sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x
^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/30*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^
4+2*x^2+1)^(1/2)+1/4*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4
)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(2+2^(1/2))
*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^
(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 501, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1221, 1178, 1197, 1103, 1195, 1216, 1706}

$$\frac{3\sqrt{2x^4+2x^2+1}\,x}{5\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(3x^2+1\right)x}{5\sqrt{2x^4+2x^2+1}}+\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{5\sqrt{15}}+\frac{\left(3+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right.\right.}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -(x*(1 + 3*x^2))/(5*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*x*Sqrt[1 + 2*x^2 + 2*x^4]
)/(5*Sqrt[2]*(1 + Sqrt[2]*x^2)) + ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x
^4]]/(5*Sqrt[15]) - (3*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*2^(3/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(35*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)
*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x
], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 + Sqrt[2])^
2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticP
i[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(210*2^(1/4)
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
```

```
c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1221

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \frac{1}{10}\int \frac{2-4x^2}{(1+2x^2+2x^4)^{3/2}}\,dx + \frac{2}{5}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}} + \frac{1}{40}\int \frac{16+24x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{35}\left(2\left(3+\sqrt{2}\right)\right)\int \frac{}{\sqrt{1+}}$$

$$= -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}} + \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{5\sqrt{15}} + \frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+}{(1+\sqrt{2}x}}}{35\sqrt[4]{2}\sqrt{1}}$$

$$= -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}} + \frac{3x\sqrt{1+2x^2+2x^4}}{5\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{5\sqrt{15}} - \frac{3\left(1+\sqrt{2}\right.}{}$$

Mathematica [C] time = 0.14, size = 199, normalized size = 0.47

$$\frac{-18x^3+(6+3i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)-9i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+}{30\sqrt{2x^4+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (-6*x - 18*x^3 - (9*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (6 + 3*I)*Sqrt[1 - I]*Sqrt[1 +
 (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
+ 2*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/
3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(30*Sqrt[1 + 2*x^2 + 2*x^4])
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{8\,x^{10}+28\,x^8+40\,x^6+32\,x^4+14\,x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 14*x^
2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

maple [C] time = 0.01, size = 366, normalized size = 0.87

$$\frac{3\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\;\mathrm{EllipticE}\left(\sqrt{-1+i}\,x, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}} - \frac{3i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\;\mathrm{EllipticE}\left(\sqrt{\right.}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] -4*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)+1/10/(-1+I)^(1/2)*(-I*x^2+x^2+1)
^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1
/2*2^(1/2)+1/2*I*2^(1/2))+3/10*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x
^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*
I*2^(1/2))+3/10/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^
4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-3/10*I
/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+2/15/(-1+I)^(1/2)*(-I
*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi((-1+
I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2+3\right)\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

```
[Out] int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^2+3\right)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(1/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

3.353 $\int \frac{1}{x^2(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=468

$$\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{15\left(\sqrt{2}x^2+1\right)}+\frac{2\left(3x^2+1\right)x}{15\sqrt{2x^4+2x^2+1}}-\frac{x}{3\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{15\sqrt{15}}+\frac{\left(3\sqrt{2}-\right.}{}$$

```
[Out] -2/225*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/3*x/(2*x^4+2
*x^2+1)^(1/2)+2/15*x*(3*x^2+1)/(2*x^4+2*x^2+1)^(1/2)-1/3*(2*x^4+2*x^2+1)^(1
/2)/x+2/15*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-2/15*(cos(2*arct
an(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(
1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(
1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-1/45*(cos(2*arctan(2^(1/4)*x))
^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-
11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*
x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
+1/6*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(
sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(-7+3*2^(1/2))*(1+x^2*2^(1/
2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^(1/2))/(2*x^4
+2*x^2+1)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 644, normalized size of antiderivative = 1.38, number of steps used = 15, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {1335, 1121, 1281, 1197, 1103, 1195, 1221, 1178, 1216, 1706}

$$\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{15\left(\sqrt{2}x^2+1\right)}+\frac{2\left(3x^2+1\right)x}{15\sqrt{2x^4+2x^2+1}}-\frac{x}{3\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{15\sqrt{15}}-\frac{\left(3+2\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -x/(3*Sqrt[1 + 2*x^2 + 2*x^4]) + (2*x*(1 + 3*x^2))/(15*Sqrt[1 + 2*x^2 + 2*x
^4]) - Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (2*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4]
)/(15*(1 + Sqrt[2]*x^2)) - (2*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]
)/(15*Sqrt[15]) - (2*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*Sqrt
[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2
*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])
/(6*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(3/4)*(3 + Sqrt[2])*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/
4)*x], (2 - Sqrt[2])/4])/(105*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 + 2*Sqrt[2])*(
1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*
ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) +
 ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x
^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(315*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1221

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \int\left(\frac{1}{3x^2\left(1+2x^2+2x^4\right)^{3/2}}-\frac{2}{3\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\right)dx$$

$$= \frac{1}{3}\int\frac{1}{x^2\left(1+2x^2+2x^4\right)^{3/2}}\,dx-\frac{2}{3}\int\frac{1}{\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx$$

$$= -\frac{x}{3\sqrt{1+2x^2+2x^4}}-\frac{1}{15}\int\frac{2-4x^2}{\left(1+2x^2+2x^4\right)^{3/2}}\,dx+\frac{1}{12}\int\frac{4-4x^2}{x^2\sqrt{1+2x^2+}}$$

$$= -\frac{x}{3\sqrt{1+2x^2+2x^4}}+\frac{2x\left(1+3x^2\right)}{15\sqrt{1+2x^2+2x^4}}-\frac{\sqrt{1+2x^2+2x^4}}{3x}-\frac{1}{60}\int\frac{1}{\sqrt{1}}$$

$$= -\frac{x}{3\sqrt{1+2x^2+2x^4}}+\frac{2x\left(1+3x^2\right)}{15\sqrt{1+2x^2+2x^4}}-\frac{\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\tan^{-1}\left(\right.}{15}$$

$$= -\frac{x}{3\sqrt{1+2x^2+2x^4}}+\frac{2x\left(1+3x^2\right)}{15\sqrt{1+2x^2+2x^4}}-\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{2\sqrt{2}\,x\sqrt{1}}{15\left(1\right.}$$

Mathematica [C] time = 0.23, size = 211, normalized size = 0.45

$$\frac{-(27-39i)\sqrt{1-i}\,x\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|\,i\right)-12i\sqrt{1-i}\,x\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i}}{90x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] ((-12*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Elliptic
E[I*ArcSinh[Sqrt[1 - I]*x], I] - (27 - 39*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)
```

```
*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 2*(15
+ 39*x^2 + 12*x^4 + 2*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I
)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(90*x*Sqrt[1 +
2*x^2 + 2*x^4])
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2\,x^{4}+2\,x^{2}+1}}{8\,x^{12}+28\,x^{10}+40\,x^{8}+32\,x^{6}+14\,x^{4}+3\,x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(8*x^12 + 28*x^10 + 40*x^8 + 32*x^6 + 14*x
^4 + 3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^{4}+2\,x^{2}+1\right)^{\frac{3}{2}}\left(2\,x^{2}+3\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2), x)
```

maple [C] time = 0.02, size = 553, normalized size = 1.18

$$-\frac{x}{3\sqrt{2x^{4}+2x^{2}+1}}-\frac{\sqrt{-ix^{2}+x^{2}+1}\;\sqrt{ix^{2}+x^{2}+1}\;\mathrm{EllipticE}\left(\sqrt{-1+i}\;x, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{5\sqrt{-1+i}\;\sqrt{2x^{4}+2x^{2}+1}}+\frac{i\sqrt{-ix^{2}+x^{2}+1}\;\sqrt{ix^{2}+}}{5\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x-1/3/(2*x^4+2*x^2+1)^(1/2)*x-1/3/(-1+I)^(1/2)*(
(1-I)*x^2+1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+
I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+(-1/3+1/3*I)/(-1+I)^(1/2)*((1-I)*x^2+
1)^(1/2)*((1+I)*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF((-1+I)^(1/2)*
x,1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(
1/2)))+8/3*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)-1/15/(-1+I)^(1/2)*(-I*x^
2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(
1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-1/5*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(
I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF((-1+I)^(1/2)*x,1/2*2^(1/
2)+1/2*I*2^(1/2))-1/5/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)
/(2*x^4+2*x^2+1)^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))+
1/5*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)
^(1/2)*EllipticE((-1+I)^(1/2)*x,1/2*2^(1/2)+1/2*I*2^(1/2))-4/45/(-1+I)^(1/2
)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi
((-1+I)^(1/2)*x,1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^{4}+2\,x^{2}+1\right)^{\frac{3}{2}}\left(2\,x^{2}+3\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \left(2\,x^2+3\right)\,\left(2\,x^4+2\,x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
[Out] int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(2x^2+3\right) \left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
[Out] Integral(1/(x**2*(2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

3.354 $\int \frac{x^7 \sqrt{d+ex^2}}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=406

$$-\frac{\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{-2a^2c^2e+4ab^2ce\cdots}{\sqrt{\cdots}}\right.}{}$$

```
[Out] -1/3*(b*e+c*d)*(e*x^2+d)^(3/2)/c^2/e^2+1/5*(e*x^2+d)^(5/2)/c/e^2+(-a*c+b^2)
*(e*x^2+d)^(1/2)/c^3-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(
b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b^2*c*d-a*c^2*d-b^3*e+2*a*b*c*e+(2*a^2*c^2*e
-4*a*b^2*c*e+3*a*b*c^2*d+b^4*e-b^3*c*d)/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)
/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+
d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b^2*c*d-a*c^2*d-b^3*e+2*a
*b*c*e+(-2*a^2*c^2*e+4*a*b^2*c*e-3*a*b*c^2*d-b^4*e+b^3*c*d)/(-4*a*c+b^2)^(1
/2))/c^(7/2)*2^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 8.59, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 897, 1287, 1166, 208}

$$-\frac{\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{-2a^2c^2e+4ab^2ce\cdots}{\sqrt{\cdots}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2 - a*c)*Sqrt[d + e*x^2])/c^3 - ((c*d + b*e)*(d + e*x^2)^(3/2))/(3*c^2*
e^2) + (d + e*x^2)^(5/2)/(5*c*e^2) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*
e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 -
4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^4*e
+ 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(b^2-ac)e}{c^3}-\frac{(cd+be)x^2}{c^2e}+\frac{x^4}{ce}-\frac{(b^2-ac)(cd^2-bde+ae^2)-(b^2cd-ac^2d-b^3e+2abce)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{d+ex \ldots}\right.}{e}$$

$$= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)\left(d+ex^2\right)^{3/2}}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}-\frac{\text{Subst}\left(\int \frac{(b^2-ac)(cd^2-bde+ae^2)+(\ldots}{\frac{cd^2-bde+ae^2}{e^2}-\ldots}\right.}{\ldots}$$

$$= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)\left(d+ex^2\right)^{3/2}}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}+\frac{\left(b^2cd-ac^2d-b^3e+2abce-\ldots\right.}{\ldots}$$

$$= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)\left(d+ex^2\right)^{3/2}}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}-\frac{\left(b^2cd-ac^2d-b^3e+2abce-\ldots\right.}{\sqrt{2}\ldots}$$

Mathematica [B] time = 10.84, size = 943, normalized size = 2.32

$$\frac{c\left(105\left(-b^3+\sqrt{b^2-4ac}\,b^2+3acb-ac\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}\,e}}\right)e^3+\sqrt{2}\sqrt{\frac{c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}\,e}}\left(105b^3e^3-35b^2\left(3\sqrt{b^2-4ac}\,e+c\left(ex^2+d\right)\right)e^2+\right.\right.}{210\sqrt{2}\sqrt{b^2-4ac}\,e^4\left(2cd+\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*(d + e*x^2)^(9/2)*(Sqrt[2]*Sqrt[(c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2
 - 4*a*c]*e)]*(105*b^3*e^3 - 35*b^2*e^2*(3*Sqrt[b^2 - 4*a*c]*e + c*(d + e*x
^2)) + 7*b*c*e*(-45*a*e^2 + (d + e*x^2)*(-5*c*d + 5*Sqrt[b^2 - 4*a*c]*e + 3
*c*(d + e*x^2))) + c*(35*a*e^2*(3*Sqrt[b^2 - 4*a*c]*e + 2*c*(d + e*x^2)) +
c*(d + e*x^2)*(7*Sqrt[b^2 - 4*a*c]*e*(5*d - 3*(d + e*x^2)) + c*(-70*d^2 + 8
4*d*(d + e*x^2) - 30*(d + e*x^2)^2)))) + 105*(-b^3 + 3*a*b*c + b^2*Sqrt[b^2
 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*e^3*ArcTanh[Sqrt[2]*Sqrt[(c*(d + e*x^2))
/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)]]))/(210*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e^4
*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)^3*(-(Sqrt[b^2 - 4*a*c]/e) - (2*c*d -
b*e)/e^2)*((c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e))^(9/2)) + (2
*c*d^3*(d + e*x^2)^(3/2)*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(-105*b^3*e^
3 + 35*b^2*e^2*(-3*Sqrt[b^2 - 4*a*c]*e + c*(d + e*x^2)) - 7*b*c*e*(-45*a*e^
2 + (d + e*x^2)*(-5*c*d - 5*Sqrt[b^2 - 4*a*c]*e + 3*c*(d + e*x^2))) + c*(35
*a*e^2*(3*Sqrt[b^2 - 4*a*c]*e - 2*c*(d + e*x^2)) + c*(d + e*x^2)*(7*Sqrt[b^
2 - 4*a*c]*e*(5*d - 3*(d + e*x^2)) + c*(70*d^2 - 84*d*(d + e*x^2) + 30*(d +
 e*x^2)^2)))))/(140*c^4*d^3*(d + e*x^2)) + (3*(b^3 - 3*a*b*c + b^2*Sqrt[b^2
 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*e^3*(d + e*x^2)^3*ArcTanh[Sqrt[2]*Sqrt[(
c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]])/(4*Sqrt[2]*d^3*(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)^3*((c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e))^(9/2))))/(3*Sqrt[b^2 - 4*a*c]*e^4*(Sqrt[b^2 - 4*a*c]/e - (2*c*d
- b*e)/e^2)))/e
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.65, size = 928, normalized size = 2.29

$$-\frac{\left(\left(\left(b^4c-5\,ab^2c^2+4\,a^2c^3\right)de-\left(b^5-6\,ab^3c+8\,a^2bc^2\right)e^2\right)c^2+2\left(b^3c^4-3\,abc^5\right)d^2-\left(3\,b^4c^3-11\,ab^2c^4+4\,a^2c^5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -(((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*
e^2)*c^2 + 2*(b^3*c^4 - 3*a*b*c^5)*d^2 - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*
```

```
c^5)*d*e - 2*((b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)
*sqrt(b^2 - 4*a*c)*d*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*abs(c
) + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^2)*arctan(2*sqrt(1/2)*sqrt(x^2*
e + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 + sqrt(-4*(c^6*d^2*e^12 - b*c^5*d*e
^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))*e^(-12)/c^6))/
((2*sqrt(b^2 - 4*a*c)*c^4*d + (b^2*c^3 - 4*a*c^4 - sqrt(b^2 - 4*a*c)*b*c^3)
*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2) + (((b^4*c - 5*a*
b^2*c^2 + 4*a^2*c^3)*d*e - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*c^2 + 2*(b^
3*c^4 - 3*a*b*c^5)*d^2 - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*c^5)*d*e + 2*((b
^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*
c)*d*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*abs(c) + (b^5*c^2 - 4
*a*b^3*c^3 + 2*a^2*b*c^4)*e^2)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*
c^6*d*e^12 - b*c^5*e^13 - sqrt(-4*(c^6*d^2*e^12 - b*c^5*d*e^13 + a*c^5*e^14
)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))*e^(-12)/c^6))/((2*sqrt(b^2 - 4
*a*c)*c^4*d - (b^2*c^3 - 4*a*c^4 + sqrt(b^2 - 4*a*c)*b*c^3)*e)*sqrt(-4*c^2*
d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2) + 1/15*(3*(x^2*e + d)^(5/2)*c^4*e
^8 - 5*(x^2*e + d)^(3/2)*c^4*d*e^8 - 5*(x^2*e + d)^(3/2)*b*c^3*e^9 + 15*sqr
t(x^2*e + d)*b^2*c^2*e^10 - 15*sqrt(x^2*e + d)*a*c^3*e^10)*e^(-10)/c^5
```

maple [C] time = 0.06, size = 496, normalized size = 1.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/5/c*x^2*(e*x^2+d)^(3/2)/e-2/15/c*d/e^2*(e*x^2+d)^(3/2)-1/3/c^2*b*(e*x^2+d
)^(3/2)/e+1/2/c^2*e^(1/2)*x*a-1/2/c^3*e^(1/2)*x*b^2-1/2/c^2*(e*x^2+d)^(1/2)
*a+1/2/c^3*(e*x^2+d)^(1/2)*b^2-1/4/c^3*sum(((-2*a*b*c*e+a*c^2*d+b^3*e-b^2*c
*d)*_R^6+(-4*a^2*c*e^2+4*a*b^2*e^2+2*a*b*c*d*e-3*a*c^2*d^2-3*b^3*d*e+3*b^2*
c*d^2)*_R^4+d*(4*a^2*c*e^2-4*a*b^2*e^2-2*a*b*c*d*e+3*a*c^2*d^2+3*b^3*d*e-3*
b^2*c*d^2)*_R^2+2*a*b*c*d^3*e-a*c^2*d^4-b^3*d^3*e+b^2*c*d^4)/(_R^7*c+3*_R^5
*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)
*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a
*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))-1/2/c^2*d/((e*x
^2+d)^(1/2)-e^(1/2)*x)*a+1/2/c^3*d/((e*x^2+d)^(1/2)-e^(1/2)*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,x^7}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 2.48, size = 11195, normalized size = 27.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(1/2)*((3*d^2)/(c*e^2) - (a*e^4 + c*d^2*e^2 - b*d*e^3)/(c^2*e^4
) + (((3*d)/(c*e^2) + (b*e^3 - 2*c*d*e^2)/(c^2*e^4))*(b*e^3 - 2*c*d*e^2))/(
c*e^2)) - (d + e*x^2)^(3/2)*(d/(c*e^2) + (b*e^3 - 2*c*d*e^2)/(3*c^2*e^4)) +
 atan(((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c
^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a
```

```
^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1/2)*(-(b^9*e
- 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d
 + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*
a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*
c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b
^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2)
- 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*
b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c
^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b
^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^
3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*
d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*
c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*
(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16
*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^8*e^4 +
 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^
2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2
 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b
^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2)
 - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^
3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*
c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(
4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^
3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((16*a^3*c^6*e^4 + 4*a*
b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4
*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d
^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c
- b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^
5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*
c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) +
5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3
 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^
4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*
a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-
(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2
*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^
2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^
8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2
*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c
*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*
b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e -
8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d +
 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*
c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*
d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3
*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) -
6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*1i)/((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 -
 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5
*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1
/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*
a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^
3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b
*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^
(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b
```

```
^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b
^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8
*e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)
^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*
e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e +
10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^
4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3
*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/
2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a
^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^
2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e
^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c -
b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*
c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*
e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*
a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
 + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (((16*a^3*c^6*
e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d
^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a
*b^2*c^6*d^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e
*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d +
 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) -
 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b
^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b
^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^
9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c
^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e +
b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4
*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1
/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 -
8*a*b^2*c^8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a
^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 -
 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3
 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(
-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b
^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3
*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*
e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2)
 - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3
)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^
7 - 8*a*b^2*c^8)))^(1/2) - (2*(a^4*b^3*e^5 - a^3*b^4*d*e^4 + a^5*c^2*d*e^4
+ a^4*c^3*d^3*e^2 - 2*a^5*b*c*e^5 - a^3*b^2*c^2*d^3*e^2 + a^4*b^2*c*d*e^4 +
 2*a^3*b^3*c*d^2*e^3 - 3*a^4*b*c^2*d^2*e^3))/c^5))*(-(b^9*e - 8*a^4*c^5*d -
 b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c
^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(
1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c -
b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a
*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2
*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2
)*2i + atan(((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2
*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3
- 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1/2)*((8
*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*
c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*
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(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3

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)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e
+ 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10
*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*
c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a
^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/2)
*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3
*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*
c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2
 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2
)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2
*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e -
 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b
^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) +
3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (((16*a^3*c^6*e^4
 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*
e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^
2*c^6*d^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(
4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*
a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*
a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1
/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^
3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c
^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*((8*a^4*c
^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d
- 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a
*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c
*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^
3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) -
 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b
^2*c^8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^
4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*
b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14
*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^
4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3
*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(
4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^
5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2
) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*
a*b^2*c^8)))^(1/2) - (2*(a^4*b^3*e^5 - a^3*b^4*d*e^4 + a^5*c^2*d*e^4 + a^4*
c^3*d^3*e^2 - 2*a^5*b*c*e^5 - a^3*b^2*c^2*d^3*e^2 + a^4*b^2*c*d*e^4 + 2*a^3
*b^3*c*d^2*e^3 - 3*a^4*b*c^2*d^2*e^3))/c^5))*((8*a^4*c^5*d - b^9*e - b^6*e*
(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d -
42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) +
11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)
^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^
2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i +
(d + e*x^2)^(5/2)/(5*c*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.355 $\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=324

$$\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)\cdots\right.}{\sqrt{2}\,c^{5/2}\sqrt{2cd-e\left(\cdots\right.}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)/c/e-b*(e*x^2+d)^(1/2)/c^2+1/2*arctanh(2^(1/2)*c^(1/2)*(
e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b*c*d-b^2*e+a*c*e+(
-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(2*
c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(
1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-
2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 3.53, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 897, 1287, 1166, 208}

$$\frac{\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)\cdots\right.}{\sqrt{2}\,c^{5/2}\sqrt{2cd-e\left(\cdots\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*Sqrt[d + e*x^2])/c^2) + (d + e*x^2)^(3/2)/(3*c*e) + ((b*c*d - b^2*e +
a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]
])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((b*c*d - b^
2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{be}{c^2}+\frac{x^2}{c}+\frac{b\left(cd^2-bde+ae^2\right)-\left(bcd-b^2e+ace\right)x^2}{c^2e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}+\frac{\text{Subst}\left(\int \frac{b\left(cd^2-bde+ae^2\right)+\left(-bcd+b^2e-ace\right)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex^2}\right)}{c^2e^2}$$

$$= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}-\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\cdots}{-\frac{\sqrt{b^2-4ac}}{2e}\cdots}\right)}{2c^2e^2}$$

$$= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}+\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{2cd-\left(b-\cdots\right.}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 7.25, size = 591, normalized size = 1.82

$$\frac{c\left(d+ex^2\right)^{7/2}\left(\frac{e^2\left(\sqrt{2}\sqrt{\frac{c(d+ex^2)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\left(5be\left(3e\sqrt{b^2-4ac}+c\left(d+ex^2\right)\right)+c\left(d+ex^2\right)\left(-5e\sqrt{b^2-4ac}+4cd-6cex^2\right)+30ace^2-15b^2e^2\right)-15e^2\left(b\sqrt{b^2-4a\cdots}\right.\right.}{\left(e\left(b-\sqrt{b^2-4ac}\right)-2cd\right)\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)^2\left(\frac{c(d+ex^2)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}\right)^{7/2}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*(d + e*x^2)^(7/2)*((e^2*(Sqrt[2]*Sqrt[(c*(d + e*x^2))/(2*c*d + (-b + Sqr
t[b^2 - 4*a*c])*e)]*(-15*b^2*e^2 + 30*a*c*e^2 + c*(d + e*x^2)*(4*c*d - 5*Sq
rt[b^2 - 4*a*c]*e - 6*c*e*x^2) + 5*b*e*(3*Sqrt[b^2 - 4*a*c]*e + c*(d + e*x^
2))) - 15*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*e^2*ArcTanh[Sqrt[2]*Sqrt[(c*
(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)]]))/((-2*c*d + (b - Sqrt[b
^2 - 4*a*c])*e)*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)^2*((c*(d + e*x^2))/(2*
c*d + (-b + Sqrt[b^2 - 4*a*c])*e))^(7/2)) - (e^2*(Sqrt[2]*Sqrt[(c*(d + e*x^
2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(-15*b^2*e^2 + 30*a*c*e^2 + c*(d +
 e*x^2)*(4*c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*c*e*x^2) - 5*b*e*(3*Sqrt[b^2 - 4
*a*c]*e - c*(d + e*x^2))) + 15*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*e^2*ArcT
anh[Sqrt[2]*Sqrt[(c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]]))/((
-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)^3*((c*(d + e*x^2))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e))^(7/2))))/(30*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.82, size = 745, normalized size = 2.30

$$\frac{\left(\left(x^2e+d\right)^{\frac{3}{2}}c^2e^2-3\sqrt{x^2e+d}\,bce^3\right)e^{(-3)}}{3c^3}+\frac{\left(\left(\left(b^3c-4abc^2\right)de-\left(b^4-5ab^2c+4a^2c^2\right)e^2\right)c^2+2\left(b^2c^4-2ac^5\right)d^2-\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/3*((x^2*e + d)^(3/2)*c^2*e^2 - 3*sqrt(x^2*e + d)*b*c*e^3)*e^(-3)/c^3 + ((
(b^3*c - 4*a*b*c^2)*d*e - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2)*c^2 + 2*(b^2*c
^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*a*b*c^4)*d*e - 2*(sqrt(b^2 - 4*a*c)*b*c^
3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*abs(
c) + (b^4*c^2 - 3*a*b^2*c^3)*e^2)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-
(2*c^4*d*e^4 - b*c^3*e^5 + sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)*
c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))*e^(-4)/c^4))/((2*sqrt(b^2 - 4*a*c)*
c^3*d + (b^2*c^2 - 4*a*c^3 - sqrt(b^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c^2*d + 2*
(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2) - (((b^3*c - 4*a*b*c^2)*d*e - (b^4 - 5*
a*b^2*c + 4*a^2*c^2)*e^2)*c^2 + 2*(b^2*c^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*
a*b*c^4)*d*e + 2*(sqrt(b^2 - 4*a*c)*b*c^3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d
*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*abs(c) + (b^4*c^2 - 3*a*b^2*c^3)*e^2)*a
rctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 - sqrt(-4*
(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)
^2))*e^(-4)/c^4))/((2*sqrt(b^2 - 4*a*c)*c^3*d - (b^2*c^2 - 4*a*c^3 + sqrt(b
^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2)
```

maple [C] time = 0.03, size = 332, normalized size = 1.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*(e*x^2+d)^(3/2)/c/e+1/2/c^2*b*e^(1/2)*x-1/2*b*(e*x^2+d)^(1/2)/c^2-1/4/c
^2*sum(((a*c*e-b^2*e+b*c*d)*_R^6+(-4*a*b*e^2+a*c*d*e+3*b^2*d*e-3*b*c*d^2)*_
R^4+d*(4*a*b*e^2-a*c*d*e-3*b^2*d*e+3*b*c*d^2)*_R^2-a*c*d^3*e+b^2*d^3*e-c*d^
4*b)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_
R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b
*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z
^2))-1/2/c^2*b*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,x^5}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.99, size = 8222, normalized size = 25.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(3/2)/(3*c*e) - atan(((((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4
*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^
3)/c^3 - (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2
)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c
*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^
2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c
^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*
a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2
) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2
)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^
2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c
^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a
*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^
2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e
 - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*
c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 +
 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 + (2*(d +
 e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6
*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^
2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*
e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8
*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d +
25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b
^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*
(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a
```

^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((2*(a^4*c*e^5 - a^3*b^2*e^5 + a^2*b^3*d*e^4 + a^3*c^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*d^2*e^3))/c^3 + (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 - (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 + (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)))*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - atan(((((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 - (2*(d + e*x^2)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +

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b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c
^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4
*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4
*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c
*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^
2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 +
2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*
b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(
4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d -
25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b
^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*
(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^
4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2
*c^4*d*e^3)/c^3 + (2*(d + e*x^2)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8
*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^
2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e
 - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*
c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d
*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*
d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^
3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^
2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 +
 b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*
a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/
2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e +
a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b
*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*
a*b^2*c^6)))^(1/2)*1i)/((2*(a^4*c*e^5 - a^3*b^2*e^5 + a^2*b^3*d*e^4 + a^3*c
^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*d^2*e^3))/c^3 + (((4*a*b^3*c^3
*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*
d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 - (2*(d + e*x^2)^(1/2)*((b^4*e*(-(4*a*c -
 b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*
b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*
d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*
c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +
b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^
6*e^3 + 32*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*
d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*
d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2
*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1
/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2
*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b
^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4
*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 2
5*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^
4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(
-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^
4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4
*d*e^3)/c^3 + (2*(d + e*x^2)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3
*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b
```

```
^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*
a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2
))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d +
18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) +
9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(
1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3
)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(
1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4
*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*
b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) -
 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*
c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3
*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^
2*c^6)))^(1/2)))*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b
^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c -
b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - ((2*
d)/(c*e) + (b*e^2 - 2*c*d*e)/(c^2*e^2))*(d + e*x^2)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.356 $\int \frac{x^3 \sqrt{d+ex^2}}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=292

$$\frac{\left(-\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-\right.}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{2}}$$

```
[Out] (e*x^2+d)^(1/2)/c+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(
-4*a*c+b^2)^(1/2)))^(1/2))*(b*c*d-b^2*e+2*a*c*e-(-b*e+c*d)*(-4*a*c+b^2)^(1/
2))/c^(3/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/
2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2)))^(1/2))*(b*c*d-b^2*e+2*a*c*e+(-b*e+c*d)*(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(
1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 3.60, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 824, 826, 1166, 208}

$$\frac{\left(-\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-\right.}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[d + e*x^2]/c + ((b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]*(c*d - b*
e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - ((b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d - b*e))*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{\sqrt{d+ex^2}}{c} + \frac{\operatorname{Subst}\left(\int \frac{-ae+(cd-be)x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx, x, x^2\right)}{2c}\\
&= \frac{\sqrt{d+ex^2}}{c} + \frac{\operatorname{Subst}\left(\int \frac{-ae^2-d(cd-be)+(cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{c}\\
&= \frac{\sqrt{d+ex^2}}{c} - \frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx\right.}{2c\sqrt{b^2-4ac}}\\
&= \frac{\sqrt{d+ex^2}}{c} + \frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{(bcd-\ldots}{\ldots}
\end{aligned}$$

Mathematica [A] time = 0.55, size = 308, normalized size = 1.05

$$\frac{\frac{\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}-2ace+b^2e-bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+\ldots\right.}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\ldots}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2] + ((b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e + b*Sq
rt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b
*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e]) + ((-(b*c*d) - c*Sqrt[b^2 - 4*a*c]*d + b^2*e
- 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a
*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/c
```

fricas [B] time = 134.69, size = 2435, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 -
 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c +
a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2
 - 2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)
*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5
*a*b^3*c + 4*a^2*b*c^2)*e - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c
^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^
6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4
*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - ((a*b^2*c^3 - 4*a^
2*c^4)*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c -
a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2)
- sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*
a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2
*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 -
2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)*e^
2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*
b^3*c + 4*a^2*b*c^2)*e - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*
d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 -
 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*
c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c
^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - ((a*b^2*c^3 - 4*a^2*c
^4)*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b
*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) + s
qrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c
^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^
2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 - 2*a
*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)*e^2)*
x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3
*c + 4*a^2*b*c^2)*e + (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2
 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*
a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4
)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)
*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + ((a*b^2*c^3 - 4*a^2*c^4)
*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^
2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) - sqrt
(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)
*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*
e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 - 2*a*b^
3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)*e^2)*x^2
 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3*c
+ 4*a^2*b*c^2)*e + (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2 -
2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c
^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*s
qrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^
2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + ((a*b^2*c^3 - 4*a^2*c^4)*e*
x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*
d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) + 4*sqrt(
e*x^2 + d))/c
```

giac [B] time = 0.78, size = 619, normalized size = 2.12

$$\frac{\sqrt{x^2e+d}}{c} - \frac{\left(2\,bc^4d^2 + \left(\left(b^2c-4\,ac^2\right)de - \left(b^3-4\,abc\right)e^2\right)c^2 - \left(3\,b^2c^3-4\,ac^4\right)de - 2\left(\sqrt{b^2-4\,ac}\,c^3d^2 - \sqrt{b^2-4\,ac}\,b\right.\right.}{\left(2\sqrt{b^2-4\,ac}\,c^2d - \left(b^2c-4\,ac^2+\sqrt{b^2-4\,ac}\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] sqrt(x^2*e + d)/c - (2*b*c^4*d^2 + ((b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)
*e^2)*c^2 - (3*b^2*c^3 - 4*a*c^4)*d*e - 2*(sqrt(b^2 - 4*a*c)*c^3*d^2 - sqrt
(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*abs(c) + (b^3*c^2 -
2*a*b*c^3)*e^2)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^2*d - b*c*e +
 sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((
2*sqrt(b^2 - 4*a*c)*c^2*d - (b^2*c - 4*a*c^2 + sqrt(b^2 - 4*a*c)*b*c)*e)*sq
rt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2) + (2*b*c^4*d^2 + ((b^2*
c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2 - (3*b^2*c^3 - 4*a*c^4)*d*e + 2
*(sqrt(b^2 - 4*a*c)*c^3*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*
c)*a*c^2*e^2)*abs(c) + (b^3*c^2 - 2*a*b*c^3)*e^2)*arctan(2*sqrt(1/2)*sqrt(x
^2*e + d)/sqrt(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^
2 + (2*c^2*d - b*c*e)^2))/c^2))/((2*sqrt(b^2 - 4*a*c)*c^2*d + (b^2*c - 4*a*
c^2 - sqrt(b^2 - 4*a*c)*b*c)*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*
c)*e)*c^2)
```

maple [C] time = 0.03, size = 275, normalized size = 0.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/c*e^(1/2)*x+1/2*(e*x^2+d)^(1/2)/c+1/4/c*sum(((-b*e+c*d)*_R^6+(-4*a*e^2
+3*b*d*e-3*c*d^2)*_R^4+d*(4*a*e^2-3*b*d*e+3*c*d^2)*_R^2+b*d^3*e-c*d^4)/(_R^
7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e
-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)
*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))+1/2/
c*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 2.34, size = 5705, normalized size = 19.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(1/2)/c - atan(((((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*
d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - (2*(d
 + e*x^2)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^
4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*
c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4
*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*
d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3
))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*
a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2)*1i - (((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3
*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + e*x^2)^(1/2)
*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3
*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*
a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/
2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*
((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*
c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a
*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2
) + (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c
^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c
^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c
*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*
d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/(((
(16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*
b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - (2*(d + e*x^2)^(1/2)*((8*a^2*c^3*d
 - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-
(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 1
2*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e
^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((8*a^2*c^3*d -
 b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(
4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12
*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x
^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*
a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e -
b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b
^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^
2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(a*c^2*d^3*e^2 -
a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4 - 2*a*b*c*d^2*e^3))/c + (((16*a^2*c^3
*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2
*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + e*x^2)^(1/2)*((8*a^2*c^3*d - b^5*e - b
^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^
2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2
*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*
c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^
2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2
)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*
e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(
b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4
 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*
a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2
) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16
*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*((8*a^2*c^3*d - b^5*e - b^2*e*(
-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^
(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8
```

```
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3*e^4
- 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2
- 16*a*b*c^3*d*e^3)/c - (2*(d + e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e
*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3
)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*
d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e
*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3
)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4
*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 -
2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*
c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2)
+ b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3*e^4 - 4*a*b^2*c^
2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3
*d*e^3)/c + (2*(d + e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c -
b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*
c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c
^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a
*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c -
b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*
c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c
^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2
*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^
3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^
(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4
*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4
*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a
*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c -
(2*(d + e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2
) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c
 - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 +
 32*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2
) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c
 - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2
*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^
2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*
c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^
3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) - (2*(a*c^2*d^3*e^2 - a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4
 - 2*a*b*c*d^2*e^3))/c + (((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2
*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d +
e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*
c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^
3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^
5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*
c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^
3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d
^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3)
)/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*
a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2)
 + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2)))*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d
 - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^
```

```
(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*
c^4)))^(1/2)*2i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**3*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.357 $\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=202

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^
2)^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2)))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)/c^(1/2)/(-
4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1247, 699, 1130, 208}

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b
^2 - 4*a*c])) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*
Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= e\,\mathrm{Subst}\left(\int \frac{x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)$$
$$= -\left(\frac{1}{2}\left(-e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex^2}\right.\right.$$
$$= -\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}{\sqrt{2}\sqrt{c}\sqrt{}}$$

Mathematica [A] time = 0.33, size = 179, normalized size = 0.89

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)-\sqrt{e\sqrt{b^2-4ac}-be+2cd}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]) + Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

fricas [B] time = 19.82, size = 1085, normalized size = 5.37

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{\frac{2\,cd-be+\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\,\log\left(\frac{be^2x^2+2\,bde-2\,ae^2+2\sqrt{\frac{1}{2}}\sqrt{ex^2+d}\left(\left(b^2-4\,ac\right)e+\left(b^3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(1/2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*
a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 + 2*sqrt(1/2
)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e + (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2
- 4*a*c^3)))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*
c^3)))/(b^2*c - 4*a*c^2)) + ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*
d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b*e +
(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*
e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e
+ (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e + (b
^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + ((b^2*c
```

```
 - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x
^2) - 1/4*sqrt(1/2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2
 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 + 2*sqr
t(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e - (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2
*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 -
 4*a*c^3)))/(b^2*c - 4*a*c^2)) - ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*
c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b
*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*lo
g((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*
c)*e - (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e
 - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) - ((
b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3
)))/x^2)
```

giac [A] time = 0.55, size = 228, normalized size = 1.13

$$-\frac{\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\,\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}\,\sqrt{x^2e+d}}{\sqrt{-\frac{2\,cd-be+\sqrt{(2\,cd-be)^2-4\left(cd^2-bde+ae^2\right)c}}{c}}}\right)}{2\,\sqrt{b^2-4\,ac}\,|c|}+\frac{\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)e}\,\mathrm{a}}{2\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sq
rt(x^2*e + d)/sqrt(-(2*c*d - b*e + sqrt((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e
+ a*e^2)*c))/c))/(sqrt(b^2 - 4*a*c)*abs(c)) + 1/2*sqrt(-4*c^2*d + 2*(b*c +
sqrt(b^2 - 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c*d - b
*e - sqrt((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/(sqrt(b^2 - 4
*a*c)*abs(c))
```

maple [C] time = 0.02, size = 177, normalized size = 0.88

$$\frac{}{4\,\mathrm{RootOf}\left(\_Z^8c+(4be-4cd)\,\_Z^6+c\,d^4+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^4+\left(4b\,d^2e-4c\,d^3\right)\_Z^2\right)^7c+12\,\mathrm{RootOf}\left(\_\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*e*sum((_R^6+_R^4*d-_R^2*d^2-d^3)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a
*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2
+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*
d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.72, size = 717, normalized size = 3.55

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{e\,x^2+d}\,\left(-2\,b^2\,c\,e^4+4\,b\,c^2\,d\,e^3-4\,c^3\,d^2\,e^2+4\,a\,c^2\,e^4\right)+\frac{\sqrt{e\,x^2+d}\,\left(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\,c^3\,e^3+64\,\cdots\right.}{8\left(16\,a^2\,\cdots\right.}\right.}{2\,c^2\,d^2\,e^3-2\,b\,c\,d\,e^4-\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] - 2*atanh((2*((d + e*x^2)^(1/2)*(4*a*c^2*e^4 - 2*b^2*c*e^4 - 4*c^3*d^2*e^2
+ 4*b*c^2*d*e^3) + ((d + e*x^2)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 3
2*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c
^2*d - 2*b^2*c*d - 4*a*b*c*e))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b
^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)/(8*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(2*c^2*d^2*e^3 + 2*a*c*e^5 - 2*b
*c*d*e^4))*(-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d -
4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh((2*((d +
 e*x^2)^(1/2)*(4*a*c^2*e^4 - 2*b^2*c*e^4 - 4*c^3*d^2*e^2 + 4*b*c^2*d*e^3) -
 ((d + e*x^2)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32*a*b*c^3*e^3 + 64
*a*c^4*d*e^2)*(e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d +
 4*a*b*c*e))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((e*(-(4*a*c - b^2)^3)
^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3
- 8*a*b^2*c^2)))^(1/2))/(2*c^2*d^2*e^3 + 2*a*c*e^5 - 2*b*c*d*e^4))*((e*(-(4
*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(8*(b^4*c
 + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.358 $\int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=281

$$\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}\right.}}\right.}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

```
[Out] -arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)/a+1/2*arctanh(2^(1/2)*c^(1/2)*(e*
x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b*d-2*a*e+d*(
-4*a*c+b^2)^(1/2))/a*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1
/2)))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))^(1/2))*c^(1/2)*(b*d-2*a*e-d*(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(-4
*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.35, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 897, 1287, 206, 1166, 208}

$$\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}\right.}}\right.}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) + (Sqrt[c]*(b*d + Sqrt[b^2
 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcT
anh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}\\
&= \frac{\operatorname{Subst}\left(\int \left(-\frac{de}{a\left(d-x^2\right)}+\frac{e\left(cd^2-bde+ae^2-cdx^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}\\
&= \frac{\operatorname{Subst}\left(\int \frac{cd^2-bde+ae^2-cdx^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a} - \frac{d\operatorname{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+e\right.}{a}\\
&= -\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\left(c\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be\right.}{2a\sqrt{b^2-4ac}}\\
&= -\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}
\end{aligned}
$$

Mathematica [A] time = 0.81, size = 241, normalized size = 0.86

$$
\frac{\frac{\sqrt{2}\left(\left(\sqrt{b^2-4ac}+b\right)\sqrt{e\sqrt{b^2-4ac}-be+2cd}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)+\left(\sqrt{b^2-4ac}-b\right)\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)\right)}{\sqrt{c}\sqrt{b^2-4ac}} - 4\sqrt{\ldots}}{4a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-4*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + (Sqrt[2]*((b + Sqrt[b^2 - 4*
a*c])*Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]] + (-b + Sqrt[b^2 - 4*
a*c])*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]]))/(Sqrt[c]*Sqrt[b^2 -
 4*a*c]))/(4*a)
```

fricas [B] time = 147.58, size = 3126, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqr
t((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)
)*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 + 4*sqr
t(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*
e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3
*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4
*a^3*c)) - ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*
d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a*sqrt(-(
a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a
^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*
d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 - 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)
*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*
sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*
d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - ((a^2*b^2 - 4*a
^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)
/(a^4*b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d -
(a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c
)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e
 - a*b*e^2)*x^2 + 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2
*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*
c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 -
4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 -
 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2
) + sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((
b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*l
og(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 - 4*sqrt(1
/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2
)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^
3*c)) + ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - 2*sqrt(d)*log(-(e*x^2
- 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2))/a, -1/4*(sqrt(1/2)*a*sqrt(-(a*b*e
- (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2
)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e +
2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 + 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(
e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-
(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e +
a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - ((a^2*b^2 - 4*a^3*c)*
e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*
b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b
^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a
^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b
```

```
*e^2)*x^2 - 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 -
 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d +
 (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*
c)))/(a^2*b^2 - 4*a^3*c)) - ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3
*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - sq
rt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2
*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 + 4*sqrt(1/2)*(a
^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4
*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt(
(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))
 + ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a
*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) + sqrt(1/2)*a*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a
^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 - 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x
^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*
b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2
*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + ((a^2*b^2 - 4*a^3*c)*e*x
^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2
 - 4*a^5*c)))/x^2) - 4*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d)))/a]
```

giac [B] time = 0.68, size = 717, normalized size = 2.55

$$\frac{d \arctan\left(\frac{\sqrt{x^2 e + d}}{\sqrt{-d}}\right)}{a\sqrt{-d}} - \frac{\left(\sqrt{-4\,c^2 d + 2\left(bc - \sqrt{b^2 - 4\,ac}\,c\right)e}\left(b^2 - 4\,ac\right)a^2 de - 2\left(\sqrt{b^2 - 4\,ac}\,acd^2 - \sqrt{b^2 - 4\,ac}\,abde\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d*arctan(sqrt(x^2*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/8*(sqrt(-4*c^2*d + 2*(b
*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e - 2*(sqrt(b^2 - 4*a*c)*a
*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2
*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2
 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
))*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c
*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 -
4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)
*abs(a)*abs(c)) + 1/8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(b^
2 - 4*a*c)*a^2*d*e + 2*(sqrt(b^2 - 4*a*c)*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d
*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*
c)*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt
(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x^2*e
 + d)/sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c +
(2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*
a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(a)*abs(c))
```

maple [C] time = 0.03, size = 294, normalized size = 1.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a*d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)+1/2/a*(e*x^2+d)^(1/2)+1/
2/a*e^(1/2)*x-1/4/a*sum((_R^6*c*d+(-4*a*e^2+4*b*d*e-3*c*d^2)*_R^4+d*(4*a*e^
```

```
2-4*b*d*e+3*c*d^2)*_R^2-c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4
*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1
/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_
Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))-1/2/a*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x), x)
```

mupad [B] time = 6.87, size = 10964, normalized size = 39.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9
+ 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e
+ a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d
 + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d +
 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2
*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a
*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c
*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*
a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*
c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^
2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8
 + 48*a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9
+ 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^
4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 -
16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*
c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a
^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^
(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))
)^(1/2) + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^
4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^
11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c
 - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e
)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(
2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a
*b*c^3*d*e^11) - ((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(
1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^
4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*
c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(
1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^
2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4
*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2
*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9
 + 480*a^3*b^3*c^3*d*e^9) + 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a
^2*b^2*c^4*d^3*e^8 + 48*a^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^
```

3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^1
0 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*
c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2
*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*
d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) -
 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)
 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4
*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10
))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(
4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))^(1/2)*1i)/(((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4
*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) - ((b^4*d +
 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2
c)))^(1/2)(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4
*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b
*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((
b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c
 - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*
a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^
3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2
*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9)
+ 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a^
2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^10) - (d + e
x^2)^(1/2)(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 1
44*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2
*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e
^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c
 - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e
)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a^2*c^4*d^2*e^10 - 4
*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e
^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d -
 a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*
a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) -
((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^
2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*
(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a
^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2
*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1
/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^
(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b
^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8
*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^
4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^
2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^
9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8 + 48*
a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a
^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^
10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5
*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^
2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2
*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2)
- 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2
) + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*
d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 2
4*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2
)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(
a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 2*c^4*d^3*e^10 - 2*b*c^3*d^2*
e^11 + 2*a*c^3*d*e^12))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^

2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*2i + atan((((d + e*x^2)^(1/2)*
(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*
a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^
(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b
^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e -
a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d +
4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^
(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*
(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*
c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^
4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2
*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3
*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8 + 48*a^2*b^2*c^4*d^3*e^8
- 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a^3*b^2*c^3*d*e^10)
- (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*
e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a
*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c
^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*
(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a
^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a*c^5*d^4*e^
8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^
4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^1
0))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-
(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^
2 - 8*a^3*b^2*c)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^
4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) - ((b^4*d
+ 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^
2*c)))^(1/2)*(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(
4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*
b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*(
(b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*
c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8
*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c
^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^
2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9)
+ 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a
^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^10) - (d +
e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 +
144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^
2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*
e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*
c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*
e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a^2*c^4*d^2*e^10 -
4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*
e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d
- a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6
*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1
i)/(((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 +
4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) - ((b^4*d + 8*a^2*c^2*d - a*b^3*e -
a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d +
4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(12*a*c^5*d^4
*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*
d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b
^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^
2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a
^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*

e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8 + 48*a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 2*c^4*d^3*e^10 - 2*b*c^3*d^2*e^11 + 2*a*c^3*d*e^12))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*2i - (d^(1/2)*atanh((20*c^4*d^(5/2)*e^10*(d + e*x^2)^(1/2))/(20*c^4*d^3*e^10 - 12*b*c^3*d^2*e^11 + (18*c^5*d^5*e^8)/a + 2*a*c^3*d*e^12 + (6*b^2*c^3*d^3*e^10)/a + (2*b^3*c^2*d^2*e^11)/a - (4*b^2*c^4*d^5*e^8)/a^2 + (6*b^3*c^3*d^4*e^9)/a^2 - (2*b^4*c^2*d^3*e^10)/a^2 - (28*b*c^4*d^4*e^9)/a) + (18*c^5*d^(9/2)*e^8*(d + e*x^2)^(1/2))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^10 + 2*a^2*c^3*d*e^12 - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^10 + 2*b^3*c^2*d^2*e^11 - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^10)/a - 12*a*b*c^3*d^2*e^11) - (28*b*c^4*d^(7/2)*e^9*(d + e*x^2)^(1/2))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^10 + 2*a^2*c^3*d*e^12 - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^10 + 2*b^3*c^2*d^2*e^11 - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^10)/a - 12*a*b*c^3*d^2*e^11) + (2*b^3*c^2*d^(3/2)*e^11*(d + e*x^2)^(1/2))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^10 + 2*a^2*c^3*d*e^12 - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^10 + 2*b^3*c^2*d^2*e^11 - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^10)/a - 12*a*b*c^3*d^2*e^11) + (6*b^2*c^3*d^(5/2)*e^10*(d + e*x^2)^(1/2))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^10 + 2*a^2*c^3*d*e^12 - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^10 + 2*b^3*c^2*d^2*e^11 - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^10)/a - 12*a*b*c^3*d^2*e^11) - (2*b^4*c^2*d^(5/2)*e^10*(d + e*x^2)^(1/2))/(18*a*c^5*d^5*e^8 + 2*a^3*c^3*d*e^12 + 20*a^2*c^4*

```
d^3*e^10 - 4*b^2*c^4*d^5*e^8 + 6*b^3*c^3*d^4*e^9 - 2*b^4*c^2*d^3*e^10 - 28*
a*b*c^4*d^4*e^9 + 6*a*b^2*c^3*d^3*e^10 + 2*a*b^3*c^2*d^2*e^11 - 12*a^2*b*c^
3*d^2*e^11) + (6*b^3*c^3*d^(7/2)*e^9*(d + e*x^2)^(1/2))/(18*a*c^5*d^5*e^8 +
 2*a^3*c^3*d*e^12 + 20*a^2*c^4*d^3*e^10 - 4*b^2*c^4*d^5*e^8 + 6*b^3*c^3*d^4
*e^9 - 2*b^4*c^2*d^3*e^10 - 28*a*b*c^4*d^4*e^9 + 6*a*b^2*c^3*d^3*e^10 + 2*a
*b^3*c^2*d^2*e^11 - 12*a^2*b*c^3*d^2*e^11) - (4*b^2*c^4*d^(9/2)*e^8*(d + e*
x^2)^(1/2))/(18*a*c^5*d^5*e^8 + 2*a^3*c^3*d*e^12 + 20*a^2*c^4*d^3*e^10 - 4*
b^2*c^4*d^5*e^8 + 6*b^3*c^3*d^4*e^9 - 2*b^4*c^2*d^3*e^10 - 28*a*b*c^4*d^4*e
^9 + 6*a*b^2*c^3*d^3*e^10 + 2*a*b^3*c^2*d^2*e^11 - 12*a^2*b*c^3*d^2*e^11) +
 (2*a*c^3*d^(1/2)*e^12*(d + e*x^2)^(1/2))/(20*c^4*d^3*e^10 - 12*b*c^3*d^2*e
^11 + (18*c^5*d^5*e^8)/a + 2*a*c^3*d*e^12 + (6*b^2*c^3*d^3*e^10)/a + (2*b^3
*c^2*d^2*e^11)/a - (4*b^2*c^4*d^5*e^8)/a^2 + (6*b^3*c^3*d^4*e^9)/a^2 - (2*b
^4*c^2*d^3*e^10)/a^2 - (28*b*c^4*d^4*e^9)/a) - (12*b*c^3*d^(3/2)*e^11*(d +
e*x^2)^(1/2))/(20*c^4*d^3*e^10 - 12*b*c^3*d^2*e^11 + (18*c^5*d^5*e^8)/a + 2
*a*c^3*d*e^12 + (6*b^2*c^3*d^3*e^10)/a + (2*b^3*c^2*d^2*e^11)/a - (4*b^2*c^
4*d^5*e^8)/a^2 + (6*b^3*c^3*d^4*e^9)/a^2 - (2*b^4*c^2*d^3*e^10)/a^2 - (28*b
*c^4*d^4*e^9)/a)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x*(a + b*x**2 + c*x**4)), x)
```

3.359 $\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=382

$$-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}\,(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-b\left(d\sqrt{b^2-4ac}+ae\right)-a\left(2\right.\right.}{\sqrt{2}\,a^2\sqrt{b^2-}}$$

```
[Out] 1/2*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/a/d^(1/2)+(-a*e+b*d)*arctanh((e*x^2+
d)^(1/2)/d^(1/2))/a^2/d^(1/2)-1/2*(e*x^2+d)^(1/2)/a/x^2-1/2*arctanh(2^(1/2)
*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b
^2*d-2*a*c*d-a*b*e+(-a*e+b*d)*(-4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(-4*a*c+b^2)^
(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e
*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b^2*d-2*a*c*
d-a*b*e-(-a*e+b*d)*(-4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 4.13, antiderivative size = 370, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 897, 1287, 199, 206, 1166, 208}

$$-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}\,(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}\,(bd-ae)-abe-\right.}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\,\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(2*a*x^2) + (e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a*Sqrt
[d]) + ((b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^2*Sqrt[d]) - (Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqr
t[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqr
t[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{de^2}{a\left(d-x^2\right)^2}-\frac{e(-bd+ae)}{a^2\left(d-x^2\right)}+\frac{e\left(-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a^2} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{\left(d-x^2\right)^2}\,dx, x, \ldots\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{2ax^2} + \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}} + \frac{e\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\right)}{2a} - \frac{\left(c\left(\ldots\right.\right.}{\ldots}$$

$$= -\frac{\sqrt{d+ex^2}}{2ax^2} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}} + \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{\sqrt{c}\left(b^2d-2acd-\ldots\right.}{\sqrt{2}\ldots}$$

Mathematica [A] time = 1.39, size = 349, normalized size = 0.91

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(\frac{\left(-bd\sqrt{b^2-4ac}+ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\ldots\right)}{\sqrt{b^2-4ac}}\ldots}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(((-(b^2*d) + 2*a*c*d - b*Sq
rt[b^2 - 4*a*c]*d + a*b*e + a*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e] - ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d +
a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]))/Sqrt[b^2 - 4*a*c] + ((-2*b*d + a*e)*Log[x])/Sqrt[d] + ((2*b*d - a*e)
*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/Sqrt[d])/(2*a^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

maple [C] time = 0.03, size = 401, normalized size = 1.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a^2*b*d^(1/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)-1/2/a^2*b*(e*x^2+d)^(
1/2)-1/2/a^2*e^(1/2)*x*b+1/4/a^2*sum((c*(-a*e+b*d)*_R^6+(-4*a*b*e^2-a*c*d*e
+4*b^2*d*e-3*b*c*d^2)*_R^4+d*(4*a*b*e^2+a*c*d*e-4*b^2*d*e+3*b*c*d^2)*_R^2+a
*c*d^3*e-b*c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3
*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootO
f(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*
e-4*c*d^3)*_Z^2))+1/2/a^2*b*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))-1/2/a/d/x^2*(e*x
^2+d)^(3/2)-1/2/a*e/d^(1/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)+1/2/a*e/d
*(e*x^2+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3), x)
```

mupad [B] time = 5.46, size = 19959, normalized size = 52.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (atan((((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e
^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*
a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) - (
((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*
e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9
 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^
10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a
^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b
*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 - ((a*e - 2*b*d)*(((d + e*x^2)^(
1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^
5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*
c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b
```

^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*
b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4) -
((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^
11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9
+ 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 12
8*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 - ((d + e*x^2)^(1/2)*(a*e
- 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 +
1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 -
1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a
^6*d^(1/2))))/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2)))*(a*e - 2*b*d))/(4*a^2*d^(1
/2)))*1i)/(4*a^2*d^(1/2)) + ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e
^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b
^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*
d^3*e^9))/(2*a^4) + (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*
e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 2
0*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 +
84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b
^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*
c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 + ((a*e - 2
*b*d)*(((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*
b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*
d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^
3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*
c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c
^4*d*e^10))/(2*a^4) + ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^1
1 - 64*a^7*b^2*c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24
*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^
5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 + ((d
+ e*x^2)^(1/2)*(a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 51
2*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a
^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7
*b^3*c^3*d*e^9))/(8*a^6*d^(1/2))))/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2)))*(a*e
- 2*b*d))/(4*a^2*d^(1/2)))*1i)/(4*a^2*d^(1/2))))/(((a^3*c^5*e^13)/2 + a*c^7*
d^4*e^9 - 2*b*c^7*d^5*e^8 + (3*a^2*c^6*d^2*e^11)/2 + 2*b^2*c^6*d^4*e^9 - 4*
a*b*c^6*d^3*e^10 - (3*a^2*b*c^5*d*e^12)/2 + a*b^2*c^5*d^2*e^11)/a^4 - ((a*e
- 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c
^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*
e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) - (((16*a^5*b*c
^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^
4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^
5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5
*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d
*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^1
0 - 68*a^4*b^2*c^4*d*e^11)/a^4 - ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(240*a^
6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^
11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9
+ 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e
^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^
10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4) - ((a*e - 2*b
d)((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^11 + 128*a^7
*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2
*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*
d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 - ((d + e*x^2)^(1/2)*(a*e - 2*b*d)*(10
24*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5
*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c
^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6*d^(1/2)))
)/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2)))*(a*e - 2*b*d))/(4*a^2*d^(1/2))))/(4*a^
2*d^(1/2)) + ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7
*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10

```
 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^
4) + (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^
3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d
^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*
d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10
 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36
*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 + ((a*e - 2*b*d)*(((d + e*
x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 -
140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^
2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432
*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 34
8*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*
a^4) + ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*
c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^
2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^1
0 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 + ((d + e*x^2)^(1/2)
*(a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e
^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*
e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9)
)/(8*a^6*d^(1/2))))/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2)))*(a*e - 2*b*d))/(4*a^
2*d^(1/2))))/(4*a^2*d^(1/2))))*(a*e - 2*b*d)*1i)/(2*a^2*d^(1/2)) - atan((((
(64*a^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^
3*e^12 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*
e^9 - 108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d
^2*e^10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^1
0 - 12*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11
 - 144*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4
*e^11 + 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 +
128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9
 - 1024*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 3
84*a^6*b^3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d
 - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d
 + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*
c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*
b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2
*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e
^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9
 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c
 - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b
^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2
 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c
^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^
8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e
^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^
2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*
e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4
*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c
 - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6
*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)
^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^
(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^
5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 +
 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*
b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a
^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^
2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a
```

^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((64*a^5*b*c
^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80
*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a
^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 3
2*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*
b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b
*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*
a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3
*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*
b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*
c^3*d*e^10)/(4*a^4) + ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(
4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*
(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*
c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^
6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512
*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^
7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*
b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^
(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2
) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^
2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 +
20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*
b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^
3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*
a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a
^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)
^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^
(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^
5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) +
a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a
*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^
(1/2) + ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*
d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^1
1 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d -
b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b
^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e
- a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8
*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((a^3*c^5*e^13 + 2*a*c^7*
d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4*e^9 - 8*a*b*
c^6*d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11)/(2*a^4) + (((64*a
^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^1
2 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 -
108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^
10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 1
2*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144
*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11
+ 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a
^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 102
4*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^
6*b^3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3
d(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*
b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(
-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 +
16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10
- 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 -

896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 96
0*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^
2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3
)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*
a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*
e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 3
2*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 -
208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9
- 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 -
648*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c
- b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^
2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2
- 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(
1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2)
+ 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2
*c)))^(1/2) - ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^
3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5
*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^
3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d +
8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*
c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((64*a^5*b*c^4*e^12
+ 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6
*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c
^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*
c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*
d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2
*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*
c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3
*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*
e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^
10)/(4*a^4) + ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c -
b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)
^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 -
8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2
*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^
4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*
d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) +
a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2)
- 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*
b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(
1/2) + ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*
b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*
d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^
3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*
c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c
^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2
) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) +
2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)
))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e
- 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^
2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) +
((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10

+ 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i - (d + e*x^2)^(1/2)/(2*a*x^2) - atan(((((64*a^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((64*a^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*

d*e^10)/(4*a^4) + ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*
c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c -
b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^
2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8
*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^
2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*
c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2
) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) -
2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)
))^(1/2) + ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*
a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*
c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^
5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*
b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b
^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^
(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2
) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^
2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b
^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) -
7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c
d(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2
) + ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*
e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 -
8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*
d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c
*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^
2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((a^3*c^5*e^13 + 2*a*c^7*d^4*
e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4*e^9 - 8*a*b*c^6*
d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11)/(2*a^4) + (((64*a^5*b
*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 +
80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108
*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 -
32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^
2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4
*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 3
2*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b
^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^
7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^
3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(
-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*
a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*
a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 5
12*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*
a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^
7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3
)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1
/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*
b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10
 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^
2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*
a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 4
8*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648
*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^

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2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3
)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*
a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2)
 + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1
/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2
*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))
)^(1/2) - ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^
6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e
^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d
- b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a
*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*
e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((64*a^5*b*c^4*e^12 + 80
*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6*d^3
*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d
^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*c^4*
d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^
11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^1
0 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*c^2*
e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8
 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^10
 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^10)/
(4*a^4) + ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)
^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^
(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^
5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3
*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^
2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^
9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^
5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7
*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*
d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
 + ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*
c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*
e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^
2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*
d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d
*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) +
a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2)
 - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*
b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(
1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 1
8*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^
3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d
+ e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4
*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*
c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*
d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-
(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 +
16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c
 - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b
^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2
 - 8*a^5*b^2*c)))^(1/2)*2i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{x^3 \left(a + bx^2 + cx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**3*(a + b*x**2 + c*x**4)), x)
```

3.360 $\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=552

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}\right.\right.}{\sqrt{2}\,a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2\right.}}}$$

```
[Out] -3/8*e^2*arctanh((e*x^2+d)^(1/2)/d^(1/2))/a/d^(3/2)-1/2*e*(-a*e+b*d)*arctan
h((e*x^2+d)^(1/2)/d^(1/2))/a^2/d^(3/2)-(-a*b*e-a*c*d+b^2*d)*arctanh((e*x^2+
d)^(1/2)/d^(1/2))/a^3/d^(1/2)-1/4*(e*x^2+d)^(1/2)/a/x^4+3/8*e*(e*x^2+d)^(1/
2)/a/d/x^2+1/2*(-a*e+b*d)*(e*x^2+d)^(1/2)/a^2/d/x^2+1/2*arctanh(2^(1/2)*c^(
1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b^3*d
-a*c*(-2*a*e+d*(-4*a*c+b^2)^(1/2))+b^2*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*b*(3*c
*d+e*(-4*a*c+b^2)^(1/2)))/a^3*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2)))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b^3*d-b^2*(a*e+d*(-4*a*c+b^2)^(1/2))
+a*c*(2*a*e+d*(-4*a*c+b^2)^(1/2))-a*b*(3*c*d-e*(-4*a*c+b^2)^(1/2)))/a^3*2^(
1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 4.24, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 897, 1287, 199, 206, 1166, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}\right.\right.}{\sqrt{2}\,a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(4*a*x^4) + (3*e*Sqrt[d + e*x^2])/(8*a*d*x^2) + ((b*d - a*
e)*Sqrt[d + e*x^2])/(2*a^2*d*x^2) - (3*e^2*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]
)/(8*a*d^(3/2)) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2*d
^(3/2)) - ((b^2*d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^3*S
qrt[d]) + (Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b
^2 - 4*a*c]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*
(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e
) - a*b*(3*c*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a
*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\mathrm{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\mathrm{Subst}\left(\int\left(-\frac{de^3}{a\left(d-x^2\right)^3}+\frac{e^2(-bd+ae)}{a^2\left(d-x^2\right)^2}+\frac{e\left(-b^2d+acd+abe\right)}{a^3\left(d-x^2\right)}+\frac{e\left(\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)\right.}{a^3\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right.\right.}{e}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a^3} - \frac{\left(de^2\right)\mathrm{Subst}\left(\int \frac{1}{\left(d-x^2\right.}\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{\left(b^2d-acd-abe\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^3\sqrt{d}}-\frac{\left(3e^2\right)\mathrm{Sul}}{}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{3e\sqrt{d+ex^2}}{8adx^2}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}}-\ldots$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{3e\sqrt{d+ex^2}}{8adx^2}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}}-\frac{e(bd-a}{}$$

Mathematica [A] time = 1.97, size = 466, normalized size = 0.84

$$\frac{\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)\left(4abde+a\left(ae^2+8cd^2\right)-8b^2d^2\right)}{d^{3/2}}-\frac{\log(x)\left(4abde+a\left(ae^2+8cd^2\right)-8b^2d^2\right)}{d^{3/2}}-\frac{4\sqrt{2}\sqrt{c}\left(\frac{\left(b^2\left(ae-d\sqrt{b^2-4ac}\right)+ab\left(e\sqrt{b^2-4ac}+3cd\right)+ac\left(d\sqrt{b^2\ldots\right.\right.}{\sqrt{e\left(\sqrt{b^2-4\ldots\right.}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((a*Sqrt[d + e*x^2]*(4*b*d*x^2 - a*(2*d + e*x^2)))/(d*x^4) - (4*Sqrt[2]*Sqr
t[c]*(((-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 - 4*
a*c]*d) + a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e] + ((b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*
c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTa
nh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/Sqrt[b^2 - 4*a*c] - ((-8*b^2*
d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*Log[x])/d^(3/2) + ((-8*b^2*d^2 + 4*a
*b*d*e + a*(8*c*d^2 + a*e^2))*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/d^(3/2))/(8
*a^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.90, size = 1055, normalized size = 1.91

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\left(\left(\left(b^4-5\,ab^2c+4\,a^2c^2\right)de-\left(ab^3-4\,a^2bc\right)e^2\right)a^2-2\left(\left(ab^2c-a^2c^2\right)\sqrt{b^2-4\,ac}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c +
4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*a^2 - 2*((a*b^2*c - a^2*c^2)*sqrt
(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^
3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
)*abs(a) - sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(2*(a^2*b^3*c -
 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a^3*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^
4*b*c)*e^2))*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e
+ sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2
))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e +
sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)) + 1/8*(sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*
b*c)*e^2)*a^2 + 2*((a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2
*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt
(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - sqrt(-4*c^2*d + 2*(b*
c + sqrt(b^2 - 4*a*c)*c)*e)*(2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a
^3*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^4*b*c)*e^2))*arctan(2*sqrt(1/2)*
sqrt(x^2*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e
 + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)
*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a
)*abs(c)) + 1/8*(8*b^2*d^2 - 8*a*c*d^2 - 4*a*b*d*e - a^2*e^2)*arctan(sqrt(x
^2*e + d)/sqrt(-d))/(a^3*sqrt(-d)*d) + 1/8*(4*(x^2*e + d)^(3/2)*b*d*e - 4*s
qrt(x^2*e + d)*b*d^2*e - (x^2*e + d)^(3/2)*a*e^2 - sqrt(x^2*e + d)*a*d*e^2)
*e^(-2)/(a^2*d*x^4)
```

maple [C] time = 0.04, size = 655, normalized size = 1.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a^2*d^(1/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)*c-1/a^3*d^(1/2)*ln((2*d
+2*(e*x^2+d)^(1/2)*d^(1/2))/x)*b^2-1/2/a^2*(e*x^2+d)^(1/2)*c+1/2/a^3*(e*x^2
+d)^(1/2)*b^2-1/2/a^2*e^(1/2)*x*c+1/2/a^3*e^(1/2)*x*b^2+1/4/a^3*sum((c*(a*b
*e+a*c*d-b^2*d)*_R^6+(-4*a^2*c*e^2+4*a*b^2*e^2+5*a*b*c*d*e-3*a*c^2*d^2-4*b^
3*d*e+3*b^2*c*d^2)*_R^4+d*(4*a^2*c*e^2-4*a*b^2*e^2-5*a*b*c*d*e+3*a*c^2*d^2+
4*b^3*d*e-3*b^2*c*d^2)*_R^2-a*b*c*d^3*e-a*c^2*d^4+b^2*c*d^4)/(_R^7*c+3*_R^5
```

```
*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)
*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^
4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))+1/2/a^2*d/(-e^
(1/2)*x+(e*x^2+d)^(1/2))*c-1/2/a^3*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))*b^2-1/4/a
/d/x^4*(e*x^2+d)^(3/2)+1/8/a*e/d^2/x^2*(e*x^2+d)^(3/2)+1/8/a*e^2/d^(3/2)*ln
((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)-1/8/a*e^2/d^2*(e*x^2+d)^(1/2)+1/2/a^2*b
/d/x^2*(e*x^2+d)^(3/2)+1/2/a^2*b*e/d^(1/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2
))/x)-1/2/a^2*b*e/d*(e*x^2+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{\left(cx^4+bx^2+a\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5), x)
```

mupad [B] time = 7.30, size = 33925, normalized size = 61.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^5*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d
^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6
*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272
*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e
^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c
^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^
2) - ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1
/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^
3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3
)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(
1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^
5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10
*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9
+ 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^
4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^
(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e +
a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)
^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)
^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^
2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28
*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*
b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352
*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8
 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*
d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7
*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10
- 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^
9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^
12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) -
 a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2
*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/
```

2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14
- 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*
c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*
d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^
6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*
a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 +
10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^
2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^
4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 93
44*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2
*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*
e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*
e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*
(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d -
25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^
4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d
*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*
b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))
^(1/2) - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^
9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8
+ 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e
^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*
d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3
*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*
b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^
8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*(
(b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^
4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +
20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4
*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4
+ 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((2048*a^12*c^4*d*e^12 + 12288*
a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 153
6*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^
8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*
d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*
a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1
024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) + ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*
c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^
3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*
b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e
+ 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^
(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 -
8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 20
48*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^
4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12
*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^
4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*
a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*
a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3
*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3
)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2
- 8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b
*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8
+ 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^
9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d
^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*

b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) - ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(

1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2
) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1
/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*
b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c
^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*
b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11
264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e
^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*
c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 614
4*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 2
28*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/
(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^
7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-
(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) +
9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a
^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a
^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d
^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e
^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5
*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b
^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 1075
2*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^1
2 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5
*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^
6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12
 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13
- 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11
+ 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*
a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^
3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(
-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4
*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c
e(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2
) - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6
*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704
*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 -
 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e
^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*
d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6
*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5
*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*
d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2
*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/
2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a
^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c
- b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*
a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6
*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^
5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168
*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11
 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c
^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11
*b^2*c^3*d*e^12)/(64*a^8*d^2) + ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d -
b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^
3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d
+ a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b
^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) -

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3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^
2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*
b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 +
15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3
*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d
- b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*
c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*
d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a
*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*
b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^1
3 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*
a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608
*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10
+ 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*
d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*
b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 -
18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e
^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*
e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d
*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d -
 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b
^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*
d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2
*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c))
)^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*
a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*
d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10
*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*
a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8
+ 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d
^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3
*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 83
6*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*
e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^
5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d
*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b
^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*
c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^
(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 2
0*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a
*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 +
16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 -
2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2
*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*
e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7
*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2
*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5
*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7
*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c
^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^
(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e +
a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)
^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)
^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (7*a^5*c^7
*d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 +
64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 1
12*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192
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*a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4 *b*c^7*d^2*e^13)/(32*a^8*d^2)))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^ 2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^ 2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b ^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4* a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i + a tan((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^ 3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6* c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272* a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^ 10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^ 2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) - ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/ 2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3 *c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3) ^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1 /2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5 *d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10* b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4 *d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a ^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^ 3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^ (1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b ^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28* a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b ^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352* a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d ^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7* b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^1 2))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2* e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^ (1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c ^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d ^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6 *c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a ^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2 *e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4 *c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 934 4*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2* e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e ^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e ^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 2 5*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4 *e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d* (-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b

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^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^
(1/2) - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9
*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 +
 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^
10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d
^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*
c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b
*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8
*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((
b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4
*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +
20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*
a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 +
 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((2048*a^12*c^4*d*e^12 + 12288*a
^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536
*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8
 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d
^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a
^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 10
24*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) + ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c
^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3
*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b
^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e
- 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8
*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 204
8*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4
*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*
b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4
*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a
^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a
*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*
e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)
^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 -
 8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*
c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8
+ 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9
 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^
3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b
^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 71
68*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*
e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*
c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c
^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d
+ b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*
c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*
d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a
*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*
b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^1
4 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a
^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a
^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9
- 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d
^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^
7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*
a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^
11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c
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^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/(((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) - ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^

6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) + ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e

^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (7*a^5*c^7*d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 + 64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 112*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4*b*c^7*d^2*e^13)/(32*a^8*d^2)))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i - (((d + e*x^2)^(1/2)*(a*e^2 + 4*b*d*e))/(8*a^2) + ((d + e*x^2)^(3/2)*(a*e^2 - 4*b*d*e))/(8*a^2*d))/((d + e*x^2)^2 - 2*d*(d + e*x^2) + d^2) + (atan(-(((((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2) - ((((a^9*c^5*e^14)/4 - (a^6*b^6*c^2*e^14)/16 + (7*a^7*b^4*c^3*e^14)/16 - (13*a^8*b^2*c^4*e^14)/16 - 12*a^6*c^8*d^6*e^8 - 12*a^7*c^7*d^4*e^10 + (a^8*c^6*d^2*e^12)/

4 - 8*a^2*b^8*c^4*d^6*e^8 + 6*a^2*b^9*c^3*d^5*e^9 + 2*a^2*b^10*c^2*d^4*e^10 + 60*a^3*b^6*c^5*d^6*e^8 - 32*a^3*b^7*c^4*d^5*e^9 - (69*a^3*b^8*c^3*d^4*e^10)/2 - (7*a^3*b^9*c^2*d^3*e^11)/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c^4*d^4*e^10 + 42*a^4*b^7*c^3*d^3*e^11 + (3*a^4*b^8*c^2*d^2*e^12)/2 + 100*a^5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^10)/2 - (327*a^5*b^5*c^4*d^3*e^11)/2 - (209*a^5*b^6*c^3*d^2*e^12)/16 + 146*a^6*b^2*c^6*d^4*e^10 + 228*a^6*b^3*c^5*d^3*e^11 + (559*a^6*b^4*c^4*d^2*e^12)/16 - (429*a^7*b^2*c^5*d^2*e^12)/16 - (33*a^8*b*c^5*d*e^13)/4 + (a^5*b^7*c^2*d*e^13)/16 - 68*a^6*b*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d*e^13)/16 - 88*a^7*b*c^6*d^3*e^11 + (109*a^7*b^3*c^4*d*e^13)/16)/(a^8*d^2) + (((((32*a^12*c^4*d*e^12 + 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^10 + 32*a^8*b^4*c^4*d^5*e^8 - 24*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^10 - 176*a^9*b^2*c^5*d^5*e^8 + 112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^10 + 6*a^9*b^5*c^2*d^2*e^11 - 320*a^10*b^2*c^4*d^3*e^10 - 48*a^10*b^3*c^3*d^2*e^11 - 64*a^10*b*c^5*d^4*e^9 + 2*a^10*b^4*c^2*d*e^12 + 96*a^11*b*c^4*d^2*e^11 - 16*a^11*b^2*c^3*d*e^12)/(a^8*d^2) - ((d + e*x^2)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(512*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(16*a^3*(d^3)^(1/2)) + ((((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2) + ((((a^9*c^5*e^14)/4 - (a^6*b^6*c^2*e^14)/16 + (7*a^7*b^4*c^3*e^14)/16 - (13*a^8*b^2*c^4*e^14)/16 - 12*a^6*c^8*d^6*e^8 - 12*a^7*c^7*d^4*e^10 + (a^8*c^6*d^2*e^12)/4 - 8*a^2*b^8*c^4*d^6*e^8 + 6*a^2*b^9*c^3*d^5*e^9 + 2*a^2*b^10*c^2*d^4*e^10 + 60*a^3*b^6*c^5*d^6*e^8 - 32*a^3*b^7*c^4*d^5*e^9 - (69*a^3*b^8*c^3*d^4*e^10)/2 - (7*a^3*b^9*c^2*d^3*e^11)/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c^4*d^4*e^10 + 42*a^4*b^7*c^3*d^3*e^11 + (3*a^4*b^8*c^2*d^2*e^12)/2 + 100*a^5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^10)/2 - (327*a^5*b^5*c^4*d^3*e^11)/2 - (209*a^5*b^6*c^3*d^2*e^12)/16 + 146*a^6*b^2*c^6*d^4*e^10 + 228*a^6*b^3*c^5*d^3*e^11 + (559*a^6*b^4*c^4*d^2*e^12)/16 - (429*a^7*b^2*c^5*d^2*e^12)/16 - (33*a^8*b*c^5*d*e^13)/4 + (a^5*b^7*c^2*d*e^13)/16 - 68*a^6*b*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d*e^13)/16 - 88*a^7*b*c^6*d^3*e^11 + (109*a^7*b^3*c^4*d*e^13)/16)/(a^8*d^2) + (((((32*a^12*c^4*d*e^12 + 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^10 + 32*a^8*b^4*c^4*d^5*e^8 - 24*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^10 - 176*a^9*b^2*c^5*d^5*e^8 + 112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^10 + 6*a^9*b^5*c^2*d^2*e^11 - 320*a^10*b^2*c^4*d^3*e^10 - 48*a^10*b^3*c^3*d^2*e^11 - 64*a^10*b*c^5*d^4*e^9 + 2*a^10*b^4*c^2*d*e^12 + 96*a^11*b*c^4*d^2*e^11 - 16*a^11*b^2*c^3*d*e^12)/(a^8*d^2) + ((d + e*

x^2)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(512*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(16*a^3*(d^3)^(1/2)))/(((7*a^5*c^7*d*e^14)/32 + (7*a^3*c^9*d^5*e^10)/4 + (63*a^4*c^8*d^3*e^12)/32 - 2*b^4*c^8*d^7*e^8 + 2*b^5*c^7*d^6*e^9 + 2*a^2*b^2*c^8*d^5*e^10 + 7*a^2*b^3*c^7*d^4*e^11 - (7*a^3*b^2*c^7*d^3*e^12)/2 + 2*a*b^2*c^9*d^7*e^8 + 2*a*b^3*c^8*d^6*e^9 - 6*a*b^4*c^7*d^5*e^10 - 3*a^2*b*c^9*d^6*e^9 - (17*a^3*b*c^8*d^4*e^11)/4 + (9*a^4*b*c^7*d^2*e^13)/32)/(a^8*d^2) - ((((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2) - ((((a^9*c^5*e^14)/4 - (a^6*b^6*c^2*e^14)/16 + (7*a^7*b^4*c^3*e^14)/16 - (13*a^8*b^2*c^4*e^14)/16 - 12*a^6*c^8*d^6*e^8 - 12*a^7*c^7*d^4*e^10 + (a^8*c^6*d^2*e^12)/4 - 8*a^2*b^8*c^4*d^6*e^8 + 6*a^2*b^9*c^3*d^5*e^9 + 2*a^2*b^10*c^2*d^4*e^10 + 60*a^3*b^6*c^5*d^6*e^8 - 32*a^3*b^7*c^4*d^5*e^9 - (69*a^3*b^8*c^3*d^4*e^10)/2 - (7*a^3*b^9*c^2*d^3*e^11)/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c^4*d^4*e^10 + 42*a^4*b^7*c^3*d^3*e^11 + (3*a^4*b^8*c^2*d^2*e^12)/2 + 100*a^5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^10)/2 - (327*a^5*b^5*c^4*d^3*e^11)/2 - (209*a^5*b^6*c^3*d^2*e^12)/16 + 146*a^6*b^2*c^6*d^4*e^10 + 228*a^6*b^3*c^5*d^3*e^11 + (559*a^6*b^4*c^4*d^2*e^12)/16 - (429*a^7*b^2*c^5*d^2*e^12)/16 - (33*a^8*b*c^5*d*e^13)/4 + (a^5*b^7*c^2*d*e^13)/16 - 68*a^6*b*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d*e^13)/16 - 88*a^7*b*c^6*d^3*e^11 + (109*a^7*b^3*c^4*d*e^13)/16)/(a^8*d^2) + (((((32*a^12*c^4*d*e^12 + 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^10 + 32*a^8*b^4*c^4*d^5*e^8 - 24*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^10 - 176*a^9*b^2*c^5*d^5*e^8 + 112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^10 + 6*a^9*b^5*c^2*d^2*e^11 - 320*a^10*b^2*c^4*d^3*e^10 - 48*a^10*b^3*c^3*d^2*e^11 - 64*a^10*b*c^5*d^4*e^9 + 2*a^10*b^4*c^2*d*e^12 + 96*a^11*b*c^4*d^2*e^11 - 16*a^11*b^2*c^3*d*e^12)/(a^8*d^2) - ((d + e*x^2)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(512*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5

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*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5
*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32
*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 460
8*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*(a^2*e^2 - 8*
b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^
2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*
a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) + ((((d + e*x^2)^(1/2)*(a^6*b^2*
c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*
a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b
^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*
a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 +
704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^
12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8
- 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 +
6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2) + ((((a^9*c^5*e^14)/4 - (a^6*b^6*c^2*e^
14)/16 + (7*a^7*b^4*c^3*e^14)/16 - (13*a^8*b^2*c^4*e^14)/16 - 12*a^6*c^8*d^
6*e^8 - 12*a^7*c^7*d^4*e^10 + (a^8*c^6*d^2*e^12)/4 - 8*a^2*b^8*c^4*d^6*e^8
+ 6*a^2*b^9*c^3*d^5*e^9 + 2*a^2*b^10*c^2*d^4*e^10 + 60*a^3*b^6*c^5*d^6*e^8
- 32*a^3*b^7*c^4*d^5*e^9 - (69*a^3*b^8*c^3*d^4*e^10)/2 - (7*a^3*b^9*c^2*d^3
*e^11)/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c
^4*d^4*e^10 + 42*a^4*b^7*c^3*d^3*e^11 + (3*a^4*b^8*c^2*d^2*e^12)/2 + 100*a^
5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^10)/2 -
 (327*a^5*b^5*c^4*d^3*e^11)/2 - (209*a^5*b^6*c^3*d^2*e^12)/16 + 146*a^6*b^2
*c^6*d^4*e^10 + 228*a^6*b^3*c^5*d^3*e^11 + (559*a^6*b^4*c^4*d^2*e^12)/16 -
(429*a^7*b^2*c^5*d^2*e^12)/16 - (33*a^8*b*c^5*d*e^13)/4 + (a^5*b^7*c^2*d*e^
13)/16 - 68*a^6*b*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d*e^13)/16 - 88*a^7*b*c^6*d
^3*e^11 + (109*a^7*b^3*c^4*d*e^13)/16)/(a^8*d^2) + (((((32*a^12*c^4*d*e^12
+ 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^10 + 32*a^8*b^4*c^4*d^5*e^8 - 2
4*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^10 - 176*a^9*b^2*c^5*d^5*e^8 +
112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^10 + 6*a^9*b^5*c^2*d^2*e^11
- 320*a^10*b^2*c^4*d^3*e^10 - 48*a^10*b^3*c^3*d^2*e^11 - 64*a^10*b*c^5*d^4*
e^9 + 2*a^10*b^4*c^2*d*e^12 + 96*a^11*b*c^4*d^2*e^11 - 16*a^11*b^2*c^3*d*e^
12)/(a^8*d^2) + ((d + e*x^2)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b
*d*e)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3
*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a
^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^
10 - 28672*a^12*b*c^4*d^3*e^9))/(512*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^
2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) - ((d + e*x^2)^(1/2)*(
32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c
^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^
5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c
^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a
^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 -
 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d
^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8
*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^
8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a
^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)
))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^
2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2))))*(a^2*e^2
 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(8*a^3*(d^3)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.361 $\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=390

$$-\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)\right.}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2c}}$$

```
[Out] 1/2*(-2*b*e+c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^2/e^(1/2)+1/2*x*(e*x^
2+d)^(1/2)/c-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(-3*a*b*c*e+2*a*c^2*d+b^
3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(3*a*b*
c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))
^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 2.92, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1291, 388, 217, 206, 1692, 377, 205}

$$-\frac{\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)\right.}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) - ((b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d
- b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b*c*d -
b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4
*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]) + ((c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(2*c^2*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1291

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[((f*x)^(m - 4)*(d +
 e*x^2)^(q - 1)*Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x])/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\int \frac{cd-be+cex^2}{\sqrt{d+ex^2}}\,dx}{c^2} - \frac{\int \frac{a(cd-be)+\left(bcd-b^2e+ace\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} - \frac{\int \left(\frac{bcd-b^2e+ace+\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{bcd-b^2e+ace-\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c^2} + \frac{(cd - \cdots}{}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(cd-2be)\operatorname{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3\cdots}{\sqrt{b^2-4a\cdots}}\right.}{}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{Subs}\cdots}{}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(bcd \cdots\right.}{}$$

Mathematica [B] time = 6.40, size = 10915, normalized size = 27.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 62.65, size = 6534, normalized size = 16.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(1/2)*c^2*e*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a
^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2
 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2
*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(((a*b^2*c^4 - 4*a
^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*
b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^
8 - 4*a*c^9)) + 2*(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e -
 ((a*b^3*c - a^2*b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^
2*b^3 - 2*a^3*b*c)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b
^2*c^5 + 8*a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*
c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)
/(b^2*c^8 - 4*a*c^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a
*b^4*c + 8*a^2*b^2*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2
*c + 2*a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*
c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4
*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(
1/2)*c^2*e*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e +
 (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c
 - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/
(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(((a*b^2*c^4 - 4*a^2*c^5)*d*x
^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2
*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9
)) + 2*(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e - ((a*b^3*c
- a^2*b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a
^3*b*c)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*
a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3
*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 -
 4*a*c^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*
a^2*b^2*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*
c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 -
2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^
2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) + sqrt(1/2)*c^2*e*
sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 -
 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*
c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 -
4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((
b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3
)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^
2*b^2*c - a^3*c^2)*d^2 + 2*(a^2*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c - a^2*b*c^
2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^
2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x
*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a
^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9))
 + ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^
2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (
b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c -
 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b
^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(1/2)*c^2*e*sqrt(-((b^
3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*
```

```
sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^
2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))
/(b^2*c^4 - 4*a*c^5))*log(-((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 -
2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b
^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^2*b^2*c -
a^3*c^2)*d^2 + 2*(a^2*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c - a^2*b*c^2)*d^2 - (
a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 - 2
*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*sqrt(((b^
4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*
d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + ((b^5*c
 - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*x)*s
qrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 -
4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c
^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4
*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) + 2*sqrt(e*x^2 + d)*c*e*x - (c*d - 2*b
*e)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/(c^2*e), 1/4*(
sqrt(1/2)*c^2*e*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2
)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(
b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*
e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(((a*b^2*c^4 - 4*a^2*c^5
)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^
2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*
a*c^9)) + 2*(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e - ((a*b
^3*c - a^2*b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3
 - 2*a^3*b*c)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5
 + 8*a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*
a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*
c^8 - 4*a*c^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c
 + 8*a^2*b^2*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2
*a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d
^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b
^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(1/2)*c
^2*e*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + (b^2*
c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a
*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c
^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqr
t(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b
*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + 2
*(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e - ((a*b^3*c - a^2*
b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c
)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^
6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c
^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^
2*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e
 + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5
*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2
)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) + sqrt(1/2)*c^2*e*sqrt(-
((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c
^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^
9)))/(b^2*c^4 - 4*a*c^5))*log(-((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^
2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e
+ (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^2*b^2*
c - a^3*c^2)*d^2 + 2*(a^2*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c - a^2*b*c^2)*d^2
 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2
 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*sqrt(
((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c
^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + ((b
```

```
^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*
x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^
4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b
^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8
 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(1/2)*c^2*e*sqrt(-((b^3*c -
3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*sqrt((
(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^
3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*
c^4 - 4*a*c^5))*log(-((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^
2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4
*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^2*b^2*c - a^3*c^
2)*d^2 + 2*(a^2*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c - a^2*b*c^2)*d^2 - (a*b^4
+ 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 - 2*sqrt(
1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*sqrt(((b^4*c^2
- 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e +
(b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + ((b^5*c - 5*a
*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*x)*sqrt(-(
(b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^
5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2
*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9
)))/(b^2*c^4 - 4*a*c^5)))/x^2) + 2*sqrt(e*x^2 + d)*c*e*x - 2*(c*d - 2*b*e)*
sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c^2*e)]
```

giac [A] time = 2.04, size = 53, normalized size = 0.14

$$-\frac{(cd - 2\,be)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2\right)}{4\,c^2} + \frac{\sqrt{x^2e + d}\,x}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(c*d - 2*b*e)*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^2 + 1/2*
sqrt(x^2*e + d)*x/c
```

maple [C] time = 0.04, size = 290, normalized size = 0.74

$$\frac{b\sqrt{e}\;\ln\left(-\sqrt{e}\;x+\sqrt{e\,x^2+d}\right)}{c^2}+\frac{d\ln\left(\sqrt{e}\;x+\sqrt{e\,x^2+d}\right)}{2c\sqrt{e}}+\frac{\sqrt{e\,x^2+d}\;x}{2c}+\frac{}{2c^2\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*x*(e*x^2+d)^(1/2)/c+1/2/c*d/e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))+1/2/c
^2*e^(1/2)*sum(((a*c*e-b^2*e+b*c*d)*_R^2+2*(-2*a*b*e^2+a*c*d*e+b^2*d*e-b*c*
d^2)*_R+e*c*d^2*a-b^2*d^2*e+b*c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e
^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln((-e^(1/2)*x+(e*x^2+d)^(1/2))^2-_
R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b
*d^2*e-4*c*d^3)*_Z+c*d^4))+1/c^2*e^(1/2)*b*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \sqrt{e x^2 + d}}{c x^4 + b x^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
[Out] int((x^4*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sqrt{d + ex^2}}{a + bx^2 + cx^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**4*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.362 $\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=324

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)/c+arctan(x*(2*c*d-e*(b-(-4*a*c+b
^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*d-b*e+(-
2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c*d-b*e+(2*a*c*e-b^
2*e+b*c*d)/(-4*a*c+b^2)^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(
-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.52, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1293, 217, 206, 1692, 377, 205}

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*
x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt
[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]) + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = -\frac{\int \frac{ae-(cd-be)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c} + \frac{e\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c}$$

$$= -\frac{\int \left(\frac{-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{-cd+be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+e}}\right.}{c}$$

$$= \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c} - \frac{\left(-cd+be\right.}{}$$

$$= \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\right.}{c}$$

$$= \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{b}}\right.}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\right.}}$$

Mathematica [B] time = 6.17, size = 7768, normalized size = 23.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
[Out] Result too large to show
```

fricas [B] time = 10.79, size = 3260, normalized size = 10.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] [-1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)
)*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2
*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*
a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((
c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c - 4*a*c^2)*d
- (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3
)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a
*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*
c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 -
4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e
^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (
b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*
x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c - 4*a
*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 -
 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c
^2 - 4*a*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^
2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^
2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e
+ b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*d*e + (b*c*d^2 + 4*a*b*
e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*
b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + ((b^2*
c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^
2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))
/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e -
(b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b
*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*d*e + (b*c*d^2 +
 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2
 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) +
 ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*
e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a
*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2
+ d)*sqrt(e)*x - d))/c, -1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e +
(b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*
b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2
+ 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^
2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5))
- ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)
*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*
a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a
*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 -
 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*
d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (
b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*
((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*
a*c^5)) - ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2
- 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*
c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c*d - (b
^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b
^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*
d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^
2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c
^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d
- (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2
)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c
```

```
*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*
e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*
x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 +
 2*a*b*d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sq
rt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)
/(b^2*c^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-
(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e +
b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) + 4*sqrt(-e)*arct
an(sqrt(-e)*x/sqrt(e*x^2 + d)))/c]
```

giac [A] time = 1.83, size = 27, normalized size = 0.08

$$-\frac{e^{\frac{1}{2}} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2 e + d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*e^(1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

maple [C] time = 0.03, size = 224, normalized size = 0.69

$$-\frac{\sqrt{e}\ \ln\left(-\sqrt{e}\ x+\sqrt{e\,x^2+d}\right)}{c}+\frac{}{2c\left(\mathrm{RootOf}\left(\_Z^4 c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*e^(1/2)/c*sum(((b*e-c*d)*_R^2+2*(2*a*e^2-b*d*e+c*d^2)*_R+b*d^2*e-c*d^3)
/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d
^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*
c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-e^(1/2)/
c*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,x^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^2*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.363 $\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4} dx$

Optimal. Leaf size=240

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(-4*a*c+b^2)^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1
/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2)))^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1174, 402, 217, 206, 377, 205}

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2
- 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}} + \frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\int \frac{}{(b-}}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}} + \frac{\left(-2cd\right.}{}$$

$$= \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\tan^{-1}}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b}}}$$

Mathematica [B] time = 5.12, size = 2585, normalized size = 10.77

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Sq
rt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[-(Sqrt[(-b + Sqrt[b^2 - 4*a*c])
/c]/Sqrt[2]) + x] - Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(2*c*d + (-b + Sqrt[
b^2 - 4*a*c])*e)*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[Sqrt[(-b + S
qrt[b^2 - 4*a*c])/c]/Sqrt[2] + x] - 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*
Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*
a*c])/c)]/Sqrt[2]) + x] + b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d
- b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sq
rt[2]) + x] + Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*
c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)
]/Sqrt[2]) + x] + 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e
+ Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] +
x] - b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] - Sqrt[b^2 -
 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4
```

```
*a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] + 2*c*Sqrt[
-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*L
og[2*d - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e
 + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - b*Sqrt[-((b + Sqrt[b^2 - 4*
a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqr
t[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*
c]*e)/c]*Sqrt[d + e*x^2]] + Sqrt[b^2 - 4*a*c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c]
)/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[(-
b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e
)/c]*Sqrt[d + e*x^2]] - 2*c*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d -
 ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a
*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^
2]] + b*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*
a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(
4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - Sqrt[b^2 - 4*a
*c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c]
)*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*
d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - 2*c*Sqrt[(-b + Sqr
t[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d -
Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^
2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] + b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*S
qrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d - Sqrt[2]*Sqrt[-((b + Sq
rt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt
[d + e*x^2]] + Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2
*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d - Sqrt[2]*Sqrt[-((b + Sqrt[b^2
 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e
*x^2]] + 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^
2 - 4*a*c]*e)/c]*Log[2*d + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x +
 Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - b*Sqrt[(-b
+ Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2
*d + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sq
rt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt
[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d + S
qrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2
 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b
^2 - 4*a*c])/c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*Sqrt[(2*c*d - b*e + Sqrt
[b^2 - 4*a*c]*e)/c]*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c])
```

fricas [B] time = 2.98, size = 985, normalized size = 4.10

$$\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{bd-2\,ae+\left(ab^2-4\,a^2c\right)\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{\left(ab^2-4\,a^2c\right)d\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}\,x^2+4\sqrt{\frac{1}{2}}\left(a^2b^2-4\,a^3c\right)\sqrt{ex^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*
a^3*c)))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 - 4
*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a^
2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^
2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x^2) -
 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4
*a^3*c)))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x^2)
```

```
+ 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 -
4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
4*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2)/x^2)
- 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 -
4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2)/x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,c]=[-72,-7,6]Evaluation time: 0.44Unable to divide, perhaps due
to rounding error%%%{18446744069414584320,[4,7,8,2,3,14,2]%%%}+%%%{-2147483
648,[3,8,10,8,3,10,1]%%%}+%%%{-12884901888,[3,8,10,7,2,12,2]%%%}+%%%{463856
467968,[3,8,10,6,1,14,3]%%%}+%%%{1924145348608,[3,8,10,5,0,16,4]%%%}+%%%{53
6870912,[3,8,9,8,5,10,0]%%%}+%%%{20401094656,[3,8,9,7,4,12,1]%%%}+%%%{-1503
23855360,[3,8,9,6,3,14,2]%%%}+%%%{-3135326126080,[3,8,9,5,2,16,3]%%%}+%%%{-
4672924418048,[3,8,9,4,1,18,4]%%%}+%%%{6047313952768,[3,8,9,3,0,20,5]%%%}+%
%%{-4294967296,[3,8,8,7,6,12,0]%%%}+%%%{-42412802048,[3,8,8,6,5,14,1]%%%}+%
%%{1046898278400,[3,8,8,5,4,16,2]%%%}+%%%{6210522710016,[3,8,8,4,3,18,3]%%%
}+%%%{-1786706395136,[3,8,8,3,2,20,4]%%%}+%%%{-11544872091648,[3,8,8,2,1,22
,5]%%%}+%%%{4398046511104,[3,8,8,1,0,24,6]%%%}+%%%{12750684160,[3,8,7,6,7,1
4,0]%%%}+%%%{-23890755584,[3,8,7,5,6,16,1]%%%}+%%%{-2103460233216,[3,8,7,4,
5,18,2]%%%}+%%%{-3324304687104,[3,8,7,3,4,20,3]%%%}+%%%{9758165696512,[3,8,
7,2,3,22,4]%%%}+%%%{1649267441664,[3,8,7,1,2,24,5]%%%}+%%%{-4398046511104,[
3,8,7,0,1,26,6]%%%}+%%%{-17985175552,[3,8,6,5,8,16,0]%%%}+%%%{161866579968,
[3,8,6,4,7,18,1]%%%}+%%%{1586990415872,[3,8,6,3,6,20,2]%%%}+%%%{-1795296329
728,[3,8,6,2,5,22,3]%%%}+%%%{-4123168604160,[3,8,6,1,4,24,4]%%%}+%%%{384829
0697216,[3,8,6,0,3,26,5]%%%}+%%%{12213813248,[3,8,5,4,9,18,0]%%%}+%%%{-1717
98691840,[3,8,5,3,8,20,1]%%%}+%%%{-212600881152,[3,8,5,2,7,22,2]%%%}+%%%{14
77468749824,[3,8,5,1,6,24,3]%%%}+%%%{-1099511627776,[3,8,5,0,5,26,4]%%%}+%%
%{-3221225472,[3,8,4,3,10,20,0]%%%}+%%%{57982058496,[3,8,4,2,9,22,1]%%%}+%%
%{-154618822656,[3,8,4,1,8,24,2]%%%}+%%%{103079215104,[3,8,4,0,7,26,3]%%%}+
%%%{1048576,[3,6,10,4,2,4,0]%%%}+%%%{8388608,[3,6,10,3,1,6,1]%%%}+%%%{16777
216,[3,6,10,2,0,8,2]%%%}+%%%{-5242880,[3,6,9,3,3,6,0]%%%}+%%%{-29360128,[3,
6,9,2,2,8,1]%%%}+%%%{-33554432,[3,6,9,1,1,10,2]%%%}+%%%{9699328,[3,6,8,2,4,
8,0]%%%}+%%%{33554432,[3,6,8,1,3,10,1]%%%}+%%%{16777216,[3,6,8,0,2,12,2]%%%
}+%%%{-7864320,[3,6,7,1,5,10,0]%%%}+%%%{-12582912,[3,6,7,0,4,12,1]%%%}+%%%{
2359296,[3,6,6,0,6,12,0]%%%}+%%%{536870912,[2,7,10,6,2,8,1]%%%}+%%%{6710886
400,[2,7,10,5,1,10,2]%%%}+%%%{18253611008,[2,7,10,4,0,12,3]%%%}+%%%{-134217
728,[2,7,9,6,4,8,0]%%%}+%%%{-5502926848,[2,7,9,5,3,10,1]%%%}+%%%{-369098752
00,[2,7,9,4,2,12,2]%%%}+%%%{-42949672960,[2,7,9,3,1,14,3]%%%}+%%%{429496729
60,[2,7,9,2,0,16,4]%%%}+%%%{956301312,[2,7,8,5,5,10,0]%%%}+%%%{18656264192,
[2,7,8,4,4,12,1]%%%}+%%%{64961380352,[2,7,8,3,3,14,2]%%%}+%%%{-8589934592,[
2,7,8,2,2,16,3]%%%}+%%%{-85899345920,[2,7,8,1,1,18,4]%%%}+%%%{-2642411520,[
2,7,7,4,6,12,0]%%%}+%%%{-27783069696,[2,7,7,3,5,14,1]%%%}+%%%{-33957085184,
[2,7,7,2,4,16,2]%%%}+%%%{73014444032,[2,7,7,1,3,18,3]%%%}+%%%{42949672960,[
2,7,7,0,2,20,4]%%%}+%%%{3556769792,[2,7,6,3,7,14,0]%%%}+%%%{17716740096,[2,
```

```
7,6,2,6,16,1]%%%}+%%%{-12884901888,[2,7,6,1,5,18,2]%%%}+%%%{-39728447488,[2
,7,6,0,4,20,3]%%%}+%%%{-2340421632,[2,7,5,2,8,16,0]%%%}+%%%{-2415919104,[2,
7,5,1,7,18,1]%%%}+%%%{12079595520,[2,7,5,0,6,20,2]%%%}+%%%{603979776,[2,7,4
,1,9,18,0]%%%}+%%%{-1207959552,[2,7,4,0,8,20,1]%%%}+%%%{2147483648,[1,8,10,
9,3,10,1]%%%}+%%%{38654705664,[1,8,10,8,2,12,2]%%%}+%%%{51539607552,[1,8,10
,7,1,14,3]%%%}+%%%{-274877906944,[1,8,10,6,0,16,4]%%%}+%%%{-536870912,[1,8,
9,9,5,10,0]%%%}+%%%{-26843545600,[1,8,9,8,4,12,1]%%%}+%%%{-188978561024,[1,
8,9,7,3,14,2]%%%}+%%%{146028888064,[1,8,9,6,2,16,3]%%%}+%%%{962072674304,[1
,8,9,5,1,18,4]%%%}+%%%{-549755813888,[1,8,9,4,0,20,5]%%%}+%%%{4294967296,[1
,8,8,8,6,12,0]%%%}+%%%{95026151424,[1,8,8,7,5,14,1]%%%}+%%%{239444426752,[1
,8,8,6,4,16,2]%%%}+%%%{-858993459200,[1,8,8,5,3,18,3]%%%}+%%%{-618475290624
,[1,8,8,4,2,20,4]%%%}+%%%{1099511627776,[1,8,8,3,1,22,5]%%%}+%%%{-127506841
60,[1,8,7,7,7,14,0]%%%}+%%%{-136633647104,[1,8,7,6,6,16,1]%%%}+%%%{62277025
792,[1,8,7,5,5,18,2]%%%}+%%%{936302870528,[1,8,7,4,4,20,3]%%%}+%%%{-5497558
13888,[1,8,7,3,3,22,4]%%%}+%%%{-549755813888,[1,8,7,2,2,24,5]%%%}+%%%{17985
175552,[1,8,6,6,8,16,0]%%%}+%%%{71940702208,[1,8,6,5,7,18,1]%%%}+%%%{-26736
1714176,[1,8,6,4,6,20,2]%%%}+%%%{-137438953472,[1,8,6,3,5,22,3]%%%}+%%%{481
036337152,[1,8,6,2,4,24,4]%%%}+%%%{-12213813248,[1,8,5,5,9,18,0]%%%}+%%%{72
47757312,[1,8,5,4,8,20,1]%%%}+%%%{103079215104,[1,8,5,3,7,22,2]%%%}+%%%{-13
7438953472,[1,8,5,2,6,24,3]%%%}+%%%{3221225472,[1,8,4,4,10,20,0]%%%}+%%%{-1
2884901888,[1,8,4,3,9,22,1]%%%}+%%%{12884901888,[1,8,4,2,8,24,2]%%%}+%%%{-1
048576,[1,6,10,5,2,4,0]%%%}+%%%{-8388608,[1,6,10,4,1,6,1]%%%}+%%%{-16777216
,[1,6,10,3,0,8,2]%%%}+%%%{8388608,[1,6,9,4,3,6,0]%%%}+%%%{62914560,[1,6,9,3
,2,8,1]%%%}+%%%{150994944,[1,6,9,2,1,10,2]%%%}+%%%{134217728,[1,6,9,1,0,12,
3]%%%}+%%%{-26476544,[1,6,8,3,4,8,0]%%%}+%%%{-163577856,[1,6,8,2,3,10,1]%%%
}+%%%{-301989888,[1,6,8,1,2,12,2]%%%}+%%%{-134217728,[1,6,8,0,1,14,3]%%%}+%
%%{41156608,[1,6,7,2,5,10,0]%%%}+%%%{178257920,[1,6,7,1,4,12,1]%%%}+%%%{167
772160,[1,6,7,0,3,14,2]%%%}+%%%{-31457280,[1,6,6,1,6,12,0]%%%}+%%%{-6920601
6,[1,6,6,0,5,14,1]%%%}+%%%{9437184,[1,6,5,0,7,14,0]%%%}+%%%{-402653184,[0,7
,10,7,2,8,1]%%%}+%%%{-5637144576,[0,7,10,6,1,10,2]%%%}+%%%{-16106127360,[0,
7,10,5,0,12,3]%%%}+%%%{100663296,[0,7,9,7,4,8,0]%%%}+%%%{4160749568,[0,7,9,
6,3,10,1]%%%}+%%%{30198988800,[0,7,9,5,2,12,2]%%%}+%%%{28991029248,[0,7,9,4
,1,14,3]%%%}+%%%{-68719476736,[0,7,9,3,0,16,4]%%%}+%%%{-687865856,[0,7,8,6,
5,10,0]%%%}+%%%{-13925089280,[0,7,8,5,4,12,1]%%%}+%%%{-48184164352,[0,7,8,4
,3,14,2]%%%}+%%%{49392123904,[0,7,8,3,2,16,3]%%%}+%%%{120259084288,[0,7,8,2
,1,18,4]%%%}+%%%{-68719476736,[0,7,8,1,0,20,5]%%%}+%%%{1845493760,[0,7,7,5,
6,12,0]%%%}+%%%{19964887040,[0,7,7,4,5,14,1]%%%}+%%%{11542724608,[0,7,7,3,4
,16,2]%%%}+%%%{-113816633344,[0,7,7,2,3,18,3]%%%}+%%%{8589934592,[0,7,7,1,2
,20,4]%%%}+%%%{68719476736,[0,7,7,0,1,22,5]%%%}+%%%{-2432696320,[0,7,6,4,7,
14,0]%%%}+%%%{-11207180288,[0,7,6,3,6,16,1]%%%}+%%%{28185722880,[0,7,6,2,5,
18,2]%%%}+%%%{34359738368,[0,7,6,1,4,20,3]%%%}+%%%{-60129542144,[0,7,6,0,3,
22,4]%%%}+%%%{1577058304,[0,7,5,3,8,16,0]%%%}+%%%{-201326592,[0,7,5,2,7,18,
1]%%%}+%%%{-14495514624,[0,7,5,1,6,20,2]%%%}+%%%{17179869184,[0,7,5,0,5,22,
3]%%%}+%%%{-402653184,[0,7,4,2,9,18,0]%%%}+%%%{1610612736,[0,7,4,1,8,20,1]%
%%}+%%%{-1610612736,[0,7,4,0,7,22,2]%%%} / %%%{-1024,[0,3,4,2,1,2,0]%%%}+%%
%{-4096,[0,3,4,1,0,4,1]%%%}+%%%{2560,[0,3,3,1,2,4,0]%%%}+%%%{4096,[0,3,3,0,
1,6,1]%%%}+%%%{-1536,[0,3,2,0,3,6,0]%%%} Error: Bad Argument Value
```

maple [C] time = 0.02, size = 161, normalized size = 0.67

$$-\frac{}{2\left(\mathrm{RootOf}\left(\_Z^4 c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^3 c+3\,\mathrm{RootC}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*e^(3/2)*sum((_R^2+2*_R*d+d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2
-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2
```

```
),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2
+(4*b*d^2*e-4*c*d^3)*_Z))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(a + b*x^2 + c*x^4),x)
[Out] int((d + e*x^2)^(1/2)/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.364 $\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=291

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{ax}$$

```
[Out] -(e*x^2+d)^(1/2)/a/x-c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x
^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2
))/a/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*
arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))/a/(b+(-4*a*c+b^2)^(1/
2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1295, 264, 1692, 377, 205}

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - (c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^
2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{bd-ae+cdx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a}+\frac{d\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax}-\frac{\int \left(\frac{cd+\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{cd-\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a}-\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{(b-}}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax}-\frac{c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{a\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{2c}}$$

Mathematica [B] time = 6.32, size = 4644, normalized size = 15.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (-1/2*(b*d*(Log[Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d -
Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt
[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*
c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2
- 4*a*c]/c]/Sqrt[2])) + (Sqrt[b^2 - 4*a*c]*d*(Log[Sqrt[-(b/c) - Sqrt[b^2 -
4*a*c]/c]/Sqrt[2] + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log
[2*d - Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c)
 - Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2
 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqr
t[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/
```

```
c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) +
Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) + (a*e*(Log[Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d -
Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt
[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*
c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2
- 4*a*c]/c]/Sqrt[2])) + (b*d*(Log[-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2]) + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2
]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt[b^2 -
 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*
e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c) - Sqrt
[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqr
t[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c
]/Sqrt[2])) - (Sqrt[b^2 - 4*a*c]*d*(Log[-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c
]/Sqrt[2]) + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d +
Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt
[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*
c]/c)*e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]
)*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*
a*c]/c]/Sqrt[2])) - (a*e*(Log[-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
 + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sq
rt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a
*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2
]))/(Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c) - Sqrt[b^2 -
 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/
c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2])) - (b*d*(Log[Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqrt[d +
((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt[2]*Sqrt[-(b/c) + Sqr
t[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sq
rt[d + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*
c*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/
Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[
b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) - (S
qrt[b^2 - 4*a*c]*d*(Log[Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqr
t[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt[2]*Sqrt[-(b/c)
 + Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)
/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sq
rt[2]*c*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*
c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) -
 Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
) + (a*e*(Log[Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqrt[d + ((-(
b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^
2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d
 + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt
[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 -
4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) + (b*d*(Lo
g[-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[d + ((-(b/c) + Sq
rt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c
]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]
]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c)
 + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sq
rt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) + (Sqrt[b^2 - 4*a
*c]*d*(Log[-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[d + ((-(
b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^
```

```
2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d
 + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*c*Sq
rt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2]) + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2
- 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) - (a*e*(
Log[-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[d + ((-(b/c) +
Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2 - 4*a
*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^
2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt[-(b/c)
 + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sq
rt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])))/a
```

fricas [B] time = 5.06, size = 2402, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b
^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^
3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c
*d*e + (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c +
 a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c
- a^2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^
2*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt
((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/
(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*
a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 -
4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a
^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sqrt(1/2)*a
*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c)*sqr
t((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)
/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c*d*e + (a^3*b^2*c
 - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2
*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 +
(4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 -
 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (
b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*
c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sq
rt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c)*sqrt((a^
2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6
*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sqrt(1/2)*a*x*sqrt(-((b^3 -
3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 +
(b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7
*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c*d*e - (a^3*b^2*c - 4*a^4*c^2)*d*x
^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2 +
(b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt
(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c +
 a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5
*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b
*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4
- 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))
/(a^3*b^2 - 4*a^4*c)))/x^2) + sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b
^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
4*a^4*c))*log((2*a^2*b*c*d*e - (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*
e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 -
```

```
4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)
*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4
*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2
*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^
3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 -
2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2
*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4
*c)))/x^2) + 4*sqrt(e*x^2 + d))/(a*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.03, size = 272, normalized size = 0.93

$$\frac{\sqrt{e}\ \ln\left(-\sqrt{e}\ x+\sqrt{e\,x^2+d}\right)}{a}+\frac{\sqrt{e}\ \ln\left(\sqrt{e}\ x+\sqrt{e\,x^2+d}\right)}{a}+\frac{}{2a\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/d/x*(e*x^2+d)^(3/2)+1/a*e/d*x*(e*x^2+d)^(1/2)+1/a*e^(1/2)*ln(e^(1/2)*x
+(e*x^2+d)^(1/2))+1/2/a*e^(1/2)*sum((_R^2*c*d+2*(-2*a*e^2+2*b*d*e-c*d^2)*_R
+c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^
2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4
*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+1
/a*e^(1/2)*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{\left(cx^4+bx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}}{x^2\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int((d + e*x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

3.365 $\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=373

$$\frac{c\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

```
[Out] -1/3*(e*x^2+d)^(1/2)/a/x^3+2/3*e*(e*x^2+d)^(1/2)/a/d/x+(-a*e+b*d)*(e*x^2+d)
^(1/2)/a^2/d/x+c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c
+b^2)^(1/2))/a^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/
2))^(1/2)+c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^2*d)/(-4*a*c+b^2)^
(1/2))/a^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1
/2)
```

Rubi [A] time = 2.53, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1295, 271, 264, 6728, 1692, 377, 205}

$$\frac{c\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(3*a*x^3) + (2*e*Sqrt[d + e*x^2])/(3*a*d*x) + ((b*d - a*e)
*Sqrt[d + e*x^2])/(a^2*d*x) + (c*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (c*(b*d - a*e - (b^2*d - 2*a*c*d -
 a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}dx = -\frac{\int \frac{bd-ae+cdx^2}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx}{a} + \frac{d\int \frac{1}{x^4\sqrt{d+ex^2}}dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} - \frac{\int \left(\frac{bd-ae}{ax^2\sqrt{d+ex^2}} + \frac{-b^2d+acd+abe-c(bd-ae)x^2}{a\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx}{a} - \frac{(2e)\int \frac{1}{x^2\sqrt{d+ex^2}}dx}{3a}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} - \frac{\int \frac{-b^2d+acd+abe-c(bd-ae)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx}{a^2} - \frac{(bd-ae)\int \frac{1}{x^2\sqrt{d+ex^2}}dx}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} - \frac{\int \left(\frac{-c(bd-ae)-\frac{c\left(b^2d-2acd-abe\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{-c}{(b+}\right.}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{\left(c\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{(b}}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{\left(c\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\text{Sub}}{}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{c\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-(}}$$

Mathematica [B] time = 6.39, size = 7777, normalized size = 20.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
[Out] Result too large to show
```

fricas [B] time = 23.17, size = 4095, normalized size = 10.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/12*(3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b
^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c
+ 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2
)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(((a^5*b^2*c^2 - 4*a
^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^
2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + 2*(a*b^4
*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*d*e - (
(b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b^2*c^3 + 4
*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e
*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^6*c + 1
```

```
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a
^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2
)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b
*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt(-((b^5 - 5*a*b
^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*
a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d
^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 -
 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6
*c)))/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d
 - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2
*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*
b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(((a^5*b^2*c^
2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*
e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + 2
*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*
d*e - ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b^2*
c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)
*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*
c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c
^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 -
4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt(-((b^5
- 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b
^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^
2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2
- 4*a^6*c)))/x^2) + 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b
*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7
- 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c +
 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(-((a^
5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b
*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11
*c)) - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 + 2*(a^2*b^3*c^2 - 2*a^3
*b*c^3)*d*e + ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*
a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 + 2*s
qrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*
a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a
^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^
3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt
(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e
+ (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(
a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)
*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2
*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3
*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*l
og(-((a^5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 -
 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2
- 4*a^11*c)) - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 + 2*(a^2*b^3*c^2
 - 2*a^3*b*c^3)*d*e + ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c
^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x
```

```
^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqr
t(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*
b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4
*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 1
3*a^3*b^3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)
*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3
*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*
b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^1
1*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) + 4*((3*b*d - a*e)*x^2 - a*d)*sqrt(e*x^2
+ d))/(a^2*d*x^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.04, size = 322, normalized size = 0.86

$$-\frac{b\sqrt{e}\,\ln\left(-\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{a^2}-\frac{b\sqrt{e}\,\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{a^2}-\frac{\sqrt{e\,x^2+d}\;bex}{a^2d}+\frac{}{2a^2\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a^2*b/d/x*(e*x^2+d)^(3/2)-1/a^2*b*e/d*x*(e*x^2+d)^(1/2)-1/a^2*b*e^(1/2)*l
n(e^(1/2)*x+(e*x^2+d)^(1/2))+1/2/a^2*e^(1/2)*sum((c*(a*e-b*d)*_R^2+2*(2*a*b
*e^2+a*c*d*e-2*b^2*d*e+b*c*d^2)*_R+a*c*d^2*e-b*c*d^3)/(_R^3*c+3*_R^2*b*e-3*
_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x
+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*
b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-1/a^2*e^(1/2)*b*ln(-e^(1/2)*x+
(e*x^2+d)^(1/2))-1/3/a/d/x^3*(e*x^2+d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{\left(cx^4+bx^2+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}}{x^4\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int((d + e*x^2)^(1/2)/(x^4*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**4*(a + b*x**2 + c*x**4)), x)
```

3.366 $\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=512

$$-\frac{\sqrt{d+ex^2}\left(-abe-acd+b^2d\right)}{a^3dx}-\frac{2e\sqrt{d+ex^2}\,(bd-ae)}{3a^2d^2x}+\frac{\sqrt{d+ex^2}\,(bd-ae)}{3a^2dx^3}-\frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd\ldots\right.}{a^3\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2c\ldots}}$$

```
[Out] -1/5*(e*x^2+d)^(1/2)/a/x^5+4/15*e*(e*x^2+d)^(1/2)/a/d/x^3+1/3*(-a*e+b*d)*(e
*x^2+d)^(1/2)/a^2/d/x^3-8/15*e^2*(e*x^2+d)^(1/2)/a/d^2/x-2/3*e*(-a*e+b*d)*(
e*x^2+d)^(1/2)/a^2/d^2/x-(-a*b*e-a*c*d+b^2*d)*(e*x^2+d)^(1/2)/a^3/d/x-c*arc
tan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2
)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(-4*
a*c+b^2)^(1/2))/a^3/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(-2*a^2*c*e+a*b^2*e+3*
a*b*c*d-b^3*d)/(-4*a*c+b^2)^(1/2))/a^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-
e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 4.94, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1295, 271, 264, 6728, 1692, 377, 205}

$$-\frac{\sqrt{d+ex^2}\left(-abe-acd+b^2d\right)}{a^3dx}-\frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{2a^2ce\ldots}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(5*a*x^5) + (4*e*Sqrt[d + e*x^2])/(15*a*d*x^3) + ((b*d - a
*e)*Sqrt[d + e*x^2])/(3*a^2*d*x^3) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^2*x) -
 (2*e*(b*d - a*e)*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - ((b^2*d - a*c*d - a*b*e)
*Sqrt[d + e*x^2])/(a^3*d*x) - (c*(b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*
d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sq
rt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(b^
2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b
^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{bd-ae+cdx^2}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a} + \frac{d\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} - \frac{\int \left(\frac{bd-ae}{ax^4\sqrt{d+ex^2}} + \frac{-b^2d+acd+abe}{a^2x^2\sqrt{d+ex^2}} + \frac{b^3d-2abcd-ab^2e+a^2ce+c\left(b^2d-acd-abe\right)x^2}{a^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx}{a} -$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} - \frac{\int \frac{b^3d-2abcd-ab^2e+a^2ce+c\left(b^2d-acd-abe\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a^3} + \frac{\left(8e^2\right)\int \frac{1}{x^2\sqrt{d}}}{15ad}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{\left(b^2d-acd-ab\right.}{a^3d}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d}}{3a^2d^2x}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d}}{3a^2d^2x}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d}}{3a^2d^2x}$$

Mathematica [B] time = 6.59, size = 10933, normalized size = 21.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
[Out] Result too large to show
```

fricas [B] time = 53.96, size = 5773, normalized size = 11.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] -1/60*(15*sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7
*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c
 + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a
^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^
2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-((a^7*b^2*c^3 - 4*a^8*
c^4)*d*x^2*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46
*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29
*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^
10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a
```

^14*b^2 - 4*a^15*c)) + 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 - 2*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e - ((b^7*c^3 - 5*a*b^5*c^4 + 6*a^2*b^3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) + ((a*b^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 + 29*a^5*b^2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3*c^3 + 12*a^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) - 15*sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) + 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 - 2*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e - ((b^7*c^3 - 5*a*b^5*c^4 + 6*a^2*b^3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) + ((a*b^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 + 29*a^5*b^2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3*c^3 + 12*a^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) + 15*sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) - 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 + 2*(a^2*b^5*

```
c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + ((b^7*c^3 - 5*a*b^5*c^4 + 6*a^2*b^
3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4
*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^2)*x^2 + 2*
sqrt(1/2)*sqrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt((
(b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40
*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c
+ 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*
c)) - ((a*b^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 + 29*a^5*b^
2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3
*c^3 + 12*a^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3
*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2
- 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46
*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29
*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^
10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a
^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) - 15*sqrt(1/2)*a^3*d^2*x^5
*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2
*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 1
0*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*
c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*
c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*
b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7
*b^2 - 4*a^8*c))*log(((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt(((b^12 - 10*a*b^
10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 +
22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^
2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) - 2*(a*b^6*
c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 + 2*(a^2*b^5*c^3 - 4*a^3
*b^3*c^4 + 3*a^4*b*c^5)*d*e + ((b^7*c^3 - 5*a*b^5*c^4 + 6*a^2*b^3*c^5 - a^3
*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4*a^4*c^6)*d
*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^2)*x^2 - 2*sqrt(1/2)*s
qrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt(((b^12 - 10*
a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^
3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^
6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) - ((a*b
^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 + 29*a^5*b^2*c^4 - 4*a
^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3*c^3 + 12*a
^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d -
 (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*
sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^
4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^
2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*
b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4
*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) - 4*((5*a*b*d*e + 2*a^2*e^2 - 15*(b^2
 - a*c)*d^2)*x^4 - 3*a^2*d^2 + (5*a*b*d^2 - a^2*d*e)*x^2)*sqrt(e*x^2 + d))/
(a^3*d^2*x^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 0.04, size = 503, normalized size = 0.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a^2/d/x*(e*x^2+d)^(3/2)*c-1/a^3/d/x*(e*x^2+d)^(3/2)*b^2-1/a^2*e/d*x*(e*x^
2+d)^(1/2)*c+1/a^3*e/d*x*(e*x^2+d)^(1/2)*b^2-1/a^2*e^(1/2)*ln(e^(1/2)*x+(e*
x^2+d)^(1/2))*c+1/a^3*e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))*b^2-1/2/a^3*e^(
1/2)*sum((c*(a*b*e+a*c*d-b^2*d)*_R^2+2*(-2*a^2*c*e^2+2*a*b^2*e^2+3*a*b*c*d*
e-a*c^2*d^2-2*b^3*d*e+b^2*c*d^2)*_R+a*b*c*d^2*e+a*c^2*d^3-b^2*c*d^3)/(_R^3*
c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(
-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z
^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-1/a^2*e^(1/2)*l
n(-e^(1/2)*x+(e*x^2+d)^(1/2))*c+1/a^3*e^(1/2)*ln(-e^(1/2)*x+(e*x^2+d)^(1/2)
)*b^2+1/3/a^2*b/d/x^3*(e*x^2+d)^(3/2)-1/5/a/d/x^5*(e*x^2+d)^(3/2)+2/15/a*e/
d^2/x^3*(e*x^2+d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{\left(cx^4+bx^2+a\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}}{x^6\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^6*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int((d + e*x^2)^(1/2)/(x^6*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**6*(a + b*x**2 + c*x**4)), x)
```

3.367 $\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=460

$$\frac{\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b\right.}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)/c+(-b*e+c*d)*(e*x^2+d)^(1/2)/c^2+1/2*arctanh(2^(1/2)*c^
(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b^3*e^2-b^2*
e*(2*c*d+e*(-4*a*c+b^2)^(1/2))+c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-
4*a*c+b^2)^(1/2)))+b*c*(c*d^2+e*(-3*a*e+2*d*(-4*a*c+b^2)^(1/2))))/c^(5/2)*2
^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*arctan
h(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(
b^3*e^2-b^2*e*(2*c*d-e*(-4*a*c+b^2)^(1/2))-c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*
(4*a*e+d*(-4*a*c+b^2)^(1/2)))+b*c*(c*d^2-e*(3*a*e+2*d*(-4*a*c+b^2)^(1/2))))
/c^(5/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 5.08, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 824, 826, 1166, 208}

$$\frac{\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b\right.}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) + ((b^3*e^2 - b
^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt
[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e))
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*
a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) - ((b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c*(c*d^2
- e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sq
rt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\left(d+ex^2\right)^{3/2}}{3c}+\frac{\text{Subst}\left(\int \frac{\sqrt{d+ex}\,(-ae+(cd-be)x)}{a+bx+cx^2}\,dx,x,x^2\right)}{2c}\\
&= \frac{(cd-be)\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3c}+\frac{\text{Subst}\left(\int \frac{-ae(2cd-be)+\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2c^2}\\
&= \frac{(cd-be)\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3c}+\frac{\text{Subst}\left(\int \frac{-ae^2(2cd-be)-d\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)+\left(c^2d^2+b^2e^2-ce\right.}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\right.}{c^2}\\
&= \frac{(cd-be)\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3c}-\frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}\,e\right)+c\left(a\sqrt{b^2-4ac}\,e^2-\right.\right.}{}\\
&= \frac{(cd-be)\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3c}+\frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}\,e\right)+c\left(a\sqrt{b^2-4ac}\,e^2-\right.\right.}{\sqrt{2}\,}
\end{aligned}$$

Mathematica [A] time = 0.97, size = 457, normalized size = 0.99

$$-\frac{\left(-bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(cd\left(d\sqrt{b^2-4ac}-4ae\right)-ae^2\sqrt{b^2-4ac}\right)+b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b\right.}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) - ((-(b^3*e^2)
+ b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d
*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 3
*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b
^2 - 4*a*c]*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt
[b^2 - 4*a*c])*e]) - ((b^3*e^2 + b^2*e*(-2*c*d + Sqrt[b^2 - 4*a*c]*e) + b*c
*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + c*(-(a*Sqrt[b^2 - 4*a*c]*e^2
) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e
*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2
- 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.25, size = 857, normalized size = 1.86

$$-\frac{\left(2\,bc^5d^3-\left(5\,b^2c^4-8\,ac^5\right)d^2e+\left(\left(b^2c^2-4\,ac^3\right)d^2e-2\left(b^3c-4\,abc^2\right)de^2+\left(b^4-5\,ab^2c+4\,a^2c^2\right)e^3\right)c^2+2\,(2\,\ldots}{\left(2\,\sqrt{b^2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -(2*b*c^5*d^3 - (5*b^2*c^4 - 8*a*c^5)*d^2*e + ((b^2*c^2 - 4*a*c^3)*d^2*e -
2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^3)*c^2 + 2*(2
*b^3*c^3 - 5*a*b*c^4)*d*e^2 - 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4
*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(
b^2 - 4*a*c)*d*e^2)*abs(c) - (b^4*c^2 - 3*a*b^2*c^3)*e^3)*arctan(2*sqrt(1/2
)*sqrt(x^2*e + d)/sqrt(-(2*c^4*d - b*c^3*e + sqrt(-4*(c^4*d^2 - b*c^3*d*e +
 a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((2*sqrt(b^2 - 4*a*c)*c^3*d
 - (b^2*c^2 - 4*a*c^3 + sqrt(b^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c^2*d + 2*(b*c
- sqrt(b^2 - 4*a*c)*c)*e)*c^2) + (2*b*c^5*d^3 - (5*b^2*c^4 - 8*a*c^5)*d^2*e
 + ((b^2*c^2 - 4*a*c^3)*d^2*e - 2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*b^
2*c + 4*a^2*c^2)*e^3)*c^2 + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^2 + 2*(sqrt(b^2 -
 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4*a*c)*a*b*c
^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*abs(c) - (b^4*c^2 - 3*a
*b^2*c^3)*e^3)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^4*d - b*c^3*e
- sqrt(-4*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c
^4))/((2*sqrt(b^2 - 4*a*c)*c^3*d + (b^2*c^2 - 4*a*c^3 - sqrt(b^2 - 4*a*c)*b
*c^2)*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2) + 1/3*((x^2*
e + d)^(3/2)*c^2 + 3*sqrt(x^2*e + d)*c^2*d - 3*sqrt(x^2*e + d)*b*c*e)/c^3
```

maple [C] time = 0.03, size = 490, normalized size = 1.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/6/c*e^(3/2)*x^3+1/8/c*e*(e*x^2+d)^(1/2)*x^2-3/4/c*e^(1/2)*x*d+1/24*(e*x^
2+d)^(3/2)/c+1/2/c^2*e^(3/2)*x*b-1/2/c^2*(e*x^2+d)^(1/2)*b*e+5/8/c*(e*x^2+d
)^(1/2)*d+1/4/c^2*sum(((-a*c*e^2+b^2*e^2-2*b*c*d*e+c^2*d^2)*_R^6+(4*a*b*e^3
-5*a*c*d*e^2-3*b^2*d*e^2+6*b*c*d^2*e-3*c^2*d^3)*_R^4+d*(-4*a*b*e^3+5*a*c*d*
e^2+3*b^2*d*e^2-6*b*c*d^2*e+3*c^2*d^3)*_R^2+a*c*d^3*e^2-b^2*d^3*e^2+2*b*c*d
^4*e-c^2*d^5)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^
3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z
^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*
c*d^3)*_Z^2))-1/2/c^2*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))*b*e+5/8/c*d^2/(-e^(1/2
)*x+(e*x^2+d)^(1/2))+1/24/c*d^3/(-e^(1/2)*x+(e*x^2+d)^(1/2))^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x^3/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 3.41, size = 16951, normalized size = 36.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(3/2)/(3*c) - atan(((((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*
a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*
b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 - (2*(d + e*
x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d
^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c
^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^
2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 +
 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2
*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^
2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3
 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 4
8*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6
*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e
 - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*
(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(
-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*
b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*
a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 14
4*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5
- 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3
 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*
e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^
5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*
e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2
 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b
^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*
x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 +
 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e
^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^
5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(((4*b^7*e^3 - 32*a^
2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^
```

4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 - (2*(d + e*x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2

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*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*
c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^
3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*
c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b
^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6
*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^
3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2
*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d
^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/
4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a
^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*
a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))
^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40
*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 -
18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 +
72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a
*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5
*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3
*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c
^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4
))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3
- 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*
e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d
*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16
*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^
3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b
*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 -
16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a
^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^
6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e -
108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (
((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4
- 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*
e^3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^
2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^
4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*
c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 2
16*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5
*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4
+ 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3
 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3
 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e
 + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e
 + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16
*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2
- 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 -
4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12
*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84
*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c
^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c
^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*
d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2
*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*
c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^
3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*
c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b
^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e
^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4
*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^
```

5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2
*c^3*d^2*e^4))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*
b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*
a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96
*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*
a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*
e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e
^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) +
2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^
3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*
c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*
c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2) + (2*(a^4*c*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*a^2*b^3*d*e^7 -
a*c^4*d^6*e^2 - a^2*c^3*d^4*e^4 + a^3*c^2*d^2*e^6 + 4*a*b*c^3*d^5*e^3 + 4*a
*b^3*c*d^3*e^5 - 6*a*b^2*c^2*d^4*e^4 + 4*a^2*b*c^2*d^3*e^5 - 5*a^2*b^2*c*d^
2*e^6))/c^3))*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^
4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^
3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4
*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^
7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 -
a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3
*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*
e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3
+ 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3
- 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^
2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/
2)*2i - (d + e*x^2)^(1/2)*(d/c + (b*e - 2*c*d)/c^2) - atan(((((4*a*b^3*c^3*
e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*
e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2
*c^4*d*e^4)/c^3 - (2*(d + e*x^2)^(1/2)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b
^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^
5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*
b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*
d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*
d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4
*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*
d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4
*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c
^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2
*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*
c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^
3 + 32*a*c^7*d*e^2))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 +
24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e +
100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e
+ 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 -
(256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c
*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*
b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/
2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3
*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a
*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a
^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2
c^6)))^(1/2) - (2(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4
*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3
*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d
^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c
^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*
a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 -
36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2

+ 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^(1/2)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 - (2*(d + e*x^2)^(1/2)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5* c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^(1/2)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4

```
*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*
c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^
3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*
e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^
6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e
^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^
4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d
^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2
*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d
*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2) + (2*(a^4*c*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*
a^2*b^3*d*e^7 - a*c^4*d^6*e^2 - a^2*c^3*d^4*e^4 + a^3*c^2*d^2*e^6 + 4*a*b*c
^3*d^5*e^3 + 4*a*b^3*c*d^3*e^5 - 6*a*b^2*c^2*d^4*e^4 + 4*a^2*b*c^2*d^3*e^5
- 5*a^2*b^2*c*d^2*e^6))/c^3))*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^
3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2
*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d
^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/
4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a
^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*
a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))
^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40
*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 +
18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 -
72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a
*b^2*c^6)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.368 $\int \frac{x\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=327

$$-\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-2ce\left(d\sqrt{b^2-\right.\right.}{}$$

```
[Out] e*(e*x^2+d)^(1/2)/c-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*
(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/c^(3/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e
*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1
/2))-2*c*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/c^(3/2)*2^(1/2)/(-4*a*c+b^2)^(1/
2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.46, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1247, 703, 826, 1166, 208}

$$-\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-2ce\left(d\sqrt{b^2-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*Sqrt[d + e*x^2])/c - ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e
*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2
])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*
a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((2*c^2*d^2 + b*(b + Sqrt[b
^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[
2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
```

```
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{e\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2c}$$

$$= \frac{e\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{-de(2cd-be)+e\left(cd^2-ae^2\right)+e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{c}$$

$$= \frac{e\sqrt{d+ex^2}}{c} + \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\text{Subst}\left(\int \frac{}{-\frac{1}{2}\sqrt{b}\ldots}\right.}{2c\sqrt{b^2-4ac}}$$

$$= \frac{e\sqrt{d+ex^2}}{c} - \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt{2cd-(\ldots}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 0.57, size = 324, normalized size = 0.99

$$\frac{\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{\left(-2ce\left(d\sqrt{b^2-4ac}+\ldots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*Sqrt[d + e*x^2])/c + ((-2*c^2*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*e^2 + 2*c
*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 -
4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + ((2*c^2*d^2 + b*(b + Sqr
t[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqr
```

```
t[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.09, size = 649, normalized size = 1.98

$$\frac{\sqrt{x^2e+d}\,e}{c}+\frac{\left(4\,c^5d^3-6\,bc^4d^2e-\left(2\left(b^2c-4\,ac^2\right)de^2-\left(b^3-4\,abc\right)e^3\right)c^2+4\left(b^2c^3-ac^4\right)de^2+2\left(\sqrt{b^2-4\,ac}\,c^3 \cdots\right.\right.}{\left(2\sqrt{b^2-4\,ac}\,c^2d-\left(b^2c-4\,ac^2+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] sqrt(x^2*e + d)*e/c + (4*c^5*d^3 - 6*b*c^4*d^2*e - (2*(b^2*c - 4*a*c^2)*d*e
^2 - (b^3 - 4*a*b*c)*e^3)*c^2 + 4*(b^2*c^3 - a*c^4)*d*e^2 + 2*(sqrt(b^2 - 4
*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e
^3)*abs(c) - (b^3*c^2 - 2*a*b*c^3)*e^3)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/
sqrt(-(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2
*d - b*c*e)^2))/c^2))/((2*sqrt(b^2 - 4*a*c)*c^2*d - (b^2*c - 4*a*c^2 + sqrt
(b^2 - 4*a*c)*b*c)*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2)
 - (4*c^5*d^3 - 6*b*c^4*d^2*e - (2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c
)*e^3)*c^2 + 4*(b^2*c^3 - a*c^4)*d*e^2 - 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - s
qrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)*abs(c) - (b^3*c
^2 - 2*a*b*c^3)*e^3)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^2*d - b*
c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2
))/((2*sqrt(b^2 - 4*a*c)*c^2*d + (b^2*c - 4*a*c^2 - sqrt(b^2 - 4*a*c)*b*c)*
e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2)
```

maple [C] time = 0.02, size = 279, normalized size = 0.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*e^(3/2)/c*x+1/2*e*(e*x^2+d)^(1/2)/c+1/4*e/c*sum(((-b*e+2*c*d)*_R^6+(-4
*a*e^2+3*b*d*e-2*c*d^2)*_R^4+d*(4*a*e^2-3*b*d*e+2*c*d^2)*_R^2+b*d^3*e-2*c*d
^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R
*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*
e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^
2))+1/2*e/c*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 4.13, size = 12392, normalized size = 37.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (e*(d + e*x^2)^(1/2))/c - atan(((((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*
c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c - (
2*(d + e*x^2)^(1/2)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*
b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c
*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4
*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 +
3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c
^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d
^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3
 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c
^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3
+ 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 -
 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5
 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d
^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^
5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*
b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*
b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^6 +
2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*
c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(((4*
b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^
2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e +
 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*
e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d
^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3)
)^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*
a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c
^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*1i - (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^
2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c + (2*(d + e*x^2)^(1/2)*(-
(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3
*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^
2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*
(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^
2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3
*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3
- 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*
a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^
2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 9
6*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a
*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*
b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*
a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b
*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*
e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*
```

a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*
d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*
c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3
 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e -
 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2
/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^
3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e
^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 +
16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*
c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^
2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^
3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^
2*e^3 - 16*a*b*c^3*d*e^4)/c - (2*(d + e*x^2)^(1/2)*(-(((4*b^5*e^3 + 32*a*c^
4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^
2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e
^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b
^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*
d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e
^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6
*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b
^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e
^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3
+ 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b
^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^
2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^
3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2)
+ 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*
d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e
 + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2
*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5
+ 12*a*b*c^2*d*e^5))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*
a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b
^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16
*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^
4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3
*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c
^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c
*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c
^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 +
 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)
/c + (2*(d + e*x^2)^(1/2)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 4
8*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12
*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 +
16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*
e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 -
 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2
*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3
*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16
*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^
3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c
*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a
^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a
*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*
b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^
3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e -
 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(1

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6*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*
e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 +
6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(
-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^
3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d
^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)
*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a
^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^
3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3
 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24
*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^
4)))^(1/2) - (2*(2*c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5
- 5*b*c^2*d^4*e^4 + 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b
*c*d^2*e^6))/c))*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c
^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*
e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^
3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*
d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3
+ 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 -
6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^
4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5
 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^
4)/c - (2*(d + e*x^2)^(1/2)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 +
48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 1
2*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 +
 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2
*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5
- 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^
2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^
3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^
2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 1
6*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^
3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c
*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a
^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a
*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*
b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^
3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e +
 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(1
6*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*
e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 +
6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(
(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3
*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^
2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*
(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^
2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3
*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3
+ 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*
a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2)*1i - (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b
^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c + (2*(d + e*x^2)^(1/
2)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2
*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^
3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c
^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 +
3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c
*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*
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e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 +
 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2
*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*
d*e^2))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3
- 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 4
8*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128
*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^
4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e -
 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^
2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c
*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 -
 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c
^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b
^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*((((4*b^5*e^3 + 32*a*c^4*d
^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e
 - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)
^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*
d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4
*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3
- 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^
3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*
c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*
c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*
d^2*e^3 - 16*a*b*c^3*d*e^4)/c - (2*(d + e*x^2)^(1/2)*((((4*b^5*e^3 + 32*a*c
^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d
^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*
e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 -
b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c
*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*
e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 -
6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*
b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*
e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((((4*b^5*e^3
+ 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b
^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^
2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^
3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2)
- 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*
d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e
 - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2
*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5
+ 12*a*b*c^2*d*e^5))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a
^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^
4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*
b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4
 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*
b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^
3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*
e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^
5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 +
16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/
c + (2*(d + e*x^2)^(1/2)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*
a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b
^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16
*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^
4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3
*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c
```

```
^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c
*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c
^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a
*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d
^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^
3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*
c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^
2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d
*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 +
 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14
*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(
16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^6
 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b
^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*((((
4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*
e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e
 + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^
3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c
*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^
3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 4
8*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b
*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) - (2*(2*c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5 - 5*
b*c^2*d^4*e^4 + 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b*c*d
^2*e^6))/c))*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^
3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 -
 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 1
28*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*
d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e
 - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*
a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4
*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.369 $\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=346

$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-cd\left(d\sqrt{b^2-4ac}+\right.\right.}{}$$

```
[Out] -d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/a-1/2*arctanh(2^(1/2)*c^(1/2)*(e*
x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(-b*(a*e^2+c*d^2)+a*e^
2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-4*a*c+b^2)^(1/2)))/a*2^(1/2)/c^(1/2)/(
-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*arctanh(2^(1/2
)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b*(a*e^2
+c*d^2)+a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))/a*2^(1/2
)/c^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.74, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 897, 1287, 206, 1166, 208}

$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-cd\left(d\sqrt{b^2-4ac}+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) - ((a*Sqrt[b^2 - 4*a*c]*e^2
 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((a
*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e
^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
```

```
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{d^2e}{a\left(d-x^2\right)}+\frac{e\left(d\left(cd^2-bde+ae^2\right)-\left(cd^2-ae^2\right)x^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{d\left(cd^2-bde+ae^2\right)+\left(-cd^2+ae^2\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\right)}{a}$$

$$= -\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b\left(cd^2+ae^2\right)\right)\text{Sub}}{2a\sqrt{b^2-4ac}}$$

$$= -\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} - \frac{\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b\left(cd^2+ae^2\right)\right)\tanh}{\sqrt{2}\,a\sqrt{c}\,\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b-\sqrt{b^2-4a\right.}}}$$

Mathematica [A] time = 1.38, size = 333, normalized size = 0.96

$$-\frac{\frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\left(cd\left(d\sqrt{b^2-4ac}-4ae\right)-ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(-\right.}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((-(((-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) + b*(
c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e
+ Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + ((a*Sq
rt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e^2)
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])/(Sqrt[2]*a*Sqrt[c]*Sqrt
[b^2 - 4*a*c])) + (d^(3/2)*Log[x])/a - (d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + e*
x^2]])/a
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.02, size = 827, normalized size = 2.39

$$\frac{d^2\arctan\left(\frac{\sqrt{x^2e+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}-\frac{\left(\left(\left(b^2c-4\,ac^2\right)d^2e-\left(ab^2-4\,a^2c\right)e^3\right)\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\,a^2-2\left(\sqrt{b^2-4\,ac}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d^2*arctan(sqrt(x^2*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/8*(((b^2*c - 4*a*c^2)
*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*
c)*e)*a^2 - 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e
+ sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)
*c)*e)*abs(a) - (2*a^2*b*c^2*d^3 + 6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2
*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*ar
ctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c*d^2
- a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c
)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2
)*abs(a)*abs(c)) + 1/8*(((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*s
qrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*a^2 + 2*(sqrt(b^2 - 4*a*c)*
a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*
sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c^2*d^3
+ 6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^
2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/
sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c
```

```
*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)
*a^2*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2)*abs(a)*abs(c))
```

maple [C] time = 0.03, size = 388, normalized size = 1.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] 7/24/a*(e*x^2+d)^(3/2)-1/a*d^(3/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)+3/
8/a*(e*x^2+d)^(1/2)*d+1/6/a*e^(3/2)*x^3-1/8/a*e*(e*x^2+d)^(1/2)*x^2+3/4/a*e
^(1/2)*x*d-1/4/a*sum(((-a*e^2+c*d^2)*_R^6+d*(-5*a*e^2+4*b*d*e-3*c*d^2)*_R^4
+d^2*(5*a*e^2-4*b*d*e+3*c*d^2)*_R^2+a*d^3*e^2-c*d^5)/(_R^7*c+3*_R^5*b*e-3*_
R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(
1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*
e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))-5/8/a*d^2/(-e^(1/2)*x+
(e*x^2+d)^(1/2))-1/24/a*d^3/(-e^(1/2)*x+(e*x^2+d)^(1/2))^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^{\frac{3}{2}}}{\left(cx^4 + bx^2 + a\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x), x)
```

mupad [B] time = 7.67, size = 28434, normalized size = 82.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10
- 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2
*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*
b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2
*c*d^2*e^14) + (-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2
*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*
c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3
*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*
e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6
*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*
a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2
*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*
c^2)))^(1/2)*(((-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2
*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*
c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3
*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*
e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6
*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*
a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2
*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*
c^2)))^(1/2)*((d + e*x^2)^(1/2)*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c
^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*
```

```
d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^
2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e
^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 -
3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*
a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3
*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b
^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256
*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*
b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*
c^3*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^4*d^2*e^10 + 48*a^2*b^2*c^4*d^
4*e^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c^2*d^2*e^10 - 16*a^3*b^2*c^3*d
^2*e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^
11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3
*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 +
 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 -
16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^
4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2
*c^2*d*e^12))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*
c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c
^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*
b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*
a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a
*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*
b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c
^2)))^(1/2) + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 9
2*a^3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*
e^10 - 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 -
36*a*b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*
a^2*b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e
^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3
- 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^
4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 +
3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a
^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*
b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*
e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^
4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(2*a^4*c
*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*
e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*
b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4
*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + (-(((4*b^4*c*d^3 -
 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*
a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2
/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^
3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d
^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*
d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2
- 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2
)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(((-(((4*b^4*c*d^3 -
 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*
a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2
/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^
3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d
^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*
d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2
- 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2
)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*(
-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^
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3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a
^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e
^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^
2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))
^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 +
48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 1
2*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(51
2*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d
^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d
*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^3*c^5*d^4*e^8
+ 192*a^4*c^4*d^2*e^10 - 48*a^2*b^2*c^4*d^4*e^8 + 64*a^2*b^3*c^3*d^3*e^9 -
16*a^2*b^4*c^2*d^2*e^10 + 16*a^3*b^2*c^3*d^2*e^10 - 64*a^4*b*c^3*d*e^11 - 2
56*a^3*b*c^4*d^3*e^9 + 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^
3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 2
24*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*
c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^
4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4
*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*(-(((4*b^4*c*d^3 -
4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a
^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/
4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3
*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^
4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d
^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 -
 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)
/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 12*a*c^5*d^7*e^8 -
4*a^4*c^2*d*e^14 + 84*a^2*c^4*d^5*e^10 + 92*a^3*c^3*d^3*e^12 + 4*b^2*c^4*d^
7*e^8 + 4*b^3*c^3*d^6*e^9 - 8*b^4*c^2*d^5*e^10 + 12*a^2*b^2*c^2*d^3*e^12 -
32*a*b*c^4*d^6*e^9 + 4*a^3*b^2*c*d*e^14 + 36*a*b^2*c^3*d^5*e^10 + 20*a*b^3*
c^2*d^4*e^11 - 160*a^2*b*c^3*d^4*e^11 - 4*a^2*b^3*c*d^2*e^13 - 16*a^3*b*c^2
*d^2*e^13))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^
2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2
*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^
2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2
 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*
b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b
^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*
c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2
)))^(1/2)*1i)/(((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*
d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^
3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^
15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 1
2*a^2*b^2*c*d^2*e^14) + (-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 -
 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e +
48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c
- 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*
d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*
d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e
 - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8
*a^3*b^2*c^2)))^(1/2)*(((-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 -
 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e +
48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c
- 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*
d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*
d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e
 - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8
*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 +
```

32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12
*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^
3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*
b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b
*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*
e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3
+ 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^
3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e
^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 -
448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480
*a^3*b^3*c^3*d*e^9) + 192*a^3*c^5*d^4*e^8 + 192*a^4*c^4*d^2*e^10 - 48*a^2*b
^2*c^4*d^4*e^8 + 64*a^2*b^3*c^3*d^3*e^9 - 16*a^2*b^4*c^2*d^2*e^10 + 16*a^3*
b^2*c^3*d^2*e^10 - 64*a^4*b*c^3*d*e^11 - 256*a^3*b*c^4*d^3*e^9 + 16*a^3*b^3
*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 17
6*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*
d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^
2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9
- 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 1
6*a^3*b^2*c^2*d*e^12))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 -
24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 4
8*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c -
128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*
c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d
^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d
^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e
- 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*
a^3*b^2*c^2)))^(1/2) - 12*a*c^5*d^7*e^8 - 4*a^4*c^2*d*e^14 + 84*a^2*c^4*d^5
*e^10 + 92*a^3*c^3*d^3*e^12 + 4*b^2*c^4*d^7*e^8 + 4*b^3*c^3*d^6*e^9 - 8*b^4
*c^2*d^5*e^10 + 12*a^2*b^2*c^2*d^3*e^12 - 32*a*b*c^4*d^6*e^9 + 4*a^3*b^2*c*
d*e^14 + 36*a*b^2*c^3*d^5*e^10 + 20*a*b^3*c^2*d^4*e^11 - 160*a^2*b*c^3*d^4*
e^11 - 4*a^2*b^3*c*d^2*e^13 - 16*a^3*b*c^2*d^2*e^13))*(-(((4*b^4*c*d^3 - 4*
a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3
*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4
- (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d
^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*
e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3
+ 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8
*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(
16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - ((d + e*x^2)^(1/2)*(2
*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*
c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10
- 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c
^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + (-(((4*b^4*c
*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2
+ 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*
e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^
6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b
^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*
b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*
d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c
*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(((-(((4*b^4*c
*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2
+ 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*
e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^
6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b
^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*
b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*
d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c
*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(

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1/2)*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 -
 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e
+ 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*
(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^
2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3
*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*
d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2
*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/
2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4
*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b
*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^3*c^5*d^
4*e^8 - 192*a^4*c^4*d^2*e^10 + 48*a^2*b^2*c^4*d^4*e^8 - 64*a^2*b^3*c^3*d^3*
e^9 + 16*a^2*b^4*c^2*d^2*e^10 - 16*a^3*b^2*c^3*d^2*e^10 + 64*a^4*b*c^3*d*e^
11 + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*
a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e
^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^
2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*
b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c
^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*(-(((4*b^4*c*
d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2
+ 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e
^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6
 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^
2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b
^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d
*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*
d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 12*a*c^5*d^7*
e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 92*a^3*c^3*d^3*e^12 - 4*b^2*
c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*e^10 - 12*a^2*b^2*c^2*d^3*e
^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 - 36*a*b^2*c^3*d^5*e^10 - 20*
a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*a^2*b^3*c*d^2*e^13 + 16*a^3
*b*c^2*d^2*e^13))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*
b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2
*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*
a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d
^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e
- 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 +
12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*
a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b
^2*c^2)))^(1/2) + 6*c^4*d^8*e^10 + 14*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 1
6*b*c^3*d^7*e^11 - 4*b^3*c*d^5*e^13 + 10*a^2*c^2*d^4*e^14 + 14*b^2*c^2*d^6*
e^12 - 24*a*b*c^2*d^5*e^13 + 10*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15))*(-(
((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*
c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2
*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6
 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*
e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(
1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48
*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*
a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2i +
atan((((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10
- 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2
*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*
b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2
*c*d^2*e^14) + ((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*
c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c
^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*
b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*
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a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a
*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*
b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c
^2)))^(1/2)*((((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c
^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^
2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b
^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a
*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*
b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b
*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^
2)))^(1/2)*((d + e*x^2)^(1/2)*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*
d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2
*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b
^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4
+ 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b
*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2
*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*
d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*
c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^
4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2
*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3
*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^4*d^2*e^10 + 48*a^2*b^2*c^4*d^4*e
^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c^2*d^2*e^10 - 16*a^3*b^2*c^3*d^2*
e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^11)
 + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*
e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16
*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*
a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c
^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^
2*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*
d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d
^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*
c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 +
 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*
c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2
*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^
2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))
)^(1/2) + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 92*a^
3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*e^10
 - 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 - 36*a
*b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*a^2*
b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 +
32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*
a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3
 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b
^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*
d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e
^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 -
 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3
 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(2*a^4*c*e^16
 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12
+ 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c
^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12
 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + ((((4*b^4*c*d^3 - 4*a^2
*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*
c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (
256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*
e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2
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- 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^3*c^5*d^4*e^8 + 192*a^4*c^4*d^2*e^10 - 48*a^2*b^2*c^4*d^4*e^8 + 64*a^2*b^3*c^3*d^3*e^9 - 16*a^2*b^4*c^2*d^2*e^10 + 16*a^3*b^2*c^3*d^2*e^10 - 64*a^4*b*c^3*d*e^11 - 256*a^3*b*c^4*d^3*e^9 + 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 12*a*c^5*d^7*e^8 - 4*a^4*c^2*d*e^14 + 84*a^2*c^4*d^5*e^10 + 92*a^3*c^3*d^3*e^12 + 4*b^2*c^4*d^7*e^8 + 4*b^3*c^3*d^6*e^9 - 8*b^4*c^2*d^5*e^10 + 12*a^2*b^2*c^2*d^3*e^12 - 32*a*b*c^4*d^6*e^9 + 4*a^3*b^2*c*d*e^14 + 36*a*b^2*c^3*d^5*e^10 + 20*a*b^3*c^2*d^4*e^11 - 160*a^2*b*c^3*d^4*e^11 - 4*a^2*b^3*c*d^2*e^13 - 16*a^3*b*c^2*d^2*e^13))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i)/(((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + ((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^

2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^3*c^5*d^4*e^8 + 192*a^4*c^4*d^2*e^10 - 48*a^2*b^2*c^4*d^4*e^8 + 64*a^2*b^3*c^3*d^3*e^9 - 16*a^2*b^4*c^2*d^2*e^10 + 16*a^3*b^2*c^3*d^2*e^10 - 64*a^4*b*c^3*d*e^11 - 256*a^3*b*c^4*d^3*e^9 + 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 12*a*c^5*d^7*e^8 - 4*a^4*c^2*d*e^14 + 84*a^2*c^4*d^5*e^10 + 92*a^3*c^3*d^3*e^12 + 4*b^2*c^4*d^7*e^8 + 4*b^3*c^3*d^6*e^9 - 8*b^4*c^2*d^5*e^10 + 12*a^2*b^2*c^2*d^3*e^12 - 32*a*b*c^4*d^6*e^9 + 4*a^3*b^2*c*d*e^14 + 36*a*b^2*c^3*d^5*e^10 + 20*a*b^3*c^2*d^4*e^11 - 160*a^2*b*c^3*d^4*e^11 - 4*a^2*b^3*c*d^2*e^13 - 16*a^3*b*c^2*d^2*e^13))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - ((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + ((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3

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+ 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3
*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^
2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*
c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*
a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((((4*b^4*c*d^3 - 4*a^2*b^3*
e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3
 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a
^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 +
 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*
a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2
*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c
*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a
^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*((((4*b^4*c*
d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2
+ 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e
^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6
 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^
2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b
^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d
*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*
d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^
10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*
a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^
2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^
4*d^2*e^10 + 48*a^2*b^2*c^4*d^4*e^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c
^2*d^2*e^10 - 16*a^3*b^2*c^3*d^2*e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4
*d^3*e^9 - 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 3
2*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d
^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10
 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 -
80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a
^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3
 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 -
12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*
c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*
a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2
*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^
3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^
3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*
c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e
^14 - 84*a^2*c^4*d^5*e^10 - 92*a^3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3
*c^3*d^6*e^9 + 8*b^4*c^2*d^5*e^10 - 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^
6*e^9 - 4*a^3*b^2*c*d*e^14 - 36*a*b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11
+ 160*a^2*b*c^3*d^4*e^11 + 4*a^2*b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*(
(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3
*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^
2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^
6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2
*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^
(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 4
8*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12
*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 6*
c^4*d^8*e^10 + 14*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 16*b*c^3*d^7*e^11 - 4
*b^3*c*d^5*e^13 + 10*a^2*c^2*d^4*e^14 + 14*b^2*c^2*d^6*e^12 - 24*a*b*c^2*d^
5*e^13 + 10*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15))*((((4*b^4*c*d^3 - 4*a^2
*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*
c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (
256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*
e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2
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- 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2i - (atanh((72*c^4*d^6*e^10*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) + (2*a^3*c*e^16*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) + (18*c^5*d^8*e^8*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*c^5*d^10*e^8 + 72*a*c^4*d^8*e^10 + 2*a^4*c*d^2*e^16 - 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^12 + 8*a^3*c^2*d^4*e^14 + 20*b^2*c^3*d^8*e^10 + 12*b^3*c^2*d^7*e^11 - (4*b^2*c^4*d^10*e^8)/a + (10*b^3*c^3*d^9*e^9)/a - (6*b^4*c^2*d^8*e^10)/a - 104*a*b*c^3*d^7*e^11 - 6*a*b^3*c*d^5*e^13 - 8*a^3*b*c*d^3*e^15 + 20*a*b^2*c^2*d^6*e^12 - 32*a^2*b*c^2*d^5*e^13 + 12*a^2*b^2*c*d^4*e^14) + (8*a^2*c^2*d^2*e^14*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) + (20*b^2*c^2*d^4*e^12*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) - (48*b*c^4*d^7*e^9*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*c^5*d^10*e^8 + 72*a*c^4*d^8*e^10 + 2*a^4*c*d^2*e^16 - 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^12 + 8*a^3*c^2*d^4*e^14 + 20*b^2*c^3*d^8*e^10 + 12*b^3*c^2*d^7*e^11 - (4*b^2*c^4*d^10*e^8)/a + (10*b^3*c^3*d^9*e^9)/a - (6*b^4*c^2*d^8*e^10)/a - 104*a*b*c^3*d^7*e^11 - 6*a*b^3*c*d^5*e^13 - 8*a^3*b*c*d^3*e^15 + 20*a*b^2*c^2*d^6*e^12 - 32*a^2*b*c^2*d^5*e^13 + 12*a^2*b^2*c*d^4*e^14) - (4*b^2*c^4*d^8*e^8*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*a*c^5*d^10*e^8 + 2*a^5*c*d^2*e^16 + 72*a^2*c^4*d^8*e^10 + 60*a^3*c^3*d^6*e^12 + 8*a^4*c^2*d^4*e^14 - 4*b^2*c^4*d^10*e^8 + 10*b^3*c^3*d^9*e^9 - 6*b^4*c^2*d^8*e^10 + 20*a^2*b^2*c^2*d^6*e^12 - 48*a*b*c^4*d^9*e^9 - 8*a^4*b*c*d^3*e^15 + 20*a*b^2*c^3*d^8*e^10 + 12*a*b^3*c^2*d^7*e^11 - 104*a^2*b*c^3*d^7*e^11 - 6*a^2*b^3*c*d^5*e^13 - 32*a^3*b*c^2*d^5*e^13 + 12*a^3*b^2*c*d^4*e^14) + (10*b^3*c^3*d^7*e^9*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*a*c^5*d^10*e^8 + 2*a^5*c*d^2*e^16 + 72*a^2*c^4*d^8*e^10 + 60*a^3*c^3*d^6*e^12 + 8*a^4*c^2*d^4*e^14 - 4*b^2*c^4*d^10*e^8 + 10*b^3*c^3*d^9*e^9 - 6*b^4*c^2*d^8*e^10 + 20*a^2*b^2*c^2*d^6*e^12 - 48*a*b*c^4*d^9*e^9 - 8*a^4*b*c*d^3*e^15 + 20*a*b^2*c^3*d^8*e^10 + 12*a*b^3*c^2*d^7*e^11 - 104*a^2*b*c^3*d^7*e^11 - 6*a^2*b^3*c*d^5*e^13 - 32*a^3*b*c^2*d^5*e^13 + 12*a^3*b^2*c*d^4*e^14) - (6*b^4*c^2*d^6*e^10*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*a*c^5*d^10*e^8 + 2*a^5*c*d^2*e^16 + 72*a^2*c^4*d^8*e^10 + 60*a^3*c^3*d^6*e^12 + 8*a^4*c^2*d^4*e^14 - 4*b^2*c^4*d^10*e^8 + 10*b^3*c^3*d^9*e^9 - 6*b^4*c^2*d^8*e^10 + 20*a^2*b^2*c^2*d^6*e^12 - 48*a*b*c^4*d^9*e^9 - 8*a^4*b*c*d^3*e^15 + 20*a*b^2*c^3*d^8*e^10 + 12*a*b^3*c^2*d^7*e^11 - 104*a^2*b*c^3*d^7*e^11 - 6*a^2*b^3*c*d^5*e^13 - 32*a^3*b*c^2*d^5*e^13 + 12*a^3*b^2*c*d^4*e^14) + (60*a*c^3*d^4*e^12*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^1

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1)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8
*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15
 - (48*b*c^4*d^9*e^9)/a) - (104*b*c^3*d^5*e^11*(d + e*x^2)^(1/2)*(d^3)^(1/2
))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*
e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2
*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*
c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 3
2*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d
^9*e^9)/a) - (6*b^3*c*d^3*e^13*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e
^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d
^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 +
(20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^
2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^
13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) + (20
*b^2*c^3*d^6*e^10*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*c^5*d^10*e^8 + 72*a*c^
4*d^8*e^10 + 2*a^4*c*d^2*e^16 - 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^12 + 8*
a^3*c^2*d^4*e^14 + 20*b^2*c^3*d^8*e^10 + 12*b^3*c^2*d^7*e^11 - (4*b^2*c^4*d
^10*e^8)/a + (10*b^3*c^3*d^9*e^9)/a - (6*b^4*c^2*d^8*e^10)/a - 104*a*b*c^3*
d^7*e^11 - 6*a*b^3*c*d^5*e^13 - 8*a^3*b*c*d^3*e^15 + 20*a*b^2*c^2*d^6*e^12
- 32*a^2*b*c^2*d^5*e^13 + 12*a^2*b^2*c*d^4*e^14) + (12*b^3*c^2*d^5*e^11*(d
+ e*x^2)^(1/2)*(d^3)^(1/2))/(18*c^5*d^10*e^8 + 72*a*c^4*d^8*e^10 + 2*a^4*c*
d^2*e^16 - 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^12 + 8*a^3*c^2*d^4*e^14 + 20
*b^2*c^3*d^8*e^10 + 12*b^3*c^2*d^7*e^11 - (4*b^2*c^4*d^10*e^8)/a + (10*b^3*
c^3*d^9*e^9)/a - (6*b^4*c^2*d^8*e^10)/a - 104*a*b*c^3*d^7*e^11 - 6*a*b^3*c*
d^5*e^13 - 8*a^3*b*c*d^3*e^15 + 20*a*b^2*c^2*d^6*e^12 - 32*a^2*b*c^2*d^5*e^
13 + 12*a^2*b^2*c*d^4*e^14) - (32*a*b*c^2*d^3*e^13*(d + e*x^2)^(1/2)*(d^3)^
(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*
d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20
*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*
b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2
 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c
^4*d^9*e^9)/a) + (12*a*b^2*c*d^2*e^14*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^
4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*
b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*
e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*
e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2
*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a
) - (8*a^2*b*c*d*e^15*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*
a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 +
 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c
^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b
^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a
*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a))*(d^3)^(1/2))/
a
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.370 $\int \frac{\left(d+ex^2\right)^{3/2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=417

$$-\frac{\sqrt{c}\left(-2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(d\sqrt{b^2-4ac}-2ae\right)+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-2\right.}{}$$

```
[Out] 1/2*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)/a+(-2*a*e+b*d)*arctanh((e*x^
2+d)^(1/2)/d^(1/2))*d^(1/2)/a^2-1/2*d*(e*x^2+d)^(1/2)/a/x^2-1/2*arctanh(2^(
1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2
)*(b^2*d^2+b*d*(-2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d^2+e*(-a*e+d*(-4*a*c+b
^2)^(1/2))))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))
)^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2
)^(1/2)))^(1/2))*c^(1/2)*(b^2*d^2-b*d*(2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d
^2-e*(a*e+d*(-4*a*c+b^2)^(1/2))))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 3.24, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1251, 897, 1287, 199, 206, 1166, 208}

$$-\frac{\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}-2ae\right)-2ae\left(d\sqrt{b^2-4ac}-ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-bd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(d*Sqrt[d + e*x^2])/(2*a*x^2) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
])/(2*a) + (Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a^2 - (
Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*e*(S
qrt[b^2 - 4*a*c]*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + 2*a*e*(S
qrt[b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x^3\left(a+bx^2+cx^4\right)}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{(d+ex)^{3/2}}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$

$$=\frac{\operatorname{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx,x,\sqrt{d+ex^2}\right)}{e}$$

$$=\frac{\operatorname{Subst}\left(\int \left(\frac{d^2e^2}{a\left(d-x^2\right)^2}-\frac{de(-bd+2ae)}{a^2\left(d-x^2\right)}+\frac{e\left(-(bd-ae)\left(cd^2-bde+ae^2\right)+cd(bd-2ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx,x,\sqrt{d+ex^2}\right.}{e}$$

$$=\frac{\operatorname{Subst}\left(\int \frac{-(bd-ae)\left(cd^2-bde+ae^2\right)+cd(bd-2ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx,x,\sqrt{d+ex^2}\right)}{a^2}+\frac{\left(d^2e\right)\operatorname{Subst}\left(\int \frac{1}{\left(d-x^2\right)^2}\right.}{a}$$

$$=-\frac{d\sqrt{d+ex^2}}{2ax^2}+\frac{\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2}+\frac{(de)\operatorname{Subst}\left(\int \frac{1}{d-x^2}\,dx,x,\sqrt{d+ex}\right.}{2a}$$

$$=-\frac{d\sqrt{d+ex^2}}{2ax^2}+\frac{\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a}+\frac{\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2}-\frac{\sqrt{c}\,\left(b^2d\right.}{}$$

Mathematica [A] time = 1.60, size = 380, normalized size = 0.91

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(\frac{\left(2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(2ae-d\sqrt{b^2-4ac}\right)-b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\left(bd\left(d\sqrt{b^2-4ac}+2ae\right)-2ae\left(d\sqrt{b^2-4ac}+ae\right)+2acd^2-b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt{2cd\ldots}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right.}{\sqrt{b^2-4ac}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*d*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(((-(b^2*d^2) + b*d*(-(Sqr
t[b^2 - 4*a*c]*d) + 2*a*e) + 2*a*(c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - a*e)))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*
c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e] - ((-(b^2*d^2) + 2*a*c*d^2
 - 2*a*e*(Sqrt[b^2 - 4*a*c]*d + a*e) + b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/Sqrt[b^2 - 4*a*c] - Sqrt[
d]*(2*b*d - 3*a*e)*Log[x] + Sqrt[d]*(2*b*d - 3*a*e)*Log[d + Sqrt[d]*Sqrt[d
+ e*x^2]])/(2*a^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.72, size = 433, normalized size = 1.04

$$-\frac{\left(2\,bd^2-3\,ade\right)\arctan\left(\frac{\sqrt{x^2e+d}}{\sqrt{-d}}\right)}{2\,a^2\sqrt{-d}}-\frac{\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\left(\left(b^2-2\,ac+\sqrt{b^2-4\,ac}\,b\right)d-\left(ab+\sqrt{b^2-\cdots\right.\right.}{4\sqrt{b^2-4\,ac}\,a^2|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d^2 - 3*a*d*e)*arctan(sqrt(x^2*e + d)/sqrt(-d))/(a^2*sqrt(-d)) -
1/4*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^2 - 2*a*c + sqrt(b
^2 - 4*a*c)*b)*d - (a*b + sqrt(b^2 - 4*a*c)*a)*e)*arctan(2*sqrt(1/2)*sqrt(x
^2*e + d)/sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3
*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(sqrt(b^2 - 4*a*c)*a^2*ab
s(c)) + 1/4*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^2 - 2*a*c
- sqrt(b^2 - 4*a*c)*b)*d - (a*b - sqrt(b^2 - 4*a*c)*a)*e)*arctan(2*sqrt(1/2
)*sqrt(x^2*e + d)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*b*d
*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(sqrt(b^2 - 4*a*c
)*a^2*abs(c)) - 1/2*sqrt(x^2*e + d)*d/(a*x^2)
```

maple [C] time = 0.04, size = 555, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -7/24/a^2*b*(e*x^2+d)^(3/2)+1/a^2*b*d^(3/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/
2))/x)-3/8/a^2*b*(e*x^2+d)^(1/2)*d-1/6/a^2*e^(3/2)*x^3*b+1/8/a^2*e*(e*x^2+d
)^(1/2)*x^2*b-3/4/a^2*e^(1/2)*x*b*d+1/2/a*e^(3/2)*x+1/a*(e*x^2+d)^(1/2)*e+1
/4/a^2*sum((c*d*(-2*a*e+b*d)*_R^6+(4*a^2*e^3-8*a*b*d*e^2+2*a*c*d^2*e+4*b^2*
d^2*e-3*b*c*d^3)*_R^4+d*(-4*a^2*e^3+8*a*b*d*e^2-2*a*c*d^2*e-4*b^2*d^2*e+3*b
*c*d^3)*_R^2+2*a*c*d^4*e-b*c*d^5)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^
2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)
^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2
)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))-1/2/a*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))*e+5/
8/a^2*d^2/(-e^(1/2)*x+(e*x^2+d)^(1/2))*b+1/24/a^2*b*d^3/(-e^(1/2)*x+(e*x^2+
d)^(1/2))^3-1/2/a/d/x^2*(e*x^2+d)^(5/2)+1/2/a*e/d*(e*x^2+d)^(3/2)-3/2/a*e*d
^(1/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^2+d\right)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^3), x)
```

mupad [B] time = 6.10, size = 35855, normalized size = 85.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d^(1/2)*atan(((d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16
 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^
4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*
d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^
3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^
8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11
 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4) - (d^(1/2)*((56*a^4*c^6*d^6*e^9 - 44*a^5
*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*
c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b
^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*
a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 +
 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a
*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c
^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/a^4 + (d^(1/2)
*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^1
2 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a
^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144
*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 +
 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e
^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d
^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*
d*e^12))/(2*a^4) + (d^(1/2)*((320*a^8*c^4*d*e^11 + 320*a^7*c^5*d^3*e^9 + 32
*a^5*b^3*c^4*d^4*e^8 - 24*a^5*b^4*c^3*d^3*e^9 - 8*a^5*b^5*c^2*d^2*e^10 + 16
*a^6*b^2*c^4*d^3*e^9 + 144*a^6*b^3*c^3*d^2*e^10 - 128*a^6*b*c^5*d^4*e^8 + 8
*a^6*b^4*c^2*d*e^11 - 448*a^7*b*c^4*d^2*e^10 - 112*a^7*b^2*c^3*d*e^11)/a^4
- (d^(1/2)*(d + e*x^2)^(1/2)*(3*a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^
4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*
d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*
d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2)
)*(3*a*e - 2*b*d))/(4*a^2))*1i)/(4*a^2) + (d^(1/2)*(3*a*e - 2*b*d)*(((d + e
*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*
a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5
*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c
^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*
c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e
^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4) + (d^(1/2)*((
56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^
3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3
*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*
a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12
- 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14
- 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3
*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3
*c^2*d*e^14)/a^4 - (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c
^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 +
736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 22
4*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10
+ 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e
^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3
*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*
d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4) - (d^(1/2)*((320*a^8*c^4*d*e^11
 + 320*a^7*c^5*d^3*e^9 + 32*a^5*b^3*c^4*d^4*e^8 - 24*a^5*b^4*c^3*d^3*e^9 -
8*a^5*b^5*c^2*d^2*e^10 + 16*a^6*b^2*c^4*d^3*e^9 + 144*a^6*b^3*c^3*d^2*e^10
- 128*a^6*b*c^5*d^4*e^8 + 8*a^6*b^4*c^2*d*e^11 - 448*a^7*b*c^4*d^2*e^10 - 1
12*a^7*b^2*c^3*d*e^11)/a^4 + (d^(1/2)*(d + e*x^2)^(1/2)*(3*a*e - 2*b*d)*(10
24*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5
```

*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c
^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6))*(3*a*e
- 2*b*d))/(4*a^2)))/(4*a^2))*(3*a*e - 2*b*d))/(4*a^2))*1i)/(4*a^2))/((3*a*c
^7*d^9*e^9 + 3*a^5*c^3*d*e^17 - 2*b*c^7*d^10*e^8 + 3*a^2*c^6*d^7*e^11 + 3*a
^4*c^4*d^3*e^15 + 4*b^2*c^6*d^9*e^9 - 2*b^3*c^5*d^8*e^10 + 2*a^2*b^2*c^4*d^
5*e^13 - (11*a^2*b^3*c^3*d^4*e^14)/2 + 11*a^3*b^2*c^3*d^3*e^15 - 8*a*b*c^6*
d^8*e^10 + 4*a*b^2*c^5*d^7*e^11 + a*b^4*c^3*d^5*e^13 - (3*a^2*b*c^5*d^6*e^1
2)/2 - 5*a^3*b*c^4*d^4*e^14 - (19*a^4*b*c^3*d^2*e^16)/2)/a^4 - (d^(1/2)*(3*
a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^
3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*
e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^
4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3
*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^
9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/
(2*a^4) - (d^(1/2)*((56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4
*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c
^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b
^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*
a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 -
28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*
c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4
*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/a^4 + (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x
^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 1
60*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2
*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*
a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 -
716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e
^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e
^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4) + (d^(1/2)
*((320*a^8*c^4*d*e^11 + 320*a^7*c^5*d^3*e^9 + 32*a^5*b^3*c^4*d^4*e^8 - 24*a
^5*b^4*c^3*d^3*e^9 - 8*a^5*b^5*c^2*d^2*e^10 + 16*a^6*b^2*c^4*d^3*e^9 + 144*
a^6*b^3*c^3*d^2*e^10 - 128*a^6*b*c^5*d^4*e^8 + 8*a^6*b^4*c^2*d*e^11 - 448*a
^7*b*c^4*d^2*e^10 - 112*a^7*b^2*c^3*d*e^11)/a^4 - (d^(1/2)*(d + e*x^2)^(1/2
)*(3*a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^
3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d
^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e
^9))/(8*a^6))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2))*(3*a*e - 2*b*d))/(4*a^2))
)/(4*a^2) + (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 +
4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d
^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5
*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d
^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e
^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 -
60*a^4*b*c^4*d^3*e^13))/(2*a^4) + (d^(1/2)*((56*a^4*c^6*d^6*e^9 - 44*a^5*c^
5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3
*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*
c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4
*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 11
1*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^
6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*
d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/a^4 - (d^(1/2)*(3
*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 -
16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*
b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^
3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 14
4*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11
+ 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*
e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e
^12))/(2*a^4) - (d^(1/2)*((320*a^8*c^4*d*e^11 + 320*a^7*c^5*d^3*e^9 + 32*a^

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5*b^3*c^4*d^4*e^8 - 24*a^5*b^4*c^3*d^3*e^9 - 8*a^5*b^5*c^2*d^2*e^10 + 16*a^
6*b^2*c^4*d^3*e^9 + 144*a^6*b^3*c^3*d^2*e^10 - 128*a^6*b*c^5*d^4*e^8 + 8*a^
6*b^4*c^2*d*e^11 - 448*a^7*b*c^4*d^2*e^10 - 112*a^7*b^2*c^3*d*e^11)/a^4 + (
d^(1/2)*(d + e*x^2)^(1/2)*(3*a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c
^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2
*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e
^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2))*(
3*a*e - 2*b*d))/(4*a^2)))/(4*a^2)))*(3*a*e - 2*b*d)*1i)/(2*a^2) - atan(((((
224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2
*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432
*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11
+ 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*
e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d
*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10
- 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12
+ 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^
3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d
^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5
*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^
3*d*e^11)/(4*a^4) - ((d + e*x^2)^(1/2)*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a
^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*
a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^
3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a
^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^
2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2
*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c
^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4
*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*
d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2
)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^
8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^
8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*((
((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^
2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d
^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 -
 (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*
d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d
^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*
b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e
^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e +
42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4
+ 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13
 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6
*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b
^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a
^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 1
32*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^
11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^1
1 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^
3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a
*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3
*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^
5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2
*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*
d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^
3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*
c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d
^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
)*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^
```

4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b
^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2
/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*
c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c
^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2)
+ 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2
*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*
e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*
b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^1
6 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c
^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4
*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c
^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d
^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^1
1 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3
*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*
b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*
b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5
*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*
d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d
^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3
*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c
*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^
2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*
1i - (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 +
160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^
11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d
^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*
c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6
*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d
^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d
^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^
7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b
^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a
^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^
7*b^2*c^3*d*e^11)/(4*a^4) + ((d + e*x^2)^(1/2)*((((4*b^6*d^3 - 4*a^3*b^3*e^
3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d
^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e
- 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2
- 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 +
3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6
*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 -
16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 -
16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^
3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c
)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 +
1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 -
1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*
a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 9
6*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12
*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^
2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 +
3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3
*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1
/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4
*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*
d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(
a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(64*a^7*b*
c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 +

736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 2
24*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10
 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*
e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^
3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4
*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3
 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^
3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e -
 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2
 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3
*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*
a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 1
6*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 1
6*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3
*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)
))^(1/2))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2
 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3
 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*
d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^
6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2
 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4)
)^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48
*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*
b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(
16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(4*a^6
*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 -
 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2
*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a
^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b
^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5
*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3
- 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3
 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e -
144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 -
 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*
a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a
*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16
*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16
*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*
b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))
)^(1/2)*1i)/((6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2
*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10
 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^
15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*
a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16)/(2*a^4)
 + (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 1
60*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11
 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4
*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^
3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b
*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5
*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3
*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*
c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5
*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6
*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*
b^2*c^3*d*e^11)/(4*a^4) - ((d + e*x^2)^(1/2)*((((4*b^6*d^3 - 4*a^3*b^3*e^3
- 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3
 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e -

144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a *b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) + ((d + e*x^2)^(1/2)*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 +

48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i - (d*(d + e*x^2)^(1/2))/(2*a*x^2) - atan(((((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*

b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*
b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*
b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2
)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160
*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b
^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^
3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 71
6*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^1
0 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^1
2 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*(-(((4*b^6*
d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2
+ 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 8
4*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4
*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2
- b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3
*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3
+ 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*
a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b
^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6
*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^
3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d
^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d
^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)
*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4
+ 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3
+ 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 -
6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 -
8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 3
6*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d +
e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 13
2*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c
^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2
*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*
b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7
*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*(-(((4*b^6
*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2
+ 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e +
84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^
4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2
- b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e -
3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3
+ 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36
*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*
b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^
6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*
e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5
*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4
*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^
2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a
^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^
6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^
5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((
1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a
^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576
*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 179
2*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) + ((d + e*x^2)^(1/2)
*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^
4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b
^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2

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/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*
c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c
^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2)
- 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2
*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*
e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*
b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2
*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e
^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9
 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3
*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*
b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*
b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5
*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*
d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d
^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3
*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c
*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^
2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
- ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c
^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e
^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4
*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4
*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^
2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*
b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4)
)*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a
^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*
b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^
2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a
*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*
c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2)
 - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^
2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2
*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4
*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 -
32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 -
 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 14
4*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 1
28*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^
2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b
*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a
^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a
*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*
c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^
(1/2) + ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*
d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 +
129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12
 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e
^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*
a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4
))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*
a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a
*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)
^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*
a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b
*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2
) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c
^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^
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2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^
4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((6*a*c^7*d^9*e^9 + 6*a^5*c^3
*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^
2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^
3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^
7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^1
4 - 19*a^4*b*c^3*d^2*e^16)/(2*a^4) + (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d
^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*
d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*
c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4
*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 44
4*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a
*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b
*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) +
(((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 9
6*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 +
576*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 -
1792*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^3*d*e^11))/(4*a^4) - ((d + e*x^2)^(1
/2)*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96
*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*
a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2
)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3
*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*
b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/
2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*
c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d
^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a
^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*
c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^
2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*
e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*
a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32
*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a
^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*
a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c
^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^
2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*
c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^
4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2
*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/
2) + ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^
3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^
5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*
d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*
c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5
*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a
^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a
^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 9
6*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12
*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^
2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 +
3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3
*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1
/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4
*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*
d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(
a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3
 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^
3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e -
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144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) + ((d + e*x^2)^(1/2)*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^

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3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 14
4*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11
 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*
e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e
^12))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4
*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*
c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*
b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^
3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d
^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^
2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^
2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3
+ 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*
e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(((4*b^6*d^3 - 4*a
^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*
b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3
*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 25
6*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d
^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*
d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b
^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c
^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*
e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*
a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8
 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c
^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4
*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*
b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5
*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*
e^13))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^
4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b
*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3
*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a
^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*
d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d
^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e
^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3
 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d
*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)))*(-(((4*b^6*d^3 - 4
*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^
2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b
^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 +
256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c
*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*
c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3
*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2
*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^
2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 -
8*a^5*b^2*c)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.371 $\int \frac{x^4\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=595

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e}\cdots}{}$$

```
[Out] 1/4*x*(e*x^2+d)^(3/2)/c+1/8*d*(-4*b*e+3*c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1
/2))/c^2/e^(1/2)-1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(b*c*d-b^2*e+a*c*e+
(-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/c^3-1/2*ar
ctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^2*d-b^
3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/c^3+1/8*(-4*b*e+3*c*d)*x*(e*x^2+d)
^(1/2)/c^2-1/2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(-3*a*b*c*e+2*a*c^2*d+
b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/c
^3/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(
3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+
b^2)^(1/2)))^(1/2)/c^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 3.28, antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1291, 388, 195, 217, 206, 1692, 402, 377, 205}

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*c^2) + (x*(d + e*x^2)^(3/2))/(4*c) -
 (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d
 - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(2*c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) + (d*(3*c*d - 4*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c^2*Sqrt[e]
) - (Sqrt[e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3) - (Sq
rt[e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1291

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[((f*x)^(m - 4)*(d +
 e*x^2)^(q - 1)*Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x])/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{\int \sqrt{d+ex^2}\left(cd-be+cex^2\right)dx}{c^2} - \frac{\int \frac{\sqrt{d+ex^2}\left(a(cd-be)+\left(bcd-b^2e+ace\right)x^2\right)}{a+bx^2+cx^4}\,dx}{c^2}$$

$$= \frac{x\left(d+ex^2\right)^{3/2}}{4c} - \frac{\int \left(\frac{\left(bcd-b^2e+ace+\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(bcd-b^2e+ace-\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}\right)\sqrt{d}}{b+\sqrt{b^2-4ac}+2cx^2}\right.}{c^2}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x\left(d+ex^2\right)^{3/2}}{4c} + \frac{(d(3cd-4be))\int \frac{1}{\sqrt{d+ex^2}}\,dx}{8c^2} - \frac{\left(bcd-b^2e+ace\right.}{}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x\left(d+ex^2\right)^{3/2}}{4c} + \frac{(d(3cd-4be))\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{8c^2} - $$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x\left(d+ex^2\right)^{3/2}}{4c} + \frac{d(3cd-4be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}} - \frac{\left(e\left(bcd-b^2e+\right.\right.}{}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x\left(d+ex^2\right)^{3/2}}{4c} - \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bcd-b^2e+ace-\right.}{2c^3\sqrt{b-\sqrt{}}}$$

Mathematica [B] time = 6.49, size = 18689, normalized size = 31.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 1.87, size = 104, normalized size = 0.17

$$\frac{1}{8}\sqrt{x^2e+d}\left(\frac{2x^2e}{c} + \frac{\left(5c^5de^2-4bc^4e^3\right)e^{(-2)}}{c^6}\right)x - \frac{\left(3c^2d^2-12bcde+8b^2e^2-8ace^2\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right.}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/8*sqrt(x^2*e + d)*(2*x^2*e/c + (5*c^5*d*e^2 - 4*b*c^4*e^3)*e^(-2)/c^6)*x
- 1/16*(3*c^2*d^2 - 12*b*c*d*e + 8*b^2*e^2 - 8*a*c*e^2)*e^(-1/2)*log((x*e^(
1/2) - sqrt(x^2*e + d))^2)/c^3
```

maple [C] time = 0.04, size = 516, normalized size = 0.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*x*(e*x^2+d)^(3/2)/c+3/8/c*d*x*(e*x^2+d)^(1/2)+3/8/c*d^2/e^(1/2)*ln(e^(1
/2)*x+(e*x^2+d)^(1/2))+1/4/c^2*e^(3/2)*b*x^2-1/4/c^2*e*b*(e*x^2+d)^(1/2)*x+
1/8/c^2*e^(1/2)*b*d-1/2/c^3*e^(1/2)*sum(((2*a*b*c*e^2-2*a*c^2*d*e-b^3*e^2+2
*b^2*c*d*e-b*c^2*d^2)*_R^2+2*(2*a^2*c*e^3-2*a*b^2*e^3+2*a*b*c*d*e^2+b^3*d*e
^2-2*b^2*c*d^2*e+b*c^2*d^3)*_R+2*a*b*c*d^2*e^2-2*a*c^2*d^3*e-b^3*d^2*e^2+2*
b^2*c*d^3*e-b*c^2*d^4)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+
3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(
_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-
4*c*d^3)*_Z))+1/c^2*e^(3/2)*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))*a-1/c^3*e^(3/2)*
ln(-e^(1/2)*x+(e*x^2+d)^(1/2))*b^2+3/2/c^2*e^(1/2)*ln(-e^(1/2)*x+(e*x^2+d)^
(1/2))*b*d-1/8/c^2*e^(1/2)*b*d^2/(-e^(1/2)*x+(e*x^2+d)^(1/2))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x^4/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(e\,x^2+d\right)}^{3/2}}{c\,x^4+b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(d + ex^{2}\right)^{\frac{3}{2}}}{a + bx^{2} + cx^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

3.372 $\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=491

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\right.}{}$$

```
[Out] 1/2*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)/c+1/2*arctanh(x*e^(1/2)/(e
*x^2+d)^(1/2))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/
c^2+1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(
-4*a*c+b^2)^(1/2))*e^(1/2)/c^2+1/2*e*x*(e*x^2+d)^(1/2)/c+1/2*arctan(x*(2*c*
d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/c^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
)*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2)))^(1/2)/c^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.80, antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1293, 195, 217, 206, 1692, 402, 377, 205}

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*x*Sqrt[d + e*x^2])/(2*c) + (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d
 - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])
])/(2*c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(2*c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqr
t[e]*x)/Sqrt[d + e*x^2]])/(2*c) + (Sqrt[e]*(c*d - b*e - (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2) + (
Sqrt[e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(
Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx = -\frac{\int \frac{\sqrt{d+ex^2}\left(ae-(cd-be)x^2\right)}{a+bx^2+cx^4}\,dx}{c} + \frac{e\int \sqrt{d+ex^2}\,dx}{c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} - \frac{\int \left(\frac{\left(-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(-cd+be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{c} + \frac{(de)\int \frac{1}{\sqrt{d+ex}}}{2c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{(de)\,\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2c}}{c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{d\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(e\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c^2} + \frac{\left(\left(2cd-\right.\right.}{}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{d\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(e\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, -\right.}{2c^2}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+e}}\right.}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [B] time = 6.26, size = 14032, normalized size = 28.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 1.97, size = 58, normalized size = 0.12

$$\frac{\sqrt{x^2e+d}\,xe}{2\,c} - \frac{\left(3\,cde-2\,be^2\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/2*sqrt(x^2*e + d)*x*e/c - 1/4*(3*c*d*e - 2*b*e^2)*e^(-1/2)*log((x*e^(1/2)
 - sqrt(x^2*e + d))^2)/c^2
```

maple [C] time = 0.03, size = 382, normalized size = 0.78

$$-\frac{e^{\frac{3}{2}} x^{2}}{4 c}+\frac{b\, e^{\frac{3}{2}} \ln \left(-\sqrt{e}\, x+\sqrt{e\,x^{2}+d}\right)}{c^{2}}+\frac{d^{2} \sqrt{e}}{8 \left(-\sqrt{e}\, x+\sqrt{e\,x^{2}+d}\right)^{2} c}-\frac{3 d \sqrt{e}\, \ln \left(-\sqrt{e}\, x+\sqrt{e\,x^{2}+d}\right)}{2 c}+\frac{\sqrt{e\,x^{2}+d}\, e}{4 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*e^(3/2)/c*x^2+1/4*e*x*(e*x^2+d)^(1/2)/c-1/8*e^(1/2)/c*d+1/2*e^(1/2)/c^
2*sum(((a*c*e^2-b^2*e^2+2*b*c*d*e-c^2*d^2)*_R^2+2*(-2*a*b*e^3+3*a*c*d*e^2+b
^2*d*e^2-2*b*c*d^2*e+c^2*d^3)*_R+a*c*d^2*e^2-b^2*d^2*e^2+2*b*c*d^3*e-c^2*d^
4)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c
*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-
4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+1/8*e^
(1/2)/c*d^2/(-e^(1/2)*x+(e*x^2+d)^(1/2))^2+e^(3/2)/c^2*ln(-e^(1/2)*x+(e*x^2
+d)^(1/2))*b-3/2*e^(1/2)/c*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}} x^2}{cx^4+bx^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x^2/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left(e\,x^2+d\right)^{3/2}}{c\,x^4+b\,x^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(d+ex^2\right)^{\frac{3}{2}}}{a+bx^2+cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

3.373 $\int \frac{\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=487

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}\right.\right.}{c\sqrt{}}$$

```
[Out] 1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(3*c*d-e*(b-(-4*a*c+b^2)^(1/2)))*e^(
1/2)/c/(-4*a*c+b^2)^(1/2)-1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(3*c*d-e*(
b+(-4*a*c+b^2)^(1/2)))*e^(1/2)/c/(-4*a*c+b^2)^(1/2)+arctan(x*(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(2*c
^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/c
/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-2*c
*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/c/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.57, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1174, 416, 523, 217, 206, 377, 205}

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}\right.\right.}{c\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c
]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sq
rt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((2*c^2*d^2 + b*(b + S
qrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[e]*(3*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c]) - (Sqrt[e]*(
3*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c*Sqrt[b^2 - 4*a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{(d+ex^2)^{3/2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^2)^{3/2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\int \frac{d\left(4cd-\left(b-\sqrt{b^2-4ac}\right)e\right)+2e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{2\sqrt{b^2-4ac}} - \frac{\int \frac{d\left(4cd-\left(b+\sqrt{b^2-4ac}\right)e\right)+2e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,c}{2\sqrt{b^2-4ac}}$$

$$= \frac{\left(e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c\sqrt{b^2-4ac}} - \frac{\left(e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c\sqrt{b^2-4ac}} +$$

$$= \frac{\left(e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}} - \frac{\left(e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right.}{2c\sqrt{b}}$$

$$= \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} -$$

Mathematica [B] time = 6.16, size = 9290, normalized size = 19.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 57.45, size = 7721, normalized size = 15.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2
 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 +
 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^
2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*
c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*
b^2 + 3*a^3*c)*d^2*e^4 + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*
b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3
*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^
2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*
b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^
3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 + 2*sqrt(1/2)*s
qrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3
)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*
e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) + ((a*b^2*c
^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*
b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*
a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*
b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2
```

```
*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c
^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2
 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c
^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*
b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^
2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 + ((a
*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2
- 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3
 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*
c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*
d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5
*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*
c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c
^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*
b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a
^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^
3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*
a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^
2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c
^2 - 4*a^2*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a
*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6
- 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5
 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2
*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^
3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 -
(a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt
((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2
*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c
^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c
^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*
x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^
3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e
^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^
3*c^5)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3
+ (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*
d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a
*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^
2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) + sqr
t(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*
c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^
2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4
 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e
 - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a
^3*c)*d^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^
2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2
+ 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a
^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (
b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^
2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2
 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sq
rt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a
^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - ((a*b^2*c^2 - 4*a^
2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)
*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^
3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3
*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*
a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) + 2*e^(3/2)*log(-2*e*x^2 - 2*sqrt
(e*x^2 + d)*sqrt(e)*x - d))/c, 1/4*(sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*
d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqr
```

t((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*

```
a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)*d^3
- (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sq
rt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a
^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b
*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2
*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5
)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*
b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3
*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*
a^3*c^5)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^
3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*
c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6
*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 +
a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) - 4
*sqrt(-e)*e*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/c]
```

giac [A] time = 1.97, size = 27, normalized size = 0.06

$$-\frac{e^{\frac{3}{2}} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2 e + d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*e^(3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

maple [C] time = 0.03, size = 217, normalized size = 0.45

$$-\frac{e^{\frac{3}{2}} \ln\left(-\sqrt{e}\, x + \sqrt{e\,x^2 + d}\right)}{c} + \frac{}{2c\left(\mathrm{RootOf}\left(\_Z^4 c + c\,d^4 + (4be - 4cd)\,\_Z^3 + \left(16a\,e^2 - 8deb + 6c\,d^2\right)\_Z^2 + \left(4b\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*e^(3/2)/c*sum(((b*e-2*c*d)*_R^2+2*e*(2*a*e-b*d)*_R+b*d^2*e-2*c*d^3)/(_R
^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*
ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)
*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-e^(3/2)/c*ln
(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^{\frac{3}{2}}}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2 + d\right)^{3/2}}{c\,x^4 + b\,x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex^2\right)^{\frac{3}{2}}}{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

3.374 $\int \frac{\left(d+ex^2\right)^{3/2}}{x^2\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=260

$$-\frac{\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{3/2}}+\frac{\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4a\right.}}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{a}}\right.}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{3/2}}$$

```
[Out] -arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+b^2)^
(1/2))^(3/2)/(-4*a*c+b^2)^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(
1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)
^(1/2)))^(3/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(3/2)-d*(e*x^2+d)^
(1/2)/a/x
```

Rubi [A] time = 0.85, antiderivative size = 432, normalized size of antiderivative = 1.66, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1295, 277, 217, 206, 1692, 402, 377, 205}

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(d-\frac{bd-2a}{\sqrt{b^2-4}}\right.}{2a\sqrt{\sqrt{b^2-4a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d*Sqrt[d + e*x^2])/(a*x)) - (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(d
+ (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(d - (b*d - 2*
a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/
(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a*Sqrt[b + Sqrt[b^2 - 4*
a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/a - (Sqrt[e]*(d -
 (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*
a) - (Sqrt[e]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]])/(2*a)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{\left(bd-ae+cdx^2\right)\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx}{a}+\frac{d\int \frac{\sqrt{d+ex^2}}{x^2}\,dx}{a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax}-\frac{\int \left(\frac{\left(cd+\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(cd-\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{a}+\frac{(de)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax}+\frac{(de)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{a}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+}}{a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a}-\frac{\left(e\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2a}-\frac{\left(\left(2cd-\right(}{}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a}-\frac{\left(e\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+}}\right.}{2a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax}-\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right.}{2a\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [B] time = 6.30, size = 7789, normalized size = 29.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
[Out] Result too large to show
```

fricas [B] time = 29.11, size = 4059, normalized size = 15.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] -1/4*(sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3
- 3*(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 -
 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*
e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4
*c))*log(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2
*(a*b^3 + 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 + ((a^3*b^2*c -
4*a^4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x
^2*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^
6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 -
 4*a^7*c)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^
4 + 6*a*b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*
a^2*b^2 + 4*a^3*c)*d^3*e^3)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 -
4*a^5*b*c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d
^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5
*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c +
 4*a^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2
```

```
*e^2)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2
- 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*
e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^
2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2)
- sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*
(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a
^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e -
3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))
*log(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*
b^3 + 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 + ((a^3*b^2*c - 4*a^
4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x^2*s
qrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 +
6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a
^7*c)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 +
6*a*b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*
b^2 + 4*a^3*c)*d^3*e^3)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^
5*b*c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e
^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2
*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a
^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2
)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*
a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4
- (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^
2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sq
rt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b
^2 - 2*a^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d
^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5
*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log
(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*b^3
+ 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 - ((a^3*b^2*c - 4*a^4*c^
2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x^2*sqrt(
-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a
*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c
)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 + 6*a*
b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*b^2
+ 4*a^3*c)*d^3*e^3)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^5*b*
c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 -
 (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2
 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c
^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2)*x)
*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*
c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b
^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 -
2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) + sqrt(1
/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 -
 2*a^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e
^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2
*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(1
2*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*b^3 + 2*
a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 - ((a^3*b^2*c - 4*a^4*c^2)*d
^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x^2*sqrt(-(18
*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3
 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) +
 (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 + 6*a*b^2*
c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*b^2 + 4*
a^3*c)*d^3*e^3)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^5*b*c)*d
 - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^
4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2
*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*
```

```
d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2)*x)*sqr
t(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d
^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 -
 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^
3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) + 4*sqrt(e*x
^2 + d)*d)/(a*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.04, size = 360, normalized size = 1.38

$$\frac{e^{\frac{3}{2}} x^{2}}{4 a}-\frac{d^{2} \sqrt{e}}{8 \left(-\sqrt{e}\, x+\sqrt{e\,x^{2}+d}\right)^{2} a}+\frac{3 d \sqrt{e}\, \ln \left(-\sqrt{e}\, x+\sqrt{e\,x^{2}+d}\right)}{2 a}+\frac{3 d \sqrt{e}\, \ln \left(\sqrt{e}\, x+\sqrt{e\,x^{2}+d}\right)}{2 a}+\frac{5 \sqrt{e\,x^{2}+d}}{4 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/d/x*(e*x^2+d)^(5/2)+1/a*e/d*x*(e*x^2+d)^(3/2)+5/4/a*e*x*(e*x^2+d)^(1/2
)+3/2/a*e^(1/2)*d*ln(e^(1/2)*x+(e*x^2+d)^(1/2))+1/4/a*e^(3/2)*x^2+1/8/a*e^(
1/2)*d-1/2/a*e^(1/2)*sum(((a*e^2-c*d^2)*_R^2+2*d*(3*a*e^2-2*b*d*e+c*d^2)*_R
+a*d^2*e^2-c*d^4)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*
c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*
c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d
^3)*_Z))-1/8/a*e^(1/2)*d^2/(-e^(1/2)*x+(e*x^2+d)^(1/2))^2+3/2/a*e^(1/2)*d*l
n(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^{2}+d\right)^{\frac{3}{2}}}{\left(cx^{4}+bx^{2}+a\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}}{x^2\,\left(c\,x^4+b\,x^2+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int((d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex^2\right)^{\frac{3}{2}}}{x^2 \left(a + bx^2 + cx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**2/(c*x**4+b*x**2+a),x)
[Out] Integral((d + e*x**2)**(3/2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

3.375 $\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=523

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\right.}{}$$

```
[Out] -1/3*(e*x^2+d)^(3/2)/a/x^3-(-a*e+b*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^
(1/2)/a^2+1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^
2*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/a^2+1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))
*(b*d-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/a^2+(-a*e+b*d)
*(e*x^2+d)^(1/2)/a^2/x+1/2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/
(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(-a*b*e-2*a*c*d+b^2*
d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/a^2/(b-(-4*a*
c+b^2)^(1/2))^(1/2)+1/2*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*
x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^2*d)/(
-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a^2/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

Rubi [A] time = 2.62, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {1295, 264, 6728, 277, 217, 206, 1692, 402, 377, 205}

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*d - a*e)*Sqrt[d + e*x^2])/(a^2*x) - (d + e*x^2)^(3/2)/(3*a*x^3) + (Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e - (b^2*d - 2*a*c*
d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b + Sqrt[
b^2 - 4*a*c]]) - (Sqrt[e]*(b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/a^2 + (Sqrt[e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a^2) + (Sqrt[e]*(b*d - a*e + (b^2*d
 - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/(2*a^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x^4\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{\left(bd-ae+cdx^2\right)\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx}{a} + \frac{d\int \frac{\sqrt{d+ex^2}}{x^4}\,dx}{a}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\int \left(\frac{(bd-ae)\sqrt{d+ex^2}}{ax^2} + \frac{\sqrt{d+ex^2}\left(-b^2d+acd+abe-c(bd-ae)x^2\right)}{a\left(a+bx^2+cx^4\right)}\right)dx}{a}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\int \frac{\sqrt{d+ex^2}\left(-b^2d+acd+abe-c(bd-ae)x^2\right)}{a+bx^2+cx^4}\,dx}{a^2} - \frac{(bd-ae)\int \frac{\sqrt{d+ex^2}}{x^2}\,dx}{a^2}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\int \left(\frac{\left(-c(bd-ae)-\frac{c\left(b^2d-2acd-abe\right)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(-c(bd-ae)+\frac{c(\cdots}{\cdots}\right.}{b+\sqrt{b\cdots}}\cdots\right.}{a^2}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{(e(bd-ae))\operatorname{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{(c\cdots}{\cdots}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\sqrt{e}\,(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(e\left(bd-ae-\cdots\right.\right.}{\cdots}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\sqrt{e}\,(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(e\left(bd-ae-\cdots\right.\right.}{\cdots}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} + \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bd-ae+\frac{b^2d-2acd\cdots}{\sqrt{b^2-4\cdots}}\right.\cdots}{2a^2\sqrt{b-\sqrt{b^2-4\cdots}}}$$

Mathematica [B] time = 6.41, size = 9321, normalized size = 17.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 144.24, size = 7830, normalized size = 14.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(1/2)*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a
*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^
3*b^2 - 2*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5
*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*
a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*
c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^
3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((2*a^5
*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a
```

```
^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*
a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2*e^4 + ((
a^5*b^2*c^2 - 4*a^6*c^3)*d^3 - (a^5*b^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b^2*c
 - 4*a^7*c^2)*d*e^2)*x^2*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2
 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^
5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(
5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5
)/(a^10*b^2 - 4*a^11*c)) + (4*a^5*b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*
c^4)*d^6 - (b^6*c + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*
a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*
b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*
a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4
 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a^7*b^3 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e^
6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a
*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20
*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b
^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2
*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^
2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d^4 - (4*a^2*b^6 - 25*a^3*b^4*c + 3
7*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c
^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*
a^6*b*c)*e^4)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*
a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2
*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^
3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c
^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*
e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b
*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)
*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*
c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*
e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^
2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a
^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*
(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^
5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a
*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e -
 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^4
*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2*e^4 + ((a^5*b^2*c^2 - 4*a
^6*c^3)*d^3 - (a^5*b^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b^2*c - 4*a^7*c^2)*d*e
^2)*x^2*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 -
2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^
5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a
^11*c)) + (4*a^5*b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c
 + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b
^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c
^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^
5*c^2)*d*e^5)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c +
8*a^8*c^2)*d - (a^7*b^3 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*
c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a
^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c
^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3
 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 13*a^3*
b^3*c^2 - 4*a^4*b*c^3)*d^4 - (4*a^2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4
```

```
*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*
a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x)*
sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3
*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 + (a^
5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4
*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d
^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^
4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*
b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)*a^2*x^3*sqrt(-((
b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^
2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 -
4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*
d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 -
 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a
^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*
a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*
b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c -
 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 -
 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2*e^4 - ((a^5*b^2*c^2 - 4*a^6*c^3)*d^3 - (a^
5*b^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b^2*c - 4*a^7*c^2)*d*e^2)*x^2*sqrt((a^6
*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6
 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b
^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 3
0*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c
^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) + (4*a^5*
b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c + 4*a*b^4*c^2 -
17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b
*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(
4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2
 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a
^7*b^3 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2
 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^
5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(
5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5
)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b
*c^3)*d^4 - (4*a^2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e +
 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*
b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x)*sqrt(-((b^5 - 5*a
*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(
a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^6*c)*
sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3
*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^
3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c
+ 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))
/(a^5*b^2 - 4*a^6*c)))/x^2) + 3*sqrt(1/2)*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3
- 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^6*c)*sqrt((a^
6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^
6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*
b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 -
30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*
c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^
2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4
)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 -
a^4*c^3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2*c -
```

```
 3*a^5*c^2)*d^2*e^4 - ((a^5*b^2*c^2 - 4*a^6*c^3)*d^3 - (a^5*b^3*c - 4*a^6*b
*c^2)*d^2*e + (a^6*b^2*c - 4*a^7*c^2)*d*e^2)*x^2*sqrt((a^6*b^2*e^6 + (b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a
^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c
 + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*
a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(
a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) + (4*a^5*b*c*e^6 + (b^5*c^
2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 +
4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2
*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a
^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2 - 2*sqrt(1/2)*sq
rt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a^7*b^3 - 4*a^8*b*
c)*e)*x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 -
2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^
5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a
^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d^4 - (4*a^
2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 1
1*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^
2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*
c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b
*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6
 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*
b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*
a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^
3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*
e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6
*c)))/x^2) + 4*((3*b*d - 4*a*e)*x^2 - a*d)*sqrt(e*x^2 + d))/(a^2*x^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.04, size = 511, normalized size = 0.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a^2*b/d/x*(e*x^2+d)^(5/2)-1/a^2*b*e/d*x*(e*x^2+d)^(3/2)-5/4/a^2*b*e*x*(e*
x^2+d)^(1/2)-3/2/a^2*b*e^(1/2)*d*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/4/a^2*e^(3
/2)*x^2*b-1/8/a^2*e^(1/2)*b*d+1/2/a^2*e^(1/2)*sum((c*d*(2*a*e-b*d)*_R^2+2*(
-2*a^2*e^3+4*a*b*d*e^2-2*b^2*d^2*e+b*c*d^3)*_R+2*a*c*d^3*e-b*c*d^4)/(_R^3*c
+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-
_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^
3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+1/a*e^(3/2)*ln(-
e^(1/2)*x+(e*x^2+d)^(1/2))-3/2/a^2*e^(1/2)*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))*b
*d+1/8/a^2*e^(1/2)*b*d^2/(-e^(1/2)*x+(e*x^2+d)^(1/2))^2-1/3/a/d/x^3*(e*x^2+
d)^(5/2)-2/3/a*e/d^2/x*(e*x^2+d)^(5/2)+2/3/a*e^2/d^2*x*(e*x^2+d)^(3/2)+1/a*
e^2/d*x*(e*x^2+d)^(1/2)+1/a*e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^{3/2}}{x^4\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**4/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.376 $\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=281

$$\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{}}{\sqrt{\sqrt{b^2-4ac}}}\right.}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[Out] -1/3*(-x^2+1)^(3/2)/c-b*(-x^2+1)^(1/2)/c^2+1/2*arctanh(2^(1/2)*c^(1/2)*(-x^
2+1)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*c+(3*a*b*c+2*a*c^2-
b^3-b^2*c)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(
1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^
(1/2))*(b^2-a*c+b*c+(-3*a*b*c-2*a*c^2+b^3+b^2*c)/(-4*a*c+b^2)^(1/2))/c^(5/2
)*2^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 7.34, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 897, 1287, 1166, 208}

$$\frac{\left(-\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{}}{\sqrt{\sqrt{b^2-4ac}}}\right.}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*Sqrt[1 - x^2])/c^2) - (1 - x^2)^(3/2)/(3*c) + ((b^2 - a*c + b*c - (b^3
 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*S
qrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b +
 2*c - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2
*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b +
 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*
c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{1-x}\,x^2}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= -\operatorname{Subst}\left(\int \frac{x^2\left(1-x^2\right)^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)$$

$$= -\operatorname{Subst}\left(\int \left(\frac{b}{c^2}+\frac{x^2}{c}-\frac{b(a+b+c)-\left(b^2-ac+bc\right)x^2}{c^2\left(a+b+c+(-b-2c)x^2+cx^4\right)}\right)dx, x, \sqrt{1-x^2}\right)$$

$$= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}+\frac{\operatorname{Subst}\left(\int \frac{b(a+b+c)+\left(-b^2+ac-bc\right)x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{c^2}$$

$$= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}-\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+ \cdots}\right.}{2c^2}$$

$$= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}+\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b+2c-\sqrt{b^2-4ac}}}+ \cdots$$

Mathematica [A] time = 0.54, size = 354, normalized size = 1.26

$$\frac{-\frac{3\sqrt{2}\left(b^2\left(\sqrt{b^2-4ac}+c\right)+bc\left(\sqrt{b^2-4ac}-3a\right)-ac\left(\sqrt{b^2-4ac}+2c\right)+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b-2c}}-\frac{3\sqrt{2}\left(b^2\left(\sqrt{b^2-4ac}-c\right)+bc\left(\sqrt{b^2-4ac}+3a\right)+ac\left(2c \cdots\right.\right.}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4 \cdots}}}}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-6*b*Sqrt[c]*Sqrt[1 - x^2] - 2*c^(3/2)*(1 - x^2)^(3/2) - (3*Sqrt[2]*(b^3 +
 b*c*(-3*a + Sqrt[b^2 - 4*a*c]) + b^2*(c + Sqrt[b^2 - 4*a*c]) - a*c*(2*c +
Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]])
 - (3*Sqrt[2]*(-b^3 + a*c*(2*c - Sqrt[b^2 - 4*a*c]) + b*c*(3*a + Sqrt[b^2 -
 4*a*c]) + b^2*(-c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x
^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c
+ Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

fricas [B] time = 16.52, size = 3615, normalized size = 12.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*
a*b^3 - b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2
)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 +
b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)
)*log(-(2*a^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*
b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4
- 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^
5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^
4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 +
 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b
^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*
b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a
^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*
x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c -
(b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*
b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a
*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 +
(a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 3*sqr
t(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4
)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(
3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 -
2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a
^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b
^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5
+ b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b
)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 -
 2*(3*a^4*b^2 - a^3*b^3)*c - sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c
^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*
b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*
c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14
*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((
b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c - (b^2*c^5 -
4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2
*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)
*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^
4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 3*sqrt(1/2)*c^2*
sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 -
 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6
- b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a^3*b^4 - (a
^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2
*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2
- 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (a^
2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 - 2*(3*a^4*b
^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*x^2*sqr
t((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b
^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^1
0 - 4*a*c^11)) - (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c
^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2
*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c^6)*sq
rt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*
b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^
```

```
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 -
(3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) + 3*sqrt(1/2)*c^2*sqrt((b^5 +
 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3
+ 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(
b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a^3*b^4 - (a^2*b^2*c^5
- 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2
 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^
6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^
4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^
3)*c - sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a
^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (
11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^1
1)) - (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^
2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a
^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (
a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 +
(11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^
11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2
- a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 2*(c*x^2 - 3*b - c)*sqrt(-x^2 + 1))/c^
2
```

giac [B] time = 4.48, size = 4637, normalized size = 16.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 6*b^5*c^5 + 24*a^2*b^2*c^6 - 40*a*b^3*c^6
+ 4*b^4*c^6 + 64*a^2*b*c^7 - 24*a*b^2*c^7 + 32*a^2*c^8 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 5*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4
 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c
)*c)*b^4*c^4 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 -
4*a*c)*c)*a^2*b*c^5 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2
 - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^6 + 20*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^7 - 2*(b^2 - 4*a*c)*b^4*c^4
 + 6*(b^2 - 4*a*c)*a*b^2*c^5 - 6*(b^2 - 4*a*c)*b^3*c^5 + 16*(b^2 - 4*a*c)*a
*b*c^6 - 4*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7 - (2*b^6*c^2 - 18*
a*b^4*c^3 + 2*b^5*c^3 + 48*a^2*b^2*c^4 - 16*a*b^3*c^4 - 32*a^3*c^5 + 32*a^2
*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)
*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)
*a*b^4*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c
)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2
- sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2
*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*
c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 33*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 20*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 20*sqrt(2)*sqr
```

```
t(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 -
4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 - 8*(
b^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2 - 2*(sqrt(2)*sqrt(-b*c
- 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt
(b^2 - 4*a*c)*c)*b^6*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c^3 - 6*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c
^3 + 3*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 2*a*b^5*c
^3 + 2*b^6*c^3 + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^4 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 11*s
qrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4
+ 7*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 16*a*b^4*c^4
 + 2*b^5*c^4 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5
 + 32*a^3*b*c^5 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2
*c^5 + 32*a^2*b^2*c^5 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*
b^3*c^5 - 16*a*b^3*c^5 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c
)*a*b*c^6 + 32*a^2*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^3 - 2*(b^2 - 4*a*c)*b^4*
c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4 + 8*(b^2 - 4*a*c)*a*b^2*c^4 - 2*(b^2 - 4*a*
c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*abs(c))*arctan(2*sqrt(1/2)*sqrt(-x^2
+ 1)/sqrt(-(b*c^3 + 2*c^4 + sqrt(-4*(a*c^3 + b*c^3 + c^4)*c^4 + (b*c^3 + 2*
c^4)^2))/c^4))/((a*b^4*c^4 + b^5*c^4 - 8*a^2*b^2*c^5 - 6*a*b^3*c^5 + 3*b^4*
c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 - 11*a*b^2*c^6 + 7*b^3*c^6 - 4*a^2*c^7 - 28*
a*b*c^7 + 5*b^2*c^7 - 20*a*c^8)*c^2) + 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 6*b^
5*c^5 + 24*a^2*b^2*c^6 - 40*a*b^3*c^6 + 4*b^4*c^6 + 64*a^2*b*c^7 - 24*a*b^2
*c^7 + 32*a^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2
- 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(
b^2 - 4*a*c)*c)*a*b^4*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 +
 sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*
c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 32*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 43*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 19*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^6
+ 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b
*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c
)*b^2*c^6 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a
*c)*c)*a*c^7 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 - 6*(b^2
 - 4*a*c)*b^3*c^5 + 16*(b^2 - 4*a*c)*a*b*c^6 - 4*(b^2 - 4*a*c)*b^2*c^6 + 8*
(b^2 - 4*a*c)*a*c^7 - (2*b^6*c^2 - 18*a*b^4*c^3 + 2*b^5*c^3 + 48*a^2*b^2*c^
4 - 16*a*b^3*c^4 - 32*a^3*c^5 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 18*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 7*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 -
 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^3 + 33*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4
*a*c)*c)*b^3*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^
2 - 4*a*c)*c)*a^2*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sq
rt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b
^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c
)*a*b*c^4)*c^2 + 2*(sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^5*
c^2 + sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 8*sqrt(2)*
sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 6*sqrt(2)*sqrt(-b*c
- 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 3*sqrt(2)*sqrt(-b*c - 2*c^2 + sq
```

```
rt(b^2 - 4*a*c)*c)*b^5*c^3 - 2*a*b^5*c^3 - 2*b^6*c^3 + 16*sqrt(2)*sqrt(-b*c
 - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 11*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 -
 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(
b^2 - 4*a*c)*c)*b^4*c^4 + 16*a*b^4*c^4 - 2*b^5*c^4 - 4*sqrt(2)*sqrt(-b*c -
2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 - 28*sqrt(2)*sqrt(-b*
c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 32*a^2*b^2*c^5 + 5*sqrt(2)*sqr
t(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 16*a*b^3*c^5 - 20*sqrt(2)*s
qrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 32*a^2*b*c^6 + 2*(b^2 - 4
*a*c)*a*b^3*c^3 + 2*(b^2 - 4*a*c)*b^4*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4 - 8*(
b^2 - 4*a*c)*a*b^2*c^4 + 2*(b^2 - 4*a*c)*b^3*c^4 - 8*(b^2 - 4*a*c)*a*b*c^5)
*abs(c))*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(b*c^3 + 2*c^4 - sqrt(-4*(
a*c^3 + b*c^3 + c^4)*c^4 + (b*c^3 + 2*c^4)^2))/c^4))/((a*b^4*c^4 + b^5*c^4
- 8*a^2*b^2*c^5 - 6*a*b^3*c^5 + 3*b^4*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 - 11*a
*b^2*c^6 + 7*b^3*c^6 - 4*a^2*c^7 - 28*a*b*c^7 + 5*b^2*c^7 - 20*a*c^8)*c^2)
- 1/3*((-x^2 + 1)^(3/2)*c^2 + 3*sqrt(-x^2 + 1)*b*c)/c^3
```

maple [B] time = 0.10, size = 2134, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3*(-x^2+1)^(3/2)/c+4/c*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/
2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^
2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2
*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)-2/c^2*a/(8*a*c-2*b^2
)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*a
rctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a
*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2*(-
4*a*c+b^2)^(1/2)-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*
b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a
+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*
a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b+8/c*a^2/(8*a*c-2*b^2)/(4*
a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan
(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*
b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b+8*a^2/(8*
a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b
)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-
2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2
))-2/c^2*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^
2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b
^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/
2)-2*a*b)^(1/2))*b^3-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*
a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x
^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*
(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2+4/c*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(
-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^
2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+
b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)-2/c^2*
a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-
2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+
2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^
(1/2))*b^2*(-4*a*c+b^2)^(1/2)-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^
2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2
)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2
)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b-8/c*a^2/(8*a*
c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^
(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b
```

```
)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*
b-8*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^
(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^
(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2
*a*b)^(1/2))+2/c^2*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*
(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*
(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c
+b^2)^(1/2)-2*a*b)^(1/2))*b^3+2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^
2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2
)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2
)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2-2/c^2*b/(2/x^2-2/x^2*(-x^2+1)^
(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}\,x^5}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^5/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.45, size = 917, normalized size = 3.26

$$\sqrt{1-x^2}\left(\frac{2}{3c}-\frac{\frac{b}{c}+1}{c}+\frac{x^2}{3c}\right)-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(b^3c+b^4-b^3\sqrt{b^2-4ac}+4a^2c^2+2ac^2\sqrt{b^2-\cdots}\right.}{4c^3\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (1 - x^2)^(1/2)*(2/(3*c) - (b/c + 1)/c + x^2/(3*c)) - (log((((x*(-(b - (b^2
 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1
)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
))*(b^3*c + b^4 - b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2 + 2*a*c^2*(b^2 - 4*a*
c)^(1/2) - b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a*b^2*c + 3*a*b*c*(b^2
 - 4*a*c)^(1/2)))/(4*c^3*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c
 - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b
+ (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (
b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*c + b^4 + b^3*(b^2 - 4*a*c)^(1/2) +
 4*a^2*c^2 - 2*a*c^2*(b^2 - 4*a*c)^(1/2) + b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*
b*c^2 - 5*a*b^2*c - 3*a*b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*((b
+ (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(
1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (
1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*c +
 b^4 + b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2 - 2*a*c^2*(b^2 - 4*a*c)^(1/2) +
b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a*b^2*c - 3*a*b*c*(b^2 - 4*a*c)^(
1/2)))/(4*c^3*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) -
(log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^3*c + b^4 - b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2
+ 2*a*c^2*(b^2 - 4*a*c)^(1/2) - b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a
```

```
*b^2*c + 3*a*b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^3*((b - (b^2 - 4*a*c)^(1/2))/(2
*c) + 1)^(1/2)*(4*a*c - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**5*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.377 $\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=229

$$-\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{\sqrt{1-x^2}}{c}$$

```
[Out] (-x^2+1)^(1/2)/c-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c-(-4*a*c+
b^2)^(1/2))^(1/2))*(b+c+(2*a*c-b^2-b*c)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)
/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2
)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(b+c+(-2*a*c+b^2+b*c)/(-4*a*c+b^2)^(1/2
))/c^(3/2)*2^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.75, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 824, 826, 1166, 208}

$$-\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{\sqrt{1-x^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[1 - x^2]/c - ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*c^(3/2)*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b + c + (b^2 - 2*a*c + b*c
)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}\,x}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{1-x^2}}{c} + \frac{\text{Subst}\left(\int \frac{a+(b+c)x}{\sqrt{1-x}\,(a+bx+cx^2)}\,dx, x, x^2\right)}{2c}$$
$$= \frac{\sqrt{1-x^2}}{c} + \frac{\text{Subst}\left(\int \frac{-a-b-c+(b+c)x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{c}$$
$$= \frac{\sqrt{1-x^2}}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right)}{2c} + \frac{\left(b+c\cdot\right.}{}$$
$$= \frac{\sqrt{1-x^2}}{c} - \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b+2c+\sqrt{b^2}}}$$

Mathematica [A] time = 0.39, size = 276, normalized size = 1.21

$$\frac{\frac{\left(b\left(c-\sqrt{b^2-4ac}\right)-c\left(\sqrt{b^2-4ac}+2a\right)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}} - \frac{\left(b\left(\sqrt{b^2-4ac}+c\right)+c\left(\sqrt{b^2-4ac}-2a\right)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}} + \sqrt{1-}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[1 - x^2] + ((b^2 + b*(c - Sqrt[b^2 - 4*a*c]) - c*(2*a + Sqrt[b^2 - 4*
a*c]))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]
]) - ((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))*ArcT
anh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]))/c
```

fricas [B] time = 6.33, size = 2053, normalized size = 8.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4
)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*sqr
t((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7))
+ (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^4*c^3
 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*
b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*b^
3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4
)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1))/
x^2) - sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a
*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*s
qrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)
) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - sqrt(1/2)*((b^4*c
^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(
a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*
b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a
*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1)
)/x^2) - sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x^2
*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^
7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^4
*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2
*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*
a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 +
1))/x^2) + sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x
^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*
c^7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - sqrt(1/2)*((b
^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 -
 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (
5*a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2
+ 1))/x^2) + 2*sqrt(-x^2 + 1))/c
```

giac [B] time = 4.10, size = 4060, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 1)/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 6*b^4*c^5 + 16*a^2*b*c^6
 - 32*a*b^2*c^6 + 4*b^3*c^6 + 32*a^2*c^7 - 16*a*b*c^7 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 5*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)
*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*
c)*c)*a^2*c^5 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 -
 4*a*c)*c)*a*b*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(
```

b^2 - 4*a*c)*c)*b^2*c^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - 6*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6 - 4*(b^2 - 4*a*c)*b*c^6 - (2*b^5*c^2 - 16*a*b^3*c^3 + 2*b^4*c^3 + 32*a^2*b*c^4 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2 - 2*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 3*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 2*a*b^4*c^3 + 2*b^5*c^3 + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 11*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*a*b^3*c^4 + 2*b^4*c^4 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + 32*a^2*b*c^5 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 16*a*b^2*c^5 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^6 + 32*a^2*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*abs(c))*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(b*c + 2*c^2 + sqrt(-4*(a*c + b*c + c^2)*c^2 + (b*c + 2*c^2)^2))/c^2))/((a*b^4*c^3 + b^5*c^3 - 8*a^2*b^2*c^4 - 6*a*b^3*c^4 + 3*b^4*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 - 11*a*b^2*c^5 + 7*b^3*c^5 - 4*a^2*c^6 - 28*a*b*c^6 + 5*b^2*c^6 - 20*a*c^7)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 6*b^4*c^5 + 16*a^2*b*c^6 - 32*a*b^2*c^6 + 4*b^3*c^6 + 32*a^2*c^7 - 16*a*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - 6*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6 - 4*(b^2 - 4*a*c)*b*c^6 - (2*b^5*c^2 - 16*a*b^3*c^3 + 2*b^4*c^3 + 32*a^2*b*c^4 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 28*sqrt(2

```
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 5*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 +
20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^4
 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^2*
c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2 + 2*(sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^
5*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*
sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 3*sqrt(2)*sqrt
(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 2*a*b^4*c^3 - 2*b^5*c^3 + 16
*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(
-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 11*sqrt(2)*sqrt(-b*c - 2*c^
2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 + 7*sqrt(2)*sqrt(-b*c -
 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 16*a*b^3*c^4 - 2*b^4*c^4 - 4*sqrt(2
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 - 28*sqrt(2
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 32*a^2*b*c^5 + 5*sqrt(
2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 16*a*b^2*c^5 - 20*sqr
t(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^6 - 32*a^2*c^6 + 2*(b^2 -
 4*a*c)*a*b^2*c^3 + 2*(b^2 - 4*a*c)*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4 - 8*(
b^2 - 4*a*c)*a*b*c^4 + 2*(b^2 - 4*a*c)*b^2*c^4 - 8*(b^2 - 4*a*c)*a*c^5)*abs
(c))*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(b*c + 2*c^2 - sqrt(-4*(a*c +
b*c + c^2)*c^2 + (b*c + 2*c^2)^2))/c^2))/((a*b^4*c^3 + b^5*c^3 - 8*a^2*b^2*
c^4 - 6*a*b^3*c^4 + 3*b^4*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 - 11*a*b^2*c^5 + 7
*b^3*c^5 - 4*a^2*c^6 - 28*a*b*c^6 + 5*b^2*c^6 - 20*a*c^7)*c^2)
```

maple [B] time = 0.06, size = 1223, normalized size = 5.34

$$-\frac{8a^2\arctan\left(\frac{-2a-2b-\frac{2\left(\sqrt{-x^2+1}-1\right)^2 a}{x^2}+2\sqrt{-4ac+b^2}}{2\sqrt{-2ab+4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2\sqrt{-4ac+b^2}\,b}}\right)}{\left(8ac-2b^2\right)\sqrt{-2ab+4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2\sqrt{-4ac+b^2}\,b}}+\frac{8a^2\arctan\left(\frac{2a+2b+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}}{2\sqrt{-2ab+4ac-2b^2-2\sqrt{-4ac}}}\right)}{\left(8ac-2b^2\right)\sqrt{-2ab+4ac-2b^2-2\sqrt{-4ac}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(
1/2)*b)^(1/2)*(-4*a*c+b^2)^(1/2)*a*b/c*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/
2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2
)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+4*a/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*
(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*(
(-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a
*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*(-4*a*c+b^2)^(1/2)-8/(8*a*c-
2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(
1/2)*a^2*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(
1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1
/2))+2/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b
^2)^(1/2)*b)^(1/2)*a*b^2/c*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^
2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*
c+b^2)^(1/2)*b)^(1/2))+2/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(
1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*(-4*a*c+b^2)^(1/2)*a*b/c*arctan(1/2*(2
*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b
^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+4*a/(8*a*c-2*b^2)/
(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*ar
ctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*
b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*(-4*a*c
+b^2)^(1/2)+8/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-
4*a*c+b^2)^(1/2)*b)^(1/2)*a^2*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/
x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*
```

```
a*c+b^2)^(1/2)*b)^(1/2))-2/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)
^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*a*b^2/c*arctan(1/2*(2*a+2*b+2*((-x^2
+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^
2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+2/c/(1/x^2*(-x^2+1)-2*(-x^2+1)^(1
/2)/x^2+1/x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}\,x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^3/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.31, size = 776, normalized size = 3.39

$$\frac{\sqrt{1-x^2}}{c}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac^2-b^2c-b^3+b^2\sqrt{b^2-4ac}+4abc-2ac\sqrt{b^2-4ac}+bc\right.}{4c^2\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (1 - x^2)^(1/2)/c - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)
*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
 (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c^2 - b^2*c - b^3 + b^2*(b
^2 - 4*a*c)^(1/2) + 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)
^(1/2)))/(4*c^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
+ (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 -
4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2)))*(b^2*c - 4*a*c^2 + b^3 + b^2*(b^2 - 4*a*c)^(1/2)
- 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*(4
*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b -
 (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c
) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^
(1/2)))*(4*a*c^2 - b^2*c - b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a*b*c - 2*a*c*
(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*((b - (b^2 - 4*a*c)^
(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/
2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1
- x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*c - 4
*a*c^2 + b^3 + b^2*(b^2 - 4*a*c)^(1/2) - 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2
) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2
))/(2*c) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**3*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.378 $\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=182

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2
))*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*
arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(b
+2*c+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1247, 699, 1130, 208}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]))
+ (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{1-x}}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= -\,\mathrm{Subst}\left(\int \frac{x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)$$
$$= \frac{1}{2}\left(-1-\frac{b+2c}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right)-\frac{1}{2}\left(1-\right.$$
$$= -\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}}{\sqrt{b+2}}\right.}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.24, size = 169, normalized size = 0.93

$$\frac{\sqrt{-\sqrt{b^2-4ac}-b-2c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)-\sqrt{\sqrt{b^2-4ac}-b-2c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/
Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]] - Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]*Ar
cTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

fricas [B] time = 2.84, size = 871, normalized size = 4.79

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+2\,c-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{bx^2+\frac{(b^2c-4\,ac^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}+\sqrt{\frac{1}{2}}\left((b^2-4\,ac)x^2+\frac{(b^3c-4\,abc^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}\right)\sqrt{\frac{b+2\,c-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}-}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(
b^2*c - 4*a*c^2))*log((b*x^2 + (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3
) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4
*a*c^3))*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) + 1/2*sqrt(1/2)*sqrt((b + 2*c
- (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2
+ (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) - sqrt(1/2)*((b^2 - 4*a*c)*
x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c - (b^2
*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1
)*a + 2*a)/x^2) - 1/2*sqrt(1/2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*
c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt(
b^2*c^2 - 4*a*c^3) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a*b*c^2)*x^2
/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 -
4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) + 1/2*sqrt(1/
2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*
c^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) - sqrt(1/2
```

```
)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqr
t((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))
- 2*sqrt(-x^2 + 1)*a + 2*a)/x^2)
```

giac [B] time = 4.24, size = 591, normalized size = 3.25

$$-\frac{\left(2\,b^2c^2 - 8\,ac^3 - \sqrt{2}\,\sqrt{b^2 - 4\,ac}\,\sqrt{-bc - 2\,c^2 - \sqrt{b^2 - 4\,ac}\,c}\,b^2 + 4\,\sqrt{2}\,\sqrt{b^2 - 4\,ac}\,\sqrt{-bc - 2\,c^2 - \sqrt{b^2 - 4\,ac}\,c}\,ac - \cdots\right.}{2\left(b^4 - 8\,ab^2c \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*b*c - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*arctan(2*sqrt(1/2)*sqrt(-x^2
 + 1)/sqrt(-(b + 2*c + sqrt((b + 2*c)^2 - 4*(a + b + c)*c))/c))/((b^4 - 8*a
*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + 5*b^2*c^2 - 20*a*c^3)*abs(c)) +
 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*b*c - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*arctan(2*sqrt(1/2)*sqrt(-x^2
 + 1)/sqrt(-(b + 2*c - sqrt((b + 2*c)^2 - 4*(a + b + c)*c))/c))/((b^4 - 8*a
*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + 5*b^2*c^2 - 20*a*c^3)*abs(c))
```

maple [B] time = 0.05, size = 1167, normalized size = 6.41

$$-\frac{4ac\arctan\left(\frac{-2a-2b-\frac{2\left(\sqrt{-x^2+1}-1\right)^2 a}{x^2}+2\sqrt{-4ac+b^2}}{2\sqrt{-2ab+4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2\sqrt{-4ac+b^2}\,b}}\right)}{\left(4ac-b^2\right)\sqrt{-2ab+4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2\sqrt{-4ac+b^2}\,b}}+\frac{4ac\arctan\left(\frac{2a+2b+\frac{2\left(\sqrt{-x^2+1}-1\right)^2 a}{x^2}\cdots}{2\sqrt{-2ab+4ac-2b^2-2\sqrt{-4ac+b^2}\cdots}}\right)}{\left(4ac-b^2\right)\sqrt{-2ab+4ac-2b^2-2\sqrt{-4ac+\cdots}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -2*a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^
(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+
b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)
*b)^(1/2))*(-4*a*c+b^2)^(1/2)-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b
^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^
(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(
1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b-4*a/(4*a*c-b^2)/
(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*ar
ctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a
*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*c+1/(4
*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b
)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1
/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/
2))*b^2-2*a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*
c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-
4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)
^(1/2)*b)^(1/2))*(-4*a*c+b^2)^(1/2)-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4
```

```
*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^
2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b
^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b+4*a/(4*a*c-
b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/
2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(
-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*c-
1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/
2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)
^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^
(1/2))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}\,x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.29, size = 649, normalized size = 3.57

$$\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac+b\sqrt{b^2-4ac}+2c\sqrt{b^2-4ac}-b^2\right)}{4c\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\left(4ac-b^2\right)}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)}{4c\left(4ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) + 2*c*(b^2 - 4*a*c)^
(1/2) - b^2))/(4*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2
)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2
 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 -
4*a*c)^(1/2))/(2*c))^(1/2)))*(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + 2*c*(b^2 - 4*
a*c)^(1/2) + b^2))/(4*c*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)
^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b -
 (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b
^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) + 2*c*(b^2
 - 4*a*c)^(1/2) - b^2))/(4*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4
*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/
((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b
 + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + 2*c
*(b^2 - 4*a*c)^(1/2) + b^2))/(4*c*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/
(2*c) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.379 $\int \frac{\sqrt{1-x^2}}{x(a+bx^2+cx^4)} dx$

Optimal. Leaf size=241

$$\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\tan$$

```
[Out] -arctanh((-x^2+1)^(1/2))/a+1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*
c-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*a+b+(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(
-4*a*c+b^2)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctanh(2^(1/2)*c^(1
/2)*(-x^2+1)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*a+b-(-4*a*c
+b^2)^(1/2))/a*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.65, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 897, 1287, 207, 1166, 208}

$$\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\tan$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(ArcTanh[Sqrt[1 - x^2]]/a) + (Sqrt[c]*(2*a + b + Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*a
+ b - Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[
b^2 - 4*a*c]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{1-x}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= -\,\mathrm{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b+c+(-b-2c)x^2+cx^4\right)}\,dx, x, \sqrt{1-x^2}\right)\\
&= -\,\mathrm{Subst}\left(\int \left(-\frac{1}{a\left(-1+x^2\right)}+\frac{-a-b-c+cx^2}{a\left(a+b+c-(b+2c)x^2+cx^4\right)}\right)dx, x, \sqrt{1-x^2}\right)\\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1-x^2}\right)}{a}-\frac{\mathrm{Subst}\left(\int \frac{-a-b-c+cx^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{a}\\
&= -\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a}+\frac{\left(c\left(2a+b-\sqrt{b^2-4ac}\right)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{\ldots}\right.}{2a\sqrt{b^2-4ac}}\\
&= -\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a}+\frac{\sqrt{c}\left(2a+b+\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{b+2c-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(2a+\ldots\right.}{\sqrt{2}\ldots}
\end{aligned}
$$

Mathematica [A] time = 0.43, size = 212, normalized size = 0.88

$$
\frac{\frac{\sqrt{2}\left(\sqrt{-\sqrt{b^2-4ac}+b+2c}\left(\sqrt{b^2-4ac}+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)+\left(\sqrt{b^2-4ac}-b\right)\sqrt{\sqrt{b^2-4ac}+b+2c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)\right)}{\sqrt{c}\sqrt{b^2-4ac}}-4\tanh^{-1}\left(\sqrt{1\ldots}\right.}{4a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-4*ArcTanh[Sqrt[1 - x^2]] + (Sqrt[2]*(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*(b
 + Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c
- Sqrt[b^2 - 4*a*c]]] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[b + 2*c + Sqrt[b^2 -
```

```
4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4
*a*c]]]))/(Sqrt[c]*Sqrt[b^2 - 4*a*c]))/(4*a)
```

fricas [B] time = 13.25, size = 1232, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 +
2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*(a
^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a
*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 -
4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b
 + b^2)/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a
*b)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 -
 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c
))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*
b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2
*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^2 - 4*a^3*c
)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)) - (a*b + b^2)*x^2 - 2*a
^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a*sqrt((a*b + b
^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*
c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a
^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 -
 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c
)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))
- (a*b + b^2)*x^2 - 2*a^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) - sq
rt(1/2)*a*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b +
 b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*(a^3*b^2
- 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^
2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c
)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)
/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a*b)*sqr
t(-x^2 + 1))/x^2) + 2*log((sqrt(-x^2 + 1) - 1)/x))/a
```

giac [B] time = 3.67, size = 3639, normalized size = 15.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(-x^2 + 1) + 1)/a + 1/2*log(-sqrt(-x^2 + 1) + 1)/a + 1/8*(4*a^
3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*
a^3*b*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*
c)*c)*a^3*b^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*
a*c)*c)*a^2*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2
- 4*a*c)*c)*a^4*b*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 10*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 4*(b^2 -
4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 -
4*(b^2 - 4*a*c)*a^2*b*c^3 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2
```

```
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^2
 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b
*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*
c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqr
t(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + sqrt(2)*sqrt(-b*c -
 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^
2 - 4*a*c)*c)*a^3*b^2*c - 6*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c
)*a^2*b^3*c + 3*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
2*a^2*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*
c)*a^4*c^2 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 -
 11*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 16*a^3*b
^2*c^2 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*
a^2*b^3*c^2 - 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)
*c)*a^3*c^3 - 32*a^4*c^3 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2
 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c + 2*(b^2 - 4*
a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^2 + 2*(b^2
 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a))*arctan(2*sqrt(1/2)*s
qrt(-x^2 + 1)/sqrt(-(a*b + 2*a*c + sqrt(-4*(a^2 + a*b + a*c)*a*c + (a*b + 2
*a*c)^2))/(a*c)))/((a^3*b^4 + a^2*b^5 - 8*a^4*b^2*c - 6*a^3*b^3*c + 3*a^2*b
^4*c + 16*a^5*c^2 + 8*a^4*b*c^2 - 11*a^3*b^2*c^2 + 7*a^2*b^3*c^2 - 4*a^4*c^
3 - 28*a^3*b*c^3 + 5*a^2*b^2*c^3 - 20*a^3*c^4)*abs(a)*abs(c)) - 1/8*(4*a^3*
b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*a^
3*b*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)
*c)*a^3*b^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*
c)*c)*a^2*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 -
4*a*c)*c)*a^4*b*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^
2 - 4*a*c)*c)*a^3*b^2*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*
c^2 - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
 - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 10*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 4*(b^2 - 4*
a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*
(b^2 - 4*a*c)*a^2*b*c^3 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 +
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c
^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b
^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)
*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 - 2*(sqrt(
2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + sqrt(2)*sqrt(-b*c - 2
*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2
 - 4*a*c)*c)*a^3*b^2*c - 6*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c + 3*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*
a^2*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)
*a^4*c^2 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 1
1*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 16*a^3*b^2
*c^2 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^
2*b^3*c^2 + 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c
)*a^3*c^3 + 32*a^4*c^3 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^3 + 32*a^3*b*c^3 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c
```

```
)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 -
 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a^2*b^2*c - 2*(b^2 - 4*a*
c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2 -
 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a))*arctan(2*sqrt(1/2)*sqr
t(-x^2 + 1)/sqrt(-(a*b + 2*a*c - sqrt(-4*(a^2 + a*b + a*c)*a*c + (a*b + 2*a
*c)^2))/(a*c)))/((a^3*b^4 + a^2*b^5 - 8*a^4*b^2*c - 6*a^3*b^3*c + 3*a^2*b^4
*c + 16*a^5*c^2 + 8*a^4*b*c^2 - 11*a^3*b^2*c^2 + 7*a^2*b^3*c^2 - 4*a^4*c^3
- 28*a^3*b*c^3 + 5*a^2*b^2*c^3 - 20*a^3*c^4)*abs(a)*abs(c))
```

maple [B] time = 0.06, size = 2099, normalized size = 8.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a*(-x^2+1)^(1/2)-1/a*arctanh(1/(-x^2+1)^(1/2))+1/(4*a*c-b^2)/(-2*a*b+4*a*
c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*(-4*a*c+b^2)^(
1/2)*b*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/
2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2
))-2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^
(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+
b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)
*b)^(1/2))*c*(-4*a*c+b^2)^(1/2)+1/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a
*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2
+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^
2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^2*(-4*a*c+b^2)^(1/2)+4/(4*a*c-b
^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2
)*a*c*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2
))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)
)-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(
1/2)*b)^(1/2)*b^2*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a
*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1
/2)*b)^(1/2))+4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-
4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2
+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c
+b^2)^(1/2)*b)^(1/2))*b*c-1/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2
)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1
/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/
2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^3+1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2
*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*(-4*a*c+b^2)^(1/2)*b*ar
ctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*
b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))-2/(4*a*
c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(
1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))
/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*
c*(-4*a*c+b^2)^(1/2)+1/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/
2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)
^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2
*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^2*(-4*a*c+b^2)^(1/2)-4/(4*a*c-b^2)/(-2*a*b+
4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*a*c*arctan
(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*
a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+1/(4*a*c-b^
2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)
*b^2*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))
/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))-
4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/
2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)
^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^
(1/2))*b*c+1/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4
```

```
*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2
*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b
^2)^(1/2)*b)^(1/2))*b^3-2/a/(2/x^2-2*(-x^2+1)^(1/2)/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}}{\left(cx^4+bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x), x)
```

mupad [B] time = 1.30, size = 669, normalized size = 2.78

$$\frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{a}+\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(4\,a\,c+2\,a\,\sqrt{b^2-4\,a\,c}+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{4\,a\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\frac{\ln\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2))/a + (log((((x*(-(b + (b^2 - 4*a*c)^(
1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (
1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c +
 2*a*(b^2 - 4*a*c)^(1/2) + b*(b^2 - 4*a*c)^(1/2) - b^2))/(4*a*(4*a*c - b^2)
*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*
a*(b^2 - 4*a*c)^(1/2) - 4*a*c + b*(b^2 - 4*a*c)^(1/2) + b^2))/(4*a*((b - (b
^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b^2
- 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)
^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2))
)*(4*a*c + 2*a*(b^2 - 4*a*c)^(1/2) + b*(b^2 - 4*a*c)^(1/2) - b^2))/(4*a*(4*
a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b -
(b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c)
 + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(
1/2)))*(2*a*(b^2 - 4*a*c)^(1/2) - 4*a*c + b*(b^2 - 4*a*c)^(1/2) + b^2))/(4*
a*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x*(a + b*x**2 + c*x**4)), x)
```

3.380 $\int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=290

$$-\frac{\sqrt{c}\left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a^2\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a^2\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(a+2b}{}$$

```
[Out] 1/2*(a+2*b)*arctanh((-x^2+1)^(1/2))/a^2-1/4/a/(1-(-x^2+1)^(1/2))+1/4/a/(1+(
-x^2+1)^(1/2))-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c-(-4*a*c+b^
2)^(1/2))^(1/2))*c^(1/2)*(a+b+(b^2+a*(b-2*c))/(-4*a*c+b^2)^(1/2))/a^2*2^(1/
2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1
/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a+b+(-b^2-a*(b-2*c))/(-4*a*c
+b^2)^(1/2))/a^2*2^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 2.36, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 897, 1287, 207, 1166, 208}

$$-\frac{\sqrt{c}\left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a^2\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a^2\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(a+2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*a*(1 - Sqrt[1 - x^2])) + 1/(4*a*(1 + Sqrt[1 - x^2])) + ((a + 2*b)*Arc
Tanh[Sqrt[1 - x^2]])/(2*a^2) - (Sqrt[c]*(a + b + (b^2 + a*(b - 2*c))/Sqrt[b
^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(a
 + b - (b^2 + a*(b - 2*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c +
Sqrt[b^2 - 4*a*c]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= -\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2\left(a+b+c+(-b-2c)x^2+cx^4\right)}\,dx, x, \sqrt{1-x^2}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{4a(-1+x)^2}+\frac{1}{4a(1+x)^2}+\frac{a+2b}{2a^2\left(-1+x^2\right)}+\frac{b(a+b+c)-(a+b)c}{a^2\left(a+b+c-(b+2c)x^2\right.}\right.\right.$$

$$= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}-\frac{\text{Subst}\left(\int \frac{b(a+b+c)-(a+b)cx^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{a^2}$$

$$= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}+\frac{\left(c\left(a+b-\frac{b^2}{\right.\right.}{}$$

$$= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}-\frac{\sqrt{c}\left(a+b+\right.}{\sqrt{2}}$$

Mathematica [A] time = 0.76, size = 292, normalized size = 1.01

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(-\frac{\left(b\left(\sqrt{b^2-4ac}+b\right)+a\left(\sqrt{b^2-4ac}+b-2c\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(b\left(\sqrt{b^2-4ac}-b\right)+a\left(\sqrt{b^2-4ac}-b+2c\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}}+(a+2b)\log\left(\right.}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*Sqrt[1 - x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(-(((b*(b + Sqrt[b^2 - 4*a*c])
 + a*(b - 2*c + Sqrt[b^2 - 4*a*c]))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])
/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (
(b*(-b + Sqrt[b^2 - 4*a*c]) + a*(-b + 2*c + Sqrt[b^2 - 4*a*c]))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c] - (a + 2*b)*Log[x] + (a + 2*b)*
Log[1 + Sqrt[1 - x^2]])/(2*a^2)
```

fricas [B] time = 38.36, size = 2799, normalized size = 9.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)
*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4
*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/
(a^4*b^2 - 4*a^5*c))*log(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b
^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b
^4)*c)/(a^8*b^2 - 4*a^9*c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^
2 - (a*b^3 + b^4)*c)*x^2 - 2*(a^2*b^2 + a*b^3)*c + sqrt(1/2)*((a^5*b^3 - 4*
a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^
2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 +
a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((a*b^3
 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*
b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*
b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*
a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a^2*x^2*
sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)
*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b
^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log
(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*
b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*
c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2
 - 2*(a^2*b^2 + a*b^3)*c - sqrt(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b
^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b
^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b
)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*
a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a
^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b
^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 +
a*b^3)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^
2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5
 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4
)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-((a^4*b^2*c - 4*a^5*c^
2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*
(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)
*c^2 - ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*
c + sqrt(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a
^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b
^2 - 4*a^9*c)) - (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*
a^2*b^3)*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a
^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^
2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^
2 - 4*a^5*c)) + 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1
))/x^2) - sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*
b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b
```

```
 + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c
)))/(a^4*b^2 - 4*a^5*c))*log(-((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 +
2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 +
2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)*c^2 - ((a^2*b + 2*a*b^
2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*c - sqrt(1/2)*((a^5*b^3
 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^
2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - (a^2*b
^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((
a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt(
(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3
*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 2*((a^3
 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) + (a + 2*b)*x^2
*log((sqrt(-x^2 + 1) - 1)/x) + sqrt(-x^2 + 1)*a)/(a^2*x^2)
```

giac [B] time = 7.12, size = 1675, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt
(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(-b*c - 2*c^2
- sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^2 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 - 16*a*b^3*c^2 + 2*b^4*c^2 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*
a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 12*a*b^2*c^3 + 16*a^2*c^4 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4 + 6*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 5*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^2
+ 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c
^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*b^2*
c^2 + 4*(b^2 - 4*a*c)*a*c^3)*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(a^2*b
 + 2*a^2*c + sqrt(-4*(a^3 + a^2*b + a^2*c)*a^2*c + (a^2*b + 2*a^2*c)^2))/(a
^2*c)))/((a^2*b^4 - 8*a^3*b^2*c + 2*a^2*b^3*c + 16*a^4*c^2 - 8*a^3*b*c^2 +
5*a^2*b^2*c^2 - 20*a^3*c^3)*abs(c)) - 1/4*(sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt
(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c + 2*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c
 + 16*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 5*sqrt(2)*sqrt(-b*c
- 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 2*b^4*c^2 - 20*sqrt
(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 12*a*
b^2*c^3 - 16*a^2*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b
^2 - 4*a*c)*c)*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sq
rt(b^2 - 4*a*c)*c)*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 +
sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^
2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 -
4*a*c)*a*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c^2 - 4*(b^2 - 4*a*c)*a*c^3)*arctan(2*
sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(a^2*b + 2*a^2*c - sqrt(-4*(a^3 + a^2*b + a^
2*c)*a^2*c + (a^2*b + 2*a^2*c)^2))/(a^2*c)))/((a^2*b^4 - 8*a^3*b^2*c + 2*a^
2*b^3*c + 16*a^4*c^2 - 8*a^3*b*c^2 + 5*a^2*b^2*c^2 - 20*a^3*c^3)*abs(c)) +
1/4*(a + 2*b)*log(sqrt(-x^2 + 1) + 1)/a^2 - 1/4*(a + 2*b)*log(-sqrt(-x^2 +
1) + 1)/a^2 - 1/2*sqrt(-x^2 + 1)/(a*x^2)
```

maple [B] time = 0.08, size = 2770, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a^2*b*(-x^2+1)^(1/2)+1/a^2*b*arctanh(1/(-x^2+1)^(1/2))+2/(4*a*c-b^2)/(-2
*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*(-4*a
*c+b^2)^(1/2)*c*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c
+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2
)*b)^(1/2))-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*
a*c+b^2)^(1/2)*b)^(1/2)*(-4*a*c+b^2)^(1/2)/a*b^2*arctan(1/2*(-2*a-2*b-2*((-
x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c
+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+3/a/(4*a*c-b^2)/(-2*a*b+4*a*c-
2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a
-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2
+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b*c*(-4*a*c+b^2)^(1/
2)-1/a^2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b
^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*
a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(
1/2)*b)^(1/2))*b^3*(-4*a*c+b^2)^(1/2)-4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(
-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*b*c*arctan(1/2*(-2*a-2*b-
2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-
4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+4/(4*a*c-b^2)/(-2*a*b+4*a
*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-
2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*
b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*c^2+1/(4*a*c-b^2)
/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)/a
*b^3*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2)
)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2))
-5/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^
(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+
b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*(-4*a*c+b^2)^(1/2)
*b)^(1/2))*b^2*c+1/a^2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)
*a+2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(-2*a-2*b-2*((-x^2+1)^(1/2)-1)^
2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*
(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^4+2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*
c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*(-4*a*c+b^2)^(1/2)*c*arctan(1/
2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c
-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))-1/(4*a*c-b^2)/
(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*(-
4*a*c+b^2)^(1/2)/a*b^2*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(
-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2
)^(1/2)*b)^(1/2))+3/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*
a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*
a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-
4*a*c+b^2)^(1/2)*b)^(1/2))*b*c*(-4*a*c+b^2)^(1/2)-1/a^2/(4*a*c-b^2)/(-2*a*b
+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/
2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c
-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^3*(-4*a*c+b^
2)^(1/2)+4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c
+b^2)^(1/2)*b)^(1/2)*b*c*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2
*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b
^2)^(1/2)*b)^(1/2))-4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*
a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*
a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-
4*a*c+b^2)^(1/2)*b)^(1/2))*c^2-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+
b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)/a*b^3*arctan(1/2*(2*a+2*b+2*((-x
```

```
^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+
b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))+5/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2
*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan(1/2*(2*a+2
*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*a*b+4*a*c-2*b^2-2
*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^2*c-1/a^2/(4*a*c-b^2
)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2)*
arctan(1/2*(2*a+2*b+2*((-x^2+1)^(1/2)-1)^2*a/x^2+2*(-4*a*c+b^2)^(1/2))/(-2*
a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*(-4*a*c+b^2)^(1/2)*b)^(1/2))*b^4+2
/a^2*b/(2/x^2-2*(-x^2+1)^(1/2)/x^2)-1/2/a/x^2*(-x^2+1)^(3/2)-1/2*(-x^2+1)^(
1/2)/a+1/2/a*arctanh(1/(-x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}}{\left(cx^4+bx^2+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^3), x)
```

mupad [B] time = 1.41, size = 825, normalized size = 2.84

$$\frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{2\,a}-\frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)(a+b)}{a^2}-\frac{\sqrt{1-x^2}}{2\,a\,x^2}-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(4\,a^2\,c-a\,b^2-b^3\cdots\right.}{4\,a^2\,\left(4\,a\,c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2))/(2*a) - (log((1/x^2 - 1)^(1/2) - (1/
x^2)^(1/2))*(a + b))/a^2 - (1 - x^2)^(1/2)/(2*a*x^2) - (log((((x*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) +
1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2
)))*(4*a^2*c - a*b^2 - b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a*b*c + a*b*(b^2 -
 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*((b + (b^2
 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/
(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^
2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(a*b^2 - 4*a^2
*c + b^3 + b^2*(b^2 - 4*a*c)^(1/2) - 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*
a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
)*(4*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*
1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a^2*c - a*b^2 - b^3 + b^2*(b^
2 - 4*a*c)^(1/2) + 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^
(1/2)))/(4*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) +
 (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4
*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*
c)^(1/2))/(2*c))^(1/2)))*(a*b^2 - 4*a^2*c + b^3 + b^2*(b^2 - 4*a*c)^(1/2) -
 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*((b
 - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.381 $\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=325

$$-\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[Out] 1/2*(2*b+c)*arcsin(x)/c^2+1/2*x*(-x^2+1)^(1/2)/c-arctan(x*(b+2*c-(-4*a*c+b^
2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*c+(
3*a*b*c+2*a*c^2-b^3-b^2*c)/(-4*a*c+b^2)^(1/2))/c^2/(b-(-4*a*c+b^2)^(1/2))^(
1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(b+2*c+(-4*a*c+b^2)^(1/2))^(
1/2)/(-x^2+1)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*c+(-3*a*b*c-2*
a*c^2+b^3+b^2*c)/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*
c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 5.39, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1291, 388, 216, 1692, 377, 205}

$$-\frac{\left(-\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(2*c) + ((2*b + c)*ArcSin[x])/(2*c^2) - ((b^2 - a*c + b*c
 - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*
c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c^
2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b^2 -
a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^
2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1291

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[((f*x)^(m - 4)*(d +
 e*x^2)^(q - 1)*Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x])/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\int \frac{b+c-cx^2}{\sqrt{1-x^2}}\,dx}{c^2} - \frac{\int \frac{a(b+c)+\left(b^2-ac+bc\right)x^2}{\sqrt{1-x^2}\left(a+bx^2+cx^4\right)}\,dx}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} - \frac{\int \left(\frac{b^2-ac+bc+\frac{-b^3+3abc-b^2c+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{b^2-ac+bc-\frac{-b^3+3abc-b^2c+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx}{c^2} + \frac{(2b+c)\int \frac{1}{\sqrt{1-x^2}}}{2c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right.}}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{b-\sqrt{b^2-4ac}-(-}\right.}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x}}\right.}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}}$$

Mathematica [B] time = 6.31, size = 10606, normalized size = 32.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 5.75, size = 2860, normalized size = 8.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c +
(b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*
b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*
c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3
*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + sqrt(1/2)*((b^6 + 4*a^2*b*
c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^
6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*
c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 +
8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4
+ (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 +
 a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 -
a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c^2*sqrt(
-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt(
(b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2
- 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a
^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*
(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^
3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*
b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2
*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 +
 a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (
4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a
*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8
 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*
b^2)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c
^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2
*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b
^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a
^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + s
qrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*
c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a
*b^4 - b^5)*c)*x + ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x -
(b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a
*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8
 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c
^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 -
4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x
^2) - sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c -
(b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*
b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*
c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3
*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*
c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^
6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x + ((b^4*
c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 +
8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4
+ (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 +
 a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 -
a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(-x^2 + 1)*c*x
+ 2*(2*b + c)*arctan((sqrt(-x^2 + 1) - 1)/x))/c^2
```

giac [B] time = 6.69, size = 1710, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^3 + 2*sqrt(2)*
sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^4 - 2*a^2*b^4 - sqrt(2)*sqrt(2*
a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*b^5 + 2*a*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*
b + sqrt(b^2 - 4*a*c)*a)*a^3*b*c - 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 -
4*a*c)*a)*a^2*b^2*c + 12*a^3*b^2*c + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2
- 4*a*c)*a)*a*b^3*c - 16*a^2*b^3*c - 16*a^4*c^2 - 16*sqrt(2)*sqrt(2*a^2 + a
*b + sqrt(b^2 - 4*a*c)*a)*a^2*b*c^2 + 32*a^3*b*c^2 - 3*sqrt(2)*sqrt(2*a^2 +
 a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 - 2*sqrt(2)*sqrt(2*a^
2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 + sqrt(2)*sqrt(2*a^2
 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 + 6*sqrt(2)*sqrt(2*a^2
+ a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c + 4*sqrt(2)*sqrt(2*a^2
 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c - 6*sqrt(2)*sqrt(2*
a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c + 8*sqrt(2)*sqrt
(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2 + 2*(b^2 - 4*
a*c)*a^2*b^2 - 2*(b^2 - 4*a*c)*a*b^3 - 4*(b^2 - 4*a*c)*a^3*c + 8*(b^2 - 4*a
*c)*a^2*b*c)*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^
2 + 1) - 1)/x)/sqrt((2*a*c^2 + b*c^2 + sqrt(-4*(a*c^2 + b*c^2 + c^3)*a*c^2
+ (2*a*c^2 + b*c^2)^2))/(a*c^2)))/(3*a^4*b^2*c^2 + 2*a^3*b^3*c^2 - a^2*b^4*
c^2 - 12*a^5*c^3 - 8*a^4*b*c^3 + 8*a^3*b^2*c^3 - 16*a^4*c^4) + 1/4*(3*sqrt(
2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^3 + 2*sqrt(2)*sqrt(2*a^2 +
 a*b - sqrt(b^2 - 4*a*c)*a)*a*b^4 + 2*a^2*b^4 - sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c)*a)*b^5 - 2*a*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2
 - 4*a*c)*a)*a^3*b*c - 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^
2*b^2*c - 12*a^3*b^2*c + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*
a*b^3*c + 16*a^2*b^3*c + 16*a^4*c^2 - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^
2 - 4*a*c)*a)*a^2*b*c^2 - 32*a^3*b*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(
b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b - sq
rt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 - sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 - 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c - 4*sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c + 6*sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c - 8*sqrt(2)*sqrt(2*a^2 + a*b
 - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2
 + 2*(b^2 - 4*a*c)*a*b^3 + 4*(b^2 - 4*a*c)*a^3*c - 8*(b^2 - 4*a*c)*a^2*b*c)
*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/
x)/sqrt((2*a*c^2 + b*c^2 - sqrt(-4*(a*c^2 + b*c^2 + c^3)*a*c^2 + (2*a*c^2 +
 b*c^2)^2))/(a*c^2)))/(3*a^4*b^2*c^2 + 2*a^3*b^3*c^2 - a^2*b^4*c^2 - 12*a^5
*c^3 - 8*a^4*b*c^3 + 8*a^3*b^2*c^3 - 16*a^4*c^4) + 1/2*sqrt(-x^2 + 1)*x/c +
 1/4*(pi*sgn(x) + 2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x
^2 + 1) - 1)))*(2*b + c)/c^2
```

maple [C] time = 0.04, size = 222, normalized size = 0.68

$$-\frac{2b\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{c^2}+\frac{\sqrt{-x^2+1}\;x}{2c}+\frac{\arcsin(x)}{2c}+\frac{}{4c^2\left(\mathrm{RootOf}\left(\_Z^8a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*x*(-x^2+1)^(1/2)/c+1/2*arcsin(x)/c+1/4/c^2*sum((a*(b+c)*_R^6+(3*a*b-a*c
+4*b^2+4*b*c)*_R^4+(3*a*b-a*c+4*b^2+4*b*c)*_R^2+a*b+a*c)/(_R^7*a+3*_R^5*a+3
*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(((-x^2+1)^(1/2)-1)/x-_R),_
R=RootOf(a*_Z^8+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))-2/c^2
*b*arctan(((-x^2+1)^(1/2)-1)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}\,x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^4/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.30, size = 1024, normalized size = 3.15

$$\operatorname{asin}(x)\left(\frac{\frac{b}{c}+1}{c}-\frac{1}{2\,c}\right)+\frac{x\,\sqrt{1-x^2}}{2\,c}-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\left(-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+a\,b\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,\cdots\right.}{2\,c\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] asin(x)*((b/c + 1)/c - 1/(2*c)) + (x*(1 - x^2)^(1/2))/(2*c) - (log((((x*(-(
b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(
2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b - (b^2
- 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/
2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(3/2)))/(2*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*
(4*a*c - b^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i
)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-
(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(
2*c))^(3/2) + a*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (
b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))
^(3/2) + b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*(4*a*c - b^2)*
((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*
c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)
 + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2
*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b - (b^2 - 4*a*c)^(1/2))
/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b
 - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(3/2)))/(2*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
- (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 -
4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a
*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1
/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-
(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*(4*a*c - b^2)*((b + (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**4*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.382 $\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=263

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sin^{-1}(x)}{c}$$

```
[Out] -arcsin(x)/c+arctan(x*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*(b+c+(2*a*c-b^2-b*c)/(-4*a*c+b^2)^(1/2))/c/(b-(-4*
a*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+arctan(x*(b+2*c+(-4*
a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+c+(-2
*a*c+b^2+b*c)/(-4*a*c+b^2)^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4
*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 2.13, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1293, 216, 1692, 377, 205}

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sin^{-1}(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(ArcSin[x]/c) + ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x
^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) +
((b + c + (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqr
t[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
```

```
] && LeQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = -\frac{\int \frac{1}{\sqrt{1-x^2}}\,dx}{c} - \frac{\int \frac{-a-(b+c)x^2}{\sqrt{1-x^2}\left(a+bx^2+cx^4\right)}\,dx}{c}$$

$$= -\frac{\sin^{-1}(x)}{c} - \frac{\int \left(\frac{-b-c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{-b-c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx}{c}$$

$$= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{c} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\int \frac{}{\sqrt{1-x}}}{c}$$

$$= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{c} + \frac{(b}{}$$

$$= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{c\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{b+}}$$

Mathematica [B] time = 6.14, size = 7543, normalized size = 28.68

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 2.72, size = 1430, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((b^2
+ 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*
c)*x^2 - 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sqrt(
-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x - ((b^3*c^2 - 4*a*b*c^3)*
sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c
^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((b^2 +
2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c)*sqr
t(-x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c
^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log
```

```
(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - sqrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b -
 b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x - ((b^3*c^2
 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c
 + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3
)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(
a*b + a*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c - (b
^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 -
 4*a*c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*a*
c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)
*x + ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqr
t((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c - (b^2
*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4
*a*c^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt(-(b^2 - (2
*a - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - sqrt(1/2
)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4
*a*b - b^2)*c)*x + ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a
*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a
 - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5))
)/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) - 4*arctan((sqr
t(-x^2 + 1) - 1)/x))/c
```

giac [B] time = 4.73, size = 3580, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(pi*sgn(x) + 2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x
^2 + 1) - 1)))/c + 1/8*((2*a^2*b^4 - 16*a^3*b^2*c + 32*a^4*c^2 + 3*sqrt(2)*
sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 + 2*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 - sqrt(2
)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 - 12*sqrt(2
)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c - 8*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c + 8*sq
rt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c - 1
6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2
 - 2*(b^2 - 4*a*c)*a^2*b^2 + 8*(b^2 - 4*a*c)*a^3*c)*c^2*abs(a) - 2*(3*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c + 5*sqrt(2)*sqrt(2*a^2
 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^3*c + sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b
^2 - 4*a*c)*a)*a*b^4*c + 2*a^2*b^4*c - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2
- 4*a*c)*a)*b^5*c + 2*a*b^5*c - 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*
a*c)*a)*a^4*c^2 - 20*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b*
c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^2*c^2 - 16*a^
3*b^2*c^2 + 10*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^3*c^2 -
16*a^2*b^3*c^2 - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*b^4*c^2 +
2*a*b^4*c^2 - 28*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*c^3 +
32*a^4*c^3 - 24*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b*c^3 +
 32*a^3*b*c^3 + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^2*c^3
 - 16*a^2*b^2*c^3 - 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*
c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a^2*b^2*c - 2*(b^2 - 4*a*c)*a*b^3*c + 8*
(b^2 - 4*a*c)*a^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c
^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a)*abs(c) + (4*a^3*b^3*c^2 + 2*a^2*b^4*c^
2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*a^3*b*c^4 + 6*sqrt(2)*sq
rt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c^2 + 7*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2*c^2 -
sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4*c^2 +
 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c
^3 + 22*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a
```

```
^2*b*c^3 + 4*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a
*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2
- 4*a*c)*b^3*c^3 + 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(
b^2 - 4*a*c)*a^2*c^4 + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sq
rt(b^2 - 4*a*c)*a*b*c^4 - 4*(b^2 - 4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b
^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*(b^2 - 4*a*c)*a^2*b*c^3)*abs(a))*arcta
n(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a*
c + b*c + sqrt(-4*(a*c + b*c + c^2)*a*c + (2*a*c + b*c)^2))/(a*c)))/((3*a^5
*b^2*c^2 + 5*a^4*b^3*c^2 + a^3*b^4*c^2 - a^2*b^5*c^2 - 12*a^6*c^3 - 20*a^5*
b*c^3 + 3*a^4*b^2*c^3 + 10*a^3*b^3*c^3 - a^2*b^4*c^3 - 28*a^5*c^4 - 24*a^4*
b*c^4 + 8*a^3*b^2*c^4 - 16*a^4*c^5)*abs(c)) - 1/8*((2*a^2*b^4 - 16*a^3*b^2*
c + 32*a^4*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2
 - 4*a*c)*a^2*b^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(
b^2 - 4*a*c)*a*b^3 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b
^2 - 4*a*c)*b^4 - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b
^2 - 4*a*c)*a^3*c - 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(
b^2 - 4*a*c)*a^2*b*c + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sq
rt(b^2 - 4*a*c)*a*b^2*c - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a
)*sqrt(b^2 - 4*a*c)*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2 + 8*(b^2 - 4*a*c)*a^3
*c)*c^2*abs(a) + 2*(3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b
^2*c + 5*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^3*c + sqrt(2
)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a*b^4*c - 2*a^2*b^4*c - sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*b^5*c - 2*a*b^5*c - 12*sqrt(2)*sqrt(
2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4*c^2 - 20*sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c)*a)*a^3*b*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*
a*c)*a)*a^2*b^2*c^2 + 16*a^3*b^2*c^2 + 10*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b
^2 - 4*a*c)*a)*a*b^3*c^2 + 16*a^2*b^3*c^2 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*b^4*c^2 - 2*a*b^4*c^2 - 28*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*a^3*c^3 - 32*a^4*c^3 - 24*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(
b^2 - 4*a*c)*a)*a^2*b*c^3 - 32*a^3*b*c^3 + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 16*sqrt(2)*sqrt(2*a^2 + a*b
- sqrt(b^2 - 4*a*c)*a)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c + 2
*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^
2 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a)*abs(c) + (4
*a^3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 -
16*a^3*b*c^4 + 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 -
 4*a*c)*a^3*b*c^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(
b^2 - 4*a*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*
sqrt(b^2 - 4*a*c)*b^4*c^2 + 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)
*a)*sqrt(b^2 - 4*a*c)*a^3*c^3 + 22*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*
a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c^3 + 4*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2
 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^3*c^3 + 16*sqrt(2)*sqrt(2*a^2 + a*b -
 sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^4 + 8*sqrt(2)*sqrt(2*a^2 + a*
b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b*c^4 - 4*(b^2 - 4*a*c)*a^3*b*
c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*(b^2 - 4*a*
c)*a^2*b*c^3)*abs(a))*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-
x^2 + 1) - 1)/x)/sqrt((2*a*c + b*c - sqrt(-4*(a*c + b*c + c^2)*a*c + (2*a*c
 + b*c)^2))/(a*c)))/((3*a^5*b^2*c^2 + 5*a^4*b^3*c^2 + a^3*b^4*c^2 - a^2*b^5
*c^2 - 12*a^6*c^3 - 20*a^5*b*c^3 + 3*a^4*b^2*c^3 + 10*a^3*b^3*c^3 - a^2*b^4
*c^3 - 28*a^5*c^4 - 24*a^4*b*c^4 + 8*a^3*b^2*c^4 - 16*a^4*c^5)*abs(c))
```

maple [C] time = 0.02, size = 175, normalized size = 0.67

$$\frac{2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{c}-\frac{}{4c\left(\mathrm{RootOf}\left(\_Z^8 a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^7 a+3\,\mathrm{R}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4/c*sum((_R^6*a+(4*c+3*a+4*b)*_R^4+(4*c+3*a+4*b)*_R^2+a)/(_R^7*a+3*_R^5*
a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(-_R+((-x^2+1)^(1/2)-1)/
x),_R=RootOf(_Z^8*a+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))+2
/c*arctan(((-x^2+1)^(1/2)-1)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}\,x^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^2/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.27, size = 870, normalized size = 3.31

$$-\frac{\mathrm{asin}(x)}{c}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(2a\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+b\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+b\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}+2c\left(-\frac{b-\cdots}{\cdots}\right.\right.}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(8ac-2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(
-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*
c))^(3/2) + 2*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2)*(8*a*c - 2*b^2)) - (log((((x*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*
a*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/
(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + 2*c*(-(b - (b^2
 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
)*(8*a*c - 2*b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1
)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x
+ (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2) + b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(3/2) + 2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/
2)))/((8*a*c - 2*b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - asin(x
)/c + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^
2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 -
 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
 + b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(3/2) + 2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/((8*a*c -
2*b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.383 $\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=220

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] arctan(x*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2))*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2)-arctan(x*(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(-4*a*c+b^2)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1174, 402, 216, 377, 205}

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]
]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{\sqrt{1-x^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\sqrt{1-x^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(b+2c-\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(b+2c+\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{1-x^2}\left(b+\sqrt{b^2\ldots}\right.}}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(b+2c-\sqrt{b^2-4ac}\right)\operatorname{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}} - \frac{\left(b+2c+\sqrt{\ldots}\right.}{}$$

$$= \frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{b+2c\ldots}}{\sqrt{b+\sqrt{b\ldots}}}\right.}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4a\ldots}}}$$

Mathematica [B] time = 5.42, size = 2266, normalized size = 10.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*(Sqrt[2]*(-b - 2*c + Sqrt[b^2 - 4*a*c])*Sqrt[-((b^2 + c*(-2*a + Sqrt[b
^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*Log[-(Sqrt[(-b + Sqrt[b^2 -
 4*a*c])/c]/Sqrt[2]) + x] + Sqrt[2]*(-((-b - 2*c + Sqrt[b^2 - 4*a*c])*Sqrt[
-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*Lo
g[Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]/Sqrt[2] + x]) + (b + 2*c + Sqrt[b^2 - 4*
a*c])*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c])
)/c^2]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2]) + x] - b*Sqrt[(-b^
2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*Log[Sqrt
[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] - 2*c*Sqrt[(-b^2 + c*(2*a + Sqr
t[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*Log[Sqrt[-((b + Sqrt[b^2
 - 4*a*c])/c)]/Sqrt[2] + x] - Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[
b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*Log[Sqrt[-((b + Sqrt[b^2 -
 4*a*c])/c)]/Sqrt[2] + x] + b*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) +
b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*
c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + 2
*c*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/
c^2)]*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b
+ 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[b^2 - 4*a*c]*Sqrt[-((b^
2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*Log[2 -
 Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[
b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - b*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*
c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*Log[2 + Sqrt[2]*Sqrt[(-b + Sqrt[b^2
- 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2
]] - 2*c*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a
*c]))/c^2)]*Log[2 + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sq
```

```
rt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + Sqrt[b^2 - 4*a*c]*Sqrt
[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*L
og[2 + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c -
 Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - b*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 -
4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt
[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1
 - x^2]] - 2*c*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2
- 4*a*c]))/c^2]*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt
[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[b^2 - 4*a*c
]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^
2]*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b +
 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + b*Sqrt[(-b^2 + c*(2*a + Sqrt[
b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*Log[2 + Sqrt[2]*Sqrt[-((b
+ Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*
Sqrt[1 - x^2]] + 2*c*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqr
t[b^2 - 4*a*c]))/c^2]*Log[2 + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x
+ Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + Sqrt[b^2 -
 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c
]))/c^2]*Log[2 + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqr
t[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]]))/(c*Sqrt[b^2 - 4*a*c]*Sq
rt[((b + 2*c - Sqrt[b^2 - 4*a*c])*(-b + Sqrt[b^2 - 4*a*c]))/c^2]*Sqrt[-(((b
 + Sqrt[b^2 - 4*a*c])*(b + 2*c + Sqrt[b^2 - 4*a*c]))/c^2)])
```

fricas [B] time = 1.26, size = 759, normalized size = 3.45

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2\,a+b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\,\log\left(-\frac{x^2+\frac{\sqrt{\frac{1}{2}}\left(\left(ab^2-4\,a^2c\right)\sqrt{-x^2+1}\,x-\left(ab^2-4\,a^2c\right)x\right)\sqrt{-\frac{2\,a+b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}}{\sqrt{a^2b^2-4\,a^3c}}+\sqrt{-x^2+1}\,-1}{x^2}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)*x
 - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4
*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) - 1)/x
^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3
*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2
+ 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b
^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1)
- 1)/x^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 -
 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt
(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt
(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2
 + 1) - 1)/x^2) + 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2
*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c
)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c
)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqr
t(-x^2 + 1) - 1)/x^2)
```

giac [B] time = 5.12, size = 641, normalized size = 2.91

$$-\frac{\left(2\,a^2b^2-8\,a^3c+3\,\sqrt{2}\sqrt{2\,a^2+ab+\sqrt{b^2-4\,ac}\,a}\,\sqrt{b^2-4\,ac}\,a^2+2\,\sqrt{2}\sqrt{2\,a^2+ab+\sqrt{b^2-4\,ac}\,a}\,\sqrt{b^2-4\,ac}\,ab\cdots\right.}{2\left(3\,a^4b^2+2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*b^2 - 8*a^3*c + 3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*
a)*sqrt(b^2 - 4*a*c)*a^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a
)*sqrt(b^2 - 4*a*c)*a*b - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*s
qrt(b^2 - 4*a*c)*b^2 + 4*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sq
rt(b^2 - 4*a*c)*a*c - 2*(b^2 - 4*a*c)*a^2)*abs(a)*arctan(-1/2*sqrt(2)*(x/(s
qrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a + b + sqrt((2*a + b)
^2 - 4*(a + b + c)*a))/a))/(3*a^4*b^2 + 2*a^3*b^3 - a^2*b^4 - 12*a^5*c - 8*
a^4*b*c + 8*a^3*b^2*c - 16*a^4*c^2) + 1/2*(2*a^2*b^2 - 8*a^3*c + 3*sqrt(2)*
sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2 + 2*sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b - sqrt(2)*sqrt
(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^2 + 4*sqrt(2)*sqrt(
2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*c - 2*(b^2 - 4*a*c)*
a^2)*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) -
 1)/x)/sqrt((2*a + b - sqrt((2*a + b)^2 - 4*(a + b + c)*a))/a))/(3*a^4*b^2
+ 2*a^3*b^3 - a^2*b^4 - 12*a^5*c - 8*a^4*b*c + 8*a^3*b^2*c - 16*a^4*c^2)
```

maple [C] time = 0.01, size = 130, normalized size = 0.59

$$-\frac{}{4\left(\mathrm{RootOf}\left(\_Z^8 a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)^7 a+3\,\mathrm{RootOf}\left(\_Z^8 a+(4a+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*sum((_R^6-_R^4-_R^2+1)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_
R^3*c+_R*a+_R*b)*ln(-_R+((-x^2+1)^(1/2)-1)/x),_R=RootOf(_Z^8*a+(4*a+4*b)*_Z
^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.27, size = 989, normalized size = 4.50

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(b^2\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+a\,b\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-2\,a\,c\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+2\,a\,c\left(-\frac{b-\sqrt{b^2\cdots}}{2\cdots}\right.\right.}{2\,a\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4\,a\,c-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b
*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b
 + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(4*a*c - b^2)*((b + (b^2 - 4*a*
c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^
(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2
)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b - (b^2 - 4*
a*c)^(1/2))/(2*c))^(1/2) + a*b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2
*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^
(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(
(b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b
- (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*
c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))
^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b*(-(b - (b^2 -
4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
 + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(3/2)))/(2*a*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4
*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/
((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b
 + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*
c))^(1/2) + a*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(
3/2) + b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(4*a*c - b^2)*((
b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.384 $\int \frac{\sqrt{1-x^2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=265

$$-\frac{c\left(\frac{2a+b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{1-x^2}}{ax}$$

```
[Out] -(-x^2+1)^(1/2)/a/x-c*arctan(x*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(1+(2*a+b)/(-4*a*c+b^2)^(1/2))/a/(b-(-4*a
*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*(b+2*c+(-4
*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(1+(-2*
a-b)/(-4*a*c+b^2)^(1/2))/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1295, 264, 1692, 377, 205}

$$-\frac{c\left(\frac{2a+b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{1-x^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[1 - x^2]/(a*x)) - (c*(1 + (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])
])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (c*(
1 - (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a
*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
```

```
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = \frac{\int \frac{1}{x^2\sqrt{1-x^2}}\,dx}{a} - \frac{\int \frac{a+b+cx^2}{\sqrt{1-x^2}\left(a+bx^2+cx^4\right)}\,dx}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{\int \left(\frac{c+\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{c-\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{\left(c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{a} - \frac{\left(c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\sqrt{1-x^2}}}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{\left(c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{a} - \frac{(c}{}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2}}{\sqrt{b+\sqrt{}}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{b+2c+}}$$

Mathematica [B] time = 4.95, size = 2661, normalized size = 10.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(4*Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c
+ Sqrt[b^2 - 4*a*c]))/c^2]*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(
c + Sqrt[b^2 - 4*a*c]))/c^2)]*Sqrt[1 - x^2] + Sqrt[2]*(2*a + b + Sqrt[b^2 -
 4*a*c])*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a
*c]))/c^2)]*x*Log[-(Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]/Sqrt[2]) + x] - Sqrt[2
]*(2*a + b + Sqrt[b^2 - 4*a*c])*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c])
+ b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]/S
qrt[2] + x] - 2*Sqrt[2]*a*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c
+ Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[
2]) + x] - Sqrt[2]*b*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqr
t[b^2 - 4*a*c]))/c^2]*x*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2]) +
 x] + Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) +
b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]
/Sqrt[2]) + x] + 2*Sqrt[2]*a*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(
-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt
[2] + x] + Sqrt[2]*b*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqr
t[b^2 - 4*a*c]))/c^2]*x*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x]
```

```
 - Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(
-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt
[2] + x] - 2*Sqrt[2]*a*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c +
Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c
]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[2
]*b*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))
/c^2)]*x*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[
(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[2]*Sqrt[b^2 - 4*a*c]
*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^
2)]*x*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b
+ 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + 2*Sqrt[2]*a*Sqrt[-((b^2 + c*
(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[2 + Sqr
t[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2
- 4*a*c])/c]*Sqrt[1 - x^2]] + Sqrt[2]*b*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 -
4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[2 + Sqrt[2]*Sqrt[(-b + Sqr
t[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1
 - x^2]] + Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a
*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[2 + Sqrt[2]*Sqrt[(-b + Sqrt[b
^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 -
x^2]] + 2*Sqrt[2]*a*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt
[b^2 - 4*a*c]))/c^2]*x*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x
 + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + Sqrt[2]*b
*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2
]*x*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b
+ 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sq
rt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x
*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2
*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - 2*Sqrt[2]*a*Sqrt[(-b^2 + c*(2*a
 + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[2 + Sqrt[2]*
Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4
*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[2]*b*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c
]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[2 + Sqrt[2]*Sqrt[-((b + Sqrt[b^
2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 -
x^2]] + Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c])
+ b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[2 + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 -
 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2
]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[((b + 2*c - Sqrt[b^2 - 4*a*c])*(-b + Sqrt[b^2
 - 4*a*c]))/c^2]*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(b + 2*c + Sqrt[b^2 - 4*a*
c]))/c^2)]*x)
```

fricas [B] time = 1.86, size = 1998, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c + sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
```

```
)*x^2 - 2*(a*b + b^2)*c - sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) + sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c + sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x + ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c - sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x + ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - 2*sqrt(-x^2 + 1))/(a*x)
```

giac [B] time = 5.04, size = 3965, normalized size = 14.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(4*a^6*b^3 + 6*a^5*b^4 + 2*a^4*b^5 - 16*a^7*b*c - 32*a^6*b^2*c - 12*a^5
*b^3*c + 32*a^7*c^2 + 16*a^6*b*c^2 + 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2
- 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*b + 13*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^
2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^3 - sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^4 - sqrt(2)*sqrt(2*a^2 + a*b - sq
rt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b
- sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*c - 6*sqrt(2)*sqrt(2*a^2 + a*b
 - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b*c + 12*sqrt(2)*sqrt(2*a^2 +
 a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^2*c + 6*sqrt(2)*sqrt(2*
a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^3*c - 16*sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*c^2 - 8*sqrt(2
)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b*c^2 - 4*(
b^2 - 4*a*c)*a^6*b - 6*(b^2 - 4*a*c)*a^5*b^2 - 2*(b^2 - 4*a*c)*a^4*b^3 + 8*
(b^2 - 4*a*c)*a^6*c + 4*(b^2 - 4*a*c)*a^5*b*c - (2*a^3*b^4 + 2*a^2*b^5 - 16
*a^4*b^2*c - 16*a^3*b^3*c + 32*a^5*c^2 + 32*a^4*b*c^2 + 3*sqrt(2)*sqrt(2*a^
2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^2 + 5*sqrt(2)*sqrt(2
*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^3 + sqrt(2)*sqrt(
2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^4 - sqrt(2)*sqrt(2
*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^5 - 12*sqrt(2)*sqrt(2
*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*c - 20*sqrt(2)*sqrt
(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c + 8*sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3*c - 16*sqrt(
2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c^2 - 16*s
```

```
qrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c^2
- 2*(b^2 - 4*a*c)*a^3*b^2 - 2*(b^2 - 4*a*c)*a^2*b^3 + 8*(b^2 - 4*a*c)*a^4*c
 + 8*(b^2 - 4*a*c)*a^3*b*c)*a^2 + 2*(3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2
- 4*a*c)*a)*a^5*b^2 + 5*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4
*b^3 + sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b^4 - 2*a^4*b^4
- sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^5 - 2*a^3*b^5 - 12*
sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^6*c - 20*sqrt(2)*sqrt(2*a
^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^5*b*c + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*a^4*b^2*c + 16*a^5*b^2*c + 10*sqrt(2)*sqrt(2*a^2 + a*b - s
qrt(b^2 - 4*a*c)*a)*a^3*b^3*c + 16*a^4*b^3*c - sqrt(2)*sqrt(2*a^2 + a*b - s
qrt(b^2 - 4*a*c)*a)*a^2*b^4*c - 2*a^3*b^4*c - 28*sqrt(2)*sqrt(2*a^2 + a*b -
 sqrt(b^2 - 4*a*c)*a)*a^5*c^2 - 32*a^6*c^2 - 24*sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c)*a)*a^4*b*c^2 - 32*a^5*b*c^2 + 8*sqrt(2)*sqrt(2*a^2 + a*b
- sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c^2 + 16*a^4*b^2*c^2 - 16*sqrt(2)*sqrt(2*a^2
 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4*c^3 - 32*a^5*c^3 + 2*(b^2 - 4*a*c)*a^4*b^
2 + 2*(b^2 - 4*a*c)*a^3*b^3 - 8*(b^2 - 4*a*c)*a^5*c - 8*(b^2 - 4*a*c)*a^4*b
*c + 2*(b^2 - 4*a*c)*a^3*b^2*c - 8*(b^2 - 4*a*c)*a^4*c^2)*abs(a))*arctan(-1
/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a^2 +
a*b + sqrt(-4*(a^2 + a*b + a*c)*a^2 + (2*a^2 + a*b)^2))/a^2))/(3*a^8*b^2 +
5*a^7*b^3 + a^6*b^4 - a^5*b^5 - 12*a^9*c - 20*a^8*b*c + 3*a^7*b^2*c + 10*a^
6*b^3*c - a^5*b^4*c - 28*a^8*c^2 - 24*a^7*b*c^2 + 8*a^6*b^2*c^2 - 16*a^7*c^
3) - 1/8*(4*a^6*b^3 + 6*a^5*b^4 + 2*a^4*b^5 - 16*a^7*b*c - 32*a^6*b^2*c - 1
2*a^5*b^3*c + 32*a^7*c^2 + 16*a^6*b*c^2 + 6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt
(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*b + 13*sqrt(2)*sqrt(2*a^2 + a*b + sq
rt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b +
 sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^3 - sqrt(2)*sqrt(2*a^2 + a*b
+ sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^4 - sqrt(2)*sqrt(2*a^2 + a*b
 + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^5 - 12*sqrt(2)*sqrt(2*a^2 +
 a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*c - 6*sqrt(2)*sqrt(2*a^2
+ a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b*c + 12*sqrt(2)*sqrt(2*
a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^2*c + 6*sqrt(2)*sq
rt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^3*c - 16*sqrt
(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*c^2 - 8*s
qrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b*c^2
- 4*(b^2 - 4*a*c)*a^6*b - 6*(b^2 - 4*a*c)*a^5*b^2 - 2*(b^2 - 4*a*c)*a^4*b^3
 + 8*(b^2 - 4*a*c)*a^6*c + 4*(b^2 - 4*a*c)*a^5*b*c - (2*a^3*b^4 + 2*a^2*b^5
 - 16*a^4*b^2*c - 16*a^3*b^3*c + 32*a^5*c^2 + 32*a^4*b*c^2 + 3*sqrt(2)*sqrt
(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^2 + 5*sqrt(2)*s
qrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^3 + sqrt(2)*
sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^4 - sqrt(2)*s
qrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^5 - 12*sqrt(2)*s
qrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*c - 20*sqrt(2)
*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c + 8*sqrt
(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3*c - 16*
sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c^2 -
 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b
*c^2 - 2*(b^2 - 4*a*c)*a^3*b^2 - 2*(b^2 - 4*a*c)*a^2*b^3 + 8*(b^2 - 4*a*c)*
a^4*c + 8*(b^2 - 4*a*c)*a^3*b*c)*a^2 - 2*(3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt
(b^2 - 4*a*c)*a)*a^5*b^2 + 5*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a
)*a^4*b^3 + sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b^4 + 2*a^4
*b^4 - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^5 + 2*a^3*b^5
- 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^6*c - 20*sqrt(2)*sqr
t(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^5*b*c + 3*sqrt(2)*sqrt(2*a^2 + a*b +
 sqrt(b^2 - 4*a*c)*a)*a^4*b^2*c - 16*a^5*b^2*c + 10*sqrt(2)*sqrt(2*a^2 + a*
b + sqrt(b^2 - 4*a*c)*a)*a^3*b^3*c - 16*a^4*b^3*c - sqrt(2)*sqrt(2*a^2 + a*
b + sqrt(b^2 - 4*a*c)*a)*a^2*b^4*c + 2*a^3*b^4*c - 28*sqrt(2)*sqrt(2*a^2 +
a*b + sqrt(b^2 - 4*a*c)*a)*a^5*c^2 + 32*a^6*c^2 - 24*sqrt(2)*sqrt(2*a^2 + a
*b + sqrt(b^2 - 4*a*c)*a)*a^4*b*c^2 + 32*a^5*b*c^2 + 8*sqrt(2)*sqrt(2*a^2 +
 a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c^2 - 16*a^4*b^2*c^2 - 16*sqrt(2)*sqrt(
```

```
2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^4*c^3 + 32*a^5*c^3 - 2*(b^2 - 4*a*c)*a
^4*b^2 - 2*(b^2 - 4*a*c)*a^3*b^3 + 8*(b^2 - 4*a*c)*a^5*c + 8*(b^2 - 4*a*c)*
a^4*b*c - 2*(b^2 - 4*a*c)*a^3*b^2*c + 8*(b^2 - 4*a*c)*a^4*c^2)*abs(a))*arct
an(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a
^2 + a*b - sqrt(-4*(a^2 + a*b + a*c)*a^2 + (2*a^2 + a*b)^2))/a^2))/(3*a^8*b
^2 + 5*a^7*b^3 + a^6*b^4 - a^5*b^5 - 12*a^9*c - 20*a^8*b*c + 3*a^7*b^2*c +
10*a^6*b^3*c - a^5*b^4*c - 28*a^8*c^2 - 24*a^7*b*c^2 + 8*a^6*b^2*c^2 - 16*a
^7*c^3) + 1/2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/a
```

maple [C] time = 0.03, size = 217, normalized size = 0.82

$$-\frac{\sqrt{-x^2+1}\,x}{a}-\frac{\arcsin(x)}{a}-\frac{2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{a}+\frac{}{4a\left(\operatorname{RootOf}\left(\_Z^8 a+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/x*(-x^2+1)^(3/2)-1/a*x*(-x^2+1)^(1/2)-1/a*arcsin(x)+1/4/a*sum(((a+b)*_
R^6+(3*a+3*b+4*c)*_R^4+(3*a+3*b+4*c)*_R^2+a+b)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*
_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(-_R+((-x^2+1)^(1/2)-1)/x),_R=RootOf(
_Z^8*a+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))-2/a*arctan(((-
x^2+1)^(1/2)-1)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^2), x)
```

mupad [B] time = 1.21, size = 1234, normalized size = 4.66

$$-\frac{\sqrt{1-x^2}}{a\,x}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^3\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+a\,b^2\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-2\,a^2\,c\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-2\,a\right.}{2\,a^2\left(4\,a\,c-b^2\right)\sqrt{\frac{b+}{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b
^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b + (b^2 - 4*a*c)^
(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b^
2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b + (b^2 - 4*a*c)
^(1/2))/(2*c))^(1/2) + a*b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*
a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (1 - x^2)^
(1/2)/(a*x) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/(
```

```
(b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b
- (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(1/2) + a*b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b -
(b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*
c))^(3/2) + b^2*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b -
 (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(3/2)))/(2*a^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)
) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2
- 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4
*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) +
 a*b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)
+ b^2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b + (b^2 - 4*
a*c)^(1/2))/(2*c))^(1/2) + a*b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))
/(2*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((
((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(
1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2)))*(b^3*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b^2*(-(
b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b - (b^2 - 4*a*c)^(1/2))
/(2*c))^(1/2) - 2*a*c^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b^2*c*(-
(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b - (b^2 - 4*a*c)^(1/2)
)/(2*c))^(1/2) + a*b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a^2*((
b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**2*(a + b*x**2 + c*x**4)), x)
```

3.385 $\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx$

Optimal. Leaf size=96

$$\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\,x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

```
[Out] -arcsin(x)-1/5*arctanh(1/2*x*(-2+2*5^(1/2))^(1/2)/(-x^2+1)^(1/2))*(-10+5*5^
(1/2))^(1/2)+1/5*arctan(1/2*x*(2+2*5^(1/2))^(1/2)/(-x^2+1)^(1/2))*(10+5*5^(
1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1293, 216, 1692, 377, 207, 203}

$$\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\,x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

```
[Out] -ArcSin[x] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1
- x^2]] - Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[(-1 + Sqrt[5])/2]*x)/Sqrt[1
- x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx = -\int \frac{1}{\sqrt{1-x^2}}\,dx - \int \frac{1-2x^2}{\sqrt{1-x^2}\left(-1+x^2+x^4\right)}\,dx$$

$$= -\sin^{-1}(x) - \int \left(\frac{-2+\frac{4}{\sqrt{5}}}{\sqrt{1-x^2}\left(1-\sqrt{5}+2x^2\right)} + \frac{-2-\frac{4}{\sqrt{5}}}{\sqrt{1-x^2}\left(1+\sqrt{5}+2x^2\right)}\right) dx$$

$$= -\sin^{-1}(x) + \frac{1}{5}\left(2\left(5-2\sqrt{5}\right)\right)\int \frac{1}{\sqrt{1-x^2}\left(1-\sqrt{5}+2x^2\right)}\,dx + \frac{1}{5}\left(2\left(5+2\sqrt{5}\right)\right)\int \frac{}{\sqrt{1-}}$$

$$= -\sin^{-1}(x) + \frac{1}{5}\left(2\left(5-2\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}-\left(-3+\sqrt{5}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right) + \frac{1}{5}\left(2\right.$$

$$= -\sin^{-1}(x) + \sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right) - \sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(-1\right.}}{\sqrt{1}}\right.$$

Mathematica [C] time = 0.50, size = 743, normalized size = 7.74

$$i\sqrt{5\left(\sqrt{5}-2\right)}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}\sqrt{1-x^2}-i\sqrt{2\left(1+\sqrt{5}\right)}\,x+2\right)+2i\sqrt{\sqrt{5}-2}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}\sqrt{1-x^2}-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

```
[Out] (-2*Sqrt[5]*ArcSin[x] + (-2 + Sqrt[5])*Sqrt[2 + Sqrt[5]]*Log[-Sqrt[(-1 + Sq
rt[5])/2] + x] + 2*Sqrt[2 + Sqrt[5]]*Log[Sqrt[(-1 + Sqrt[5])/2] + x] - Sqrt
[5*(2 + Sqrt[5])]*Log[Sqrt[(-1 + Sqrt[5])/2] + x] - (2*I)*Sqrt[-2 + Sqrt[5]
]*Log[(-I)*Sqrt[(1 + Sqrt[5])/2] + x] - I*Sqrt[5*(-2 + Sqrt[5])]*Log[(-I)*S
qrt[(1 + Sqrt[5])/2] + x] + (2*I)*Sqrt[-2 + Sqrt[5]]*Log[I*Sqrt[(1 + Sqrt[5
])/2] + x] + I*Sqrt[5*(-2 + Sqrt[5])]*Log[I*Sqrt[(1 + Sqrt[5])/2] + x] + (2
*I)*Sqrt[-2 + Sqrt[5]]*Log[2 - I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt
[5])]*Sqrt[1 - x^2]] + I*Sqrt[5*(-2 + Sqrt[5])]*Log[2 - I*Sqrt[2*(1 + Sqrt[
5])]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] - (2*I)*Sqrt[-2 + Sqrt[5]]*Lo
g[2 + I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] - I*
Sqrt[5*(-2 + Sqrt[5])]*Log[2 + I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt
[5])]*Sqrt[1 - x^2]] + 2*Sqrt[2 + Sqrt[5]]*Log[2 - Sqrt[2*(-1 + Sqrt[5])]*x
 + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]] - Sqrt[5*(2 + Sqrt[5])]*Log[2 -
 Sqrt[2*(-1 + Sqrt[5])]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]] - 2*Sq
rt[2 + Sqrt[5]]*Log[2 + Sqrt[2*(-1 + Sqrt[5])]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[
5])*(-1 + x^2)]] + Sqrt[5*(2 + Sqrt[5])]*Log[2 + Sqrt[2*(-1 + Sqrt[5])]*x +
 Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]])/(2*Sqrt[5])
```

fricas [B] time = 1.09, size = 290, normalized size = 3.02

$$\frac{2}{5}\sqrt{5}\sqrt{\sqrt{5}+2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{-x^2+1}\left(\sqrt{5}-3\right)+\sqrt{5}-3\right)\sqrt{\sqrt{5}+2}\sqrt{\frac{x^4-4x^2-\sqrt{5}\left(x^4-2x^2\right)-2\left(\sqrt{5}x^2-x^2+2\right)\sqrt{-x^2+1}}{x^4}}}{4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*(sqrt(2)*(sqrt(-x^2 + 1)*(sqrt(5)
- 3) + sqrt(5) - 3)*sqrt(sqrt(5) + 2)*sqrt((x^4 - 4*x^2 - sqrt(5)*(x^4 - 2*
x^2) - 2*(sqrt(5)*x^2 - x^2 + 2)*sqrt(-x^2 + 1) + 4)/x^4) + 2*sqrt(-x^2 + 1
)*sqrt(sqrt(5) + 2)*(sqrt(5) - 3))/x) + 1/10*sqrt(5)*sqrt(sqrt(5) - 2)*log(
-(2*x^2 + (sqrt(-x^2 + 1)*(sqrt(5)*x + x) - sqrt(5)*x - x)*sqrt(sqrt(5) - 2
) + 2*sqrt(-x^2 + 1) - 2)/x^2) - 1/10*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(2*x^2
 - (sqrt(-x^2 + 1)*(sqrt(5)*x + x) - sqrt(5)*x - x)*sqrt(sqrt(5) - 2) + 2*s
qrt(-x^2 + 1) - 2)/x^2) + 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [B] time = 0.72, size = 209, normalized size = 2.18

$$-\frac{1}{2}\pi\operatorname{sgn}(x)-\frac{1}{5}\sqrt{5\sqrt{5}+10}\arctan\left(-\frac{\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}}{\sqrt{2\sqrt{5}+2}}\right)-\frac{1}{10}\sqrt{5\sqrt{5}-10}\log\left(\left|\sqrt{2\sqrt{5}-2}-\frac{x}{\sqrt{-x^2+1}-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*pi*sgn(x) - 1/5*sqrt(5*sqrt(5) + 10)*arctan(-(x/(sqrt(-x^2 + 1) - 1) -
 (sqrt(-x^2 + 1) - 1)/x)/sqrt(2*sqrt(5) + 2)) - 1/10*sqrt(5*sqrt(5) - 10)*l
og(abs(sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/
x)) + 1/10*sqrt(5*sqrt(5) - 10)*log(abs(-sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2
 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x)) - arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)
^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

maple [B] time = 0.09, size = 160, normalized size = 1.67

$$-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{\sqrt{-x^2+1}-1}{\sqrt{2+\sqrt{5}}\ x}\right)}{5\sqrt{2+\sqrt{5}}}+\frac{\sqrt{\sqrt{5}-2}\ \sqrt{5}\ \operatorname{arctanh}\left(\frac{\sqrt{-x^2+1}-1}{\sqrt{\sqrt{5}-2}\ x}\right)}{5}+2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\frac{\sqrt{2+\sqrt{5}}\ \sqrt{5}\ \operatorname{arcta}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x)
```

```
[Out] -1/5*(2+5^(1/2))^(1/2)*5^(1/2)*arctan(((-x^2+1)^(1/2)-1)/x/(2+5^(1/2))^(1/2
))+1/5*(5^(1/2)-2)^(1/2)*5^(1/2)*arctanh(((-x^2+1)^(1/2)-1)/x/(5^(1/2)-2)^(
1/2))-1/5*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh(((-x^2+1)^(1/2)-1)/x/(2+5^(1/2)
)^(1/2))-1/5*5^(1/2)/(5^(1/2)-2)^(1/2)*arctan(((-x^2+1)^(1/2)-1)/x/(5^(1/2)
-2)^(1/2))+2*arctan(((-x^2+1)^(1/2)-1)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}\,x^2}{x^4+x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^2/(x^4 + x^2 - 1), x)
```

mupad [B] time = 1.50, size = 383, normalized size = 3.99

$$-\operatorname{asin}(x)-\frac{\ln\left(\frac{\frac{\left(x\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}-1\right)1\mathrm{i}}{\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}-2\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}-1\right)1\mathrm{i}}{\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}+2\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - x^2)^(1/2))/(x^2 + x^4 - 1),x)
```

```
[Out] (log((((x*(- 5^(1/2)/2 - 1/2)^(1/2) - 1)*1i)/(5^(1/2)/2 + 3/2)^(1/2) - (1 -
 x^2)^(1/2)*1i)/(x - (- 5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) + 2))/((2*(- 5^(1
/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 3/2)^(1/2))
- (log((((x*(5^(1/2)/2 - 1/2)^(1/2) - 1)*1i)/(3/2 - 5^(1/2)/2)^(1/2) - (1 -
 x^2)^(1/2)*1i)/(x - (5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) - 2))/((2*(5^(1/2)/
2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(3/2 - 5^(1/2)/2)^(1/2)) - asin
(x) + (log((((x*(5^(1/2)/2 - 1/2)^(1/2) + 1)*1i)/(3/2 - 5^(1/2)/2)^(1/2) +
(1 - x^2)^(1/2)*1i)/(x + (5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) - 2))/((2*(5^(1
/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(3/2 - 5^(1/2)/2)^(1/2)) -
(log((((x*(- 5^(1/2)/2 - 1/2)^(1/2) + 1)*1i)/(5^(1/2)/2 + 3/2)^(1/2) + (1 -
 x^2)^(1/2)*1i)/(x + (- 5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) + 2))/((2*(- 5^(1
/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 3/2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\sqrt{-(x-1)(x+1)}}{x^4+x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+1)**(1/2)/(x**4+x**2-1),x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(x**4 + x**2 - 1), x)
```

3.386 $\int \frac{x^8}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=479

$$-\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c^3\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

```
[Out] 3/8*d^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(5/2)+1/2*b*d*arctanh(x*e^(1
/2)/(e*x^2+d)^(1/2))/c^2/e^(3/2)+(-a*c+b^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/
2))/c^3/e^(1/2)-3/8*d*x*(e*x^2+d)^(1/2)/c/e^2-1/2*b*x*(e*x^2+d)^(1/2)/c^2/e
+1/4*x^3*(e*x^2+d)^(1/2)/c/e-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2
)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^3-2*a*b*c+(-2*a^2*c^2+4*
a*b^2*c-b^4)/(-4*a*c+b^2)^(1/2))/c^3/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^3-2*a*b*c+(2*a^2*c^2-4*
a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2))/c^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.86, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1303, 217, 206, 321, 1692, 377, 205}

$$-\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c^3\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-3*d*x*Sqrt[d + e*x^2])/(8*c*e^2) - (b*x*Sqrt[d + e*x^2])/(2*c^2*e) + (x^3
*Sqrt[d + e*x^2])/(4*c*e) - ((b^3 - 2*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]
]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b^3 - 2*a*b*c + (b^4 - 4*a*b
^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b +
Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (3*d^2*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c*e^(5/2)) + (b*d*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/(2*c^2*e^(3/2)) + ((b^2 - a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]])/(c^3*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^8}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx = \int \left(\frac{b^2-ac}{c^3\sqrt{d+ex^2}} - \frac{bx^2}{c^2\sqrt{d+ex^2}} + \frac{x^4}{c\sqrt{d+ex^2}} - \frac{a\left(b^2-ac\right)+b\left(b^2-2ac\right)x^2}{c^3\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right) d$$

$$= -\frac{\int \frac{a\left(b^2-ac\right)+b\left(b^2-2ac\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx}{c^3} - \frac{b\int \frac{x^2}{\sqrt{d+ex^2}}dx}{c^2} + \frac{\int \frac{x^4}{\sqrt{d+ex^2}}dx}{c} + \frac{\left(b^2-ac\right)\int \frac{1}{\sqrt{d+ex^2}}}{c^3}$$

$$= -\frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} - \frac{\int \left(\frac{b\left(b^2-2ac\right)+\frac{-b^4+4ab^2c-2a^2c^2}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{b\left(b^2-2ac\right)-\frac{-b^4+4ab^2c-2}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d}}\right.}{c^3}$$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} + \frac{\left(b^2-ac\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}} - $$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}} + \frac{\left(b^2-a\right.}{}$$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} - \frac{\left(b^3-2abc-\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\tan}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd}}$$

Mathematica [A] time = 1.87, size = 461, normalized size = 0.96

$$\frac{-\frac{8\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} - \frac{8\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{8\left(b^2-ac\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex}}\right.}{\sqrt{e}}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*b*c*x*Sqrt[d + e*x^2])/e + (2*c^2*x^3*Sqrt[d + e*x^2])/e - (8*(b^3 - 2
*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
]) - (8*(b^3 - 2*a*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*
c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e]) + (4*b*c*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(3/2)
+ (8*(b^2 - a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e] + (3*c^2*d*(
-(Sqrt[e]*x*Sqrt[d + e*x^2]) + d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]))/e^(
5/2))/(8*c^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 2.03, size = 105, normalized size = 0.22

$$\frac{1}{8}\sqrt{x^2e+d}\left(\frac{2x^2e^{(-1)}}{c}-\frac{\left(3c^5de+4bc^4e^2\right)e^{(-3)}}{c^6}\right)x-\frac{\left(3c^2d^2+4bcde+8b^2e^2-8ace^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+\right.\right.}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^2*e + d)*(2*x^2*e^(-1)/c - (3*c^5*d*e + 4*b*c^4*e^2)*e^(-3)/c^6)
*x - 1/16*(3*c^2*d^2 + 4*b*c*d*e + 8*b^2*e^2 - 8*a*c*e^2)*e^(-5/2)*log((x*e
^(1/2) - sqrt(x^2*e + d))^2)/c^3
```

maple [C] time = 0.03, size = 377, normalized size = 0.79

$$\frac{\sqrt{e x^2+d}\,x^3}{4ce}-\frac{a\ln\left(\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{c^2\sqrt{e}}+\frac{b^2\ln\left(\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{c^3\sqrt{e}}+\frac{bd\ln\left(\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{2c^2e^{\frac{3}{2}}}+\frac{3d^2\ln\left(\sqrt{e}\,x+\right.}{8c\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/4*x^3*(e*x^2+d)^(1/2)/c/e-3/8*d*x*(e*x^2+d)^(1/2)/c/e^2+3/8/c*d^2/e^(5/2)
*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/2*b*x*(e*x^2+d)^(1/2)/c^2/e+1/2/c^2*b*d/e^
(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/c^2*a*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e
^(1/2)+1/c^3*b^2*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)-1/2/c^3*e^(1/2)*sum(
(b*(2*a*c-b^2)*_R^2+2*(2*a^2*c*e-2*a*b^2*e-2*a*b*c*d+b^3*d)*_R+2*a*b*c*d^2-
b^3*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d
^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(
4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(x^8/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{d+ex^2}\,\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**8/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.387 $\int \frac{x^6}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=366

$$\frac{\left(-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\tanh^{-1}}{c^2}$$

```
[Out] -1/2*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(3/2)-b*arctanh(x*e^(1/2)/(e*
x^2+d)^(1/2))/c^2/e^(1/2)+1/2*x*(e*x^2+d)^(1/2)/c/e+arctan(x*(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2
-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))
)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*(-3*a*c+
b^2)/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a
*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.17, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1303, 217, 206, 321, 1692, 377, 205}

$$\frac{\left(-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\tanh^{-1}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^
2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c*e^(3/2)) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^2*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(-\frac{b}{c^2\sqrt{d+ex^2}}+\frac{x^2}{c\sqrt{d+ex^2}}+\frac{ab+\left(b^2-ac\right)x^2}{c^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{ab+\left(b^2-ac\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c^2} - \frac{b\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c^2} + \frac{\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{c}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce} + \frac{\int \left(\frac{b^2-ac+\frac{b\left(-b^2+3ac\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{b^2-ac-\frac{b\left(-b^2+3ac\right)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c^2} - \frac{b\,\mathrm{Sub}\ldots}{\ldots}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce} - \frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}} + \frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+e\ldots}}\ldots}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce} - \frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}} - \frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}} + \frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\mathrm{S}\ldots}{\ldots}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce} + \frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(b^2-ac+\frac{b(b\ldots}{\sqrt{b\ldots}}\ldots\right.}{c^2\sqrt{b+\sqrt{b^2\ldots}}}$$

Mathematica [A] time = 1.03, size = 355, normalized size = 0.97

$$\frac{\frac{2\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{2\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{cd\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}\cdots}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((c*x*Sqrt[d + e*x^2])/e + (2*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e]) + (2*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e]) - (c*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(3/2
) - (2*b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e])/(2*c^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 2.14, size = 55, normalized size = 0.15

$$\frac{\sqrt{x^2e+d}\,xe^{(-1)}}{2c}+\frac{(cd+2be)e^{\left(-\frac{3}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2*e + d)*x*e^(-1)/c + 1/4*(c*d + 2*b*e)*e^(-3/2)*log((x*e^(1/2)
- sqrt(x^2*e + d))^2)/c^2
```

maple [C] time = 0.03, size = 269, normalized size = 0.73

$$-\frac{b\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{c^2\sqrt{e}}-\frac{d\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{2c\,e^{\frac{3}{2}}}+\frac{\cdots}{2c^2\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\_Z^3+\left(16a\,e^2-\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/2*x*(e*x^2+d)^(1/2)/c/e-1/2/c*d/e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/c
^2*b*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)+1/2/c^2*e^(1/2)*sum(((a*c-b^2)*_
R^2+2*(-2*a*b*e-a*c*d+b^2*d)*_R+a*c*d^2-b^2*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*
c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x
^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e
+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\sqrt{e\,x^2 + d}\,\left(c\,x^4 + b\,x^2 + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(x^6/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{d + ex^2}\,\left(a + bx^2 + cx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**6/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.388 $\int \frac{x^4}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=298

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(1/2)-arctan(x*(2*c*d-e*(b-(-4*a*c+b
^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b
^2)/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c
/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1303, 217, 206, 1692, 377, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^2
 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]]/(c*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule steps

$$\int \frac{x^4}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx = \int\left(\frac{1}{c\sqrt{d+ex^2}} - \frac{a+bx^2}{c\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{1}{\sqrt{d+ex^2}}dx}{c} - \frac{\int \frac{a+bx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx}{c}$$

$$= -\frac{\int\left(\frac{b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{b-\frac{-b^2+2ac}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c} + \frac{\text{Subst}\left(\int \frac{1}{1-ex^2}dx,\right.}{c}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}dx, x\right.}{c}$$

$$= -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-}}{\sqrt{b+}}\right.}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(\right.}}$$

Mathematica [A] time = 0.59, size = 292, normalized size = 0.98

$$\frac{-\frac{\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[
b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b
```

```
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^
2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[b + Sqrt[b^2 -
4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqrt
[d + e*x^2]]/Sqrt[e])/c
```

fricas [B] time = 44.77, size = 11094, normalized size = 37.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*
c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^
2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4
- 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a
^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*
c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 - 4
*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)
*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3
- a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e +
(b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d
*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b
*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 + 2*sqrt(1/2)
*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a
*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d
*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^
2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*
(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 -
 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^
5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e
+ (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c
)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4
*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b
^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e
 + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5
)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*
c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2)*c*e*
sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 -
 (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2
 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4
*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^
2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c
^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^
2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b
^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^
2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^
2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c
^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4
- 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*
b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((
b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c
^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2
- 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 -
 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6
)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^
2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 - 5*a*b^3*c + 4*a^
2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b
```

```
*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*
a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt(
(a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(
(b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^
2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c
^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e
 + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*
c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^
3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
 + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3
*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(
a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3
 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))
*log((2*a^3*b*d*e - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3
)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2
*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4
 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*
e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) -
 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^
2 - 4*a^3*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^
4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4
*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*
x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)
*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 -
 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^
2*b^2*c^4 - 4*a^3*c^5)*e^4)) - ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*
b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((
b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^
2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4
 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*
d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d
^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)
))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^
2*c^3)*e^2)))/x^2) + sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^
2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2
 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(
a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^
3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*
c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b
^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e -
((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c
^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*
d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*
b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 -
 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)
*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^
2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 -
 (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3
 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2
+ (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*
a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2
*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^
5)*e^4)) - ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c +
4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (
a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e +
 (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^
2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4
*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^
4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c
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^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) +
 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/(c*e), 1/4*(sqr
t(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a
*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((
a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((
b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2
*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^
4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e
+ (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 - 4*a^2*c^4)
*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x
^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4
- 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a
^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a
*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x
^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3
+ 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a
^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2
*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5
- 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3
*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 - 5*a*b
^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^
3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b
^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)
*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*
b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c
^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 +
 (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a
*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2)*c*e*sqrt(-((b
^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2
 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 -
 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d
^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^
2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4))
)/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2
*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 -
4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2
 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4
*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^
2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c
^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2
- (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 -
 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e
 + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b
*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3
 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e +
 (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*
d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*
d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*
x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^
2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*
e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6
- 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8
*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^
3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2
*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a
*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e +
(a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2
)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*
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a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4
 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^
4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a
^3*b*d*e - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e +
 (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3
*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(
a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b
^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3
*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2
*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*
a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a
^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b
^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*
c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4
 - 4*a^3*c^5)*e^4)) - ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a
^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a
*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b
*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2
*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(
b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 -
2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*
c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^
2)))/x^2) + sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e +
((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c
^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a
^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^
4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^
3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 -
4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e - ((a*b^2*c
^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^
3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(
a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^
3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*
c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4
*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqr
t(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2
 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*
c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 -
2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^
4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2
*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4))
- ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2
)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2
*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c
^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 -
2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)
*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2
*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 -
 (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - 4*sqrt(-
e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*e)]
```

giac [A] time = 2.05, size = 27, normalized size = 0.09

$$-\frac{e^{\left(-\frac{1}{2}\right)} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2 e + d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

maple [C] time = 0.02, size = 200, normalized size = 0.67

$$\frac{}{2c\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^3c+3\,\mathrm{RootO}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/c*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)+1/2/c*e^(1/2)*sum((_R^2*b+2*(2*a*
e-b*d)*_R+b*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c
*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c
+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^
3)*_Z))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(x^4/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{d+ex^2}\,\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.389 $\int \frac{x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] -arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2)^(1/2))^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(
1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(
1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1303, 377, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Arc
Tan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]
]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx = \int \left(\frac{1-\frac{b}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{1+\frac{b}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right.$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx + \left(1+\frac{b}{\sqrt{b^2-4ac}}\right.$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\right.$$

$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\ldots\right.}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b\ldots\right.}}$$

Mathematica [A] time = 0.48, size = 227, normalized size = 0.95

$$\frac{\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c
]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/Sqrt[2*c*d + (-b +
Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])
])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 10.22, size = 3395, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)
*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c
- 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a
^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 -
4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log((((b^2*c - 4*a*c^2)*d^3 - (b^3 -
 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^
4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2
*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 + 2*a*d^2 - (b*d
^2 - 4*a*d*e)*x^2 + 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^3*c - 4*a*b*c^2)
*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 -
2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4
*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*
b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*
e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*s
```

```
qrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a
*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^
3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c
)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 -
 (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)
*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2
- 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^
2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log((((b^2*c - 4*a*c
^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c
^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*
c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2
+ 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2 - 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^
3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a
^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^
4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2
*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*s
qrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 -
 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^
3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 +
 (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e +
(a*b^2 - 4*a^2*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - ((b^2*c
- 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^
2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a
^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/
((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(
-(((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)
*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2
*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*
a^3*c)*e^4))*x^2 - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2 + 2*sqrt(1/2)*((b^2 - 4*
a*c)*d^2*x + ((b^3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
 + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*
c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*
sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*
c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*
c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4
*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3
- 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*d -
 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e
^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4
- 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 -
 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*
a^2*c)*e^2))*log(-(((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2
- 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)
*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^
3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2 - 2*sqr
t(1/2)*((b^2 - 4*a*c)*d^2*x + ((b^3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c -
 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3
)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 -
2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4
*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^
2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c
^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a
*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,c]=[-72,-7,6]Evaluation time: 0.44Unable to divide, perhaps due
to rounding error%%%{18446744069414584320,[4,7,8,2,3,14,2]%%%}+%%%{-2147483
648,[3,8,10,8,3,10,1]%%%}+%%%{-12884901888,[3,8,10,7,2,12,2]%%%}+%%%{463856
467968,[3,8,10,6,1,14,3]%%%}+%%%{1924145348608,[3,8,10,5,0,16,4]%%%}+%%%{53
6870912,[3,8,9,8,5,10,0]%%%}+%%%{20401094656,[3,8,9,7,4,12,1]%%%}+%%%{-1503
23855360,[3,8,9,6,3,14,2]%%%}+%%%{-3135326126080,[3,8,9,5,2,16,3]%%%}+%%%{-
4672924418048,[3,8,9,4,1,18,4]%%%}+%%%{6047313952768,[3,8,9,3,0,20,5]%%%}+%
%%{-4294967296,[3,8,8,7,6,12,0]%%%}+%%%{-42412802048,[3,8,8,6,5,14,1]%%%}+%
%%{1046898278400,[3,8,8,5,4,16,2]%%%}+%%%{6210522710016,[3,8,8,4,3,18,3]%%%
}+%%%{-1786706395136,[3,8,8,3,2,20,4]%%%}+%%%{-11544872091648,[3,8,8,2,1,22
,5]%%%}+%%%{4398046511104,[3,8,8,1,0,24,6]%%%}+%%%{12750684160,[3,8,7,6,7,1
4,0]%%%}+%%%{-23890755584,[3,8,7,5,6,16,1]%%%}+%%%{-2103460233216,[3,8,7,4,
5,18,2]%%%}+%%%{-3324304687104,[3,8,7,3,4,20,3]%%%}+%%%{9758165696512,[3,8,
7,2,3,22,4]%%%}+%%%{1649267441664,[3,8,7,1,2,24,5]%%%}+%%%{-4398046511104,[
3,8,7,0,1,26,6]%%%}+%%%{-17985175552,[3,8,6,5,8,16,0]%%%}+%%%{161866579968,
[3,8,6,4,7,18,1]%%%}+%%%{1586990415872,[3,8,6,3,6,20,2]%%%}+%%%{-1795296329
728,[3,8,6,2,5,22,3]%%%}+%%%{-4123168604160,[3,8,6,1,4,24,4]%%%}+%%%{384829
0697216,[3,8,6,0,3,26,5]%%%}+%%%{12213813248,[3,8,5,4,9,18,0]%%%}+%%%{-1717
98691840,[3,8,5,3,8,20,1]%%%}+%%%{-212600881152,[3,8,5,2,7,22,2]%%%}+%%%{14
77468749824,[3,8,5,1,6,24,3]%%%}+%%%{-1099511627776,[3,8,5,0,5,26,4]%%%}+%%
%{-3221225472,[3,8,4,3,10,20,0]%%%}+%%%{57982058496,[3,8,4,2,9,22,1]%%%}+%%
%{-154618822656,[3,8,4,1,8,24,2]%%%}+%%%{103079215104,[3,8,4,0,7,26,3]%%%}+
%%%{1048576,[3,6,10,4,2,4,0]%%%}+%%%{8388608,[3,6,10,3,1,6,1]%%%}+%%%{16777
216,[3,6,10,2,0,8,2]%%%}+%%%{-5242880,[3,6,9,3,3,6,0]%%%}+%%%{-29360128,[3,
6,9,2,2,8,1]%%%}+%%%{-33554432,[3,6,9,1,1,10,2]%%%}+%%%{9699328,[3,6,8,2,4,
8,0]%%%}+%%%{33554432,[3,6,8,1,3,10,1]%%%}+%%%{16777216,[3,6,8,0,2,12,2]%%%
}+%%%{-7864320,[3,6,7,1,5,10,0]%%%}+%%%{-12582912,[3,6,7,0,4,12,1]%%%}+%%%{
2359296,[3,6,6,0,6,12,0]%%%}+%%%{536870912,[2,7,10,6,2,8,1]%%%}+%%%{6710886
400,[2,7,10,5,1,10,2]%%%}+%%%{18253611008,[2,7,10,4,0,12,3]%%%}+%%%{-134217
728,[2,7,9,6,4,8,0]%%%}+%%%{-5502926848,[2,7,9,5,3,10,1]%%%}+%%%{-369098752
00,[2,7,9,4,2,12,2]%%%}+%%%{-42949672960,[2,7,9,3,1,14,3]%%%}+%%%{429496729
60,[2,7,9,2,0,16,4]%%%}+%%%{956301312,[2,7,8,5,5,10,0]%%%}+%%%{18656264192,
[2,7,8,4,4,12,1]%%%}+%%%{64961380352,[2,7,8,3,3,14,2]%%%}+%%%{-8589934592,[
2,7,8,2,2,16,3]%%%}+%%%{-85899345920,[2,7,8,1,1,18,4]%%%}+%%%{-2642411520,[
2,7,7,4,6,12,0]%%%}+%%%{-27783069696,[2,7,7,3,5,14,1]%%%}+%%%{-33957085184,
[2,7,7,2,4,16,2]%%%}+%%%{73014444032,[2,7,7,1,3,18,3]%%%}+%%%{42949672960,[
2,7,7,0,2,20,4]%%%}+%%%{3556769792,[2,7,6,3,7,14,0]%%%}+%%%{17716740096,[2,
7,6,2,6,16,1]%%%}+%%%{-12884901888,[2,7,6,1,5,18,2]%%%}+%%%{-39728447488,[2
,7,6,0,4,20,3]%%%}+%%%{-2340421632,[2,7,5,2,8,16,0]%%%}+%%%{-2415919104,[2,
7,5,1,7,18,1]%%%}+%%%{12079595520,[2,7,5,0,6,20,2]%%%}+%%%{603979776,[2,7,4
,1,9,18,0]%%%}+%%%{-1207959552,[2,7,4,0,8,20,1]%%%}+%%%{2147483648,[1,8,10,
9,3,10,1]%%%}+%%%{38654705664,[1,8,10,8,2,12,2]%%%}+%%%{51539607552,[1,8,10
,7,1,14,3]%%%}+%%%{-274877906944,[1,8,10,6,0,16,4]%%%}+%%%{-536870912,[1,8,
9,9,5,10,0]%%%}+%%%{-26843545600,[1,8,9,8,4,12,1]%%%}+%%%{-188978561024,[1,
8,9,7,3,14,2]%%%}+%%%{146028888064,[1,8,9,6,2,16,3]%%%}+%%%{962072674304,[1
,8,9,5,1,18,4]%%%}+%%%{-549755813888,[1,8,9,4,0,20,5]%%%}+%%%{4294967296,[1
,8,8,8,6,12,0]%%%}+%%%{95026151424,[1,8,8,7,5,14,1]%%%}+%%%{239444426752,[1
,8,8,6,4,16,2]%%%}+%%%{-858993459200,[1,8,8,5,3,18,3]%%%}+%%%{-618475290624
,[1,8,8,4,2,20,4]%%%}+%%%{1099511627776,[1,8,8,3,1,22,5]%%%}+%%%{-127506841
60,[1,8,7,7,7,14,0]%%%}+%%%{-136633647104,[1,8,7,6,6,16,1]%%%}+%%%{62277025
792,[1,8,7,5,5,18,2]%%%}+%%%{936302870528,[1,8,7,4,4,20,3]%%%}+%%%{-5497558
13888,[1,8,7,3,3,22,4]%%%}+%%%{-549755813888,[1,8,7,2,2,24,5]%%%}+%%%{17985
175552,[1,8,6,6,8,16,0]%%%}+%%%{71940702208,[1,8,6,5,7,18,1]%%%}+%%%{-26736
```

```
1714176,[1,8,6,4,6,20,2]%%%}+%%%{-137438953472,[1,8,6,3,5,22,3]%%%}+%%%{481
036337152,[1,8,6,2,4,24,4]%%%}+%%%{-12213813248,[1,8,5,5,9,18,0]%%%}+%%%{72
47757312,[1,8,5,4,8,20,1]%%%}+%%%{103079215104,[1,8,5,3,7,22,2]%%%}+%%%{-13
7438953472,[1,8,5,2,6,24,3]%%%}+%%%{3221225472,[1,8,4,4,10,20,0]%%%}+%%%{-1
2884901888,[1,8,4,3,9,22,1]%%%}+%%%{12884901888,[1,8,4,2,8,24,2]%%%}+%%%{-1
048576,[1,6,10,5,2,4,0]%%%}+%%%{-8388608,[1,6,10,4,1,6,1]%%%}+%%%{-16777216
,[1,6,10,3,0,8,2]%%%}+%%%{8388608,[1,6,9,4,3,6,0]%%%}+%%%{62914560,[1,6,9,3
,2,8,1]%%%}+%%%{150994944,[1,6,9,2,1,10,2]%%%}+%%%{134217728,[1,6,9,1,0,12,
3]%%%}+%%%{-26476544,[1,6,8,3,4,8,0]%%%}+%%%{-163577856,[1,6,8,2,3,10,1]%%%
}+%%%{-301989888,[1,6,8,1,2,12,2]%%%}+%%%{-134217728,[1,6,8,0,1,14,3]%%%}+%
%%{41156608,[1,6,7,2,5,10,0]%%%}+%%%{178257920,[1,6,7,1,4,12,1]%%%}+%%%{167
772160,[1,6,7,0,3,14,2]%%%}+%%%{-31457280,[1,6,6,1,6,12,0]%%%}+%%%{-6920601
6,[1,6,6,0,5,14,1]%%%}+%%%{9437184,[1,6,5,0,7,14,0]%%%}+%%%{-402653184,[0,7
,10,7,2,8,1]%%%}+%%%{-5637144576,[0,7,10,6,1,10,2]%%%}+%%%{-16106127360,[0,
7,10,5,0,12,3]%%%}+%%%{100663296,[0,7,9,7,4,8,0]%%%}+%%%{4160749568,[0,7,9,
6,3,10,1]%%%}+%%%{30198988800,[0,7,9,5,2,12,2]%%%}+%%%{28991029248,[0,7,9,4
,1,14,3]%%%}+%%%{-68719476736,[0,7,9,3,0,16,4]%%%}+%%%{-687865856,[0,7,8,6,
5,10,0]%%%}+%%%{-13925089280,[0,7,8,5,4,12,1]%%%}+%%%{-48184164352,[0,7,8,4
,3,14,2]%%%}+%%%{49392123904,[0,7,8,3,2,16,3]%%%}+%%%{120259084288,[0,7,8,2
,1,18,4]%%%}+%%%{-68719476736,[0,7,8,1,0,20,5]%%%}+%%%{1845493760,[0,7,7,5,
6,12,0]%%%}+%%%{19964887040,[0,7,7,4,5,14,1]%%%}+%%%{11542724608,[0,7,7,3,4
,16,2]%%%}+%%%{-113816633344,[0,7,7,2,3,18,3]%%%}+%%%{8589934592,[0,7,7,1,2
,20,4]%%%}+%%%{68719476736,[0,7,7,0,1,22,5]%%%}+%%%{-2432696320,[0,7,6,4,7,
14,0]%%%}+%%%{-11207180288,[0,7,6,3,6,16,1]%%%}+%%%{28185722880,[0,7,6,2,5,
18,2]%%%}+%%%{34359738368,[0,7,6,1,4,20,3]%%%}+%%%{-60129542144,[0,7,6,0,3,
22,4]%%%}+%%%{1577058304,[0,7,5,3,8,16,0]%%%}+%%%{-201326592,[0,7,5,2,7,18,
1]%%%}+%%%{-14495514624,[0,7,5,1,6,20,2]%%%}+%%%{17179869184,[0,7,5,0,5,22,
3]%%%}+%%%{-402653184,[0,7,4,2,9,18,0]%%%}+%%%{1610612736,[0,7,4,1,8,20,1]%
%%}+%%%{-1610612736,[0,7,4,0,7,22,2]%%%} / %%%{-1024,[0,3,4,2,1,2,0]%%%}+%%
%{-4096,[0,3,4,1,0,4,1]%%%}+%%%{2560,[0,3,3,1,2,4,0]%%%}+%%%{4096,[0,3,3,0,
1,6,1]%%%}+%%%{-1536,[0,3,2,0,3,6,0]%%%} Error: Bad Argument Value
```

maple [C] time = 0.02, size = 161, normalized size = 0.67

$$-\frac{}{2\left(\mathrm{RootOf}\left(\_Z^4 c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^3 c+3\,\mathrm{RootOf}\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -1/2*e^(1/2)*sum((_R^2-2*_R*d+d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2
-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2
),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2
+(4*b*d^2*e-4*c*d^3)*_Z))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(x^2/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{d+ex^2}\,\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.390 $\int \frac{1}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=243

$$\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

```
[Out] 2*c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2*c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1174, 377, 205}

$$\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]
*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (2*c*ArcTan[(Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && !IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{2c\,\mathrm{ta}}{\sqrt{b^2-4ac}\sqrt{b+}}$$

Mathematica [A] time = 0.41, size = 229, normalized size = 0.94

$$\frac{2c\left(\frac{\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} - \frac{\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*(ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e]) - ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b + Sqrt[b^2 - 4*a*
c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 23.40, size = 4557, normalized size = 18.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 -
 (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d
*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*
d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*
c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 -
 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e
 + ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*
c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^
2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b
^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^
5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 + 2*sqrt
(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*
a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4
*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3
*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*
a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)
```

) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e + ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e - ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) + ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e - ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 -

```
 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*
c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c
)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 - 2*sqrt(1/
2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3
*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^
4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^
3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4
*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) +
 ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b
^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2
*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4
*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c
- 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*
e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*
a^3*c)*e^2)))/x^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.02, size = 151, normalized size = 0.62

$$-\frac{}{\mathrm{RootOf}\left(\_Z^4c + c\,d^4 + (4be - 4cd)\,\_Z^3 + \left(16a\,e^2 - 8deb + 6c\,d^2\right)\_Z^2 + \left(4b\,d^2e - 4c\,d^3\right)\_Z\right)^3 c + 3\,\mathrm{RootOf}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -2*e^(3/2)*sum(_R/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*
c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*
c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d
^3)*_Z))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,x^2 + d}\,\left(c\,x^4 + b\,x^2 + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(1/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\,\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.391 $\int \frac{1}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx$

Optimal. Leaf size=280

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{adx}$$

```
[Out] -(e*x^2+d)^(1/2)/a/d/x-c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e
*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))/a/(2*c
*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1303, 264, 1692, 377, 205}

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*d*x)) - (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b + Sqrt
[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
```

```
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx = \int \left(\frac{1}{ax^2\sqrt{d+ex^2}} + \frac{-b-cx^2}{a\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{1}{x^2\sqrt{d+ex^2}}dx}{a} + \frac{\int \frac{-b-cx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} + \frac{\int \left(\frac{-c-\frac{bc}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{-c+\frac{bc}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx}{a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4a}}\right.\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}dx,\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} - \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{c\left(1-\frac{b}{\sqrt{b^2-4}}\right.}{a\sqrt{b+\sqrt{b^2-}}}$$

Mathematica [A] time = 1.01, size = 271, normalized size = 0.97

$$-\frac{\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{\sqrt{d+ex^2}}{dx}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[d + e*x^2]/(d*x) + (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
- b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])
 + (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2
- 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a)
```

fricas [B] time = 46.07, size = 6431, normalized size = 22.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a
^2*c^2)*e - ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4
*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c
 - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/
((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b
^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^
8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c
)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log((((a^3*b^2*c^3 - 4*a^4*c^4)*d^3 - (a^
3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2)*x^2*sqrt(
((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c
^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)
*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c
^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)) + 2
*(a*b^2*c^3 - a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 2*a^2*b*c^3)*d*e - ((b^3*c^3 -
a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e + 4*(a*b^3*c^2 - 2*a
^2*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3*c^2 - 4*a^5*b*c
^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*d^2*e + (a^4*b^5 - 5*a^
5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6*a^6*b^2*c + 8*a^7*c^2)*e^3)*x*s
qrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2
*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*
c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a
^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4))
 + ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 - (2*a*b^5*c - 11*a^2*b^3*c
^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4*c + 8*a^3*b^2*c^2)*e^2)*x)*sqrt
(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - ((a^3*b^2*c -
4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(
((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c
^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)
*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c
^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((
a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c
)*e^2)))/x^2) - sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b
^2*c + 2*a^2*c^2)*e - ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*
d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2
- 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*
c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*
e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d
*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 -
 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log((((a^3*b^2*c^3 - 4*a^4*c^4)
*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2)
*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 -
4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c
 - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)
*e^4)) + 2*(a*b^2*c^3 - a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 2*a^2*b*c^3)*d*e - ((
b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e + 4*(a*b^3
*c^2 - 2*a^2*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3*c^2 -
 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*d^2*e + (a^4*
b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6*a^6*b^2*c + 8*a^7*c^2
)*e^3)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c
^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^
2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b
^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^
9*c)*e^4)) + ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 - (2*a*b^5*c - 11
*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4*c + 8*a^3*b^2*c^2)*e^
2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - ((a^
```

```
3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*
e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 -
4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c
 - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)
*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2
 - 4*a^5*c)*e^2)))/x^2) + sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b
^4 - 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4
*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2
*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c +
4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b
*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*
a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 -
(a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log(-(((a^3*b^2*c^3 -
 4*a^4*c^4)*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*
c^3)*d*e^2)*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*
a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6
*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 -
2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2
 - 4*a^9*c)*e^4)) - 2*(a*b^2*c^3 - a^2*c^4)*d^2 + 2*(a*b^3*c^2 - 2*a^2*b*c^
3)*d*e + ((b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e
 + 4*(a*b^3*c^2 - 2*a^2*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^
4*b^3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*d^
2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6*a^6*b^2*c
+ 8*a^7*c^2)*e^3)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c
- 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(
(a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^
4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8
*b^2 - 4*a^9*c)*e^4)) - ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 - (2*a
*b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4*c + 8*a^3*
b^2*c^2)*e^2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^
2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2
- 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*
a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6
*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 -
2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2
 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e
 + (a^4*b^2 - 4*a^5*c)*e^2)))/x^2) - sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*
c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (
a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^
2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4
*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*
c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a
^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*
c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log(-(((a^
3*b^2*c^3 - 4*a^4*c^4)*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c
^2 - 4*a^5*c^3)*d*e^2)*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*
(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e +
(a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3
 + (a^8*b^2 - 4*a^9*c)*e^4)) - 2*(a*b^2*c^3 - a^2*c^4)*d^2 + 2*(a*b^3*c^2 -
 2*a^2*b*c^3)*d*e + ((b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a
^2*c^4)*d*e + 4*(a*b^3*c^2 - 2*a^2*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2
 + d)*(((a^4*b^3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*
a^6*c^3)*d^2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6
*a^6*b^2*c + 8*a^7*c^2)*e^3)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2
- 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*
c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*
e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d
```

```
*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)) - ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)
*d^2 - (2*a*b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4
*c + 8*a^3*b^2*c^2)*e^2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e
+ (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*
(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e +
(a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3
 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a
^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)))/x^2) + 4*sqrt(e*x^2 + d))/(a*d*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 0.02, size = 197, normalized size = 0.70

$$\overline{2a\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^3c+3\,\mathrm{RootO}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
[Out] -(e*x^2+d)^(1/2)/a/d/x+1/2/a*e^(1/2)*sum((c*_R^2+2*(2*b*e-c*d)*_R+c*d^2)/(_
R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)
*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d
)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.392 $\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=341

$$\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{b\sqrt{d+ex^2}}{a^2dx}+$$

```
[Out] -1/3*(e*x^2+d)^(1/2)/a/d/x^3+b*(e*x^2+d)^(1/2)/a^2/d/x+2/3*e*(e*x^2+d)^(1/2
)/a/d^2/x+c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/(2*c
*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+c*arctan(x*
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/(b+(-4*a*c+b^2)^(1/2))^(1/
2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1303, 271, 264, 1692, 377, 205}

$$\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{b\sqrt{d+ex^2}}{a^2dx}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(3*a*d*x^3) + (b*Sqrt[d + e*x^2])/(a^2*d*x) + (2*e*Sqrt[d
+ e*x^2])/(3*a*d^2*x) + (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2]])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 -
 4*a*c])*e]) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
]])])/(a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{ax^4\sqrt{d+ex^2}} - \frac{b}{a^2x^2\sqrt{d+ex^2}} + \frac{b^2-ac+bcx^2}{a^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{b^2-ac+bcx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a^2} + \frac{\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{a} - \frac{b\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{\int \left(\frac{bc+\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{bc-\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ \dots}}\right.}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2 \dots\right.}}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{}{b+\sqrt{b^2 \dots}}\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-(b \dots}}{\sqrt{b-\sqrt{b^2 \dots}}}\right.}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b- \dots\right.}}$$

Mathematica [A] time = 0.69, size = 320, normalized size = 0.94

$$\frac{\frac{3c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{3c\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} - \frac{a\left(d-2ex^2\right)\sqrt{d+ex^2}}{d^2x^3} + \frac{3b\sqrt{d+ex^2}}{dx}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((3*b*Sqrt[d + e*x^2])/(d*x) - (a*(d - 2*e*x^2)*Sqrt[d + e*x^2])/(d^2*x^3)
+ (3*c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqr
t[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[
b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (3*c*(b
+ (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[
b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(3*a^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.03, size = 248, normalized size = 0.73

$$-\frac{}{2a^2\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^3c+3\,\mathrm{Roo}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] b*(e*x^2+d)^(1/2)/a^2/d/x-1/2/a^2*e^(1/2)*sum((b*c*_R^2+2*(-2*a*c*e+2*b^2*e
-b*c*d)*_R+b*c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_
R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^
4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c
*d^3)*_Z))-1/3*(e*x^2+d)^(1/2)/a/d/x^3+2/3*e*(e*x^2+d)^(1/2)/a/d^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(1/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.393 $\int \frac{1}{x^6\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=443

$$-\frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx}-\frac{c\left(\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd\cdots}}{\sqrt{\sqrt{b^2\cdots}}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2\cdots}\right.}}$$

```
[Out] -1/5*(e*x^2+d)^(1/2)/a/d/x^5+1/3*b*(e*x^2+d)^(1/2)/a^2/d/x^3+4/15*e*(e*x^2+
d)^(1/2)/a/d^2/x^3-(-a*c+b^2)*(e*x^2+d)^(1/2)/a^3/d/x-2/3*b*e*(e*x^2+d)^(1/
2)/a^2/d^2/x-8/15*e^2*(e*x^2+d)^(1/2)/a/d^3/x-c*arctan(x*(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c
+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^3/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c-b*(-3*a*c+b^
2)/(-4*a*c+b^2)^(1/2))/a^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.43, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1303, 271, 264, 1692, 377, 205}

$$-\frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx}-\frac{c\left(\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd\cdots}}{\sqrt{\sqrt{b^2\cdots}}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2\cdots}\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(5*a*d*x^5) + (b*Sqrt[d + e*x^2])/(3*a^2*d*x^3) + (4*e*Sqr
t[d + e*x^2])/(15*a*d^2*x^3) - ((b^2 - a*c)*Sqrt[d + e*x^2])/(a^3*d*x) - (2
*b*e*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^3*x)
- (c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]
)])/(a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
]) - (c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{ax^6\sqrt{d+ex^2}} - \frac{b}{a^2x^4\sqrt{d+ex^2}} + \frac{b^2-ac}{a^3x^2\sqrt{d+ex^2}} + \frac{-b\left(b^2-2ac\right)-c\left(b^2\right.}{a^3\sqrt{d+ex^2}\left(a+bx^2\right.}\right.$$

$$= \frac{\int \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a^3} + \frac{\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{a} - \frac{b\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{a^2} + \frac{\left(b^2-ac\right)\int \frac{}{x^2}}{a^3}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} - \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx} + \frac{\int \left(\frac{-\frac{bc\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-c\left(b^2-ac\right)}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx} - \frac{2be\sqrt{d+e}}{3a^2d^2x}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx} - \frac{2be\sqrt{d+e}}{3a^2d^2x}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx} - \frac{2be\sqrt{d+e}}{3a^2d^2x}$$

Mathematica [A] time = 1.64, size = 383, normalized size = 0.86

$$-\frac{\frac{a^2\sqrt{d+ex^2}\left(3d^2-4dex^2+8e^2x^4\right)}{d^3x^5}+\frac{15\left(b^2-ac\right)\sqrt{d+ex^2}}{dx}+\frac{15c\left(\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{15c\left(-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\cdots\right.}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-\cdots}}}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/15*((15*(b^2 - a*c)*Sqrt[d + e*x^2])/(d*x) - (5*a*b*(d - 2*e*x^2)*Sqrt[d
 + e*x^2])/(d^2*x^3) + (a^2*Sqrt[d + e*x^2]*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4)
)/(d^3*x^5) + (15*c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*
Sqrt[d + e*x^2])]])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2
 - 4*a*c])*e]) + (15*c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c
]]*Sqrt[d + e*x^2])]])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]))/a^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.04, size = 350, normalized size = 0.79

$$-\frac{\cdots}{2a^3\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4c\,d^3\right)\_Z\right)^3c+3\,\mathrm{Roo}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -(-a*c+b^2)*(e*x^2+d)^(1/2)/a^3/d/x-1/2/a^3*e^(1/2)*sum((c*(a*c-b^2)*_R^2+2
*(4*a*b*c*e-a*c^2*d-2*b^3*e+b^2*c*d)*_R+a*c^2*d^2-b^2*c*d^2)/(_R^3*c+3*_R^2
*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^
(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a
*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+1/3*b*(e*x^2+d)^(1/2)/a
^2/d/x^3-2/3*b*e*(e*x^2+d)^(1/2)/a^2/d^2/x-1/5*(e*x^2+d)^(1/2)/a/d/x^5+4/15
*e*(e*x^2+d)^(1/2)/a/d^2/x^3-8/15*e^2*(e*x^2+d)^(1/2)/a/d^3/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)\sqrt{ex^2 + d}\, x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\, \sqrt{e\,x^2 + d}\, \left(c\,x^4 + b\,x^2 + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \sqrt{d + ex^2}\, \left(a + bx^2 + cx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(1/(x**6*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.394 $\int \frac{x^6}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=350

$$\frac{2\left(-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}+\frac{2\left(\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\,\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}-\frac{\ldots}{e\sqrt{d+\ldots}}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(3/2)-d^2*x/e/(a*e^2-b*d*e+c*d^2)/(e
*x^2+d)^(1/2)+2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/
2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+2
*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2))*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 4.33, antiderivative size = 507, normalized size of antiderivative = 1.45, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1297, 288, 217, 206, 1692, 377, 205}

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\left(ae^2-bde+cd^2\right)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan\ldots}{c\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}\ldots\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d^2*x)/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + ((b^2*d - a*c*d -
a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*S
qrt[d + e*x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a*b*e + (b^3*d -
 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c
*d^2 - b*d*e + a*e^2)) + (d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(e^(3/2
)*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(c*Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1297

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(d^2*f^4)/(c*d^2 - b*d*e + a*e^2), Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(
(f*x)^(m - 4)*(d + e*x^2)^(q + 1)*Simp[a*d + (b*d - a*e)*x^2, x])/(a + b*x^
2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^6}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{x^2\left(ad+(bd-ae)x^2\right)}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2} + \frac{d^2\int \frac{x^2}{\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{\int \left(\frac{bd-ae}{c\sqrt{d+ex^2}} - \frac{a(bd-ae)+\left(b^2d-acd-abe\right)x^2}{c\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx}{cd^2-bde+ae^2}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\int \frac{a(bd-ae)+\left(b^2d-acd-abe\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c\left(cd^2-bde+ae^2\right)} + \frac{d^2\,\mathrm{Subst}\left(\cdots\right.}{e\left(c\cdots\right.}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)} + \frac{\int\left(\frac{b^2d-acd-abe+\cdots}{\left(b-\sqrt{b^2-4a\cdots}\right.}\cdots\right.}{\cdots}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)} - \frac{(bd-ae)\tanh^{\cdots}}{c\sqrt{e}\left(cd^2-b\cdots\right.}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)} - \frac{(bd-ae)\tanh^{\cdots}}{c\sqrt{e}\left(cd^2-b\cdots\right.}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\mathrm{t}\cdots}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-\cdots}\right.}}$$

Mathematica [B] time = 11.26, size = 10968, normalized size = 31.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 2.25, size = 75, normalized size = 0.21

$$-\frac{c^2d^2x}{\left(c^3d^2e-bc^2de^2+ac^2e^3\right)\sqrt{x^2e+d}} - \frac{e^{\left(-\frac{3}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -c^2*d^2*x/((c^3*d^2*e - b*c^2*d*e^2 + a*c^2*e^3)*sqrt(x^2*e + d)) - 1/2*e^
(-3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

maple [C] time = 0.04, size = 480, normalized size = 1.37

$$\frac{8ab\,e^{\frac{3}{2}}}{\left(4a\,e^{2}-4deb+4c\,d^{2}\right)\left(2e\,x^{2}-2\sqrt{e\,x^{2}+d}\,\sqrt{e}\,x+2d\right)c^{2}}+\frac{8ad\sqrt{e}}{\left(4a\,e^{2}-4deb+4c\,d^{2}\right)\left(2e\,x^{2}-2\sqrt{e\,x^{2}+d}\,\sqrt{e}\,x+2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/c*x/e/(e*x^2+d)^(1/2)+1/c/e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/c^2*b*
x/d/(e*x^2+d)^(1/2)+2/c*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum(((a*b*e+a*c*d
-b^2*d)*_R^2+2*(2*a^2*e^2-3*a*b*d*e-a*c*d^2+b^2*d^2)*_R+a*b*d^2*e+a*c*d^3-b
^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^
2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4
*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+8
/c^2*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x
+2*d)*a*b+8/c*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d
)^(1/2)*x+2*d)*a*d-8/c^2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/
2)*(e*x^2+d)^(1/2)*x+2*d)*b^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(cx^{4}+bx^{2}+a\right)\left(ex^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{6}}{\left(e\,x^{2}+d\right)^{3/2}\,\left(c\,x^{4}+b\,x^{2}+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(d+ex^{2}\right)^{\frac{3}{2}}\left(a+bx^{2}+cx^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**6/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.395 $\int \frac{x^4}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=360

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\cdots}}{\sqrt{\sqrt{b^2-4a\cdots}}}\right.}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\cdots}$$

```
[Out] d*x/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+
2*a*c*d-b^2*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(2*c*d-e*(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e
+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.27, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1297, 191, 1692, 377, 205}

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\cdots}}{\sqrt{\sqrt{b^2-4a\cdots}}}\right.}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (d*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((b*d - a*e - (b^2*d - 2*
a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^
2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)
) - ((b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d
+ e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1297

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(d^2*f^4)/(c*d^2 - b*d*e + a*e^2), Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(
(f*x)^(m - 4)*(d + e*x^2)^(q + 1)*Simp[a*d + (b*d - a*e)*x^2, x])/(a + b*x^
2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{ad+(bd-ae)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2}+\frac{d^2\int \frac{1}{\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{dx}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{\int \left(\frac{bd-ae+\frac{-b^2d+2acd+abe}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{bd-ae-\frac{-b^2d+2acd+a}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d}}\right.}{cd^2-bde+ae^2}$$

$$= \frac{dx}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+e}}}{cd^2-bde+ae^2}$$

$$= \frac{dx}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{b-\sqrt{b^2-4ac}-(-}\right.}{cd^2-bde+ae}$$

$$= \frac{dx}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-(b-}}{\sqrt{b-\sqrt{b^2-4}}}\right.}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(}$$

Mathematica [C] time = 7.90, size = 2162, normalized size = 6.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x/(c*d*Sqrt[d + e*x^2]) - ((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(45*Sqr
t[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d +
 e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((-
b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2)
)/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[-(((2*c*d
 + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))
]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*
(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(
```

```
-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2
)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[-(((2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]]
)/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (-b + Sqrt[b^2
- 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b
+ Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*H
ypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d
*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d + (-b + Sqrt[b
^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((
-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))
]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)
/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*c*(b - Sqrt[b^2 - 4*a*c]
)*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x
^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2))]) - ((b - (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((b +
Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d
^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b^2
 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*Ar
cSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sq
rt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + Sqr
t[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2
)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e*x
^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*c*(b + S
qrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))*Sq
rt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2))])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root o
```

f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,c]=[-72,-7,6]Evaluation time: 0.6Unable to divide, perhaps due t
o rounding error%%%{-2147483648,[3,8,10,8,3,10,1]%%%}+%%%{-12884901888,[3,8
,10,7,2,12,2]%%%}+%%%{463856467968,[3,8,10,6,1,14,3]%%%}+%%%{1924145348608,
[3,8,10,5,0,16,4]%%%}+%%%{536870912,[3,8,9,8,5,10,0]%%%}+%%%{20401094656,[3
,8,9,7,4,12,1]%%%}+%%%{-150323855360,[3,8,9,6,3,14,2]%%%}+%%%{-313532612608
0,[3,8,9,5,2,16,3]%%%}+%%%{-4672924418048,[3,8,9,4,1,18,4]%%%}+%%%{60473139
52768,[3,8,9,3,0,20,5]%%%}+%%%{-4294967296,[3,8,8,7,6,12,0]%%%}+%%%{-424128
02048,[3,8,8,6,5,14,1]%%%}+%%%{1046898278400,[3,8,8,5,4,16,2]%%%}+%%%{62105
22710016,[3,8,8,4,3,18,3]%%%}+%%%{-1786706395136,[3,8,8,3,2,20,4]%%%}+%%%{-
11544872091648,[3,8,8,2,1,22,5]%%%}+%%%{4398046511104,[3,8,8,1,0,24,6]%%%}+
%%%{12750684160,[3,8,7,6,7,14,0]%%%}+%%%{-23890755584,[3,8,7,5,6,16,1]%%%}+
%%%{-2103460233216,[3,8,7,4,5,18,2]%%%}+%%%{-3324304687104,[3,8,7,3,4,20,3]
%%%}+%%%{9758165696512,[3,8,7,2,3,22,4]%%%}+%%%{1649267441664,[3,8,7,1,2,24
,5]%%%}+%%%{-4398046511104,[3,8,7,0,1,26,6]%%%}+%%%{-17985175552,[3,8,6,5,8
,16,0]%%%}+%%%{161866579968,[3,8,6,4,7,18,1]%%%}+%%%{1586990415872,[3,8,6,3
,6,20,2]%%%}+%%%{-1795296329728,[3,8,6,2,5,22,3]%%%}+%%%{-4123168604160,[3,
8,6,1,4,24,4]%%%}+%%%{3848290697216,[3,8,6,0,3,26,5]%%%}+%%%{12213813248,[3
,8,5,4,9,18,0]%%%}+%%%{-171798691840,[3,8,5,3,8,20,1]%%%}+%%%{-212600881152
,[3,8,5,2,7,22,2]%%%}+%%%{1477468749824,[3,8,5,1,6,24,3]%%%}+%%%{-109951162
7776,[3,8,5,0,5,26,4]%%%}+%%%{-3221225472,[3,8,4,3,10,20,0]%%%}+%%%{5798205
8496,[3,8,4,2,9,22,1]%%%}+%%%{-154618822656,[3,8,4,1,8,24,2]%%%}+%%%{103079
215104,[3,8,4,0,7,26,3]%%%}+%%%{1048576,[3,6,10,4,2,4,0]%%%}+%%%{8388608,[3
,6,10,3,1,6,1]%%%}+%%%{16777216,[3,6,10,2,0,8,2]%%%}+%%%{-5242880,[3,6,9,3,
3,6,0]%%%}+%%%{-29360128,[3,6,9,2,2,8,1]%%%}+%%%{-33554432,[3,6,9,1,1,10,2]
%%%}+%%%{9699328,[3,6,8,2,4,8,0]%%%}+%%%{33554432,[3,6,8,1,3,10,1]%%%}+%%%{
16777216,[3,6,8,0,2,12,2]%%%}+%%%{-7864320,[3,6,7,1,5,10,0]%%%}+%%%{-125829
12,[3,6,7,0,4,12,1]%%%}+%%%{2359296,[3,6,6,0,6,12,0]%%%}+%%%{-536870912,[2,
7,10,6,2,8,1]%%%}+%%%{18446744062703697920,[2,7,10,5,1,10,2]%%%}+%%%{-18253
611008,[2,7,10,4,0,12,3]%%%}+%%%{134217728,[2,7,9,6,4,8,0]%%%}+%%%{55029268
48,[2,7,9,5,3,10,1]%%%}+%%%{36909875200,[2,7,9,4,2,12,2]%%%}+%%%{4294967296
0,[2,7,9,3,1,14,3]%%%}+%%%{-42949672960,[2,7,9,2,0,16,4]%%%}+%%%{-956301312
,[2,7,8,5,5,10,0]%%%}+%%%{-18656264192,[2,7,8,4,4,12,1]%%%}+%%%{-6496138035
2,[2,7,8,3,3,14,2]%%%}+%%%{8589934592,[2,7,8,2,2,16,3]%%%}+%%%{85899345920,
[2,7,8,1,1,18,4]%%%}+%%%{2642411520,[2,7,7,4,6,12,0]%%%}+%%%{27783069696,[2
,7,7,3,5,14,1]%%%}+%%%{33957085184,[2,7,7,2,4,16,2]%%%}+%%%{-73014444032,[2
,7,7,1,3,18,3]%%%}+%%%{-42949672960,[2,7,7,0,2,20,4]%%%}+%%%{-3556769792,[2
,7,6,3,7,14,0]%%%}+%%%{-17716740096,[2,7,6,2,6,16,1]%%%}+%%%{12884901888,[2
,7,6,1,5,18,2]%%%}+%%%{39728447488,[2,7,6,0,4,20,3]%%%}+%%%{2340421632,[2,7
,5,2,8,16,0]%%%}+%%%{2415919104,[2,7,5,1,7,18,1]%%%}+%%%{-12079595520,[2,7,
5,0,6,20,2]%%%}+%%%{-603979776,[2,7,4,1,9,18,0]%%%}+%%%{1207959552,[2,7,4,0
,8,20,1]%%%}+%%%{2147483648,[1,8,10,9,3,10,1]%%%}+%%%{38654705664,[1,8,10,8
,2,12,2]%%%}+%%%{51539607552,[1,8,10,7,1,14,3]%%%}+%%%{-274877906944,[1,8,1
0,6,0,16,4]%%%}+%%%{-536870912,[1,8,9,9,5,10,0]%%%}+%%%{-26843545600,[1,8,9
,8,4,12,1]%%%}+%%%{-188978561024,[1,8,9,7,3,14,2]%%%}+%%%{146028888064,[1,8
,9,6,2,16,3]%%%}+%%%{962072674304,[1,8,9,5,1,18,4]%%%}+%%%{-549755813888,[1
,8,9,4,0,20,5]%%%}+%%%{4294967296,[1,8,8,8,6,12,0]%%%}+%%%{95026151424,[1,8
,8,7,5,14,1]%%%}+%%%{239444426752,[1,8,8,6,4,16,2]%%%}+%%%{-858993459200,[1
,8,8,5,3,18,3]%%%}+%%%{-618475290624,[1,8,8,4,2,20,4]%%%}+%%%{1099511627776
,[1,8,8,3,1,22,5]%%%}+%%%{-12750684160,[1,8,7,7,7,14,0]%%%}+%%%{-1366336471
04,[1,8,7,6,6,16,1]%%%}+%%%{62277025792,[1,8,7,5,5,18,2]%%%}+%%%{9363028705
28,[1,8,7,4,4,20,3]%%%}+%%%{-549755813888,[1,8,7,3,3,22,4]%%%}+%%%{-5497558
13888,[1,8,7,2,2,24,5]%%%}+%%%{17985175552,[1,8,6,6,8,16,0]%%%}+%%%{7194070
2208,[1,8,6,5,7,18,1]%%%}+%%%{-267361714176,[1,8,6,4,6,20,2]%%%}+%%%{-13743
8953472,[1,8,6,3,5,22,3]%%%}+%%%{481036337152,[1,8,6,2,4,24,4]%%%}+%%%{-122
13813248,[1,8,5,5,9,18,0]%%%}+%%%{7247757312,[1,8,5,4,8,20,1]%%%}+%%%{10307
9215104,[1,8,5,3,7,22,2]%%%}+%%%{-137438953472,[1,8,5,2,6,24,3]%%%}+%%%{322
1225472,[1,8,4,4,10,20,0]%%%}+%%%{-12884901888,[1,8,4,3,9,22,1]%%%}+%%%{128
84901888,[1,8,4,2,8,24,2]%%%}+%%%{-1048576,[1,6,10,5,2,4,0]%%%}+%%%{-838860

```
8,[1,6,10,4,1,6,1]%%%}+%%%{-16777216,[1,6,10,3,0,8,2]%%%}+%%%{8388608,[1,6,
9,4,3,6,0]%%%}+%%%{62914560,[1,6,9,3,2,8,1]%%%}+%%%{150994944,[1,6,9,2,1,10
,2]%%%}+%%%{134217728,[1,6,9,1,0,12,3]%%%}+%%%{-26476544,[1,6,8,3,4,8,0]%%%
}+%%%{-163577856,[1,6,8,2,3,10,1]%%%}+%%%{-301989888,[1,6,8,1,2,12,2]%%%}+%
%%{-134217728,[1,6,8,0,1,14,3]%%%}+%%%{41156608,[1,6,7,2,5,10,0]%%%}+%%%{17
8257920,[1,6,7,1,4,12,1]%%%}+%%%{167772160,[1,6,7,0,3,14,2]%%%}+%%%{-314572
80,[1,6,6,1,6,12,0]%%%}+%%%{-69206016,[1,6,6,0,5,14,1]%%%}+%%%{9437184,[1,6
,5,0,7,14,0]%%%}+%%%{402653184,[0,7,10,7,2,8,1]%%%}+%%%{5637144576,[0,7,10,
6,1,10,2]%%%}+%%%{16106127360,[0,7,10,5,0,12,3]%%%}+%%%{-100663296,[0,7,9,7
,4,8,0]%%%}+%%%{-4160749568,[0,7,9,6,3,10,1]%%%}+%%%{-30198988800,[0,7,9,5,
2,12,2]%%%}+%%%{-28991029248,[0,7,9,4,1,14,3]%%%}+%%%{68719476736,[0,7,9,3,
0,16,4]%%%}+%%%{687865856,[0,7,8,6,5,10,0]%%%}+%%%{13925089280,[0,7,8,5,4,1
2,1]%%%}+%%%{48184164352,[0,7,8,4,3,14,2]%%%}+%%%{-49392123904,[0,7,8,3,2,1
6,3]%%%}+%%%{-120259084288,[0,7,8,2,1,18,4]%%%}+%%%{68719476736,[0,7,8,1,0,
20,5]%%%}+%%%{-1845493760,[0,7,7,5,6,12,0]%%%}+%%%{-19964887040,[0,7,7,4,5,
14,1]%%%}+%%%{-11542724608,[0,7,7,3,4,16,2]%%%}+%%%{113816633344,[0,7,7,2,3
,18,3]%%%}+%%%{-8589934592,[0,7,7,1,2,20,4]%%%}+%%%{-68719476736,[0,7,7,0,1
,22,5]%%%}+%%%{2432696320,[0,7,6,4,7,14,0]%%%}+%%%{11207180288,[0,7,6,3,6,1
6,1]%%%}+%%%{-28185722880,[0,7,6,2,5,18,2]%%%}+%%%{-34359738368,[0,7,6,1,4,
20,3]%%%}+%%%{60129542144,[0,7,6,0,3,22,4]%%%}+%%%{-1577058304,[0,7,5,3,8,1
6,0]%%%}+%%%{201326592,[0,7,5,2,7,18,1]%%%}+%%%{14495514624,[0,7,5,1,6,20,2
]%%%}+%%%{-17179869184,[0,7,5,0,5,22,3]%%%}+%%%{402653184,[0,7,4,2,9,18,0]%
%%}+%%%{-1610612736,[0,7,4,1,8,20,1]%%%}+%%%{1610612736,[0,7,4,0,7,22,2]%%%
} / %%%{1024,[0,3,4,2,1,2,0]%%%}+%%%{4096,[0,3,4,1,0,4,1]%%%}+%%%{-2560,[0,
3,3,1,2,4,0]%%%}+%%%{-4096,[0,3,3,0,1,6,1]%%%}+%%%{1536,[0,3,2,0,3,6,0]%%%}
 Error: Bad Argument Value
```

maple [C] time = 0.03, size = 338, normalized size = 0.94

$$-\frac{8a\,e^{\frac{3}{2}}}{\left(4a\,e^{2}-4deb+4c\,d^{2}\right)\left(2e\,x^{2}-2\sqrt{e\,x^{2}+d}\,\sqrt{e}\,x+2d\right)c}+\frac{8bd\sqrt{e}}{\left(4a\,e^{2}-4deb+4c\,d^{2}\right)\left(2e\,x^{2}-2\sqrt{e\,x^{2}+d}\,\sqrt{e}\,x+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/c*x/d/(e*x^2+d)^(1/2)-2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum(((a*e-b*d)*
_R^2+2*d*(-3*a*e+b*d)*_R+a*d^2*e-b*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*
a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/
2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)
*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-8/c*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x
^2-2*(e*x^2+d)^(1/2)*e^(1/2)*x+2*d)*a+8/c*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)
/(2*e*x^2-2*(e*x^2+d)^(1/2)*e^(1/2)*x+2*d)*b*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(cx^4+bx^2+a\right)\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(d+ex^2\right)^{\frac{3}{2}}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.396 $\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=333

$$\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\left(ae^2-bde+cd^2\right)}+\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4a\ldots\right.}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{\ldots}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,\left(\ldots\right)}$$

```
[Out] -e*x/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)+c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2
)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(d+(2*a*e-b*d
)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(d+(-2*a*e+b*d)/(-4
*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1299, 191, 1692, 377, 205}

$$\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\left(ae^2-bde+cd^2\right)}+\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4a\ldots\right.}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{\ldots}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,\left(\ldots\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((e*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + (c*(d - (b*d - 2*a*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (c*(d + (b*
d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1299

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> -Dist[(d*e*f^2)/(c*d^2 - b*d*e + a*e^2), Int[(
f*x)^(m - 2)*(d + e*x^2)^q, x], x] + Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[
((f*x)^(m - 2)*(d + e*x^2)^(q + 1)*Simp[a*e + c*d*x^2, x])/(a + b*x^2 + c*x
^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !Int
egerQ[q] && LtQ[q, -1] && GtQ[m, 1] && LeQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \frac{\int \frac{ae+cdx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2} - \frac{(de)\int \frac{1}{\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\int \left(\frac{cd+\frac{c(-bd+2ae)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{cd-\frac{c(-bd+2ae)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\cdots}\cdots}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,d\cdots}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\cdots\right.}\cdots\right.}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2\cdots}\right.}}{\sqrt{b-\sqrt{b^2-4ac}}\cdots}\cdots\right.}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e\cdots}}$$

Mathematica [C] time = 6.72, size = 2119, normalized size = 6.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((1 - b/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] +
2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + S
qrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
)^2))])/d - 45*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*
(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])
/d - (45*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/
(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*
```

```
a*c])*e)*x^4*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-
b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^
2)) + 4*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4
*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-
b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]
+ (4*e*x^2*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2
- 4*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d
*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))
)])/d))/(15*(b - Sqrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])
*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e
*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]) + ((1 + b/Sqrt[b^2 - 4*a*c]
)*x*(45*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d
 + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[
((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2))]] - (30*e*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30
*e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])
*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2
, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2))] + (4*e*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
))])/d))/(15*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,c]=[-72,-7,6]Evaluation time: 0.57Unable to divide, perhaps due
```

to rounding error%%%{-2147483648,[3,8,10,8,3,10,1]%%%}+%%%{-12884901888,[3,
8,10,7,2,12,2]%%%}+%%%{463856467968,[3,8,10,6,1,14,3]%%%}+%%%{1924145348608
,[3,8,10,5,0,16,4]%%%}+%%%{536870912,[3,8,9,8,5,10,0]%%%}+%%%{20401094656,[
3,8,9,7,4,12,1]%%%}+%%%{-150323855360,[3,8,9,6,3,14,2]%%%}+%%%{-31353261260
80,[3,8,9,5,2,16,3]%%%}+%%%{-4672924418048,[3,8,9,4,1,18,4]%%%}+%%%{6047313
952768,[3,8,9,3,0,20,5]%%%}+%%%{-4294967296,[3,8,8,7,6,12,0]%%%}+%%%{-42412
802048,[3,8,8,6,5,14,1]%%%}+%%%{1046898278400,[3,8,8,5,4,16,2]%%%}+%%%{6210
522710016,[3,8,8,4,3,18,3]%%%}+%%%{-1786706395136,[3,8,8,3,2,20,4]%%%}+%%%{
-11544872091648,[3,8,8,2,1,22,5]%%%}+%%%{4398046511104,[3,8,8,1,0,24,6]%%%}
+%%%{12750684160,[3,8,7,6,7,14,0]%%%}+%%%{-23890755584,[3,8,7,5,6,16,1]%%%}
+%%%{-2103460233216,[3,8,7,4,5,18,2]%%%}+%%%{-3324304687104,[3,8,7,3,4,20,3
]%%%}+%%%{9758165696512,[3,8,7,2,3,22,4]%%%}+%%%{1649267441664,[3,8,7,1,2,2
4,5]%%%}+%%%{-4398046511104,[3,8,7,0,1,26,6]%%%}+%%%{-17985175552,[3,8,6,5,
8,16,0]%%%}+%%%{161866579968,[3,8,6,4,7,18,1]%%%}+%%%{1586990415872,[3,8,6,
3,6,20,2]%%%}+%%%{-1795296329728,[3,8,6,2,5,22,3]%%%}+%%%{-4123168604160,[3
,8,6,1,4,24,4]%%%}+%%%{3848290697216,[3,8,6,0,3,26,5]%%%}+%%%{12213813248,[
3,8,5,4,9,18,0]%%%}+%%%{-171798691840,[3,8,5,3,8,20,1]%%%}+%%%{-21260088115
2,[3,8,5,2,7,22,2]%%%}+%%%{1477468749824,[3,8,5,1,6,24,3]%%%}+%%%{-10995116
27776,[3,8,5,0,5,26,4]%%%}+%%%{-3221225472,[3,8,4,3,10,20,0]%%%}+%%%{579820
58496,[3,8,4,2,9,22,1]%%%}+%%%{-154618822656,[3,8,4,1,8,24,2]%%%}+%%%{10307
9215104,[3,8,4,0,7,26,3]%%%}+%%%{1048576,[3,6,10,4,2,4,0]%%%}+%%%{8388608,[
3,6,10,3,1,6,1]%%%}+%%%{16777216,[3,6,10,2,0,8,2]%%%}+%%%{-5242880,[3,6,9,3
,3,6,0]%%%}+%%%{-29360128,[3,6,9,2,2,8,1]%%%}+%%%{-33554432,[3,6,9,1,1,10,2
]%%%}+%%%{9699328,[3,6,8,2,4,8,0]%%%}+%%%{33554432,[3,6,8,1,3,10,1]%%%}+%%%
{16777216,[3,6,8,0,2,12,2]%%%}+%%%{-7864320,[3,6,7,1,5,10,0]%%%}+%%%{-12582
912,[3,6,7,0,4,12,1]%%%}+%%%{2359296,[3,6,6,0,6,12,0]%%%}+%%%{-536870912,[2
,7,10,6,2,8,1]%%%}+%%%{18446744062703697920,[2,7,10,5,1,10,2]%%%}+%%%{-1825
3611008,[2,7,10,4,0,12,3]%%%}+%%%{134217728,[2,7,9,6,4,8,0]%%%}+%%%{5502926
848,[2,7,9,5,3,10,1]%%%}+%%%{36909875200,[2,7,9,4,2,12,2]%%%}+%%%{429496729
60,[2,7,9,3,1,14,3]%%%}+%%%{-42949672960,[2,7,9,2,0,16,4]%%%}+%%%{-95630131
2,[2,7,8,5,5,10,0]%%%}+%%%{-18656264192,[2,7,8,4,4,12,1]%%%}+%%%{-649613803
52,[2,7,8,3,3,14,2]%%%}+%%%{8589934592,[2,7,8,2,2,16,3]%%%}+%%%{85899345920
,[2,7,8,1,1,18,4]%%%}+%%%{2642411520,[2,7,7,4,6,12,0]%%%}+%%%{27783069696,[
2,7,7,3,5,14,1]%%%}+%%%{33957085184,[2,7,7,2,4,16,2]%%%}+%%%{-73014444032,[
2,7,7,1,3,18,3]%%%}+%%%{-42949672960,[2,7,7,0,2,20,4]%%%}+%%%{-3556769792,[
2,7,6,3,7,14,0]%%%}+%%%{-17716740096,[2,7,6,2,6,16,1]%%%}+%%%{12884901888,[
2,7,6,1,5,18,2]%%%}+%%%{39728447488,[2,7,6,0,4,20,3]%%%}+%%%{2340421632,[2,
7,5,2,8,16,0]%%%}+%%%{2415919104,[2,7,5,1,7,18,1]%%%}+%%%{-12079595520,[2,7
,5,0,6,20,2]%%%}+%%%{-603979776,[2,7,4,1,9,18,0]%%%}+%%%{1207959552,[2,7,4,
0,8,20,1]%%%}+%%%{2147483648,[1,8,10,9,3,10,1]%%%}+%%%{38654705664,[1,8,10,
8,2,12,2]%%%}+%%%{51539607552,[1,8,10,7,1,14,3]%%%}+%%%{-274877906944,[1,8,
10,6,0,16,4]%%%}+%%%{-536870912,[1,8,9,9,5,10,0]%%%}+%%%{-26843545600,[1,8,
9,8,4,12,1]%%%}+%%%{-188978561024,[1,8,9,7,3,14,2]%%%}+%%%{146028888064,[1,
8,9,6,2,16,3]%%%}+%%%{962072674304,[1,8,9,5,1,18,4]%%%}+%%%{-549755813888,[
1,8,9,4,0,20,5]%%%}+%%%{4294967296,[1,8,8,8,6,12,0]%%%}+%%%{95026151424,[1,
8,8,7,5,14,1]%%%}+%%%{239444426752,[1,8,8,6,4,16,2]%%%}+%%%{-858993459200,[
1,8,8,5,3,18,3]%%%}+%%%{-618475290624,[1,8,8,4,2,20,4]%%%}+%%%{109951162777
6,[1,8,8,3,1,22,5]%%%}+%%%{-12750684160,[1,8,7,7,7,14,0]%%%}+%%%{-136633647
104,[1,8,7,6,6,16,1]%%%}+%%%{62277025792,[1,8,7,5,5,18,2]%%%}+%%%{936302870
528,[1,8,7,4,4,20,3]%%%}+%%%{-549755813888,[1,8,7,3,3,22,4]%%%}+%%%{-549755
813888,[1,8,7,2,2,24,5]%%%}+%%%{17985175552,[1,8,6,6,8,16,0]%%%}+%%%{719407
02208,[1,8,6,5,7,18,1]%%%}+%%%{-267361714176,[1,8,6,4,6,20,2]%%%}+%%%{-1374
38953472,[1,8,6,3,5,22,3]%%%}+%%%{481036337152,[1,8,6,2,4,24,4]%%%}+%%%{-12
213813248,[1,8,5,5,9,18,0]%%%}+%%%{7247757312,[1,8,5,4,8,20,1]%%%}+%%%{1030
79215104,[1,8,5,3,7,22,2]%%%}+%%%{-137438953472,[1,8,5,2,6,24,3]%%%}+%%%{32
21225472,[1,8,4,4,10,20,0]%%%}+%%%{-12884901888,[1,8,4,3,9,22,1]%%%}+%%%{12
884901888,[1,8,4,2,8,24,2]%%%}+%%%{-1048576,[1,6,10,5,2,4,0]%%%}+%%%{-83886
08,[1,6,10,4,1,6,1]%%%}+%%%{-16777216,[1,6,10,3,0,8,2]%%%}+%%%{8388608,[1,6
,9,4,3,6,0]%%%}+%%%{62914560,[1,6,9,3,2,8,1]%%%}+%%%{150994944,[1,6,9,2,1,1

```
0,2]%%%}+%%%{134217728,[1,6,9,1,0,12,3]%%%}+%%%{-26476544,[1,6,8,3,4,8,0]%%
%}+%%%{-163577856,[1,6,8,2,3,10,1]%%%}+%%%{-301989888,[1,6,8,1,2,12,2]%%%}+
%%%{-134217728,[1,6,8,0,1,14,3]%%%}+%%%{41156608,[1,6,7,2,5,10,0]%%%}+%%%{1
78257920,[1,6,7,1,4,12,1]%%%}+%%%{167772160,[1,6,7,0,3,14,2]%%%}+%%%{-31457
280,[1,6,6,1,6,12,0]%%%}+%%%{-69206016,[1,6,6,0,5,14,1]%%%}+%%%{9437184,[1,
6,5,0,7,14,0]%%%}+%%%{402653184,[0,7,10,7,2,8,1]%%%}+%%%{5637144576,[0,7,10
,6,1,10,2]%%%}+%%%{16106127360,[0,7,10,5,0,12,3]%%%}+%%%{-100663296,[0,7,9,
7,4,8,0]%%%}+%%%{-4160749568,[0,7,9,6,3,10,1]%%%}+%%%{-30198988800,[0,7,9,5
,2,12,2]%%%}+%%%{-28991029248,[0,7,9,4,1,14,3]%%%}+%%%{68719476736,[0,7,9,3
,0,16,4]%%%}+%%%{687865856,[0,7,8,6,5,10,0]%%%}+%%%{13925089280,[0,7,8,5,4,
12,1]%%%}+%%%{48184164352,[0,7,8,4,3,14,2]%%%}+%%%{-49392123904,[0,7,8,3,2,
16,3]%%%}+%%%{-120259084288,[0,7,8,2,1,18,4]%%%}+%%%{68719476736,[0,7,8,1,0
,20,5]%%%}+%%%{-1845493760,[0,7,7,5,6,12,0]%%%}+%%%{-19964887040,[0,7,7,4,5
,14,1]%%%}+%%%{-11542724608,[0,7,7,3,4,16,2]%%%}+%%%{113816633344,[0,7,7,2,
3,18,3]%%%}+%%%{-8589934592,[0,7,7,1,2,20,4]%%%}+%%%{-68719476736,[0,7,7,0,
1,22,5]%%%}+%%%{2432696320,[0,7,6,4,7,14,0]%%%}+%%%{11207180288,[0,7,6,3,6,
16,1]%%%}+%%%{-28185722880,[0,7,6,2,5,18,2]%%%}+%%%{-34359738368,[0,7,6,1,4
,20,3]%%%}+%%%{60129542144,[0,7,6,0,3,22,4]%%%}+%%%{-1577058304,[0,7,5,3,8,
16,0]%%%}+%%%{201326592,[0,7,5,2,7,18,1]%%%}+%%%{14495514624,[0,7,5,1,6,20,
2]%%%}+%%%{-17179869184,[0,7,5,0,5,22,3]%%%}+%%%{402653184,[0,7,4,2,9,18,0]
%%%}+%%%{-1610612736,[0,7,4,1,8,20,1]%%%}+%%%{1610612736,[0,7,4,0,7,22,2]%%
%} / %%%{1024,[0,3,4,2,1,2,0]%%%}+%%%{4096,[0,3,4,1,0,4,1]%%%}+%%%{-2560,[0
,3,3,1,2,4,0]%%%}+%%%{-4096,[0,3,3,0,1,6,1]%%%}+%%%{1536,[0,3,2,0,3,6,0]%%%
} Error: Bad Argument Value
```

maple [C] time = 0.03, size = 252, normalized size = 0.76

$$-\frac{8d\sqrt{e}}{\left(4a\,e^{2}-4deb+4c\,d^{2}\right)\left(2e\,x^{2}-2\sqrt{e\,x^{2}+d}\,\sqrt{e}\,x+2d\right)}-\frac{}{\left(4a\,e^{2}-4deb+4c\,d^{2}\right)\left(\mathrm{RootOf}\left(\_Z^{4}c+c\,d^{4}+(4be-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum((_R^2*c*d+2*(2*a*e^2-c*d^2)*_R+c*d
^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-
c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e
-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-8*e^(
1/2)*d/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^(1/2)*e^(1/2)*x+2*d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(cx^4+bx^2+a\right)\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(e\,x^2+d\right)^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(d+ex^2\right)^{\frac{3}{2}}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**2/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.397 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=341

$$-\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\left(ae^2-bde+cd^2\right)}-\frac{c\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

```
[Out] e^2*x/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)-c*arctan(x*(2*c*d-e*(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*
c*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*c*d)/
(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d
-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1172, 191, 1692, 377, 205}

$$-\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\left(ae^2-bde+cd^2\right)}-\frac{c\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - (c*(e - (2*c*d - b*e)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(e + (2
*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x^2)^q, x], x] + Dist[1
/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x^2)^(q + 1)*(c*d - b*e - c*e*x^2))/(
a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q] && LtQ[q, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \frac{\int \frac{cd-be-cex^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\int \left(\frac{-ce-\frac{c(-2cd+be)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{-ce+\frac{c(-2cd+b\ldots}{\sqrt{b^2-4a\ldots}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\ldots}\right.}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,d\ldots}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\operatorname{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(\ldots\right.\right.}\right.}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2\ldots}\right.}}{\sqrt{b-\sqrt{b^2-4ac}}\ldots}\right.}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e\ldots}}$$

Mathematica [C] time = 7.15, size = 2112, normalized size = 6.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*
x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x
^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[
Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sq
rt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c]
```

```
- 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5
/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))
^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*
c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*Sqrt[b^2 -
 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*
x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2
))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]) - (2*c*x*(45*Sqrt[-(((b + Sqrt[
b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b^2 - 4*
a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*ArcSin[
Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[((
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + Sqrt[b^2
 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e*x^2*((
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*Sqrt[b^2 - 4*
a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(
d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2))])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,c]=[-72,-7,6]Evaluation time: 0.58Unable to divide, perhaps due
to rounding error%%%{-2147483648,[3,8,10,8,3,10,1]%%%}+%%%{-12884901888,[3,
```

8,10,7,2,12,2]%%%}+%%%{463856467968,[3,8,10,6,1,14,3]%%%}+%%%{1924145348608
,[3,8,10,5,0,16,4]%%%}+%%%{536870912,[3,8,9,8,5,10,0]%%%}+%%%{20401094656,[
3,8,9,7,4,12,1]%%%}+%%%{-150323855360,[3,8,9,6,3,14,2]%%%}+%%%{-31353261260
80,[3,8,9,5,2,16,3]%%%}+%%%{-4672924418048,[3,8,9,4,1,18,4]%%%}+%%%{6047313
952768,[3,8,9,3,0,20,5]%%%}+%%%{-4294967296,[3,8,8,7,6,12,0]%%%}+%%%{-42412
802048,[3,8,8,6,5,14,1]%%%}+%%%{1046898278400,[3,8,8,5,4,16,2]%%%}+%%%{6210
522710016,[3,8,8,4,3,18,3]%%%}+%%%{-1786706395136,[3,8,8,3,2,20,4]%%%}+%%%{
-11544872091648,[3,8,8,2,1,22,5]%%%}+%%%{4398046511104,[3,8,8,1,0,24,6]%%%}
+%%%{12750684160,[3,8,7,6,7,14,0]%%%}+%%%{-23890755584,[3,8,7,5,6,16,1]%%%}
+%%%{-2103460233216,[3,8,7,4,5,18,2]%%%}+%%%{-3324304687104,[3,8,7,3,4,20,3
]%%%}+%%%{9758165696512,[3,8,7,2,3,22,4]%%%}+%%%{1649267441664,[3,8,7,1,2,2
4,5]%%%}+%%%{-4398046511104,[3,8,7,0,1,26,6]%%%}+%%%{-17985175552,[3,8,6,5,
8,16,0]%%%}+%%%{161866579968,[3,8,6,4,7,18,1]%%%}+%%%{1586990415872,[3,8,6,
3,6,20,2]%%%}+%%%{-1795296329728,[3,8,6,2,5,22,3]%%%}+%%%{-4123168604160,[3
,8,6,1,4,24,4]%%%}+%%%{3848290697216,[3,8,6,0,3,26,5]%%%}+%%%{12213813248,[
3,8,5,4,9,18,0]%%%}+%%%{-171798691840,[3,8,5,3,8,20,1]%%%}+%%%{-21260088115
2,[3,8,5,2,7,22,2]%%%}+%%%{1477468749824,[3,8,5,1,6,24,3]%%%}+%%%{-10995116
27776,[3,8,5,0,5,26,4]%%%}+%%%{-3221225472,[3,8,4,3,10,20,0]%%%}+%%%{579820
58496,[3,8,4,2,9,22,1]%%%}+%%%{-154618822656,[3,8,4,1,8,24,2]%%%}+%%%{10307
9215104,[3,8,4,0,7,26,3]%%%}+%%%{1048576,[3,6,10,4,2,4,0]%%%}+%%%{8388608,[
3,6,10,3,1,6,1]%%%}+%%%{16777216,[3,6,10,2,0,8,2]%%%}+%%%{-5242880,[3,6,9,3
,3,6,0]%%%}+%%%{-29360128,[3,6,9,2,2,8,1]%%%}+%%%{-33554432,[3,6,9,1,1,10,2
]%%%}+%%%{9699328,[3,6,8,2,4,8,0]%%%}+%%%{33554432,[3,6,8,1,3,10,1]%%%}+%%%
{16777216,[3,6,8,0,2,12,2]%%%}+%%%{-7864320,[3,6,7,1,5,10,0]%%%}+%%%{-12582
912,[3,6,7,0,4,12,1]%%%}+%%%{2359296,[3,6,6,0,6,12,0]%%%}+%%%{-536870912,[2
,7,10,6,2,8,1]%%%}+%%%{18446744062703697920,[2,7,10,5,1,10,2]%%%}+%%%{-1825
3611008,[2,7,10,4,0,12,3]%%%}+%%%{134217728,[2,7,9,6,4,8,0]%%%}+%%%{5502926
848,[2,7,9,5,3,10,1]%%%}+%%%{36909875200,[2,7,9,4,2,12,2]%%%}+%%%{429496729
60,[2,7,9,3,1,14,3]%%%}+%%%{-42949672960,[2,7,9,2,0,16,4]%%%}+%%%{-95630131
2,[2,7,8,5,5,10,0]%%%}+%%%{-18656264192,[2,7,8,4,4,12,1]%%%}+%%%{-649613803
52,[2,7,8,3,3,14,2]%%%}+%%%{8589934592,[2,7,8,2,2,16,3]%%%}+%%%{85899345920
,[2,7,8,1,1,18,4]%%%}+%%%{2642411520,[2,7,7,4,6,12,0]%%%}+%%%{27783069696,[
2,7,7,3,5,14,1]%%%}+%%%{33957085184,[2,7,7,2,4,16,2]%%%}+%%%{-73014444032,[
2,7,7,1,3,18,3]%%%}+%%%{-42949672960,[2,7,7,0,2,20,4]%%%}+%%%{-3556769792,[
2,7,6,3,7,14,0]%%%}+%%%{-17716740096,[2,7,6,2,6,16,1]%%%}+%%%{12884901888,[
2,7,6,1,5,18,2]%%%}+%%%{39728447488,[2,7,6,0,4,20,3]%%%}+%%%{2340421632,[2,
7,5,2,8,16,0]%%%}+%%%{2415919104,[2,7,5,1,7,18,1]%%%}+%%%{-12079595520,[2,7
,5,0,6,20,2]%%%}+%%%{-603979776,[2,7,4,1,9,18,0]%%%}+%%%{1207959552,[2,7,4,
0,8,20,1]%%%}+%%%{2147483648,[1,8,10,9,3,10,1]%%%}+%%%{38654705664,[1,8,10,
8,2,12,2]%%%}+%%%{51539607552,[1,8,10,7,1,14,3]%%%}+%%%{-274877906944,[1,8,
10,6,0,16,4]%%%}+%%%{-536870912,[1,8,9,9,5,10,0]%%%}+%%%{-26843545600,[1,8,
9,8,4,12,1]%%%}+%%%{-188978561024,[1,8,9,7,3,14,2]%%%}+%%%{146028888064,[1,
8,9,6,2,16,3]%%%}+%%%{962072674304,[1,8,9,5,1,18,4]%%%}+%%%{-549755813888,[
1,8,9,4,0,20,5]%%%}+%%%{4294967296,[1,8,8,8,6,12,0]%%%}+%%%{95026151424,[1,
8,8,7,5,14,1]%%%}+%%%{239444426752,[1,8,8,6,4,16,2]%%%}+%%%{-858993459200,[
1,8,8,5,3,18,3]%%%}+%%%{-618475290624,[1,8,8,4,2,20,4]%%%}+%%%{109951162777
6,[1,8,8,3,1,22,5]%%%}+%%%{-12750684160,[1,8,7,7,7,14,0]%%%}+%%%{-136633647
104,[1,8,7,6,6,16,1]%%%}+%%%{62277025792,[1,8,7,5,5,18,2]%%%}+%%%{936302870
528,[1,8,7,4,4,20,3]%%%}+%%%{-549755813888,[1,8,7,3,3,22,4]%%%}+%%%{-549755
813888,[1,8,7,2,2,24,5]%%%}+%%%{17985175552,[1,8,6,6,8,16,0]%%%}+%%%{719407
02208,[1,8,6,5,7,18,1]%%%}+%%%{-267361714176,[1,8,6,4,6,20,2]%%%}+%%%{-1374
38953472,[1,8,6,3,5,22,3]%%%}+%%%{481036337152,[1,8,6,2,4,24,4]%%%}+%%%{-12
213813248,[1,8,5,5,9,18,0]%%%}+%%%{7247757312,[1,8,5,4,8,20,1]%%%}+%%%{1030
79215104,[1,8,5,3,7,22,2]%%%}+%%%{-137438953472,[1,8,5,2,6,24,3]%%%}+%%%{32
21225472,[1,8,4,4,10,20,0]%%%}+%%%{-12884901888,[1,8,4,3,9,22,1]%%%}+%%%{12
884901888,[1,8,4,2,8,24,2]%%%}+%%%{-1048576,[1,6,10,5,2,4,0]%%%}+%%%{-83886
08,[1,6,10,4,1,6,1]%%%}+%%%{-16777216,[1,6,10,3,0,8,2]%%%}+%%%{8388608,[1,6
,9,4,3,6,0]%%%}+%%%{62914560,[1,6,9,3,2,8,1]%%%}+%%%{150994944,[1,6,9,2,1,1
0,2]%%%}+%%%{134217728,[1,6,9,1,0,12,3]%%%}+%%%{-26476544,[1,6,8,3,4,8,0]%%

```
%}+%%%{-163577856,[1,6,8,2,3,10,1]%%%}+%%%{-301989888,[1,6,8,1,2,12,2]%%%}+
%%%{-134217728,[1,6,8,0,1,14,3]%%%}+%%%{41156608,[1,6,7,2,5,10,0]%%%}+%%%{1
78257920,[1,6,7,1,4,12,1]%%%}+%%%{167772160,[1,6,7,0,3,14,2]%%%}+%%%{-31457
280,[1,6,6,1,6,12,0]%%%}+%%%{-69206016,[1,6,6,0,5,14,1]%%%}+%%%{9437184,[1,
6,5,0,7,14,0]%%%}+%%%{402653184,[0,7,10,7,2,8,1]%%%}+%%%{5637144576,[0,7,10
,6,1,10,2]%%%}+%%%{16106127360,[0,7,10,5,0,12,3]%%%}+%%%{-100663296,[0,7,9,
7,4,8,0]%%%}+%%%{-4160749568,[0,7,9,6,3,10,1]%%%}+%%%{-30198988800,[0,7,9,5
,2,12,2]%%%}+%%%{-28991029248,[0,7,9,4,1,14,3]%%%}+%%%{68719476736,[0,7,9,3
,0,16,4]%%%}+%%%{687865856,[0,7,8,6,5,10,0]%%%}+%%%{13925089280,[0,7,8,5,4,
12,1]%%%}+%%%{48184164352,[0,7,8,4,3,14,2]%%%}+%%%{-49392123904,[0,7,8,3,2,
16,3]%%%}+%%%{-120259084288,[0,7,8,2,1,18,4]%%%}+%%%{68719476736,[0,7,8,1,0
,20,5]%%%}+%%%{-1845493760,[0,7,7,5,6,12,0]%%%}+%%%{-19964887040,[0,7,7,4,5
,14,1]%%%}+%%%{-11542724608,[0,7,7,3,4,16,2]%%%}+%%%{113816633344,[0,7,7,2,
3,18,3]%%%}+%%%{-8589934592,[0,7,7,1,2,20,4]%%%}+%%%{-68719476736,[0,7,7,0,
1,22,5]%%%}+%%%{2432696320,[0,7,6,4,7,14,0]%%%}+%%%{11207180288,[0,7,6,3,6,
16,1]%%%}+%%%{-28185722880,[0,7,6,2,5,18,2]%%%}+%%%{-34359738368,[0,7,6,1,4
,20,3]%%%}+%%%{60129542144,[0,7,6,0,3,22,4]%%%}+%%%{-1577058304,[0,7,5,3,8,
16,0]%%%}+%%%{201326592,[0,7,5,2,7,18,1]%%%}+%%%{14495514624,[0,7,5,1,6,20,
2]%%%}+%%%{-17179869184,[0,7,5,0,5,22,3]%%%}+%%%{402653184,[0,7,4,2,9,18,0]
%%%}+%%%{-1610612736,[0,7,4,1,8,20,1]%%%}+%%%{1610612736,[0,7,4,0,7,22,2]%%
%} / %%%{1024,[0,3,4,2,1,2,0]%%%}+%%%{4096,[0,3,4,1,0,4,1]%%%}+%%%{-2560,[0
,3,3,1,2,4,0]%%%}+%%%{-4096,[0,3,3,0,1,6,1]%%%}+%%%{1536,[0,3,2,0,3,6,0]%%%
} Error: Bad Argument Value
```

maple [C] time = 0.02, size = 246, normalized size = 0.72

$$\frac{}{\left(16a\,e^2-16deb+16c\,d^2\right)\left(\mathrm{RootOf}\left(\_Z^4c+c\,d^4+(4be-4cd)\,\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4b\,d^2e-4\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 8*e^(3/2)/(16*a*e^2-16*b*d*e+16*c*d^2)*sum((_R^2*c+2*(2*b*e-3*c*d)*_R+c*d^2
)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*
d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4
*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+32*e^(3
/2)/(16*a*e^2-16*b*d*e+16*c*d^2)/(2*e*x^2-2*(e*x^2+d)^(1/2)*e^(1/2)*x+2*d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^{\frac{3}{2}}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(1/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.398 $\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=339

$$-\frac{2c^2\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}-\frac{2c^2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}+\frac{}{ad\sqrt{d+e}}$$

```
[Out] e*(-b*e+c*d)*x/a/d/(c*d^2+e*(a*e-b*d))/(e*x^2+d)^(1/2)+(-2*e*x^2-d)/a/d^2/x
/(e*x^2+d)^(1/2)-2*c^2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x
^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))/a/(2*c*d
-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2*c^2*arctan(
x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))/a/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(3/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 2.84, antiderivative size = 462, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1301, 271, 191, 6728, 264, 1692, 377, 205}

$$-\frac{c\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{c\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2}}{\sqrt{\sqrt{}}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(e^2/(d*(c*d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2])) - (2*e^3*x)/(d^2*(c*d^
2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((c*d - b*e)*Sqrt[d + e*x^2])/(a*d*(c
*d^2 - b*d*e + a*e^2)*x) - (c*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - S
qrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(c*d - b*e -
 (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sq
rt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 -
b*d*e + a*e^2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(
d + e*x^2)^q, x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^m*(d + e*
x^2)^(q + 1)*Simp[c*d - b*e - c*e*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && L
tQ[q, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \frac{\int \frac{cd-be-cex^2}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \frac{\int\left(\frac{cd-be}{ax^2\sqrt{d+ex^2}} + \frac{-bcd+b^2e-ace-c(cd-be)x^2}{a\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right.}{cd^2-bde+ae^2}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\int}{}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{(}{ad}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{(}{ad}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{(}{ad}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{(}{ad}$$

Mathematica [C] time = 6.77, size = 2158, normalized size = 6.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d + 2*e*x^2)/(a*d^2*x*Sqrt[d + e*x^2])) - ((c + (b*c)/Sqrt[b^2 - 4*a*c])
*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(
d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt
[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] -
2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b + Sq
rt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)
*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[-
(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*
c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (-b +
 Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*
Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*
c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*
e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/
```

```
2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] -
 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*a*(b - Sqrt[b^
2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]))*Sq
rt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2))]) + ((-c + (b*c)/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((b
+ Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/
(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b
^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*
ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[
Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + S
qrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x
^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e
*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*a*(b +
 Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))*
Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2))])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.04, size = 387, normalized size = 1.14

$$-\frac{8b e^{\frac{3}{2}}}{\left(4a e^2-4deb+4c d^2\right)\left(2e x^2-2\sqrt{e x^2+d}\,\sqrt{e}\,x+2d\right)a}+\frac{8cd\sqrt{e}}{\left(4a e^2-4deb+4c d^2\right)\left(2e x^2-2\sqrt{e x^2+d}\,\sqrt{e}\,x+2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/d/x/(e*x^2+d)^(1/2)-2/a*e/d^2*x/(e*x^2+d)^(1/2)-2/a*e^(1/2)/(4*a*e^2-4
*b*d*e+4*c*d^2)*sum((c*(b*e-c*d)*_R^2+2*(-2*a*c*e^2+2*b^2*e^2-3*b*c*d*e+c^2
*d^2)*_R+b*c*d^2*e-c^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b
*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=Ro
otOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d
^2*e-4*c*d^3)*_Z))-8/a*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+
d)^(1/2)*e^(1/2)*x+2*d)*b+8/a*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*
(e*x^2+d)^(1/2)*e^(1/2)*x+2*d)*c*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)\left(ex^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\left(d+ex^{2}\right)^{\frac{3}{2}}\left(a+bx^{2}+cx^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(1/(x**2*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.399 $\int \frac{1}{x^4\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=419

$$\frac{2c^2\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}+\frac{2c^2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}-\frac{ex\left(ace\cdots\right.}{a^2d\sqrt{d+ex^2}\cdots}$$

```
[Out] -1/3/a/d/x^3/(e*x^2+d)^(1/2)+1/3*(4*a*e+3*b*d)/a^2/d^2/x/(e*x^2+d)^(1/2)+2/
3*e*(4*a*e+3*b*d)*x/a^2/d^3/(e*x^2+d)^(1/2)-e*(a*c*e-b^2*e+b*c*d)*x/a^2/d/(
c*d^2+e*(a*e-b*d))/(e*x^2+d)^(1/2)+2*c^2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)
/(-4*a*c+b^2)^(1/2))/a^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2)+2*c^2*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*
x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2
))/a^2/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 5.57, antiderivative size = 647, normalized size of antiderivative = 1.54, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1301, 271, 191, 6728, 264, 1692, 377, 205}

$$\frac{c\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}+\frac{c\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-\cdots\right.}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -e^2/(3*d*(c*d^2 - b*d*e + a*e^2)*x^3*Sqrt[d + e*x^2]) + (4*e^3)/(3*d^2*(c*
d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2]) + (8*e^4*x)/(3*d^3*(c*d^2 - b*d*e +
 a*e^2)*Sqrt[d + e*x^2]) - ((c*d - b*e)*Sqrt[d + e*x^2])/(3*a*d*(c*d^2 - b*
d*e + a*e^2)*x^3) + (2*e*(c*d - b*e)*Sqrt[d + e*x^2])/(3*a*d^2*(c*d^2 - b*d
*e + a*e^2)*x) + ((b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x^2])/(a^2*d*(c*d^2 -
b*d*e + a*e^2)*x) + (c*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*
e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqr
t[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*
e^2)) + (c*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(S
qrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[b^2 - 4*a*
c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(
d + e*x^2)^q, x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^m*(d + e*
x^2)^(q + 1)*Simp[c*d - b*e - c*e*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && L
tQ[q, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \frac{\int \frac{cd-be-cex^2}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{x^4\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{\int\left(\frac{cd-be}{ax^4\sqrt{d+ex^2}} + \frac{-bcd+b^2e-ace}{a^2x^2\sqrt{d+ex^2}} + \frac{b^2cd-ac\ldots}{\ldots}\right.}{cd^2-bde+\ldots}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \ldots$$

Mathematica [C] time = 6.80, size = 2218, normalized size = 5.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (b*(d + 2*e*x^2))/(a^2*d^2*x*Sqrt[d + e*x^2]) - (d^2 - 4*d*e*x^2 - 8*e^2*x^
4)/(3*a*d^3*x^3*Sqrt[d + e*x^2]) + ((b*c + (c*(b^2 - 2*a*c))/Sqrt[b^2 - 4*a
*c])*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*
x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x
^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[
Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sq
rt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5
/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*
```

```
c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))
^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*
c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*a^2*(b - S
qrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b +
Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2))]) + ((b*c - (c*(b^2 - 2*a*c))/Sqrt[b^2 - 4*a*c]
)*x*(45*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d
 + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[
((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2))]] - (30*e*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30
*e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])
*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2
, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2))] + (4*e*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
))])/d))/(15*a^2*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 0.04, size = 541, normalized size = 1.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
[Out] 1/a^2*b/d/x/(e*x^2+d)^(1/2)+2/a^2*b*e/d^2*x/(e*x^2+d)^(1/2)-2/a^2*e^(1/2)/(
4*a*e^2-4*b*d*e+4*c*d^2)*sum((c*(a*c*e-b^2*e+b*c*d)*_R^2+2*(4*a*b*c*e^2-3*a
```

```
*c^2*d*e-2*b^3*e^2+3*b^2*c*d*e-b*c^2*d^2)*_R+a*c^2*d^2*e-b^2*d^2*e*c+b*c^2*
d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e
-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*
e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-8/a*
e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^(1/2)*e^(1/2)*x+2*d)
*c+8/a^2*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^(1/2)*e^(1/
2)*x+2*d)*b^2-8/a^2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^
(1/2)*e^(1/2)*x+2*d)*b*c*d-1/3/a/d/x^3/(e*x^2+d)^(1/2)+4/3/a*e/d^2/x/(e*x^2
+d)^(1/2)+8/3/a*e^2/d^3*x/(e*x^2+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)\left(ex^2 + d\right)^{\frac{3}{2}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(d + ex^{2}\right)^{\frac{3}{2}} \left(a + bx^{2} + cx^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(1/(x**4*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.400 $\int \frac{(fx)^m\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=243

$$\frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{m+1}{2};1,-q;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\right.}{f(m+1)\sqrt{b^2-4ac}\left(\right.}$$

```
[Out] 2*c*(f*x)^(1+m)*(e*x^2+d)^q*AppellF1(1/2+1/2*m,1,-q,3/2+1/2*m,-2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)),-e*x^2/d)/f/(1+m)/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2)-2*c*(f*x)^(1+m)*(e*x^2+d)^q*AppellF1(1/2+1/2*m,1,-q,3/2
+1/2*m,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/f/(1+m)/((1+e*x^2/d)^q)/(-
4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.65, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1305, 511, 510}

$$\frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{m+1}{2};1,-q;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\right.}{f(m+1)\sqrt{b^2-4ac}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^2)^q*AppellF1[(1 + m)/2, 1, -q, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt
[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^2)/d)^q) - (2*c*(f*x)^(1 + m)*(d + e*x^2
)^q*AppellF1[(1 + m)/2, 1, -q, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 +
 (e*x^2)/d)^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1305

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q, 1/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2
- 4*a*c, 0] &&  !IntegerQ[q] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(fx)^m \left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx = \int \left(\frac{2c(fx)^m \left(d+ex^2\right)^q}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} - \frac{2c(fx)^m \left(d+ex^2\right)^q}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right) dx$$
$$= \frac{(2c)\int \frac{(fx)^m\left(d+ex^2\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(fx)^m\left(d+ex^2\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{\left(2c\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{(fx)^m\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{(fx)^m\left(1+\right.}{b+\sqrt{b^2-4a}}}{\sqrt{b^2-4ac}}$$
$$= \frac{2c(fx)^{1+m}\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1+m}{2};1,-q;\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)} - \frac{2c(fx)^{1+m}\left(d\right.}{}$$

Mathematica [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
[Out] Integrate[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q \left(fx\right)^m}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q \left(fx\right)^m}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m \left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q \left(fx\right)^m}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x\right)^m\,\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int(((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.401 $\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=313

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,\ldots\right.}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-\ldots}\right.\right.}$$

```
[Out] -1/2*(b*e+c*d)*(e*x^2+d)^(1+q)/c^2/e^2/(1+q)+1/2*(e*x^2+d)^(2+q)/c/e^2/(2+q
)+1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2))))*(a-1/c*b^2+b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))/c/(1+q)
/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))+1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2
+q],2*c*(e*x^2+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(a-1/c*b^2-b*(-3*a*c+b^
2)/c/(-4*a*c+b^2)^(1/2))/c/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] time = 0.94, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1251, 1628, 68}

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,\ldots\right.}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-\ldots}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((c*d + b*e)*(d + e*x^2)^(1 + q))/(2*c^2*e^2*(1 + q)) + (d + e*x^2)^(2 + q
)/(2*c*e^2*(2 + q)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c])
)*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e)*(1 + q)) + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(d +
e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1
+ q))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^7\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(-cd-be)(d+ex)^q}{c^2e} + \frac{\left(\frac{b^2}{c^2}-\frac{a}{c}-\frac{b\left(b^2-3ac\right)}{c^2\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(\frac{b^2}{c^2}-\frac{a}{c}+\frac{b\left(b^2-3ac\right)}{c^2\sqrt{b^2-4ac}}\right.}{b+\sqrt{b^2-4ac}}\right.\right.$$

$$= -\frac{(cd+be)\left(d+ex^2\right)^{1+q}}{2c^2e^2(1+q)} + \frac{\left(d+ex^2\right)^{2+q}}{2ce^2(2+q)} - \frac{\left(a-\frac{b^2}{c}-\frac{b\left(b^2-3ac\right)}{c\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\,dx\right.}{2c}$$

$$= -\frac{(cd+be)\left(d+ex^2\right)^{1+q}}{2c^2e^2(1+q)} + \frac{\left(d+ex^2\right)^{2+q}}{2ce^2(2+q)} + \frac{\left(a-\frac{b^2}{c}+\frac{b\left(b^2-3ac\right)}{c\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}{}_2F_1\left(1,1+q\right.}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)}$$

Mathematica [A] time = 0.79, size = 272, normalized size = 0.87

$$\frac{\left(d+ex^2\right)^{q+1}\left(\frac{c\left(\frac{b\left(b^2-3ac\right)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{(q+1)\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)} + \frac{c\left(-\frac{b\left(b^2-3ac\right)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)} - \frac{be+cd}{e^2(q+1)}\right.}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*(-((c*d + b*e)/(e^2*(1 + q))) + (c*(d + e*x^2))/(e^2*(
2 + q)) + (c*(a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeo
metric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)])/((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*(a - b^2/c - (
b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeometric2F1[1, 1 + q, 2 + q,
(2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*(1 + q))))/(2*c^2)
```

fricas [F] time = 1.59, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(ex^2+d\right)^q x^7}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q x^7}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^{7} \left(e\,x^{2}+d\right)^{q}}{c\,x^{4}+b\,x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^{2}+d\right)^{q} x^{7}}{cx^{4}+bx^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7} \left(e\,x^{2}+d\right)^{q}}{c\,x^{4}+b\,x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.402 $\int \frac{x^5\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=256

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+\right.}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}\right.\right.}$$

```
[Out] 1/2*(e*x^2+d)^(1+q)/c/e/(1+q)+1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],
2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^2)
^(1/2))/c/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))+1/2*(e*x^2+d)^(1+q)*hyperg
eom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(b+(-2*a
*c+b^2)/(-4*a*c+b^2)^(1/2))/c/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] time = 0.54, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1251, 1628, 68}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+\right.}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (d + e*x^2)^(1 + q)/(2*c*e*(1 + q)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c]
)*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e)*(1 + q)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*H
ypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^5\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2(d+ex)^q}{a+bx+cx^2}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{(d+ex)^q}{c}+\frac{\left(-\frac{b}{c}+\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(-\frac{b}{c}-\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\right)dx,x,x^2\right)$$

$$= \frac{\left(d+ex^2\right)^{1+q}}{2ce(1+q)}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx,x,x^2\right)}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{}{b}\right.}{2c}$$

$$= \frac{\left(d+ex^2\right)^{1+q}}{2ce(1+q)}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}{}_2F_1\left(1,1+q;2+q;\frac{2c\left(d+ex^2\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}+\frac{\left(b+\frac{b^2}{\sqrt{b^2}}\right.}{}$$

Mathematica [A] time = 0.34, size = 211, normalized size = 0.82

$$\frac{\left(d+ex^2\right)^{q+1}\left(\frac{\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}+\frac{1}{e}\right)}{2c(q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*(e^(-1) + ((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*Hype
rgeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 -
4*a*c])*e)])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e) + ((b + (b^2 - 2*a*c)/Sqr
t[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(2*c*
(1 + q))
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(ex^2+d\right)^q x^5}{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q x^5}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

maple [**F**] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(e\,x^{2}+d\right)^{q}}{c\,x^{4}+b\,x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(e x^{2} + d\right)}^{q} x^{5}}{c x^{4} + b x^{2} + a}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(e\,x^2+d\right)}^q}{c\,x^4+b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.403 $\int \frac{x^3\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=210

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\right.}{2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right.\right.}$$

```
[Out] -1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4
*a*c+b^2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))-1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/(1+q)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2)))
```

Rubi [A] time = 0.33, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1251, 830, 68}

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\right.}{2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q,
 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*(2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - ((1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^
2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3 \left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\right)dx, x, x^2\right)$$
$$= \frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx, x, x^2\right) + \frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Sub}$$
$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q} {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+\right.}{2\left(2c\right.}$$

Mathematica [A] time = 0.20, size = 183, normalized size = 0.87

$$-\frac{\left(d+ex^2\right)^{q+1}\left(\left(d\sqrt{b^2-4ac}+2ae-bd\right){}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)+\left(d\sqrt{b^2-4ac}-2ae+bd\right){}_2F_1\right.}{4(q+1)\sqrt{b^2-4ac}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/4*((d + e*x^2)^(1 + q)*((-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*Hypergeom
etric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)] + (b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, 1 + q, 2
+ q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]))/(Sqrt[b^2 - 4
*a*c]*(c*d^2 + e*(-(b*d) + a*e))*(1 + q))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q x^3}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.404 $\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=198

$$\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

```
[Out] -c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a
*c+b^2)^(1/2))))/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+
c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))/(1+q)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] time = 0.36, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1247, 711, 68}

$$\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2
))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)*(1 + q))) + (c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1
, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sq
rt[b^2 - 4*a*c]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 711

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{2c(d+ex)^q}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c(d+ex)^q}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)dx,$$
$$= \frac{c\operatorname{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx, x, x^2\right)}{\sqrt{b^2-4ac}} - \frac{c\operatorname{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\,dx, x, x^2\right)}{\sqrt{b^2-4ac}}$$
$$= -\frac{c\left(d+ex^2\right)^{1+q}{}_2F_1\left(1,1+q;2+q;\frac{2c\left(d+ex^2\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{c\left(d+ex^2\right)^{1+q}{}_2F_1\left(1,1+q;2+q;\right.}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4a\right.\right.}$$

Mathematica [A] time = 0.26, size = 168, normalized size = 0.85

$$\frac{c\left(d+ex^2\right)^{q+1}\left(\frac{{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)} - \frac{{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}\right)}{(q+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*(d + e*x^2)^(1 + q)*(-(Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^
2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])
*e)) + Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(Sqrt[b^2 - 4*a
*c]*(1 + q))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(ex^2+d\right)^q x}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.405 $\int \frac{\left(d+ex^2\right)^q}{x\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=262

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^{q+1}{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}{}_2F_1\left(1,q+1;q+2\cdots\right.}{2a(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+\cdots\right.\right.}$$

```
[Out] -1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],1+e*x^2/d)/a/d/(1+q)+1/2*c*(e
*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/a/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))+1/2*c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b
+(-4*a*c+b^2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/a/(1+q)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))
```

Rubi [A] time = 0.50, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1251, 960, 65, 830, 68}

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^{q+1}{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}{}_2F_1\left(1,q+1;q+2\cdots\right.}{2a(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q
, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*a*(2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d +
 e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1
 + q)) - ((d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^2
)/d])/(2*a*d*(1 + q))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(d+ex)^q}{x(a+bx+cx^2)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(d+ex)^q}{ax} + \frac{(-b-cx)(d+ex)^q}{a(a+bx+cx^2)}\right) dx, x, x^2\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{(d+ex)^q}{x}\,dx, x, x^2\right)}{2a} + \frac{\operatorname{Subst}\left(\int \frac{(-b-cx)(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2ad(1+q)} + \frac{\operatorname{Subst}\left(\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(-c+\frac{b}{\sqrt{b^2}}\right.}{b+\sqrt{b^2}}\right.\right.}{2a}$$

$$= -\frac{(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2ad(1+q)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\operatorname{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2}\right.}{2a}$$

$$= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)}{2}$$

Mathematica [A] time = 0.35, size = 218, normalized size = 0.83

$$\frac{(d+ex^2)^{q+1}\left(\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right){}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)} - \frac{{}_2F_1\left(1,q+1;q+2;\frac{ex^2}{d}+\right.}{d}\right.}{2a(q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric
2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) - Hypergeometric2F1[1, 1 + q, 2 + q,
1 + (e*x^2)/d]/d))/(2*a*(1 + q))
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q}{cx^5+bx^3+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q/(c*x^5 + b*x^3 + a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q}{\left(cx^4+bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)^q}{\left(c\,x^4+b\,x^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x)
[Out] int((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q}{\left(cx^4+bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^q}{x\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^q}{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**q/(x*(a + b*x**2 + c*x**4)), x)
```

3.406 $\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=322

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+\right.}{2a^2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}\right.\right.}$$

```
[Out] 1/2*b*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],1+e*x^2/d)/a^2/d/(1+q)+1/2*e
*(e*x^2+d)^(1+q)*hypergeom([2, 1+q],[2+q],1+e*x^2/d)/a/d^2/(1+q)-1/2*c*(e*x
^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a*c+b^2)
^(1/2))))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/(1+q)/(2*c*d-e*(b-(-4*a*c
+b^2)^(1/2)))-1/2*c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/(1
+q)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] time = 0.66, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 960, 65, 830, 68}

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+\right.}{2a^2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometri
c2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
])/(2*a^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - (c*(b - (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 +
q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*a^2*(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (b*(d + e*x^2)^(1 + q)*Hypergeometric
2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a^2*d*(1 + q)) + (e*(d + e*x^2)^(1
+ q)*Hypergeometric2F1[2, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a*d^2*(1 + q))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^q}{x^3\left(a+bx^2+cx^4\right)}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{(d+ex)^q}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{(d+ex)^q}{ax^2}-\frac{b(d+ex)^q}{a^2x}+\frac{\left(b^2-ac+bcx\right)(d+ex)^q}{a^2\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)$$
$$=\frac{\text{Subst}\left(\int \frac{\left(b^2-ac+bcx\right)(d+ex)^q}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2}+\frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x^2}\,dx,x,x^2\right)}{2a}-\frac{b\,\text{Subst}\left(\int \frac{(d+e}{x}\right.}{2a^2}$$

$$=\frac{b\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2a^2d(1+q)}+\frac{e\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(2,1+q;2+q;1+\right.}{2ad^2(1+q)}$$
$$=\frac{b\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2a^2d(1+q)}+\frac{e\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(2,1+q;2+q;1+\right.}{2ad^2(1+q)}$$
$$=-\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c\left(d+ex^2\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right.}{}$$

Mathematica [A] time = 0.35, size = 259, normalized size = 0.80

$$\frac{\left(d+ex^2\right)^{q+1}\left(-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}-\frac{c\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}+\frac{ae\,{}_2F_1\left(2,q+1;q+2;\right.}{d^2}\right.}{2a^2(q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*(-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Hypergeome
tric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)) - (c*(b + (-b^2 + 2*a*c)/Sqrt[
```

```
b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) + (b*Hype
rgeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])/d + (a*e*Hypergeometric2F1[2
, 1 + q, 2 + q, 1 + (e*x^2)/d])/d^2))/(2*a^2*(1 + q))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^2+d)^q}{cx^7+bx^5+ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q/(c*x^7 + b*x^5 + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^q}{(c\,x^4+b\,x^2+a)\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)^q}{x^3\,(c\,x^4+b\,x^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int((d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.407 $\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=339

$$\frac{x\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q}{c^2(}$$

```
[Out] -b*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/c^2/((1+e*x^2/d)^q)+1/
3*x^3*(e*x^2+d)^q*hypergeom([3/2, -q],[5/2],-e*x^2/d)/c/((1+e*x^2/d)^q)+x*(
e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)*
(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/((1+e*x^2/d)^q)/(b-(-4*a*c+
b^2)^(1/2))+x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)),-e*x^2/d)*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/((1+e*x^2/d)
^q)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.63, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1303, 246, 245, 365, 364, 1692, 430, 429}

$$\frac{x\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q}{c^2(}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1
[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c^2*(
b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + ((b^2 - a*c + (b*(b^2 - 3*a*c))
/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*
x^2)/d)^q) - (b*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d
)])/(c^2*(1 + (e*x^2)/d)^q) + (x^3*(d + e*x^2)^q*Hypergeometric2F1[3/2, -q,
 5/2, -((e*x^2)/d)])/(3*c*(1 + (e*x^2)/d)^q)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^6 \left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx = \int \left(-\frac{b\left(d+ex^2\right)^q}{c^2}+\frac{x^2\left(d+ex^2\right)^q}{c}+\frac{\left(ab+\left(b^2-ac\right)x^2\right)\left(d+ex^2\right)^q}{c^2\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{\left(ab+\left(b^2-ac\right)x^2\right)\left(d+ex^2\right)^q}{a+bx^2+cx^4}dx}{c^2} - \frac{b\int \left(d+ex^2\right)^q dx}{c^2} + \frac{\int x^2\left(d+ex^2\right)^q dx}{c}$$

$$= \frac{\int \left(\frac{\left(b^2-ac+\frac{b\left(-b^2+3ac\right)}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(b^2-ac-\frac{b\left(-b^2+3ac\right)}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{c^2} - \frac{\left(b\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \left(1+\right.}{c^2}$$

$$= -\frac{bx\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c^2} + \frac{x^3\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(\frac{3}{2},-q;\frac{5}{2};-\right.}{3c}$$

$$= -\frac{bx\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c^2} + \frac{x^3\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(\frac{3}{2},-q;\frac{5}{2};-\right.}{3c}$$

$$= \frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)x\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)} + \frac{\left(b^2-ac+\right.}{}$$

Mathematica [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
[Out] Integrate[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q x^6}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q x^6}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q x^6}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.408 $\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=273

$$-\frac{x\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}-\frac{x\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F}{c\left(\sqrt{b^2-4ac}\right.}$$

```
[Out] x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/c/((1+e*x^2/d)^q)-x*(e*x^
2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)*(b+(
2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2))-x*(
e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)*
(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c/((1+e*x^2/d)^q)/(b+(-4*a*c+b^2)^(1/2)
)
```

Rubi [A] time = 0.53, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1303, 246, 245, 1692, 430, 429}

$$-\frac{x\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}-\frac{x\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F}{c\left(\sqrt{b^2-4ac}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -
q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c*(b - Sqrt[b^2
 - 4*a*c])*(1 + (e*x^2)/d)^q)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(
d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
-((e*x^2)/d)])/(c*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (x*(d + e*x^
2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(c*(1 + (e*x^2)/d)^q)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
```

```
Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4 (d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \left(\frac{(d+ex^2)^q}{c} - \frac{(a+bx^2)(d+ex^2)^q}{c(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int (d+ex^2)^q\,dx}{c} - \frac{\int \frac{(a+bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx}{c}$$

$$= -\frac{\int \left(\frac{\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(b-\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right) dx}{c} + \frac{\left((d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right) \int \left(1+\frac{ex^2}{d}\right)^q}{c}$$

$$= \frac{x (d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{c} - \frac{\left(b+\right.}{}$$

$$= \frac{x (d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c} - \frac{\left(\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right)\int}{c}$$

$$= -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) x (d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right.}{}$$

Mathematica [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x^4 (d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^2+d)^q x^4}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.409 $\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=162

$$\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}-\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{\sqrt{b^2-4ac}}$$

```
[Out] -x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2
/d)/((1+e*x^2/d)^q)/(-4*a*c+b^2)^(1/2)+x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/((1+e*x^2/d)^q)/(-4*a*c+b^2)^(1/2
)
```

Rubi [A] time = 0.31, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1303, 430, 429}

$$\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}-\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)^q)) + (x*(d + e*x^2
)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)
/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)^q)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2 \left(d+ex^2\right)^q}{a+bx^2+cx^4} \, dx = \int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b+\sqrt{b^2-4ac}+2cx^2} \right) dx$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right) \int \frac{\left(d+ex^2\right)^q}{b-\sqrt{b^2-4ac}+2cx^2} \, dx + \left(1+\frac{b}{\sqrt{b^2-4ac}}\right) \int \frac{\left(d+ex^2\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}$$

$$= \left(\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right) \int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2} \, dx + \left(\left(1+\frac{b}{\sqrt{b^2-4ac}}\right.\right.$$

$$= -\frac{x\left(d+ex^2\right)^q \left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}} + \frac{x\left(d+ex^2\right)^q \left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2}\right.}{\sqrt{b^2-\right.}}$$

Mathematica [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(d+ex^2\right)^q}{a+bx^2+cx^4} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
[Out] Integrate[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q x^2}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q x^2}{cx^4+bx^2+a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q x^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.410 $\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=190

$$-\frac{2cx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right.}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] -2*c*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e
*x^2/d)/((1+e*x^2/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*(e*x^2+d)^q*
AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/((1+e*x^2/d
)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.29, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1174, 430, 429}

$$-\frac{2cx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right.}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 -
4*a*c]), -((e*x^2)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d
)^q) - (2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), -((e*x^2)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*
x^2)/d)^q)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)}{b+\sqrt{b^2-4a}}}{\sqrt{b^2-4ac}}$$

$$= -\frac{2cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)}{b^2-4}$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)^q/(a + b*x^2 + c*x^4),x]
[Out] Integrate[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x]
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^2+d)^q}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int((e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^q/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^q/(a + b*x^2 + c*x^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.411 $\int \frac{\left(d+ex^2\right)^q}{x^2\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=264

$$-\frac{cx\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right.}{a\left(\sqrt{b^2-}\right.}$$

```
[Out] -(e*x^2+d)^q*hypergeom([-1/2, -q],[1/2],-e*x^2/d)/a/x/((1+e*x^2/d)^q)-c*x*(
e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)*
(1+b/(-4*a*c+b^2)^(1/2))/a/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2))-c*x*(e*x^
2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)*(1-b
/(-4*a*c+b^2)^(1/2))/a/((1+e*x^2/d)^q)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.55, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1303, 365, 364, 1692, 430, 429}

$$-\frac{cx\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right.}{a\left(\sqrt{b^2-}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((c*(1 + b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b - Sqrt[b^2 - 4*a*c])
*(1 + (e*x^2)/d)^q)) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF
1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b
 + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e*x^2)^q*Hypergeometric2F1
[-1/2, -q, 1/2, -((e*x^2)/d)])/(a*x*(1 + (e*x^2)/d)^q)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
```

```
Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex^2\right)^q}{x^2\left(a+bx^2+cx^4\right)}\,dx &= \int \left(\frac{\left(d+ex^2\right)^q}{ax^2}+\frac{\left(-b-cx^2\right)\left(d+ex^2\right)^q}{a\left(a+bx^2+cx^4\right)}\right)dx \\
&= \frac{\int \frac{\left(d+ex^2\right)^q}{x^2}\,dx}{a}+\frac{\int \frac{\left(-b-cx^2\right)\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx}{a} \\
&= \frac{\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(-c+\frac{bc}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{a}+\frac{\left(\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{x^2}
\end{aligned}$$

$$= -\frac{\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\left(d+ex^2\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{a}$$

$$= -\frac{\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right.}{a}$$

$$= -\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)x\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}-\frac{c\left(1-\right.}{}$$

Mathematica [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^q}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x]
[Out] Integrate[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)), x]
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q}{cx^6+bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q/(c*x^6 + b*x^4 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q}{\left(cx^4+bx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)^q}{\left(c\,x^4+b\,x^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x)
[Out] int((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q}{\left(cx^4+bx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^q}{x^2\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/x**2/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.412 $\int \frac{\left(d+ex^2\right)^q}{x^4\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=328

$$\frac{cx\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{cx\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F}{a^2\left(\sqrt{b^2-4ac\right.}}$$

```
[Out] -1/3*(e*x^2+d)^q*hypergeom([-3/2, -q],[-1/2],-e*x^2/d)/a/x^3/((1+e*x^2/d)^q
)+b*(e*x^2+d)^q*hypergeom([-1/2, -q],[1/2],-e*x^2/d)/a^2/x/((1+e*x^2/d)^q)+
c*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^
2/d)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2
)^(1/2))+c*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)),-e*x^2/d)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/((1+e*x^2/d)^q)/(b+(-4
*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.62, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1303, 365, 364, 1692, 430, 429}

$$\frac{cx\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{cx\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F}{a^2\left(\sqrt{b^2-4ac\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -
q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a^2*(b - Sqrt[b
^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*
x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), -((e*x^2)/d)])/(a^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e
*x^2)^q*Hypergeometric2F1[-3/2, -q, -1/2, -((e*x^2)/d)])/(3*a*x^3*(1 + (e*x
^2)/d)^q) + (b*(d + e*x^2)^q*Hypergeometric2F1[-1/2, -q, 1/2, -((e*x^2)/d)]
)/(a^2*x*(1 + (e*x^2)/d)^q)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)^q}{x^4\left(a+bx^2+cx^4\right)}\,dx &= \int \left(\frac{\left(d+ex^2\right)^q}{ax^4} - \frac{b\left(d+ex^2\right)^q}{a^2x^2} + \frac{\left(b^2-ac+bcx^2\right)\left(d+ex^2\right)^q}{a^2\left(a+bx^2+cx^4\right)}\right)dx \\
&= \frac{\int \frac{\left(b^2-ac+bcx^2\right)\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx}{a^2} + \frac{\int \frac{\left(d+ex^2\right)^q}{x^4}\,dx}{a} - \frac{b\int \frac{\left(d+ex^2\right)^q}{x^2}\,dx}{a^2} \\
&= \frac{\int \left(\frac{\left(bc+\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(bc-\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{a^2} + \frac{\left(\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right.}{x^4}}{a} \\
&= -\frac{\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(-\frac{3}{2},-q;-\frac{1}{2};-\frac{ex^2}{d}\right)}{3ax^3} + \frac{b\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(-\frac{1}{2},\right.}{a^2x} \\
&= -\frac{\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(-\frac{3}{2},-q;-\frac{1}{2};-\frac{ex^2}{d}\right)}{3ax^3} + \frac{b\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(-\frac{1}{2},\right.}{a^2x} \\
&= \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)} + \frac{c\left(b-\right.}{}
\end{aligned}
$$

Mathematica [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^q}{x^4\left(a+bx^2+cx^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x]
[Out] Integrate[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)), x]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q}{cx^8+bx^6+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q/(c*x^8 + b*x^6 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q}{\left(cx^4+bx^2+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)^q}{\left(c\,x^4+b\,x^2+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x)
[Out] int((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^q}{\left(cx^4+bx^2+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^q}{x^4\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/x**4/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.413 $\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=40

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{1-c^4x^4}}{cx\sqrt{\frac{1}{c^2x^2}+1}}\right)}{c}$$

```
[Out] -arctanh((-c^4*x^4+1)^(1/2)/c/x/(1+1/c^2/x^2)^(1/2))/c
```

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1448, 1252, 848, 63, 208}

$$-\frac{x\sqrt{\frac{1}{c^2x^2}+1}\,\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 + c^2*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1448

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Dist[(e^IntPart[q]*(d + e*x^mn)^FracPart[q])/(x^(mn*FracPart[q])*(1
 + d/(x^mn*e))^FracPart[q]), Int[x^(mn*q)*(1 + d/(x^mn*e))^q*(a + c*x^n2)^p
, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&  !Integer
Q[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx &= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\int \frac{\sqrt{1+c^2x^2}}{x\sqrt{1-c^4x^4}}\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x\sqrt{1-c^4x^2}}\,dx, x, x^2\right)}{2\sqrt{1+c^2x^2}} \\
&= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{2\sqrt{1+c^2x^2}} \\
&= -\frac{\left(\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\mathrm{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x, \sqrt{1-c^2x^2}\right)}{c^2\sqrt{1+c^2x^2}} \\
&= -\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 44, normalized size = 1.10

$$
-\frac{x\sqrt{\frac{1}{c^2x^2}+1}\,\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{c^2x^2+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 + c^2*x^2])
```

fricas [B] time = 1.41, size = 120, normalized size = 3.00

$$
-\frac{\log\left(\frac{c^2x^2+\sqrt{-c^4x^4+1}\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)-\log\left(-\frac{c^2x^2-\sqrt{-c^4x^4+1}\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)}{2\,c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c^2*x^2 + 1)) - log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)))/c
```

giac [A] time = 0.22, size = 42, normalized size = 1.05

$$
-\frac{\left(\log\left(\sqrt{-c^2x^2+1}\,+1\right)-\log\left(-\sqrt{-c^2x^2+1}\,+1\right)\right)|c|}{2\,c^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(log(sqrt(-c^2*x^2 + 1) + 1) - log(-sqrt(-c^2*x^2 + 1) + 1))*abs(c)/c^
2
```

maple [C] time = 0.06, size = 101, normalized size = 2.52

$$-\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,\sqrt{-c^4x^4+1}\,x\,\mathrm{csgn}\left(\frac{1}{c}\right)\ln\left(\frac{2\sqrt{-\frac{c^2x^2-1}{c^2}}\,c\,\mathrm{csgn}\left(\frac{1}{c}\right)+2}{c^2x}\right)}{\left(c^2x^2+1\right)\sqrt{-\frac{c^2x^2-1}{c^2}}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] -((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(-c^4*x^4+1)^(1/2)*csgn(1/c)*ln(2*(csgn(1/c)
*c*(-1/c^2*(c^2*x^2-1))^(1/2)+1)/x/c^2)/(c^2*x^2+1)/(-1/c^2*(c^2*x^2-1))^(1
/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{c^2x^2}+1}}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(1/(c^2*x^2) + 1)/sqrt(-c^4*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{c^2\,x^2}+1}}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(c^2*x^2) + 1)^(1/2)/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((1/(c^2*x^2) + 1)^(1/2)/(1 - c^4*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{-(cx-1)(cx+1)\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c**2/x**2)**(1/2)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 + 1/(c**2*x**2))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```